1. (Optimal provision of unemployment insurance with incentive problems.) Suppose there is a unit measure of identical firms and time consists of one period. Firms are competitive and have access to the following technology for converting a labor input, $H$, into a homogeneous output good, $y$:

$$y = H.$$ 

Firms enter competitive labor markets and hire labor at a given wage rate, $W$.

There is a unit measure of households. The only source of randomness is an idiosyncratic shock to households’ disutility of labor (perhaps a health shock). At the start of the period each household independently draws $l$ from a uniform $[0, 1]$ distribution. Households can either work one unit of time or not at all and a household’s choice of employment status is determined by its realized value of $l$. A working household enjoys utility

$$\log c^w - \psi l^\sigma, \quad \sigma > 0, \quad \psi > 1,$$

where $l^\sigma$ is the household’s labor disutility and $c^w$ denotes the working household’s level of consumption. A working household has the following budget constraint:

$$c^w \leq (1 - \tau) W,$$

where $\tau$ denotes the labor tax rate. A non-working household enjoys utility

$$\log c^{nw},$$

where $c^{nw}$ denotes the level of consumption of a non-working household that is eligible for a government-supplied unemployment benefit, $T$. The budget constraint of such a household is:

$$c^{nw} \leq T.$$
Households that do not work and are ineligible for the unemployment benefit and must consume zero. Households make their work and consumption decision to maximize utility subject to their budget constraints and subject to the eligibility requirements for $T$.

There are two versions of this model economy.

**Full information**: the government observes each household’s idiosyncratic shock, $l$. The government sets a value, $l^*$, where $0 < l^* < 1$. To be eligible to receive $T$, a household must not be working and must have $l > l^*$. Households that work, or who draw $l \leq l^*$ are not eligible for $T$.

**Partial information**: $l$ is private information to the household. The government can make $T$ contingent on a household’s employment status, but not on the household’s realized value of $l$.

In both versions of the model, the government observes the household’s employment status and level of consumption. We shall only consider ‘interior equilibria’, those in which aggregate labor supply is greater than zero and less than unity. We define aggregate labor supply as the measure of households that choose to work in the market. Thus, let $g(l)$ be an indicator function, equal to unity for households that work and zero for households that do not work. Let $f(l)$ (= 1 in our Uniform case) denote the density of households with idiosyncratic shock, $l$. Then,

$$\text{aggregate labor supply} \equiv h = \int_0^1 f(l) g(l) \, dl.$$ 

(a) Consider the full information version of the model. Government policy is characterized by $\tau$, $T$, $l^*$.

i. Prove that an interior equilibrium has the property, $W = 1$.

ii. Let $0 \leq h \leq 1$ denote the aggregate supply of labor. Suppose

$$1 - \tau \leq T.$$  (1)

Derive $g(l)$ for each $l \in [0, 1]$. Show that $h = l^*$. Show that if (1) does not hold, then some households forego $T$ even though they are eligible for it. Display an expression for $h$ that is valid in this case.
iii. Display the government’s budget constraint and prove ‘Walras’ law’: the sum of the government and household budget constraints imply the economy-wide resource constraint. In answering this question, you may suppose that households with $0 < l < h$ work and households with $l > h$ do not work, for some $h$.

iv. Provide a formal definition of the household optimization problem and define an equilibrium for the economy, conditional on a given specification of government policy. Derive an expression for aggregate, equally weighted utility across all households that is a function of $c_w$, $c_{nw}$ and $h$ only.

v. Consider the set of interior equilibria in which the government budget constraint is satisfied as a strict equality. Prove that the equilibrium in this set which has the highest aggregate utility has the property, $c_w = c_{nw}$. That is, optimal government unemployment policy equates the consumption of the employed and of the unemployed.

(b) Consider the partial information version of the model. Government policy selects $\tau$ and $T$. Eligibility to receive $T$ is contingent on employment status, but not on a household’s realized value of $l$.

i. Show that there is no equilibrium with $c_{nw} = c_w = c > 0$. (Hint: think about the government budget constraint.)

ii. If an equilibrium is to be characterized by positive employment, $h > 0$, then $c_w/c_{nw}$ must exceed unity. Derive a simple formula which determines how big $c_w/c_{nw}$ must be, to provide the right incentives for a measure, $h$, of households to choose to work. This formula is the ‘incentive constraint’.

2. Consider example 1 in the notes on “Financial Frictions Under Asymmetric Information and Moral Hazard”. Consider the model parameter values listed at the end of that section.

(a) How does the contract compare for $a = 2$ and $a = 3$? What happens to the multiplier on the incentive compatibility constraint? Provide intuition.
(b) How does the contract compare for $\gamma = 2$ and $\gamma = 1.5$. Provide
intuition.

(c) How does the contract compare for $\beta = 1$ and $\beta = 0.98$. Provide
intuition.

3. (Financial friction theory of Solow residual). Consider an economy
in which a final good is produced using the following Dixit-Stiglitz
aggregator:

$$y = \left[ \int_0^1 y_i^{\frac{1}{\lambda}} \, di \right]^\lambda, \quad 0 < \lambda < 1. $$

A representative, competitive firm produces $y$ taking the price of $y$
(normalized to equal unity) and the prices of the intermediate inputs,
$p_i$, as given. The $i^{th}$ intermediate good is produced by a monopolist
using the following production function:

$$y_i = k_i^{\alpha} l_i^{1-\alpha}, \quad 0 < \alpha < 1,$$

$i \in (0,1)$. Here, $l_i$ and $k_i$ denote the quantity of labor and capital,
respectively, hired by the $i^{th}$ monopolist. The $i^{th}$ monopolist is com-
petitive in factor markets, where it can rent as much or as little capital
and labor as it wishes, at factor prices, $r$ and $w$, respectively.

Consider the following financial friction, for which I do not provide a
structural explanation. A subset of $\nu$ firms must borrow their factor
costs in advance. In particular, suppose that firms with $i \in (0, \nu)$ must
go to a loan market to borrow their labor and capital bills, respectively,
before production commences. That is, for a firm with $i \in (0, \nu)$, the
cost of labor at the end of the period when it computes its profits is
$Rw_i$. Similarly, its end of period cost of capital is $Rk_i$. Here, $R > 1$
denotes the gross interest rate. That is, $R$ is the interest and principal
on the beginning of period loans, $rk_i$ and $wl_i$, respectively. Thus, for
firms $i \in (0, \nu)$ profits are

$$p_i y_i - Rrk_i - Rw_i.$$

Firms, $i \in (\nu, 1)$, pay their capital and labor costs out of end of period
revenues and so their end of period profits are:

$$p_i y_i - rk_i - wl_i.$$
Denote aggregate capital and labor by $k$ and $l$, respectively, where
\[ k = \int_0^1 k_i \, di, \quad l = \int_0^1 l_i \, di. \]

Show that aggregate final output can be expressed as
\[ y = q(R) k^\alpha l^{1-\alpha}, \]
and display an explicit expression for $q(R)$. Explain why $q(1) = 1$ and $q < 1$ for $R > 1$.

The ‘Solow residual’ is a variable computed using aggregate data, and is defined as follows:
\[ \frac{GDP}{K^a L^{1-a}}. \]

Here, $GDP$ denotes real gross domestic product, $K$ denotes a measure of the aggregate stock of capital, $L$ a measure of the aggregate labor input. Finally, $a$ is a measure of the share of income going to labor based on data from the National Income and Product Accounts. During the period when the real business cycle model was popular, the Solow residual was interpreted as an exogenous shock to technology. Note that under our financial friction assumption, the Solow residual is actually a function of an endogenous variable, the interest rate.