Adversarial Coordination and Public Information Design

Additional Material

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Abstract

This document contains additional results for the manuscript "Adversarial Coordination and Public Information Design." All numbered items (i.e., sections, subsections, lemmas, conditions, propositions, and equations) in this document contain the prefix "AM." Any numbered reference without the prefix "AM" refers to an item in the main text. Please refer to the main text for notation and definitions.

Section AM1 expands the discussion in the main text of the role played by the multiplicity of the receivers and by their exogenous private information for the optimality of monotone rules.

Section AM2 studies comparative statics of the optimal policy in a family of economies in which banks issue equity, or debt, to fund their short-term liquidity obligations, and where the (market-clearing) price of the securities is endogenous and depends on the information revealed through the stress tests. In particular, it investigates the effects of an increase in market uncertainty on the toughness of the optimal stress tests, and shows how the latter depends on the type of security issued by the banks.

Section AM3 extends the result in Theorem 1 in the main text (about the optimality of perfectly coordinating the market response) to a class of economies in which (a) agents' prior beliefs need not be consistent with a common prior, nor be generated by signals drawn independently across agents, conditionally on θ, (b) the number of agents is arbitrary (in particular, finitely many agents), (c) payoffs can be heterogenous across agents, (d) agents have a level-K degree of sophistication, (e) the policy maker may possess imperfect information about the payoff state and/or the agents' beliefs, (f) the policy maker may disclose different information to different agents.

Finally, Section AM4 discusses the benefits to discriminatory disclosures, when the latter are feasible.

Section AM1: Role of Multiplicity of Receivers and of Exogenous Private Information

Subsection AM1.1. Single Receiver

To appreciate the role that the multiplicity of the receivers plays for the results in the main text, consider the following variant of the economy of Section 2 in the main text.

Timing. At $t = 0$, the policy maker chooses a disclosure policy $\{\pi, S\}$ that, for each fundamental θ, sends a signal s from a distribution $\pi(\theta) \in \Delta(S)$.[1]At $t = 1$, a single receiver with signal x drawn from a log-supermodular distribution $p(x|\theta)$ has to decide whether to take a "friendly" action, $a = 1$, or an adversarial" action, $a = 0$.

Payoffs. The policy maker's payoff is equal to $W > 0$ in case of no default and $L < 0$ in case of default. Default occurs if and only if $\theta \leq 1 - a$. Hence,

$$U^{PM}(\theta, a) \equiv W \times \mathbf{1}\{\theta > 1 - a\} + L \times \mathbf{1}\{\theta \leq 1 - a\}.$$

As for the receiver's payoff, we consider two cases. The first one corresponds to a market where the receiver's payoff is aligned with the policy maker's payoff, as in the baseline model of Section 2. The second case, instead, corresponds to a market where the receiver's payoff is misaligned with the policy maker's payoff, over the critical region of fundamentals $(0, 1)$ where the fate of the bank depends on the receiver's behavior.

Case 1: Aligned Preferences

The receiver's payoff differential between taking the friendly action (interpreted as "pledging" to the bank) and the adversarial action (interpreted as "refraining from pledging") is given by $u^I(\theta, 1) - u^I(\theta, 0) = b < 0$ in case of default, and by $u^I(\theta, 1) - u^I(\theta, 0) = g > 0$ in case of no default, with $g > 0 > b$. In this case, it is immediate to see that the following pass/fail policy is optimal: the policy maker gives a pass to all banks with fundamentals $\theta > 0$ and a fail to all banks with fundamentals $\theta \leq 0$.

Case 2: Misaligned Preferences

The receiver's payoff differential between taking the friendly action (interpreted as "refraining from speculating" against the bank) and the adversarial action (interpreted as "speculating" against the bank) is given by $u^I(\theta, 1) - u^I(\theta, 0) = -g < 0$ for any $\theta \leq 1$, and by $u^I(\theta, 1) - u^I(\theta, 0) = -b > 0$ for any $\theta > 1$, with $g > 0 > b$. That is, the receiver obtains a payoff equal to 0 when she abstains from speculating against the bank (the friendly action). When, instead, she speculates against the

[1] Here we accommodate for stochastic disclosure rules, although this is not essential to the results, as explained below.

bank (the adversarial action), she obtains a payoff equal to $g > 0$ in case speculation is successful (i.e., in case of default) and a payoff equal to $b < 0$ in case the bank survives the attack.

We start by showing that, in this case, Assumptions 2 and 3 in Guo & Shmaya (2019) are satisfied. In fact, note that, for any realization $x \in \mathbb{R}$ of the receiver's signal, the ratio between the receiver's and the sender's payoff differential is equal to

$$\varphi(\theta) \equiv \frac{u^I(\theta,1) - u^I(\theta,0)}{U^{PM}(\theta,1) - U^{PM}(\theta,0)} = \begin{cases} -\infty & \theta \leq 0 \\ \frac{-g}{W-L} & \theta \in (0,1] \\ +\infty & \theta > 1 \end{cases}$$

and is increasing in θ, which implies that Assumption 2 in Guo & Shmaya (2019) holds. That Assumption 3 also holds follows from noting that the receiver's payoff differential changes from negative to positive at $\theta = 1$, for any $x \in \mathbb{R}$. By virtue of Theorem 3.1 in Guo & Shmaya (2019), the optimal policy is thus a deterministic cutoff mechanism that recommends to take action $a = 1$ on intervals $(\underline{\pi}(x), \bar{\pi}(x)) \subset \Theta$, with $\underline{\pi}(x)$ decreasing in x, and $\bar{\pi}(x)$ increasing in x.

Next observe, when there is a continuum of receivers with the same payoffs as the representative speculator above, under an adversarial/robust design, the optimal policy satisfies the same properties as in Theorems 1-3 in the baseline model of Section 2 in the main body, despite the misalignment in payoffs. This is because, under MARP, independently of whether the payoffs are aligned or misaligned, all agents play the adversarial action unless it is iteratively dominant for them to play the friendly action, exactly as in the baseline model.

The optimal policy with a single receiver is thus fundamentally different from the optimal policy with multiple receivers. First, with a single receiver, the optimal policy cannot be implemented with a simple pass/fail announcement. It requires sending multiple (in fact a continuum) of grades. Each grade is associated with a different cut-off $x^*(s)$ such that, given the announced grade s, the receiver plays the friendly action only if $x > x^*(s)$. With multiple receivers, instead, when $p(x|\theta)$ is log-supermodular, as assumed here, the optimal policy is a simple pass/fail test (Theorem 2 in the main text).

Second, observe that, with a single receiver, the optimal policy has the *interval structure*. That is, for any x, the optimal policy induces the receiver to play the friendly action over an interval $(\underline{\pi}(x), \bar{\pi}(x)) \subset \Theta$ of states. With multiple receivers, instead, the optimal policy has the interval structure only when it is monotone (in this case, the interval is $(\underline{\pi}(x), \bar{\pi}(x)) = (\theta^*, +\infty)$ for all x).

We conclude that the structure of the optimal policy with a single (privately informed) receiver is fundamentally different from the one with multiple (privately informed) receivers.

Subsection AM1.2. Multiple Receivers with No Exogenous Private Information

Next, consider an economy with a continuum of investors/receivers, of measure 1, but assume that they do not possess any exogenous private information. All investors share the policy maker's prior

F about θ. As in the main body, denote by $A \in [0,1]$ the measure of investors taking the friendly action and let $u^I(\theta, A)$ denote the representative investor's payoff differential between taking the friendly and the adversarial action when fundamentals are θ and the fraction of investors choosing the friendly action is A.

When investors do not possess exogenous private information, the optimal policy is a monotone binary policy, irrespective of whether the agents' payoffs are aligned or not with the policy maker's payoff. To see this, denote by $\mu_s^\pi \in \Delta(\Theta)$ the common posterior generated by the observation of signal realization $s \in S$ under the policy π. When investors play according to MARP, the only way the policy maker can induce an investor to play the friendly action is to convince him that the friendly action is strictly dominant for him. That is, each investor plays the friendly action at s if, and only if,[2]

$$\int_\Theta u^I(\theta, 0)\, \mu_s^\pi(\mathrm{d}\theta) > 0.$$

As a result, under the adversarial/robust design, the game with multiple receivers who possess no exogenous private information is isomorphic to a game with a single receiver with payoff differential equal to $u^I(\theta, 0)$. That the optimal policy in such a case is monotone follows from Mensch (2018) and Inostroza (2021).

The optimal policy is thus again fundamentally different from the optimal policy for the economy with multiple receivers possessing heterogenous private information.

Section AM2: Micro-foundations and Comparative Statics

In this section, we use the generalizations in Section S4 in the Online Appendix to conduct comparative statics analysis in a family of simple economies in which banks issue equity, or debt, to fund their short-term liquidity obligations, and where the (market-clearing) price of the securities is endogenous and depends on the information revealed through the stress tests. In particular, we investigate the effects of an increase in risk about the bank's fundamentals on the toughness of the optimal stress tests, and show how the latter depend on the type of security issued by the banks.

Subsection AM2.1: Micro-foundations

Consider a representative bank that, at the beginning of period 1, has former liabilities in the amount of D which need to be repaid by the end of the period for the bank to continue operating. The bank has legacy assets that deliver liquid funds $l(\theta) \in \mathbb{R}_+$ at the end of period 1 and, conditional on the bank repaying its period-1 liabilities, a cash flow $C(\theta) \in \mathbb{R}_+$ in period 2. In addition, in case of default, the liquidation of the bank's assets in period 1 delivers an extra cash flow equal to $\gamma(\theta)$,

[2] To see this, note that, because the game is supermodular, when the above inequality is reversed, playing the aggressive action becomes a best response to the conjecture that everyone else plays the aggressive action.

where the functions C and γ are bounded, differentiable, and Lipschitz continuous. Additionally, the bank has outstanding shares whose total amount is normalized to 1.

In order to pay for its former liabilities, the bank can either issue new shares or new short-term debt. We study each of these two cases separately. In both cases, we assume that each potential investor is endowed with 1 unit of capital and has to decide whether to "invest" by purchasing the security issued by the bank, "bet against" the bank by short-selling the security, or do nothing. Depending on the case of interest, the decision to do nothing may correspond to the decision to invest in other securities or, in case of an existing stakeholder, to maintain the existing portfolio. To keep the portfolio decision simple, we assume that each investor is constrained in the position he can take and let that position be normalized to 1. That is, each investor can either buy or sell at most one unit of the security issued (see Albagli et al. (2015) and Brunnermeier and Pedersen (2005) for similar assumptions). We also simplify the analysis by assuming that investors submit market orders. This allows us to abstain from the role of the market as an aggregator of the investors' information which is beyond the scope of the analysis here.

Each investor $i \in [0,1]$ is endowed with an exogenous private signal $x_i = \theta + \sigma\epsilon_i$ of the bank's underlying fundamentals θ, with the noise ϵ_i drawn independently across investors (and independently of θ) from a log-concave distribution. For simplicity, assume here that $supp[P(\cdot|\theta)] = \mathbb{R}$ for all θ, in which case $\varrho_\theta = \mathbb{R}$ for all θ. We also assume that the policy maker confines attention to monotone policies, which, by virtue of Theorem S4-1 in the Online Appendix, is without loss of optimality provided that the agents' expected payoffs differentials $u(\theta, 1 - P(x|\theta))$ between purchasing and selling the bank's securities are log-supermodular over $\left\{(\theta, x) \in [\underline{\theta}, \bar{\theta}] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\right\}$ and the policy maker's payoff satisfies Condition S4-PC in the Online Appendix and part 3 of Condition M in the main text.

Finally, to simplify the exposition, we assume that θ is drawn from an improper uniform prior over $\mathbb{R}$. This assumption is inconsequential to our results. The agents' hierarchies of beliefs over θ (and hence their expected payoffs) are well defined despite the improperness of the prior. Furthermore, given the focus on monotone rules, optimal policies are also well defined (a monotone policy is optimal if and only if its threshold θ^* satisfies Condition (3) in the main text and such a condition is well defined despite the improperness of the prior).

Equity issuances

The bank issues $q > 1$ new shares at a price p which is determined in equilibrium. After observing their private signals, all investors simultaneously decide whether to submit a *market order* to purchase one share of the bank ($a_i = 1$), short-sell the bank's equity ($a_i = 0$), or do nothing ($a_i = \emptyset$). Let A denote the amount of investors who decide to purchase the shares. We assume that the aggregate demand for the bank's shares is given by $A + Y_E(p, z)$, where $Y_E(p, z)$ represents additional demand coming from sources exogenous to the model (e.g., a combination of high-frequency traders submitting limit-orders and of short-term liquidity traders submitting market orders). The variable z parametrizes residual uncertainty that may correlate with the bank's fundamentals (e.g., the "amount" of liquidity traders, and/or the short-term value the high-frequency traders derive from purchasing the shares). We assume that $Y_E(\cdot,\cdot)$ is a non-increasing function of the price of the bank's shares, p, and a non-decreasing function of z.

Investors are risk-neutral. Along with the fact that investors submit market orders and face constraints on their positions, this last assumption implies that doing nothing is dominated by either purchasing or short-selling a share.[3] Because each investor who does not purchase a share, short-sells one, the total supply of shares is thus given by $1-A+q$, where $1-A$ is the amount of shares shorted by the investors. It follows that the equilibrium price of the shares, $p_E^\star(A,z)$, is implicitly determined by the market-clearing condition

$$1-A+q = A + Y_E(p,z)\,. \tag{AM1}$$

Given the monotonicities of Y_E, $p_E^\star(A,z)$ is increasing in A and in z, and decreasing in q. Hereafter, we assume that a solution to (AM1) exists for any (z,A) and is bounded over $(A,z)\in[0,1]\times[\underline{z},\bar{z}]$.

The bank avoids default as long as the proceeds from the period-1 equity issuance are sufficient to cover the bank's liabilities D, that is, if and only if,

$$R_E(\theta,A,z) \equiv l(\theta) + \rho_S q p_E^\star(A,z) - D > 0,$$

where ρ_S is the short-term return on the cash $qp_E^\star$ collected through the equity issuance.

The investors' payoff differential (between buying and short-selling a share) in case of no default is then equal to

$$\hat{g}_E(\theta,A,z) \equiv 2\left(\frac{C(\theta)+\rho_L\left(l(\theta)+\rho_S q p_E^\star(A,z)-D\right)}{1+q} - p_E^\star(A,z)\right)$$

whereas the payoff differential in case of default is equal to

$$\hat{b}_E(\theta,A,z) \equiv -2p_E^\star(A,z)\,,$$

where ρ_L is the long-term return on the extra cash $l(\theta)+\rho_S q p_E^\star - D$ available to the bank at the end of period 1, after the bank pays its liabilities D. Note that, in writing $\hat{g}_E$ and $\hat{b}_E$, we used the fact that, in case of no default, the long-term equilibrium price of equity is equal to to the long-term cash flow $C(\theta)$ augmented by the long-term return on the funds $l(\theta)+\rho_S q p_E^\star - D$ invested at the end of period 1, divided by the amount of outstanding shares, $1+q$. In case of default, instead, the long-term price of equity is equal to zero.[4] The payoff from short-selling the bank's shares is equal to

[3] Whether an investor sells a share he already owns or short-sells one that he borrows makes no difference in this setting. For simplicity, hereafter we focus on the case of short-selling.

[4] That, in case of default, the long-term price of equity is equal to zero reflects the fact that equity is junior to all other existing claims and the assets' liquidation value $\gamma(\theta)$ is small and hence insufficient to provide any funds to the equity holders.

the negative of the payoff from purchasing the shares and, therefore, the payoff differential between the two actions is equal to twice the payoff from purchasing the shares.

This economy is thus a special case of the general model of Section S4 in the Online Appendix, with the agents' expected payoff differential taking the form of

$$u_E(\theta, A) \equiv -2\int_{\underline{z}}^{\bar{z}} p_E^\star(A,z)\,\mathrm{d}Q_\theta(z) + 2\int_{\hat{z}_E(\theta,A)}^{\bar{z}} \left[\frac{C(\theta)+\rho_L\big(l(\theta)+\rho_S q p_E^\star(A,z)-D\big)}{1+q}\right]\mathrm{d}Q_\theta(z) \qquad \text{(AM2)}$$

with $\hat{z}_E(\theta, A)$ denoting the critical level of z below which the bank defaults.[5] Provided that $u_E(\theta, A)$ is non-decreasing in A, and that, for any x, $u_E(\theta, 1 - P(x|\theta))$ has the single-crossing property, all the conclusions from Section S4 in the Online Appendix apply.[6]

Debt issuances

Next, consider the case of debt issuances. The bank issues $q > 1$ bonds at the beginning of period 1. Each bond is a contract that specifies a payment of F_D in period 2, in case the bank does not default, and covenants L_D that discipline the way the proceeds from liquidation will be divided between old and new debt-holders in case of default. Investors either purchase ($a_i = 1$) or short-sell ($a_i = 0$) one unit of the bond by submitting a market order.[7]

Letting A denote the fraction of investors purchasing the bond and p its price, we then have that the total demand for the bond is equal to $A+Y_D(p,z)$, where $Y_D(p,z)$ represents the exogenous (net) demand for the bonds by high-frequency and noisy traders. As in the case of equity issuances, Y_D is assumed to be non-increasing in p and non-decreasing in z. We also assume that, for all $z \in [\underline{z}, \bar{z}]$, if $p > F_D$, then $Y_D(p,z) < 0$, which guarantees that the equilibrium price of debt, $p_D^\star$, is smaller than its face value F_D.

As in the case of equity issuances, the bank avoids default as long as its liquid funds at the end of period 1, $l(\theta) + \rho_S q p_D^\star$, exceed the amount of former liabilities, D. Formally, default is avoided if, and only if,

$$R_D(\theta, A, z) \equiv l(\theta) + \rho_S q p_D^\star(A, z) - D > 0,$$

[5] Formally, $\hat{z}_E(\theta, A)$ is implicitly defined by the solution to $l(\theta) + \rho_S q p_E^\star(A, z) = D$ whenever the equation has a solution, is equal to $\underline{z}$ when $l(\theta) + \rho_S q p_E^\star(A, \underline{z}) > D$, and is equal to $\bar{z}$ when $l(\theta) + \rho_S q p_E^\star(A, \bar{z}) < D$.

[6] Note that the first term in (AM2) is decreasing in A, as $p_E^\star(A, z)$ is increasing in A, for any z. However, the second term is increasing in A (the integrand function is increasing in A and the threshold $\hat{z}(\theta, A)$ is decreasing in A). Hence, provided that the effects from the second term prevail, $u_E(\theta, A)$ is increasing in A. When θ and z are independent, the first term is invariant in θ whereas the second term is increasing in θ. When, instead θ and z are positively correlated, the first term may be decreasing in θ (this is because a higher a θ implies a FOSD shift in the distribution of z, i.e., $Q_\theta(z)$ is weakly decreasing in θ, for any z). However, provided that the dependence of z on θ is small, $u_E(\theta, A)$ is increasing in θ. Clearly $u_E(\theta, A)$ being monotone in (θ, A) suffices for $u_E(\theta, 1 - P(x|\theta))$ to have the single-crossing property, but is not necessary.

[7] Investors may also do nothing ($a_i = \emptyset$). As in the case of equity issuances, such a decision is dominated by either purchasing or short-selling one unit of the bond. The arguments are the same as with equity issuances.

where the equilibrium price for the newly issued bonds $p_D^\star(A,z)$ is implicitly given by the market-clearing condition

$$q+1-A=A+Y_D\left(p_D^\star,z\right). \tag{AM3}$$

As in the case of equity issuances, we assume that a solution to (AM3) exists for any (z,A) and is bounded over $(z,A)\in[\underline{z},\bar{z}]\times[0,1]$.

The payoff differential between purchasing the bond versus short-selling it is then equal to

$$\hat{g}_D\left(\theta,A,z\right)\equiv 2\left(\min\left\{F_D,\frac{C\left(\theta\right)+\rho_L\left(l(\theta)+\rho_S q p_D^\star\left(A,z\right)-D\right)}{q}\right\}-p_D^\star\left(A,z\right)\right)$$

in case the bank does not default, and is equal to

$$\hat{b}_D\left(\theta,A,z\right)\equiv 2\left(\frac{L_D}{qL_D+D}\left(\gamma\left(\theta\right)+l(\theta)+\rho_S q p_D^\star\left(A,z\right)\right)-p_D^\star\left(A,z\right)\right)$$

in case of default. That is, in case the bank is able to repay its short-term liabilities, investors that purchased the bond receive in period 2 the minimum between the bond's face value, F_D, and the bank's period-2 net cash-flows, $C\left(\theta\right)+\rho_L\left(l(\theta)+\rho_S q p_D^\star\left(A,z\right)-D\right)$, divided by the amount of bonds issued, q. If, instead, the bank is unable to repay its short-term liabilities, and hence defaults, the amount that each debt-holder receives is equal to a fraction $qL_D/[qL_D+D]$ of the total cash $l(\theta)+\rho_S q p_D^\star$ available at the end of period 1, augmented by the additional funds $\gamma(\theta)$ obtained by liquidating the bank's assets, and divided by the amount of bonds issued, q. In other words, the available cash is divided between old and new debt holders in a pro-rated manner. Hereafter, we assume that, in case the bank does not default, the amount of cash $C(\theta)$ generated by the bank's legacy asset in period 2 is sufficiently large to cover the bond's face value F_D for all (θ,A,z).

This economy too is thus a special case of the general model of Section S4 in the Online Appendix, with the agents' expected payoff differential between purchasing and short-selling the bond taking the form of

$$\begin{aligned}u_D(\theta,A)=&-2\textstyle\int_{\underline{z}}^{\bar{z}}p_D^\star\left(A,z\right)\mathrm{d}Q_\theta(z)+2F_D\int_{\hat{z}_D(\theta,A)}^{\bar{z}}\mathrm{d}Q_\theta(z)\\&+2\textstyle\int_{\underline{z}}^{\hat{z}_D(\theta,A)}\frac{L_D}{qL_D+D}\left(\gamma\left(\theta\right)+l(\theta)+\rho_S q p_D^\star\left(A,z\right)\right)\mathrm{d}Q_\theta(z)\end{aligned} \tag{AM4}$$

where $\hat{z}_D(\theta,A)$ is the critical level below which the bank defaults, defined in the same way as in the case of equity issuances. Provided that $u_D(\theta,A)$ is non-decreasing in A and that, for any x, $u_D\left(\theta,1-P(x|\theta)\right)$ has the single-crossing property, all the conclusions from Section S4 in the Online Appendix apply.[8]

[8] As in the case of equity issuances, the properties that $u_D(\theta,A)$ is non-decreasing in A and $u_D\left(\theta,1-P(x|\theta)\right)$ has the single-crossing property may be consistent with the first term in (AM4) being decreasing in A.

Subsection AM2.2: Effects of market uncertainty on toughness of optimal stress tests

In this subsection, we show how the toughness of the optimal stress tests is affected by an increase in risk about the bank's fundamentals. Specifically, we use the two micro-foundations in the previous subsection to investigate how an increase in risk (formally captured by an increase in the parameter σ scaling the noise in the agents' signals $x_i = \theta + \sigma\epsilon_i$) affects the critical threshold θ^* below which the policy maker fails the bank under examination.

Let $\theta^*_E(\sigma)$ and $\theta^*_D(\sigma)$ denote the thresholds characterizing the optimal monotone policies when the precision of the agents' exogenous information is σ^{-2} and the bank funds itself with equity and debt, respectively. Also let θ^{MS}_E and θ^{MS}_D be the Laplacian thresholds for the two economies under consideration (defined by $\int_0^1 u_h(\theta^{MS}_h, A)dA = 0$, $h = E, D$) and recall that, in the absence of any public information disclosure, when $\sigma \to 0^+$, purchasing (alternatively, short-selling) security h is the unique rationalizable action for $x > \theta^{MS}_h$ (alternatively, for $x < \theta^{MS}_h$). Lastly, let $\theta^{\triangle}_h$ be defined by the solution to $u_h(\theta^{\triangle}_h, 1/2) = 0$, $h = D, E$, and note that the threshold $\theta^{\triangle}_h$ is the critical value of the fundamentals at which an investor who knows the fundamentals and expects 1/2 of the investors to purchase security h and 1/2 to short-sell it is indifferent between purchasing and short-selling the security, for $h = E, D$. Hereafter, we assume that the problem the policy maker faces is "severe" in the sense that $\theta^{MS}_h \geq \theta^{\triangle}_h$, $h = E, D$.

We then have the following result:

Proposition AM2-1. *Suppose that (a) θ and z are independent, (b) $\rho_S = 1$, (c) $x_i = \theta + \sigma\epsilon_i$, with ϵ_i drawn from a standard Normal distribution, independently of (θ, z), (d) there exists $l \in \mathbb{R}_+$ such that $l(\theta) = l$ for all θ, and (e) $\gamma(\theta)$ and $C(\theta)$ are strictly increasing. There exists $\sigma^\dagger > 0$ such that, for any $\sigma, \sigma' \in (0, \sigma^\dagger]$, with $\sigma' > \sigma$, $\theta^*_E(\sigma') < \theta^*_E(\sigma)$ and $\theta^*_D(\sigma') > \theta^*_D(\sigma)$.*

The result in the proposition says the following. Take an economy in which the precision of the agents' exogenous information is sufficiently high (that is, σ is small) and consider the effects of an increase in risk (formally captured by the transition to $\sigma' > \sigma$) on the toughness of the optimal stress test (formally captured by the threshold θ^*_h below which the policy maker fails the bank, $h = E, D$). More risk leads to a reduction in the toughness of the optimal stress test when the bank finances itself with equity and to an increase in the toughness of the optimal stress test when the bank finances itself with debt.

Intuitively, the reason why, under the assumed specification, risk is beneficial to the bank in case of equity financing but not in case of debt financing is the following. Under equity financing, investors are exposed to variations in fundamentals primarily through upside risk. Their payoff differential (between purchasing and short-selling equity) is increasing in θ in case of no default and is equal to $-2p^\star_E$ in case of default. Provided that the price of equity does not vary much with (θ, z), which is the case under the assumed specification, an increase in risk then makes investors more willing to purchase equity. The policy maker can then decrease the critical threshold θ_E^* below which she fails the bank while guaranteeing that, after announcing that the bank passed the test, the unique rationalizable profile continues to feature all investors pledging by purchasing equity.

Under debt financing, instead, investors are exposed to variations in fundamentals primarily through downside risk. When the liquidation value $\gamma(\theta)$ is increasing in θ and $p_D^\star$ is not very sensitive to (θ, z), the investors' payoff differential (between purchasing and short-selling debt) is increasing in θ in case of default but constant in fundamentals in case the bank survives. An increase in risk then makes investors less willing to pledge. The policy maker must then increase the critical threshold θ_D^* below which she fails the bank if she wants to guarantee that, after announcing that the bank passed the test, the unique rationalizable profile features all investors pledging by purchasing the newly issued debt.

That risk is beneficial to the bank in case of equity financing but detrimental in case of debt financing need not extend to alternative specifications of the investors' payoffs under the two securities. What appears to be true more generally is the following single-crossing property. Whenever more risk is beneficial to the bank in case of debt financing, the same tends to be true under equity financing.[9]

Proof of Proposition AM2-1 . Given any threshold $\hat{\theta}$, and any signal x, let

$$\psi_h(x, \hat{\theta}, \sigma) \equiv \int_\Theta u_h\left(\theta, 1 - \Phi\left(\frac{x-\theta}{\sigma}\right)\right) \mathrm{d}\Lambda\left(\theta | x, 1; \sigma\right)$$

denote the payoff of an agent with signal x, of precision σ^{-2}, who, after hearing that the bank passed the test, learns that $\theta > \hat{\theta}$, and who expects all other agents to buy security h when their signal exceeds x and short-sell it otherwise, with $h = D$ in case the bank finances itself with debt, and $h = E$ in the case the bank finances itself with equity. Here $\Lambda(\cdot|x, 1; \sigma)$ represents the posterior belief over Θ for an agent with exogenous signal x of precision σ^{-2} who learns that the bank passed the test (and hence that $\theta > \hat{\theta}$).

We start with the following result:

Lemma AM2-1. *For any $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$, any $x > \hat{\theta}$,*

$$\lim_{\sigma \to 0^+} \frac{\partial}{\partial \sigma} \psi_E(x, \hat{\theta}, \sigma) > 0 > \lim_{\sigma \to 0^+} \frac{\partial}{\partial \sigma} \psi_D(x, \hat{\theta}, \sigma).$$

[9] Clearly, for this single-crossing result to hold, one needs to make sure that the two cases are comparable. This requires, among other things, that the exogenous demand for the bank's securities is the same in the two cases, i.e., that $Y_D(p, z) = Y_E(p, z)$, for any (p, z).

Proof of Lemma AM2-1. Note that, for any $x > \hat{\theta}$,

$$\begin{aligned}
\tfrac{\partial}{\partial\sigma}\psi_h(x,\hat{\theta},\sigma) &= \tfrac{\partial}{\partial\sigma}\int_{\hat{\theta}}^{\infty} u_h\left(\theta, 1-\Phi\left(\tfrac{x-\theta}{\sigma}\right)\right)\frac{\phi\left(\frac{x-\theta}{\sigma}\right)}{\sigma\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\mathrm{d}\theta \\
&= \tfrac{\partial}{\partial\sigma}\left\{\frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\int_{1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}^{1} u_h\left(x-\sigma\Phi^{-1}\left(1-A\right),A\right)\mathrm{d}A\right\} \\
&= \frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\int_{1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}^{1}\frac{\partial u_h\left(x-\sigma\Phi^{-1}(1-A),A\right)}{\partial\theta}\left(-\Phi^{-1}\left(1-A\right)\right)\mathrm{d}A \\
&+\frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\left[\psi\left(x,\hat{\theta},\sigma\right)-u_h\left(\hat{\theta},1-\Phi\left(\tfrac{x-\hat{\theta}}{\sigma}\right)\right)\right]\phi\left(\tfrac{x-\hat{\theta}}{\sigma}\right)\left(\tfrac{x-\hat{\theta}}{\sigma^2}\right).
\end{aligned}\tag{AM5}$$

The first equality follows from the change in variables, $A = 1-\Phi\left(\left(x-\theta\right)/\sigma\right)$, whereas the second equality follows from the chain rule of differentiation.

The proof proceeds in two steps. Step 1 shows that, for any $x > \hat{\theta}$, when $\sigma \to 0^+$, the second term in the right-hand side of the last equality in (AM5) vanishes. Step 2 shows that, for any $x > \hat{\theta}$, when $\sigma \to 0^+$, the first term in the right-hand side of the last equality in (AM5) is positive for equity but negative for debt.

Step 1. Because $g_h(\cdot)$ and $b_h(\cdot)$ are bounded, for any σ, $\psi_h\left(x,\hat{\theta},\sigma\right) - u_h\left(\hat{\theta},1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)\right)$ is also bounded. Furthermore, for any $x > \hat{\theta}$, and any σ, $\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right) \in [1/2, 1)$. Finally, use L'Hopital's rule to observe that, for any $x > \hat{\theta}$, $\lim_{\sigma\to 0^+}\phi\left(\frac{x-\hat{\theta}}{\sigma}\right)\left(\frac{x-\hat{\theta}}{\sigma^2}\right) = 0$. Jointly, the above properties imply that, for any $x > \hat{\theta}$, the second term in the right-hand side of the last equality in (AM5) vanishes as $\sigma \to 0^+$.

Step 2. Because z and θ are independent,

$$u_h\left(\theta,A\right) = \int_{\underline{z}}^{\hat{z}_h(A)}\hat{b}_h\left(\theta,A,z\right)\mathrm{d}Q\left(z\right) + \int_{\hat{z}_h(A)}^{\bar{z}}\hat{g}_h\left(\theta,A,z\right)\mathrm{d}Q\left(z\right),$$

where $\hat{z}_h\left(A\right)$ is a shortcut for $\hat{z}_h(\theta,A)$ and is independent of θ because $l(\theta)$ is invariant in θ.[10] This means that

$$\frac{\partial u_h\left(\theta,A\right)}{\partial\theta} = \int_{\underline{z}}^{\hat{z}_h(A)}\frac{\partial\hat{b}_h\left(\theta,A,z\right)}{\partial\theta}\mathrm{d}Q\left(z\right) + \int_{\hat{z}_h(A)}^{\bar{z}}\frac{\partial\hat{g}_h\left(\theta,A,z\right)}{\partial\theta}\mathrm{d}Q\left(z\right)$$

where, for $h = E$,[11] $\partial\hat{b}_E\left(\theta,A,z\right)/\partial\theta = 0$ and $\partial\hat{g}_E\left(\theta,A,z\right)/\partial\theta = C'\left(\theta\right)/(1+q)$, implying that

$$\frac{\partial u_E\left(\theta,A\right)}{\partial\theta} = \frac{C'\left(\theta\right)}{1+q}\left(1-Q\left(\hat{z}_E\left(A\right)\right)\right),$$

whereas, for $h = D$, $\partial\hat{b}_D\left(\theta,A,z\right)/\partial\theta = L_D\gamma'(\theta)/(qL_D+D)$ and $\partial\hat{g}_D\left(\theta,A,z\right)/\partial\theta = 0$, implying that

$$\frac{\partial u_D\left(\theta,A\right)}{\partial\theta} = \frac{L_D}{qL_D+D}\gamma'\left(\theta\right)Q\left(\hat{z}_D\left(A\right)\right).$$

[10] Recall that $\hat{z}_h(\theta,A)$ is implicitly defined by the solution to $l(\theta)+\rho_S qp_h^{\star}\left(A,z\right) = D$ whenever the equation has a solution, is equal to $\underline{z}$ when $l(\theta)+\rho_S qp_h^{\star}\left(A,\underline{z}\right) > D$, and is equal to $\bar{z}$ when $l(\theta)+\rho_S qp_h^{\star}\left(A,\bar{z}\right) < D$.

[11] Note that we used the assumption that $\rho_S = 1$.

Hence, the above results imply that, for any $x > \hat{\theta}$,

$$\begin{aligned}\lim_{\sigma\to 0^+}\frac{\partial}{\partial\sigma}\psi_h(x,\hat{\theta},\sigma) &= \lim_{\sigma\to 0^+}\frac{1}{\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\int_{1-\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}^{1}\frac{\partial u_h\left(x-\sigma\Phi^{-1}(1-A),A\right)}{\partial\theta}\left(-\Phi^{-1}\left(1-A\right)\right)\mathrm{d}A \\ &= \int_0^1\frac{\partial u_h(x,A)}{\partial\theta}\left(-\Phi^{-1}\left(1-A\right)\right)\mathrm{d}A\end{aligned}$$

where the second equality follows from Lebesgue dominated convergence theorem (that $|\partial u_h\left(\theta,A\right)/\partial\theta|$ is uniformly bounded follows from the derivations above along with the fact that C and γ are Lipschitz continuous).

Next, use the change in variables $\omega = -\Phi^{-1}\left(1-A\right)$ and the fact that, for any x, $\phi(x) = \phi(-x)$, to note that

$$\int_0^1\left(-\Phi^{-1}\left(1-A\right)\right)\mathrm{d}A = \int_{-\infty}^{\infty}\omega\phi\left(\omega\right)\mathrm{d}\omega = 0.$$

The last property implies that

$$\lim_{\sigma\to 0^+}\frac{\partial}{\partial\sigma}\psi_h(x,\hat{\theta},\sigma) = \int_0^1\frac{\partial u_h\left(x,A\right)}{\partial\theta}\left(-\Phi^{-1}\left(1-A\right)\right)\mathrm{d}A = \mathrm{cov}\left(\frac{\partial u_h\left(x,A\right)}{\partial\theta},-\Phi^{-1}\left(1-A\right)\right),$$

where, for any pair of random variables X and Y, $\mathrm{cov}\left(X,Y\right)$ denotes the covariance between the two variables.

Equity. Using the properties above, we have that, for any $x > \hat{\theta}$,

$$\mathrm{cov}\left(\frac{\partial u_E(x,A)}{\partial\theta},-\Phi^{-1}\left(1-A\right)\right) = \frac{C'(x)}{1+q}\mathrm{cov}\left(1-Q\left(\hat{z}_E\left(A\right)\right),-\Phi^{-1}\left(1-A\right)\right) > 0,$$

where the inequality follows from the fact that $\hat{z}_E\left(A\right)$ is decreasing in A.

Debt. Using the properties above, we have that, for any $x > \hat{\theta}$,

$$\mathrm{cov}\left(\frac{\partial u_D(x,A)}{\partial\theta},-\Phi^{-1}\left(1-A\right)\right) = \frac{L_D}{qL_D+D}\gamma'\left(\theta\right)\mathrm{cov}\left(Q\left(\hat{z_D}\left(A\right)\right),-\Phi^{-1}\left(1-A\right)\right) < 0,$$

where the inequality follows again from the fact that $\hat{z}_D\left(A\right)$ is decreasing in A.

The lemma follows from combining the results from Step 1 with those from Step 2. ■

Now observe that, for any precision of private information σ^{-2} and any monotone pass/fail policy with threshold $\hat{\theta}$, after the pass grade $s = 1$ is announced, purchasing security h is the unique rationalizable action for all investors if, and only if, $\psi_h\left(x,\hat{\theta},\sigma\right) > 0$ for all $x \in \mathbb{R}$ (the arguments are analogous to the ones in the proof of Theorem 2 in the main text). From the discussion following Theorem 3 in the main text, then observe that the threshold $\theta_h^*(\sigma)$ defining the optimal monotone policy when the precision of the investors' information is σ^{-2} and the bank funds itself by issuing security $h = D, E$ is given by $\theta_h^*\left(\sigma\right) = \inf\left\{\hat{\theta} : \psi_h\left(x,\hat{\theta},\sigma\right) \geq 0 \text{ for all } x \in \mathbb{R}\right\}$.

Next, for any $\sigma > 0$, let $x_h^*\left(\sigma\right) \equiv \arg\min_{x\in\mathbb{R}}\ \psi_h\left(x,\theta_h^*\left(\sigma\right),\sigma\right)$ and note that $x_h^*\left(\sigma\right)$ is a solution to the equation $\psi_h\left(x_h^*\left(\sigma\right),\theta_h^*\left(\sigma\right),\sigma\right) = 0$. Next, for any $\hat{\theta} \in (\underline{\theta},\bar{\theta})$, any $\tilde{\sigma} > 0$, let $\Psi_h(\hat{\theta},\tilde{\sigma}) \equiv \inf_{x\in\mathbb{R}}\ \psi_h(x,\hat{\theta},\tilde{\sigma})$ and, for any $(\hat{\theta},\tilde{\sigma})$ such that

$$\arg\min_{x\in\mathbb{R}}\ \psi_h(x,\hat{\theta},\tilde{\sigma}) \neq \emptyset,$$

let

$$x_h^{**}(\hat{\theta},\tilde{\sigma}) \in \arg\min_{x\in\mathbb{R}} \psi_h(x,\hat{\theta},\tilde{\sigma}).$$

Note that, when $\hat{\theta} = \theta_h^*(\sigma)$ and $\tilde{\sigma} = \sigma$, $x_h^{**}(\theta_h^*(\sigma),\sigma) = x_h^*(\sigma)$.

Now observe that $\lim_{\sigma\to 0^+} \theta_h^*(\sigma) = \theta_h^{MS}$, $h = E, D$. The definition of $\theta_h^{\triangle}$, along with the strict monotonicity of $u_h(\theta, 1/2)$ in θ, imply that, for any $\theta > \theta_h^{\triangle}$, $u_h(\theta, 1/2) > 0$. Hence, for any $\hat{\theta} > \theta_h^{\triangle}$, $\sigma > 0$, and $x \leq \hat{\theta}$,

$$\psi_h\left(x,\hat{\theta},\sigma\right) = \int_{\hat{\theta}}^{\infty} u_h\left(\theta, 1-\Phi\left(\frac{x-\theta}{\sigma}\right)\right)\frac{\phi\left(\frac{x-\theta}{\sigma}\right)}{\sigma\Phi\left(\frac{x-\hat{\theta}}{\sigma}\right)}\mathrm{d}\theta > 0. \tag{AM6}$$

The assumption that $\theta_h^{MS} > \theta_h^{\triangle}$, $h = E, D$, along with the continuity of $\theta_h^*(\sigma)$ in σ,[12] then imply that there exist $\hat{\sigma} > 0$ such that, for any $\sigma \in (0,\hat{\sigma})$, $\theta_h^*(\sigma) > \theta_h^{\triangle}$, for $h = E, D$. The result in (AM6) then implies that, for any $\sigma, \tilde{\sigma} \in (0,\hat{\sigma})$, $x_h^*(\sigma), x_h^{**}(\theta_h^*(\sigma),\tilde{\sigma}) > \theta_h^*(\sigma)$.

Now observe that the arguments in the proof of Lemma AM2-1 above imply that there exists $\sigma^{\#}, K > 0$ such that, for any $\sigma, \tilde{\sigma} \in (0,\sigma^{\#})$, with $\tilde{\sigma} \geq \sigma$, and any $x > \theta_h^*(\sigma)$, $\psi_h(x,\theta_h^*(\sigma),\tilde{\sigma})$ is partially differentiable in its third argument, $\tilde{\sigma}$, with the partial derivative continuous in $(x,\hat{\theta},\tilde{\sigma})$ and uniformly bounded over

$$\left\{(\sigma,\tilde{\sigma},x) \in (0,\sigma^{\#})\times(0,\sigma^{\#})\times\mathbb{R} : \tilde{\sigma} \geq \sigma,\ x-\theta_h^*(\sigma) \in (0,K)\right\}, \quad h = E, D.$$

Also observe that, for any $\varepsilon > 0$, there exists σ_ε such that, for any $\sigma, \tilde{\sigma} \in (0,\sigma_\varepsilon)$,[13]

$$|\theta_h^*(\sigma) - \theta_h^{MS}|, |x_h^*(\sigma) - \theta_h^{MS}|, |x_h^{**}(\theta_h^*(\sigma),\tilde{\sigma}) - \theta_h^{MS}| < \varepsilon. \tag{AM7}$$

Now let $\bar{\sigma} \equiv \min\{\sigma^{\#}, \hat{\sigma}, \sigma_{K/2}\}$. The properties above, along with the envelope theorem of Milgrom and Segal (2002), imply that, for any $\sigma, \tilde{\sigma} \in (0,\bar{\sigma})$, with $\tilde{\sigma} \geq \sigma$, and $h = E, D$,

$$\frac{\partial}{\partial\tilde{\sigma}}\Psi_h(\theta_h^*(\sigma),\tilde{\sigma}) = \frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right),\theta_h^*(\sigma),\tilde{\sigma}\right).$$

The last property, along with the fact that $\Psi_h(\theta_h^*(\sigma),\sigma) = 0$, imply that, for any $\sigma, \sigma' \in (0,\bar{\sigma})$, with $\sigma' > \sigma$,

$$\Psi_h(\theta_h^*(\sigma),\sigma') = \int_{\tilde{\sigma}=\sigma}^{\tilde{\sigma}=\sigma'} \frac{\partial}{\partial\tilde{\sigma}}\Psi_h(\theta_h^*(\sigma),\tilde{\sigma})\mathrm{d}\tilde{\sigma} = \int_{\tilde{\sigma}=\sigma}^{\tilde{\sigma}=\sigma'} \frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right),\theta_h^*(\sigma),\tilde{\sigma}\right)\mathrm{d}\tilde{\sigma}.$$

The continuity of $\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x,\hat{\theta},\tilde{\sigma}\right)$ in $(x,\hat{\theta},\tilde{\sigma})$ around $\left(\theta_h^{MS},\theta_h^{MS},0\right)$, along with Condition (AM7), imply that there exists $\sigma^{\dagger} \in (0,\bar{\sigma})$ such that, for any $\sigma, \sigma' \in (0,\sigma^{\dagger})$,

$$\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^{**}\left(\theta_h^*(\sigma),\tilde{\sigma}\right),\theta_h^*(\sigma),\tilde{\sigma}\right) \overset{sgn}{=} \lim_{\sigma\to 0^+}\left.\frac{\partial}{\partial\tilde{\sigma}}\psi_h\left(x_h^*(\sigma),\theta_h^*(\sigma),\tilde{\sigma}\right)\right|_{\tilde{\sigma}=\sigma}.$$

[12] The continuity of $\theta_h^*(\sigma)$ in σ in turn follows from the fact that $\theta_h^*(\sigma)$ and $x_h^*(\sigma)$ are such that $\psi_h(x_h^*(\sigma),\theta_h^*(\sigma),\sigma) = 0$, along with the continuity of $\psi_h\left(x,\hat{\theta},\sigma\right)$ in $\left(x,\hat{\theta},\sigma\right)$ and the strict monotonicity of $\psi_h\left(x,\hat{\theta},\sigma\right)$ in $\hat{\theta}$ at any $\left(x,\hat{\theta},\sigma\right)$ such that $\psi_h\left(x,\hat{\theta},\sigma\right) = 0$.

[13] The arguments for this claim are similar to those in other global-games papers and omitted for brevity.

The above properties, along with Lemma AM3-1 above, thus imply that, for any $\sigma, \sigma' \in (0, \sigma^\dagger)$, with $\sigma' > \sigma$,

$$\Psi_E(\theta_E^*(\sigma), \sigma') > 0 > \Psi_D(\theta_D^*(\sigma), \sigma').$$

The result in the proposition then follows from the above conclusions along with the monotonicity of $\Psi_h(\cdot, \sigma')$ in the truncation point θ_h^*, $h = E, D$. Q.E.D.

Section AM3: Generalization of Perfect Coordination Property

Consider the following amendment of the model in Section 2 in the main text.

Agents and exogenous information. Let N denote the set of agents; N is assumed to be measurable and can be finite or infinite. For each $i \in N$, let X_i denote a measurable set and define $\mathcal{X} \equiv \prod_{i \in N} X_i$. The set $\mathcal{X}$ is endowed with the product topology. For each $i \in N$, let $\Lambda_i : X_i \to \Delta(\Theta \times \mathcal{X})$ be a measurable function (with respect to the Borel sigma-algebra associated with X_i). The profile $\mathbf{x} = (x_i)_{i \in N} \in \mathcal{X}$ indexes the hierarchy of the agents' exogenous beliefs about θ and the beliefs of other agents.

The state of Nature in this environment is denoted by $\omega = (\theta, \mathbf{x}) \in \Omega \equiv \Theta \times \mathcal{X}$ and comprises the realization of the payoff fundamental θ and the exogenous profile of the agents' beliefs $\mathbf{x}$. Note that no restriction on the agents' belief profile $\mathbf{x}$ is imposed. In particular, the agents' beliefs need not be consistent with a common prior, nor be generated by signals drawn independently conditionally on θ.

Payoffs. Each agent's expected payoff differential (between pledging and not pledging) is given by

$$u_i(\theta, A) = \begin{cases} g_i(\theta, A) & \text{if } r = 1 \\ b_i(\theta, A) & \text{if } r = 0, \end{cases}$$

$i \in N$, where A denotes the aggregate size of the pledge (in case of finitely many agents, A coincides with the number of agents pledging). The functions g_i and b_i are continuously differentiable and satisfy the same monotonicity assumptions as in the main text. In other words, for any $i \in N$, any (θ, A): (a) $\frac{\partial}{\partial \theta} g_i(\theta, A), \frac{\partial}{\partial \theta} b_i(\theta, A) \geq 0$, (b) $\frac{\partial}{\partial A} g_i(\theta, A), \frac{\partial}{\partial A} b_i(\theta, A) \geq 0$; and (c) $g_i(\theta, A) > 0 > b_i(\theta, A)$. Default occurs if and only if $R(\theta, A) \leq 0$, where R is increasing in (θ, A).

For simplicity, and to better highlight the novel effects, we abstract from the possibility that the regime outcome (i.e., default), as well as the agents' payoffs, may depend on variables z only imperfectly correlated with θ. As explained in Section S4 in the Online Appendix (see the discussion around Theorem S4-1), the possibility of increasing the agents' expected payoffs while coordinating them on the same course of action extends to economies in which the regime outcome is a stochastic function of (θ, A). The optimality of policies satisfying the perfect coordination property also extends to these more general economies provided the distribution from which the agents' signals are drawn satisfies the MLRP property and the planner's payoff satisfies Condition S4-PC in Section S4 in the Online Appendix.

Disclosure Policies. Let $\mathcal{S}$ be a compact metric space defining the set of possible disclosures to the agents. Let $m : N \to \mathcal{S}$ denote a *message function,* specifying, for each individual $i \in N$, the endogenous signal $m_i \in \mathcal{S}$ disclosed to the individual. Let $M(\mathcal{S})$ denote the set of all possible message functions with codomain $\mathcal{S}$. Let $\mathcal{P}$ be a partition of Ω and $h(\omega)$ the information set (equivalently, the cell) in $\mathcal{P}$ containing the state $\omega \in \Omega$. A *disclosure policy* $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ consists of a set $\mathcal{S}$ along with a mapping $\pi : \Omega \to \Delta(M(\mathcal{S}))$ measurable with respect to the σ-algebra defined by the partition $\mathcal{P}$.[14] For each ω, $\pi(\omega)$ denotes the lottery whose realization yields the message function used by the policy maker to communicate with the agents. The case in which the partition $\mathcal{P}$ coincides with Ω corresponds to the case in which the policy maker is able to distinguish any two states in Ω (in this case the σ-algebra associated with Ω is the Borel σ-algebra).

Solution Concept. Agents have a level-K degree of sophistication. The policy maker adopts a conservative approach and evaluates the performance of any given policy on the basis of the "worst outcome" consistent with the agents playing (interim correlated) level-K rationalizable strategies. That is, for any given selected policy Γ, the policy maker expects the market to play according to the "most aggressive level-K rationalizable profile" defined as follows:

Definition AM3-1. Given any policy Γ, any $K \in \mathbb{N} \cup \{\infty\}$, the most aggressive level-K rationalizable profile (MARP-K) associated with Γ is the strategy profile $a^{\Gamma}_{(K)} \equiv (a^{\Gamma}_{(K),i})_{i\in[0,1]}$ that minimizes the policy maker's ex-ante expected payoff, among all profiles surviving K rounds of *iterated deletion of interim strictly dominated strategies.*

Hereafter we use IDISDS to refer to the process of iterated deletion of interim strictly dominated strategies.

Definition AM3-2. A policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ satisfies the **perfect-coordination property (PCP)** if, for any $\omega \in \Omega$, any message function $m \in supp[\pi(\omega)]$, any $i, j \in N$, $a^{\Gamma}_{(K),i}(x_i, m_i) = a^{\Gamma}_{(K),j}(x_j, m_j)$.

Fix an arbitrary policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$. For any $\omega \in \Omega$, any message function $m \in supp[\pi(\omega)]$, let $r(\omega, m; a^{\Gamma}_{(K)}) \in \{0, 1\}$ denote the regime outcome that prevails at ω when the distribution of endogenous signals is m, and agents play according to the strategy profile $a^{\Gamma}_{(K)}$.

Definition AM3-3. The disclosure policy $\Gamma = (\mathcal{S}, \mathcal{P}, \pi)$ is **regular** if for any $\omega', \omega'' \in \Omega$ for which $h(\omega') = h(\omega'')$ and any $m \in supp\ [\pi(\omega')] = supp\ [\pi(\omega'')]$, $r(\omega', m; a^{\Gamma}_{(K)}) = r(\omega'', m; a^{\Gamma}_{(K)})$.

[14]That is, by the collection of $\mathcal{P}$-saturated sets. Let $\mathcal{B}$ be the standard Borel σ-algebra associated with the primitive set Ω. A set $A \in \mathcal{B}$ is $\mathcal{P}$-saturated if $\omega \in A$ implies $h(\omega) \subseteq A$. Thus $A = \cup_{\omega \in A} h(\omega)$.

A disclosure policy is thus regular if the default outcome induced by MARP-K compatible with Γ is measurable with respect to the policy maker's information (as captured by the partition $\mathcal{P}$).[15] With an abuse of notation, when we find it convenient to highlight the measurability restriction implied by the regularity of the policy, we will denote by $r(h(\omega), m; a^{\Gamma}_{(K)}) \in \{0, 1\}$ the regime outcome that prevails *at any state in* $h(\omega)$ under the message function m. Observe that, when the policy maker can perfectly distinguish between any two states, then any policy is regular.

Theorem AM3-1. *For any regular policy* Γ*, there exists another regular policy* Γ^* *satisfying* ***PCP*** *and such that, for any* ω*, the probability of default under* Γ^* *is the same as under* Γ.

Proof of Theorem AM3-1. Let $\mathcal{A}^{\Gamma} \equiv \{(a_i(\cdot) : X_i \times \mathcal{S} \to [0, 1])_{i \in N}\}$ denote the entire set of strategy profiles in the continuation game among the agents that starts with the policy maker announcing the policy Γ. For any $n \in \mathbb{N}$, let $T^{\Gamma}_{(n)}$ denote the set of strategies surviving n rounds of IDISDS under the original policy Γ, with $T^{\Gamma}_{(0)} = \mathcal{A}^{\Gamma}$. Denote by $\bar{a}^{\Gamma}_{(n)} \equiv (\bar{a}^{\Gamma}_{(n),i}(\cdot))_{i \in [0,1]} \in T^{\Gamma}_{(n)}$ the profile in $T^{\Gamma}_{(n)}$ that minimizes the policy maker's ex-ante payoff. Such a profile also minimizes the policy maker's interim payoff, as it will become clear from the arguments below. Hereafter, we refer to the profile $\bar{a}^{\Gamma}_{(n)}$ as the most aggressive profile surviving n rounds of IDISDS. The profiles $(\bar{a}^{\Gamma}_{(n)})_{n \in \mathbb{N}}$ can be constructed inductively as follows. The profile $\bar{a}^{\Gamma}_{(0)} \equiv (\bar{a}^{\Gamma}_{(0),i}(\cdot))_{i \in [0,1]})$ prescribes that all agents refrain from pledging irrespective of their exogenous and endogenous signals; that is, each $\bar{a}^{\Gamma}_{(0),i}(\cdot)$ is such that $\bar{a}^{\Gamma}_{(0),i}(x_i, s) = 0$, for all $(x_i, s) \in X_i \times \mathcal{S}$.[16] Given any strategy profile $a \in \mathcal{A}^{\Gamma}$, any $i \in N$, let $U^{\Gamma}_i(x_i, m_i; a)$ denote the payoff that agent i with exogenous signal x_i and endogenous signal m_i obtains from pledging, when all other agents follow the behavior specified by the strategy profile a. For any $n \geq 1$, the most aggressive strategy profile surviving n rounds of IDISDS is the one specifying, for each agent i, each $(x_i, m_i) \in X_i \times \mathcal{S}$, $\bar{a}^{\Gamma}_{(n),i}(x_i, m_i) = 1$ if $U^{\Gamma}_i(x_i, m_i; \bar{a}^{\Gamma}_{(n-1)}) > 0$ and $\bar{a}^{\Gamma}_{(n),i}(x_i, m_i) = 0$ if $U^{\Gamma}_i(x_i, m_i; \bar{a}^{\Gamma}_{(n-1)}) \leq 0$. The most aggressive level-K rationalizable strategy profile (MARP-K) consistent with the policy Γ is thus the profile $\bar{a}_{(K)}{}^{\Gamma} = (\bar{a}^{\Gamma}_{(K),i}(\cdot))_{i \in N} \in T^{\Gamma}_{K}$. The case of fully rational agents in the main text corresponds to the limit in which $K \to \infty$. To be consistent with the notation in the main text, we denote MARP consistent with Γ by dropping the subscript K and denoting such profile by $\bar{a}^{\Gamma} \equiv ((\bar{a}^{\Gamma}_i(\cdot))_{i \in N}$, with $\bar{a}^{\Gamma}_i(\cdot) \equiv \lim_{\mathrm{K} \to \infty} \bar{a}^{\Gamma}_{(K),i}(\cdot)$, all $i \in N$.

Now, consider the policy $\Gamma^+ = (\mathcal{S}^+, \mathcal{P}, \pi^+)$, $\mathcal{S}^+ = \mathcal{S} \times \{0, 1\}$, obtained from the original policy Γ by replacing each message function $m : N \to \mathcal{S}$ in the support of each $\pi(\omega)$ with the message function $m^+ : N \to \mathcal{S}^+$ that discloses to each agent $i \in N$ the same message m_i disclosed by the original policy m, along with the regime outcome $r(\omega, m; \bar{a}^{\Gamma}_{(K)})$ that would have prevailed at (ω, m) under Γ when all agents play according to the most aggressive level-K rationalizable strategy profile $\bar{a}^{\Gamma}_{(K)}$ consistent with the original policy Γ. That is, for each $\omega \in \Omega$, each $m \in supp[\pi(\omega)]$, the policy Γ^+ selects the message function m^+ obtained from the original message function m by

[15] Note that regularity is violated in the two-state-two-receiver model in Alonso and Zachariadis (2021)

[16] Note that, to ease the notation, we let each individual strategy prescribe an action for all $(x_i, m_i) \in \mathbb{R} \times \mathcal{S}$, including those that may be inconsistent with the policy Γ.

adding to its codomain the regime outcome $r(\omega, m; \bar{a}^{\Gamma}_{(K)})$ that would have prevailed at (ω, m) under MARP-K $\bar{a}^{\Gamma}_{(K)}$, with the same probability that Γ would have selected the original message function m. Hereafter, we denote by $m_i^+ = (m_i, r(\omega, m; \bar{a}^{\Gamma}_{(K)}))$ the message sent to agent i under the new policy Γ^+ when the exogenous state is ω and the message function selected under the original policy Γ is m. Note that the assumption that Γ is regular implies that Γ^+ is measurable with respect to the σ-algebra generated by $\mathcal{P}$ and hence also regular.

Let $\mathcal{A}^{\Gamma^+} \equiv \{(a_i(\cdot) : X_i \times \mathcal{S} \times \{0, 1\}\} \to [0, 1])_{i \in N}\}$ denote the set of strategy profiles in the continuation game among the agents that starts with the policy maker announcing the new policy Γ^+. For any $n \in \mathbb{N}$, let $T^{\Gamma^+}_{(n)} \subset \mathcal{A}^{\Gamma^+}$ denote the set of strategies surviving n rounds of IDISDS under the new policy Γ^+, with $T^{\Gamma^+}_{(0)} = \mathcal{A}^{\Gamma^+}$. Denote by $\bar{a}^{\Gamma^+}_{(n)} \equiv (\bar{a}^{\Gamma^+}_{(n),i}(\cdot))_{i \in N} \in T^{\Gamma^+}_{(n)}$ the profile in $T^{\Gamma^+}_{(n)}$ that minimizes the policy maker's ex-ante payoff, and observe that $\bar{a}^{\Gamma^+}_{(0)} \equiv (\bar{a}^{\Gamma^+}_{(0),i}(\cdot))_{i \in N}$ prescribes that all agents refrain from pledging, irrespective of their exogenous and endogenous signals.

Step 1. First, we prove that, for any $i \in N$,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \forall a \in \mathcal{A}^{\Gamma}\}$$

$$\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 1); a) > 0 \ \forall a \in \mathcal{A}^{\Gamma^+}\}.$$

That is, any agent who finds it dominant to pledge under Γ after receiving information (x_i, m_i) also finds it dominant to pledge under Γ^+ after receiving information $(x_i, (m_i, 1))$. To see this, first use the fact that the game is supermodular to observe that, given any policy Γ,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \ \forall a \in \mathcal{A}^{\Gamma}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; \bar{a}^{\Gamma}_{(0)}) > 0\}.$$

Likewise,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 0); a) > 0 \ \forall a \in \mathcal{A}^{\Gamma^+}\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i, 0); \bar{a}^{\Gamma^+}_{(0)}) > 0\}.$$

Next, observe that, because under both $\bar{a}^{\Gamma}_{(0)}$ and $\bar{a}^{\Gamma^+}_{(0)}$ all agents refrain from pledging, regardless of their exogenous and endogenous information, under both $\bar{a}^{\Gamma}_{(0)}$ and $\bar{a}^{\Gamma^+}_{(0)}$, default occurs if, and only if, $\theta \leq \bar{\theta}$ (with $\bar{\theta}$ defined by $R(\bar{\theta}, 0) = 0$). Then, note that, under Γ^+, for any $i \in N$, any $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\partial \Lambda_i^{\Gamma^+}(\omega, m | x_i, (m_i, 1)) = \frac{\mathbf{1}_{\{r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}}}{\Lambda_i^{\Gamma}(1 | x_i, m_i)} \partial \Lambda^{\Gamma}(\omega, m | x_i, m_i), \tag{AM8}$$

where

$$\Lambda_i^{\Gamma}(1 | x_i, m_i) \equiv \int_{\{(\omega, m) : r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}} \mathrm{d}\Lambda^{\Gamma}(\omega, m | x_i, m_i)$$

is the total probability that, under the policy Γ, agent i with information (x_i, m_i) assigns to the event $\{(\omega, m) \in \Omega \times M(\mathcal{S}) : r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1\}$. Under Bayesian learning, the agents' beliefs under the new policy policy Γ^+ thus correspond to "truncations" of their beliefs under the original policy Γ. In turn, this property of Bayesian updating implies that, for any $(x_i, m_i) \in X_i \times \mathcal{S}$ such that

$$U_i^{\Gamma}(x_i, m_i; \bar{a}_{(0)}^{\Gamma}) = \int_{(\omega,m)} \Big(b_i(\theta,1)\mathbf{1}_{\{\theta\leq\bar{\theta}\}} + g_i(\theta,1)\mathbf{1}_{\{\theta>\bar{\theta}\}}\Big)\, \mathrm{d}\Lambda_i^{\Gamma}(\omega, m|x_i, m_i) > 0,$$

it must be that

$$\begin{aligned} U_i^{\Gamma^+}(x_i, (m_i,1); \bar{a}_{(0)}^{\Gamma^+}) &= \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)} \int_{(\omega,m)} \Big(b_i(\theta,1)\mathbf{1}_{\{\theta\leq\bar{\theta}\}} + g_i(\theta,1)\mathbf{1}_{\{\theta>\bar{\theta}\}}\Big) \times \\ &\quad \times \mathbf{1}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=1\}} \mathrm{d}\Lambda_i^{\Gamma}(\omega, m|x_i, m_i) \\ &> \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)} \int_{(\omega,m)} \Big(b_i(\theta,1)\mathbf{1}_{\{\theta\leq\bar{\theta}\}} + g_i(\theta,1)\mathbf{1}_{\{\theta>\bar{\theta}\}}\Big)\, \mathrm{d}\Lambda_i^{\Gamma}(\omega, m|x_i, m_i) \\ &= \frac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)} U_i^{\Gamma}(x_i, m_i; \bar{a}_{(0)}^{\Gamma}) \\ &> 0, \end{aligned}$$

where the first equality follows from the truncation property of Bayesian updating, the first inequality from the fact that, for all $(\omega, m) \in \Omega \times M(\mathcal{S})$ such that $r(\omega, m; \bar{a}_{(K)}^{\Gamma}) = 0$, $\theta \leq \bar{\theta}$, and hence $r(\omega, m; \bar{a}_{(0)}^{\Gamma}) = 0$, implying that

$$b_i(\theta,1)\mathbf{1}_{\{\theta\leq\bar{\theta}\}} + g_i(\theta,1)\mathbf{1}_{\{\theta>\bar{\theta}\}} = b_i(\theta,1) < 0,$$

the second equality from the definition of $U_i^{\Gamma}(x_i, m_i; \bar{a}_{(0)}^{\Gamma})$, and the second inequality from the fact that $U_i^{\Gamma}(x_i, m_i; \bar{a}_{(0)}^{\Gamma}) > 0$.

The above result implies that any an agent who, under Γ, finds it dominant to pledge after receiving information (x_i, m_i) also finds it dominant to pledge under Γ^+ after receiving information $(x_i, (m_i, 1))$, as claimed.

Step 2. We now show that a property analogous to the one established in Step 1 applies to any other round of the IDISDS procedure. The result is established by induction. Take any round $n \in \{1, 2, ..., K\}$ and assume that, for any $0 \leq k \leq n-1$, any $i \in [0,1]$,

$$\begin{aligned} &\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \;\; \forall a \in T_{(k-1)}^{\Gamma}\} \\ &\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i,1); a) > 0, \;\; \forall a \in T_{(k-1)}^{\Gamma^+}\}. \end{aligned} \tag{AM9}$$

Recall that this means that any agent who, under Γ, finds it optimal to pledge when his opponents play *any* strategy surviving k rounds of IDISDS under Γ continues to find it optimal to pledge when expecting his opponents to play *any* strategy surviving k rounds of IDISDS under Γ^+. Below we show that that the same property extends to strategies surviving n rounds of IDISDS. That is,

$$\begin{aligned} &\{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma}(x_i, m_i; a) > 0 \;\; \forall a \in T_{(n-1)}^{\Gamma}\} \\ &\subseteq \{(x_i, m_i) \in X_i \times \mathcal{S} : U_i^{\Gamma^+}(x_i, (m_i,1); a) > 0, \;\; \forall a \in T_{(n-1)}^{\Gamma^+}\}. \end{aligned} \tag{AM10}$$

To see this, use again the fact that the game is supermodular to observe that

$$\{(x_i,m_i)\in X_i\times\mathcal{S}: U_i^{\Gamma}(x_i,m_i;a)>0 \;\; \forall a\in T_{(n-1)}^{\Gamma}\}=\{(x_i,m_i)\in X_i\times\mathcal{S}: U_i^{\Gamma}(x_i,m_i;\bar{a}_{(n-1)}^{\Gamma})>0\} \tag{AM11}$$

and, likewise,

$$\begin{aligned}&\{(x_i,m_i)\in X_i\times\mathcal{S}: U_i^{\Gamma^+}(x_i,(m_i,1);a)>0, \quad \forall a\in T_{(n-1)}^{\Gamma^+}\}\\ &=\{(x_i,m_i)\in X_i\times\mathcal{S}: U_i^{\Gamma^+}(x_i,m_i;\bar{a}_{(n-1)}^{\Gamma^+})>0\},\end{aligned} \tag{AM12}$$

where recall that $\bar{a}_{(n-1)}^{\Gamma}$ (alternatively, $\bar{a}_{(n-1)}^{\Gamma^+}$) is the most aggressive profile surviving $n-1<K$ rounds of IDSIDS under Γ (alternatively, Γ^+).

Now let $A(\omega,m;a)$ denote the aggregate size of the pledge that, under Γ, prevails at (ω,m), when agents play according to $a\in\mathcal{A}^{\Gamma}$. Then take any $i\in N$ and any $(x_i,m_i)\in X_i\times\mathcal{S}$ such that

$$U_i^{\Gamma}(x_i,m_i;\bar{a}_{(n-1)}^{\Gamma})=\int_{(\omega,m)}u_i(\theta,A(\omega,m;\bar{a}_{(n-1)}^{\Gamma}))\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i)>0.$$

Because $\bar{a}_{(n-1)}^{\Gamma}$ is more aggressive than $\bar{a}_{(K)}^{\Gamma}$, in the sense that, for any $i\in N$, any $(x_i,m_i)\in X_i\times\mathcal{S}$, $\bar{a}_{(n-1),i}^{\Gamma}(x_i,m_i)\le\bar{a}_{K,i}^{\Gamma}(x_i,m_i)$, then for all (ω,m),

$$r(\omega,m;\bar{a}_{(K)}^{\Gamma})=0\Rightarrow r(\omega,m;\bar{a}_{(n-1)}^{\Gamma})=0.$$

This implies that

$$\begin{aligned}\int_{(\omega,m)}u_i(\theta,A(\omega,m;\bar{a}_{(n-1)}^{\Gamma}))\mathbf{1}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=0\}}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i) &= \\ \int_{(\omega,m)}b_i(\theta,A(\omega,m;\bar{a}_{(n-1)}^{\Gamma}))\mathbf{1}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=0\}}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i) &< 0\end{aligned} \tag{AM13}$$

This observation, together with the truncation property in (AM8), implies that, for any $i\in N$, any $(x_i,m_i)\in X_i\times\mathcal{S}$ such that $U_i^{\Gamma}(x_i,m_i;\bar{a}_{(n-1)}^{\Gamma})>0$,

$$\begin{aligned}U_i^{\Gamma^+}(x_i,(m_i,1);\bar{a}_{(n-1)}^{\Gamma}) &= \int_{(\omega,m)}u_i(\theta,A(\omega,m;\bar{a}_{(n-1)}^{\Gamma}))\mathrm{d}\Lambda_i^{\Gamma^+}(\omega,m|x_i,m_i)\\ &= \tfrac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}\int_{(\omega,m)}u(\theta,A(\omega,m;\bar{a}_{(n-1)}^{\Gamma}))\mathbf{1}_{\{r(\omega,m;\bar{a}_{(K)}^{\Gamma})=1\}}\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i)\\ &> \tfrac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}\int_{(\omega,m)}u(\theta,A(\omega,m;\bar{a}_{(n-1)}^{\Gamma}))\mathrm{d}\Lambda_i^{\Gamma}(\omega,m|x_i,m_i)\\ &= \tfrac{1}{\Lambda_i^{\Gamma}(1|x_i,m_i)}U_i^{\Gamma}((x_i,m_i);\bar{a}_{(n-1)}^{\Gamma})\\ &> 0\end{aligned} \tag{AM14}$$

where the first and third equalities are by definition, the second equality follows from (AM8), the first inequality follows from (AM13), and the last inequality from the fact that $U_i^{\Gamma}(x_i,m_i;\bar{a}_{(n-1)}^{\Gamma})>0$, by assumption.

Next, note that $\bar{a}_{(n-1)}^{\Gamma}$ and $\bar{a}_{(n-1)}^{\Gamma^+}$ are such that, for all $i\in N$, all $(x_i,m_i)\in X_i\times\mathcal{S}$, $\bar{a}_{(n-1),i}^{\Gamma}(x_i,m_i)$, $\bar{a}_{(n-1),i}^{\Gamma^+}(x_i,(m_i,0))\in\{0,1\}$ and

$$\{(x_i,m_i)\in X_i\times\mathcal{S}:\bar{a}_{(n-1),i}^{\Gamma}(x_i,m_i)=1\}=\{(x_i,m_i)\in X_i\times\mathcal{S}:U_i^{\Gamma}(x_i,m_i;\bar{a}_{(n-2)}^{\Gamma})>0\}$$

and, likewise,

$$\{(x_i, m_i) \in X_i \times \mathcal{S} : \bar{a}^{\Gamma+}_{(n-1),i}(x_i, (m_i, 1)) = 1\} = \{(x_i, m_i) \in X_i \times \mathcal{S} : U^{\Gamma+}_i(x_i, (m_i, 1); \bar{a}^{\Gamma+}_{(n-2)}) > 0\}.$$

Together properties (AM9), (AM11) and (AM12) imply that $\bar{a}^{\Gamma}_{(n-1)}$ and $\bar{a}^{\Gamma+}_{(n-1)}$ are such that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma}_{(n-1),i}(x_i, m_i) = 1 \Rightarrow \bar{a}^{\Gamma+}_{(n-1),i}(x_i, (m_i, 1)) = 1. \tag{AM15}$$

Condition (AM15), along with the fact that the game is supermodular, implies that

$$U^{\Gamma+}_i(x_i, (m_i, 1); \bar{a}^{\Gamma}_{(n-1)}) > 0 \Rightarrow U^{\Gamma+}_i(x_i, (m_i, 1); \bar{a}^{\Gamma+}_{(n-1)}) > 0. \tag{AM16}$$

Together (AM14) and (AM16) imply the property in (AM10).

Step 3. Equipped with the results in steps 1 and 2 above, we now prove that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma+}_{(K),i}(x_i, (m_i, 1)) = 1.$$

This follows directly from the fact that, for all $i \in N$, all $(x_i, m_i) \in X_i \times \mathcal{S}$,

$$\bar{a}^{\Gamma}_{(K),i}(x_i, m_i) = 1 \Rightarrow \bar{a}^{\Gamma+}_{(K),i}(x_i, (m_i, 1)) = 1, \tag{AM17}$$

which, in turn implies that, for any (ω, m),

$$r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 1 \Rightarrow r(\omega, m; \bar{a}^{\Gamma+}_{(K)}) = 1.$$

Under Γ^+, the announcement that $r = 1$ thus reveals to the agents that (ω, m) is such that $r(\omega, m; \bar{a}^{\Gamma+}_{(K)}) = 1$. Because the payoff from pledging is strictly positive when the bank survives, any agent i receiving a signal $(m_i, 1)$ thus necessarily pledges. Under MARP-K consistent with the new policy Γ^+ thus all agents pledge, regardless of their exogenous and endogenous private signals, when the policy publicly announces $r = 1$. That they all refrain from pledging, irrespective of (x_i, m_i), when the policy announces $r = 0$ follows from the fact that $r = 0$ makes it common certainty among the agents that (ω, m) is such that $r(\omega, m; \bar{a}^{\Gamma}_{(K)}) = 0$ and hence that $\theta \leq \bar{\theta}$. But then, irrespective of (x_i, m_i), any agent $i \in N$ receiving exogenous information x_i and endogenous information $m^+_i = (m_i, 0)$ finds it optimal to refrain from pledging when expecting all other agents to abstain from pledging no matter their exogenous and endogenous information. This implies that under MARP-K consistent with the new policy Γ^+, all agents refrain from pledging when hearing that $r = 0$.

We conclude that the new policy Γ^+ satisfies the perfect coordination property and is such that, for any $(\theta, \mathbf{x}) \in \Theta \times \mathcal{X}$, the probability of default under Γ^+ is the same as under Γ. Q.E.D.

Section AM4: Discriminatory Disclosures

In this section, we consider an extension in which the policy maker can disclose different information to different market participants. The purpose of the section is to illustrate the possible benefits of discriminatory disclosures, when the latter are feasible. To maintain the analysis as simple as possible, we assume that the environment satisfies the conditions in Theorem 3, implying that, if the policy maker were to restrict attention to non-discriminatory policies, the optimal policy would be a simple monotone pass/fail test failing with certainty all institutions with fundamentals below a cut-off θ^* and passing with certainty all the others.

We start by explaining that the benefits of discriminatory disclosures stem from the possibility of increasing the uncertainty each agent faces about the beliefs that rationalize other agents' behavior. We then consider a parametric setting in which the policy maker can engineer any public disclosure of her choice, but is constrained to use Gaussian signals when communicating privately with the agents. The advantage of such a parametric approach is that the combination of the exogenous and the endogenous private information can be conveniently summarized in a uni-dimensional sufficient statistics. This in turn permits us to relate the benefits of discriminatory disclosures to the type of securities issued by the banks (more generally, to the sensitivity of the agents' payoffs to the underlying fundamentals).[17]

In this section, to simplify the exposition, we assume away the shocks z imperfectly correlated with θ.

Subsection AM4.1: Benefits of Discriminatory Disclosures

Perhaps surprisingly, the reason why discriminatory disclosures may improve upon non-discriminatory ones has little to do with the possibility of tailoring the information disclosed to the agents to their prior beliefs. Discriminatory disclosures may outperform non-discriminatory ones because, by enhancing the dispersion of posterior beliefs, they make it harder for the agents to refrain from pledging, thus permitting the policy maker to save a larger set of institutions.

To illustrate the point in the simplest possible way, consider an economy in which the agents' prior beliefs are homogenous (formally, this amounts to assuming the exogenous private signals x are completely uninformative). Next let $u(\theta, A)$ denote the payoff from pledging when the fundamentals are θ and the aggregate size of the pledge is A. Notice that, for any $\hat{\theta}$ such that

$$\int u(\theta, 0)\mathrm{d}F(\theta|\theta > \hat{\theta}) > 0,$$

the most aggressive rationalizable strategy profile following the public announcement that $\theta > \hat{\theta}$ is

[17] See also Li et al (2021) and Morris et al (2020) for the characterization of the optimal *discriminatory* policy when agents do not possess any exogenous private information.

such that every agent pledges.[18] Under the assumptions of Theorem 3 in the main text, the optimal non-discriminatory policy is then a threshold rule with cut-off equal to[19]

$$\hat{\theta}^* = \inf\{\hat{\theta} \in \mathbb{R}\; s.t. \int u(\theta, 0)\mathrm{d}F(\theta|\theta > \hat{\theta}) > 0\}. \tag{AM18}$$

Suppose now the policy maker, instead of announcing whether θ is above or below some threshold $\hat{\theta}$, sends to each individual a private signal of the form $m_i = \theta + \sigma\xi_i$, where $\sigma \in \mathbb{R}_+$ is a scalar, and where the idiosyncratic terms ξ_i are drawn from a smooth distribution over the entire real line (e.g., a standard Normal distribution), independently across agents, and independently from θ. From standard results in the global games literature, we know that, as the private messages become highly precise (formally, as $\sigma \to 0^+$), in the absence of any public disclosure, under the most aggressive rationalizable profile, each agent pledges if, and only if, his endogenous private signal falls above the threshold $\theta^{MS} \in (\underline{\theta}, \bar{\theta})$ implicitly defined by the unique solution to

$$\int_0^1 u(\theta^{MS}, A)\mathrm{d}A = 0. \tag{AM19}$$

As explained in the main text, the threshold θ^{MS} corresponds to the value of the fundamentals at which an agent who knows θ and holds *Laplacian* beliefs with respect to the size of the pledge[20] is indifferent between pledging and not pledging. Importantly, θ^{MS} *is independent of the initial common prior* and of the distribution of the noise terms ξ in the agents' signals. The above result thus implies that, with discriminatory disclosures, the policy maker can always guarantee that default never occurs for any $\theta > \theta^{MS}$. We then have the following result:[21]

Proposition AM4-1. *Assume the agents possess no exogenous private information about the underlying fundamentals. Let $\hat{\theta}^*$ be the threshold in (AM18) and θ^{MS} be the threshold in (AM19). Whenever $\theta^{MS} < \hat{\theta}^*$, discriminatory disclosures strictly improve upon non-discriminatory ones.*

The result follows directly from the arguments preceding the proposition. Because $\hat{\theta}^*$ can be arbitrarily close to $\overline{\theta}$ for particular prior distributions, and because θ^{MS} is invariant in the prior distribution from which θ is drawn, the result in Proposition AM4-1 is relevant in many cases of interest.

As anticipated above, the reason why discriminatory disclosures can improve upon non-discriminatory ones is that they permit the policy maker to enhance the dispersion of the agents higher-order beliefs.

[18] The notation $F(\theta|\theta > \hat{\theta})$ stands for the common posterior obtained from the prior F by conditioning on the event that $\theta > \hat{\theta}$.

[19] Here we follow the same abuse as in the main text and refer to the optimal non-discriminatory policy as the monotone policy whose threshold is given by $\hat{\theta}^*$.

[20] This means that the agent believes that the size of the aggregate pledge is uniformly distributed over $[0, 1]$.

[21] The proposition shows that the condition $\theta^{MS} < \hat{\theta}^*$ is sufficient for discriminatory policies to strictly improve upon non-discriminatory ones. When, instead, $\theta^{MS} \geq \hat{\theta}^*$, whether or not the optimal policy is discriminatory depends on the prior F and on the sensitivity of the agents' payoffs to θ. See Li et al (2021) and Morris et al (2020) for a characterization of the optimal discriminatory policy when payoffs are constant in θ.

A higher dispersion in turn makes it more difficult for the agents to play adversarially to the policy maker (i.e, to refrain from pledging). Formally speaking, when beliefs are sufficiently dispersed, an agent receiving a private signal indicating that the bank may collapse under a sufficiently large attack (i.e, in case few agents pledge) may nonetheless pledge if he expects many other agents to have received extreme signals indicating that the fundamentals are strong enough for the bank not to collapse, no matter the size of the attack. In this case, pledging may become *iteratively dominant* for this individual. The optimality of discriminatory policies thus follows from a "divide-and-conquer" logic reminiscent of the one in the vertical contracting literature (see, e.g., Segal (2003) and the references therein). Importantly, when discriminatory policies outperform non-discriminatory ones, this is not because they mis-coordinate the response by the market (recall that, by virtue of Theorem S4-1 in the Online Appendix, the optimal policy always satisfies the perfect-coordination property, irrespectively of whether or not it is discriminatory), but because, by enhancing the heterogeneity in structural uncertainty, they make it difficult for market participants to coordinate on an adversarial course of action when the planner recommends that they pledge.

Subsection AM4.2: Payoff Sensitivity and the Optimality of Discriminatory Policies

We conclude by showing how the optimality of discriminatory policies may depend to the sensitivity of the agents' payoffs to the underlying fundamentals and relate such sensitivity to the type of securities issued by the banks under scrutiny. To gain on tractability, we consider an environment in which the prior distribution F from which θ is drawn is an improper uniform distribution over the entire real line and where the agents' exogenous private signals are given by $x_i = \theta + \sigma_\eta \eta_i$, with $\eta_i \sim \mathcal{N}(0, 1)$.[22] Furthermore, to facilitate the aggregation of the agents' exogenous and endogenous signals into a uni-dimensional statistics, we restrict attention to the following parametric structure. The policy maker can engineer any public disclosure of her choice but is constrained to sending signals of the Gaussian form $\widetilde{m}_i = \theta + \sigma_\xi \xi_i$, with $\xi_i \sim \mathcal{N}(0, 1)$, when communicating privately with the agents. The restriction to Gaussian private applies only to the information the policy maker discloses *privately* to the agents, over and above the information conveyed by the public test. In each state θ, the endogenous information $m_i = (\widetilde{s}, \widetilde{m}_i)$ disclosed to each agent i thus comprises a public signal $\widetilde{s}$, along with a private signal $\widetilde{m}_i$. Under such a structure, the quality of the private signals is then conveniently parametrized by the variance $\sigma_\xi^2 > 0$ of the endogenous noise terms.

We also assume the agents' payoff from pledging depends on the aggregate size of the pledge A only through the effects of the latter on the default outcome. In other words, we assume that there exist strictly increasing functions $\bar{g}(\theta)$ and $\bar{b}(\theta)$ such that the payoff of each agent pledging to the bank is equal to $\bar{g}(\theta)$ in case the bank does not default and equal to $\bar{b}(\theta)$, in case the bank default.

[22] The assumption that F is an improper uniform distribution is standard in the global-game literature. It simplifies the formulas below, without any serious effect on the results. Also observe that the entire hierarchy of the agents' beliefs is well defined, despite the prior being improper.

The payoff from not pledging is equal to zero. Finally, we assume that the function R determining the default outcome takes the same linear form $R(\theta, A) = \theta - 1 + A$ as in the baseline model.[23]

Then observe that the information contained in each pair $(x_i, \widetilde{m}_i)$ of exogenous and endogenous private signals is the same as the information contained in the sufficient statistics

$$z_i \equiv \frac{\sigma_\xi^2 x_i + \sigma_\eta^2 \tilde{m}_i}{\sigma_\eta^2 + \sigma_\xi^2},$$

which, given θ, is normally distributed with mean θ and variance $\sigma_z^2 \equiv \left(\sigma_\eta^2 \sigma_\xi^2\right) / \left(\sigma_\eta^2 + \sigma_\xi^2\right)$. Hence, the policy maker's choice of the discriminatory component of her disclosure policy can be conveniently reduced to the choice of the standard deviation σ_z of the above sufficient statistics, with $\sigma_z \in (0, \sigma_\eta]$.

Arguments analogous to those establishing Lemma 1 in the Appendix in the main document then imply that, for any realization $\widetilde{s}$ of the endogenous public signal, the most aggressive rationalizable strategy profile a^Γ is characterized by a unique cut-off $\bar{z}(\widetilde{s})$ (whose value depends on the distribution from which the public signal is drawn) such that, for all $i \in [0, 1]$, $a_i^\Gamma(x_i, (\widetilde{s}, \tilde{m}_i)) = \mathbf{1}\{z_i > \bar{z}(\widetilde{s})\}$. Moreover, arguments similar to those establishing Theorem 2 in the main text imply that, for any given choice of σ_z^2, the optimal public announcement is binary with $\tilde{s} \in \{0, 1\}$ — that is, the public test has a pass/fail structure. Finally, from Theorem S4-1 in the Online Appendix, the optimal policy has the perfect-coordination property which means that, given σ_z^2, $\bar{z}(0) = +\infty$, and $\bar{z}(1) = -\infty$. That is, all agents pledge when $\tilde{s} = 1$, and they all refrain from pledging when $\widetilde{s} = 0$.

Next, let Φ denotes the cdf of the standard Normal distribution, and, for any $\theta \in [0, 1]$, define

$$z_{\sigma_z}^*(\theta) \equiv \theta + \sigma_z \Phi^{-1}(\theta),$$

to be the private statistics threshold such that, when all agents refrain from pledging when $z_i < z_{\sigma_z}^*(\theta)$ and pledge when $z_i > z_{\sigma_z}^*(\theta)$, default occurs when the fundamentals fall below θ and does not occur when they are above θ.[24]

For any $(\theta_0, \hat{\theta}, \sigma_z)$, let $\psi(\theta_0, \hat{\theta}, \sigma_z)$ denote the payoff from pledging of an agent with private statistics $z_{\sigma_z}^*(\theta_0)$, when regime change occurs for all $\theta \le \theta_0 \in [0, 1]$, public information reveals that $\theta \ge \hat{\theta}$, and the total precision of private information is σ_z^{-2}. Then let

$$\theta_{\sigma_z}^{inf} \equiv \inf\left\{\hat{\theta} : \ \psi(\theta_0, \hat{\theta}, \sigma_z) > 0 \ \text{ all } \ \theta_0 \in [0, 1]\right\}.$$

Note that, for any $\hat{\theta} > \theta_{\sigma_z}^{inf}$, under the most aggressive rationalizable strategy profile, all agents pledge after the public signal reveals that $\theta \ge \hat{\theta}$. Hereafter, we assume that all agents pledge also when public disclosures reveal that $\theta \ge \theta_{\sigma_z}^{inf}$. This simplifies the exposition below by permitting us to talk about the "optimal policy." As discussed in the main body, the latter does not formally exist

[23]The results below extend to more general payoff functions, as long as the agents' exogenous signals x are sufficiently precise.

[24]Given that $R(\theta, A) = \theta - 1 + A$, $z_{\sigma_z}^*(\theta)$ is implicitly defined by the equation $\Phi\left(\frac{z^* - \theta}{\sigma_z}\right) = \theta$.

when agents are expected to play according to the most aggressive rationalizable profile. However, because the policy maker can always guarantee that, no matter the selection of the rationalizable strategy profile, each agent pledges for any $\theta > \theta^{inf}_{\sigma_z}$, we find the abuse justified.

Proposition AM4-2. *Suppose the policy maker is constrained to using Gaussian signals when communicating privately with the agents. Let*

$$\sigma_z^* \equiv \arg\min_{\sigma_z \in (0,\sigma_\eta]} \theta^{inf}_{\sigma_z}.$$

The optimal disclosure policy has the following structure. The policy maker publicly announces whether $\theta < \theta^{inf}_{\sigma_z^*}$, or whether $\theta \geq \theta^{inf}_{\sigma_z^*}$. In addition, when $\theta \geq \theta^{inf}_{\sigma_z^*}$, the policy maker sends a Gaussian private signal to each agent of precision $\sigma_\xi^{-2} = [\sigma_\eta^2 - (\sigma_z^*)^2]/(\sigma_z^*)^2\sigma_\eta^2$.

The result follows from the arguments preceding the proposition – note that the precision of the endogenous private information σ_ξ^{-2} in the proposition is the one that, together with the precision of the exogenous signals σ_η^{-2} yields a total precision σ_z^{-2} for the sufficient statistics z_i that minimizes the threshold $\theta^{inf}_{\sigma_z}$ defining the default outcome.

Equipped with the result in Proposition AM4-2, we can then identify primitive conditions under which the optimal policy is non-discriminatory. By virtue of Proposition AM4-2, discriminatory disclosures strictly dominate non-discriminatory ones if, and only if, $\sigma_z^* < \sigma_\eta$ (equivalently, if, and only if, there exists $\sigma_z < \sigma_\eta$ such that $\theta^{inf}_{\sigma_z} < \theta^{inf}_{\sigma_\eta}$). For any precision σ_z^{-2} of the agents' private statistics, let $\theta^{\#}_{\sigma_z}$ denote the unique solution to the equation $\psi(\theta^{\#}_{\sigma_z}, \theta^{inf}_{\sigma_z}, \sigma_z) = 0$. Note that, under MARP, $\theta^{\#}_{\sigma_z}$ identifies the fundamental threshold below which regime change occurs when the total precision of the agents' private information is σ_z^{-2} , and the endogenous disclosure of public information reveals that $\theta \geq \theta^{inf}_{\sigma_z}$. Let[25]

$$D(\theta, \theta^{\#}_{\sigma_z}) \equiv \begin{cases} \bar{b}'(\theta) \text{ if } \theta < \theta^{\#}_{\sigma_z} \\ \bar{g}'(\theta) \text{ if } \theta \geq \theta^{\#}_{\sigma_z}. \end{cases}$$

Proposition AM4-3. *Suppose that, for any $\sigma_z \in (0, \sigma_\eta]$,*

$$\mathbb{E}[D(\theta, \theta^{\#}_{\sigma_z})(\theta - \theta^{\#}_{\sigma_z}) | z^*(\theta^{\#}_{\sigma_z}), \theta \geq \theta^{inf}_{\sigma_z}; \sigma_z] \geq 0. \qquad \text{(AM20)}$$

Then the optimal policy is non-discriminatory.

The formal proof is below. Here we first discuss the intuition behind the result and its implications. The condition in Proposition AM5-3 is a measure of the sensitivity of the marginal agent's

[25]Here $\bar{b}'$ and $\bar{g}'$ denote the derivatives of the $\bar{b}$ and $\bar{g}$ functions, respectively.

net payoff from pledging to the underlying fundamentals.[26] To see this, note that the condition is equivalent to[27]

$$\frac{\mathbb{E}[\bar{g}'(\theta)(\theta-\theta^{\#}_{\sigma_z})|z^*(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{\#}_{\sigma_z};\sigma_z]}{\mathbb{E}[\bar{g}(\theta)|z^*(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{\#}_{\sigma_z};\sigma_z]}\geq\frac{\mathbb{E}[\bar{b}'(\theta)(\theta^{\#}_{\sigma_z}-\theta)|z^*(\theta^{\#}_{\sigma_z}),\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z});\sigma_z]}{\mathbb{E}[|\bar{b}(\theta)||z^*(\theta^{\#}_{\sigma_z}),\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z});\sigma_z]}.$$

The left-hand side is the elasticity of the marginal agent's expected net payoff from pledging with respect to the underlying fundamentals, in case of no default. The right-hand side is the corresponding elasticity in case of default.[28]

To gather some intuition, consider the case in which, when default occurs, the agents' payoff differential between pledging and not pledging is constant in the underlying fundamentals (i.e., $\bar{b}'(\theta)=0$ for all θ). In this case, the marginal agent faces only *upside risk*. Hence, when the quality of private information decreases (which amounts to a mean-preserving increase in risk), the agent's expected net payoff from pledging increases. Starting from any policy that discloses private information to the agents (i.e., for which $\sigma_z<\sigma_\eta$), the policy maker can then do better by reducing the precision of the agents' private information. In this case, the optimal policy is non-discriminatory.

The value of Proposition AM4-3 is in indicating how the optimality of discriminatory disclosures relates to the sensitivity of the agents' payoffs to the underlying fundamentals. In turn, such sensitivity typically depends on the type of security issued by the banks. For example, the above condition is more likely to hold when investors are *equity holders*. In this case, when the bank defaults, their claims are junior (i.e., subordinated) with respect to those from other stake holders with higher seniority (e.g., bond holders). In case of default, the agents' payoff then amount to a liquidation value that is typically little sensitive to the exact amount of the bank's performing loans (the bank's fundamentals). On the contrary, when the bank does not default (i.e., when the government succeeds in persuading the bank's equity holders to stay put), the value of the equity-holders' claims reflect the bank's long-term profitability, which is sensitive to the amount of the bank's performing loans. The result in Proposition AM4-3 thus indicates that discriminatory disclosures are more likely to be beneficial when the banks are seeking external funding by issuing bonds than when they do so by issuing equity.

Proof of Proposition AM4-3. We establish the result by showing that, when Condition (AM20) holds, for any fixed $\hat{\theta}$, the function $\Psi(\hat{\theta},\sigma_z)\equiv\min_{\theta_0\in[0,1]}\psi(\theta_0,\hat{\theta},\sigma_z)$ is increasing in σ_z. Moreover, in this case, the regime threshold in the absence of any public disclosure, $\theta^*_{\sigma_z}$, implicitly defined by $\psi(\theta^*_{\sigma_z},-\infty,\sigma_z)=0$, is decreasing in σ_z, with $\lim_{\sigma_z\to 0^+}\theta^*_{\sigma_z}=\theta^{MS}$.

[26]The marginal agent is the one with signal $z^*_{\sigma_z}(\theta^{\#}_{\sigma_z})$.

[27]See also Iachan and Nenov (2015) for a similar condition in a related class of games of regime change.

[28]Observe that, for the marginal agent with signal $z^*_{\sigma_z}(\theta^{\#}_{\sigma_z})$,

$$Pr(\theta\geq\theta^{\#}_{\sigma_z}|z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{inf}_{\sigma_z};\sigma_z)\mathbb{E}[\bar{g}(\theta)|z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{\#}_{\sigma_z};\sigma_z]=$$
$$Pr(\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z})|z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\geq\theta^{inf}_{\sigma_z};\sigma_z)\mathbb{E}[|\bar{b}(\theta)||z^*_{\sigma_z}(\theta^{\#}_{\sigma_z}),\theta\in(\theta^{inf}_{\sigma_z},\theta^{\#}_{\sigma_z});\sigma_z].$$

To ease the notation, let $\sigma = \sigma_z$. By the envelope theorem, we have that $\frac{\partial}{\partial \sigma}\Psi(\hat{\theta}, \sigma) = \frac{\partial}{\partial \sigma}\psi(\bar{\theta}_\sigma, \hat{\theta}, \sigma)$, with $\bar{\theta}_\sigma \in \arg\min_{\theta_0 \in [0,1]} \psi(\theta_0, \hat{\theta}, \sigma)$. Note that, for any $\theta_0 > \hat{\theta}$, any σ,

$$\frac{\partial}{\partial \sigma}\psi(\theta_0, \hat{\theta}, \sigma) = \frac{\partial}{\partial \sigma}\int_{\hat{\theta}}^{\infty} \left(\bar{b}(\theta)\mathbf{1}_{\theta<\theta_0} + \bar{g}(\theta)\mathbf{1}_{\theta\geq\theta_0}\right) \frac{\phi\left(\frac{z_\sigma^*(\theta_0)-\theta}{\sigma}\right)}{\sigma\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)} \mathrm{d}\theta$$

$$= \frac{\partial}{\partial \sigma}\frac{\int_{1-\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)}^{1} \left(\bar{b}(z_\sigma^*(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A<1-\theta_0} + \bar{g}(z_\sigma^*(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A>1-\theta_0}\right)\mathrm{d}A}{\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

$$= \frac{\int_{1-\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)}^{1} \left(\bar{b}'(z_\sigma^*(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A<1-\theta_0} + \bar{g}'(z_\sigma^*(\theta_0)-\sigma\Phi^{-1}(1-A))\mathbf{1}_{A>1-\theta_0}\right)(\Phi^{-1}(\theta_0)-\Phi^{-1}(1-A))\mathrm{d}A}{\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

$$+\frac{(\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)(\theta_0-\hat{\theta})}{\sigma^2\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)}$$

where the second equality follows from the change of variables $A = 1-\Phi\left(\frac{z_\sigma^*(\theta_0)-\theta}{\sigma}\right)$ along with the fact that, by definition, $1-\Phi\left(\frac{z_\sigma^*(\theta_0)-\theta_0}{\sigma}\right) = 1-\theta_0$, while the third equality from using $z_\sigma^*(\theta) = \theta+\sigma\Phi^{-1}(\theta)$. Lastly, by reverting the change of variables, and letting

$$D(\theta, \theta_0) \equiv \begin{cases} \bar{b}'(\theta) \text{ if } \theta < \theta_0 \\ \bar{g}'(\theta) \text{ if } \theta \geq \theta_0, \end{cases}$$

we have that

$$\begin{aligned} \frac{\partial}{\partial \sigma}\psi(\theta_0, \hat{\theta}, \sigma) &= \frac{\int_{\hat{\theta}}^{\infty} D(\theta, \theta_0)(\theta-\theta_0)\phi\left(\frac{z_\sigma^*(\theta_0)-\theta}{\sigma}\right)\mathrm{d}\theta + (\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)\left(\theta_0-\hat{\theta}\right)}{\sigma^2\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)} \\ &= \sigma^{-1}\mathbb{E}[D(\theta,\theta_0)(\theta-\theta_0)|z_\sigma^*(\theta_0), \theta \geq \hat{\theta}] + \frac{(\psi(\theta_0,\hat{\theta},\sigma)-b(\hat{\theta}))\phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)\left(\theta_0-\hat{\theta}\right)}{\sigma^2\Phi\left(\frac{z_\sigma^*(\theta_0)-\hat{\theta}}{\sigma}\right)}. \end{aligned}$$

When evaluated at $\hat{\theta} = \theta_\sigma^{inf}$ and $\theta_0 = \theta_\sigma^{\#}$, because $\psi(\theta_\sigma^{\#}, \theta_\sigma^{inf}, \sigma) = 0$, we have that the above expression becomes

$$\frac{\partial}{\partial \sigma}\psi(\theta_\sigma^{\#}, \theta_\sigma^{inf}, \sigma) = \sigma^{-1}\mathbb{E}[D(\theta,\theta_\sigma^{\#})(\theta-\theta_\sigma^{\#})|z_\sigma^*(\theta_\sigma^{\#}), \theta \geq \theta_\sigma^{inf}] + \frac{|b(\theta_\sigma^{inf})|\phi\left(\frac{z_\sigma^*(\theta_\sigma^{\#})-\theta_\sigma^{inf}}{\sigma}\right)\left(\theta_\sigma^{\#}-\theta_\sigma^{inf}\right)}{\sigma^2\Phi\left(\frac{z_\sigma^*(\theta_\sigma^{\#})-\theta_\sigma^{inf}}{\sigma}\right)}. \tag{AM21}$$

It is now easy to see that Condition (AM20) implies that $\frac{\partial}{\partial \sigma}\psi(\theta_\sigma^{\#}, \theta_\sigma^{inf}, \sigma) > 0$.

The above property implies that, fixing $\theta_{\sigma_z}^{inf}$, a marginal increase in the standard deviation of the agents' private information at σ_z increases $\Psi(\theta_{\sigma_z}^{inf}, \sigma_z)$. Furthermore, because the threshold $\theta_{\sigma_z}^{\#}$ solves $\psi(\theta_{\sigma_z}^{\#}, \theta_{\sigma_z}^{inf}, \sigma_z) = 0$, we have that, by increasing σ_z while keeping the threshold $\theta_{\sigma_z}^{inf}$ fixed, the policy maker guarantees that, for any $\theta > \theta_{\sigma_z}^{inf}$, $\psi(\theta, \theta_{\sigma_z}^{inf}, \sigma_z) > 0$. Next, note that $\theta_{\sigma_z}^{inf}$ is decreasing in σ_z. This follows from the fact that, for any σ_z, any $\theta > \hat{\theta}$, $\psi(\theta, \hat{\theta}, \sigma_z)$ is strictly increasing in $\hat{\theta}$ (this last property in turn follows from Lemma 2 in Angeletos at al. (2007)). From the above results, we thus have that, starting from any discriminatory policy, a reduction in the precision of the agents' private information (i.e., a marginal increase in σ_z) lowers the fundamental threshold $\theta_{\sigma_z}^{inf}$ below which regime default occurs, thus improving the policy maker's payoff. Q.E.D.

References

Alonso, R. and K. Zachariadis, 2021. Persuading Large Investors. WP, LSE.

Angeletos, G.-M., Hellwig, C., Pavan, A., 2007. Dynamic global games of regime change: Learning, multiplicity, and the timing of attacks. *Econometrica* 75 (3), 711–756.

Brunnermeier, M.K., Pedersen, L.H., 2005. Predatory trading. *The Journal of Finance* 60, 1825–1863.

Guo, Y., Shmaya, E., 2019. The interval structure of optimal disclosure. *Econometrica* 87 (2), 653– 675.

Iachan, F. S., Nenov, P. T., 2015. Information quality and crises in regime-change games. *Journal of Economic Theory* 158, 739–768.

Inostroza, N., 2021. Persuading Multiple Audiences: An Information Design Approach to Banking Regulation. WP, University of Toronto.

Li, F., Yangbo, S., Zhao, M., 2021. Global manipulation by local obfuscation. WP, UNC Chapel Hill.

Mensch, J., 2018. Monotone Persuasion. WP, Hebrew University of Jerusalem.

Milgrom, P., Segal, I., 2002. Envelope theorems for arbitrary choice sets. Econometrica 70, 583–601.

Morris, S., Daisuke, O., Takahashi, S., 2021. Information design in binary-action supermodular games. WP, MIT.

Segal, I., 2003. Coordination and discrimination in contracting with externalities: Divide and conquer? *Journal of Economic Theory* 113 (2), 147–181.