Technical Appendix for ‘Financial Factors in Business Cycles’ (Preliminary)

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1 Appendix A: Equilibrium Conditions of the Model

The equations that characterize the model’s equilibrium are derived below. The model has two sources of growth: a deterministic trend in the price of investment goods, and a stochastic trend in neutral technology. Our model solution algorithm requires that the model variables be stationary, so the first section below describes how we scaled the variables in order to induce stationarity. After discussing the scaling, we discuss equilibrium conditions associated with each section of the model.

1.1 Scaling of the Variables

To solve the model, we first scale the variables and exploit the fact that, in terms of scaled variables, the model has a steady state. Real variables are scaled as follows:

\[
\begin{align*}
\bar{k}_{t+1} &= \frac{\bar{K}_{t+1}}{z_t^*\Upsilon_t}, \quad i_t = \frac{I_t}{z_t^*\Upsilon_t}, \quad Y_{zt} = \frac{Y_t}{z_t^*}, \quad z_t^* = z_t\Upsilon(\frac{-1}{\alpha_1}), \\
c_t &= \frac{C_t}{z_t^*}, \quad w_t = \frac{W_t}{z_t^*P_t}, \quad \theta_t = \frac{\theta_t^*}{z_t^*}, \quad g_t = \frac{G_t}{z_t^*},
\end{align*}
\]

where \(u_{c,t}\) denotes the derivative of present discounted utility with respect to \(C_t\) and \(z_t^*\) is defined in (??). The scaling here indicates that the capital stock and investment grow at a faster rate than does the output of goods and of consumption. Also, the marginal utility of consumption is falling at the same rate as output and consumption grow.

Prices are scaled as follows:

\[
q_t = \Upsilon^t \frac{Q_{t^*}}{P_t^*}, \quad r_t^k = \Upsilon^t r_t^{*k}, \quad \bar{w}_t = \frac{W_t}{z_t^*P_t^*}.
\]

This indicates that price and rental rate of capital, both expressed in units of consumption goods, are trending down with the growth rate of investment-specific technical change. At the same time, the real wage grows at the same rate as output and consumption.

Monetary and financial variables are scaled as follows:

\[
\begin{align*}
m_{3t} &= \frac{M_{3t}}{z_t^*P_t}, \quad m_{4t+1} = \frac{M_{4t+1}}{z_t^*P_t}, \quad m_{t} = \frac{M_t}{M_t^b}, \\
D_t^h &= M_{t}^b - M_t = M_t^b (1 - m_t) = m_t^b z_{t-1}^* P_{t-1} (1 - m_t), \\
d_{t}^m &= \frac{D_t^m}{M_t^b}, \quad b_t^{Tot} = \frac{B_t^{Tot}}{P_t z_t^*}, \quad n_{t+1} = \frac{N_{t+1}}{z_t^* P_t}, \quad \lambda_{z,t} = \lambda_t P_t z_t^*, \\
x_t &= \frac{X_t}{M_t^b}, \quad \nu_t = \frac{V_t}{z_t^* P_t}, \quad d_t = \frac{\mu G(\bar{\omega}_t, \sigma_{t-1}) (1 + R_t^k) Q_{K_{t-1}^*}}{z_t^* P_t}.
\end{align*}
\]
All these variables, when expressed in real terms, grow at the same rate as output.

Other scaling conventions used are:

\[ \tilde{p}_t = \frac{\tilde{P}_t}{P_t}, \quad p_{i,t+j} = \frac{P_{i,t+j}}{P_{t+j}}, \quad \mu_{z,t} = \mu_{z,t} \gamma^{\frac{\alpha}{1-\alpha}}, \]

\[ p^*_t = \frac{P^*_t}{P_t}, \quad w_t = \frac{\tilde{W}_t}{W_t}, \quad w^*_t = \frac{W^*_t}{W_t}, \quad w^{+}_t = \frac{W^+_t}{W_t}. \]

1.2 Equations Associated with Firms

The production function of a representative final good firm is provided in (??). The first order necessary for profit maximization is:

\[ P_{it} = P_t \left( \frac{Y_{it}}{Y_t} \right)^{\frac{\lambda_{f,t}}{1-\alpha}}. \]

The price of final goods satisfies the following relation:

\[ P_t = \left[ \int_0^1 P_{it}^{\frac{1-\lambda_{f,t}}{\lambda_{f,t}}} \, dl \right]^{1-\lambda_{f,t}}. \]

The production function of the \( j^{th} \) intermediate good producer is provided in (??). Marginal cost divided by \( P_t \) is:

\[ s_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^{1-\alpha} \left( \frac{r^k_t \left[ 1 + \psi_{k,t} R_t \right]}{\tilde{w}_t \left[ 1 + \psi_{l,t} R_t \right]} \right) \left( \frac{W_t}{P_t} \right)^{1-\alpha}. \]

In scaled terms,

\[ s_t = \left( \frac{1}{1-\alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^{1-\alpha} \left( \frac{r^k_t \left[ 1 + \psi_{k,t} R_t \right]}{\tilde{w}_t \left[ 1 + \psi_{l,t} R_t \right]} \right) \left( \frac{W_t}{P_t} \right)^{1-\alpha}. \]  

In addition, real marginal cost must be equal to the cost of renting one unit of capital divided by the marginal productivity of capital:

\[ s_t = \frac{\tilde{r}^k_t \left[ 1 + \psi_{k,t} R_t \right]}{\alpha \epsilon_k \left( \frac{z_{l,t}}{K_t} \right)^{1-\alpha}} = \frac{r^k_t \left[ 1 + \psi_{k,t} R_t \right]}{\alpha \epsilon_k \left( \frac{Y_{\mu,t} \gamma^{\frac{\alpha}{1-\alpha}}}{w_{ik,t}^+} \right)^{1-\alpha}}, \]
In (2), we have imposed that the share of aggregate homogeneous labor, say $\nu^l_t$, and the share of aggregate capital, say $\nu^k_t$, used in goods production are equal. That is, $\nu^l_t = \nu^k_t$.

This property of equilibrium reflects that the production function in the firm sector is the same as the (value-added) production function in the banking sector.

The $i^{th}$ firm that has the opportunity to reoptimize its price in the current period does so to maximize (2.2). Since the firm must satisfy demand, we can substitute out the demand curve in their objective function:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j \lambda_{t+j} \left[ (X_{t,j}\tilde{p}_t)^{1-\frac{\lambda_{f,t+j}}{\lambda_{f,t+j-1}}} Y_{t+j} - s_{t+j} Y_{t+j} (X_{t,j}\tilde{p}_t)^{\frac{\lambda_{f,t+j}}{\lambda_{f,t+j-1}}} \right],$$

where

$$X_{t,j} \equiv \frac{\tilde{\pi}_{t+j} \cdots \tilde{\pi}_{t+1}}{\pi_{t+j} \cdots \pi_{t+1}}, j > 0$$

$$= 1, \; j = 0.$$

The $i^{th}$ firm maximizes this expression by choice of $\tilde{p}_t$. The fact that this variable does not have an index, $i$, reflects that all firms that have the opportunity to reoptimize in period $t$ solve the same problem, and hence have the same solution. The first order condition is:

$$E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \tilde{p}_t^{\frac{\lambda_{f,t+j}}{\lambda_{f,t+j-1}}-1} [\tilde{p}_t X_{t,j} - \lambda_{f,t+j} s_{t+j}] = 0,$$

where $A_{t+j}$ is exogenous from the point of view of the firm:

$$A_{t+j} = \lambda_{z,t+j} Y_{z,t+j} (X_{t,j})^{\frac{\lambda_{f,t+j}}{\lambda_{f,t+j-1}}}.$$

After rearranging the first order condition for prices:

$$\tilde{p}_t = \frac{E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda_{f,t+j} s_{t+j}}{E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} X_{t,j}} = \frac{K_{p,t}}{F_{p,t}},$$

say, where

$$K_{p,t} \equiv E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} \lambda_{f,t+j} s_{t+j}$$

$$F_{p,t} = E_t \sum_{j=0}^{\infty} (\beta \xi_p)^j A_{t+j} X_{t,j}$$
When $\lambda_{f,t}$ is non-stochastic, $K_{p,t}$ and $F_{p,t}$ have convenient recursive representations:

$$E_t \left[ \lambda_{z,t} Y_{z,t} + \left( \frac{\hat{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{\lambda_{f,t}}} \beta \xi_p F_{p,t+1} - F_{p,t} \right] = 0 \quad (3)$$

$$E_t \left[ \lambda_f \lambda_{z,t} Y_{z,t} s_t + \beta \xi_p \left( \frac{\hat{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{\lambda_{f,t}-1}} K_{p,t+1} - K_{p,t} \right] = 0. \quad (4)$$

Turning to the aggregate price index:

$$P_t = \left[ \int_0^1 P_{it}^{\frac{1}{1-\lambda_{f,t}}} \frac{1}{1-\lambda_{f,t}} \right]^{(1-\lambda_{f,t})}$$

$$= \left[ (1 - \xi_p) \int_0^1 P_{it}^{\frac{1}{1-\lambda_{f,t}}} d\xi + \xi_p (\hat{\pi}_t P_{t-1})^{\frac{1}{1-\lambda_{f,t}}} \right]^{(1-\lambda_{f,t})}$$

After dividing by $P_t$, substituting out for $\hat{\pi}_t$, and rearranging:

$$\frac{1 - \xi_p \left( \frac{\hat{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_{f,t}}} \xi_{p,t} P_{t-1}}{(1 - \xi_p)} = \left( \frac{F_{p,t}}{K_{p,t}} \right)^{\frac{1}{1-\lambda_{f,t}}}$$

To construct the resource constraint, we explain below that the following price-distortion measure is required:

$$P_t^* = \left[ \int_0^1 \left( \frac{P_t(f)}{P_t} \right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}} df \right]^{\frac{1}{1-\lambda_{f,t}}}$$

Writing this expression explicitly:

$$P_t^* = \left( 1 - \xi_p \right) \hat{P}_t^{\frac{1}{1-\lambda_{f,t}}} + \xi_p \left( \frac{\hat{\pi}_t P_{t-1}^*}{P_t} \right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}$$

$$= \left( 1 - \xi_p \right) \left( 1 - \xi_p \left( \frac{\hat{\pi}_t P_{t-1}^*}{\hat{\pi}_t P_{t-1}} \right)^{\frac{1}{1-\lambda_{f,t}}} \right)^{\lambda_{f,t}} + \xi_p \left( \frac{\hat{\pi}_t P_{t-1}^*}{\hat{\pi}_t P_{t-1}} \right)^{\frac{\lambda_{f,t}}{1-\lambda_{f,t}}}$$

$$= h_p \left( \frac{\hat{\pi}_t}{\hat{\pi}_t P_{t-1}^*} \right).$$
say. In setting up our equilibrium conditions, the five equations are (1), (2), (3), (4), and (5).

Following Yun (1996), when we work with a first order linear approximation about a steady state in which there are no price distortions and \( \lambda_f \) is stochastic, then the equations associated with price setting reduce to just one:

\[
\hat{\pi}_t - (\pi_t^{target} + (1 - \iota) \hat{n}_{t-1}) = \beta E_t \left[ \hat{\pi}_{t+1} - (\pi_{t+1}^{target} + (1 - \iota) \hat{n}_t) \right] + \frac{(1 - \beta \xi_p)(1 - \xi_p)}{\xi_p} \left( \hat{\lambda}_{f,t} + \hat{s}_t \right).
\]

(6)

### 1.3 Capital Producers

The production function for new capital is given by:

\[
x' = x + (1 - S(\zeta_{i,t} I_t/I_{t-1})) I_t,
\]

where \( x \) denotes the end-of-period \( t \) stock of existing, installed capital and \( x' \) denotes the beginning-of-period \( t+1 \) stock of installed capital. The capital producing firm’s time \( t \) profits are:

\[
\Pi^K_t = Q_{K',t} \left[ x + (1 - S(\zeta_{i,t} I_t/I_{t-1})) I_t \right] - Q_{K',t} x - \frac{P_t I_t}{\mu_{Y,t}}.
\]

Since the choice of \( I_t \) influences profits in period \( t + 1 \), the firm must incorporate that into the objective as well. But, that term involves \( I_{t+1} \) and \( x_{t+1} \). Evidently, the problem choosing \( x_t \) and \( I_t \) expands into the problem of solving an infinite horizon optimization problem:

\[
\max_{\{I_{t+j}, x_{t+j}\}} \sum_{j=0}^{\infty} \beta^j \lambda_{t+j} \left[ Q_{K',t+j} (x_{t+j} + F(I_{t+j}, I_{t+j-1}, \zeta_{i,t+j}) \right] - Q_{K',t+j} x_{t+j} - \frac{P_{t+j} I_{t+j}}{\mu_{Y,t+j}}
\]

where it is understood that \( I_{t+j} \) and \( x_{t+j} \) are functions of all the shocks up to period \( t + j \), and

\[
F(I_t, I_{t-1}, \zeta_{i,t}) \equiv (1 - S(\zeta_{i,t} I_t/I_{t-1})) I_t.
\]

From this problem it is evident that any value of \( x_{t+j} \) whatsoever is profit maximizing. Thus, setting \( x_{t+j} = (1 - \delta) \hat{K}_{t+j} \) is consistent with both profit maximization by firms and with market clearing.
The first order necessary condition for maximization of $I_t$ is:

$$E_t \left[ \lambda_t \Upsilon^{-t} P_t q_t F_{1,t} - \lambda_t \frac{P_t}{\Upsilon^{t+1} \mu_{t+1}} + \beta \lambda_{t+1} P_{t+1} q_{t+1} \Upsilon^{-t-1} F_{2,t+1} \right] = 0,$$

or, after multiplication by $z_{t+1} \Upsilon^t$:

$$E \left[ \lambda_{z,t} q_t F_{1,t} - \lambda_{z,t} \frac{1}{\mu_{z,t}} + \beta \frac{\lambda_{z,t+1}}{\mu_{z,t+1}} q_{t+1} F_{2,t+1} \Omega_t \right] = 0. \quad (7)$$

The law of motion of capital is:

$$K_{t+1} = (1 - \delta) K_t + \left[ 1 - S \left( \frac{\zeta_{z,t} I_t}{I_{t-1}} \right) \right] I_t,$$

and $S'' > 0$ is a model parameter. Also, $I/I_{t-1}$ is the the growth rate of investment along a steady state growth path:

$$\frac{I}{I_{t-1}} = \Upsilon^* \mu_z$$

or, after scaling,

$$\bar{k}_{t+1} = (1 - \delta) \frac{1}{\mu_{z,t}} \Upsilon \bar{k}_t + \left[ 1 - S \left( \frac{\zeta_{z,t} I_t}{I_{t-1}} \right) \right] \bar{k}_t. \quad (8)$$

### 1.4 Entrepreneurs

The following derives the equilibrium conditions associated with entrepreneurs. As noted in the body of the paper, we assume that the standard deviation of $\log(\omega)$ at date $t$ is $\sigma_t$, which is the realization of a stochastic process. Although the realization of $\omega$ is not known at the time the entrepreneur receives a loan from the bank, the value of $\sigma_t$ is known. We write the distribution function of $\omega$ as $F_t:

$$\Pr [\omega \leq x] = F_t(x).$$

After observing the time-$t + 1$ aggregate shocks, the entrepreneur decides on the time-$t + 1$ level of capital utilization, $u_{t+1}$, and then rents out capital services, $\omega K_{t+1} = u_{t+1} \omega \bar{K}_{t+1}.$ Higher rates of utilization are associated with higher costs, in currency units, as follows:

$$P_{t+1} \Upsilon^{-(t+1)} \tau_{t+1} a(u_{t+1}) \omega \bar{K}_{t+1}.$$ 

Here,

$$\tau_{t+1} a(u_{t+1}) \Upsilon^{-(t+1)} \omega \bar{K}_{t+1}$$
denotes the quantity of final output goods the firm must purchase if it is to utilize capital at the rate, \( u_{t+1} \), where \( a \) is an increasing and convex function. Also, \( \tau_{oil}^{t+1} \) is a unit-mean stochastic process that perturbs the capital utilization cost. We interpret this as a shock to the price of oil.

In period \( t + 1 \), the entrepreneur chooses \( u_{t+1} \) to solve:

\[
\max_{u_{t+1}} \left[ u_{t+1} r_{t+1} - \tau_{oil}^{t+1} a(u_{t+1}) \right] \omega \bar{K}_{t+1} P_{t+1} T^{-(t+1)},
\]

and maximization of \( u_{t+1} \) implies:

\[
r_{t+1}^{k} = \tau_{oil}^{t+1} a'(u_{t+1}).
\]

(9)

Entrepreneurs purchase physical capital at the end of period \( t \) at price \( Q_{\bar{K},t} \) and sell the undepreciated component at the end of period \( t + 1 \) at price \( Q_{\bar{K},t+1} \). The entrepreneur pays tax rate, \( \tau^{k} \), on income earned from renting capital, subject to being permitted to deduct depreciation. For an entrepreneur who receives idiosyncratic productivity shock, \( \omega \), the gross rate of return on capital purchased in time-\( t \) is

\[
1 + R_{t+1}^{k,\omega} = \left\{ \frac{(1 - \tau^{k}) \left[ u_{t+1} r_{t+1}^{k} - \tau_{oil}^{t+1} a(u_{t+1}) \right]}{Q_{\bar{K},t}} T^{-(t+1)} P_{t+1} + (1 - \delta) Q_{\bar{K}',t+1} + \tau^{k} \delta Q_{\bar{K}',t} \right\} \omega
\]

\[
= (1 + R_{t+1}^{k}) \omega.
\]

Here, \( R_{t+1}^{k} \) is the average rate of return on capital across all entrepreneurs.

We suppose that \( N_{t+1} < Q_{\bar{K}',t} \bar{K}_{t+1} \), where \( Q_{\bar{K}',t} \bar{K}_{t+1} \) is the cost of the capital purchased by entrepreneurs with net worth, \( N_{t+1} \). The part of the capital stock that cannot be financed with net worth must be financed with bank loans, \( B_{t+1} \):

\[
B_{t+1} = Q_{\bar{K}',t} \bar{K}_{t+1} - N_{t+1} \geq 0.
\]

We suppose that the entrepreneur receives a standard debt contract from the bank. This specifies a loan amount, \( B_{t+1} \), and a gross rate of interest, \( Z_{t+1} \), to be paid if \( \omega \) is high enough. Entrepreneurs who draw \( \omega \) below a cutoff level, \( \bar{\omega}_{t+1} \), are bankrupt and must give everything they have to the bank. The cutoff satisfies:

\[
\bar{\omega}_{t+1} (1 + R_{t+1}^{k}) Q_{\bar{K}',t} \bar{K}_{t+1} = Z_{t+1} B_{t+1}.
\]

(10)

The bank finances its time-\( t \) loans to entrepreneurs, \( B_{t+1} \), by borrowing from households. We assume the bank pays households a nominal rate of return, \( R_{t+1}^{e} \), that is not contingent upon the realization of \( t + 1 \) shocks.
We suppose that the only source of cash to banks in each $t + 1$ state of nature is the receipts from entrepreneurs. In particular, they do not have access to other state-contingent markets for cash. As a result, the cash banks pay out to households in each period $t + 1$ state of nature cannot exceed the cash they receive from entrepreneurs in that state of nature. This condition, together with the ex ante period $t$ zero profit condition resulting from free entry by banks implies that net cash flow must be zero in each period $t + 1$ state of nature:

$$ [1 - F_t(\bar{\omega}_{t+1})] Z_{t+1} B_{t+1} + (1 - \mu) \int_{0}^{\bar{\omega}_{t+1}} \omega dF_t(\omega) (1 + R^k_{t+1}) Q_{R',t} \bar{K}_{t+1} = (1 + R^e_{t+1}) B_{t+1}. $$

(11)

The first term corresponds to the revenues received from entrepreneurs with idiosyncratic shock, $\omega$, above the cutoff. The second term before the equality is the revenues, after monitoring costs, of bankrupt entrepreneurs. The right side corresponds to the funds that must be paid to households. Substituting out for $Z_{t+1} B_{t+1}$ using (10), dividing the result by $(1 + R^k_{t+1}) Q_{R',t} \bar{K}_{t+1}$ and rewriting,

$$ \Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{1 + R^e_{t+1}}{1 + R^k_{t+1} Q_{R',t} \bar{K}_{t+1}} B_{t+1} $$

(12)

where

$$ G_t(\bar{\omega}_{t+1}) = \int_{0}^{\bar{\omega}_{t+1}} \omega dF_t(\omega). $$

(13)

BGG argue that, given a mild regularity condition on $F_t$, the expression on the left of the equality in (12) has an inverted $U$ shape in $\bar{\omega}_{t+1}$. There is some unique interior maximum, $\bar{\omega}^*_{t+1}$. It is increasing for $\bar{\omega}^N_{t+1} < \bar{\omega}^*_{t+1}$ and decreasing for $\bar{\omega}^N_{t+1} \bar{\omega}^*_{t+1}$. Conditional on a given ratio, $B_{t+1} / (Q_{R',t} \bar{K}_{t+1})$, the right side is a function of the period $t + 1$ shocks, because $R^k_{t+1}$ is. Evidently, the situation resembles the usual Laffer-curve setup, with the right side playing the role of the ‘government financing requirement’ and the left the role of tax revenues as a function of function of the ‘tax rate’, $\bar{\omega}_{t+1}$. So, we see that if there is any $\bar{\omega}_{t+1}$ that solves the above equation for given $B^N_{t+1} / (Q_{R',t} \bar{K}^N_{t+1})$, then generically there are two solutions. Between these two, the smaller one is preferred by entrepreneurs, so this is a candidate standard debt contract. These considerations indicate that $\bar{\omega}_{t+1}$ is a function of the realized period $t + 1$ uncertainty. From this it follows from (10) that $Z_{t+1}$ is too. In addition, we infer that any shock which drives up $R^k_{t+1}$ will simultaneously drive down $\bar{\omega}_{t+1}$ and, therefore, the rate of bankruptcy.

The standard debt contract can be characterized in terms of a loan amount, $B_{t+1}$, and a cut off level, $\bar{\omega}_{t+1}$. Equation (12) can be used to compute $\bar{\omega}_{t+1}$ for a given value of $B_{t+1}$. Another relationship which can be used to determine $B_{t+1}$, is the first order condition associated with the optimal loan contract.
As noted above, competition implies that the loan contract is the best possible one, from the point of view of the entrepreneur. The entrepreneur’s utility is linear in its net worth at the end of the loan contract:

$$E_t \left\{ \int_{\omega_{t+1}}^{\infty} \left[ (1 + R_{t+1}^k) \omega Q_{K',t} \bar{K}_{t+1} - Z_{t+1} B_{t+1} \right] dF_t(\omega) \right\}$$

$$= E_t \left\{ \int_{\omega_{t+1}^N}^{\infty} \left[ \omega - \omega_{t+1}^N \right] dF_t(\omega) (1 + R_{t+1}^k) \right\} Q_{K',t} \bar{K}_{t+1},$$

after substituting from (10). Dividing by $N_{t+1} (1 + R_{t+1}^e)$, the last expression can be written in compact form as follows:

$$E_t \left\{ \left[ 1 - \Gamma_t(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} \right\} k_{t+1}, \quad (14)$$

where

$$k_{t+1} = \frac{Q_{K',t} \bar{K}_{t+1}}{N_{t+1}}.$$

The standard debt contract is found by choosing $k_{t+1}, \bar{\omega}_{t+1}$ to maximize (14) subject to (12). In Lagrangian form, this problem is:

$$\max_{\bar{\omega},k} E_t \left\{ \left[ 1 - \Gamma_t(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} k + \lambda_{t+1} \left[ k \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} (\Gamma_t(\bar{\omega}) - \mu G_t(\bar{\omega})) - k + 1 \right] \right\},$$

where $\lambda_{t+1}$ is a multiplier. The first order conditions for $k$ and $\bar{\omega}$ are, respectively:

$$E_t \left\{ \left[ 1 - \Gamma_t(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} + \lambda_{t+1} \left[ \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} (\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})) - 1 \right] \right\} = 0$$

$$\Gamma_t'(\bar{\omega}_{t+1}) = \lambda_{t+1} \left[ \Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1}) \right]$$

Using the second expression to define the multiplier, we conclude that the necessary conditions that determine the two parameters of the optimal debt contract are (12) and:

$$E_t \left\{ \left[ 1 - \Gamma_t(\bar{\omega}_{t+1}) \right] \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} + \frac{\Gamma_t'(\bar{\omega}_{t+1})}{\Gamma_t'(\bar{\omega}_{t+1}) - \mu G_t'(\bar{\omega}_{t+1})} \left[ \frac{1 + R_{t+1}^k}{1 + R_{t+1}^e} (\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})) - 1 \right] \right\} = 0.$$  \quad (15)
The derivatives in the above expression are straightforward:

\[
\Gamma_t'(\bar{\omega}_{t+1}) = 1 - F_t(\bar{\omega}_{t+1}) - \bar{\omega}_{t+1} F_t' (\bar{\omega}_{t+1}) + G_t'(\bar{\omega}_{t+1}) \\
= 1 - F_t(\bar{\omega}_{t+1}^N) \\
G_t'(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1} F_t'(\bar{\omega}_{t+1}).
\]

The law of motion of aggregate net worth is given by the following equation:

\[
N_{t+1} = \gamma_t V_t + W_t^e, \tag{16}
\]

where \(V_t\) is the net worth of entrepreneurs at the end of the period, just prior to the time when \(1 - \gamma_t\) are selected to exit. In (16), \(\gamma_t\) captures the fact that at the end of the period, after the entrepreneur has sold his capital, paid off his debt, and earned rental income, he exits the economy with probability \(1 - \gamma_t\). At the same time, a fraction, \(1 - \gamma_t\), of new entrepreneurs enters. The fraction, \(\gamma_t\), who survive and the fraction, \(1 - \gamma_t\), who enter both receive a transfer, \(W_t^e\). Without this transfer, new entrepreneurs would not have any net worth, and they would not be able to buy any capital. In addition, among the \(\gamma_t\) entrepreneurs who survive there are some who are bankrupt and have no net worth. Without a transfer they too would never again be able to buy capital. In (16), \(V_t\) is

\[
V_t = (1 + R_t^k) Q_{K',t-1} K_t - \left[1 + R_t^e + \frac{\mu \int_0^{\bar{\omega}_t} \omega d F_t(\omega) (1 + R_t^k) Q_{K',t-1} K_t}{Q_{K',t-1} K_t - N_t} \right] (Q_{K',t-1} K_t - N_t)
\]

The first term in braces in (16) represents the revenues from selling capital, plus the rental income of capital, net of the costs of utilization, averaged across all entrepreneurs. The object in square brackets is the average gross rate of return paid by all entrepreneurs on time \(t-1\) loans. As indicated by equation (11), this must be the sum of what is owed by banks to households, plus monitoring costs associated with bankruptcy.

The \(1 - \gamma_t\) entrepreneurs who exit in period \(t\) consume a fraction, \(\Theta\), of their net worth:

\[
P_t C_t^e = (1 - \gamma_t) \Theta V_t. \tag{17}
\]

The complementary fraction, \(1 - \Theta\), is transferred, in lump-sum form, to households.

The key equilibrium conditions associated with the entrepreneur are (??), (??), (12), (??) and (16). Another equilibrium relation, (17), will be addressed in our discussion of the resource constraint. Equations (??) and (??) are in scaled form, and need not be transformed further. Equation (??), in terms of scaled variables, is:

\[
1 + R_{t+1}^k = \frac{(1 - \tau^k) [u_{t+1}^k u_{t+1}^k - \tau^a_{t+1} (u_{t+1})] + (1 - \delta) q_{t+1} \pi_{t+1} + \tau^k \delta}{\pi_{t+1} + \tau^k \delta}. \tag{18}
\]
Equation (12) after transforming the variables, becomes:

\[
\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{1 + R^e_t}{1 + R^k_t} \left( 1 - \frac{n_{t+1}}{q_t k_{t+1}} \right).
\] (19)

Multiply by \((q_t k_{t+1}/n_{t+1}) (1 + R^k_{t+1}) / (1 + R^e_{t+1})\), to obtain:

\[
\frac{q_t k_{t+1}}{n_{t+1}} = \frac{q_t k_{t+1}}{n_{t+1}} \frac{1 + R^k_{t+1}}{1 + R^e_{t+1}} \left[ \Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) \right] + 1.
\]

Rewriting (16), and expressing the result in terms of scaled variables:

\[
n_{t+1} = \frac{\gamma_t}{\mu \pi^{\ast}_t n^{\ast}_t} q_{t+1} k_t \left\{ R^k_t - R^e_t - \mu \int_0^{\bar{\omega}_t} \omega dF_{t-1}(\omega) \left( 1 + R^k_t \right) \right\} + w^e_t + \frac{\gamma_t}{\mu \pi^{\ast}_t} (1 + R^e_t) n_t.
\] (20)

### 1.5 Banking

Following is the Lagrangian representation of the bank problem, after substituting out for \(T_t\) using (??):

\[
\max_{\lambda_t, \mu_t, \alpha, \beta_t, \gamma_t, \delta_t, \zeta_t, \xi_t} \lambda_t \left\{ R_t S^w_t \left[ (1 + R^e_t) B_t - (1 + R^m_t) D_{m,t} - (1 + R^T_t) T_{t-1} \right] - [B_{t+1} - T_t - D_{t+1}^m] - R_{at} A_t - (1 + \psi_{k,t} R_t) P_t \tau^k_t K^b_t - (1 + \psi_{l,t} R_t) W_t L^b_t \right\} + \beta E_t \lambda_{t+1} [R_{t+1} S^w_{t+1} + \left( (1 + R^e_{t+1}) B_{t+1} - (1 + R^m_{t+1}) D_{m,t+1} - (1 + R^T_{t+1}) T_t \right] - [B_{t+2} - T_{t+1} - D_{t+2}^m] - R_{at+1} A_{t+1} - (1 + \psi_{k,t+1} R_{t+1}) P_{t+1} \tau^k_{t+1} K^b_{t+1} - (1 + \psi_{l,t+1} R_{t+1}) W_{t+1} L^b_{t+1} \right\} + \lambda_t \left[ h(x^b_t, K^b_t, L^b_t, A_t - \tau (A_t + S^w_t)/P_t, \xi_t, x^b_t, z_t) - (A_t + S^w_t + \delta D_t^m)/P_t \right] + \beta E_t \lambda_{t+1} [h(x^b_{t+1}, K^b_{t+1}, L^b_{t+1}, A_{t+1} - \tau (A_{t+1} + S^w_{t+1})/P_{t+1}, \xi_{t+1}, x^b_{t+1}, z_{t+1}) - (A_{t+1} + S^w_{t+1} + \delta D_{t+1}^m)/P_{t+1} \right] + \mu_t \left[ T_t + D_{t+1}^m - B_{t+1} \right] + \beta E_t \mu_{t+1} \left[ T_{t+1} + D_{t+2}^m - B_{t+2} \right]
\]

Here,

\[
h(x^b_t, K^b_t, L^b_t, \epsilon_t, \xi_t, x^b_t, z_t) = x^b_t \left( (K^b_t)^{\alpha} (z_t)^{1-\alpha} \right)^{\xi_t} (\epsilon_t)^{1-\xi_t},
\]

\[
e_t^b = \frac{E_t^b}{P_t} \frac{A_t - \tau_t (A_t + S^w_t)}{P_t}
\]
Here, $\mu_t$ and $\mu_{t+1}$ denote the Lagrange multipliers on (??). Differentiate with respect to $T_t$ and $D_{t+1}^m$:

$$\lambda_t + \mu_t = \beta E_t \lambda_{t+1} (1 + R_{t+1}^T)$$

$$\lambda_t + \mu_t = \beta E_t \lambda_{t+1} (1 + R_{t+1}^m) + \beta E_t \lambda_{t+1}^b \frac{S}{P_{t+1}}$$

Subtracting these:

$$E_t \left[ \beta \lambda_{t+1} (R_{t+1}^m - R_{t+1}^T) + \beta \lambda_{t+1}^b \frac{S}{P_{t+1}} \right] = 0 \quad (22)$$

The first term in braces represents the gains from increasing $D_{t+1}^m$. It reflects that when $D_{t+1}^m$ increases and $T_t$ therefore must decrease, profits rise because interest charges fall (we assume $R_{t+1}^T > R_{t+1}^m$). These profits are discounted to $t$ using $\beta \lambda_{t+1}$. The next term reflects that increasing $D_{t+1}^m$ requires increasing capital and labor inputs to provide the implied increase in banking services.

The first order conditions are, for $A_t$, $S_t^w$, $K_t^b$, $l_t^b$, respectively:

$$-\lambda_t R_t^a + \lambda_t^b \frac{1}{P_t} [(1 - \tau) h_{e^r,t} - 1] = 0 \quad (23)$$

$$\lambda_t R_t - \lambda_t^b \frac{1}{P_t} \tau h_{e^r,t} + 1 = 0 \quad (24)$$

$$-\lambda_t (1 + \psi_{k,t} R_t) P_t \tilde{r}_t^k + \lambda_t^b h_{K^b,t} = 0 \quad (25)$$

$$-\lambda_t (1 + \psi_{l,t} R_t) W_t + \lambda_t^b h_{l^b,t} = 0 \quad (26)$$

$$\lambda_t R_t^b - \lambda_t^b \frac{h_{e^r,t}}{P_t} = 0, \quad (27)$$

where $h_i$ denotes the partial derivative of $h$ with respect to its $i^{th}$ argument. Substituting for $\lambda_t^b$ in (25) and (26) from (24), we obtain:

$$(1 + \psi_{k,t} R_t) \tilde{r}_t^k = \frac{R_t h_{K^b,t}}{1 + \tau h_{e^r,t}},$$

and

$$(1 + \psi_{l,t} R_t) \frac{W_t}{P_t} = \frac{R_t h_{l^b,t}}{1 + \tau h_{e^r,t}}. \quad (28)$$

These are the first order conditions associated with the bank’s choice of capital and labor. Each says that the bank attempts to equate the marginal product - in terms of extra loans - of an additional factor of production, with the associated marginal cost. The marginal
product in producing loans must take into account two things: an increase in $S^w$ requires an equal increase in deposits and an increase in deposits raises required reserves. The first raises loans by the marginal product of the factor in $h$, while the reserve implication works in the other direction.

After scaling, (28) reduces to:

$$0 = \frac{R_t h_{z,l_t} \psi_t}{1 + \tau h_{e_r,t}} - (1 + \psi_t R_t) w_t,$$

where

$$h_{z,l_t} \equiv h_{\psi_t} / z_t^*.$$

Substituting from (24) for $\lambda_t^b$ into (22):

$$E_t \left[ \lambda_{t+1} \left( R_{t+1}^T - R_{t+1}^m \right) - \frac{\lambda_{t+1} \xi \kappa_{t+1}}{h_{e_r,t+1} \tau + 1} \right] = 0.$$

Expressing this in terms of scaled variables:

$$E_t \frac{\lambda_{z,t+1}}{\mu_{z,t+1} \pi_{t+1}} \left[ R_{t+1}^T - R_{t+1}^m - \frac{\xi \kappa_{t+1}}{h_{e_r,t+1} \tau + 1} \right] = 0,$$

(30)

Taking the ratio of (24) to (23), we obtain:

$$R_t^a = \frac{(1 - \tau) h_{e_r,t}^i - 1}{\tau h_{e_r,t}^i + 1} R_t.$$

This can be thought of as the first order condition associated with the bank’s choice of $A_t$. The object multiplying $R_t$ is the increase in $S^w$ the bank can offer for one unit increase in $A$. The term on the right of the equality indicates the net interest earnings from those loans. The term on the left indicates the cost. Recall that $R_t$ represents net interest on loans, because the actual interest is $R_t + R_t^a$, so that $R_t$ represents the spread between the interest rate charged by banks on their loans and the cost to them of the underlying funds. Since loans are made in the form of deposits, and deposits earn $R_t^a$ in interest, the net cost of a loan to a borrower is $R_t$.

The derivative of $h$ with respect to $l_t^b$ is:

$$h_{l_t^b} = \xi_t a^b x_t^b \left( (K_t^b)^{\alpha} (z_t l_t^b)^{1-\alpha} \right) \xi_t^{-1} (e_t^r)^{1-\xi_t} (1 - \alpha) \left( \frac{K_t^b}{z_t l_t^b} \right)^{\alpha} z_t,$$

which, after scaling, is

$$h_{l_t^b} = \xi_t a^b x_t^b (e_t^r)^{1-\xi_t} (1 - \alpha) \left( \frac{k_t}{\Upsilon_{z^*,t} l_t} \right)^{\alpha} z_t^*.$$
Here, we have used the fact, $k_t^b/l_t^b = k_t/l_t$. Also,

$$e_{v,t} = \frac{e_t^r}{(K_t^b)^{\alpha} (z_t l_t^b)^{1-\alpha}}.$$  

The derivative of $h$ with respect to excess reserves, $e_t^r$, is:

$$h_{e_t^r} = (1 - \xi_t) a^b x^b_t (e_{v,t})^{-\xi_t}.$$  

The production function for deposits is:

$$a^b x^b_t (e_{v,t})^{-\xi_t} e_t^r = \frac{M_t^b - M_t + S^w_t + \zeta D_{m,t}}{\bar{P}_t},$$  

which, after scaling, reduces to:

$$a^b x^b_t (e_{v,t})^{-\xi_t} e_t^r = \frac{M_t^b - M_t + (\psi_{l,t} W_t l_t + \psi_{k,t} k_t r_t^k K_t) + \zeta D_{m,t}}{z_t^* \bar{P}_t} = m_{1t} + m_{2t},$$  

where

$$m_{1t} = \frac{m_t^b (1 - m_t + \zeta d_{m,t})}{\bar{P}_t z_t^* \bar{P}_t},$$

$$m_{2t} = \psi_{l,t} w_t l_t + \psi_{k,t} r_t^k k_t \frac{\bar{P}_t}{\bar{P}_t \bar{Y}_t}.$$  

The ratio of real excess reserves to value-added, denote by $e_{v,t}$, is:

$$e_{v,t} = \frac{A_{k} - \tau (A_{k} + S_{w}^w)}{P_t} \frac{e_t^r}{(K_t^b)^{\alpha} (z_t l_t^b)^{1-\alpha}} \left(1 - \xi_t\right) a^b x^b_t (e_{v,t})^{-\xi_t} e_t^r = \frac{1}{\bar{P}_t} \left(1 - \nu_t^k\right) k_t \left(1 - \nu_t^l\right) l_t \left(1 - \nu_t^l\right) l_t$$

In practice, we set $\nu_t^k = \nu_t^l$. Here, value-added is expressed in terms of aggregate homogeneous labor, $l_t$. This is related to the differentiated labor of individual households by $(??)$. We denote the unweighted integral of differentiated household labor - what we assume is measured in the data - by $L_t$. In subsection 1.7 below, we show that $L_t$ and $l_t$ are related by
\[ l_t = (w_t^*)^{\frac{1}{\lambda w}} L_t, \] where \( w_t^* \) is a variable discussed there. Expressing the last relationship in terms of \( L_t \), we obtain:

\[
e_{v,t} \equiv \frac{\lambda w}{\lambda w - 1} L_t,
\]

1.6 Households

Our discussion is divided into two parts. First, we consider the non-wage decisions of the household. We then turn to the equilibrium conditions associated with wage setting. Finally, we derive the scaled representation of the utilitarian welfare function for our model.

1.6.1 Non Wage-setting Decisions

We consider the Lagrangian representation of the household problem, in which \( \lambda_t \geq 0 \) is the multiplier on \((??)\). The consumption and the wage decisions are taken before the realization of the financial market shocks. The other decisions, \( M_{t+1}^b, M_t \) and \( T_t \) are taken after the realization of all shocks during the period. The period \( t \) multipliers are functions of all the date \( t \) shocks. We now consider the first order conditions associated with this maximization problem.

\[
E^0_t \sum_{t=0}^{\infty} \beta^t \zeta_{c,t} \{ [u(C_t - bC_{t-1}) - \zeta_t z(h_{j,t}) - v_t \left[ \left( \frac{(1+\tau^c)P_{t+1}C_{t+1}}{M_{t+1}} \right)^{1-\chi_{t+1}} \left( \frac{(1+\tau^c)P_{t+1}C_{t+1}}{D_{t+1}^m} \right)^{1-\chi_{t+1}} \right]^{1-\sigma_q} - H \left( \frac{M_{t+1}}{M_{t+1-1}} \right) + \lambda_t \left[ (1 + R^m_t) (M_t^b - M_t) + X_t - T_t - D_{t+1}^m - (1 + \tau^c) P_t C_t \right]
\]

deleting terms in the budget constraint that are not in the household’s control. We now consider the various first order conditions associated with this maximization problem.
The first order condition with respect to time deposits, \( T_t \), is:

\[
E_t \left\{ -\lambda_t + \beta \lambda_{t+1} (1 + R^T_{t+1}) \right\} = 0,
\]

which, after scaling, becomes:

\[
E_t \left\{ -\lambda_{z,t} + \frac{\beta}{\mu_{z,t+1}\mu_{z, t+1}} \lambda_{z,t+1} (1 + R^T_{t+1}) \right\} = 0. \quad (34)
\]

Although the capital decision is made by the entrepreneur in the benchmark model, we also explore a more standard formulation in which that decision is made by the household (see Appendix B for further discussion). In this formulation, we drop the variables, \( \bar{\omega}_t \) and \( N_t \), and the three equations, (15), (19), and (20), which pertain to the standard debt contract and the law of motion of net worth. We replace these three equations with an intertemporal equation for the household:

\[
E \left\{ -\lambda_t + \beta \lambda_{t+1} (1 + R^k_{t+1}) \right\} \mid \Omega_t = 0,
\]

where \( R^k_{t+1} \), the after tax rate of return on capital, is defined in (18). Expressing this in terms of scaled variables:

\[
E \left\{ -\lambda_{zt} + \frac{\beta}{\pi_{t+1}\mu_{z,t+1}} \lambda_{zt+1} (1 + R^k_{t+1}) \right\} = 0. \quad (35)
\]

The first order condition with respect to \( M_t \) is:

\[
E_t \{ \zeta_{c,t} v_t \left[ \left( \frac{(1 + \tau^c) P_t C_t}{M_t} \right)^{(1-\chi_t)\theta_t} \left( \frac{(1 + \tau) P_t C_t}{M^p_t - M_t} \right)^{(1-\chi)(1-\theta_t)} \left( \frac{(1 + \tau^c) P_t C_t}{D^a_t} \right) \right]^{1-\sigma_q} 
\times \left[ \frac{(1 - \chi_t) \theta_t}{M_t} - \frac{(1 - \chi_t)(1 - \theta_t)}{M^p_t - M_t} - \zeta_{c,t} H'(\frac{M_t}{M_{t-1}}) \frac{1}{M_{t-1}} + \beta \zeta_{c,t+1} H'(\frac{M_{t+1}}{M_t}) \right] - \lambda_t R^a_t \} = 0 \quad (36)
\]

Equation (36) is the ‘money demand equation’ in this model. The term to the right of the last minus sign before the equality indicates the net increase in interest earnings in period \( t \) that occur with a unit decrease in transactions balances, \( M_t \). Multiplication by \( \lambda_t \) converts this gain into utility units. The rest of the expression indicates the utility cost of the reduction
in transactions services. Expressing this in scaled terms:

\[
E_t\{\zeta_{c,t}v_t\left[ (1 + \tau^c)c_t \left( \frac{1}{m_t} \right)^{(1-\chi_t)\theta_t} \left( \frac{1}{1 - m_t} \right)^{(1-\chi_t)(1-\theta_t)} \left( \frac{1}{d_t^m} \right)^{x_t} \right]^{1-\sigma_q} \times \\
\left( \frac{\pi_t \mu_z^*}{m_t^b} \right)^{2-\sigma_q} \left( \frac{(1 - \chi_t) \theta_t - (1 - \chi_t)(1 - \theta_t)}{1 - m_t} \right) - \zeta_{c,t} H'(\frac{m_t m_t^b \pi_{t-1} \mu_z^*_{t-1}}{m_{t-1}^b m_{t-1}^b}) \pi_t \mu_z^* \pi_{t-1} \mu_z^*_{t-1} \right. \\
+ \beta \zeta_{c,t+1} H'\left( \frac{m_{t+1} m_{t+1}^b \pi_{t+1} \mu_z^*_{t+1}}{m_t m_t^b} \right) \frac{(\pi_{t+1} \mu_z^*_{t+1})^2}{(m_t m_t^b)^2} \left( \frac{1}{\mu_{t+1} - M_{t+1}} \right) + \beta \lambda_{t+1} \left( 1 + R_{t+1}^a \right) - \lambda_t \}
\]

The first order condition with respect to \( M_{t+1}^b \) is:

\[
E_t\{\beta \zeta_{c,t+1} v_{t+1} \left( 1 - \theta_{t+1} \right) \left( 1 - \chi_{t+1} \right) \left[ (1 + \tau^c) C_{t+1} \left( \frac{1}{M_{t+1}} \right)^{(1-\chi_{t+1})\theta_{t+1}} \left( \frac{1}{M_{t+1}^b - M_{t+1}} \right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left( \frac{1}{\mu_{t+1} - M_{t+1}} \right)^{x_{t+1}} \right]^{1-\sigma_q} \\
\times \frac{1}{\mu_{t+1} - M_{t+1}} + \beta \lambda_{t+1} \left( 1 + R_{t+1}^a \right) - \lambda_t \} = 0
\]

This expression can be understood as follows. Suppose in period \( t \) the household reduces consumption by one unit of currency. The utility cost of this is \( \lambda_t \), the last term before the equality. The other two terms capture benefits. One is that the extra base at \( t+1 \) generates, holding \( M_{t+1} \) constant, more deposits in \( t+1 \). This generates extra interest in \( t+1 \) which can be consumed in \( t+1 \) for a time \( t \) utility benefit of \( \beta \lambda_{t+1} \). The other term captures the increased utility generated by the greater deposits. Rewriting this expression in terms of the scaled variables,

\[
E_t\{\beta \zeta_{c,t+1} v_{t+1} \left( 1 - \theta_{t+1} \right) \left( 1 - \chi_{t+1} \right) \left[ (1 + \tau^c) C_{t+1} \left( \frac{1}{M_{t+1}} \right)^{(1-\chi_{t+1})\theta_{t+1}} \left( \frac{1}{M_{t+1}^b - M_{t+1}} \right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left( \frac{1}{\mu_{t+1} - M_{t+1}} \right)^{x_{t+1}} \right]^{1-\sigma_q} \\
\times \frac{1}{\mu_{t+1} - M_{t+1}} + \beta \lambda_{t+1} \left( 1 + R_{t+1}^a \right) - \lambda_t \} = 0
\]
The first order condition with respect to $D^m_{t+1}$ is:

$$E_t\{\beta \zeta_{c,t+1} u_{t+1} \chi_{t+1} [(1 + \tau^C) P_{t+1} z_{t+1} c_{t+1} \left( \frac{1}{M^b_{t+1} m_{t+1}} \right)^{(1-\chi_{t+1})\theta_{t+1}} \times \left( \frac{1}{M^b_{t+1} (1 - m_{t+1})} \right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left( \frac{1}{M^m_{t+1} d_{t+1}^m} \right)^{(1-\sigma_q)} \frac{1}{M^m_{t+1} d_{t+1}^m} + \beta \lambda_{t+1} (1 + R^m_{t+1}) - \lambda_t \} = 0,$$

which, in terms of scaled variables reduces to:

$$E_t\{\beta \zeta_{c,t+1} u_{t+1} \chi_{t+1} [(1 + \tau^C) c_{t+1} \left( \frac{1}{m_{t+1}} \right)^{(1-\chi_{t+1})\theta_{t+1}} \times \left( \frac{1}{1 - m_{t+1}} \right)^{(1-\chi_{t+1})(1-\theta_{t+1})} \left( \frac{1}{d_{t+1}^m} \right)^{(1-\sigma_q)} \frac{1}{d_{t+1}^m} \frac{1}{m_{t+1}} \left( \frac{1}{m_{t+1}} \right)^{2-\sigma_q} (\pi_{t+1} \mu^z_{z,t+1})^{1-\sigma_q} + \frac{\beta}{\pi_{t+1} \mu^z_{z,t+1}} \lambda_{t+1} (1 + R^m_{t+1}) - \lambda_{zt} \} = 0 \tag{39}$$

We now consider $C_t$. It is useful to define $u_{c,t}$ as the derivative of the present discounted value of utility with respect to $C_t$:

$$E_t\left[u_{c,t} - \zeta_{c,t} u'(C_t - bC_{t-1}) + b\beta \zeta_{c,t+1} u'(C_{t+1} - bC_t)\right] = 0,$$

which, in terms of scaled variables corresponds to:

$$E_t\left[u_{c,t}^z - \frac{\mu^z_{z,t} \zeta_{c,t}}{c_t \mu^z_{z,t} - bc_{t-1}} + b\beta \frac{\zeta_{c,t+1}}{c_{t+1} \mu^z_{z,t+1} - bc_t}\right] = 0 \tag{40}$$

The first order condition associated with $C_t$ is:

$$E_t\{u_{c,t} - \zeta_{c,t} u_t C_t^{-\sigma_q} \left[ (1 + \tau^C) \left( \frac{P_t}{M_t} \right)^{(1-\chi_{t})\theta_t} \left( \frac{P_t}{M^b_t - M_t} \right)^{(1-\chi_{t})(1-\theta_t)} \left( \frac{P_t}{D^m_t} \right)^{\chi_t} \right]^{1-\sigma_q} - (1 + \tau^C) P_t \lambda_t \} = 0,$$

which, in terms of scaled variables, corresponds to

$$-\zeta_{c,t} u_t c_t^{-\sigma_q} \left[ (1 + \tau^C) \left( \frac{1}{m_t} \right)^{(1-\chi_{t})\theta_t} \left( \frac{1}{1 - m_t} \right)^{(1-\chi_{t})(1-\theta_t)} \left( \frac{1}{m_t^\zeta} \right)^{\chi_t} \right]^{1-\sigma_q} \left( \frac{\pi_t \mu^z_{z,t}}{m_t^\zeta} \right)^{1-\sigma_q} - (1 + \tau^C) \zeta_{c,t} \lambda_{z,t} \right].$$
1.6.2 Household Wage Decision

Suppose the $j^{th}$ household has the opportunity to reoptimize its wage at time $t$. We denote this wage rate by $\tilde{W}_t$. This is not indexed by $j$ because the situation of each household that optimizes its wage is the same. In choosing $\tilde{W}_t$, the household considers the discounted utility (neglecting currently irrelevant terms in the household objective) of future histories when it cannot reoptimize:

$$E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \{ -\zeta c_{t+i} \zeta z_t \bar{z}(h_{j,t+i}) + \lambda_{t+i}(1 - \tau^i_{t+i})W_{j,t+i} h_{j,t+i} \},$$

Here,

$$z(h) = \psi_{L_i} h^{1+\sigma_L} \frac{1}{1+\sigma_L},$$

and $\lambda_{t+i}$ is the multiplier on the household’s period $t+i$ budget constraint. The demand for the $j^{th}$ household’s labor services, conditional on it having optimized in period $t$ and not again since, is:

$$h_{j,t+i} = \left( \frac{\tilde{W}_{t+i}}{W_{t+i}} \right)^{\bar{\pi}}_{w} l_{t+i},$$

where $l_{t+i}$ denotes homogeneous labor in period $t+i$. Given our assumptions about the evolution of the wage of non-optimizing households, we have

$$\frac{\tilde{W}_{t+i}}{W_{t+i}} = \frac{\tilde{W}_t}{w_{t+i} z_t} X_{t,i} = \frac{w_t \tilde{w}_t}{w_{t+i}} X_{t,i},$$

where

$$X_{t,i} = \tilde{\pi}_w, t+i \cdots \tilde{\pi}_w, t+1 (\mu_{*i}^{1-\theta}) (\mu_{*,t+i} \cdots \mu_{*,t+1})^\theta, \ i > 0$$

$$= 1, \ i = 0,$$

and

$$\tilde{\pi}_w, t+1 \equiv (\pi_{t+1})^{\bar{\pi}_w, 1} (\pi_t)^{\bar{\pi}_w, 2} \frac{1}{\bar{\pi}_w}.$$

Also,

$$\frac{W_{j,t+i}}{P_{t+i} z^*_{t+i}} = X_{t,i} \frac{W_{j,t}}{P_t z^*_t} = X_{t,i} \frac{\tilde{W}_t}{W_t} \frac{W_t}{P_t z^*_t} = X_{t,i} w_t \tilde{w}_t.$$
Substituting out for \( h_{j,t+1} \) using the demand curve, and using the scaled variables:

\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left\{ -\zeta_{c,t+i} \zeta_{t+i} z \left( \frac{\tilde{W}_{t+i}}{W_{t+i}} \right)^{1-\lambda_w} l_{t+i} \right\} + \lambda_{t+i}(1 - \tau_{t+i}^l) W_{j,t+i} \left( \frac{\tilde{W}_{t+i}}{W_{t+i}} \right)^{1-\lambda_w} l_{t+i},
\]

and substituting out for the wages,

\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left\{ -\zeta_{c,t+i} \zeta_{t+i} z \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i}} \right)^{1-\lambda_w} l_{t+i} \right\} + \lambda_{t+i}(1 - \tau_{t+i}^l) \tilde{w}_{t+i} \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i}} \right)^{1-\lambda_w} l_{t+i}.\]

Differentiate this expression with respect to \( w_t \), rearrange we obtain the first order necessary condition for household optimization of \( w_t \):

\[
E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i} \right) \left( \frac{\lambda_{t+i} \tilde{w}_t}{\lambda_w} \right)^{1-\lambda_w} l_{t+i} \{ \frac{\lambda_{t+i}}{\lambda_w} w_t \tilde{w}_t X_{t,i} - \zeta_{c,t+i} \zeta_{t+i} z'_{t+i} \} = 0,
\]

where

\[
z'_{t+j} = \psi_L \left[ \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+j}} X_{t,j} \right) \right]^{\sigma_L}, \quad j > 0
\]

\[
z'_{t+0} = \psi_L \left[ w_t^{1-\lambda_w} l_t \right]^{\sigma_L}, \quad j = 0.
\]

The first order condition can be solved for \( w_t \) as follows:

\[
w_t = \left[ \frac{\psi_L K_{w,t}}{w_t F_{w,t}} \right]^{\frac{\lambda_w-1}{\lambda_w(1+\sigma_L)-1}},
\]

where

\[
K_{w,t} = E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i} \right) \left( \frac{\lambda_{t+i} \tilde{w}_t}{\lambda_w} \right)^{1+\sigma_L} l_{t+i} \zeta_{c,t+i} \zeta_{t+i}
\]

\[
F_{w,t} = E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \left( \frac{w_t \tilde{w}_t}{\tilde{w}_{t+i}} X_{t,i} \right) \left( \frac{\lambda_{t+i} \tilde{w}_t}{\lambda_w} \right)^{1-\lambda_w} l_{t+i} \lambda_{t+i} \tilde{w}_t X_{t,i}.
\]

We obtain a second restriction on \( w_t \) using the relation between the aggregate wage rate and the wage rates of individual households:

\[
W_t = \left[ (1 - \xi_w) \left( \tilde{W}_t \right)^{1-\lambda_w} + \xi_w \left( \tilde{\pi}_{w,t} (\mu_{z^*})^{1-\vartheta} (\mu_{z^*,t})^{\vartheta} W_{t-1} \right)^{1-\lambda_w} \right]^{1-\lambda_w}.
\]
Dividing both sides by $W_t$ and rearranging,

$$
\begin{bmatrix}
1 - \xi_w \left( \frac{\hat{w}_{w,t}}{\tilde{\pi}_{w,t}} (\mu_{z^*})^{1-\vartheta} (\mu_{z^*,t}^{\vartheta}) \right)^{\frac{1}{1-\lambda_w}} \\
1 - \xi_w
\end{bmatrix}^{1-\lambda_w} = w_t,
$$

where

$$
\pi_{w,t} \equiv \frac{W_t}{W_{t-1}} = \frac{\hat{w}_{t-1} z_{t-1}^* P_{t-1}}{\hat{w}_{t-1} z_{t-1}^* P_{t-1}}.
$$

Substituting, out for $w_t$ from the household’s first order condition for wage optimization:

$$
\frac{1}{\psi_L} \begin{bmatrix}
1 - \xi_w \left( \frac{\hat{w}_{w,t}}{\tilde{\pi}_{w,t}} (\mu_{z^*})^{1-\vartheta} (\mu_{z^*,t}^{\vartheta}) \right)^{\frac{1}{1-\lambda_w}} \\
1 - \xi_w
\end{bmatrix}^{1-\lambda_w (1+\sigma_L)} = \hat{w}_t F_{w,t} = K_{w,t}
$$

We move now to express $K_{w,t}$ and $F_{w,t}$ in recursive form. Making use of the facts,

$$
\frac{\hat{w}_t}{\hat{w}_{t+j}} X_{t,j} = \frac{\hat{w}_{t+1} \cdots \hat{w}_{t+j}}{\hat{w}_{t+1} \cdots \hat{w}_{t+j}} (\mu_{z^*})^{1-\vartheta} (\mu_{z^*,t+j} \cdots \mu_{z^*,t+1})^{\vartheta},
$$

$$
\frac{X_{t,i}}{X_{t,i-l}} = X_{t+l-i,i}, \quad l = 1, 2, ..., i - 1,
$$

it is straightforward to verify:

$$
E_t\left\{ \zeta_{c,t} \left( w_t^* \right)^{\frac{L_t}{\lambda_w}} (1 - \tau_1) L_t \right\} = 0
$$

(42)

$$
E_t\left\{ \left( w_t^* \right)^{\frac{L_t}{\lambda_w}} (1 + \sigma_L) \zeta_{c,t} \right\} = 0
$$

(43)

These equations make use of the equilibrium relation between $L_t$, the aggregate level of household employment, and $\tau_t$, the quantity of homogeneous labor. This relation can obtained by using the labor demand curve and the definition of $L_t$

$$
L_t \equiv \int_0^1 h_t (j) \, dj = l_t \times (w_t^*)^{\frac{\lambda_w}{1-\lambda_w}},
$$

(44)
where \( w^*_t \equiv W^*_t/W_t \) and:
\[
W^*_t = \left[ \int_0^1 W_t(j) \frac{\lambda_w}{1-\lambda_w} \, dj \right]^{\frac{1-\lambda_w}{\lambda_w}}.
\]

To obtain a law of motion for \( w^*_t \) divide both sides of the last equation by \( W_t \):

\[
w^*_t = \begin{bmatrix}
(1 - \xi_w) \left( \frac{\tilde{W}_t}{W_t} \right)^{\frac{\lambda_w}{1-\lambda_w}} + \xi_w \left( \frac{\tilde{\pi}_{w,t} (\mu_{z^*})^{1-\theta} (\mu_{z^*})^{\theta} \tilde{W}_{t-1} W^*_t}{W_t W_{t-1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \\
(1 - \xi_w) \left( \frac{\tilde{\pi}_{w,t} (\mu_{z^*})^{1-\theta} (\mu_{z^*})^{\theta} \tilde{W}_{t-1} W^*_t}{W_t W_{t-1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \\
\end{bmatrix}^{\frac{1-\lambda_w}{\lambda_w}} \]

say.

Following Yun (1996), as well as Erceg, Henderson and Levin, (2000), when we linearize the system about a non-stochastic steady state in which wages and prices are not distorted, then \( \dot{w}^*_t = 0 \) and the equation characterizing wage dynamics reduces to:

\[
\eta_0 \dot{\tilde{w}}_{t-1} + \eta_1 \tilde{w}_t + \eta_2 \tilde{E}_t \tilde{w}_{t+1} + \eta_3 \tilde{\pi}_{t-1} + \eta_4 \tilde{\pi}_t + \eta_5 \tilde{L}_t + \eta_6 \tilde{z}_{t+1} + \eta_7 \tilde{\mu}_{z^*,t} + \eta_8 \tilde{E}_t \tilde{\mu}_{z^*,t+1} + \eta_9 \tilde{\pi}_t^{\text{tar} et} + \eta_{10} \tilde{E}_t \tilde{\pi}_t^{\text{tar} et} = 0,
\]

where

\[
\begin{align*}
\eta_0 &= b_w \xi_w, \\
\eta_1 &= -b_w (1 + \xi_w^2 \beta) + \sigma_L \lambda_w, \\
\eta_2 &= b_w \xi_w \beta, \\
\eta_3 &= -b_w \xi_w (1 + \beta t^2), \\
\eta_4 &= b_w \xi_w \beta, \\
\eta_5 &= -\sigma_L (1 - \lambda_w), \\
\eta_6 &= (1 - \lambda_w), \\
\eta_7 &= b_w \xi_w (\vartheta - 1), \\
\eta_8 &= -b_w \xi_w \beta (\vartheta - 1), \\
\eta_9 &= b_w \xi_w t, \\
\eta_{10} &= -b_w \xi_w \beta t,
\end{align*}
\]

\[
\begin{align*}
b_w &= \frac{\sigma_L \lambda_w - \lambda_w}{(1 - \beta \xi_w)(1 - \xi_w)}.
\end{align*}
\]
1.6.3 Household Utility

In this subsection, we derive an expression for average household utility, in terms of scaled variables. The objects in (??) are written in terms of unscaled variables. Writing them directly in terms of scaled variables:

\[ E^j_\infty \beta^l_\infty \{ \log \left( z_{t+t-1}^* \right) + \log(c_{t+t} \mu z_{t+t} - bc_{t+t-1}) - \zeta_{t+t} z(h_{j,t+t}) \}
\]

\[ - v_{t+t} \left[ \frac{\left( 1+\tau c \right) c_{t+t} \mu z_{t+t} - bc_{t+t-1}}{m_{t+t}^2} \left( \frac{1}{m_{t+t}^2} \right)^{(1-\chi_{t+t}) \theta_{t+t}} \left( \frac{1}{(1-m_{t+t}^2)} \right)^{(1-\chi_{t+t}) (1-\theta_{t+t})} \left( \frac{1}{m_{t+t}^2} \right)^{\chi_{t+t}} \right]^{1-\sigma_q} \].

Since \( \log \left( z_{t+t-1}^* \right) \) is exogenous, there is no loss in simply dropping it. So, the period utility function is simply

\[ \zeta_{c,t} \log(c_{t} \mu z_{t} - bc_{t-1}) - \zeta_{t} z(h_{j,t}) \]

\[ - v_{t} \left[ \frac{\left( 1+\tau c \right) c_{t} \mu z_{t} - bc_{t-1}}{m_{t}^2} \left( \frac{1}{m_{t}^2} \right)^{(1-\chi_{t}) \theta_{t}} \left( \frac{1}{(1-m_{t}^2)} \right)^{(1-\chi_{t}) (1-\theta_{t})} \left( \frac{1}{m_{t}^2} \right)^{\chi_{t}} \right]^{1-\sigma_q} \].

Note that utility is a function of the \( j^{th} \) household’s level of effort. We adopt a social welfare function which weights each household equally. So, we must integrate:

\[ \int_0^1 z(h_{j,t}) dh_{j,t} = \frac{\psi_L}{1+\sigma_L} \int_0^1 h_{j,t}^{1+\sigma_L} dh \]

\[ = \frac{\psi_L}{1+\sigma_L} \int_0^1 \left( \frac{W_{j,t} w}{W_t} \right) \left( \frac{\lambda_w}{1-\lambda_w} \right) \frac{l_t}{l_t}^{1+\sigma_L} \]

\[ = \frac{\psi_L}{1+\sigma_L} \left( \frac{1}{W_t} \right) \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} \left( l_t \right)^{1+\sigma_L} \left( W_t^+ \right)^{\lambda_w(1+\sigma_L)} \]

\[ = \frac{\psi_L}{1+\sigma_L} \left( w_t^+ \right) \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} \left( l_t \right)^{1+\sigma_L}, \]

where the demand curve for \( h_{j,t} \) has been used:

\[ h_{j,t} = \left( \frac{W_{j,t}}{W_t} \right) \frac{\lambda_w}{1-\lambda_w} l_t \]

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Also,

\[ W_t^+ = \left[ \int_0^1 (W_{j,t}) \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} dj \right] \frac{1-\lambda_w}{\lambda_w(1+\sigma_L)} \]

\[ = \left[ (1 - \xi_w) \left( W_t \right) \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} + \xi_w \left( \frac{\hat{\pi}_{w,t} (\mu_{z*,t})^{\theta} (\mu_{z*,t})^{\vartheta} W_t^{+} - 1)}{1 - \lambda_w} \right) \right] \frac{1-\lambda_w}{\lambda_w(1+\sigma_L)} \]

Divide on both sides by \( W_t \):

\[ w_t^+ = \left[ (1 - \xi_w) w_t \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} + \xi_w \left( \frac{\hat{\pi}_{w,t} (\mu_{z*,t})^{\theta} (\mu_{z*,t})^{\vartheta} w_{t-1}^+)}{1 - \lambda_w} \right) \right] \frac{1-\lambda_w}{\lambda_w(1+\sigma_L)} \]

Then, substituting out for \( w_t \),

\[ w_t^+ = \left[ (1 - \xi_w) \left( 1 - \xi_w \left( \frac{1}{1 - \xi_w} \right) \right) \right] \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} \]

\[ + \xi_w \left( \frac{\hat{\pi}_{w,t} (\mu_{z*,t})^{\theta} (\mu_{z*,t})^{\vartheta} w_{t-1}^+)}{1 - \lambda_w} \right) \]

\[ = h^+ \left( \frac{\hat{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z*,t}, w_{t-1}^+ \right) \]

Recall,

\[ (w_t^*)^{\lambda_w \sigma_L^{-1}} \frac{L_t}{L_t} = l_t \]

so that, in terms of total household employment, \( L_t \):

\[ \int_0^1 z(h_{j,t}) dj = \frac{\psi_L}{1+\sigma_L} \left( w_t^+ \right) \frac{\lambda_w(1+\sigma_L)}{1-\lambda_w} \left( w_t^+ \right) \frac{\lambda_w(1+\sigma_L)}{\lambda_w - 1} L_t^{1+\sigma_L} \]

\[ = \frac{\psi_L}{1+\sigma_L} \left( \frac{w_t^*}{w_t^+} \right) \frac{\lambda_w(1+\sigma_L)}{\lambda_w - 1} L_t^{1+\sigma_L} \]

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We conclude that the scaled, period utility function is:

$$u(c_t, c_{t-1}, w^*, w^+_t, L_t, m_t, m_{t-1}, x_{t-1}, \mu_{z^*, t}, \zeta_c, \zeta_t, \tau_c^t, \chi_t, \theta_t) = \zeta_c [\log(c_t \mu_{z^*} - b c_{t-1})] - \zeta_t \psi_L \left( \frac{w^*_t}{w^+_t} \right)^{\frac{\lambda_w (1 + \sigma_L)}{\lambda_w - 1}} \frac{L_t^{1 + \sigma_L}}{L_t^{1 + \sigma_L}}$$

$$- v_t \left[ \frac{(1 + t^* \pi_c \mu_{z^*}, t \pi_t \mu_{z^*}, t)}{1 - \sigma_q} \right].$$

1.7 Resource Constraint and Zero Profit Condition

Following the approach of Yun (1996), we now develop the aggregate resource constraint for this economy, in terms of the aggregate stock of capital and the aggregate supply of labor by households. Let $Y^*$ denote the unweighted integral of output of the intermediate good producers (we assume that production is non-negative in each firm):

$$Y^*_t = \int_0^1 Y_{j,t} dj$$

$$= \int_0^1 [\epsilon_t K_{j,t}^\alpha (z_{l,t})^{1-\alpha} - \Phi z^*_t] dj$$

$$= \epsilon_t \left( \frac{K_t}{l_t} \right)^\alpha (z_t)^{1-\alpha} \int_0^1 l_{j,t} dj - \Phi z^*_t,$$

where $K_t$ is the economy-wide stock of capital services and $l_t$ is the economy-wide level of homogenous labor. This expression exploits the fact that all firms - intermediate good firms as well as banks - confront the same factor prices, and so they adopt the same capital services to homogeneous labor ratio. In equilibrium, this ratio must coincide with the economy-wide aggregate capital to homogeneous labor ratio. Let $v^*_t$ denote the share of economy-wide homogeneous labor used by intermediate goods firms. Then,

$$Y^*_t = v^*_t \epsilon_t (K_t)^\alpha (z_t)^{1-\alpha} - \Phi z^*_t.$$

Recall that the demand for $Y_{j,t}$ is

$$\left( \frac{P_{j,t}}{P_t} \right)^{\lambda_{j,t} - 1} = \frac{Y_{j,t}}{Y_t},$$
so that

\[ Y_t^* = \int_0^1 Y_{j,t} dj = \int_0^1 Y_t \left( P_t \right)^{\frac{\lambda_{j,t}}{1-\lambda_{j,t}}} dj = Y_t \left( P_t \right)^{\frac{\lambda_{j,t}}{1-\lambda_{j,t}}} P_t^{\frac{1}{1-\lambda_{j,t}}}, \]

say, where

\[ P_t^* = \left[ \int_0^1 P_t^{\frac{\lambda_{j,t}}{1-\lambda_{j,t}}} dj \right]^{1-\lambda_{j,t}} \]

Then,

\[ Y_t = (p_t^*)^{\frac{\lambda_{j,t}}{1-\lambda_{j,t}}} \left[ u_t^j z_t^{1-\alpha} (K_t)^\alpha \left( (w_t^*)^{\frac{\lambda_w}{1-\lambda_w}} L_t \right)^{1-\alpha} - \Phi z_t^j \right], \tag{50} \]

where

\[ p_t^* = \frac{P_t^*}{P_t}. \]

The law of motion of \( p_t^* \) is provided in (5). In (50), we have written aggregate homogeneous labor in terms of the aggregate of household differentiated labor, using (44). The law of motion for \( w_t^* \) is provided in (45). The first equality uses the results derived above, and the second can be confirmed by taking into account the demand curve and integrating.

Evaluating the resource constraint, (??) at equality, replacing \( Y_t \) by (50), and scaling by \( z_t^* \):

\[ d_t + \tau_{o,\Delta} a(u_t) \frac{\bar{k}_t}{\mu_t} + g_t + c_t + \frac{i_t}{\mu_{Y,t}} + \Theta(1-\gamma)v_{nt} \]

\[ = (p_t^*)^{\frac{\lambda_{j,t}}{1-\lambda_{j,t}}} \left\{ \epsilon_t \left[ \frac{\bar{k}_t}{\mu_t} \right]^{\alpha} \left[ (w_t^*)^{\frac{\lambda_w}{1-\lambda_w}} L_t \right]^{1-\alpha} - \Phi \right\}, \tag{51} \]

where,

\[ d_t = \frac{\mu G(\bar{\omega}_t, \sigma_{t-1}) \left( 1 + R_{K_{t-1}}^k \right) Q_{K_{t-1}} \bar{k}_t}{z_t^* P_t} \]

\[ = \frac{\mu G(\bar{\omega}_t, \sigma_{t-1}) \left( 1 + R_{K_{t-1}}^k \right) q_{t-1} \bar{k}_t}{\mu_{z,t}^*} \frac{1}{\pi_t}. \]
We will use a steady state zero profit condition to determine a value for the constant term in production. Economy-wide average period $t$ profits are:

$$\int_0^1 P_t Y_t dj - P_t s_t \left( \int_0^1 Y_t dj + \Phi z_t^* \right) = \int_0^1 P_t Y_t dj - P_t s_t \left( Y_t \left( p_t^* \right)^{\lambda_f,t} + \Phi z_t^* \right) = P_t Y_t - P_t s_t \left( Y_t \left( p_t^* \right)^{\lambda_f,t} + \Phi z_t^* \right). \tag{52}$$

### 1.8 Other Variables

Other variables that are of interest in our analysis are the laws of motion of the monetary base, $M_1^b$, $M_3^b$, the external finance premium and total loans. The monetary base evolves as follows:

$$M_{t+1}^b = M_t^b (1 + x_t),$$

where $x_t$ is the net growth rate of the monetary base ($x_t \equiv X_t / M_t^b$). In terms of scaled variables, this law of motion is:

$$m_{t+1}^b = \frac{1}{\pi t \mu^*_t} m_t^b (1 + x_t). \tag{53}$$

$M_3^b$ is defined as the monetary base, plus household demand deposits, plus firm demand deposits, which consists of working capital loans:

$$M_3^b = M_t^b + D_t^m + \psi_{l,t} W_t l_t + \psi_{k,t} P_t r_t K_t.$$

Our definition of $M_3^b$ is designed to correspond to the end of period $t$ stock of $M_3$. The monetary base, $M_t^b$, is defined as the base available at the beginning of time $t$. Similarly, the savings deposits, $D_t^m$, are made at the start of period $t$. After scaling,

$$m_{3t} = m_{t+1}^b \left( 1 + d_t^m \right) + \psi_{l,t} w_t l_t + \psi_{k,t} r_t^k u_t \kappa_t.$$

Total loans are defined as working capital loans plus loans to entrepreneurs:

$$B_{t+1}^{Tot} = \psi_{l,t} W_t l_t + \psi_{k,t} P_t r_t^k K_t + Q_{K,t} \tilde{K}_{t+1} - N_{t+1},$$

or, in scaled terms:

$$b_{t+1}^{Tot} = \psi_{l,t} w_t l_t + \psi_{k,t} \frac{r_t^k u_t \kappa_t}{\mu^*_t} + q_t \tilde{K}_{t+1} - n_{t+1}. \tag{55}$$

The external finance premium is defined as follows:

$$P_t^e = Z_t - (1 + R_t^e) = \bar{\omega}_t \left( 1 + R_t^e \right) \frac{q_{t-1} \tilde{K}_t}{q_{t-1} \tilde{K}_t - n_t} - (1 + R_t^e), \tag{56}$$

after making use of (10) and after scaling.
1.9 Pulling all the Equations Together

Following is a concise listing of all the equilibrium conditions we have derived, expressed in scaled form.

Equation (1) is a measure of marginal cost:

$$s_t = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right) \left( \frac{1}{\epsilon_t} \right) \left( \frac{1 + \psi_{k,t} R_t}{1 + \psi_{k,t} R_t} \right)^{\alpha} \left( \frac{\bar{w}_t [1 + \psi_{l,t} R_t]}{\epsilon_t} \right)^{1-\alpha}.$$  

Equation (2) is another measure of marginal cost:

$$s_t = \frac{\tau_t^k \left[ 1 + \psi_{k,t} R_t \right]}{\epsilon_t} \left( \frac{\bar{w}_t [1 + \psi_{l,t} R_t]}{\epsilon_t} \right)^{1-\alpha}.$$  

Equation (7) is the first order condition for investment by capital producers:

$$E_t \left[ \lambda_{zt} q_{t+1} F_{1,t} - \lambda_{zt} \frac{1}{\mu_{r,t}} + \beta \frac{\lambda_{zt} \mu_{r,t+1}}{\mu_{r,t} \mu_{t+1}} q_{t+1} F_{2,t+1} \right] \Omega_t = 0.$$  

Equation (9) is entrepreneurs’ first order condition for capital utilization:

$$r_t^k = \tau_t^{oil} a(d(u_t)).$$  

Equation (15) is the condition for the standard debt contract offered to entrepreneurs to be optimal (subject to constraints):

$$E_t \left[ \left[ 1 - \Gamma_t(\bar{w}_{t+1}) \right] \left[ 1 + \frac{R_{t+1}^e}{R_{t+1}^e} \right] + \frac{\Gamma_t(\bar{w}_{t+1})}{\Gamma_t(\bar{w}_{t+1})} \left( \frac{1 + R_{t+1}^e}{1 + R_{t+1}^e} \left( \Gamma_t(\bar{w}_{t+1}) - \mu G_t(\bar{w}_{t+1}) \right) - 1 \right) \right] = 0.$$  

Equation (19) is the zero profit condition associated with lending to entrepreneurs:

$$\Gamma_t(\bar{w}_{t+1}) - \mu G_t(\bar{w}_{t+1}) = \frac{1 + R_{t+1}^e}{1 + R_{t+1}^e} \left( 1 - \frac{n_{t+1}}{q_{t+1}} \right)$$  

Equation (20) is the law of motion for net worth:

$$n_{t+1} = \frac{\gamma_t}{\pi_t \mu_{s,t}} \left( R_t^k - R_t^e - \mu \int \omega dF(\omega) \left( 1 + R_t^k \right) \right) + \bar{k}_t q_{t-1} + w_t^e + \gamma_t \left( \frac{1 + R_t^e}{\pi_t} \right) \frac{1}{\mu_{s,t}} n_t.$$  

Equation (18) is the definition of the after tax rate of return on capital:

$$R_{t+1}^k = \frac{[u_{t+1} R_{t+1}^k - \tau_t^{oil} a(u_{t+1})] + (1 - \delta) q_{t+1} \pi_{t+1} + \tau^k \delta - 1}{\tilde{T}_q_t}.$$  

29
Equation (33) is the ratio of bank excess reserves to their value-added:

\[
e_{v,t} = \frac{(1 - \tau_t) \frac{m^b_t}{\pi_t \mu^*_z} (1 - \zeta_t) - \tau_t \left( \psi_{k,t} \frac{r^k_t}{\mu^*_z} \right) k_t}{\left( \frac{1}{\mu^*_z Y} (1 - \nu^k_t) k_t \right)^{\alpha} \left( (1 - \nu^l_t) k_t \right)^{1-\alpha}}.
\]

Equation (29) is the necessary condition for optimal choice of labor by firms:

\[
0 = \frac{R^t h_{z,p,t}}{1 + \tau_t h_{e^r,t}} - (1 + \psi_{l,t} R^t) w_t,
\]

\[h_{z,p,t} = h_{l^*,t}/z^*_t.\]

Equation (32) is the banking services production function:

\[
a^b h^b_b (e_{v,t})^{-\xi_t} \frac{e^r_t}{z^*_t} = m_{1t} + m_{2t},
\]

where

\[
m_{1t} = \frac{m^b_t (1 - \zeta_t + \gamma_d m_t)}{\pi_t \mu^*_z}
\]

\[
m_{2t} = \psi_{l,t} w_t + \psi_{k,t} \frac{r^k_t k_t}{\mu^*_z Y}.
\]

Equation (31) is a relation between net interest on bank loans, \( R^*_t \), and interest on deposits, \( R^a_t \), dictated by banking efficiency:

\[
R^a_t = \frac{(1 - \tau_t) h_{e^r,t} - 1}{\tau_t h_{e^r,t} + 1} R^t.
\]

Equation (40) is the marginal discounted utility of household consumption:

\[
E_t \left[ u_{z,t} - \frac{\mu^*_c c^*_{z,t}}{c_t \mu^*_z} \frac{\kappa_{c,t}}{b c_{t-1}^*} + b \beta \frac{\kappa_{c,t+1}}{c_{t+1} \mu^*_z} \frac{\kappa_{c,t+1}}{b c_{t+1}^*} \right] = 0.
\]

Equation (34) is the intertemporal efficiency condition associated with the household time deposit decision:

\[
E_t \left\{ -\lambda_{z,t} + \frac{\beta}{\mu^*_z \pi_{t+1}} \lambda_{z,t+1} (1 + R^T_{t+1}) \right\} = 0.
\]
Equation (37) is the efficiency condition associated with the household cash decision:

\[ E_t \{ \zeta_{c,t} c_t \left[ (1 + \tau^c) \frac{1}{m_t} (1 - \chi_t) \beta_t \left( \frac{1}{1 - m_t} (1 - \chi_t) (1 - \theta_t) \right) \mu_t \right] ^{1 - \sigma_q} \times \left( \frac{\pi_t \mu_t^*}{m^b_t} \right) ^{1 - \sigma_q} \left[ (1 - \chi_t) \beta_t - \left( 1 - \chi_t \right) (1 - \theta_t) \right] - \zeta_{c,t} H' \left( \frac{m_t m^b_{t-1} \pi_t - \mu^*_{t-1}}{m_{t-1} m^b_{t-1}} \pi_t \mu^*_t \pi_{t-1} \mu^*_{t-1} \right) \right] = 0 \]

Equation (38) is the efficiency condition associated with the household \( M^b_{t+1} \) decision:

\[ E_t \{ \beta \zeta_{c,t+1} v_{t+1} (1 - \theta_{t+1}) (1 - \chi_{t+1}) \times \left[ (1 + \tau^c) \frac{1}{m_{t+1}} (1 - \chi_{t+1}) \beta_{t+1} \left( \frac{1}{1 - m_{t+1}} (1 - \chi_{t+1}) (1 - \theta_{t+1}) \right) \mu_{t+1} \right] ^{1 - \sigma_q} \times \left( \frac{1}{m^b_{t+1}} \right) ^{1 - \sigma_q} \left( \frac{1}{1 - m_{t+1}} \right) ^{1 - \sigma_q} \right] \times \left( \frac{1}{\pi_{t+1} \mu^*_{t+1}} \lambda_{t+1} (1 + R_{t+1}^a) - \lambda_{t, t} \right) = 0 \]

Equation (41) is the efficiency condition associated with the household consumption decision:

\[ - \zeta_{c,t} c_t (1 + \tau^c) \left[ (1 - \chi_t) \beta_t \left( \frac{1}{1 - m_t} (1 - \chi_t) (1 - \theta_t) \right) \mu_t \right] ^{1 - \sigma_q} \left( \frac{1}{m^b_t} \right) ^{1 - \sigma_q} \left( \frac{\pi_t \mu^*_t}{m^b_t} \right) ^{1 - \sigma_q} \left( \lambda_{t, t} \right) = 0 \]

Equation (41) is the resource constraint:

\[ d_t + \tau^o \frac{\mu_t}{\lambda_{t, t}} + g_t + c_t + \frac{i_t}{\mu_{t, t}} + \Theta \frac{1 - \gamma_t}{\gamma_t} [n_{t+1} - w_t^c] = \left( p^*_t \right) ^{\lambda_t} \left\{ \epsilon_t \mu^*_t \left( \frac{k_t}{\mu^*_{t, t}} \right) ^{\alpha} \left[ (w_t^c)^{\lambda_t} L_t \right] ^{1 - \alpha} \right\} - \phi \]

Equation (51) is the capital evolution equation:

\[ k_{t+1} = (1 - \delta) \frac{\mu^*_t}{\mu^*_{t, t}} \bar{k}_t + \left[ 1 - S \left( \frac{\zeta_{t, t} i_t \mu^*_{t, t}}{\lambda_{t-1}} \right) \right] i_t. \]
Equation (53) is the law of motion of the monetary base:

$$m^{b}_{t+1} = \frac{1}{\pi_{t}^{*}\mu_{z,t}^{*}}m^{b}_{t}(1+x_{t}).$$

Equation (35) is the efficiency condition associated with household capital accumulation in the version of the model in which there are no entrepreneurs:

$$E_{t}\left\{-\lambda_{zt} + \frac{\beta}{\pi_{t+1}^{*}\mu_{z,t+1}^{*}}\lambda_{zt+1}(1+R^{k}_{t+1})\right\} = 0.$$  

Equation (30) is a banking efficiency condition:

$$E_{t}^{*}\left\{R_{t+1}^{T} - R^{m}_{t+1} - \frac{sR_{t+1}}{h_{e^*,t+1}\tau_{t}^{*}+1}\right\} = 0.$$  

Equation (39) is the household efficiency condition associated with the choice of $D^{m}_{t+1}$:

$$E_{t}\left\{\beta\zeta_{c,t+1}v_{t+1}\chi_{t+1}(1+C)\ c_{t+1}\left(1 - \frac{1}{m_{t+1}}\right)^{(1-\chi_{t+1})\theta_{t+1}}\right.\times\left(1 - \frac{1}{m_{t+1}}\right)^{(1-\chi_{t+1})(1-\theta_{t+1})}\left(1 - \frac{1}{d^{m}_{t+1}}\right)^{\chi_{t+1}}\beta_{t}^{*}\lambda_{zt} + (1 + R^{m}_{t+1}) - \lambda_{zt}\right\} = 0.$$  

Equation (54) is the law of motion for $M3_{t}$:

$$m_{3t} = m^{b}_{t+1}(1 + d^{m}_{t+1}) + \psi_{l,t}w_{t}l_{t} + \psi_{k,t}^{*}\frac{r^{k}_{t}u_{t}k_{t}}{\mu_{z,t}^{*}}.$$  

Equation (55) is the law of motion for total bank loans (working capital plus entrepreneurial loans):

$$b^{Tot}_{t} = \psi_{l,t}w_{t}l_{t} + \psi_{k,t}^{*}\frac{r^{k}_{t}u_{t}k_{t}}{\mu_{z,t}^{*}} + (q_{t}k_{t+1} - n_{t+1}).$$  

Equations (5), (45), (3) and (4), respectively, are the equilibrium conditions associated
with Calvo sticky prices:

\[ p_t^* - h_p \left( \frac{\tilde{\pi}_t}{\pi_t}, p_{t-1}^* \right) = 0 \]

\[ w_t^* - h_w \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z^*,t}, w_{t-1}^* \right) = 0 \]

\[ E_t \left\{ \lambda_{z, t} Y_{z,t} + \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{1-\lambda_f} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0 \]

\[ E_t \left\{ \lambda_{f,t} \lambda_{z, t} Y_{z,t} s_t + \beta \xi_p \left( \frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{-\lambda_f} K_{p,t+1} - K_{p,t} \right\} = 0. \]

The variable, \( K_{p,t} \), is a function of \( F_{p,t} \) via the following relationship:

\[ K_{p,t} = F_{p,t} \left[ 1 - \frac{\xi_p \left( \frac{\tilde{\pi}_t}{\pi_t} \right)^{1-\lambda_f}}{(1 - \xi_p)} \right]^{1-\lambda_f}. \]

Also,

\[ \tilde{\pi}_t - (\pi_t^{\text{target}})^{1-\epsilon} = 0. \]

Equations (42), (43) and (48), respectively are the equilibrium conditions associated with Calvo sticky wages:

\[ E_t \left\{ (w_t^*)^{\lambda_w/(\lambda_w-1)} L_t \left( 1 - \tau_t \right) \frac{\lambda_{z,t}}{\lambda_w} \right\} = 0 \]

\[ + \beta \xi_w (\mu_{z^*})^{\lambda_w/(\lambda_w-1)} \left( \frac{1}{\pi_{w,t+1}} \right) \left( \frac{\lambda_w}{\lambda_w-1} \right) \frac{1}{\lambda_w} \tilde{\pi}_{w,t+1} F_{w,t+1} - F_{w,t} \}

\[ E_t \left\{ \left( w_t^* \right)^{\lambda_w/(1-\sigma_t)} L_t \right\}^{1+\sigma_t} \lambda_{c,t} \xi_t \]

\[ + \beta \xi_w E_t \left( \frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} (\mu_{z^*})^{1-\vartheta} (\mu_{z^*,t+1})^{\vartheta} \right) \frac{\lambda_w}{\lambda_w} \left( 1+\sigma_t \right) K_{w,t+1} - K_{w,t} \}

\[ w_t^+ = h^+ \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}, \mu_{z^*,t}, w_{t-1}^+ \right). \]
where

\[
\begin{align*}
\frac{1}{\psi_L} \left[ \frac{1 - \xi_w \left( \frac{\pi_{w,t}^* (\mu_\sigma^*)}{\pi_{w,t}^*} (\mu_{z^*}^*)^{1-\vartheta} (\mu_{z^*}^*)^{1-\vartheta} \right)^{1-\lambda_w}}{1 - \xi_w} \right]^{1-\lambda_w (1+\sigma_L)} 
\tilde{w}_{t} F_{w,t} - K_{w,t} &= 0 \\
\tilde{\pi}_{w,t} - \left( \pi_t^{\text{target}} \right)^{i_w} (\pi_{t-1})^{1-i_w} &= 0
\end{align*}
\]

When we work with a first order expansion of the model about a steady state in which wages and prices are not distorted, then the seven wage and price equations, (5) - (48), reduce to simply (46) and (6). In addition, the unknowns associated with these equations reduce to just \( \tilde{w}_t \) and \( \pi_t \).

When we linearize the system about a steady state in which wages and prices are not distorted, our system is composed of 27 equations in 27 variables. Because the growth rate of \( M_{1t} \) and \( M_{3t} \) require lagged capital, in putting our equations into matrix form, it is
convenient to work with the following 29-dimensional column vector:

$$Z_t = \begin{pmatrix}
\pi_t \\
s_t \\
r^k_t \\
i_t \\
u_t \\
\bar{\omega}_t \\
R^k_t \\
n_{t+1} \\
q_t \\
u^l_t \\
e_{\nu,t} \\
m^b_{t+1} \\
R^c_t \\
u^c_{c,t} \\
\lambda_{z,t} \\
m_t \\
R_{t,t} \\
c_t \\
\bar{\bar{w}}_t \\
L_t \\
\bar{k}_{t+1} \\
R^c_{t+1} \\
x_t \\
d^m_{t+1} \\
R^m_{t+1} \\
m_{3t} \\
\hat{m}^b_t \\
\hat{b}^l_{\text{tot}} \\
\hat{k}_t
\end{pmatrix}$$

(57)

The vector, $Z_t$, includes the 27 variables discussed above, as well as one lag of two of the variables. Our basic 27 equations are composed of, first, the following 24: (1), (2), (7), (9), (15), (19), (20), (18), (33), (29), (32), (31), (40), (34), (37), (38), (41), (51), (8), (53), (30), (39), (54), (55). In addition, we include the price/wage equations, (6), (46), and the monetary policy rule, (??). Finally, we require two equations which capture that two variables $Z_t$ and $Z_{t-1}$ coincide. We thus have 29 equations in 29 unknowns.
Our system also includes equation (48), which determines the variable \( w_t^+ \), which is required in the computation of average utility, (49). This is only used in the computation of optimal policy.

2 Appendix B: Alternative Versions of the Model

By dropping and/or replacing equations in the previous appendix, we obtain alternative versions of the model. We consider three versions of the model: a version without entrepreneurs, a version without banks (‘the financial accelerator model’), a version without entrepreneurs and banks (‘the simple model’), and the real business cycle version of the model. Tables B.1 and B.2 report the priors and posterior distributions for the parameters of the simple model and the financial accelerator model.

2.1 Model Without Entrepreneurs

The equations of the model dealing with the entrepreneur are the Euler equation associated with utilization, (9), the optimality condition for debt contracts, (15), the zero-profit condition on associated with entrepreneurial loans, (19), the law of motion of entrepreneurial net worth, (20), and the definition of the rate of return on capital, (18). We replace the three equations, (15), (19) and (20), dealing with the entrepreneur with the single equation (35), associated with the household decision to accumulate capital. So, when the entrepreneur is dropped, we lose two equations, net. Corresponding to this, we drop the two variables, \( \bar{\omega}_t, n_{t+1} \).

2.2 Model Without Entrepreneurs and Banks

To drop the entrepreneur, we make the adjustments in the previous subsection. That is, we replace (15), (19) and (20) with (35) and we drop the two variables, \( \bar{\omega}_t, n_{t+1} \).

To also drop the banks, we make additional adjustments which make the model similar to the one in Christiano, Eichenbaum and Evans (2005). Households begin the period with the beginning-of-period monetary base, \( M^b_t \), and they choose to hold part of it, \( M_t \leq M^b_t \), in the form of currency. They do so because \( M_t \) generates utility. The part of \( M^b_t \) that households do not hold in the form of \( M_t \) is deposited in a financial intermediary. If either \( \psi_l > 0 \) or \( \psi_k > 0 \), then there is a demand for loans by firms, for working capital. An equilibrium condition of the model is that this demand equal supply, \( M^b_t - M_t \). We allow for the possibility that \( \psi_l = \psi_k = 0 \), in which case the equilibrium condition is \( M^b_t - M_t = 0 \). Apart from demand deposits, there are no other financial assets in this version of the model. In addition, the financial intermediary requires no resources to do intermediation. As a
result, the random variables, \( x_b \) and \( \xi_t \) do not appear in this version of the model. The rates of return on time and savings deposits respectively, \( R_T \) and \( R_m \), do not exist in this version of the model. Similarly, the rate of interest earned by the bank from entrepreneurs, \( R_e \), no longer appears. The interest rate on household deposits with the financial intermediary is \( R_a \). The assumption that banks require no resources implies that \( R_t = R_a \), where \( R_t \) is the interest rate paid by firms for working capital loans. We drop the following five banking equations: (33), (29), (32), (31), (30). We also drop the household efficiency conditions associated with time, \( T_t \), and savings, \( D_m \), deposits, respectively, (34) and (39). In addition, we drop the equation pertaining to \( m_{3t} \), (54) and the equation pertaining to \( b_t^{Tot} \), (55). We add the money market clearing condition:

\[
\psi_{l,t} W_t l_t + \psi_{k,t} P_t \tilde{r}^k t K_t = M^b_t - M_t.
\]

Scale the money market clearing condition by \( P_t z^*_t \) and express employment in terms of the sum of household employment, \( L_t \), using (44):

\[
\psi_{l} \tilde{w}_t L_t (w^*_t)^{\lambda_w - 1} + \psi_{k} \tilde{r}^k_t u_t \frac{\tilde{k}_t}{\tilde{\mu}_{z,t}^*} = \frac{m^b_t}{\tilde{\pi}_{t} z_{z,t}^*} (1 - m_t) .
\]

Thus, dropping the banking system leads us to drop 8 equations, net. We drop the following 8 variables from \( Z_t \) in (57): \( e_{i,t}, R_t^{m+1}, \, \hat{g}_t^{l}, \, R_t^{a}, \, \hat{g}_t^{m}, \, \hat{g}_t^{e}, \, \hat{g}_3_t, \, m_{3t}, \, b_t^{Tot} \).

When we drop the entrepreneurs and banks, the system is reduced by 10 equations and 10 unknowns (not counting the two identity equations in the full system.). Thus, the complete system of equations is now composed in part of the following 14: (1), (2), (7), (9), (18), (40), (37), (38), (41), (51), (8), (53), (35), (58). There are, in addition, the equations corresponding to sticky prices and wages, (6) and (46) and the monetary policy rule, (??). In (??) it is understood that \( \hat{R}_t^{target} \) corresponds to the single nominal rate of interest in the system, \( R_t \). Also, \( \hat{g}_3_t \) does not appear in the Taylor rule in the version of the model without banks and entrepreneurs. An implication of this is that we do not now need to include lags of \( \tilde{k}_t \) and \( m_t^b \). The reduced system has a total of 17 equations and 17 unknowns.
endogenous variables, $Z_t$, are:

$$Z_t = \begin{pmatrix}
\pi_t \\
s_t \\
r_k^k \\
i_t \\
u_t \\
R_k \\
q_t \\
m_{b,t+1} \\
R_t \\
\tilde{u}_{c,t} \\
\lambda_{z,t} \\
m_t \\
c_t \\
\tilde{w}_t \\
L_t \\
k_{t+1} \\
x_t
\end{pmatrix}_{17 \times 1}$$ (59)

### 2.3 Model with No Entrepreneurs, Banks or Money (‘Simple Model’)

Following is a description of the version of the model without financial frictions, banks or money. We list the equations.

Equation (1) is a measure of marginal cost:

$$s_t = \left(\frac{1}{1 - \alpha}\right) \left(\frac{1}{\alpha}\right) \frac{r^k_t \left[1 + \psi_{k,t} R_t\right]}{\epsilon_t} \frac{\left[1 + \psi_{l,t} R_t\right]^{1-\alpha}}{\epsilon_t}. \quad (60)$$

Equation (2) is another measure of marginal cost:

$$s_t = \frac{r^k_t \left[1 + \psi_{k,t} R_t\right]}{\alpha \epsilon_t \left(\Gamma \frac{\mu_{z,t} L_t (w^*) \lambda_{w,-1}}{\mu_{z,t}}\right)^{1-\alpha}} \quad (61)$$

Equation (7) is the first order condition for investment by capital producers:

$$E \left[\zeta_{c,t} \lambda_{zt} q_t F_{1,t} - \zeta_{c,t} \lambda_{zt} \frac{1}{\mu_{r,t}} + \beta \zeta_{c,t+1} \lambda_{zt+1} \frac{1}{\mu_{r,t+1}} \frac{\zeta_{c,t+1} \lambda_{zt+1} q_{t+1} F_{2,t+1} | \Omega_t}{q_{t+1} F_{2,t+1} | \Omega_t}\right] = 0. \quad (62)$$
Equation (9) is entrepreneurs’ first order condition for capital utilization:

\[ r_t^k = \tau_t^{oil} a'(u_t). \]  (63)

Equation (18) is the definition of the after tax rate of return on capital:

\[ R_{t+1}^k = \frac{(1 - \tau^k) [u_{t+1} r_{t+1}^k - \tau_{t+1}^{oil} a(u_{t+1})] + (1 - \delta) q_{t+1}}{\tau_{t+1} + \tau^k \delta - 1}. \]  (64)

Equation (40) is the marginal discounted utility of household consumption:

\[
E_t \left[ u_{c,t}^z - \frac{\mu_{z,t}^* \zeta_{c,t}}{c_t \mu_{z,t}^* - bc_{t-1}} + b \beta \frac{\zeta_{c,t+1}}{c_{t+1} \mu_{z,t+1}^* - bc_t} \right] = 0 \]  (65)

Equation (38) is the efficiency condition associated with the household \( M_{t+1}^b \) decision:

\[
E_t \left\{ \frac{1}{\pi_{t+1} \mu_{z,t+1}^*} \zeta_{c,t+1} \lambda_{z,t+1} (1 + R_{t+1}) - \zeta_{c,t} \lambda_{z,t} \right\} = 0 \]  (66)

A relation connection marginal utility and the multiplier:

\[ u_{c,t}^z = (1 + \tau^C) \zeta_{c,t} \lambda_{z,t} \]

Equation (51) is the resource constraint:

\[
\tau_t^{oil} a(u_t) \frac{\lambda_{t}}{\mu_{z,t}^*} + g_t + c_t + \frac{i_t}{\mu_{x,t}} = \left( p_t^* \right)^{\lambda_t} \left\{ e_t \left( u_t \frac{\lambda_{t}}{\mu_{z,t}^*} \right)^\alpha \left[ \left( w_t^* \right)^{\lambda_{w,t}^*} L_t \right]^{1-\alpha} - \phi \right\} \]  (67)

Equation (8) is the capital evolution equation:

\[ \bar{k}_{t+1} = (1 - \delta) \frac{1}{\mu_{z,t}^*} \bar{k}_t + \left[ 1 - s \left( \frac{\zeta_{i,t} i_t \mu_{z,t}^*}{i_{t-1}} \right) \right] i_t. \]  (68)

Equation (35) is the efficiency condition associated with household capital accumulation:

\[
E_t \left\{ -\zeta_{c,t} \lambda_{z,t} + \frac{\beta}{\pi_{t+1} \mu_{z,t+1}^*} \zeta_{c,t+1} \lambda_{z,t+1} (1 + R_{t+1}) \right\} = 0. \]  (69)
Equations (5), (45), (3) and (4), respectively, are the equilibrium conditions associated with Calvo sticky prices (and, in one case, wages):

\[ p^*_t - h_p \left( \frac{\pi_t}{\pi_{t-1}} \right) = 0 \]  \hspace{1cm} (70)

\[ w^*_t - h_w \left( \frac{\pi_{w,t}}{\pi_{w,t-1}} \right) = 0 \]  \hspace{1cm} (71)

\[ E_t \left\{ \lambda_{z,t} Y_{z,t} + \left( \frac{\pi_{t+1}}{\pi_{t+1}} \right)^{1-\lambda_f} \beta \xi_p F_{p,t+1} - F_{p,t} \right\} = 0 \]  \hspace{1cm} (72)

\[ E_t \left\{ \lambda_{f,t} \lambda_{z,t} Y_{z,t} s_t + \beta \xi_p \left( \frac{\pi_{t+1}}{\pi_{t+1}} \right)^{1-\lambda_f} K_{p,t+1} - K_{p,t} \right\} = 0, \]  \hspace{1cm} (73)

where \( h_w \) is defined in (45) and

\[ \tilde{\pi}_t = \left( \pi_{t, \text{target}} \right)^{1-\lambda_f} \pi_{t-1}^{1-\lambda_f} = 0 \]

\[ K_{p,t} - F_{p,t} \left[ 1 - \xi_p \left( \frac{\tilde{\pi}_t}{\pi_t} \right)^{1-\lambda_f} \right]^{1-\lambda_f} = 0 \]

\[ (p^*_t)^{\lambda_f} \left\{ \varepsilon_t \left( u_t^+ \frac{\tilde{\mu}_{z,t}}{\tilde{\mu}_{z,t}} \right)^{\alpha} \left[ (w^*_t)^{\lambda_w - \tau_t} L_t \right]^{1-\alpha} \right\} = Y_{z,t} \]

Equations (42), (43) and (48), respectively are the equilibrium conditions associated with Calvo sticky wages.

\[ E_t \left\{ \xi_{c,t} \left( w^*_t \right)^{\lambda_w - \tau_t} L_t \left( 1 - \tau_t \right)^{\lambda_{z,t}} \right\} = 0 \]  \hspace{1cm} (74)

\[ + \beta \xi_w (\mu_{z^*})^{1-\lambda_w} \left( \mu_{z^*,t+1} \right)^{1-\lambda_w} \left( \frac{1}{\pi_{w,t+1}} \right) \left( \frac{\pi_{w,t+1}}{\pi_{t+1}} \right)^{1-\lambda_w} F_{w,t+1} - F_{w,t} \right\} = 0 \]

\[ E_t \left\{ \left( w^*_t \right)^{\lambda_w} L_t \right\}^{1+\lambda_w \xi_w} + \xi_{c,t} \xi_t \]  \hspace{1cm} (75)

\[ + \beta \xi_w E_t \left( \frac{\pi_{w,t+1}}{\pi_{w,t+1}} \right)^{1-\lambda_w} (\mu_{z^*,t+1})^{1-\lambda_w} K_{w,t+1} - K_{w,t} \right\} = 0 \]

\[ w^*_t = h^+ \left( \frac{\pi_{w,t}}{\pi_{w,t}}, \mu_{z^*,t}, w^*_t \right) \]  \hspace{1cm} (76)
where $h^+$ is defined in (48) and

$$
\frac{1}{\psi_L} \left[ \frac{1 - \xi_w \left( \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} (\mu_{z*}) \right)^{1-\varrho}}{1 - \xi_w} \right]^{1-\lambda_w (1+\sigma_L)} \tilde{w}_t F_{w,t} - K_{w,t} = 0
$$

$$
\tilde{\pi}_{w,t} - \left( \pi_t^{\text{target}} \right)^{t_w,1} (\pi_{t-1})^{t_w,2} \frac{1-\epsilon_{w,1} - \epsilon_{w,2}}{\rho} = 0.
$$

Relative to the model with no entrepreneurs and banks, but with money, we drop four variables, $m, m^b, \lambda_z, x$. Here, there are 18 unknowns:

$s, r^k_t, R_t, \tilde{w}_t, L_t, \tilde{k}_t, u_t, q_t, \bar{u}_{w,\ell}, \pi_t, c_t, i_t, p^*_t, w^*_t, R^k_t, F_{p,t}, F_{w,t}, w^+_t$

and 17 equations, (60)-(76). Actually, the five variables, $p^*_t, w^*_t, F_{p,t}, F_{w,t}, w^+_t$, are only used for computing optimal policy. For our positive model, we drop these 5 variables and replace the 7 equations, (70)-(76), with the two, (6) and (46). In addition, we use the Taylor rule, (??). This gives us 13 unknowns and 13 equations.

### 2.3.1 Steady State

We now turn to computing the steady state for these equations. We have set $u_t = 1$ here. The expression for real marginal cost is:

$$
s = \left( \frac{1}{1 - \alpha} \right)^{1-\alpha} \left( \frac{1}{\alpha} \right)^{\alpha} \frac{(r^k [1 + \psi_k R])^\alpha (\delta [1 + \psi_k R])^{1-\alpha}}{\epsilon},
$$

or,

$$
\tilde{w} = \frac{1}{1 + \psi_k R} \left[ (\alpha) s \left( (1 - \alpha)^{1-\alpha} \right)^{s \epsilon} \right]^{\frac{1}{1-\alpha}}.
$$

(77)

The expression corresponding to equation (??) is:

$$
s = \frac{r^k [1 + \psi_k R]}{\alpha \epsilon \left( \gamma \mu_z (w^*) \frac{\lambda_w}{\lambda_w - 1} \frac{1}{k} \right)^{1-\alpha}},
$$

or,

$$
\frac{L}{k} = \frac{1}{\gamma \mu_z (w^*) \frac{\lambda_w}{\lambda_w - 1} \frac{1}{s \alpha \epsilon}} \left[ \frac{r^k [1 + \psi_k R]}{s \alpha \epsilon} \right]^{\frac{1}{1-\alpha}}.
$$

(78)
Equation (??) is the first order condition for investment by capital producers:

$$\lambda_z q F_1 - \lambda_z \frac{1}{\mu_T} + \beta \frac{\lambda_z}{\mu_z^* T} q F_2 = 0.$$ 

Now, $F_1 = 1$, $F_2 = 0$ in steady state, so that the latter equation implies, after dividing by $\lambda_z$:

$$q = \frac{1}{\mu_T}. \quad (79)$$

Equation (??) is entrepreneurs’ first order condition for capital utilization:

$$r^k = \tau^{oil} a', \quad (80)$$

where $a'$ is the value of $a'(u_t)$ when $u_t = 1$. This is an equation that is simply used to pin down $a'$. Equation (??) is the definition of the after tax rate of return on capital:

$$R^k = \frac{(1 - \tau^k) r^k + (1 - \delta)q \pi^{ss} + \tau^k \delta - 1}{\tau q},$$

or

$$r^k = \frac{(R^k - \tau^k \delta + 1) \frac{\tau q}{\pi^{ss}} - (1 - \delta)q}{1 - \tau^k}, \quad (81)$$

where $a(u_t) = 0$ for $u_t = 1$ has been used. Equation (??) is the marginal discounted utility of household consumption:

$$u^z_{c} = \frac{\mu_z^* \zeta_c}{c \mu_z^* - bc} - \frac{b \beta}{c \mu_z^* - bc} \zeta_c = \frac{1}{c} \left( \frac{\mu_z^* - b \beta}{\mu_z^* - b} \right) \zeta_c \quad (82)$$

Equation (??) is the efficiency condition associated with the household $M^b_{t+1}$ decision:

$$\beta \frac{1}{\pi^{ss} \mu_z^*} (1 + R) = 1$$

or,

$$R = \frac{\pi^{ss} \mu_z^*}{\beta} - 1. \quad (83)$$

Equation (??) is the efficiency condition associated with the household consumption decision, $u^z_{c} = (1 + \tau^C) \lambda_z$, so that, after substituting from (82):

$$\frac{1}{c} \left( \frac{\mu_z^* - b \beta}{\mu_z^* - b} \right) \zeta_c = (1 + \tau^C) \zeta_c \lambda_z, \quad (84)$$

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Equation (8) is the capital evolution equation:

$$\bar{k} = (1 - \delta) \frac{1}{\mu^*_z} \bar{k} + i_t,$$

(85)

where we have used that adjustment costs are zero in steady state. Equation (86) is the law of motion of the monetary base:

$$\pi^{ss} = \frac{1 + x}{\mu^*_z}.$$  

(86)

Here, we have cancelled \(m^b\) from both sides of the equation. Equation (87) is the efficiency condition associated with household capital accumulation:

$$R^k = \frac{\pi^{ss} \mu^*_z}{\beta} - 1,$$

(87)

where \(\lambda_z\) has been cancelled from both sides.

Equations (88), (89), (90) and (91), respectively, are the equilibrium conditions associated with Calvo sticky prices and these reduce to:

$$p^* = \frac{\left(1 - \xi_p\right) \left(\frac{1 - \xi_p}{\frac{\pi}{\pi^{ss}}\frac{1 - \xi_p}{\lambda_f}}\right)^{\frac{1 - \lambda_f}{\lambda_f}}}{1 - \xi_p \left(\frac{\pi}{\pi^{ss}}\right)^{\frac{1}{1 - \lambda_f}}}$$  

(88)

$$w^* = \frac{\left(1 - \xi_w\right) \left(\frac{1 - \xi_w}{\frac{\pi}{\pi^{ss}}\frac{1 - \xi_w}{\lambda_w}}\right)^{\frac{1 - \lambda_w}{\lambda_w}}}{1 - \xi_w \left(\frac{\pi}{\pi^{ss}}\right)^{\frac{\lambda_w}{1 - \lambda_w}}}$$  

(89)

$$F_p = \frac{\lambda_z Y}{1 - \left(\frac{\pi}{\pi^{ss}}\right)^{\frac{1 - \xi_p}{\lambda_f}}} \beta \xi_p$$  

(90)

$$K_p = \frac{\lambda_z Y}{1 - \left(\frac{\pi}{\pi^{ss}}\right)^{\frac{1 - \lambda_f}{\lambda_f}}} \beta \xi_p$$  

(91)

where \(F_p\) and \(K_p\) have the following relationship:

$$K_p = F_p \left[\frac{1 - \xi_p \left(\frac{\pi}{\pi^{ss}}\right)^{\frac{1 - \lambda_f}{\lambda_f}}}{1 - \xi_p \frac{x}{\lambda_f}}\right]^{1 - \lambda_f}.$$
The latter expression, given (92) and (90), implies

\[
\lambda_f s = \left[ \frac{1 - \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}}{1 - \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}} \right] \left[ \frac{1 - \xi_p \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}}{1 - \xi_p} \right]^{1 - \lambda_f}. \tag{92}
\]

Equation (??) is the resource constraint:

\[
g + c + \frac{i}{\mu_x} = Y_z, \quad Y_z \equiv (p^*)^\frac{\lambda_f}{1 - \lambda_f} \left\{ \epsilon \left( \frac{k}{\Upsilon \mu_z^*} \right)^\alpha \left[ (w^*)^{\lambda_w \mu_z^*} L \right]^{1 - \alpha} - \Phi \right\}. \tag{93}
\]

Here, we have used that \(a(u) = 0\) and \(u = 1\) in steady state. We can simplify (93) by substituting out for \(g\) with our assumption, \(g = \eta Y_z\), and we can substitute out for \(\Phi\) using the steady state zero profit condition on firms. Dividing (52) by \(P_z z^*_t\), the steady state zero profit condition is:

\[
Y_z = s \left[ Y_z (p^*)^\frac{\lambda_f}{1 - \lambda_f} + \Phi \right]
\]

This is output net of fixed costs (i.e., the quantity sold) minus total cost (net output plus fixed cost, times the constant marginal cost). Divide both sides by \(s\) and use (92) to substitute out for \(s\), to obtain:

\[
\left( \lambda_f \begin{pmatrix}
\frac{1 - \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}}{1 - \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}} & \frac{1 - \xi_p \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}}{1 - \xi_p}
\end{pmatrix}
\right) Y_z = \Phi.
\]

Now,

\[
Y_z = (p^*)^{\frac{\lambda_f}{1 - \lambda_f}} \epsilon \left( \frac{k}{\Upsilon \mu_z^*} \right)^\alpha \left[ (w^*)^{\lambda_w \mu_z^*} L \right]^{1 - \alpha} - \left( \lambda_f \begin{pmatrix}
\frac{1 - \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}}{1 - \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}} & \frac{1 - \xi_p \left( \frac{\beta \xi_p}{\pi s} \right)^{1 - \lambda_f}}{1 - \xi_p}
\end{pmatrix}
\right) Y_z.
\]
so that

\[ Y_z = \frac{1}{\lambda_f} \left[ 1 - \left( \frac{\pi_{ss}}{\pi_{ss}} \right)^{1-\lambda_f} \beta \xi_p \right] \left[ 1 - \epsilon \left( \frac{\pi_{ss}}{\pi_{ss}} \right)^{1-\lambda_f} \right]^{1-\lambda_f} \epsilon \left( \frac{1}{\Upsilon \mu_z^*} \right)^{\alpha} \left[ (w^*)^{\frac{\lambda_w}{\lambda_w-1}} \frac{L}{k} \right]^{1-\alpha} \]

by (??). The resource constraint becomes, after using \( g = \eta_g Y_z \) and making use of (??):

\[
c + \frac{1 - (1 - \delta) \frac{1}{\mu_z^*}}{\mu_T} \bar{k} = (1 - \eta_g) \epsilon \left( \frac{1}{\Upsilon \mu_z^*} \right)^{\alpha} \left[ (w^*)^{\frac{\lambda_w}{\lambda_w-1}} \frac{L}{k} \right]^{1-\alpha} \bar{k},
\]

or,

\[
c = A \bar{k}
\]

\[
A \equiv (1 - \eta_g) \epsilon \left( \frac{1}{\Upsilon \mu_z^*} \right)^{\alpha} \left[ (w^*)^{\frac{\lambda_w}{\lambda_w-1}} \frac{L}{k} \right]^{1-\alpha} - \frac{1 - (1 - \delta) \frac{1}{\mu_z^*}}{\mu_T}.
\]

In (??)-(??):

\[
\bar{\pi} - (\pi_{ss})^t \bar{\pi}^{1-t} = 0 \quad \tag{95}
\]

\[
\pi_w = \mu_z^* \pi_{ss}^t. \quad \tag{96}
\]

Equations (??), (??) and (??), respectively are the equilibrium conditions associated
with Calvo sticky wages.

\[
F_w = \frac{\zeta_c \left( w^* \right)^{\lambda_w - 1} L (1 - \tau^L) \lambda_z}{1 - \beta \xi_w \left( \frac{\mu_* \pi_w}{\pi} \right)^{\lambda_w - 1} \pi_w^{1 - \lambda_w}}
\]  
(97)

\[
K_w = \frac{\left( w^* \right)^{\lambda_w - 1} L (1 + \sigma_L) \zeta_c}{1 - \beta \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{\lambda_w - 1} \lambda_w (1 + \sigma_L)}
\]  
(98)

\[
w^+ = \left[ \frac{1 - \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{1 - \lambda_w}}{1 - \lambda_w} \right] ^{1 - \lambda_w (1 + \sigma_L)} \left( 1 - \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{1 - \lambda_w} \right)^{1 - \lambda_w (1 + \sigma_L)} \psi_L \zeta_c \left( w^* \right)^{\lambda_w - 1} L ^{\sigma_L}.
\]  
(99)

Here, \( K_w \) is related to \( F_w \) as follows:

\[
\frac{1}{\psi_L} \left[ \frac{1 - \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{1 - \lambda_w}}{1 - \xi_w} \right] ^{1 - \lambda_w (1 + \sigma_L)} \tilde{w} F_w - K_w = 0.
\]

Also,

\[
\tilde{\pi}_w - \left( \frac{\pi^{ss}}{\pi} \right)^{\omega_{w1}} = 0.
\]  
(100)

Substituting out for \( F_w \) and \( K_w \) from (97) and (98), we obtain, after simplifying:

\[
\left[ \frac{1 - \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{1 - \lambda_w}}{1 - \xi_w} \right] ^{1 - \lambda_w (1 + \sigma_L)} \left[ \frac{1 - \beta \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{1 - \lambda_w}}{1 - \beta \xi_w \left( \frac{\pi_w \mu_*}{\pi} \right)^{1 - \lambda_w}} \right] ^{1 - \lambda_w (1 + \sigma_L)} (1 - \tau^L) \tilde{w} = \lambda_w \psi_L \zeta_c \left( w^* \right)^{\lambda_w - 1} L ^{\sigma_L}.
\]  
(101)

Ignoring \( w^* \) and the two objects in square brackets before the equality sign, this says that the after tax real wage is a markup, \( \lambda_w \), over the marginal cost of leisure, measured in consumption units.

The equations can be solved, conditional on a given value of the steady state money base growth rate, \( x \). The variables, \( q \), \( \pi^{ss} \), and \( R^s \) can be solved using (??), (??) and (??), respectively. Then, \( r^k \) is obtained from (??) and \( a' \) from (??). \( R \) is solved using (??). With \( \tilde{\pi} / \pi^{ss} \) from (??), solve (??) for \( p^* \) (if \( \pi^{ss} = \tilde{\pi} \), then \( p^* = 1 \)). With \( \pi_w \) and \( \tilde{\pi}_w \) from (??) and (100), respectively, solve (??) for \( w^* \) (note, if \( \pi^{ss} = \tilde{\pi} \), then \( w^* = 1 \)). Solve (99) for \( w^+ \). Solve
(??) for s. Given s it is possible to compute $\bar{w}$ from (??). Then, $L/\bar{k}$ can be computed from (??).

Solve (101) can be solved for $\lambda_z$:

$$
\lambda_z = \lambda_w \frac{\psi_L \zeta_c \left((w^*)^{\frac{\lambda_w}{1-\lambda_w}} L\right)^{\sigma_L}}{\left[1 - \xi_w \left(\frac{\psi_L \zeta_c}{\bar{k}}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w(1+\sigma_L)} \left[1 - \beta \xi_w \left(\frac{\psi_L \zeta_c}{\bar{k}}\right)^{\frac{1}{1-\lambda_w}}\right]} (1 - \tau^t) \bar{w}.
$$

Substituting out for $\lambda_z$ from (84) and for $c$ from (94), we obtain

$$
\frac{1}{Ak} \frac{(\mu^*_z - b\beta)}{\mu^*_z - b} = \frac{(1 + \tau^C)\lambda_w \psi_L \zeta_c \left((w^*)^{\frac{\lambda_w}{1-\lambda_w}} L\right)^{\sigma_L}}{\left[1 - \xi_w \left(\frac{\psi_L \zeta_c}{\bar{k}}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w(1+\sigma_L)} \left[1 - \beta \xi_w \left(\frac{\psi_L \zeta_c}{\bar{k}}\right)^{\frac{1}{1-\lambda_w}}\right]} (1 - \tau^t) \bar{w},
$$

so that

$$
\bar{k} = D \frac{1}{\tau^C},
$$

where

$$
D = \frac{1}{A} \frac{(\mu^*_z - b\beta)}{\mu^*_z - b} \frac{\left[1 - \xi_w \left(\frac{\psi_L \zeta_c}{\bar{k}}\right)^{\frac{1}{1-\lambda_w}}\right]^{1-\lambda_w(1+\sigma_L)} \left[1 - \beta \xi_w \left(\frac{\psi_L \zeta_c}{\bar{k}}\right)^{\frac{1}{1-\lambda_w}}\right]}{(1 + \tau^C)\lambda_w \psi_L \zeta_c \left((w^*)^{\frac{\lambda_w}{1-\lambda_w}} L\right)^{\sigma_L}} (1 - \tau^t) \bar{w}.
$$

This completes the description of an algorithm for computing the steady state of the model without entrepreneurs, banks or money.

### 3 Appendix C: Estimation

We solved the model by log-linearizing the equilibrium conditions about steady state, using the strategy in Christiano (2002). The 27 equilibrium conditions are summarized in Appendix A. There are 27 endogenous variables whose values are determined at time $t$, and these are contained in a $27 \times 1$ vector denoted $Z_t$. Given values for the parameters of the

---

1The computations require finding the matrix root of a matrix polynomial. For this, we used the AIM algorithm provided by Gary Anderson of the Federal Reserve Board of Governors.

2See equation (57) in the appendix.
model, we compute steady state values for each variable in $Z_t$. We then construct the $27 \times 1$
vector, $z_t$ as follows. If $Z_{it}$ is the $i^{th}$ element of $Z_t$ and $Z_i$ is the corresponding steady state,
then the $i^{th}$ element of $z_t$ is $z_{it} = (Z_{it} - Z_i)/Z_i$. Given the shocks described in the previous
section, we can write the equilibrium conditions in the following form:

$$E_t \begin{bmatrix} \alpha_0 z_{t+2} + \alpha_1 z_{t+1} + \alpha_2 z_t + \beta_0 s_{t+1} + \beta_1 s_t \end{bmatrix} = 0,$$

where $\alpha_i$ are $27 \times 27$ matrices, $i = 0, 1, 2$, and $\beta_i$ are $27 \times 14$ matrices, $i = 0, 1$. The solution
to this system, which takes into account (105) is:

$$z_t = Az_{t-1} + Bs_t,$$

(102)

where $A$ is a $27 \times 27$ matrix with eigenvalues less than unity and $B$ is a $27 \times 14$ matrix.

The variables in $z_t$ are chosen partly for computational convenience, and not at all with
the variables in mind that we wish to use in estimation. These variables appear as the solid
lines in Figures 2a (EA data) and 2b (US). To derive our model’s implications for these
variables, we log-linearize the mapping from $X_t$ to $z_t$ and $s_t$:

$$X_t = \alpha + \tau z_t + \tau^s s_t + \bar{\tau} z_{t-1}. \quad (103)$$

The real oil price in our model corresponds to $\tau_t^{oil}$, discussed in section ??.

Equations (??), (102) and (103) represent a complete description of the joint (linearized)
distribution of the variables, $X_t$. We make use of this for purposes of model estimation.

For convenience, we describe our system using the notation in Hamilton (1994, chapter
13). Let the state vector, $\xi_t$, be:

$$\xi_t = \begin{pmatrix} z_t \\ z_{t-1} \\ s_t \end{pmatrix}.$$

Then, the state equation, which summarizes (??) and (102), is

$$\begin{pmatrix} z_{t+1} \\ z_t \\ s_{t+1} \end{pmatrix} = \begin{bmatrix} A & 0 & BP \\ I & 0 & 0 \\ 0 & 0 & P \end{bmatrix} \begin{pmatrix} z_t \\ z_{t-1} \\ s_t \end{pmatrix} + \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix} u_{t+1},$$

or, in obvious, compact notation:

$$\xi_{t+1} = F\xi_t + v_{t+1}, \quad Ev_{t+1}v_{t+1}' = Q,$$

(104)

where

$$v_{t+1} = \begin{pmatrix} B \\ 0 \\ I \end{pmatrix} u_{t+1}. \quad (105)$$

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The variance covariance matrix, $Q$, has the following structure:

$$
Q \equiv EVtv'_t = \begin{bmatrix}
BDB' & 0 & BD \\
0 & 0 & 0 \\
DB' & 0 & D \\
\end{bmatrix}.
$$

The observation equation is:

$$
y_t = H\xi_t + w_t, \quad Ew_tw'_t = R,
$$

where $R$ is diagonal and $w_t$ is iid over time. Also,

$$
H = \begin{bmatrix} \tau & \bar{\tau} & \tau^s \end{bmatrix}.
$$

Note from (103) that $H\xi_t = X_t$, apart from the constant vector, $\alpha$.

The state-space, observer representation is a function of $(F, H, R, Q)$. These objects are themselves functions of the model parameters. We form the Gaussian likelihood function in the way described in Hamilton (1994), section 13.4. In particular, let

$$
f_t = \left(2\pi\right)^{-n/2} |HP_{t|t-1}H' + R|^{-1/2} \times \exp \left\{ -\frac{1}{2} (y_t - H\xi_{t|t-1})' \left(HP_{t|t-1}H' + R\right)^{-1} (y_t - H\xi_{t|t-1}) \right\},
$$

for $t = 1, 2, ..., T$. Here, $t = 1$ corresponds to 1981Q1 and $t = T$ corresponds to 2006Q4. The variable, $n$, denotes the dimension of $\xi_t$, and

$$
\xi_{t|t-1} = E [\xi_t | y_{t-1}, ..., y_1],
$$

$t = 1, 2, ..., t_{1|0} = E (\xi_t), \quad t_{1|0} = E (\xi_t)$, the unconditional expectation of $\xi_t$. Also,

$$
P_{t+1|t} = E \left[ \left( \xi_{t+1} - \xi_{t+1|t} \right) \left( \xi_{t+1} - \xi_{t+1|t} \right)' | y_t, ..., y_1 \right]

= F \left[ P_{t|t-1} - P_{t|t-1}H' \left(HP_{t|t-1}H' + R\right)^{-1} HP_{t|t-1} \right] F' + Q,
$$

for $t = 1, 2, ..., T$, with

$$
P_{1|0} = E (\xi_1 - E\xi_1) (\xi_1 - E\xi_1)'.
$$

Finally,

$$
\xi_{t+1|t} = F\xi_{t|t-1} + FP_{t|t-1}H' \left(HP_{t|t-1}H' + R\right)^{-1} (y_t - H\xi_{t|t-1}).
$$

Then, the log likelihood function is:

$$
\sum_{t=t_1}^{T} \log f_t,
$$

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where \( t_1 \) corresponds to 1985Q1. Our Bayesian estimation criterion is:

\[
L = \sum_{i=1}^{N} \log p_i(\theta_i) + \sum_{t=t_1}^{T} \log f_t, \tag{108}
\]

where \( p_i(\theta_i) \) is the density of the prior distribution associated with the \( i^{th} \) model parameter being estimated, \( \theta_i \). In selecting \( p_i \)'s, we choose from three density functions: normal, inverted gamma and beta.

4 Appendix D: Variance Decomposition

To obtain the variance decomposition for our model, we first summarize the basic equations:

\[
\begin{align*}
\xi_{t+1} &= F\xi_t + G\varepsilon_{t+1}, \quad E\varepsilon_{t+1}\varepsilon'_{t+1} = V^z \\
D(L)y_t &= D(L)H\xi_t + D(L)w_t, \quad Ew_tw'_t = R,
\end{align*}
\]

where \( D(L) \) is a possible transformation on the observed data, \( y_t \). For example, we may be interested in levels of some of the variables which appear in growth rate in \( y_t \). In this case the relevant diagonal term of \( D(L) \) would have \( 1/(1 - L) \).

Note:

\[
\tilde{y}_t \equiv D(L)y_t = D(L)H[I - FL]^{-1}G\varepsilon_t + D(L)w_t.
\]

Define the ‘spectral density’ of \( \tilde{y}_t \):

\[
S_{\tilde{y}}(z) = D(z)H([I - Fz]^{-1})GV^zG'([I - Fz^{-1}]^{-1})'H'D(z^{-1})' + D(z)RD(z^{-1})',
\]

where \( z \) is a complex variable:

\[
z = e^{-i\omega}, \quad \omega \in [0, \pi].
\]

The conversion from \( \omega \) to time is

\[
\Delta t = \frac{2\pi}{\omega}.
\]

Recall:

\[
E\tilde{y}_t\tilde{y}'_t = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{\tilde{y}}(e^{-i\omega}) \, d\omega.
\]

Consider the following decomposition:

\[
S_{\tilde{y}}(z) = \sum_{i=1}^{N} S_{\tilde{y}}^i(z) + D(z)RD(z^{-1})',
\]

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where
\[ S_i^j(z) = D(z) H \left( [I - Fz]^{-1} \right) GV_i^c G' \left( [I - Fz^{-1}]^{-1} \right)' H' D(z^{-1})' \]
and \( Q_i \) is the original \( Q \) matrix with only the \( i^{th} \) diagonal element of \( Q \) and zeros everywhere else.

Say we are interested in the \( j^{th} \) variable in \( \tilde{y}_t \), \( j = 1, \ldots, N \). Define
\[ \tau_j = [0, \ldots, \underline{j}^{th} \text{ entry} 1, \ldots, 0]' \]
Then,
\[ \tau_j' S_i^j(z) \tau_j = \sum_{i=1}^{N} \tau_j' S_i^j(z) \tau_j + \tau_j' D(z) RD(z^{-1})' \tau_j, \]
and the percent variance of the \( j^{th} \) variable due to the \( i^{th} \) shock is:
\[ P_{i,j}(z) = 100 \times \frac{\tau_j' S_i^j(z) \tau_j}{\tau_j' S_i^j(z) \tau_j}, \quad i, j = 1, \ldots, N. \]
The percent variance of the \( j^{th} \) variable due to the measurement error is:
\[ 100 \times \frac{\tau_j' D(z) RD(z^{-1})' \tau_j}{\tau_j' S_i^j(z) \tau_j}. \]

5 Appendix E: Comparing Models Based on Forecasts

We evaluate the fit of our model by comparing its out of sample forecasting properties with those of a Bayesian Vector Autoregression, as well as with versions of the model without banks and without financial frictions and banks. Our model comparisons are implemented in two ways. First, we do a direct comparison of out of sample root mean square errors (RMSE). The RMSE comparison ignores covariances in forecast errors, and this motivates our second method of comparison. This computes the optimal weight assigned to our model in a portfolio of two models that includes ours and the BVAR. The optimality criterion is out-of-sample RMSE.

5.1 Computing RMSE’s

To compute out of sample forecasts from our model, we use a procedure analogous to our estimation strategy. In particular, when we compute a forecast as of time \( t \), we remove
the sample mean (computed from data \( t \) and earlier) from the data. We then re-estimate the model parameters and use standard Kalman filter formulas to compute forecasts. (In practice, we only re-estimate the model every other period.) We take the model forecast to be the forecasts produced by the Kalman filter, plus the sample mean. Alternative possible strategies would involve adding back the sample mean over the previous year or two, but we have not explored these.

5.2 Statistical Test of Differences Between RMSE’s

Following is a Generalized Method of Moment strategy for determining the statistical significance of the difference between the RMSE’s of different models.\(^3\) Let \( e^i_t \) denote forecast error at a particular date, \( t \), from model \( i \), \( i = 1, 2 \). The horizon of the forecast error is denoted, \( n \). That is,

\[
e^i_t = x_t - P^i_{t-n},
\]

where \( P^i_{t-n} \) is the forecast based on model \( i \), computed at \( t - n \). Define

\[
\gamma = \sigma_1 - \sigma_2,
\]

where \( \sigma_i \) is the RMSE of forecasting procedure, \( i \):

\[
(\sigma_i)^2 = E\left[e^i_t\right]^2
\]

Note

\[
\gamma (\gamma + (\sigma_2)^2) = (\sigma_1)^2 - (\sigma_2)^2.
\]

Consider

\[
h_t(\gamma, \sigma_2) = \begin{pmatrix} (\gamma + 2\sigma_2) - \left[ (\sigma_1)^2 - (\sigma_2)^2 \right] \\ (\sigma_2)^2 - (\sigma_1)^2 \end{pmatrix},
\]

and note that

\[
Eh_t(\gamma, \sigma_2) = \begin{pmatrix} 0 \\ 0 \end{pmatrix},
\]

when \( \gamma \) and \( \sigma_2 \) are evaluated at their true values. Define

\[
g_T(\gamma, \sigma_2) = \frac{1}{T} \sum_{t=1}^{T} h_t(\gamma, \sigma_2),
\]

and let \( \hat{\gamma}, \hat{\sigma}_2 \) denote the (unique) values of \( \gamma, \sigma_2 \) which set \( g_T(\gamma, \sigma_2) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \). Note that \( \hat{\gamma} \) is just the standard estimator of the difference between the RMSE’s of the two procedures.

\(^3\)This statistic was proposed in Christiano (1990).
Define the 2 by 2 matrix, $D$:

$$D = \frac{\partial g_T (\gamma, \sigma_2)}{\partial (\gamma, \sigma_2)} = 2 \begin{bmatrix} \gamma + \sigma_2 & \gamma \\ 0 & \sigma_2 \end{bmatrix},$$
evaluated at the true values of $\gamma, \sigma_2$. Define

$$S = \sum_{k=-\infty}^{\infty} Eh_t h_{t-k}',$$
where $h_t$ is evaluated at the true values of $\gamma, \sigma_2$. Then, according to $GMM$,

$$\sqrt{T} \left( \begin{array}{c} \hat{\gamma} - \gamma \\ \hat{\sigma}_2 - \sigma_2 \end{array} \right) \sim N(0, V),$$

where

$$V = D^{-1} S \left( D^{-1} \right)' .$$

Thus, we use the following sampling theory for the variables that interest us:

$$\left( \begin{array}{c} \hat{\gamma} - \gamma \\ \hat{\sigma}_2 - \sigma_2 \end{array} \right) \sim N(0, \frac{V}{T}).$$

Note that $V$ is a function of the true parameters because $S$ and $D$ are. The matrix $D$ is replaced by $D_T$, which is $D$ with the parameters replaced by their point estimate. The matrix $S$ is replaced by $S_T$, where $S_T$ is an estimator of the frequency zero spectral density of $h_t$. One estimator of this is:

$$S_T = \sum_{k=-r}^{r} g(k) \hat{C}(k), \quad g(k) = \begin{cases} 1 - \frac{|k|}{r} & |k| \leq r \\ 0 & |k| > r \end{cases},$$

where

$$\hat{C}(k) = \frac{1}{T} \sum_{t=k+1}^{T} h_t h'_{t-k}.$$ 

In practice, we set $r = 5$. We found that the results are not sensitive to alternative values of $r$. 

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5.3 An Alternative Strategy for Comparing Models

An alternative comparison of two models computes the optimal linear combination of the model forecasts. Thus, let \( y_{t+k}^i \) be the forecast computed at time \( t \) using model \( i, i = 1, 2 \). The forecast errors are:

\[
e_{i,t+k} = y_{t+k} - y_{t+k}^i.
\]

The forecast error of the weighted combination of the two models is:

\[
e_{\lambda,t+k} = \lambda e_{1,t+k} + (1 - \lambda) e_{2,t+k}.
\]

A measure of the value of model 1 compared with model 2 is given by the value of \( \lambda_k \) which minimizes the root mean square error of \( e_{\lambda,t+k} \):

\[
\hat{\lambda}_k = \arg\min_{\lambda} \frac{1}{T} \sum_{i=1}^{T} (e_{\lambda,t+k})^2,
\]

where \( T \) is the number of observations on the forecast error. In practice, this is a function of \( k \). The solution to the optimization problem is the coefficient on \( y_1^t \) in the bivariate regression of \( y_t \) on \( y_1^t \) and \( y_2^t \), when the coefficients are constrained to sum to unity. Although we have not yet done so, it would be interesting to consider the case where the coefficients are not constrained to sum to unity. In addition, it would be useful to also investigate the inclusion of past forecast errors at various horizons.

We derive a sampling theory for \( \hat{\lambda}_k \) by considering the first order condition (or, regression normal equation):

\[
\frac{1}{T} \sum_{t=1}^{T} h_t (\lambda_k, e_{1,t}, e_{2,t}) = 0,
\]

where

\[
h_t (\lambda_k, e_{1,t}, e_{2,t}) = \lambda_k (e_{2,t} - e_{1,t})^2 - e_{2,t} [e_{2,t} - e_{1,t}].
\]

Let

\[
D = E (e_{2,t} - e_{1,t})^2
\]

\[
S = \sum_{k=-\infty}^{\infty} Eh_t h'_{t-k},
\]

where both \( S \) and \( D \) are evaluated in population, at the true parameter values. Our estimator of the sampling uncertainty in \( \hat{\lambda} \) is based on

\[
\sqrt{T} (\hat{\lambda}_k - \lambda_k) \sim N (0, V),
\]

\[
V = \frac{S}{D^2}.
\]

(109)
In practice, we estimate $S$ using the strategy in the previous subsection, and we evaluate $D$ using the sample analogs of the population moments in $D$. This strategy does not take into account the sampling uncertainty in the estimated model parameters used to construct $y_{it}^j$, $i = 1, 2$. A more complete analysis would take this sampling uncertainty into account.

Figures Ea and Eb display our results for $\lambda_k$ for the EA and the US, respectively. Note that in many cases the confidence intervals are very wide. We view it as evidence in favor of our model if $\lambda_k > 0.5$ cannot be rejected. In the case of the EA the null hypothesis, $\lambda_k > 0.5$, cannot be rejected in all cases except for M1 and hours worked. Interestingly, our other measure of fit based on RMSE’s also finds evidence of poor forecasting performance in these cases. In the case of the US, the null hypothesis, $\lambda_k > 0.5$, is rejected in the case of inflation, the risk premium and M3. This is consistent with the negative assessment of the forecasting performance of our model for these variables based on our RMSE-based statistic, in Figure 4b. Interestingly, the two measures of forecasting performance differ in the case of the spread between long and short rates. Our RMSE-based metric finds that our model forecasts of the spread are significantly worse than the BVAR’s forecasts of that variable. However, the regression-based measure we consider here says that there is not a significant difference between the two models on this dimension.

We also consider a multivariate analog of $\lambda_k$. Let $e_{i,t}^j$ denote the $k$ step ahead forecast error in variable $j$ using procedure $i$. Then,

$$\hat{\lambda}_k = \arg\min_{\lambda_k} \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{N} \frac{[\lambda_k e_{1,t}^j + (1 - \lambda_k) e_{2,t}^j]^2}{\sigma_j^2},$$

where $\sigma_j$ is the variance of the underlying variable being forecast. The first order condition for this problem is:

$$\sum_{t=1}^{T} h_t (\lambda_k, e_{1,t}^j, e_{2,t}^j; \text{all } j) = 0,$$

where

$$h_t (\lambda_k, e_{1,t}^j, e_{2,t}^j; \text{all } j) = \sum_{j=1}^{N} \frac{[\lambda_k e_{1,t}^j + (1 - \lambda_k) e_{2,t}^j] [e_{1,t}^j - e_{2,t}^j]}{\sigma_j^2}.$$  

Let

$$D = E \sum_{j=1}^{N} \frac{[e_{1,t}^j - e_{2,t}^j]^2}{\sigma_j^2}$$

$$S = \sum_{k=-\infty}^{\infty} Eh_{t+k}^t h_{t-k}^t.$$
As before, we estimate $D$ by replacing population moments with sample analogs. We estimate $S$ as discussed above. The distribution of our multivariate statistic (abstracting from sampling uncertainty in the underlying model parameters) is given by (109).

6 Appendix F: Laplace Approximation

Although these calculations are standard, for completeness we include the derivation of the Laplace approximation for the posterior distribution of our parameters. Let $\theta \in \mathbb{R}^N$ denote the $N$-dimensional vector of parameters. Let $g(\theta) \equiv \log f(y|\theta) f(\theta)$, where $f(y|\theta)$ denotes the likelihood of the data and $f(\theta)$ denotes the prior on the parameters. Let $\theta^*$ denote the mode of $g(\theta)$. Write out the second order Taylor series expansion about $\theta = \theta^*$:

$$g(\theta) \approx g(\theta^*) + g_{\theta}(\theta^*)(\theta - \theta^*) - \frac{1}{2} (\theta - \theta^*)' g_{\theta\theta}(\theta^*)(\theta - \theta^*),$$

where

$$g_{\theta\theta}(\theta^*) = -\frac{\partial^2 \log f(y|\theta) f(\theta)}{\partial \theta \partial \theta}|_{\theta = \theta^*}.$$

Note that the fact that $\theta^*$ is the mode implies $g_{\theta\theta}(\theta^*)$ is positive definite and $g_\theta = 0$. Then,

$$f(y|\theta) f(\theta) \approx f(y|\theta^*) f(\theta^*) \exp \left\{ -\frac{1}{2} (\theta - \theta^*)' g_{\theta\theta}(\theta^*)(\theta - \theta^*) \right\},$$

after imposing $g_\theta(\theta^*) = 0$. Note that

$$\frac{1}{(2\pi)^{\frac{N}{2}}} |g_{\theta\theta}(\theta^*)|^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\theta - \theta^*)' g_{\theta\theta}(\theta^*)(\theta - \theta^*) \right\}$$

is the multinormal density for an $N$-dimensional random variable, $\theta$, with variance covariance matrix, $g_{\theta\theta}(\theta^*)^{-1}$, and mean $\theta^*$. As a result, its integral over all values of $\theta$ is unity:

$$\int \frac{1}{(2\pi)^{\frac{N}{2}}} |g_{\theta\theta}(\theta^*)|^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\theta - \theta^*)' g_{\theta\theta}(\theta^*)(\theta - \theta^*) \right\} d\theta = 1.$$
Then,

$$\int f(y|\theta) f(\theta) d\theta \simeq \int f(y|\theta^*) f(\theta^*) \exp \left\{ -\frac{1}{2} (\theta - \theta^*)' g_{\theta \theta} (\theta^*) (\theta - \theta^*) \right\} d\theta$$

$$= \frac{f(y|\theta^*) f(\theta^*)}{|g_{\theta \theta} (\theta^*)|^\frac{1}{2}} \int \frac{1}{(2\pi)^\frac{N}{2}} |g_{\theta \theta} (\theta^*)|^\frac{1}{2} \exp \left\{ -\frac{1}{2} (\theta - \theta^*)' g_{\theta \theta} (\theta^*) (\theta - \theta^*) \right\} d\theta$$

$$= \frac{f(y|\theta^*) f(\theta^*)}{|g_{\theta \theta} (\theta^*)|^\frac{1}{2}}.$$

Thus,

$$f(y) = \int f(y|\theta) f(\theta) d\theta \simeq (2\pi)^\frac{N}{2} f(y|\theta^*) f(\theta^*) |g_{\theta \theta} (\theta^*)|^\frac{1}{2}.$$ 

7 Appendix G: News Shocks

We now modify our environment to allow the possibility that there are advance ‘news’ signals about some future variable, say $x_t$. The model, proposed by Josh Davis in his thesis, is as follows:

$$x_t = \rho x_{t-1} + \varepsilon_t + \eta_{t-1} + \eta_{t-2} + \ldots + \eta_{t-p}. \quad (110)$$

When $\eta_{t-1} = \eta_{t-2} = \ldots = \eta_{t-p} = 0$, then this is the usual scalar first order autoregressive representation. The variable, $\eta_{t-j}$ is realized at time $t-j$ and represents news about $x_t$. The superscript on the variable indicates the date of $x_t$ that the news is relevant for, while the subscript indicates the date that the news is realized. The model with news in effect has $p$ additional parameters:

$$\sigma_1^2 = Var(\eta_{t-1}) , \quad \sigma_2^2 = Var(\eta_{t-2}) , \ldots , \sigma_p^2 = Var(\eta_{t-p}).$$

These parameters are all independent and it is possible, for example, that the econometrics would prefer a subset to be zero. For example, it is possible that only $\sigma_p^2 > 0$. This means that at time $t$, there is news about $x_{t+p}$, and no further news arrives until time $t+p$ when $x_{t+p}$ is realized.

To better understand the nature of the news time series representation, it is interesting to note that the addition of news has no impact on the univariate time series representation of $x_t$. Put differently, whether there is news in a variable cannot be determined by examining the scalar time series representation of that variable. Consider, the pure moving average
representation of \( x_t \):

\[
x_t = \left[ \varepsilon_t + \eta_{t-1}^t + \eta_{t-2}^t + \ldots + \eta_{t-p}^t \right]
+ \rho \left[ \varepsilon_{t-1} + \eta_{t-2}^{t-1} + \eta_{t-3}^{t-1} + \ldots + \eta_{t-p}^{t-1} \right]
+ \rho^2 \left[ \varepsilon_{t-2} + \eta_{t-3}^{t-2} + \eta_{t-4}^{t-2} + \ldots + \eta_{t-p}^{t-2} \right]
+ \ldots
\]

Note that the objects in square brackets at different dates are not correlated with each other, but each has the same variance. As a result, \( x_t \) has a first order autoregressive representation, regardless of the pattern of \( \sigma_j^2 \), \( j = 1, \ldots, p \). Manipulating the pure moving average representation, or the original representation above, we find:

\[
E x_t x_{t-1} = \rho E x_{t-1} x_{t-1} + E \left[ \varepsilon_t + \eta_{t-1}^t + \eta_{t-2}^t + \ldots + \eta_{t-p}^t \right] x_{t-1}
= \rho Var \left( x_t \right),
\]

because

\[
E \left[ \varepsilon_t + \eta_{t-1}^t + \eta_{t-2}^t + \ldots + \eta_{t-p}^t \right] x_{t-1} = 0.
\]

The latter result reflects that there is no overlap between the shocks determining \( x_{t-1} \) and the object in square brackets. Similarly, it is easy to show that:

\[
E x_t x_{t-j} = \rho^j Var \left( x_t \right), j = 1, \ldots.
\]

This establishes that the number of signals in \( x_t \) is not identified from observations on \( x_t \) alone. However, the cross equation restrictions delivered by an economic model can deliver identification of the \( \sigma_j^2 \)'s.

We now set this process up in state space/observer form. Suppose, to begin, that \( p = 2 \). Then,

\[
x_t = \rho x_{t-1} + \varepsilon_t + \eta_{t-1}^t + \eta_{t-2}^t. \tag{111}
\]

It is useful to set up some auxiliary variables, \( u_{t-1}^{t+1} \) and \( u_{t-2}^t \). Write

\[
\begin{bmatrix}
  x_t \\
  u_{t+2}^t \\
  u_{t+1}^t
\end{bmatrix}
= \begin{bmatrix}
  \rho & 0 & 1 \\
  0 & 0 & 0 \\
  0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  x_{t-1} \\
  u_{t+1}^{t-1} \\
  u_{t-1}^t
\end{bmatrix}
+ \begin{bmatrix}
  \varepsilon_t \\
  \eta_t^{t+2} \\
  \eta_t^t
\end{bmatrix}. \tag{112}
\]

It is easy to confirm that this is the same as (111). Write the first equation:

\[
x_t = \rho x_{t-1} + u_{t-1}^{t+1} + \varepsilon_t. \tag{113}
\]
To determine \( u_{t-1} \) evaluate (112) at the previous date:

\[
\begin{align*}
    u_{t-1}^{t+1} &= \eta_{t-1}^t \\
    u_{t-1}^t &= u_{t-2}^t + \eta_{t-1}^t.
\end{align*}
\]

The second of the above two expressions indicates that we must evaluate (112) at an earlier date:

\[
\begin{align*}
    u_{t-2}^t &= \eta_{t-2}^t \\
    u_{t-2}^{t-1} &= u_{t-3}^{t-1} + \eta_{t-2}^{t-1}.
\end{align*}
\]

Combining the first of these equations with the second of the previous set of two equations, we obtain:

\[
    u_{t-1}^t = \eta_{t-2}^t + \eta_{t-1}^t.
\]

Substituting this into (113), we obtain (111), which is the result we sought. We can refer to \( u_{t-1}^t \) as the “state of signals about \( x_t \) as of \( t-1 \)”.

We can refer to \( \eta_{t-2}^t \) as the “signal about \( x_t \) that arrives at time \( t-2 \)”. We can refer to \( \eta_{t-1}^t \) as the “signal about \( x_t \) that arrives at time \( t-1 \)”.

We now consider the case of general \( p \). Thus, we have

\[
\begin{align*}
    x_t &= \rho x_{t-1} + \varepsilon_t + u_{t-1}^t \\
    u_{t-1}^t &= u_{t-2}^t + \eta_{t-1}^t \\
    u_{t-2}^t &= u_{t-3}^t + \eta_{t-2}^t \\
    \vdots \\
    u_{t-p+1}^t &= u_{t-p}^t + \eta_{t-p+1}^t \\
    u_{t-p}^t &= \eta_{t-p}^t.
\end{align*}
\]

According to this setup, there are \( p \) signals about \( x_t \). The first arrives in \( t-p \), the second in \( t-p+1 \) and the \( p^{th} \) in \( t-1 \). This is set up in state space form as follows:

\[
\begin{bmatrix}
    x_t \\
    u_{t-1}^{t+p} \\
    u_{t-1}^{t+p-1} \\
    \vdots \\
    u_t^{t+1}
\end{bmatrix}
= \begin{bmatrix}
    \rho & 0 & \cdots & 0 & 1 \\
    0 & 0 & \cdots & 0 & 0 \\
    0 & 1 & \cdots & 0 & 0 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    0 & 0 & \cdots & 1 & 0
\end{bmatrix}
\begin{bmatrix}
    x_{t-1}^{t} \\
    u_{t-1}^{t} \\
    u_{t-1}^{t-1} \\
    \vdots \\
    u_{t-1}^{t-1}
\end{bmatrix}
+ \begin{bmatrix}
    \varepsilon_t^{t+p} \\
    \eta_{t}^{t+p} \\
    \eta_{t}^{t+p-2} \\
    \vdots \\
    \eta_{t}^{t+1}
\end{bmatrix}
\]

We can write this in compact notation as follows:

\[
\Psi_{x,t} = P_x \Psi_{x,t-1} + \varepsilon_{x,t},
\]

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where

\[
\Psi_{x,t} = \begin{bmatrix}
x_t \\
\varepsilon_{t}^{t+p} \\
\varepsilon_{t}^{t+p-1} \\
\vdots \\
\varepsilon_{t}^{t+1}
\end{bmatrix}, \quad P_x = \begin{bmatrix}
\rho & 0 & \cdots & 0 & 1 \\
0 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \cdots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0
\end{bmatrix}, \quad \varepsilon_{x,t} = \begin{bmatrix}
\varepsilon_t \\
\eta_{t}^{t+p} \\
\eta_{t}^{t+p-1} \\
\vdots \\
\eta_{t}^{t+1}
\end{bmatrix},
\]

\[
E\varepsilon_{x,t} \varepsilon_{x,t}' = \begin{bmatrix}
\sigma_{\varepsilon}^2 & 0 & \cdots & 0 \\
0 & \sigma_{\varepsilon}^2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \sigma_p^2
\end{bmatrix}.
\]
Figure Eb: US - Weight ($\lambda$) on Model versus (1-$\lambda$) on BVAR. +/- 1 std Confidence Interval.
### Table B.1. Simple Model Parameter Estimates: EA and US

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Mode</th>
<th>Std. dev. (Hess.)</th>
<th>Mode</th>
<th>Std. dev. (Hess.)</th>
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</thead>
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<tr>
<td>$\xi_p$</td>
<td>Beta</td>
<td>0.75*</td>
<td>0.05</td>
<td>0.7263</td>
<td>0.0361</td>
<td>0.5388</td>
<td>0.0425</td>
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<td>$\xi_w$</td>
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<td>0.1</td>
<td>0.6498</td>
<td>0.0444</td>
<td>0.7398</td>
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<td>$t$</td>
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<td>0.15</td>
<td>0.8846</td>
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<td>0.15</td>
<td>0.3195</td>
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<td>$\vartheta$</td>
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<td>0.15</td>
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<td>$S''$</td>
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<td>3.5</td>
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<td>2.7404</td>
<td>1.1031</td>
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<td>$\sigma_a$</td>
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<th>Std. dev.</th>
<th>Std. dev.</th>
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<td>(Hess.)</td>
<td>US</td>
</tr>
<tr>
<td>$\sigma$ Investm. specific shock ($\mu_{\tau,i}$)</td>
<td>Inv. Gamma 0.003</td>
<td>5 d</td>
<td>0.0033</td>
<td>0.0002</td>
<td>0.0032</td>
</tr>
<tr>
<td>$\sigma$ Government consumption shock ($g_{i}$)</td>
<td>Inv. Gamma 0.01</td>
<td>5 d</td>
<td>0.0216</td>
<td>0.0016</td>
<td>0.0239</td>
</tr>
<tr>
<td>$\sigma$ Persistent product. shock ($\mu_{z,i}$)</td>
<td>Inv. Gamma 0.01</td>
<td>5 d</td>
<td>0.0053</td>
<td>0.0005</td>
<td>0.0078</td>
</tr>
<tr>
<td>$\sigma$ Transitory product. shock ($\epsilon_{i}$)</td>
<td>Inv. Gamma 0.01</td>
<td>5 d</td>
<td>0.0040</td>
<td>0.0003</td>
<td>0.0041</td>
</tr>
<tr>
<td>$\sigma$ Consump. prefer. shock ($\zeta_{c,i}$)</td>
<td>Inv. Gamma 0.01</td>
<td>5 d</td>
<td>0.0153</td>
<td>0.0021</td>
<td>0.0162</td>
</tr>
<tr>
<td>$\sigma$ Margin. effic. of invest. shock ($\zeta_{i,i}$)</td>
<td>Inv. Gamma 0.01</td>
<td>5 d</td>
<td>0.0245</td>
<td>0.0020</td>
<td>0.0978</td>
</tr>
<tr>
<td>$\sigma$ Oil price shock ($\tau_{oil,i}$)</td>
<td>Inv. Gamma 0.1</td>
<td>5 d</td>
<td>0.1555</td>
<td>0.0120</td>
<td>0.1327</td>
</tr>
<tr>
<td>$\sigma$ Monetary policy shock ($\epsilon_{i}$)</td>
<td>Inv. Gamma 0.25</td>
<td>5 d</td>
<td>0.5578</td>
<td>0.0477</td>
<td>0.5501</td>
</tr>
<tr>
<td>$\sigma$ Price markup shock ($\lambda_{f,i}$)</td>
<td>Inv. Gamma 0.01</td>
<td>5 d</td>
<td>0.0093</td>
<td>0.0019</td>
<td>0.0066</td>
</tr>
</tbody>
</table>

Notes: *Upper numbers refer to EA, lower numbers to US. The US priors was taken from LOWW. The EA prior for prices is consistent with the results produced by the Inflation Persistent Network (see Altissimo et al., 2006). Simple model: version of main model without banking sector or financial frictions. This version basically corresponds to CEE or SW.
Table B.2. Financial Accelerator Parameter Estimates: Euro area and US

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Type</th>
<th>Prior Mean</th>
<th>Prior Std. dev.</th>
<th>Posterior Mode</th>
<th>Posterior Std. dev. (Hess.)</th>
<th>Posterior Mode</th>
<th>Posterior Std. dev. (Hess.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_p$</td>
<td>Beta</td>
<td>0.75*</td>
<td>0.375</td>
<td>0.7302</td>
<td>0.0363</td>
<td>0.4992</td>
<td>0.0368</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Beta</td>
<td>0.75*</td>
<td>0.375</td>
<td>0.655</td>
<td>0.0416</td>
<td>0.7823</td>
<td>0.0288</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.8587</td>
<td>0.0669</td>
<td>0.2498</td>
<td>0.1019</td>
</tr>
<tr>
<td>$\alpha_w$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.2857</td>
<td>0.0845</td>
<td>0.2706</td>
<td>0.1090</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.15</td>
<td>0.8901</td>
<td>0.0471</td>
<td>0.9263</td>
<td>0.0333</td>
</tr>
<tr>
<td>$S''$</td>
<td>Normal</td>
<td>7.7</td>
<td>3.5</td>
<td>21.154</td>
<td>2.8468</td>
<td>18.075</td>
<td>2.5415</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Gamma</td>
<td>6</td>
<td>5</td>
<td>25.711</td>
<td>7.5877</td>
<td>23.001</td>
<td>6.0091</td>
</tr>
<tr>
<td>$\alpha_x$</td>
<td>Normal</td>
<td>1.75</td>
<td>0.1</td>
<td>1.8877</td>
<td>0.0824</td>
<td>1.7932</td>
<td>0.0913</td>
</tr>
<tr>
<td>$\alpha_y$</td>
<td>Normal</td>
<td>0.1</td>
<td>0.05</td>
<td>0.1126</td>
<td>0.0498</td>
<td>0.1083</td>
<td>0.0613</td>
</tr>
<tr>
<td>$\alpha_{dx}$</td>
<td>Normal</td>
<td>0.3</td>
<td>0.1</td>
<td>0.2204</td>
<td>0.0970</td>
<td>0.2044</td>
<td>0.1086</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Beta</td>
<td>0.8</td>
<td>0.05</td>
<td>0.8408</td>
<td>0.0154</td>
<td>0.8625</td>
<td>0.0139</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9693</td>
<td>0.0166</td>
<td>0.9856</td>
<td>0.0060</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.8546</td>
<td>0.0977</td>
<td>0.9159</td>
<td>0.0334</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.9432</td>
<td>0.0281</td>
<td>0.9750</td>
<td>0.0172</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
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<td>0.2</td>
<td>0.7212</td>
<td>0.0535</td>
<td>0.9727</td>
<td>0.0098</td>
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<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.8150</td>
<td>0.0316</td>
<td>0.9312</td>
<td>0.0228</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9525</td>
<td>0.0152</td>
<td>0.9476</td>
<td>0.0165</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5398</td>
<td>0.0998</td>
<td>0.9768</td>
<td>0.0076</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9157</td>
<td>0.0282</td>
<td>0.9234</td>
<td>0.0251</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9334</td>
<td>0.0276</td>
<td>0.9808</td>
<td>0.0122</td>
</tr>
</tbody>
</table>
Table B.2, continued

<table>
<thead>
<tr>
<th>Type</th>
<th>Mode</th>
<th>Df.</th>
<th>Prior</th>
<th>Mode</th>
<th>Std. dev. (Hess.)</th>
<th>Mode</th>
<th>Std. dev. (Hess.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$ Investm. specific shock ($\mu_{Y,t}$)</td>
<td>Inv. Gamma</td>
<td>0.003</td>
<td>5 d</td>
<td>0.0033</td>
<td>0.0003</td>
<td>0.0032</td>
<td>0.0002</td>
</tr>
<tr>
<td>$\sigma$ Government consumption shock ($g_{r,t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0150</td>
<td>0.0013</td>
<td>0.0205</td>
<td>0.0016</td>
</tr>
<tr>
<td>$\sigma$ Persistent product. shock ($\mu_{z,t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0054</td>
<td>0.0005</td>
<td>0.0076</td>
<td>0.0006</td>
</tr>
<tr>
<td>$\sigma$ Transitory product. shock ($\epsilon_{t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0041</td>
<td>0.0004</td>
<td>0.0040</td>
<td>0.0004</td>
</tr>
<tr>
<td>$\sigma$ Financial wealth shock ($\gamma_{t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0167</td>
<td>0.0024</td>
<td>0.0056</td>
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<tr>
<td>$\sigma$ Riskiness shock ($\sigma_{t}$)</td>
<td>Inv. Gamma</td>
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<td>5 d</td>
<td>0.0780</td>
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</tr>
<tr>
<td>$\sigma$ Consump. prefer. shock ($\zeta_{c,t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0265</td>
<td>0.0055</td>
<td>0.0243</td>
<td>0.0051</td>
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<tr>
<td>$\sigma$ Margin. effic. of invest. shock ($\zeta_{i,t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0289</td>
<td>0.0028</td>
<td>0.1774</td>
<td>0.0556</td>
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<tr>
<td>$\sigma$ Oil price shock ($\tau_{oil}^{i}$)</td>
<td>Inv. Gamma</td>
<td>0.1</td>
<td>5 d</td>
<td>0.1557</td>
<td>0.0120</td>
<td>0.1328</td>
<td>0.0101</td>
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<tr>
<td>$\sigma$ Monetary policy shock ($\epsilon_{t}$)</td>
<td>Inv. Gamma</td>
<td>0.25</td>
<td>5 d</td>
<td>0.4985</td>
<td>0.0408</td>
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</tr>
<tr>
<td>$\sigma$ Price markup shock ($\lambda_{f,t}$)</td>
<td>Inv. Gamma</td>
<td>0.01</td>
<td>5 d</td>
<td>0.0104</td>
<td>0.0019</td>
<td>0.0071</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

- Upper numbers refer to EA, lower numbers to US. The US priors were taken from LOWW. The EA prior for prices is consistent with the results produced by the Inflation Persistent Network (see Altissimo et al., 2006).
- Financial Accelerator refers to the version of our model without banks, but with financial frictions.