1. Consider the version of the neoclassical growth model in which preferences have the following form:

$$\sum_{t=0}^{\infty} \beta^t u(c_t - \bar{c}),$$

where $\bar{c} > 0$ and $c_t$ denotes consumption. The resource constraint is unaffected:

$$c_t + k_{t+1} \leq f(k_t).$$

The non-negativity constraints are:

$$c_t \geq \bar{c}, \ k_{t+1} \geq 0,$$

and $k_0$ is given. Consider the assumptions made on $u$ and $f$ by S-L to guarantee that $A4.3 - A4.9$ are satisfied in the standard version of the model (i.e., the one in which $\bar{c} = 0$).

(a) Display a graph of the highest value of $k_{t+1}$ for each value of $k_t$ that is consistent with the non-negativity constraints. This graph represents the feasible set, $\Gamma(k_t)$, for the model economy. We say that the economy is viable if there exist sequences, $c_t, k_{t+1}, t = 0, 1, ..., $ that satisfy the non-negativity constraints for some $k_0 > 0$.

(b) Explain that failure of viability does not necessarily imply the non-negativity constraints are violated for small values of $t$, but it does imply that the non-negativity constraints are violated for sufficiently large values of $t$.

(c) Explain why viability requires the upper contour of $\Gamma$ to be greater than, or equal to, the 45 degree line for at least one value of $k_t$. To make the analysis interesting, one would want the upper contour to be substantially higher than the 45 degree line for a substantial interval of $k$. From here on, suppose that the latter is the case.
(d) Show graphically that there is a lowest value of $k_0$ for which the economy is viable. Denote this by $k_*$. Explain why there is a highest feasible value of the capital stock, $\bar{k}$, and indicated where that is in the graph of $\Gamma$. Let $K$ denote the set, $[k_*, \bar{k}]$, of possible values for the capital stock. We restrict $k_t \in K$ for all $t$.

(e) Verify that $A4.3 - A4.4$ are satisfied in the model economy with $\bar{c} > 0$. Let $g : K \to R$ denote the policy rule. Show that (i) $g(k_*) = k_*$, (ii) if $k \in \text{int}[K]$ then $g(k) \in \text{int}[\Gamma(k)]$, (iii) $g(k)$ is strictly increasing for $k \in \text{int}[K]$, (iv) there exists a unique steady state, $k > k_*$, denoted by $k^*$ and $f'(k^*) = 1/\beta$ (verify that $k^* \in \text{int}[K]$), (v) $g$ cuts the 45 degree line from above.

(f) Suppose $k_0 > k_*$. Do we get convergence to $k^*$? Is that convergence monotone?

2. Problem 6.3, page 139 in S-L.

3. Exercise 6.7a-e, pages 157-158 in S-L. As discussed in class, the efficient allocations reflect a balance between the properties of preferences and technology. In 6.7e you are asked to study a parameterization and set of initial capital stocks for the two-sector model in which it is efficient for consumption to cycle. Presumably, the technology implies that there are gains to be had in cycling. Otherwise it would not be efficient to cycle, given the preference for smooth consumption implied by concavity of utility. What are those gains? What is the intuition behind the result that it is desirable for consumption to cycle?

4. Consider the neoclassical growth model studied in class, with $\beta = 1/1.03$, $\alpha = 1/3$, $\delta = 0.10$, $\gamma = 1$, where preferences are given by:

\[ \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma}, \]

and the aggregate resource constraint is given by:

\[ c_t + k_{t+1} - (1 - \delta)k_t \leq k_t^0. \]

(a) What is the steady state value of $k$?
(b) Let $k' = g(k)$ denote the policy rule in the recursive formulation of the model. Compute $g'$, the derivative of $g$ at the steady state value of $k$.

(c) According to the first order Taylor expansion of $g$ about steady state,

$$\frac{k_t - k}{k_0 - k} = (g')^t,$$

where $k$ denotes the steady state value of $k$, $k_0$ is the initial value of the capital stock, and $k_t$ is the value of the capital stock in period $t \geq 0$. From the above expression, one can compute how much time, $t$, it takes to close, say, 95 percent of an initial gap, $k_0 - k$, between the initial capital stock and its steady state. That is, one can compute the value of $t$ required for the value of $k_t - k$ to be 5 percent of the value of $k_0 - k$. Compute how much time (i.e., the value of $t$) is required to close 95 percent of a gap between an initial value of the capital stock, $k_0$, and the steady state value.

(d) Recompute the time needed to close 95 percent of the gap when the value of $\delta$ is changed to 0.99. Then, set $\delta$ back to 0.10 and instead change the value of $\gamma$ to 4. In each case, provide the economic intuition behind the change in the time needed to close 95 percent of the gap.

(e) Consider the Solow model, which uses the resource constraint described above, but which assumes that people save and invest a fixed fraction, $s$, of gross output, $k^o$:

$$k_{t+1} - (1-\delta)k_t = sk_t^o$$

What value of $s$ is required in order for the steady states of the neoclassical and Solow models to coincide? How much time does it take for 95 percent of the gap between $k_0$ and steady state capital to be closed in the Solow model? Provide economic intuition behind the different amounts of time required, in the Solow and neoclassical growth models, to close 95 percent of the gap.