1. (Financial friction theory of Solow residual). Consider an economy in which a final good is produced using the following Dixit-Stiglitz aggregator:

\[ y = \left[ \int_0^1 y_i^\lambda di \right]^\lambda, \quad 0 < \lambda < 1. \]

A representative, competitive firm produces \( y \) taking the price of \( y \) (normalized to equal unity) and the prices of the intermediate inputs, \( p_i \), as given. The \( i^{th} \) intermediate good is produced by a monopolist using the following production function:

\[ y_i = k_i^\alpha l_i^{1-\alpha}, \quad 0 < \alpha < 1, \]

\( i \in (0, 1). \) Here, \( l_i \) and \( k_i \) denote the quantity of labor and capital, respectively, hired by the \( i^{th} \) monopolist. The \( i^{th} \) monopolist is competitive in factor markets, where it can rent as much or as little capital and labor as it wishes, at factor prices, \( r \) and \( w \), respectively.

Consider the following financial friction, for which I do not provide a structural explanation. A subset of \( \nu \) firms must borrow their factor costs in advance. In particular, suppose that firms with \( i \in (0, \nu) \) must go to a loan market to borrow their labor and capital bills, respectively, before production commences. That is, for a firm with \( i \in (0, \nu) \), the cost of labor at the end of the period when it computes its profits is \( Rwl_i \). Similarly, its end of period cost of capital is \( Rwk_i \). Here, \( R > 1 \) denotes the gross interest rate. That is, \( R \) is the interest and principal on the beginning of period loans, \( rk_i \) and \( wl_i \), respectively. Thus, for firms \( i \in (0, \nu) \) profits are

\[ p_iy_i - Rrk_i - Rwl_i. \]

Firms, \( i \in (\nu, 1) \), pay their capital and labor costs out of end of period revenues and so their end of period profits are:

\[ p_iy_i - rk_i - wl_i. \]
Denote aggregate capital and labor by \( k \) and \( l \), respectively, where
\[
k = \int_0^1 k_i di, \quad l = \int_0^1 l_i di.
\]
Show that aggregate final output can be expressed as
\[
y = q(R) k^{\alpha} l^{1-\alpha},
\]
and display an explicit expression for \( q(R) \). Explain why \( q(1) = 1 \) and \( q < 1 \) for \( R > 1 \).

The ‘Solow residual’ is a variable computed using aggregate data, and is defined as follows:
\[
\frac{GDP}{K^a L^{1-a}}.
\]
Here, \( GDP \) denotes real gross domestic product, \( K \) denotes a measure of the aggregate stock of capital, \( L \) a measure of the aggregate labor input. Finally, \( a \) is a measure of the share of income going to labor based on data from the National Income and Product Accounts. During the period when the real business cycle model was popular, the Solow residual was interpreted as an exogenous shock to technology. Note that under our financial friction assumption, the Solow residual is actually a function of an endogenous variable, the interest rate.

2. The next two questions concern the model in the handout on “Notes on Financial Frictions Under Asymmetric Information and Costly State Verification”, which appears on the course website.\(^1\)

(a) Compute the steady state of the model as \( \mu \) gets very small, relative to its value in the calculations in the handout. What happens to the wedge, the bankruptcy rate and leverage? Provide intuition.

(b) Do the same as in (a), for the case where \( \gamma \) is increased relative to its value in the handout.

3. Replace the household utility function used there with \( u(c, l) = \log(c) + \psi \log(1-l) \), where a value for \( \psi \) is chosen so that \( l = 1/3 \) in steady

\(^1\)http://faculty.wcas.northwestern.edu/~lchrist/d11/d1111/economics_411-2011.htm
state. Hold $\psi$ fixed at this value throughout your work on this question. Use all the other parameter values reported in the handout. Use the same (imperfect) measure of the welfare cost of the financial system that is applied in the handout. The idea is that the economy is presented with an option to stay permanently in the steady state with financial frictions, or shifting (in one step!) to the steady state without financial frictions. Let $l^{*} (=1/3)$ and $c$ denote consumption in the initial steady state and let $l^{*}, c^{*}$ denote the corresponding quantities in the steady state without financial frictions. Then, the welfare measure is the value of $\lambda$ that solves:

$$
\log (c^{*} (1 - \lambda)) + \psi \log (1 - l^{*}) = \log (c) + \psi \log (1 - l) .
$$

Is the assessment of the welfare cost of financial frictions used here very different from the one reported in the handout?