1. Consider the ‘Moral Hazard I’ model in the manuscript, ‘Government Policy, Credit Markets and Economic Activity’. Consider the following parameter values:

\[ N = 1, \ y = 5, \ \gamma = 1.1, \ \beta = 0.97, \ \theta = 0.38, \ R^k = 1.10. \]

(a) Compute a benchmark equilibrium for this model (i.e., an equilibrium that ignores the banker incentive constraint). In particular, report the values for \( c, C, d \) and bank profits. Verify that this is not a Financial Equilibrium (i.e., the incentive constraint is violated). In particular, report the value of the incentive for a bank to default.

(b) Compute the minimum tax-financed equity transfer to banks that eliminates the interest rate spread.

2. Entrepreneurs have access to a technology for converting capital, \( k \), into output

\[ \omega k^{\alpha}, \]

where \( \omega \) is a technology shock drawn independently by each entrepreneur from a distribution with \( E\omega = 1 \) and cumulative distribution function \( F(x) \equiv \text{prob} [\omega \leq x] \). The realization of \( \omega \) is observed by the entrepreneur, and can be seen by a lender only if the lender pays a monitoring cost, \( \mu k^{\alpha} \). The depreciation rate on capital is \( \delta \), so that after production, entrepreneurs have \( (1 - \delta) k \) units of capital left, which they can sell at a price of unity (the price of output is the numeraire). Thus, after production the entrepreneur who draws \( \omega \) has the following resources:

\[ y(\omega) = \omega k^{\alpha} + (1 - \delta) k, \ 0 < \delta, \alpha \leq 1, \]

At the beginning of the period, entrepreneurs have no capital, but they do have net worth, \( n \), that they can use for purchasing \( k \). Suppose there are many entrepreneurs with each possible level of \( n \).
Consider an entrepreneur with net worth, $n$, who purchases an amount of capital, $k > n$. The entrepreneur borrows $b \equiv k - n$ at gross rate of interest, $Z$, from a bank. There is a large number of banks that specialize in lending to entrepreneurs with each level of net worth, $n$, and there is free entry into banking. In case the entrepreneur’s revenue, $y(\omega)$, falls below the required payment to the bank, $Z(k - n)$, the entrepreneur declares bankruptcy and is monitored. In addition, the bank takes whatever the entrepreneur has. Let $\tilde{\omega}$ be defined by

$$\tilde{\omega}k^{\alpha} + (1 - \delta)k = Z(k - n).$$

Prior to making loans to entrepreneurs, banks have access to a competitive market in which they can borrow as much or as little as they want, at gross rate of interest, $R$. At the end of the period, when the banks have to repay household loans, the only source of funds available to them is the funds given to them by entrepreneurs. Entrepreneurial utility prior to production is proportional to their expected end-of-period resources, net of bank costs.

(a) Show that the expected profits of an entrepreneur with net worth $n$, interest rate $Z$, and loan amount $k - n$ can be written

$$[1 - \Gamma(\tilde{\omega})]k^{\alpha},$$

where

$$\Gamma(\tilde{\omega}) = [1 - F(\tilde{\omega})] \tilde{\omega} + \int_{0}^{\tilde{\omega}} \omega dF(\omega).$$

(b) Suppose that each bank deals with a large, randomly selected set of entrepreneurs. Show that the average revenues, across all loans to entrepreneurs with net worth $n$, is

$$[\Gamma(\tilde{\omega}) - \mu F(\tilde{\omega})]k^{\alpha} + (1 - \delta)k.$$

(c) Display the zero profit condition for the banks that lend to entrepreneurs with net worth, $n$. Write this in terms of the variables, $b, R, Z$ and $n$ only (not $\tilde{\omega}$ or $k$). Can there be an equilibrium in which banks offer an interest rate $Z$ and banks allow entrepreneurs to borrow as much as they want at that interest rate? Explain.
(d) Let the bank zero profit condition define a menu of contracts, \((b, Z)\), that banks who lend to entrepreneurs with net worth \(n\) offer in equilibrium. Display a constrained optimization problem that characterizes which \(b, Z\) combination entrepreneurs with net worth, \(n\), select from this menu. You may assume that the chosen contract is interior and is uniquely characterized by the first order conditions evaluated at equality.

i. Will the interest rate in the selected contract vary with entrepreneurial net worth, \(n\)? Does your answer to the last question change across the cases, \(\alpha = 1\) and \(\alpha < 1\)? Establish your answer carefully.

ii. Define leverage, \(L\), as

\[
L = \frac{b + n}{n}. 
\]

Show that \(L\) is independent of \(n\) when \(\alpha = 1\).


Let \(p(e)\) have the following representation:

\[
p(e) = a + be, \ b > 0.
\]

(a) The second period budget constraint of the representative household is \(C = Rd + \pi\), where \(R\) denotes the interest rate on deposits, \(d\), and \(\pi\) denotes profits returned to the household by bankers. In the household’s decision about \((d, c, C)\), \(\pi\) is treated as exogenous. Show that when the equilibrium expression for \(\pi\) is substituted into the household budget constraint, then the second period budget constraint reduces to:

\[
C = R(N + d).
\]

(b) The observable effort equilibrium can be solved by solving one nonlinear equation in \(R\). Given \(R\) one can compute \(c, C, d\) by solving the household problem. Effort, \(e\), can then be solved using
the efficiency condition for effort. This in turn allows one to solve for $p(e)$. Finally, adjust $R$ until

$$R = p(e) R^g + (1 - p(e)) R^b$$

is satisfied. Note that $R^d_b$ and $R^d_g$ are not pinned down, though they must satisfy the zero profit condition of mutual funds and the cash constraint. Consider the following parameterization of the model:

$$a = 0.75, \ b = 0.20, \ y = 10, \ N = 6, \ R^g = 1.2, \ R^b = 0.3, \ \gamma = 0.9, \ \beta = 0.97.$$ 

Compute the equilibrium values of $c, C, d, R$. Show that for this numerical example, $R^d_g = R^d_b = R$ is consistent with the cash constraint being satisfied by banks whose assets perform badly. That is, each bank has enough net worth, so the creditors of the banks (i.e., the mutual funds) that perform poorly do not share in those banks’ losses. Explain why this is also an unobservable effort equilibrium.

(c) Consider the lower level of net worth, $N = 5$. Compute the observed effort equilibrium. Explain why this is not an unobserved effort equilibrium. Banks do not have enough net worth to insulate creditors from losses when things go wrong (i.e., their portfolio performs poorly). Though it is not required for this homework, you may want to compute the unobserved effort equilibrium in this case. Because bankers’ payments to creditors must be low when things go right, creditors require that those payments are high when things are fine. As a result, bankers’ incentive to exert effort is reduced. This results in a reduction in the overall return on bank deposits (and, an increase in variance across banks). As a result, the return on household deposits is low and they save less than is efficient. Because it now processes less intermediation than is efficient, we can say that the banking system is ‘dysfunctional’.