Exposure to the Unexpected, Duty of Disclosure, and Contract Design∗

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Abstract

We introduce the concept of “relative exposure to the unexpected” and show that, when parties to a contract covertly engage in information acquisition prior to designing the contract, the concept underlies a variety of phenomena such as expectation conformity (the parties’ tendency to conform to the intensity of information gathering that is expected of them), excessive investment in information acquisition, and the welfare merits of mandatory disclosure laws. The paper develops a simple framework that captures the above phenomena in a unified manner and sheds light on a few policy implications.

Keywords: information acquisition, contracts, adverse selection, endogenous contractual incompleteness, caveat emptor, mandatory disclosures, expectation conformity.

JEL numbers: C72; C78; D82; D83; D86.

1 Introduction

A key activity of public and private decision-makers is to design, negotiate, and enter contractual agreements, such as private contracts, laws, or international treaties. To this purpose, they hire engineering, financial, or legal experts, and set aside other activities in order to gather information about the implications of alternative designs. Information gathering simultaneously determines the degree of contractual incompleteness and influences transactional frictions that arise from asymmetric information. It is therefore central to the functioning of markets, regulation, and political decisions.

This paper studies a simple contracting environment in which the parties covertly engage in information acquisition prior to designing the contract, and the information they gather changes the nature of the contract between the two parties. Its contribution is three-fold. First, it brings together

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within a unified framework and generalizes several otherwise disconnected contributions. Second, it demonstrates that a sufficient statistic, the “relative exposure to the unexpected,” underlies a variety of concepts and phenomena that are relevant for this type of situations. Third, it analyzes the welfare implications of mandatory disclosure laws.

In the model, described in Section 2, parties can opt for a standard contract. This contract may or may not be adequate. If it is not adequate, the parties are exposed to a loss of efficiency, whose relative incidence to the players, which we label “relative exposure to the unexpected,” plays a key role both in contract design and in the subsequent contract renegotiation. For example, the physical design may turn out to be inappropriate for the buyer’s needs or unexpectedly costly for the seller to produce; alternatively, the parties may omit to index the contract to some key contingencies, with serious impact on risk-sharing or behavior. Information gathering prior to signing the contract can allow the parties to prevent such inefficiencies and write a better contract. However, in addition to its effect on efficiency, information also involves rent-seeking aspects. Consequently, the resulting contract may be too complete or too incomplete, depending on the situation under consideration.

Section 3 analyses the case in which only one party can engage in information gathering. The information that is acquired is “hard” and hence verifiable. Both the incentive to acquire information and to disclose it increases with the information acquirer’s relative exposure to the unexpected. We show that expectation conformity (namely, a player’s incentives to conform to what is expected out of them), and its corollary (the possibility of multiple equilibria) obtains if and only if the information acquirer is relatively more exposed to the unexpected; in this case, the party gathering information is always better off in the equilibrium with the lowest intensity of information acquisition.

Section 4 studies mandatory disclosure. The notion of mandatory disclosure is complex, and has been the object of tensions in contract law for a long time. For example, in Macquarie International Health Clinic Pty Ltd vs Sydney South West Area Health Service (2010, NSWCA 268), the Court held that the obligation of “good faith” does not require parties to compromise their own commercial interests, but that parties must cooperate, including disclosing information, in a reasonable way to achieve the contract’s objectives. The Macquarie decision allows for the possibility of delay in disclosure; in some cases, indeed, a party may refrain from disclosing at the contractual stage, but then disclose and renegotiate before news about contractual inefficiencies publicly accrue. Our results identify precise conditions under which a party has incentives to delay disclosure and show that such conditions are related to a party’s relative exposure to the unexpected. The possibility of delay in disclosure raises the issue of the welfare merits of making disclosure a legal requirement, which we also investigate in Section 4.

Section 4.1 shows that, in the absence of disclosure regulation, strategic dissimulation of information at the contracting stage occurs when the other party to the contract is relatively more exposed to the unexpected and post-contract renegotiation does a good job at limiting the damage associated with a wrong design. In this case, mandatory disclosure, studied in Section 4.2, has both

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1See Kronman (1978) seminal paper on the topic.
positive and negative welfare effects. On the positive side, it prevents efficiency losses associated with delayed voluntary disclosure. On the negative side, it involves two costs. First, the party gathering information, having fewer options on what to do with it, is less keen on gathering socially useful information in the first place (we show that late disclosures occur only when information gathering is inefficiency low from a social standpoint). Second, the information acquirer is deterred from disclosing information that is acquired after the contract is signed whenever, as is likely, it cannot be proven whether this information was acquired prior to or after the contracting date. So, while mandatory disclosure is always optimal for exogenously acquired information, its optimality when information is endogenously acquired requires strong conditions. In that respect, the optimal treatment of contract-relevant information bears resemblance to intellectual property law.\(^2\)

Finally, Section 4.3 investigates the optimal mechanism design when the (balanced-budget) court (a) either observes whether a party discloses information post contract, (b) or observes that the party concealed information prior to contracting (observing only that the contract is renegotiated does not alter the equilibrium outcomes, as the parties can undo any penalty that one pays to the other because of renegotiation). We show that, in general, optimally designed penalties do better than simple mandatory disclosure rules, but the key trade-offs in their design are similar to those associated with the choice of whether or not to make disclosure mandated.

Section 5 extends the analysis to two-sided information gathering and analyses contract completeness. It finds that the pattern of information acquisition is skewed (only one party acquires information) if and only if the information acquirer socially over-invests in information acquisition. We use these results to discuss the broader impacts of disclosure laws. Section 6 concludes.

**Relationship to the literature.** The paper is related to several strands of the literature. The informal literature on transaction costs economics (e.g., Williamson (1975, 1985)) and the more formal literature on ownership (e.g., Grossman and Hart (1986), Hart and Moore (1990)) identifies the difficulty of foreseeing actions and contingencies as a foundation for otherwise unsatisfactory contracts. More recently and more closely related to this work, Gabaix and Laibson (2006) emphasizes the strategic lack of disclosure in an environment with “shrouded attributes,” but does not analyze information acquisition. The information-gathering approach to incomplete contracts is taken up in Bolton and Faure-Grimaud (2010), Tirole (2009), Von Thadden and Zhao (2012), and Zhao (2015). Relative to these papers, we bring a unifying framework and obtain new insights regarding mandatory disclosure, regulation, the design of optimal penalties, and two-sided information acquisition.

The idea of modeling contractual incompleteness as an investment determining the probability that players learn the need to modify a default contract traces back to Bajari and Tadelis (2001). A key contribution of our paper is in relating such investment to the subsequent decision of whether to disclose hard information proving the need to change the contract, and in using the analysis do

\(^2\)The latter specifies that an inventor is entitled to benefit from their innovation if the latter is novel, non-obvious, and useful. The goal of intellectual property law is to reward inventors without generating socially costly “underserved” rents.
discuss the welfare merits of mandatory disclosure laws and other related policy interventions.

The paper is also related to Tirole (2009). It shares with that paper the idea that pre-contractual information gathering can be insufficient or excessive (as it reflects rent-seeking) compared to what is efficient, and the focus on how information acquisition impacts the degree of contractual incompleteness.\(^3\) That earlier paper, set in a context akin to that of Example 1 in the present paper, goes in some directions that are not (but could be) explored in the present paper: the role of competition, and the effect of vertical integration and relationship contracting. The current paper’s contribution is different. First, it identifies a summary statistic, the relative exposure to the unexpected, that encapsulates the various results on the extent of disclosure, expectation conformity, excess- or under-investment in information acquisition, or the impact of disclosure regulation. Second, it analyses the welfare implications of disclosure laws.

Disclosure games, in which a sender holds hard (verifiable) information and decides whether to reveal it to a receiver who then takes an action affecting both parties, have received significant attention.\(^4\) The corresponding literature has investigated factors that prevent unraveling (uncertainty as to whether the sender has received information, or costly disclosure; see Dekel et al (2018) for recent developments in this literature) or lead to the disclosure of bad news (various forms of reputation; see Bourjade and Jullien (2011), Dziuda (2011), Grubb (2011), and Ispano (2018)). Most papers on disclosure games (e.g., Okuno-Fujiwara et al (1990)) take the information structure as given; exceptions include Matthews and Postlewaite (1985), Shavell (1994) and Dang (2008).

In parallel, the information transmission literature pioneered by Crawford-Sobel (1982) posits that the sender holds soft (unverifiable) information and transmits a message to the receiver, who then takes an action affecting both.\(^5\) Reviewing this equally sizable literature lies outside the scope

\(^{3}\)Related angles are examined, among others, in Spier (1992), Gabaix and Laibson (2006), and Bolton and Faure–Grimaud (2010).

\(^{4}\)See Sobel (2013) for a survey. Okuno-Fujiwara et al (1990) assume that evidence is exogenous and contains a general analysis of when voluntary disclosure leads to full disclosure. Shavell (1994) provides the basic analysis of costly information acquisition prior to disclosure (earlier work on the topic includes Farrell (1985) and Sobel (1989)). The paper, however, does not emphasize strategic complementarities between anticipated and optimal information gathering. Rather, it provides a careful analysis of an informal idea of Kronman (1978), according to which mandated information disclosure is likely to reduce incentives for information acquisition. The paper further shows that the impact of required disclosure is rather asymmetric: It discourages buyers but not sellers from acquiring socially valuable information. Kartik et al (2017) consider a multi-sender model of disclosure of hard information; among other things, the paper shows that disclosure strategies are strategic substitutes under a disclosure cost and strategic complements under a concealment cost. Another recent contribution to the literature is Hoffmann et al (2020) in which the sender secretly collects information about the receiver’s preferences and selectively discloses it subject to absorption capacity constraints limiting the number of dimensions over which information can be disclosed (which is another impediment to unraveling).

\(^{5}\)Much of the attention has focused on the polar cases of hard and soft information. In general, how hard (per se informative) communicated information is depends on the – endogenous – communication efforts exerted by the sender and the receiver (see Dewatripont and Tirole (2005)).
of this paper. Most contributions again take the information structure as given.6

A rich literature discusses the implications of the possibility of acquiring information during bargaining (after a contract offer is on the table) rather than before bargaining. In Cremer and Khalil (1992), a contract is designed so as to alter the incentives to the other party’s information acquisition (see also Lewis and Sappington (1997), and Szalay (2009) for a more general treatment). A principal/buyer wants to buy an amount of input from an agent/seller. The agent learns her marginal cost for free when starting the production process, i.e., after signing the contract; the agent can alternatively pay a fixed cost to learn her marginal cost prior to accepting the principal’s contract. So, information acquisition is purely wasteful, and the optimal contract is designed so as to deter any information acquisition. More generally, a contract offer may be used to induce or deter information acquisition, but, in practice, much information is acquired prior to bargaining, vindicating the approach in the present paper. Cremer and Khalil (1994) and Cremer et al. (1998a,b) study information acquisition prior to contracting. For example, in Cremer et al. (1998a), the agent can learn her marginal cost of production, or choose not to learn it, prior to contracting (she learns it costlessly after contracting). The principal, who does not observe the agent’s information acquisition strategy and can only guess it, then offers a contract. The principal, who moves second, cannot prevent socially wasteful information acquisition. The menu offered by the principal embodies three options if the are two cost levels, as the agent in equilibrium plays a mixed strategy and may not acquire information. Our work complements this line of research by providing a more flexible framework that we use to investigate the welfare effects of mandatory disclosure laws.

The present paper also connects to a large legal literature on caveat emptor (“Let the buyer beware”). Caveat emptor, which has a long tradition dating back to the Roman times, provides in common law a safe harbor for a seller not to disclose information to the buyer.7 Several arguments have been made in its favor. The first is that, under symmetric information, the seller’s liability may alter consumer choices when the buyer’s relative tastes for price and defects are heterogenous (Buchanan (1970)). The main justification is to avoid frivolous lawsuits by having buyers bear the risk of loss and thereby imposing upon them a duty to inspect. In particular, caveat emptor holds it that the seller has no duty to disclose patent or obvious defects to the buyer. The possibility of asymmetric information has over time led courts and legislative enactments to what Johnson (2008) calls “caveat emptor light”, more in line with Kronman (1978)’s influential theory of information as a property right. Kronman (1978) makes a distinction between casually and deliberately acquired

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6 An exception is Pei (2015), who allows the sender to choose her information structure from a “rich set” (if a partition belongs to the sender’s choice set, then any coarser partition also belongs to this set). The paper shows that, in contrast with Crawford-Sobel (1982), all equilibria are such that the sender communicates all she knows to the receiver, and characterizes the equilibrium outcomes. Gentzkow and Kamenica (2017) obtain a related result in a game with hard information. They show that, if information acquisition is costly and observable by the decision maker (who observes the selected experiment but not its realization), then disclosure requirements have no effect on the set of pure-strategy equilibria.

7 French law, by contrast, tends to view dissimulation as contrary to good faith bargaining.
information. Information acquired by the homeowner as a by-product of living in the house (the presence of termites, the occasional flooding of the basement) should be disclosed, while that deliberately acquired by the seller is akin to a property and should not be subject to a disclosure obligation. If the presence of termites or the occasional flooding are just redistributive (they affect the sale price, but not who should own the house for what purpose), this precept runs counter Cooter and Ulen (2016), who distinguish between “productive” and “redistributive” facts, and argue that the redistributive facts should not be subject to a disclosure duty. The present paper’s contribution relative to this legal literature is to offer a formal framework in which this legal debate can be analyzed. It considers a situation that is more general than a sale (in the model, both parties have a post-contractual stake, and indeed may want to renegotiate the contract to their mutual advantage), and it shows the role played by a sufficient statistic (a player’s relative exposure to the unexpected) in determining bargaining behavior and outcomes, and in assessing the merits of alternative disclosure laws.

2 Model

2.1 Description

Players and contingencies. Two risk-neutral players engage in a contractual relationship with unknown payoffs. The state space is binary, with \( \Omega = \{ \omega, \hat{\omega} \} \), and it is common knowledge that each party assigns prior probabilities \( q \) and \( \hat{q} \) to \( \omega \) and \( \hat{\omega} \), respectively. In state \( \omega \), the players’ initial (pre-search) and final (post-search) information is \( \emptyset \) regardless of the intensity of information acquisition (there is no snag/flaw to be discovered, say). In state of nature \( \hat{\omega} \), instead, player \( i \) learns the state \( \hat{\omega} \) with probability \( \rho_i \), whereas, with probability \( 1 - \rho_i \), he learns nothing (i.e., her information remains equal to \( \emptyset \) ), where \( \rho_i \) measures the intensity of information acquisition. The draws are independent between the two players, conditionally on the state (this assumption plays no role until Section 5.2). The information that player \( i \) receives proving the state is \( \hat{\omega} \) is hard and therefore can be disclosed in a verifiable manner to player \( j \neq i \) if player \( i \) decides so. We provide a few examples below. The cost of information acquisition is \( C_i(\rho_i) \), with \( C_i'(\rho_i) > 0 \) for \( \rho_i > 0 \), \( C''_i(\rho_i) > 0 \) for all \( \rho_i \), \( C_i'(0) = C_i(0) = 0 \), and \( C_i'(1) = \infty \).

A prominent interpretation of this information structure goes as follows: Knowing the state of nature allows a trade, a technological choice, or a contract design to match the state. In state \( \omega \), a known (“business as usual” or “boiler plate”) choice, \( a \), is optimal. This choice is also available in state \( \hat{\omega} \). However, state \( \hat{\omega} \) optimally requires an original response, \( \hat{a} \), which becomes known to the players only after learning the state. The very act of learning the state \( \hat{\omega} \) reveals the nature of \( \hat{a} \).

There is little loss of generality in assuming that designing a solution comes for free with the discovery that the standard design is not the appropriate one. Suppose that the first search only shows the party that there is a snag with the standard design (i.e., reveals that the state is not \( \omega \)). One could then envision a second search in which the party looks for the appropriate solution. The resulting search cost could be subsumed in the overall cost of learning.
don’t know. Accordingly, beliefs are a martingale.\textsuperscript{9}

An alternative interpretation of the model is in terms of contract incompleteness. Contracts often fail to describe some states of nature or actions to be undertaken in certain states of nature. Incompleteness is often motivated by the presence of “unforeseen contingencies.” The question as to whether omitted contingencies are truly unforeseeable or just too costly to conceive and include in the contract can be sidestepped under an approach in terms of “I did not think about/I did not have this in mind when making the decision”.

\textit{Payoffs.} The players can contract on an action and transfer money between themselves. As mentioned already, action \( a \) (alternatively, \( \hat{a} \)) is jointly optimal, i.e., maximizes the sum of the two players’ gross surplus, in state \( \omega \) (alternatively, \( \hat{\omega} \)). Initially, only the state space and action \( a \) are known to the players. Searching for information leads each player to either learn nothing, or to learn both \( \hat{\omega} \) and \( \hat{a} \): As we noted, learning that the state is \( \hat{\omega} \) also reveals to a player what’s to be done in that state of nature (that is, action \( \hat{a} \)) and conversely.

We let \( U_i \) (alternatively, \( \hat{U}_i \)) denote player \( i \)'s gross surplus in state \( \omega \) (alternatively, \( \hat{\omega} \)) when the optimal action for that state is taken (\( a \) when the state is \( \omega \) and \( \hat{a} \) when the state is \( \hat{\omega} \)). We also denote by \( U = \Sigma_i U_i \) and \( \hat{U} = \Sigma_i \hat{U}_i \). We then denote by \( \delta \geq 0 \) the deadweight loss associated with choosing action \( a \) in state \( \hat{\omega} \) (recall that, in state \( \omega \), only action \( a \) is available; see footnote 9). Such a loss can then be decomposed into the respective losses to the two players, \( \delta_i, i = 1, 2 \), with \( \Sigma_i \delta_i = \delta \). These payoffs should be interpreted as post-renegotiation payoffs in case the initial contract is eventually renegotiated, in which case \( \delta \) can be interpreted as the “adjustment cost,” which is incurred in case the parties fail to specify the right design \( \hat{a} \) upfront at the initial contractual stage. These payoffs are gross of monetary transfers between the two players and of the information-gathering costs.

The assumption that payoffs are renegotiation-inclusive deserves some comment. An improper design, by definition, leads to a higher cost or to a lower customer satisfaction (or both). When the state of nature is realized, it may be too late to change the design, and the deadweight loss is fully incurred (the “no-renegotiation case”). Alternatively, the loss may be reduced, or even eliminated, by altering the design and incurring some associated adjustment cost (the “renegotiation case”). One would then expect that the earlier the state of nature is publicly revealed, the lower the adjustment cost (for example, fewer investments will be sunk by the seller and the buyer).

Figure 1 represents the players’ joint payoffs as a function of the state and the action specified at the contractual stage.

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\textsuperscript{9}The binary-choice structure can alternatively be derived by assuming that the action set is composed of the status-quo action \( a \), and of a large number of known, but ex-ante indistinguishable alternative actions. With probability \( q \), all these other actions give very negative gross payoffs to the players. With probability \( \hat{q} = 1 - q \), instead, one of the alternative actions gives a higher total surplus than action \( a \), whereas the remaining alternatives to the status quo still give very negative gross payoffs. The “relevant alternative” can be labeled \( \hat{a} \) and the state in which there is such a relevant alternative is labelled \( \hat{\omega} \).
This formalism exhibits flexibility also along another dimension, which we will later make use of. In a first interpretation of the model, the information held by the parties can be disclosed voluntarily prior to contracting or is not disclosed at all until the state of nature is publicly realized. As we clarify in due time, the incentives the players face in such a situation are the same as under a mandatory disclosure regime in which any post-contractual disclosure is deemed by courts a violation of good-faith bargaining (a party disclosing information after a contract was signed but before the state is publicly revealed may be interpreted as concealing information that was already available at the contractual stage, even when this is not the case). A different situation arises when a party can conceal information at the contractual stage and reveal it after the contract is signed but before the state of nature is fully revealed, without courts ruling against such a behavior (we will refer to such a possibility as delayed disclosure). Formally, such a situation is similar to the first one that arises when players can disclose only prior to contracting, but with a lower realized deadweight loss in case of delayed disclosure. We will come back to this situation in Section 4.

We assume that each player’s outside option (that is, her payoff in case of no trade with the other party) is equal to zero. Finally, we assume that $U, \hat{U} \geq 0$, which means that, under the appropriate design, there are gains from trade (at least weakly) in either state.\footnote{The assumption that $\hat{U} \geq 0$ is without loss of generality; once the parties learn that the state is $\hat{\omega}$, because they face no residual uncertainty anymore, they can always decide to leave the relationship and enjoy their outside options, if this is the efficient course of action. The assumption that $U \geq 0$, instead, is with loss of generality; it rules out the possibility that the parties stay in the relationship when, under the default design $a$, their total surplus is below their outside options, because they hope that the state is $\hat{\omega}$ and that this event will eventually become known to them. This possibility, however, does not seem particularly relevant for the applications the analysis is meant for.}

**Bargaining outcome.** Let $w_i$ denote player $i$’s bargaining power, or weight, in any negotiation between the two players, with $\Sigma_i w_i = 1$. We assume that transfers negotiated between the two players are such that player $i$ (alternatively, $j$) reaps a fraction $w_i$ (alternatively, $w_j$) of the total gains from trade expected by any third-party (e.g., an arbitrator), given the information disclosed by the two players, and given the parties’ (equilibrium) investments in information acquisition. For example, suppose that, in equilibrium, player $i$ is expected to invest $\rho_i$ in information acquisition...
and disclose with probability \( d_i \) in state \( \hat{\omega} \) (as explained above, disclosure is possible only in this state) and, likewise, that player \( j \) is expected to invest \( \rho_j \) and disclose with probability \( d_j \). Let \( q' \) (alternatively, \( \hat{q}' \)) denote the posterior probability of the state being \( \omega \) (alternatively, \( \hat{\omega} \)) in the absence of any disclosure, given the players’ equilibrium strategies.\(^{11}\) Using Bayes’ rule

\[
q' = \frac{q}{q + q(1 - \rho_i d_i)(1 - \rho_j d_j)} = 1 - \hat{q}'.
\]

In case of no disclosure, the transfer \( t_i \) to player \( i \) (which is equal to \(-t_j\) where \( t_j \) is the transfer to player \( j \)) is thus given by the unique solution to

\[
q'U_i + \hat{q}'(\hat{U}_i - \delta_i) + t_i = w_i\left[q'U + \hat{q}'(\hat{U} - \delta)\right]
\]

and yields player \( i \) a fraction \( w_i \) of the total surplus \( q'U + \hat{q}'(\hat{U} - \delta) \) expected under the equilibrium strategies. Note that the term in square brackets in the right-hand side is the total surplus expected by a third party in case of no disclosure. The transfer \( t_i \) is thus determined so as to give to player \( i \) a fraction \( w_i \) of the expected total surplus.

Likewise, when one of the two players discloses, thus revealing that the state is \( \hat{\omega} \) (recall that the disclosed information is hard), then the transfer to player \( i \) is set to satisfy

\[
\hat{U}_i + t_i = w_i\hat{U}.
\]

The same division of surplus is used to determine transfers ex-post, in case parties renegotiate the initial contract. That is, ex-post transfers are determined so that each player \( i \) receives a fraction \( w_i \) of the total surplus from renegotiating the initial contract. We also assume that the gross payoffs in each of the two states are such that, when the transfers between the two players are determined by the rule described above, each player prefers contracting with the other player to her outside option (accounting for the fact that, off path, beliefs may differ from equilibrium beliefs \( q' \) and \( \hat{q}' \)). In the Appendix, we identify the precise conditions that guarantee that this is the case and show that these conditions are always met when, given \( \delta \) and \( U_i - \hat{U}_i \), the gross surplus \( U_i \) is large enough for all \( i \).

The advantage of the protocol above is that it permits us to capture the incentives the two players face when it comes to the collection of hard information and the choice of whether or not to disclose it in the simplest possible way. In particular, it eliminates both signaling through the choice of \( t_i \) (with the associated multiplicity of equilibrium offers driven by the players’ out-of-equilibrium beliefs) and the possibility that a player collects information not to disclose it, but to reject offers by the opponent that deliver a payoff below their outside option. While both possibilities are certainly of interest, they are not novel and distract from what the paper delivers. Importantly, the outcomes of the above protocol coincide with those under the familiar Rubinstein-Stahl sequential bargaining game when (a) offers are frequent (i.e., when the delay between offers vanishes), (b) the players’ participation

\(^{11}\)Hereafter, we use the “prime” sign to denote a third party’s posterior beliefs, in the absence of disclosure, computed using the players’ equilibrium strategies.
constraints under the above protocol are satisfied, and (c) players hold passive beliefs. The last assumption, which amounts to the refinement that players do not change their beliefs over the information held by the opponent when they receive an off-path offer, seems justified in our setting, given that all types of each player have the same preferences over the negotiated price, which makes signaling implausible.

The following concept will play an important role in our analysis:

**Definition 1.** Player $i$’s relative exposure to the unexpected is given by

$$
\sigma_i \equiv \left[ U_i - (\hat{U}_i - \delta_i) \right] - w_i \left[ U - (\hat{U} - \delta) \right].
$$

Player $i$ is relatively more exposed to the unexpected if $\sigma_i > 0$.

Intuitively, player $i$ loses (or gains) gross surplus $U_i - (\hat{U}_i - \delta_i)$ when the default design $a$ is specified in the contract and the realized state is $\hat{\omega}$ rather than $\omega$. From an ex-ante viewpoint, the player may be able to appropriate a fraction of the total loss $U - (\hat{U} - \delta)$, the magnitude of which depends on the player’s bargaining weight $w_i$. Note that the measure $\sigma_i$ is indeed a relative exposure measure as $\Sigma_i \sigma_i = 0$. Accordingly, we say that player $i$ is relatively more exposed if $\sigma_i > 0$.

The parameter $\sigma_i$ will play an important role as a sufficient statistic underlying our various results. It allows us to give a formal meaning to Cooter and Ulen (2016) distinction between “productive and redistributive facts”. The authors acknowledge that information acquisition usually unveils both productive and redistributive elements. In our model, $\sigma_i$ has a clear redistributive/zero-sum nature, whereas $\hat{q}\delta$ captures the productive stake in information acquisition (equivalently, the expected losses that the two players can avoid by discovering that the state is $\hat{\omega}$ and writing a contract specifying the appropriate action $\hat{a}$ for that state).

**Timing.** In Section 3, we will assume that disclosure occurs either prior to contracting or never until the state of nature is publicly realized (as anticipated above, this situation will be shown to be equivalent to one in which disclosure is mandatory by law). This assumption does not rule out renegotiation after the state of nature is publicly disclosed, as shown by the two illustrations discussed in Subsection 2.2. By contrast, the case of delayed disclosure will be studied in Section 4. The timing, in the case of pre-contractual disclosure, is summarized in Figure 2 and goes as follows:

1. The parties secretly and non-cooperatively choose how much information to acquire (formally captured by the probability $\rho_i$ of learning the state $\hat{\omega}$, when relevant);

2. Parties either learn that the state is $\hat{\omega}$ or do not receive any information (i.e., receive the null signal $\emptyset$);

3. Parties simultaneously choose whether to disclose hard information proving to the other party that the state is $\hat{\omega}$, when this is the case;

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12 The proof follows from familiar arguments and is thus omitted; it is, however, available upon request.
In the absence of any disclosure, the transfer $t_i$ between the two parties is determined by a third party (e.g., an arbitrator) according to (1) in conjunction with action $a$. If one of the parties discloses, in which case the state $\hat{\omega}$ is revealed, the action $\hat{a}$ is chosen and the transfer $t_i$ is determined according to (2). In either case, the transfer is determined at stage 4 and is not part of a pre-determined contract.

The state of nature is publicly realized.\textsuperscript{13} Note that one can, without loss of generality, assume that, if exerted, the outside option is exerted at stage (4) as a party can always choose to not invest in information acquisition, thus incurring no cost.

![Figure 2: Timing](image)

2.2 Illustrations

**Example 1: Buyer-seller game.**
The seller’s cost of supplying the buyer is known and equal to $c$. The buyer’s gross surplus is $B$ if the design matches the state of nature, but only $b < B$ if design $a$ is chosen in state $\hat{\omega}$. If design $a$ is chosen and state $\hat{\omega}$ is revealed, the buyer can still enjoy the full surplus $B$ but only if the seller incurs some adjustment cost $\alpha \geq 0$ (such a cost may reflect the changes to the product necessary to deliver the value $B$ to the buyer in state $\hat{\omega}$). The information disclosed in state $\hat{\omega}$ then represents evidence that the initial is inappropriate to meet the buyer’s needs or more generally to deliver utility $B$ to the buyer.

Figures 3 and 4 summarize the gross payoffs of the two parties without and with renegotiation, respectively.

1. **No renegotiation** (the ex-post adjustment cost is large: $\alpha \geq B - b$).

In this case, the gross payoffs satisfy the following conditions:

$$U = \hat{U} = B - c, \quad \delta_B = B - b, \quad \delta_S = 0, \quad \delta = B - b.$$\textsuperscript{11}

\textsuperscript{13}As noted above, once the state is publicly revealed, the parties can still renegotiate to their mutual advantage. This is the case if action $a$ is specified in the original contract, the state of nature is $\hat{\omega}$, and the adjustment cost to replace action $a$ with action $\hat{a}$ is not too large: see the examples in Section 2.2.
Ex-post renegotiation implying that the buyer is relatively more exposed to the unexpected. As a result, and hence and of nature model. Contract incompleteness is then related to the amount of pre-contractual

\( \rho \) assumes exogenous one-sided cognition, with \( c \) are state independent provided that the action matches the state of nature. More generally, one can allow these payoffs to depend on the state of nature; a case in point is the shrouded environment. “Contract incompleteness” is then related to the amount of pre-contractual

Example 1: the buyer-seller game

Consider the celebrated buyer-seller paradigm. The seller’s cost of supplying the buyer is \( \alpha < B \) without and with renegotiation, respectively. Then

\[ \delta_B = (B - b) - w_B[(B - b) - \alpha] = w_S(B - b) + w_B\alpha \]

and

\[ \delta_S = -w_S(B - b - \alpha) = \alpha - \delta_B \]

and hence

\[ \delta = \alpha \quad \text{and} \quad \sigma_B = w_S(B - b). \]

Note that, for \( \alpha = B - b \), the expressions are the same as for the no-renegotiation case.

This model allows one to get at the notion of contract incompleteness in an otherwise familiar modeling environment. Contract incompleteness is then related to the amount of pre-contractual information gathering and is measured by the probability that the wrong design is adopted in state of nature \( \tilde{\omega} \); this is a rather natural definition, especially when some adjustment/renegotiation occurs in that configuration.
Figures 3 and 4 describe symmetric versions of the buyer-seller game: The players’ payoffs are state independent provided that the optimal (state-dependent) action is specified in the original contract. More generally, one can allow these payoffs to depend on the state; a case in point is the shrouded attributes model, to which we turn next.

**Example 2: Generalized Gabaix and Laibson (2006) shrouded attributes model**

A seller may disclose to a buyer that the satisfactory consumption of a basic good may require the supply of an add-on also provided by the seller. The cost to the seller of the basic good is equal to \( c \). The add-on may or may not be needed for the buyer to be able to enjoy the basic good. Let \( \omega \) correspond to the state in which the add-on is not needed and \( \tilde{\omega} \) the state in which it is. The cost of the add-on to the seller is \( \hat{c} \). Gabaix and Laibson (2006) assume an exogenous information structure in which the seller is perfectly informed whereas the buyer is unable to acquire information. Although their model is phrased in terms of boundedly rational agents, the key insights can be illustrated in the context of our framework. The situation examined in Gabaix and Laibson (2006) corresponds to a special version of our model in which stages (1) and (2) are irrelevant (and hence omitted in the timing described in Figure 5), \( \rho_S = 1 \) and \( \rho_B = 0 \).

**Figure 5: Timing in the exogenous-information shrouded-attributes model**

For simplicity, assume that, in state \( \tilde{\omega} \) in which the add-on is needed, the basic good brings no value to the buyer, unless combined with the add-on. The value the buyer assigns to the good (in its basic configuration in state \( \omega \) or together with the add-on in state \( \tilde{\omega} \)) is \( v \), with the latter drawn from \( \mathbb{R}_+ \) according to the distribution \( F \) and privately observed by the buyer ex-post, i.e., at stage (5). The motivation for a random value is that it generates a downward-sloping demand (and hence a monopoly distortion) in case of ex-post contracting (i.e., in case the need for the add-on is disclosed only ex-post, after the initial contract is signed) without introducing adverse selection on the buyer side at the ex-ante contracting stage. Let \( r^m \) denote the monopoly price for the add-on in state \( \tilde{\omega} \) when the state is revealed only ex-post, after the initial contract is signed. Because, in this state, the value for the basic good alone is zero, the value the buyer assigns to the add-on coincides
with the value \( v \) the buyer assigns to the basic good together with the add-on (equivalently, one can think of \( v \) as the buyer’s value for the “satisfactory” version of the good, which coincides with the basic specification in state \( \omega \) and to the combination of the basic configuration with the add-on in state \( \hat{\omega} \)). Hence, the ex-post monopoly price for the add-on in state \( \hat{\omega} \) is the value of \( r \) for which \((r - \hat{c})[1 - F(r)]\) is the highest. The lack of ex-ante adverse selection implies that if the need for the add-on is disclosed at the ex-ante stage, then it is optimal for the seller to price it at marginal cost, in which case there is no ex-post distortion. Let \( S(r) = \int_0^\infty (v - r)dF(v) \) denote the buyer’s expected surplus when the price for the add-on is \( r \), gross of the price of the basic good. Gabaix and Laibson (2006)’s model thus corresponds to a buyer-seller game in which the payoffs are asymmetric across the two states when \( \hat{c} > 0 \). The two players’ payoffs (gross of the price paid by the buyer for the basic good) are represented in Figure 6.

<table>
<thead>
<tr>
<th>( \omega )</th>
<th>( \hat{\omega} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>( \int_0^\infty vdF(v), -c )</td>
</tr>
<tr>
<td>( \hat{a} )</td>
<td>( \int_0^\infty (v - r^m)dF(v), -c + (r^m - \hat{c})[1 - F(r^m)] )</td>
</tr>
<tr>
<td></td>
<td>( \int_\hat{c}^\infty (v - \hat{c})dF(v), -c )</td>
</tr>
</tbody>
</table>

Figure 6: Payoffs in the Gabaix and Laibson (2006) model (gross of the ex-ante transfer)

Hence, modulo the exogeneity of the information structure, this model is also a special case of our general model in which the following conditions hold:

- \( w_S = 1 \) (the seller is a price setter);
- \( \delta = [S(\hat{c}) - S(r^m)] - (r^m - \hat{c})[1 - F(r^m)] \) is the deadweight loss associated with non-disclosure and ex-post monopoly pricing (so the “wrong design” de facto corresponds to a contractual failure in which the seller, by not disclosing, fails to commit to the cost-based add-on price);
- \( \sigma_B = \int_0^{r^m} v dF(v) + [1 - F(r^m)]r^m \) (the buyer’s relative exposure to the unexpected is equal to the buyer’s loss of utility under ex-post monopoly pricing and combines the loss from foregone consumption (in case \( v \in [0, r^m] \)) and the extra payment (in case \( v > r^m \)).
2.3 Special cases of information gathering

The analysis in the next sections allows for fairly flexible payoff specifications and for different forms of information gathering. The following special cases, however, may be particularly salient in applications.

1. **Inner-looking** information gathering: The uncertainty that player $i$ faces is about her own payoff only: $U_j = \hat{U}_j = \hat{\delta}_j$. For example, in a natural variant of the buyer-seller game described above, the unknown state of nature might raise the production cost from $c$ to $C > c$ (or possibly lower it), without affecting the buyer’s valuation (i.e., $b = B$).

2. **Outer-looking** information gathering: The uncertainty that player $i$ faces is about player $j$’s payoff only: $U_i = \hat{U}_i = \hat{\delta}_i$. This situation is precisely the one described in the buyer-seller game of Example 1 above, where the seller may learn about whether the design of the good fits the buyer’s needs, and where the seller’s cost is fixed at $c$.

3. **Mixed case**: Player $i$’s information gathering uncovers information that is payoff-relevant for both players. For example, in the Gabaix and Laibson (2006) model considered above, ex-post renegotiation makes the state of nature relevant for both parties.

3 One-sided information gathering under voluntary disclosure

Suppose that only player $i$ can gather information (player $i$ will then be referred to as the information-acquiring player, whereas $j$ will be referred to as “the other player”). For example, only the seller is able to find out whether a design flaw may prevent the buyer from fully enjoying the consumption experience (outer-looking information gathering), or only the buyer may invest in learning whether the design matches her own needs (inner-looking information gathering).

3.1 Disclosure decision

In this subsection, we fix the investment in information gathering by player $i$ at $\rho_i = \rho^*$ and investigate player $i$’s incentive to disclose.$^{14}$

When player $i$ is expected to disclose with probability $d^* \in (0,1)$, the posterior probability that player $j$ assigns to the state being $\omega$ in the absence of any disclosure (which coincides with the probability assigned to the same event by the third party, e.g., an arbitrator negotiating the transfer on behalf of the two players) is equal to

$$q'(\rho^*, d^*) = \frac{q}{1 - (1 - q)\rho^*d^*}. \tag{3}$$

$^{14}$Because player $i$ is the only player to acquire and disclose information, we drop the subscript $i$ from $\rho_i$ and $d_i$ to ease the notation.
Suppose first that player \( i \) is expected to disclose with certainty when she finds that the state is \( \tilde{\omega} \) (i.e., \( d^* = 1 \)) (anticipating the analysis in the next subsection, note that this is necessarily the case when information is endogenous and costly to acquire, as assumed here, for there is no point in acquiring information if it is weakly suboptimal to disclose it).

Suppose player \( i \) learns that the state is \( \tilde{\omega} \). By disclosing this information, she obtains a payoff equal to \( w_i\tilde{U} \), that is, her share \( w_i \) of the (complete-information) total surplus \( \tilde{U} \). By not disclosing, instead, she obtains

\[
\tilde{U}_i - \delta_i + t_i,
\]

where \( t_i \) is the transfer associated with the default design \( a \), as given in Condition (1). Simple algebra reveals that player \( i \) prefers disclosing to not disclosing if and only if

\[
w_i\delta \geq q' (\rho^*, 1) \sigma_j, \tag{4}
\]

where \( \sigma_j \) is player \( j \)'s relative exposure to the unexpected.

Condition (4) says that player \( i \) prefers to disclose whenever her share of the deadweight loss, \( w_i\delta \), exceeds the cross-subsidy embodied in the transfer \( t_i \), namely \( q' (\rho^*, 1) \sigma_j \). This cross-subsidy is proportional to the posterior probability \( q' (\rho^*, 1) \) that player \( j \) (as well as the third party negotiating the transfer \( t_i \) on behalf of the two players) assigns to the erroneous state, \( \omega \), in the absence of disclosure, when player \( i \) is expected to search with intensity \( \rho^* \) and disclose with certainty. The right-hand-side of (4) is thus a rent that player \( i \) obtains thanks to the mis-pricing induced by the third party’s incorrect beliefs.

If Condition (4) is violated, disclosing information when expected to do so is not sequentially rational for player \( i \). In particular, when \( w_i\delta \leq q\sigma_j \), because, for any probability \( d^* > 0 \) by which player \( i \) discloses, \( q' (\rho^*, d^*) > q = q' (\rho^*, 0) \), there is no equilibrium in which player \( i \) discloses information with positive probability after learning that the state is \( \tilde{\omega} \). If, instead,

\[
q\sigma_j < w_i\delta < q' (\rho^*, 1) \sigma_j,
\]

then, we have that, while disclosing with certainty cannot be part of an equilibrium, disclosing with probability \( d^* \in (0, 1) \), with the latter appropriately defined, can be consistent with player \( i \)'s rationality. In fact, using again Condition (1), we have that, when player \( j \) and the third party expect player \( i \) do disclose with probability \( d^* \), player \( i \) is indifferent between disclosing and not disclosing.

---

15 There are in general two possible motivations for player \( i \) to acquire information. The first (on which we focus here) is to disclose it so as to avoid the deadweight loss of a wrong contract design. The second is to decide whether or not to trade at all with player \( j \) (this alternative motivation is ruled out by the assumption that, given the transfer \( t_i \) proposed by the third party, player \( i \)’s expected surplus is higher than her outside option, no matter the player’s posterior beliefs). See also the discussion in the Appendix.
if and only if \( q'(\rho^*, d^*) \sigma_j = w_i \delta \), which implies that \( d^* \) must be equal to the unique solution to

\[
\frac{q}{1 - (1 - q) \rho^* d^*} = \frac{w_i \delta}{\sigma_j}.
\] (5)

The following proposition summarizes the above results:

**Proposition 1.** [incentive to disclose prior to contracting] Suppose that player \( i \)'s investment in information acquisition is fixed at \( p_i = \rho^* \). In the voluntary disclosure game, player \( i \)'s disclosure behavior is unique.

1. **Player \( i \) discloses with certainty if** \( w_i \delta > q'(\rho^*, 1) \sigma_j \);
2. **Player \( i \) does not disclose if** \( w_i \delta \leq q \sigma_j \);
3. **Player \( i \) randomizes between disclosing and not disclosing if** \( q \sigma_j < w_i \delta \leq q'(\rho^*, 1) \sigma_j \). **In this case, the probability** \( d^* \) **with which player \( i \) discloses is given by the unique solution to** (5).

### 3.1.1 Applications

- **Full bargaining power.**

In some applications, the informed player has full bargaining power so that \( w_i = 1 \). Condition (4) then simplifies to

\[
\delta \geq q'(\rho^*, 1)[U_j - (\hat{U}_j - \delta_j)].
\]

The left-hand side is the total deadweight loss from not changing the design when it is optimal to do so, whereas the right-hand side is the surplus that \( i \) can extract from \( j \) because of the latter’s misperception of the state. In turn, this is equal to the extra utility \( U_j - (\hat{U}_j - \delta_j) \) that \( j \) expects to derive from the state being \( \omega \) instead of \( \hat{\omega} \), scaled by the probability \( q'(\rho^*, 1) \) that \( j \) assigns to the state being \( \omega \) when \( i \) is expected to disclose with certainty. In other words, the left-hand side is the efficiency gain from taking the correct action in state \( \hat{\omega} \) whereas the right-hand side is the speculative gain from taking advantage of the opponent’s misperception of the state.

- **Symmetric buyer-seller game.**

In Example 1, \( \sigma_B = w_S(B - b) \). So, in the absence of renegotiation (for example because the adjustment costs are high), Condition (4) boils down to \( w_B \geq -q'(\rho^*, 1) w_S \) if \( i = B \), and \( w_S \geq q'(\rho^*, 1) w_S \) if \( i = S \). In other words, when renegotiation is too costly, player \( i \) always discloses, no matter whether she is a buyer or a seller.

In the presence of renegotiation, instead, the buyer always discloses, whereas the seller discloses only if the deadweight loss (then equal to the adjustment cost \( \alpha \)) is large enough: \( \alpha \geq q'(\rho^*, 1)(B - b) \).

---

\(^{16}\)In this case, the ex-ante expected deadweight loss is equal to

\[
DWL \equiv \hat{q}(1 - \rho^* d^*) \delta = q\left[ \frac{\sigma_j}{w_i \delta} - 1 \right] \delta.
\]

17
The result in Proposition 1 implies that the seller opts for shrouded attributes (that is, does not disclose the need for the add-on when the latter is needed) if \( w_S \delta \leq q \sigma_B \). Applying the specific expressions for these variables and using the fact that \( w_S = 1 \) in the Gabaix and Laibson (2006) model, we have that the seller conceals only if

\[
\int_{\hat{\omega}}^{r_m} (v - \hat{c}) dF(v) \leq q \left[ \int_0^{r_m} v dF(v) + \left[ 1 - F(r_m) \right] r_m \right].
\]

The left-hand side of (6) is the deadweight loss associated with non-disclosure and the concomitant monopoly pricing; this loss is entirely borne by the seller who has full bargaining power as a price setter. The right-hand side is the product of the posterior probability of the good state (which is equal to the prior probability in a no-disclosure equilibrium) and the buyer’s loss of utility when the state is \( \hat{\omega} \) instead of \( \omega \). This loss is decomposed into foregone gross utility from consuming the good in its adequate specification (\( v \leq r_m \)) and extra payment (\( v > r_m \)).

### 3.2 Information gathering

We now turn to player \( i \)'s information-gathering choice. Let \( t_i(\rho^*, d^*) \) be the transfer that player \( i \) obtains in the absence of any disclosure when she is expected to exert effort \( \rho^* \) in acquiring information and disclose with probability \( d^* \). Using Condition (1), we have that

\[
t_i(\rho^*, d^*) = w_i U - U_i + \hat{q}(\rho^*, d^*) \sigma_i,
\]

where \( \hat{q}(\rho^*, d^*) \equiv 1 - q'(\rho^*, d^*) \).

Let us first look for an equilibrium in which player \( i \) does not acquire information, i.e., \( \rho^* = 0 \). Because \( C'_i(0) = 0 \), such an equilibrium exists if and only if there is no net gain from disclosure: \( w_i \delta \leq q \sigma_j \) (see Proposition 1).

We therefore focus on the more interesting case in which, in equilibrium, player \( i \) exerts effort \( \rho^* > 0 \) in acquiring information and discloses with certainty upon learning that the state is \( \hat{\omega} \). Note that the mixed-strategy region in part 3 of Proposition 1 can exist only if information is exogenous. As explained above, player \( i \) puts zero effort in acquiring information if the latter is costly and if it is weakly optimal ex-post not to disclose.

Let \( V_i(\rho, \rho^*) \) denote player \( i \)'s gross payoff when she chooses effort \( \rho \) and is expected to choose effort \( \rho^* \) and disclose with certainty in case she finds that the state is \( \hat{\omega} \). Using the result in Proposition 1, for \( i \) to strictly prefer to disclose after learning that the state is \( \hat{\omega} \), it must be that \( w_i \delta > q'(\rho^*, 1) \sigma_j \). Clearly, the same condition also implies that player \( i \) strictly prefers to disclose when her actual effort is \( \rho \) (recall that information gathering is covert). Hence, when \( w_i \delta > q'(\rho^*, 1) \sigma_j \),

\[
V_i(\rho, \rho^*) = \hat{q} \left[ \rho w_i \hat{U} + (1 - \rho) \left( \hat{U}_i - \delta_i + t_i(\rho^*, 1) \right) \right] + q \left[ U_i + t_i(\rho^*, 1) \right].
\]
For $\rho^*$ to be selected in equilibrium, it must be that

$$\rho^* \in \arg \max_{\rho \in [0,1]} \{V_i(\rho, \rho^*) - C_i(\rho)\}$$

and hence $\rho^*$ must satisfy the first-order-condition

$$C'_i(\rho^*) = \hat{q}\left[w_i\hat{U} - (\hat{U}_i - \delta_i + t_i(\rho^*,1))\right].$$

Equivalently, plugging the formula for $t_i(\rho^*,1)$ from (7) and using $\sigma_i = -\sigma_j$, we have that $\rho^*$ must satisfy

$$C'_i(\rho^*) = \hat{q}\left[w_i\delta - q'(\rho^*,1)\sigma_j\right]. \tag{9}$$

As explained above, the right-hand side of Condition (9) is the benefit of disclosing information, net of its opportunity cost. Consistently with the result in Proposition 1, Condition (9) admits a solution $\rho^* > 0$ if and only if $w_i\delta > q\sigma_j$, as $q'(\rho^*,1)$ is increasing in $\rho^*$ and is such that $q'(0,1) = q$. Furthermore, in this case, $\rho^*$ is unique if $\sigma_j \geq 0$, as $q'(\rho^*,1)$ is strictly increasing in $\rho^*$.

Comparing the equilibrium effort level to its efficient counterpart, we have that player $i$ over-invests in information gathering if the private benefit of investing in information acquisition (the right-hand side of (9)) exceeds the social benefit ($\hat{q}\delta$). This happens if and only if party $i$ is heavily exposed, in the sense that

$$q'(\rho^*,1)\sigma_i > w_j\delta. \tag{10}$$

**Proposition 2. [over-investment in information gathering]** Player $i$ over-invests in information acquisition (relative to what is socially efficient) only if player $i$ is relatively more exposed to the unexpected than player $j$, that is, only if $\sigma_i > 0$.

Next, we turn to expectation conformity, which we define as follows. Consider two effort levels $\rho$ and $\rho'$. We say that expectation conformity holds if player $i$ has more incentives to choose $\rho'$ than $\rho$ when expected to do so.\(^{17}\) That is, expectation conformity holds if and only if

$$\Gamma_i^{EC}(\rho, \rho') \equiv [V_i(\rho', \rho') - V_i(\rho, \rho')] - [V_i(\rho', \rho) - V_i(\rho, \rho)] > 0.$$ 

Now suppose that player $i$ finds it optimal to disclose information both when she is expected to exert effort $\rho$ and when she is expected to exert effort $\rho'$ (recall that there is no point for $i$ to acquire any information if she then conceals it). Simple computations then show that

$$\Gamma_i^{EC}(\rho, \rho') = q\hat{q}(\rho' - \rho)\left[\frac{1}{1 - \hat{q}\rho} - \frac{1}{1 - \hat{q}\rho'}\right] \sigma_i.$$ 

Thus expectation conformity holds if and only if player $i$ is relatively more exposed to the unexpected, i.e. $\sigma_i \geq 0$. In this case, the higher the effort expected from him, the lower the probability of

\(^{17}\)See Pavan and Tirole (2022) for a more general analysis of expectation conformity in strategic cognition.
the bad state in the absence of disclosure and so the less favorable the agreement to player \( i \) whenever she does not disclose any information.\(^{18}\) This raises player \( i \)'s incentives to acquire information.

Finally, note that the exposure to the unexpected is also the key to the existence of multiple equilibria and to the possibility that player \( i \) is better off in a low-effort equilibrium. To see this, first note that, when \( \Gamma_i^{EC} < 0 \), i.e., when \( \sigma_i < 0 \), the equilibrium effort is unique.\(^{19}\) Next, observe that, when \( \Gamma_i^{EC} > 0 \), i.e., when \( \sigma_i > 0 \), for any pair \((\rho, \rho')\), one can construct cost functions such that both \( \rho \) and \( \rho' \) are equilibrium effort levels. Finally, observe that

\[
\text{sgn} \left( \frac{dV_i(\rho^*, \rho^*)}{d\rho^*} \right) = \text{sgn} \left( \frac{\partial t_i(\rho^*)}{\partial \rho^*} \right) = -\text{sgn}(\sigma_i),
\]

which implies that, when multiple equilibria are possible (which is the case only if \( \sigma_i > 0 \)) player \( i \) is better off in the low-effort equilibrium. Summarizing the analysis above, we have the following result:

**Proposition 3.** \([\text{equilibrium investment in information acquisition}]\)

(i) Effort \( \rho^* > 0 \) can be sustained in equilibrium if and only if it satisfies Condition (9).\(^{20}\)

(ii) In any equilibrium in which \( \rho^* > 0 \), disclosure occurs with certainty.

(iii) If and only if player \( i \) is relatively more exposed to the unexpected \((\sigma_i > 0)\), then player \( j \)'s anticipation of a higher effort by player \( i \) increases the latter player’s value to acquire information (expectation conformity), thus raising the possibility of multiple equilibria.

(iv) In case of multiple equilibria, player \( i \) is better off in a low-effort equilibrium.

### 4 Strategic delay and mandatory disclosure

#### 4.1 Delayed disclosure

So far, we have assumed that a party acquiring information discloses it before contracting if she discloses it at all; in the absence of intent of pre-contractual disclosure, the party would not acquire any information. This property no longer holds if the information can be withheld and disclosed at some stage between the contracting date and the date at which the state of nature is publicly revealed. In this subsection, instead, we consider an environment where there is no penalty for withholding information other than the deadweight loss of embarking on the wrong design.

To illustrate, we modify the course of events in Figure 2 by allowing for interim renegotiation at stage (5).\(^{21}\) If the state of nature is \( \hat{\omega} \) and action \( a \) specified by the contract is renegotiated into

\(^{18}\)Use (7) to verify that, when \( \sigma_i > 0 \), in case of no disclosure, the transfer \( t_i(\rho^*, 1) \) to player \( i \) is decreasing in the effort \( \rho^* \) that player \( j \) expects from \( i \).

\(^{19}\)As anticipated above, when \( \sigma_i < 0 \) (equivalently, when \( \sigma_j > 0 \)), Condition (9) has a unique solution.

\(^{20}\)Note that, given \( \rho^* \), the function \( V_i(\rho, \rho^*) - C_i(\rho) \) is globally concave in \( \rho \).

\(^{21}\)Such interim renegotiation is not to be confused with the ex-post renegotiation that may occur after the state of nature is publicly revealed (such as those considered in the buyer-seller or in the shrouded-attributes games of Subsection 2.2).
at stage (5), the total deadweight loss is only $\beta \delta$ where $\beta \in [0,1]$: Some useless investments can be avoided, while others have already been sunk prior to this early renegotiation stage. We will not need to specify whose investments have been sunk by stage (5), as we will see that only the total deadweight loss $\beta \delta = \beta_i \delta_i + \beta_j \delta_j$ matters.

The possibility of interim renegotiation raises the informed party’s incentive to disguise her information at the contracting stage because, when concealing that the state is $\omega$, she is then able to limit the damage done by concealment while still taking advantage of the mis-pricing at stage (4) – if party $j$ is relatively more exposed, that is, if $\sigma_j > 0$.

We also allow party $i$ to exogenously receive information at stage (5): She learns the state $\omega$ (if it occurs and she did not learn it earlier) with probability $\mu$. Namely, we assume that (a) party $i$’s pre-contracting probability of being informed in state $\omega$, $\rho_i$, belongs to $[0, 1 - \mu]$ and the cost function satisfies the same properties as earlier, with $\lim_{\rho \rightarrow 1 - \mu} C_i'(\rho) = +\infty$, and (b) the total probability of learning state $\omega$ when it prevails is $\rho + \mu$.\footnote{In a multi-period extension, one would expect the probability of receiving information, $\mu$, and the remaining deadweight loss, $\beta$, to be positively correlated: As time elapses, more information is acquired and more investments are sunk, thus making the deadweight loss larger.} As we show below, the possibility for the players of receiving fortuitous information after signing the contract plays a role in determining the structure of the optimal regulation. We represent these changes in the environment in Figure 7, which describes only the contractual/post-contractual events (the pre-contractual ones are the same as in the previous sections).

\begin{figure}[h]
\centering
\begin{tabular}{c|c|c}
  (4) & (5) & (6) \\
  \hline
  Negotiate a monetary & [If voluntary disclosure] interim & State of nature realizes. \\
  transfer and action & renegotiation: $i$ may disclose $\omega$ & If the state of nature is $\omega$ and action $a$ has been \\
  $\check{a}$ if it is the only & if either she has concealed it & specified at stage (4) and not renegotiated at stage \\
  common knowledge & prior to contract design or she & (5), the deadweight loss is $\delta = \delta + \delta_i$. \\
  action & learns it at stage (5) (probability & \\
  $\check{\omega}$ if $\omega$ is common & $\mu$). & \\
  knowledge. & Disclosure of $\omega$ and & \\
 & renegotiation reduce the & \\
 & deadweight loss to $\beta \delta$ where & \\
 & $\beta \in [0,1]$. & \\
\end{tabular}
\caption{Interim renegotiation}
\end{figure}

Suppose that party $i$ is expected to invest $\rho^*$ in information acquisition and disclose with probability $\rho^*$ at stage (3). When, at stage (2), party $i$ learns that the state is $\omega$ but does not disclose

\footnote{One could alternatively assume independence between the two events (the total probability of receiving information being equal to $\rho + (1 - \rho)\mu$) or various forms of positive or negative correlation between the two events, without any significant change in the qualitative results. One could also assume that stage-(5) information acquisition, which here is assumed to be exogenous, is subject to moral hazard, again without any significant change to the insights.}
this information till stage (5), she receives a payoff equal to

\[(\hat{U}_i - \delta_i) + t_i + w_i(1 - \beta)\delta,\]

where \(t_i = t_i(\rho^*, d^*)\) now solves

\[
t_i + q'(\rho^*, d^*)U_i + \hat{q}'(\rho^*, d^*)[\hat{U}_i - \delta_i + \frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}w_i(1 - \beta)\delta]
\]

\[
= w_i \left\{ q'(\rho^*, d^*)U + \hat{q}'(\rho^*, d^*) \left[ (\hat{U} - \delta) + \frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*} (1 - \beta)\delta \right] \right\} \tag{11}
\]

with \(q'(\rho^*, d^*) = 1 - \hat{q}'(\rho^*, d^*)\) as defined in the previous section. One can verify that the transfer \(t_i\) that solves (11) is the same as in the previous section (as given in (1) or, equivalently, by (7)).

The condition for player \(i\) to disclose for sure at stage (3) \((d^* = 1)\) then becomes

\[w_i\hat{U} \geq \hat{U}_i - \delta_i + t_i + w_i(1 - \beta)\delta\]

which, after substituting for \(t_i = t_i(\rho^*, 1)\), we can rewrite as

\[w_i\beta\delta \geq q'(\rho^*, 1)\sigma_j,\] \[\tag{12}\]

where, as earlier, \(\sigma_j \equiv U_j - (\hat{U}_j - \delta_j) - w_j[U - (\hat{U} - \delta)]\) denotes player \(j\)'s relative exposure to the unexpected.

Player \(i\)'s equilibrium investment in information gathering \(\rho^*\), when the latter is positive and player \(i\) discloses with certainty at stage (3), then continues to be given by the solution to\textsuperscript{24}

\[C'_i(\rho^*) = \hat{q}[w_i\delta - q'(\rho^*, 1)\sigma_j].\] \[\tag{13}\]

In the special case in which renegotiation can take place at no cost before any investment is sunk (i.e., \(\beta = 0\)), it is optimal for player \(i\) to disclose prior to contracting if and only if she is relatively more exposed to the unexpected (i.e., if and only if \(\sigma_i \geq 0\)). In this case, the specification of the contractual terms is just a zero-sum game. More generally, we have the following result:

**Proposition 4.** \([\text{delayed disclosure}]\) Let \(\rho^{ND}(\beta)\) and \(\rho^D\) solve \(C'_i(\rho^{ND}) = \hat{q}w_i(1 - \beta)\delta\) and \(C'_i(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]\), respectively.\textsuperscript{25} There exist thresholds \(\beta^*, \beta^{**} \in [0, 1]\), with \(\beta^* \leq \beta^{**}\), such that the following are true:

1. when \(\beta \leq \beta^*\), \(\rho^* = \rho^{ND}(\beta)\) and disclosure at stage (3) occurs with probability zero;

\textsuperscript{24}The reason why the condition for the equilibrium value of \(\rho^*\) is unaffected by the possibility of a late disclosure is that the increase in the probability of finding information at stage (3) comes with a reduction in the probability of not finding any information at all (the probability \(\mu\) of receiving information at stage (5) is fixed).

Note also that for a given \(d^*\),

\[V(\rho, \rho^*) = q[U_i + t_i(\rho^*, d^*)] + \hat{q}[(1 - \mu - \rho)[\hat{U}_i - \delta_i + t_i(\rho^*, d^*)] + \rho d^*(w_i \hat{U}) + [\mu + \rho(1 - d^*)][\hat{U}_i - \delta_i + t_i(\rho^*, d^*)] + w_i(1 - \beta)\delta].\]

\textsuperscript{25}The assumption that \(\lim_{\rho \to 1 - \mu} C'_i(\rho) = +\infty\) implies that \(\rho^{ND}(\beta), \rho^D < 1 - \mu\) for all \(\beta \in [0, 1]\).
2. when $\beta^* < \beta < \beta^{**}$, $\rho^* = \rho^{ND}(\beta)$ and disclosure occurs with probability $d^*(\beta) \in (0, 1)$ given by the unique solution to $q\sigma_j/[1 - (1 - q)\rho^{ND}(\beta)d^*] = w_i\beta\delta$;

3. when $\beta \geq \beta^{**}$, $\rho^* = \rho^D$ and disclosure occurs with certainty.\textsuperscript{26}

**Proof of Proposition 4.** From the discussion preceding the proposition, we have that, when $w_i\beta\delta \leq q\sigma_j$, player $i$ never discloses at stage (3). That, in this case, the equilibrium investment in information gathering is given by $\rho^{ND}(\beta)$ follows from the fact that the benefit of cognition comes from the possibility to disclose at stage (5), thus reducing the deadweight loss from $\delta$ to $\beta\delta$ (the marginal benefit of cognition is thus equal to $\hat{q}w_i(1 - \beta)\delta$).

When, instead, $w_i\beta\delta > q\sigma_j$, then disclosure at stage (3) occurs with positive probability under any equilibrium. In particular, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, which requires that $\sigma_j > 0$, in the unique equilibrium, player $i$ invests $\rho^{ND}(\beta)$ and then discloses with probability $d^*(\beta) \in (0, 1)$ given by the unique solution to $q\sigma_j/[1 - (1 - q)\rho^{ND}(\beta)d] = w_i\beta\delta$. To see why this is the case, note that, when $\rho = \rho^{ND}(\beta)$, if player $i$ were expected to disclose with certainty, the net benefit of disclosure would be equal to $w_i\beta\delta - q'(\rho^{ND}(\beta), 1)\sigma_j < 0$, whereas, if she were expected to disclose with probability zero, the net benefit of disclosing at stage (3) would be equal to $w_i\beta\delta - q\sigma_j > 0$, implying that, in either case, player $i$ would have a profitable deviation. Hence, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, in any equilibrium in which $\rho^* = \rho^{ND}(\beta)$, the equilibrium probability of disclosure, $d^*(\beta)$, is given by the unique solution to $w_i\beta\delta - q'(\rho^{ND}(\beta), d^*)\sigma_j = 0$. Note that the last condition implies that, when player $j$ and the third party expect player $i$ to invest $\rho^{ND}(\beta)$ in information gathering and discloses with probability $d^*(\beta)$, player $i$ is indifferent between disclosing and not disclosing at stage (3). Also note that, because $\rho^{ND}(\beta)$ is decreasing in $\beta$, $d^*(\beta)$ is increasing in $\beta$.

To see that, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, there exists no equilibrium in which $\rho$ is different from $\rho^{ND}(\beta)$, first recall that, because $q\sigma_j < w_i\beta\delta$, in any equilibrium, disclosure must occur with positive probability. Next, observe that, in any equilibrium in which player $i$ discloses with probability less than 1, player $i$ must be indifferent between disclosing and concealing at stage (3), implying that the marginal benefit from information acquisition is equal to $\hat{q}w_i(1 - \beta)\delta$.\textsuperscript{27} Because player $i$’s net payoff is strictly concave in $\rho_i$, player $i$’s investment in information gathering must necessarily be equal to $\rho^{ND}(\beta)$. Finally, observe that there exists no equilibrium in which disclosure occurs with certainty. To see this, note that, if such an equilibrium existed, then player $i$’s equilibrium investment in information gathering $\rho^D$ would have to solve $C'_i(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$, with $q'(\rho^D, 1)\sigma_j \leq w_i\beta\delta$. Because $q'(\rho, 1)$ is increasing in $\rho$, and because $w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, it must be that $\rho^D < \rho^{ND}(\beta)$. However, the definitions of $\rho^D$ and $\rho^{ND}(\beta)$ imply that

$$C'_i(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j] \geq \hat{q}[w_i\delta - w_i\beta\delta] = C'_i(\rho^{ND}(\beta))$$

\textsuperscript{26} As discussed above, equation $C'_i(\rho^D) = \hat{q}[w_i\delta - q'(\rho^D, 1)\sigma_j]$ can have multiple solutions only when $\sigma_j < 0$. In this case, $\beta^* = \beta^{**} = 0$ as shown in the proof of the proposition.

\textsuperscript{27} Note that this is the marginal benefit in case player $i$ conceals at stage (3).
which is inconsistent with $p^D < \rho^{ND}(\beta)$. Hence, when $q\sigma_j < w_i\beta\delta < q'(\rho^{ND}(\beta), 1)\sigma_j$, in equilibrium, disclosure occurs with probability less than 1.

Lastly, suppose that $w_i\beta\delta \geq q'(\rho^{ND}(\beta), 1)\sigma_j$. Then the arguments above imply that, in equilibrium, $\rho^* = \rho^D$ and disclosure occurs with certainty at stage (3).

The result in the proposition then follows from the arguments above by observing that, when $\sigma_j \leq 0$, $\beta^* = \beta^{**} = 0$. When, instead, $\sigma_j > 0$, $\beta^*$ is given by the unique solution to

$$w_i\beta^*\delta = q\sigma_j$$

whereas $\beta^{**} \geq \beta^*$ is given by the unique solution to

$$w_i\beta^{**}\delta = q'(\rho^{ND}(\beta^{**}), 1)\sigma_j.$$ 

Note that, for $\beta = \beta^{**}$, $\rho^{ND}(\beta^{**}) = \rho^D$. That $\rho^{ND}(\beta)$ is strictly decreasing in $\beta$ then implies that, for $0 \leq \beta < \beta^{**}$, $\rho^{ND}(\beta) > \rho^D$. Q.E.D.

### 4.2 Mandatory disclosure

We now investigate the effects of mandatory disclosure, by which we mean policies that severely punish any late disclosure. As in the previous subsection, we assume that the court is unable to determine whether information disclosed at stage (5) was received at stage (5) or it was received at stage (2) and disclosed only at stage (5) for strategic/opportunistic reasons. Disclosures at stage (5) are severely punished, no matter when the information was received. Hence, when disclosure is mandatory, party $i$ either discloses at stage (3) or never.

Note that mandatory disclosure is irrelevant in the absence of interim renegotiation (that is, when delayed disclosure at stage (5) is either not feasible or not useful to reduce the welfare losses). This is because, as stated in Proposition 3, in that case, in equilibrium, party $i$ either acquires no information or discloses it with certainty prior to contracting. Things are different when interim renegotiation is possible. As shown above, in this case, party $i$ may find it optimal to acquire information and then conceal it till stage (5). Mandatory disclosure then has a bite, as it affects both the incentives for $i$ to disclose information at stage (3) and the incentives to acquire information in the first place.

Our next result summarizes the implications of mandatory disclosure. Let $\rho^M$ and $d^M$ denote the equilibrium investment in information acquisition and probability of disclosure under a mandatory disclosure law.

**Proposition 5.** [mandatory disclosure] Under mandatory disclosure, $\rho^M = d^M = 0$ if $w_i\delta \leq q\sigma_j$, and $\rho^M = \rho^D$ and $d^M = 1$ if $w_i\delta > q\sigma_j$, with $\rho^D$ satisfying $C_i'(\rho^D) = q[w_i\delta - q'(\rho^D, 1)\sigma_j]$. If the equilibrium under voluntary disclosure involves disclosure with probability less than one (i.e., if $\beta < \beta^{**}$) and disclosure becomes mandatory, the investment in information acquisition is reduced.\(^{28}\)

\(^{28}\)Note that $\beta^{**} > 0$ only when $\sigma_j > 0$ in which case $\rho^D$ is unique.
There exists $\mu^*$ such that, for all $\mu \geq \mu^*$, a mandatory disclosure law reduces welfare. Similarly, for $\beta$ either small or close to (but smaller than) $\beta^{**}$, a mandatory disclosure law reduces welfare.

**Proof of Proposition 5.** When disclosure is mandatory, the equilibrium cognition is the same as in the game of Section 3 in which interim disclosure (i.e., after the contract is signed but before the state is publicly revealed) is not feasible. This means that $\rho^M = 0$ if $w_i \delta \leq q \sigma_j$ and $\rho^D = \rho^D$, with $\rho^D$ satisfying $C'_i(\rho^D) = \tilde{q}[w_i \delta - q'(\rho^D, 1) \sigma_j]$ if $w_i \delta > q \sigma_j$. Because $\rho^M < \rho^{ND}(\beta)$ when $\beta < \beta^{**}$, we thus have that, in this case, mandatory disclosure reduces the investment in information acquisition. The net benefit of a mandatory-disclosure law (when it has an effect, i.e., when $\beta < \beta^{**}$) is the difference $\Delta W$ between the welfare deadweight loss under voluntary disclosure and the welfare deadweight loss under mandatory disclosure:

$$\Delta W(\beta, \mu) = \tilde{q} \delta \left[ (1 - \rho^{ND}(\beta) - \mu) + \mu \beta + \rho^{ND}(\beta)(1 - d^*(\beta)) \beta - (1 - \rho^M) \right]$$
$$= \tilde{q} \delta [\rho^{ND}(\beta)(1 - d^*(\beta)) \beta - (\rho^{ND}(\beta) - \rho^M) - (1 - \beta)\mu].$$

(14)

The benefit of the law is to avoid the deadweight loss $\beta \delta$ from delay when, under voluntary disclosure, early disclosure happens with probability $d^*(\beta) < 1$, where $d^*(\beta)$ is the value in part 2 of Proposition 4.

The law, however, has two costs. The first one is that it reduces the investment in information acquisition as $\rho^M < \rho^{ND}(\beta)$. The second one is that it deters player $i$ from disclosing information that is received exogenously with probability $\mu$ post contract, i.e., at stage (5).  

Because $\Delta W(\beta, \mu)$ is decreasing in $\mu$, there exists $\mu^*$ such that, for all $\mu \geq \mu^*$, $\Delta W(\beta, \mu) < 0$. It is also easy to see that, when $\beta = 0$, $\Delta W(\beta, \mu) = -\tilde{q} \delta [\rho^{ND}(0) - \rho^M + \mu] < 0$. In this case, interim renegotiation entails no deadweight loss and, perhaps unexpectedly, the mandatory-disclosure requirement reduces welfare by disincentivizing information acquisition (it also makes player $i$ reluctant to disclose information acquired after the contract, but, as we argued earlier, $\mu$ is likely to be close to 0 if $\beta$ is). By continuity of $\Delta W(\beta, \mu)$ in $\beta$, mandatory disclosure reduces welfare for $\beta$ strictly positive but small. Lastly, observe that, when $\beta = \beta^{**}$, $d^*(\beta) = 1$, in which case $\Delta W(\beta, \mu) < 0$. By continuity, $\Delta W(\beta, \mu) < 0$ also for $\beta$ strictly below $\beta^{**}$ but close to $\beta^{**}$. Q.E.D.

Figure 8 summarizes the analysis in the last two propositions for the case in which $0 < \sigma_j < w_i \delta/q$. Note that this case is the most interesting one. When $\sigma_j \leq 0$, there is always disclosure, whether mandated or not (i.e., $\beta^* = \beta^{**} = 0$) and hence mandatory disclosure has no bite. When, instead, $\sigma_j \geq w_i \delta/q$, then a fortiori $\sigma_j \geq w_i \beta \delta/q$ and so no-disclosure prevails whenever disclosure is voluntary ($\beta^* = \beta^{**} = 1$). Furthermore, there is no information acquisition under mandatory disclosure ($\rho^M = 0$) in which case mandatory disclosure is clearly welfare-reducing as it discourages party $i$ from disclosing information exogenously received at stage (5): $\Delta W < 0$.

---

29 Recall that the court is unable to distinguish between information disclosed at stage (5) that is received at stage (5) and information disclosed at stage (5) that is received at stage (3) and concealed for strategic reasons. As a result, any late disclosure is punished by the court, no matter when the information was received.

30 Recall that, because $\lim_{\rho \rightarrow 1 - \mu} C_i(\rho) = +\infty$, $\rho^M, \rho^{ND}(\beta) \leq 1 - \mu$. 

25
The corollary below summarizes the implications of the above results for the structure of optimal disclosure laws. To relate the results to the policy debate, we find it useful to amend our maintained assumption about the cost of cognition as follows: there exists \( \rho_{\text{ex}} \) such that \( C_i(\rho_i) = 0 \) for all \( \rho_i \leq \rho_{\text{ex}} \), \( C_i'(\rho_i), C_i''(\rho_i) > 0 \) for all \( \rho_i > \rho_{\text{ex}} \), \( C_i'(\rho_{\text{ex}}) = 0 \), and \( C_i'(1 - \mu) = \infty \) (note that the baseline model is nested with \( \rho_{\text{ex}} = 0 \)). The idea is that player \( i \) may be able to collect some information for free prior to contracting. We then have the following result (the proof follows from inspection of the formula for \( \Delta W \) in (14)):

**Corollary 1.** \([\text{optimal disclosure law}]\) The optimal disclosure law is:

- mandatory disclosure when the arrival of exogenous pre-contractual information is frequent (\( \rho_{\text{ex}} \) large), the arrival of post-contractual information is rare (\( \mu \) small), and \( \beta \) is close to \( \beta^* \);

- voluntary disclosure when the arrival of exogenous pre-contractual information is rare (\( \rho_{\text{ex}} \) small), the arrival of post-contractual information is frequent (\( \mu \) large) and \( \beta \) is either close to zero or to \( \beta^{**} \).

The corollary formalizes Kronman (1978)’s and Eisenberg (2003)’s informal argument that mandatory-disclosure laws must distinguish between the cases of information casually acquired prior to contracting and information that results from deliberate search (which, according to Kronman, must benefit
from a legal no-disclosure privilege, in effect a property right). When player $j$ is relatively more exposed to the unexpected (i.e., $\sigma_j > 0$) in which case mandatory disclosure matters, mandatory disclosure reduces the incentive of party $i$ to acquire information. It may also make party $i$ reluctant to disclose information received post contracting.

It would seem reasonable to protect particularly exposed parties (those for whom $\sigma_j > 0$) against concealment of contract-relevant information; that insight is indeed correct when information is exogenous (a person skilled in the art has casually acquired information prior to contracting through the unfolding of past professional relationships), but not if information is endogenous: When $\sigma_j$ is large (party $j$ is very exposed to the unexpected), the information-acquiring party does not acquire any information under mandatory disclosure, as she has to reveal the bad news prior to contracting. A mandatory disclosure law then de facto mandates disclosure of non-existing information. In contrast, it deters disclosure of information casually acquired after the contract is signed and it prevents the pre-contracting acquisition of useful information that would have occurred in the absence of a disclosure requirement.

4.3 Broader class of mechanisms

Can other mechanisms be employed that make contracts more efficient? Let us discuss in more detail the objective function of, and the information and instruments available to, the court or arbitrator. We assume that the court has no redistributive purposes and only aims at raising the efficiency of the relationship; so the court is preoccupied with the investment in information acquisition and the avoidance of deadweight losses, not with the monetary transfers paid or received by the two parties. In terms of instruments, the court can impose a penalty $p \geq 0$ on party $i$; the mechanism is balanced in that the penalty is paid to party $j$.

One can envision alternative information structures for the court, ranked by their fineness:

(a) At the very least, the court observes that the initial contract is renegotiated at stage (5) (implying that party $i$ has disclosed information revealing that the state is $\hat{\omega}$ at that stage), but nothing more. In particular, it does not know whether the information that gave rise to the renegotiation was received at stage (2) or at stage (5).

(b) The court can directly observe that party $i$ disclosed relevant information at stage (5). However, it cannot determine whether the information disclosed at stage (5) was received at stage (2) or at stage (5). This is the information structure considered when studying mandatory disclosure laws.\footnote{In richer settings in which disclosure can occur over an interval of time, the court can also set a deadline between the contracting date and the time at which information becomes public such that the party disclosing information is charged a penalty when disclosing after the contracting date but before the deadline. The optimal choice of such a deadline is then determined by the speed by which the deadweight loss from specifying a wrong design at the contracting date increases with time, relative to the speed by which exogenous information is expected to arrive after the contracting date.}

27
(c) When party $i$ conceals information at stage (3), the court receives evidence of this behavior with some strictly positive probability; in this case, $p$ stands for the expected penalty paid by party $i$ (nominal penalty times probability of detection) for non-disclosure at stage (3).

Below we consider the role of penalties under each of these information structures.

(a) Suppose that the court can only verify that renegotiation took place (the new contract differs from the initial one). In case renegotiation occurs, party $i$ is charged a penalty $p$ that is paid to party $j$. Such a penalty is neutral (as in Tirole 1986) because its payment is passed through in the contract renegotiation. To see this, let $t_i'$ denote the transfer received by party $i$ as part of the renegotiation; it is given by $(1 - \beta_i)\delta_i + t_i' - p = w_i(1 - \beta)\delta$. Because the penalty is paid only when the two parties agree to renegotiate the initial contract, the negotiated ex-post transfer $t_i'$ offsets one-for-one the penalty, making the latter irrelevant.

(b) Next, suppose that the court can directly observe that party $i$ disclosed relevant information at stage (5) that proves that the state is $\hat{\omega}$. It can then force party $i$ to pay a penalty $p \geq 0$ to party $j$ irrespective of whether renegotiation occurs. As a result, the penalty does not impact the renegotiation process. This implies that party $i$’s net benefit from disclosing information that is still private at stage (5) is, regardless of the stage of accrual, equal to $w_i(1 - \beta)\delta - p$. So if $p > w_i(1 - \beta)\delta$ there is never any disclosure at stage (5). The outcome is then the one under mandatory disclosure discussed in Section 4. When, instead, $p = 0$, the outcome is the one under the voluntary-disclosure regime discussed in Section 4.

Clearly, any two levels of the penalty $p$ and $p'$ such that $p, p' > w_i(1 - \beta)\delta$ are equivalent (both in terms of payoffs and welfare) because they discourage party $i$ from disclosing information at stage (5) no matter when it was received. Next, observe that, for any $p \leq w_i(1 - \beta)\delta$, when party $i$ is expected to invest $\rho^*$ in information acquisition, disclose at stage (3) with probability $d^*$, and disclose with certainty at stage (5) (both the information received and withheld at stage (3) and the one received exogenously at stage (5)) the transfer $t_i(\rho^*, d^*; p)$ that party $i$ receives at stage (4) in the absence of any disclosure now solves

\[
t_i + q'(\rho^*, d^*)U_i + \hat{q}'(\rho^*, d^*)\left[\hat{U}_i - \delta_i + \frac{\mu + \rho^*(1-d^*)}{1-\rho^*d^*}[w_i(1 - \beta)\delta - p]\right]
= w_i \left\{q'(\rho^*, d^*)U_i + \hat{q}'(\rho^*, d^*)\left[\hat{U} - \delta_i + \frac{\mu + \rho^*(1-d^*)}{1-\rho^*d^*}(1 - \beta)\delta\right]\right\}.
\]

That is, relative to the absence of the penalty (i.e., to $p = 0$), the stage-(4) transfer $t_i(\rho^*, d^*; p)$ to party $i$ increases by $\hat{q}'(\rho^*, d^*)\frac{\mu + \rho^*(1-d^*)}{1-\rho^*d^*}p$:

\[
t_i(\rho^*, d^*; p) = t_i(\rho^*, d^*; 0) + \hat{q}'(\rho^*, d^*)\frac{\mu + \rho^*(1-d^*)}{1-\rho^*d^*}p.
\]

Party $i$ then finds it optimal to disclose at stage (3) only if

\[
w_i\hat{U} \geq \hat{U}_i - \delta_i + t_i(\rho^*, d^*; p) + w_i(1 - \beta)\delta - p
\]
which, using the expression for \( t_i(\rho^*, d^*; p) \) derived above, can be rewritten as

\[
w_i\beta\delta \geq q'(\rho^*, d^*)\sigma_j - \left[1 - q'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}\right] p.
\]

Holding \( \rho^* \) and \( d^* \) fixed, we thus have that the net reduction of player \( i \)'s payoff due to the penalty, in case she withholds information at stage (3), is equal to

\[
\left[1 - q'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}\right] p > 0.
\]

Thus, for any \( p \leq w_i(1 - \beta)\delta \), an equilibrium in which party \( i \) invests \( \rho^* \) in information acquisition, discloses with probability \( d^* > 0 \) at stage (3) and discloses with certainty at stage (5) (no matter when the information available at stage (5) was received) exists if and only if Condition (15) is satisfied (with the condition holding as an equality when \( d^* \in (0, 1) \)) and

\[
C_i'(\rho^*) = \hat{q}\left[w_i(1 - \beta)\delta - q'(\rho^*, d^*)\sigma_j - q'(\rho^*, d^*)\frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*}p\right].
\]

Note that, when \( d^* \in (0, 1) \), because party \( i \) is indifferent between disclosing and not disclosing at stage (3), the marginal benefit of cognition (the right-hand-side of (16)) is also equal to

\[
\hat{q}\left[w_i(1 - \beta)\delta - p\right]
\]

which is the benefit in case player \( i \) does not plan to disclose at stage (3).

The following result then holds:

**Proposition 6.** [optimal penalty when court unable to detect information dissimulation]

Suppose that the court can verify whether or not party \( i \) discloses relevant information at stage (5) but cannot determine whether the information was received at stage (2) or at stage (5).

1. Small penalties for late disclosures (formally, \( p \leq w_i(1 - \beta)\delta \)) come with the same trade-offs as mandatory-disclosure laws (which are formally equivalent to large penalties \( p > w_i(1 - \beta)\delta \)): They increase the incentives for disclosure at stage (3) but reduce the marginal benefits to information acquisition.

2. Under the welfare-maximizing equilibrium, welfare is always higher under a penalty equal to \( p = w_i(1 - \beta)\delta \) than under a mandatory-disclosure law, with the comparison strict for example when, under a mandatory-disclosure law, there is no information acquisition in equilibrium.

3. When, in the absence of any policy intervention, the equilibrium features disclosure with certainty at stage (3), the unique optimal penalty is \( p^* = 0 \).

**Proof of Proposition 6.** The result in part 1 follows from the discussion preceding the proposition. The result in part 2 follows from the fact that any equilibrium when \( p > w_i(1 - \beta)\delta \) is also an equilibrium when \( p = w_i(1 - \beta)\delta \). When, under a policy of mandatory disclosure, the unique equilibrium features no investment in information acquisition, then it must be that \( w_i\delta \leq q'\sigma_j \). In
this case, when \( p = w_i(1 - \beta)\delta \), an equilibrium exists in which there is no investment in information acquisition but party \( i \) discloses with certainty the information received exogenously at stage (5). The equilibrium when \( p = w_i(1 - \beta)\delta \) then Pareto-dominates the one under a mandatory disclosure law. The result in part 3 is an immediate implication of part 1: Any \( p > 0 \) reduces the investment in information acquisition without augmenting the probability of disclosure. Q.E.D.

(c) Finally, suppose that the court can detect with some strictly positive probability whether party \( i \) withheld information at stage (3) and, in case it receives evidence of such a behavior, charges party \( i \) a penalty. Then, interpret \( p \) as the penalty that party \( i \) expects to pay in case information is withheld at stage (3) (thus \( p \) is the product of the probability of detection and the fine). The probability the court finds out that party \( i \) withheld information at stage (3) is independent of whether or not party \( i \) discloses at stage (5). As a result, party \( i \) always discloses the information she possess at stage (5), irrespective of when it was received and of the magnitude of \( p \). This is the key difference with respect to the scenario (b) considered above.

Because of this difference, when party \( i \) is expected to invest \( \rho^* \) in information acquisition and disclose at stage (3) with probability \( d^* \), the transfer that she receives at stage (4) in case of no disclosure at stage (3) now solves

\[
t_i + q'(\rho^*, d^*)U_i + \tilde{q}'(\rho^*, d^*) \left[ \tilde{U}_i - \delta_i + \frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*} w_i (1 - \beta)\delta - \frac{\rho^*(1 - d^*)}{1 - \rho^*d^*} p \right]
\]

\[
= w_i \left\{ q'(\rho^*, d^*)U_i + \tilde{q}'(\rho^*, d^*) \left[ \tilde{U} - \delta \right] + \frac{\mu + \rho^*(1 - d^*)}{1 - \rho^*d^*} (1 - \beta)\delta \right\}.
\]

Equivalently,

\[
t_i(\rho^*, d^*; p) = w_iU - U_i - \tilde{q}'(\rho^*, d^*)\sigma_j + \tilde{q}'(\rho^*, d^*)\rho^*(1 - d^*) \frac{1}{1 - \rho^*d^*} p.
\]

For party \( i \) to find it optimal to disclose at stage (3) with strictly positive probability, it must be that

\[
w_i\tilde{U} \geq \tilde{U}_i - \delta_i + t_i(\rho^*, d^*; p) + w_i(1 - \beta)\delta - p.
\]

Using the above expression for \( t_i(\rho^*, d^*; p) \), we can rewrite the above condition for disclosure as follows:

\[
w_i\beta\delta \geq q'(\rho^*, d^*)\sigma_j - \left[ 1 - \tilde{q}'(\rho^*, d^*)\rho^*(1 - d^*) \frac{1}{1 - \rho^*d^*} \right] p.
\]

An equilibrium in which party \( i \) invests \( \rho^* \) in information acquisition and discloses with probability \( d^* > 0 \) at stage (3) then exists if and only if Condition (17) is satisfied (with the condition holding as an equality when \( d^* \in (0, 1) \)) and

\[
C_i'(\rho^*) = \tilde{q} \left[ w_i\delta - q'(\rho^*, d^*)\sigma_j - \tilde{q}'(\rho^*, d^*)\rho^*(1 - d^*) \frac{1}{1 - \rho^*d^*} p \right].
\]

As in scenario (b) above, when \( d^* \in (0, 1) \), because party \( i \) must be indifferent between disclosing and not disclosing at stage (3), the return to investing in information acquisition in case party \( i \)
discloses (the right-hand-side of (18)) must coincide with the return
\[
\hat{q} \left[ w_i (1 - \beta) \delta - p \right]
\]
to information gathering in case party \(i\) conceals at stage (3). We then have the following result:

**Proposition 7. [optimal penalty when information dissimilation is partly detectable]**

Suppose that, in case party \(i\) conceals information at stage (3), the court finds evidence of such a behavior with strictly positive probability and then charges party \(i\) a fine.

1. **Penalties for concealing information involve the same trade-offs as mandatory-disclosure laws:** They increase the incentives for early disclosure but reduce the benefits to investing in information acquisition.

2. **A policy of mandatory-disclosure is strictly dominated by a large penalty that is paid by party \(i\) only when the court finds evidence of information concealment.**

3. **When, in the absence of any regulation, the equilibrium features disclosure with certainty at stage (3), any penalty \(p \geq 0\) is optimal.**

**Proof of Proposition 7.** The result in part 1 follows from the arguments preceding the proposition. The result of part 2 follows from the fact that investment in information acquisition and stage-(3) disclosure are the same under the two policies, but information received exogenously at stage (5) is disclosed with probability 1 under a penalty for concealment and with probability zero under a mandatory-disclosure law. Finally, Part 3 is an immediate implication of the fact that, by disclosing with probability one at stage (3), party \(i\) does not pay any penalty. Q.E.D.

The take-away message from the last two propositions is that appropriately-designed penalties (either for late disclosure or for concealment of information prior to contracting, when the latter can be detected with positive probability) do better than mandatory disclosure laws as they do not disincentivize the relevant parties from disclosing information received exogenously after the signing of the contract.

5 Two-sided information gathering

5.1 Over-investment in information acquisition implies one-sided information gathering

Under unilateral information gathering, no matter whether disclosure is mandated or voluntary, player \(i\) over-invests in information acquisition if and only if Condition (10) holds.\(^{32}\) The same

\(^{32}\)To see this, recall that, under voluntary disclosure, if player \(i\) discloses with certainty at stage (4), then her investment in information acquisition is given by \(\rho^D = \rho^*\) with the latter satisfying \(C'_i(\rho^*) = \hat{q}w_i \delta - \hat{q}'(\rho^*, 1) \sigma\), whereas, if she discloses with probability less than 1 then her investment is given by \(\rho^{ND}(\beta)\) with the latter satisfying \(C'_i(\rho^{ND}) = \hat{q}w_i (1 - \beta) \delta\). The social marginal benefit of information acquisition is equal to \(\hat{q} \delta\). Because \(\rho^{ND}(\beta) \geq \rho^D\),
condition implies that, given player $i$’s investment in information acquisition, player $j$ would not want to disclose that the state is $\hat{\omega}$ if she knew it, and therefore would not want to acquire information, however small the cost of doing so: see Condition (4). The intuition for this result is that excess investment by player $i$ occurs when the private benefit of information exceeds the social benefit. Put it differently, information, at the margin, reduces $j$’s welfare; and so player $j$ has no incentive to acquire this information, however cheap.

A simple implication of the above results is that there can be over-investment in information acquisition only by the player who is relatively more exposed to the unexpected (i.e., $\sigma_i > 0$), for example the buyer in the buyer-seller game.\(^{33}\)

**Proposition 8.** *over-investment and two-sided information gathering* \(^{33}\)Irrespective of whether disclosure is mandatory or voluntary:

(i) Under one-sided information gathering, player $i$ over-invests in information acquisition if and only if Condition (10) is satisfied;

(ii) Under two-sided information gathering, player $j$ does not invest in information acquisition, even when $C_j'(0) = 0$, if and only if (10) is satisfied.

### 5.2 Actual information gathering by both players

Next, we look for an equilibrium in which both parties invest in information acquisition (and disclose information), which, as we saw, requires that $w_i\delta + q^{'}\sigma_i > 0$ for all $i$, where $q^{'}$ is the posterior belief that the state is $\omega$ conditional on none of the parties having disclosed that the state is $\hat{\omega}$. Assume that the search outcomes are independent.\(^{34}\) Such an equilibrium must satisfy the following properties.

For all $i$, under mandatory disclosure, or if $\sigma_i \geq 0$, either $\rho_i = 0$ if $w_i\delta \leq q^{'}\sigma_j$, or $\rho_i$ is given by the solution to

$$
C_i'(\rho_i) = \hat{q}(1 - \rho_j)(w_i\delta - q^{'}\sigma_j),
$$

where the probability that the state is $\omega$ in case neither player discloses is given by

$$
q^{'} = \frac{q}{q + \hat{q}(1 - \rho_i)(1 - \rho_j)}.
$$

we then have that, when Condition (10) holds, player $i$ necessarily over-invests. When, instead, Condition (10) is violated, because $\hat{q}w_i(1 - \beta)\delta < \hat{q}\delta$, then necessarily player $i$ underinvests.

\(^{33}\)Condition (10) applied to the buyer-seller game when it is the buyer to acquire information takes the following form:

- in the absence of ex-post renegotiation: $q^{'}(\rho^*, 1) > 1$, which is impossible, irrespective of the equilibrium level of $\rho^*$, so that there is never over-investment;

- under ex-post renegotiation: $q^{'}(\rho^*, 1)(B - b) > \alpha$, which is satisfied if, for example, the adjustment cost $\alpha$ is close to zero — to see this, note that, in this application, $\rho^*$ solves $C_B'(\rho^*) = \hat{q} [w_B\alpha + \frac{q(1 - w_B)(B - b)}{1 - (1 - q)\rho^*}]$ along with the fact that $q^{'}(\rho^*, 1) = q/[1 - (1 - q)\rho^*]$.

\(^{34}\)In the symmetric case, for example, the socially optimal level of cognition per party, $\rho^{FB}$, is then given by $C'(\rho^{FB}) = \hat{q}(1 - \rho^{FB})\delta$.  

32
In the absence of mandatory disclosure and if $\sigma_i < 0$, $\rho_i$ is given by the solution to to
\[ C_i'(\rho_i) = \hat{q}(1 - \rho_j)w_i\delta. \] (21)

Note that the first-order condition (19) can be rewritten as
\[ C_i'(\rho_i) = \hat{q} \left[ (1 - \rho_j)w_i\delta + (-\sigma_j)\frac{q}{1 - \rho_j} + \hat{q}(1 - \rho_i) \right]. \] (22)

If $\sigma_j \leq 0$, that is, if player $i$ is relatively more exposed to the unexpected, player $i$’s reaction curve ($\rho_i$ as a function of $\rho_j$) is downward-sloping. Strategic substitutability in information gathering holds also if $\sigma_j > 0$. In fact, the derivative of the right-hand-side of Condition (22) with respect to $\rho_j$ is
\[ \hat{q} \left[ -w_i\delta + \sigma_j (q')^2 \right], \] which is negative when both players engage in information acquisition.

Assuming that the functions $C_i$ are sufficiently convex so that the reaction curves cross only once, thus defining a stable equilibrium, we then have the following result:

**Proposition 9.** [actual two-sided information gathering] Assume independent searches and a stable equilibrium. Then, when both players invest in information acquisition, their investments are locally strategic substitutes, reflecting the public good nature of information. Suppose that player $i$ is relatively more exposed to the unexpected ($\sigma_i > 0$); then lifting the mandatory disclosure requirement increases $j$’s investment and reduces $i$’s.

### 6 Concluding remarks

Information acquisition, broadly defined, is at the core of informational asymmetries, and therefore frictions in contracting. This paper introduces the concept of *relative exposure to the unexpected* and shows that the latter is a unifying factor underlying players’ incentives to acquire information, the benefits to align their investments to other players’ expectations, the incentives to disclose hard information, and the welfare implications of pre-contractual disclosure obligations.

Disclosure is a central topic in law and economics. We have formalized some of the relevant trade-offs and used the model to assess the welfare merits of various policy proposals and identify novel policy recommendations. Clearly, applications abound beyond the framework developed here. A case in point is patenting, as the search for prior art embodies both efficiency and rent-seeking aspects. As in this paper, the inventor may casually or deliberately acquire information about relevant prior art and choose whether or not to disclose this information to the patent office when applying for a patent. By concealing prior art, the intellectual property (IP) owner can exploit the monopoly power conferred by the patent beyond what motivated by the necessity to incentivize R&D toward novel content. In this context, the patent office does not negotiate a monetary transfer, but chooses the patent’s coverage and scope, both of which impact the inventor’s incentives and total welfare.

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35Note that in this case player $j$ always discloses.
Disclosure rules play an important role also for the diligence of search, the opposition process, and the choice of what to include in a patent. Assessing the welfare merits of various policy interventions requires developing a proper framework.

Another case in point is disclosure in standard-setting processes. Many standard setting organizations (SSOs) require that any IP owner participating in working group discussions disclose any potentially relevant patent rights they own that they know of, or reasonably should know of. Such requirements allow the SSO members to assess the nature of IP ownership covering the proposed standard. On the other hand, search and disclosure are costly for IP owners; their patent portfolios may have thousands of items, and they may need to search for “the needle in the haystack”. Furthermore, early disclosure of plans may limit the IP owner’s ability to get future patent awards and may convey information to rivals about what the IP owner intends to do with its existing patents. Again, the terms of the disclosure requirement are complex (type of disclosures, timing of information release) and the duty of disclosure is worth theoretical and empirical investigations.

Finally, many contracts (such as laws, international agreements) involve more than two protagonists and their negotiation obeys certain protocols which are likely to influence information acquisition. We leave these topics and many other issues broadly related to information acquisition in contracting not covered in the present paper for future research.

7 Appendix: Participation Constraints

In this Appendix, we show that the assumption that $U_i$ is large (for given $U_i - \hat{U}_i$), $i = 1, 2$, guarantees that, under the assumed bargaining protocol, all the equilibria in the baseline model in which participation is compulsory are also equilibria when players can reject the third party’s offers. The analysis below considers the case in which $\beta = 1$ (the model of Sections 2 and 3). Because each player’s payoff is weakly higher when disclosure can be delayed to stage (5) than when it must be done at stage (3) or never thereafter, the conditions below guarantee participation also in the model of Section 4 in which $\beta < 1$. Furthermore, because $\hat{U} \geq 0$ and because, under the assumed protocol, each player gets a payoff equal to $w_i\hat{U}$, $i = 1, 2$, in case of disclosure, to establish the result, it suffices to consider the payoff that each player expects in the absence of disclosure.

7.1 One-sided information gathering

Consider first the case where only player $i$ can acquire information. 

*Player j*. In the absence of any disclosure by player $i$, when the proposed transfer is the one in (1), player $j$’s expected payoff when accepting the offer is equal to $w_j[q'U + \hat{q'(\hat{U} - \delta)}]$, where $q'$ and $\hat{q'}$ are shortcuts for $q'(\rho^*, d^*)$ and $\hat{q'(\rho^*, d^*)} = 1 - q'(\rho^*, d^*)$, respectively, with $q'(\rho^*, d^*)$ as defined in (3). Because $U$ and $\hat{U}$ are non-negative and because $\hat{q'} \leq \hat{q}$, a sufficient condition for player $j$’s participation constraint to be satisfied is that the expected aggregate surplus under the common
prior \((q, \hat{q})\) is non-negative:

\[ qU + \hat{q}(\hat{U} - \delta) \geq 0. \tag{23} \]

Clearly, because in any equilibrium in which player \(i\) does not acquire information, \(q' = q\), Condition (23) is also necessary to guarantee participation in equilibria involving no information acquisition.

**Player \(i\).** The analysis is more complex for the player engaging in information acquisition, because the decision of whether or not to accept the third party’s offer depends on whether the player’s actual investment in information acquisition is equal to the equilibrium one and on the information received by the player. After choosing \(\rho_i = \rho^*\), where \(\rho^*\) is the equilibrium investment level, player \(i\) clearly prefers accepting the mediator’s contract to her outside option. To see this, consider first the case in which player \(i\) discovers that the state is \(\hat{\omega}\). By disclosing, player \(i\) obtains \(w_i\hat{U}\) which is strictly higher than her outside option given that \(U, \hat{U} \geq 0\). If not disclosing (followed by accepting the mediator’s offer) gives player \(i\) a payoff even higher than the one under disclosure, such payoff must be higher than the one under the outside option. Hence, no matter whether on path player \(i\) discloses or not, upon learning that the state is \(\hat{\omega}\), player \(i\)’s equilibrium payoff is always higher than her outside option.\(^{36}\)

Next, consider the case in which player \(i\) does not find evidence that the state is \(\hat{\omega}\). Her equilibrium payoff is then equal to \(w_i[q'(U) + \hat{q}'(\hat{U} - \delta)]\), where again \(q'\) and \(\hat{q}'\) are shortcuts for \(q'(\rho^*, d^*)\) and \(\hat{q}'(\rho^*, d^*) = 1 - q'(\rho^*, d^*)\), respectively. The latter’s payoff is thus weakly greater than 0 under the same assumptions that guarantee that player \(j\)’s equilibrium payoff is non-negative (e.g., when (23) is satisfied).

Now suppose that player \(i\) deviates and chooses cognition \(\rho_i \neq \rho^*\). Suppose first that \(w_i\delta \leq q\sigma_j\). The analysis in Section 3 implies that, when participation is mandatory, in the (unique) equilibrium, player \(i\) exerts no cognition, so that \(\rho^* = 0\). Thus consider the situation that player \(i\) faces when she deviates to \(\rho_i > 0 = \rho^*\). If, after deviating, player \(i\) learns that the state is \(\hat{\omega}\), condition \(w_i\delta \leq q\sigma_j\) implies that, by concealing and then accepting the mediator’s offer, player \(i\) obtains more than by disclosing and then accepting the mediator’s offer. Because the latter’s strategy gives him a payoff equal to \(w_i\hat{U} \geq 0\), concealing and then accepting the mediator’s offer is at least as profitable than dropping out and taking the outside option.

When, instead, player \(i\) does not receive evidence that the state is \(\hat{\omega}\), because

\[ t_i = w_i[q'(U) + \hat{q}'(\hat{U} - \delta)] - qU_i - \hat{q}(\hat{U}_i - \delta_i), \]

her payoff from accepting the mediator’s offer is equal to

\[ q'U_i + \hat{q}'(\hat{U}_i - \delta_i) + t_i = w_i[q'(U) + \hat{q}'(\hat{U} - \delta)] + (q' - q)[U_i - (\hat{U}_i - \delta_i)], \tag{24} \]

\(^{36}\)When, on path, player \(i\) discloses, it is possible that, by not disclosing, player \(i\) obtains a payoff \(\hat{U}_i - \delta_i + t_i < 0\) when learning that the state is \(\hat{\omega}\). This, however, has no implications for the results.
where here we abuse notation and let \( q' = q'(\rho_i) \) denote player \( i \)'s posterior that the state is \( \omega \) when investing \( \rho_i \) in information acquisition and not finding any evidence that the state is \( \omega \); similarly, we let \( q' = 1 - q'(\rho_i) \) denote player \( i \)'s posterior that the state is \( \omega \) when player \( i \)'s investment is \( \rho_i \) and she finds no evidence that the state is \( \omega \).

Under Condition (23), the above payoff is positive for any \( q' \) (equivalently, for all \( \rho_i \)) if and only if
\[
 w_i[qU + \hat{q}(\hat{U} - \delta)] + (1 - q)[U_i - (\hat{U}_i - \delta_i)] \geq 0. \tag{25}
\]
Hence, jointly, Conditions (23) and (25) imply that, when, in the game with compulsory participation, player \( i \) strictly positive in any equilibrium. Now take any equilibrium in the game with compulsory participation in which participation is compulsory:

Next, suppose that \( w_i \delta > q \sigma_j \). Recall that, when participation is compulsory, player \( i \)'s investment in information acquisition \( i \) strictly positive in any equilibrium. Now take any equilibrium in the game with compulsory participation in which player \( i \) invests \( \rho^* > 0 \) and discloses with certainty and let \( q'(\rho^*, 1) \) and \( q'(\rho^*, 1) = 1 - q'(\rho^*, 1) \) denote the posterior probability that player \( j \) and the third party assign to the state being \( \omega \) and \( \omega \), respectively, in the absence of any disclosure, when player \( i \) is expected to invest \( \rho^* \) and disclose with certainty (clearly, the same probabilities also coincide with player \( i \)'s beliefs when player \( i \) invests \( \rho^* \) and does not receive any evidence that the state is \( \omega \)). In any such equilibrium, player \( i \)'s ex-ante expected payoff, net of the cost of acquiring information, is equal to
\[
 (1 - \hat{q}\rho^*)w_i[q'(\rho^*, 1)U + (1 - q'(\rho^*, 1))(\hat{U} - \delta)] + \hat{q}\rho^* w_i \hat{U} - C_i(\rho^*)
\]
In the game with voluntary participation, the following is thus a necessary condition for player \( i \) to find it optimal to follow the same strategy as in the game in which participation is compulsory:
\[
 (1 - \hat{q}\rho^*)w_i[q'(\rho^*, 1)U + (1 - q'(\rho^*, 1))(\hat{U} - \delta)] + \hat{q}\rho^* w_i \hat{U} - C_i(\rho^*) \geq 0. \tag{26}
\]
Note that, by the law of iterated expectation, the expression in the left-hand-side of (26) is bounded from below by
\[
 w_i[qU + (1 - q)(\hat{U} - \delta)] - C_i(\rho^*)
\]
Next, recall that, in the game in which participation is compulsory, in any equilibrium in which player \( i \) invests \( \rho^* \) in information acquisition, necessarily \( w_i \delta > q'(\rho^*, 1) \sigma_j \). Now suppose that, in the game with voluntary participation, player \( i \) deviates and invests \( \rho_i \neq \rho^* \). When player \( i \) learns that the state is \( \omega \), disclosing and then accepting the third party’s offer yields him a gross payoff \( w_i \hat{U} \geq 0 \) which is higher than the payoff that player \( i \) can obtain by either concealing and then accepting the mediator’s offer or dropping out and enjoying the outside option. When, instead, player \( i \) does not receive evidence that the state is \( \omega \), accepting the third party’s offer yields a gross payoff equal to
\[
 q'(\rho_i, 1)U_i + \hat{q}'(\rho_i, 1)(\hat{U}_i - \delta_i) + t_i = w_i[q'(\rho^*, 1)U + \hat{q}'(\rho^*, 1)(\hat{U} - \delta)] + (\hat{q}'(\rho_i, 1) - \hat{q}'(\rho^*, 1))[U_i - (\hat{U}_i - \delta_i)],
\]
where \( \hat{q}'(\rho_i, 1) - \hat{q}'(\rho^*, 1) \in \left[-(1-q), 1-q \right]. \) Player \( i \) then prefers accepting the third party’s offer to her outside option if and only if

\[
    w_i[q'(\rho^*, 1)U + \hat{q}'(\rho^*, 1)(\hat{U} - \delta)] + (\hat{q}'(\rho_i, 1) - \hat{q}'(\rho^*, 1))[U_i - (\hat{U}_i - \delta_i)] \geq 0. 
\]

(27)

Note that a sufficient condition for (27) to hold is that

\[
    w_i[q'U + \hat{q}'(\hat{U} - \delta)] \geq (1-q)\left| U_i - (\hat{U}_i - \delta_i) \right|. 
\]

(28)

We conclude that Conditions (23), (25), (26), and (27), with the latter holding for all \( \rho_i \), are sufficient to guarantee that all the equilibria of the game in which participation is compulsory are also equilibria in the game in which participation is voluntary. These conditions are always satisfied when, given \( U_i - \hat{U}_i, U_i \) is large, \( i = 1, 2 \). Also note that, Conditions (25) and (27) are implied by Condition (23) in the following special cases:

- Outer-looking information gathering: \( U_i = \hat{U}_i = \hat{U}_i - \delta_i \).
- Inner-looking information gathering with unknown opportunity: \( U_i = \hat{U}_i - \delta_i \). The idea is that the known design has also known consequences so that \( U_i = \hat{U}_i - \delta_i \). But a better design in state \( \hat{\omega} \) might improve player \( i \)’s utility (say, a cost reduction equal to \( \delta_i \)).

### 7.2 Two-sided information gathering

Next, consider equilibria in settings in which both players can invest in information acquisition. As the analysis in Section 5 reveals, these equilibria involve strategic effects similar to those in the case of one-sided information gathering. Conditions (23), (25) and (27) above thus guarantee that the equilibria in the game with voluntary participation coincide with those in the game in which participation is mandatory also under two-sided information gathering.
References


