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Information Acquisition and Welfare

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We study information acquisition in a flexible framework with strategic complementarity or substitutability in actions and a rich set of externalities that are responsible for possible wedges between the equilibrium and the efficient acquisition of information. First, we relate the (in)efficiency in the acquisition of information to the (in)efficiency in the use of information and explain why efficiency in the use is no guarantee of efficiency in the acquisition. Next, we show how the acquisition of private information affects the social value of public information (*i.e.* the comparative statics of equilibrium welfare with respect to the quality of public information). Finally, we illustrate the implications of the results in a monetary economy with price rigidities and dispersed information about productivity shocks.

Keywords: Endogenous information, strategic complementarity/substitutability, externalities, efficiency, welfare, price-setting complementarities

JEL Codes: C72, D62, D83, E50

1. INTRODUCTION

Many economic environments feature a large group of agents taking decisions under dispersed information about relevant economic fundamentals affecting individual preferences and/or the profitability of investment opportunities. In such environments, the information that the agents choose to collect about the underlying fundamentals is determined by their desire to align their actions with the fundamentals, as well as with other agents' actions. Furthermore, because acquiring information is costly, the precision of information acquired in equilibrium depends on the quality of public information provided by policy makers, statistics bureaus, and the like.

In this article, we investigate how the precision of private information acquired in equilibrium differs from the socially optimal one and relate the discrepancy between the two to the primitives of the environment, as well as to the way information is used in equilibrium. We then use such a characterization to examine how the social value of public information is affected by the endogenous response in the acquisition of private information.

To abstract from specific institutional details and identify general principles while retaining tractability, in the first part of the article we conduct our analysis within the flexible family of

Gaussian-quadratic economies studied in the literature (see, among others, Morris and Shin, 2002, Angeletos and Pavan, 2007, Hellwig and Veldkamp, 2009, and Myatt and Wallace, 2012; see also Vives, 2008, for an excellent overview of this literature and its applications). This framework allows for either strategic complementarity or substitutability in actions as well as for a rich set of externalities that are responsible for possible wedges between the equilibrium and the efficient use of information. Contrary to the previous literature, however, we allow the agents to choose the precision of their private information before they commit their actions. We assume that the acquisition of private information is costly and allow for an arbitrary cost function. Importantly, and realistically, we allow the agents to adjust the precision of their private information in response to variations in the quality of information provided by policy makers, statistics bureaus, and other sources of public information. Controlling for the endogenous response in the acquisition of private information has implications for the social value of public information and may revert some of the conclusions identified in the previous literature, as we explain in due course below.

Our first result characterizes the precision of private information acquired in equilibrium. It establishes that the latter is decreasing in the precision of public information, with a degree of substitutability between the two that is increasing in the importance that the agents assign to aligning their actions with those of other agents (that is, in the equilibrium degree of coordination).[1] Importantly, we show that, while the intensity of the substitution effect depends on the strength of the coordination motive, its sign does not: irrespective of whether the economy features strategic complementarity or substitutability in actions, an increase in the precision of public information always crowds out the private information acquired in equilibrium.

We then proceed by characterizing the precision of private information that a benevolent planner would like the agents to acquire so as to maximize welfare, defined as the ex ante expected utility of a representative agent. Such a characterization is one of the distinctive contributions of the article. We start by showing that the precision of private information acquired in equilibrium is typically inefficient, even in those economies in which the use of information (*i.e.* the mapping from information to actions) is efficient. The reason why efficiency in the use does not guarantee efficiency in the acquisition of information is that agents may suffer, or benefit, from the dispersion of individual actions in the cross section of the population and such dispersion naturally depends on the precision of private information. When such externality has a direct, non-strategic, effect on individual utilities (as is the case, for example, in beauty contests, as well as in economies with price-setting complementarities) it is unlikely to be internalized in equilibrium and is responsible for a wedge between the equilibrium and the efficient acquisition of private information, despite the economy responding efficiently to the information it collects. More precisely, we show that in economies in which the use of information is efficient, the precision of private information acquired in equilibrium is inefficiently high if agents benefit from the dispersion of individual actions in the population, whereas it is inefficiently low if agents suffer from such a dispersion.

Moving away from economies in which the use of information is efficient, we then consider separately two families of economies in which the inefficiency in the acquisition of information originates in the use of information. The first family comprises economies in which the equilibrium under complete information is first-best efficient, but in which the equilibrium degree of coordination (*i.e.* the importance that agents assign to aligning their actions with the actions of others) differs from the socially optimal degree of coordination (defined to be the importance that the planner assigns to such alignment in actions). The second family comprises economies in which the equilibrium degree of coordination coincides with the socially optimal one, but

1. Although established in a different setting, the substitutability between public and private information is also documented in a recent paper by Myatt and Wallace (2012). The focus of that paper is, however, different from the one in the present study, as discussed below in the related literature section.

in which the complete-information equilibrium fails to be first-best efficient. In both cases, we assume away externalities from the dispersion of individual actions so as to isolate the novel effects brought in by the inefficiency in the use of information.

Consider first economies in which the inefficiency in the acquisition of information originates in the discrepancy between the equilibrium and the socially optimal degrees of coordination. We show that the precision of private information acquired in equilibrium is inefficiently low when the equilibrium degree of coordination exceeds the socially optimal one, whereas it is inefficiently high when the equilibrium degree of coordination falls short of the socially optimal level. This is because economies in which agents coordinate too much are also economies in which the sensitivity of equilibrium actions to private information relative to public is inefficiently low. Because agents expect to respond to their private information less than what is efficient, they also underinvest in information acquisition. The opposite is true in economies in which the equilibrium degree of coordination falls short of the socially optimal one: in this case, agents overrespond to private information and, as a result, overinvest in information acquisition. Note that these results hold irrespective of whether the economy features strategic complementarity or substitutability in actions and irrespective of the cost of information acquisition.

Next, consider economies in which the inefficiency in the acquisition of information originates in the inefficiency of the complete-information equilibrium actions. We show that the precision of private information acquired in equilibrium is inefficiently low if and only if the sensitivity of the complete-information equilibrium actions to the fundamentals falls short of the sensitivity of the first-best allocation. In other words, the direction in which the economy fails to respond to changes in fundamentals under complete information determines the direction in which it fails to invest efficiently in information acquisition, under dispersed information.

Clearly, the cases considered above do not cover all possibilities. There are economies in which the inefficiency in the acquisition of information comes from a combination of the three channels discussed above: (i) the presence of externalities from the cross-sectional dispersion of individual actions, (ii) the discrepancy between the equilibrium and the efficient degree of coordination, and (iii) the discrepancy between the complete-information equilibrium actions and the first-best allocation. By isolating the source of the inefficiency, the economies discussed above represent useful benchmarks that one can then use to examine more complex economies.

Importantly, understanding what primitive sources are responsible for inefficiencies in the acquisition of private information has implications for the social value of public information. While previous research focused on the *partial* effect that more precise public information has on welfare holding fixed the precision of private information, here we investigate the *total* effect, taking into account how variations in the quality of public information trigger variations in the precision of private information acquired in equilibrium. In particular, we show that, irrespective of whether the economy features strategic complementarity or substitutability in actions, the crowding-out effects of public information on private information increase the social value of public information in economies that overinvest in the acquisition of private information, while the opposite is true in economies that underinvest in information acquisition. Interestingly, such crowding-out effects may change not only the size but also the sign of the social value of public information. More precisely, we show that there exists a critical threshold $\Delta > 0$ for the discrepancy between the equilibrium degree of coordination, α, and the socially optimal degree of coordination, α^*, such that, whenever $\alpha - \alpha^* < \Delta$, such crowding-out effects can turn the social value of public information from negative to positive, but never from positive to negative. Note that this result is true irrespective of whether or not the economy is efficient under complete information, of whether or not agents benefit from the dispersion of individual actions in the population, and of the details of the cost function for the acquisition of private information. In other words, whenever the importance that the agents assign to aligning their individual decisions does not exceed by too much the socially optimal level, the crowding-out effects of public information on private information are never strong enough to turn negative the social value of public information.[2]

In the second part of the article, we show how the insights from the Gaussian-quadratic model may help also outside this model. We consider a fully microfounded economy with price rigidities, monopolistic competition, and isoelastic preferences, along the lines of those studied in Hellwig (2005) and Roca (2010). Contrary to these papers, however, we endogenize the acquisition of private information. In these economies, agents (in the role of price setters) respond not only to variations in the average action (the average price) but also to variations in the volatility of the average price and in the dispersion of prices in the cross-section of the population. As a result, the equilibrium best responses are more complex than in the Gaussian-quadratic model. Nonetheless, the key insights from the Gaussian-quadratic model about the sources of inefficiency in the acquisition of information extend to this richer economy. We also identify primitive conditions for this economy to overinvest in the acquisition of private information in terms of risk-aversion, technology, and substitutability of goods across producers.

The rest of the article is organized as follows. We briefly review the pertinent literature in Section 2 below. Section 3 contains all results for the Gaussian-quadratic model, while Section 4 contains the results for the monetary economy with price-setting complementarities. Section 5 concludes. All proofs for the Gaussian-quadratic model are in Appendix A, whereas the derivations for the monetary economy are in Appendix B.

2. RELATED LITERATURE

Private information acquisition in coordination settings: The role of private information acquisition in coordination settings has been explored recently in Hellwig and Veldkamp (2009), Maćkowiak and Wiederholt (2009), and Myatt and Wallace (2012, 2013). Hellwig and Veldkamp (2009) show that strategic complementarities in actions induce strategic complementarity in private information acquisition (*i.e.* "agents who want to do what others do, want to know what others know"). A similar point is made in Maćkowiak and Wiederholt (2009): in a framework with rational inattention, they show that strategic complementarity in price-setting decisions leads to strategic complementarity in the price setters' allocation of attention. Myatt and Wallace (2012) and Myatt and Wallace (2013) consider information acquisition in, respectively, a beauty-contest framework and a Cournot model with differentiated goods. In these two papers, agents have access to an arbitrarily large number of information sources and the precision of each source depends on both the accuracy of the source (the "sender" noise) and the attention that an agent devotes to the source (the "receiver" noise). As in our model, and in contrast to Hellwig and Veldkamp (2009), the attention allocated to each source is a continuous choice. This difference has implications for the equilibrium determinacy: while there are multiple symmetric equilibria in Hellwig and Veldkamp (2009), the symmetric equilibrium is unique in the present paper as well as in Myatt and Wallace (2012, 2013).

Related are also two independent and contemporaneous papers that contrast the equilibrium acquisition of information to the efficient acquisition of information in business-cycle models: Llosa and Venkateswaran (2013) and Maćkowiak and Wiederholt (2013). The paper by

2. This condition is satisfied, for example, in the beauty contest model of Morris and Shin (2002). We show that the crowding-out effects of public information on private information can either increase or decrease the social value of public information in their model, depending on whether the economy collects too much or too little private information in equilibrium. However, while such effects may turn the social value of public information from negative to positive, they can never turn it from positive to negative.

Llosa and Venkateswaran (2013) considers three specifications of the business cycle. The first specification is an RBC model in the spirit of Angeletos and La'O (2010), in which firms make employment decisions under dispersed information about the productivity of labour. The second specification is a business-cycle model with dispersed information on a nominal shock, as in Hellwig (2005). The third specification is a monetary economy with dispersed information on a productivity shock, similar to the one considered in the second part of this article. The key differences between our work and Llosa and Venkateswaran (2013) are that we (a) allow for more general preferences, and (b) consider a setting with both private and public information. Allowing for a more general information and preference structure (in particular, permitting the households to be risk averse in their consumption choices) has important implications for the conclusions about the (in)efficiency in the acquisition of information. The two papers are also fundamentally different in their objectives. While Llosa and Venkateswaran (2013) seeks to compare the equilibrium acquisition of private information (with and without policy intervention) across the aforementioned specifications of the business cycle, here we seek to identify general channels that are responsible for inefficiencies in the acquisition of private information. For this purpose, the Gaussian-quadratic model in the first part of the paper remains the ideal framework, because of its simplicity and flexibility. We then use the monetary economy application as an illustration of how the insights from the Gaussian-quadratic model may help also in fully microfounded applications.[3]

The paper by Maćkowiak and Wiederholt (2013) compares the equilibrium allocation of attention to the efficient allocation of attention in a business-cycle model with rationally inattentive producers. In addition to certain differences in the information structure, that paper differs from ours in the payoff structure. In particular, it considers a linear-quadratic specification along the lines of those considered in Angeletos and Pavan (2007) and in the first part of the present study. However, payoffs are assumed to be independent of the dispersion of actions in the cross-section of the population. As we show in the present study, such a dependence is a natural feature of many microfounded models. Importantly, abstracting from such a dependence has important implications for the comparison of the equilibrium and the efficient allocation of attention (alternatively, for the (in)efficiency of the acquisition of private information). For example, the conclusion that economies in which the equilibrium degree of coordination exceeds the socially optimal one are also economies in which agents acquire too little information (or pay too little attention to relevant events) may not hold when payoffs depend positively on the dispersion of individual actions, as is the case, for example, in the business cycle application we consider in Section 4.

Another dimension in which the present study differs from all the papers cited above is that here we also investigate how inefficiencies in the acquisition of private information affect the social value of public information.

Related are also recent papers by Szkup and Trevino (2012) and Yang (2012). Contrary to this article and the papers cited above, these works study binary-action global games of regime change and focus on how the determinacy of equilibria is affected by the endogeneity of private information. Other differences are that these papers do not study the sources of inefficiency in the acquisition of private information and do not relate them to the social value of public information.

A crucial feature of our model is that agents respond to variations in the quality of public information by changing the precision of private information they acquire in equilibrium. A similar

3. Another recent paper that studies endogenous information acquisition in a business-cycle framework is Angeletos and La'O (2013). That paper shows how policies that induce efficiency in the use of information also induce efficiency in the acquisition of information. That conclusion, however, rests on the assumption that agents can perfectly insure their consumption at no cost, an assumption that we do not make here.

timing is considered in Dewan and Myatt (2008, 2012). In a model of political leadership, party activists decide how much attention to pay to different leaders in order to coordinate their actions. Taking this into account, party leaders may decide to obfuscate the clarity of their communications. The focus of these papers is, however, very different from ours: party leaders maximize the probability of winning the election, whereas the planner maximizes welfare in our environment.

Information acquisition is also studied in Demerzis and Hoeberichts (2007), Wong (2008), and Colombo and Femminis (2008). The first paper assumes that agents choose the quality of their private information before observing the quality of public information. The second paper considers a model in which agents decide whether or not to perfectly learn the underlying state of nature taking into account the quality of information provided by a policy maker. In turn, the policy maker learns from observing the agents' average action. Colombo and Femminis (2008) consider a model in which the timing of information acquisition is the same as in the present work. However, that paper restricts attention to a beauty-contest environment *à la* Morris and Shin (2002) and assumes a linear cost for both private and public information, thus leading to the prediction that, in equilibrium, only one type of information is acquired. The key difference between the present work and the papers mentioned above is that here we seek to characterize the sources of inefficiency in the acquisition of private information and then use such a characterization to explain how the social value of public information is affected by the acquisition of private information.

Crowding-out effects of public information: Our study is also related to the literature that documents the crowding-out effects of exogenous public information on the endogenous public information aggregated by prices and other public statistics (*e.g.* Vives, 1993; Morris and Shin, 2005; Amato and Shin, 2006 and, more recently, Amador and Weill, 2010, and Vives, 2013). In these papers, more precise public information, by inducing agents to rely less on their exogenous private information, has the perverse effect of reducing the informativeness of endogenous public signals. In contrast, in the present study, more precise public information exerts a crowding-out effect on the agents' acquisition of private information. Whether such a crowding-out effect contributes positively or negatively to the social value of public information is then shown to depend on whether agents underinvest or overinvest in the acquisition of private information. In this article, we identify primitive conditions for each case.

Social value of public information: Morris and Shin (2002) show that public information may have a detrimental effect on welfare in economies resembling Keynes' beauty contests. Our study contributes to this literature by showing that, because in these economies agents may collect too much private information relative to what is efficient, more precise public information, by inducing agents to cut on their acquisition of private information, may have a net positive effect on welfare in situations in which it was shown to have a negative effect ignoring the endogeneity of private information.

Following up on Morris and Shin (2002), Cornand and Heinemann (2008), again in a beauty contest framework, show that more precise public information may have a positive effect on welfare when it reaches only a fraction of market participants.[4] A similar information structure is considered in Morris and Shin (2007); they show that, in the absence of direct externalities from dispersion, the fragmentation of information always leads to welfare losses. The welfare implications of both papers rest on the fact that some relevant information reaches only a share of market participants. In contrast, in our framework, the welfare effects of public information depend on the strategic substitutability between private and public information.

4. Morris and Shin (2002) also stimulated a debate on the empirical plausibility of their result, questioned by Svensson (2006) and reaffirmed by Morris *et al.* (2006).

The analysis of the social value of information has been extended by Angeletos and Pavan (2007) to a large class of Gaussian-quadratic economies featuring either strategic complementarity or substitutability in actions and a rich set of externalities. The analysis in Section 3 in the present study considers the same family of Gaussian-quadratic economies as in that paper. The contribution of the present study is in endogenizing the acquisition of private information and in relating the inefficiency in the acquisition process to the sources of inefficiency in the use of information. As mentioned above, this characterization has also important implications for the social value of public information.

Related is also the work by Myatt and Wallace (2014) who consider a Lucas–Phelps island economy with several (imperfectly correlated) information sources. They show that it is never optimal for a benevolent planner to provide a perfectly public or perfectly private signal. In the same spirit, Chahrour (2012) shows that a central bank may optimally choose to withhold information when private agents may miscoordinate on the sources of information they pay attention to.

The welfare implications of more transparency in public disclosures is also the theme of a few recent contributions to the macroeconomic literature on monetary policy with monopolistic competition and dispersed information (*e.g.* Hellwig, 2005; Lorenzoni, 2010; Roca, 2010; Angeletos *et al.*, 2011). None of these papers looks at how the social value of public information is affected by the inefficiency in the acquisition of private information.

3. GAUSSIAN-QUADRATIC ECONOMIES

3.1. *The model*

Agents, Actions, and Payoffs: The economy is populated by a (measure-1) continuum of agents, indexed by i, and uniformly distributed over $[0,1]$. Let $k_i \in \mathbb{R}$ denote agent i's action, K and σ_k^2, respectively, the average action and the dispersion of individual actions in the cross section of the population, and θ a random variable parametrizing the economic fundamentals, which we assume to be Normally distributed, with mean μ and variance π_θ^{-1}. Each agent i's payoff is represented by the function

$$u(k_i, K, \sigma_k, \theta).$$

As standard in the literature, we assume that (a) u is approximated by a second-order polynomial and (b) that the dispersion of individual actions σ_k^2 has only a second-order non-strategic effect on u. In other words, u is additively separable in σ_k^2 with coefficient $u_{\sigma\sigma}/2$, so that $u_{k\sigma} = u_{K\sigma} = u_{\theta\sigma} = 0$ and $u_\sigma(k, K, 0, \theta) = 0$, for all (k, K, θ).[5] In addition to the above conditions, we assume that the partial derivatives of u satisfy the following conditions: (i) $u_{kk} < 0$, (ii) $\alpha \equiv -u_{kK}/u_{kk} < 1$, (iii) $u_{kk} + 2u_{kK} + u_{KK} < 0$, (iv) $u_{kk} + u_{\sigma\sigma} < 0$, and (v) $u_{k\theta} \neq 0$. Condition (i) imposes concavity at the individual level, so that best responses are well defined, while Condition (ii) implies that the slope of best responses with respect to K is less than one, which in turn guarantees uniqueness of the equilibrium. Conditions (iii) and (iv) guarantee that the first-best allocation is unique and bounded. Finally, Condition (v) ensures that the fundamental variable θ affects equilibrium behaviour.

Timing and Information Acquisition: Each agent i observes a combination of noisy private and public signals about the fundamental θ. We summarize all public signals into a single statistics

$$y = \theta + \varepsilon$$

5. The notation u_k denotes the partial derivative of u with respect to k, whereas the notation u_{kK} denotes the cross derivative with respect to k and K. Similar notation applies to the other arguments of the payoff function.

and all private signals into the statistics

$$x_i = \theta + \xi_i.$$

The correlated noise ε is drawn from a Normal distribution, independently of θ, with mean zero and precision π_y. Each idiosyncratic noise ξ_i is drawn from a Normal distribution, independent of θ, ε, and ξ_j $(j \neq i)$, with mean zero and precision π_{x_i}. Note that, while y is common knowledge among the agents, x_i is private information to agent i. Hereafter, we refer to $\pi_z \equiv \pi_\theta + \pi_y$ as to the total precision of public information. The latter combines the precision of the common prior with the precision of the public signal y.

The economy described above is the same as in Angeletos and Pavan (2007). To this economy we add an initial stage in which agents choose the precision of their private information. In particular, we assume that, before observing the realization of the public signal y and before committing her action k_i, each agent i chooses the precision π_{x_i} of her private information. After observing the signals (x_i, y), each agent i then chooses her action k_i simultaneously with any other agent.

We denote by $C(\pi_x)$ the cost of private information acquisition and assume that C is a continuously differentiable function satisfying $C^{'}(\pi_x), C^{''}(\pi_x) > 0$, for all $\pi_x > 0$, $C'(0) = 0$ and $\lim_{\pi_x \to \infty} C'(\pi_x) = \infty$. These conditions on the cost function are not crucial for the results but facilitate the exposition by guaranteeing interior solutions. Each agent i's payoff, net of the cost of information acquisition, is then given by $u(k_i, K, \sigma_k, \theta) - C(\pi_{x_i})$.

As standard in this literature, we do not model the cost of public information. The reason is that our interest is in the inefficiencies in the acquisition of private information and on how such inefficiencies affect the marginal benefit of public information. The cost of public information is irrelevant for our analysis. It becomes relevant only if one wants to characterize the "optimal" supply of public information. This can be done by combining our results about the marginal benefit of more precise public information with a specific cost function. However, this is beyond the interest of this study.

3.2. *Equilibrium acquisition of private information*

We focus on symmetric equilibria whereby all agents acquire information of the same quality and then follow the same rule to map the information they receive into their actions. As shown in Angeletos and Pavan (2007), in any such equilibrium, given the precision of private and public information (π_x, π_z), for any (x, y), the action that each agent takes is given by

$$k(x, y; \pi_x, \pi_z) = \mathbb{E}[(1-\alpha)\kappa + \alpha K \mid x, y; \pi_x, \pi_z], \tag{1}$$

where $\kappa \equiv \kappa_0 + \kappa_1 \theta$ denotes the complete-information equilibrium action,[6] $K = \mathbb{E}[k(x, y; \pi_x, \pi_z) \mid \theta, y; \pi_x, \pi_z]$ denotes the incomplete-information average action, and $\alpha \equiv u_{kK}/|u_{kk}|$ denotes the "equilibrium degree of coordination", or, equivalently, the slope of individual best responses to variations in aggregate activity over and above the complete-information level.[7]

Now let $k(\cdot; \pi_x, \pi_z)$ denote the unique rule that solves the above functional equation. Taking into account how information is used in equilibrium, we then have that the marginal benefit that each individual assigns to more accurate private information, evaluated at the equilibrium level, is given by the following result.

6. The scalars κ_0 and κ_1 are given by $\kappa_0 \equiv \frac{-u_k(0,0,0,0)}{u_{kk}+u_{kK}}$ and $\kappa_1 \equiv \frac{-u_{k\theta}}{u_{kk}+u_{kK}}$, respectively.

7. Note that the expression in (1) does not depend on the information being Gaussian. It follows directly from the assumption that u is quadratic.

Proposition 1. *Let $\hat{\pi}_x$ be the precision of private information acquired in equilibrium. The (gross) marginal private benefit that each agent assigns to an increase in the precision of her private information (evaluated at $\pi_x = \hat{\pi}_x$) is given by*

$$\frac{u_{kk}}{2}\frac{\partial}{\partial \pi_x} Var[k-K \mid k(\cdot;\hat{\pi}_x,\pi_z),\hat{\pi}_x,\pi_z], \tag{2}$$

where $\frac{\partial}{\partial \pi_x} Var[k-K \mid k(\cdot;\hat{\pi}_x,\pi_z),\hat{\pi}_x,\pi_z]$ denotes the marginal reduction in the dispersion of individual actions around the mean action that obtains when one changes π_x around $\hat{\pi}_x$, holding fixed the equilibrium use of information (that is, holding fixed the rule $k(\cdot;\hat{\pi}_x,\pi_z)$).

In other words, in equilibrium, the marginal benefit that each agent assigns to an increase in the precision of her private information coincides with the marginal reduction in the dispersion of her action around the mean action, weighted by the importance $|u_{kk}|/2$ that the individual assigns to such a reduction. Importantly, this marginal reduction is computed holding fixed the strategy $k(\cdot;\hat{\pi}_x,\pi_z)$ that each agent plans to use after collecting the information. This is intuitive given that, from usual envelope arguments, the individual expects her information to be used optimally once acquired. As we see below, this interpretation helps us understand the sources of inefficiency in the acquisition of private information.

When applied to the Gaussian information structure described above, the result in the previous proposition leads to the following equilibrium characterization.

Proposition 2. *In the unique symmetric equilibrium, each agent acquires private information of precision $\hat{\pi}_x$ implicitly given by*

$$\hat{\pi}_x = \sqrt{\frac{|u_{kk}|\kappa_1^2}{2C'\left(\hat{\pi}_x\right)}} - \frac{\pi_z}{1-\alpha}, \tag{3}$$

where $\pi_z \equiv \pi_\theta + \pi_y$ denotes the total precision of public information.

Obviously, the precision of private information acquired in equilibrium decreases with the marginal cost of acquiring information. More interestingly, the precision of private information acquired in equilibrium increases with (i) the sensitivity of the complete-information action κ_1 to the fundamental and (ii) the curvature of individual payoffs $|u_{kk}|$. Both effects should be expected, for they imply a higher value for aligning individual actions to the fundamental. Note also that the precision of private information acquired in equilibrium, $\hat{\pi}_x$, is decreasing in the degree of strategic complementarity in the agents' actions, α. This follows from the fact that a higher degree of strategic complementarity increases the value that each agent assigns to aligning her actions with the actions of others, and hence it reduces the value that each agent assigns to learning the fundamental θ. The opposite result is obtained under strategic substitutability, *i.e.* for $\alpha < 0$.

The most interesting implication of Condition (3) is, however, the one highlighted in the following Corollary, which is instrumental to the analysis in the subsequent sections.

Corollary 1. (Crowding-out effects of public information) *(i) An increase in the precision of public information reduces the precision of private information acquired in equilibrium: $-\frac{1}{1-\alpha} \leq \frac{\partial \hat{\pi}_x}{\partial \pi_z} \leq 0$. (ii) The substitutability between public and private information is increasing in the equilibrium degree of coordination: $\frac{\partial^2 \hat{\pi}_x}{\partial \alpha \partial \pi_z} \leq 0$.*

Part (i) highlights the fact that more precise public information crowds out the precision of private information acquired in equilibrium, whereas part (ii) highlights how such crowding out increases with the equilibrium degree of coordination, α.

The intuition for part (i) is quite straightforward: when agents possess more precise public information, they can better forecast both the fundamental θ and the aggregate action K, in which case there is less value in acquiring private information. The intuition for part (ii) is that the value of public information, relative to private, is in permitting each agent to align her action with the actions of others. The higher the value that each agent assign to such alignment, the higher the value of public information relative to private, and hence the stronger the substitutability between the two sources of information.[8]

3.3. *Efficient acquisition of private information*

We now turn to the efficient acquisition of private information. We distinguish between two scenarios. In the first one, the planner can control the way the agents use the information they acquire, without however being able to transfer information from one agent to another. In the second one, we assume that the planner is unable to change the way the agents use their available information and ask the question of what precision of private information maximizes welfare when information is processed according to the equilibrium rule. In either case, the welfare criterion we adopt is the one considered in the rest of the literature—the ex ante expected utility of a representative agent.

3.3.1. Welfare-maximizing acquisition under efficient use. From Angeletos and Pavan (2007), we know that, for any (π_x, π_z), efficiency in the use of information requires that all agents follow the unique rule $k^*(\cdot;\pi_x,\pi_y)$ that solves the functional equation

$$k(x,y;\pi_x,\pi_z) = \mathbb{E}[(1-\alpha^*)\kappa^* + \alpha^* K \mid x,y;\pi_x,\pi_z],$$

where $\kappa^* \equiv \kappa_0^* + \kappa_1^*\theta$ denotes the first-best allocation,[9] $K = \mathbb{E}[k(x,y;\pi_x,\pi_z) \mid \theta, y;\pi_x,\pi_z]$ denotes the incomplete-information average action, and $\alpha^* \equiv [u_{\sigma\sigma} - 2u_{kK} - u_{KK}]/[u_{kk} + u_{\sigma\sigma}]$ denotes the "socially optimal degree of coordination". The latter is the fictitious degree of coordination that the planner would like the agents to perceive for their equilibrium actions to maximize ex ante welfare.

We then have the following result.

Proposition 3. *Suppose that the planner can control the way the agents use their available information. For any (π_x,π_z), the (gross) marginal social benefit of inducing the agents to acquire more precise private information is given by*

$$\frac{u_{kk}+u_{\sigma\sigma}}{2}\frac{\partial}{\partial \pi_x} Var[k-K \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z], \tag{4}$$

where $\frac{\partial}{\partial \pi_x} Var[k-K \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ denotes the marginal reduction in the dispersion of individual actions around the mean action that obtains when one changes π_x holding fixed the efficient use of information (that is, holding fixed the rule $k^*(\cdot;\pi_x,\pi_z)$).

8. The results in Corollary 1 are consistent with those in Propositions 1 and 2 in Myatt and Wallace (2012), who consider a more restrictive payoff specification but a more general information structure. A similar substitutability result can also be found in Hellwig and Veldkamp (2009), and in Wong (2008).

9. The scalars κ_0^* and κ_1^* are given by $\kappa_0^* = \frac{u_k(0,0,0)+u_K(0,0,0)}{-(u_{kk}+2u_{kK}+u_{KK})}$ and $\kappa_1^* = \frac{u_{k\theta}+u_{K\theta}}{-(u_{kk}+2u_{kK}+u_{KK})}$, respectively.

As with the equilibrium, the social (marginal) benefit of more precise private information is simply the reduction in the dispersion of individual actions around the mean action, weighted by the social aversion to dispersion $|u_{kk}+u_{\sigma\sigma}|/2$. Importantly, use (2) and (4) to note that possible discrepancies between the social and the private benefit of more precise private information originate in either (i) the inefficiency of the equilibrium use of information (*i.e.* the discrepancy between the rule $k(\cdot;\pi_x,\pi_z)$ and the rule $k^*(\cdot;\pi_x,\pi_z)$), or (ii) the external effect $u_{\sigma\sigma}$ that the dispersion of individual actions has on payoffs, which is not internalized in equilibrium. We then have the following result.

Proposition 4. *Suppose that the planner can control the way the agents use their available information. The precision of private information π_x^* that maximizes welfare is given by the unique solution to*

$$\pi_x^* = \sqrt{\frac{|u_{kk}+u_{\sigma\sigma}|\kappa_1^{*2}}{2C'\left(\pi_x^*\right)}} - \frac{\pi_z}{1-\alpha^*}.$$

Comparing the results in Propositions 2 and 4 we arrive to the following conclusion.

Proposition 5. *Let $\hat{\pi}_x$ denote the precision of private information acquired in equilibrium and π_x^* the precision of private information that maximizes welfare when the planner can control the way the agents use their available information. Then $\hat{\pi}_x < \pi_x^*$ (resp., $\hat{\pi}_x > \pi_x^*$) if and only if*

$$|u_{kk}|\left(\frac{\kappa_1(1-\alpha)}{\pi_z+(1-\alpha)\hat{\pi}_x}\right)^2 < |u_{kk}+u_{\sigma\sigma}|\left(\frac{\kappa_1^*(1-\alpha^*)}{\pi_z+(1-\alpha^*)\hat{\pi}_x}\right)^2 \tag{5}$$

(resp., if and only if the sign of the inequality in (5) is reversed).

To understand the result, recall from the analysis above that both the private and the social marginal benefit of an increase in the precision of private information come from the marginal reduction in the cross-sectional dispersion of individual actions around the mean action. The magnitude of this reduction depends on the sensitivity of individual actions to private information, which is given by

$$\frac{\kappa_1(1-\alpha)\pi_x}{\pi_z+(1-\alpha)\pi_x}$$

under the equilibrium strategy, and by

$$\frac{\kappa_1^*\left(1-\alpha^*\right)\pi_x}{\pi_z+(1-\alpha^*)\pi_x}$$

under the efficient strategy. The weight that the planner assigns to reducing the cross-sectional dispersion of individual actions is $|u_{kk}+u_{\sigma\sigma}|/2$, while the weight that each individual assigns to reducing the dispersion of her own action around the mean action is $|u_{kk}|/2$. We thus have that the precision of private information acquired in equilibrium falls short of the efficient level if and only if the reduction of the dispersion of individual actions under the equilibrium strategy, weighted by the importance that each individual assigns to such a reduction, falls short of the reduction under the efficient strategy, weighted by the importance that the planner assigns to such a reduction. The following two corollaries are then implications of the previous result.

Corollary 2. *Efficiency in the acquisition of private information requires (i) efficiency in the use of information and (ii) alignment between the private and the social benefit of reducing the dispersion of individual actions in the cross-section of the population, which obtains if and only if there are no external effects from dispersion, i.e. if and only if* $u_{\sigma\sigma}=0$.[10]

Corollary 3. *Let* $\hat{\pi}_x$ *denote the precision of private information acquired in equilibrium and* π_x^* *the precision of private information that maximizes welfare when the planner can control the way the agents use their available information.*

(i) Consider economies that are efficient in their use of information ($\kappa=\kappa^*$ *and* $\alpha=\alpha^*$)*. Then* $\hat{\pi}_x<\pi_x^*$ *(resp.,* $\hat{\pi}_x>\pi_x^*$*) if and only if* $u_{\sigma\sigma}<0$ *(resp., if and only if* $u_{\sigma\sigma}>0$*).*

(ii) Consider economies that are efficient under complete information and in which there are no externalities from the dispersion of actions in the cross-section of the population ($\kappa=\kappa^*$ *and* $u_{\sigma\sigma}=0$)*. Then* $\hat{\pi}_x<\pi_x^*$ *(resp.,* $\hat{\pi}_x>\pi_x^*$*) if and only if* $\alpha>\alpha^*$*(resp., if and only if* $\alpha<\alpha^*$*).*

(iii) Consider economies in which the equilibrium degree of coordination coincides with the socially optimal degree of coordination and in which there are no externalities from the dispersion of actions in the cross-section of the population ($\alpha=\alpha^*$ *and* $u_{\sigma\sigma}=0$)*. Then* $\hat{\pi}_x<\pi_x^*$ *(resp.,* $\hat{\pi}_x>\pi_x^*$*) if and only if* $\kappa_1<\kappa_1^*$ *(resp., if and only if* $\kappa_1>\kappa_1^*$*).*

Let us start with the economies in part (i) of Corollary 3. Because in these economies the equilibrium use of information is efficient, the marginal reduction in the dispersion of individual actions under the equilibrium rule $k(\cdot;\pi_x,\pi_z)$ coincides with the marginal reduction under the efficient rule $k^*(\cdot;\pi_x,\pi_z)$. As emphasized in Corollary 2, that the equilibrium use of information is efficient, however, does not guarantee that the private and the social marginal benefit of more precise private information coincide. The reason is that the private benefit fails to take into account that other agents' payoffs depend on the dispersion of individual actions in the population, and that the latter naturally decreases with the precision of information acquired in equilibrium. Because this externality has no strategic effects, it is not internalized and is thus a source of a wedge between the private and the social value of more precise private information. In particular, the precision of private information acquired in equilibrium falls short of the efficient level in the presence of a negative externality from dispersion, $u_{\sigma\sigma}<0$, while the opposite is true for economies in which the externality is positive.

Next, consider part (ii) of Corollary 3 and take an economy in which $\alpha>\alpha^*$. Because there are no direct externalities from dispersion (*i.e.* $u_{\sigma\sigma}=0$), the weight that each individual agent assigns to a reduction in the dispersion of her action around the mean action coincides with the weight that the planner assigns to such a reduction. The discrepancy between the private and the social marginal benefit of more precise private information then originates in the use of information. More precisely, it originates in the fact that, in equilibrium, agents respond too little to private sources of information when choosing their actions (recall that economies in which the equilibrium degree of coordination, α, exceeds the socially optimal one, α^*, are economies in which agents overrespond to public sources of information and underrespond to private ones). This implies that the reduction in the dispersion of individual actions under the equilibrium rule $k(\cdot;\pi_x,\pi_z)$ is smaller than under the efficient rule $k^*(\cdot;\pi_x,\pi_z)$. In turn, this makes the private benefit smaller than the social benefit, thus contributing to underinvestment in the acquisition of private information.

10. Note that, in principle, efficiency in the acquisition of private information can obtain even without efficiency in the use of information. However, this can happen only in the knife-edge case where the discrepancy between $\kappa_1(1-\alpha)\pi_x/[\pi_z+(1-\alpha)\pi_x]$ and $\kappa_1^*(1-\alpha^*)\pi_x/[\pi_z+(1-\alpha^*)\pi_x]$ is perfectly offset by the discrepancy between $|u_{kk}|$ and $|u_{kk}+u_{\sigma\sigma}|$.

The same logic explains part (iii) of Corollary 3. As in part (ii), the inefficiency in the acquisition of private information originates in the inefficiency in the use of information. However, the latter now comes from the discrepancy between the sensitivity of the complete-information equilibrium actions to fundamentals, κ_1, and the sensitivity of the first-best allocation to fundamentals, κ_1^*, opposed to the gap between the equilibrium and the socially optimal degree of coordination.

3.3.2. Welfare-maximizing acquisition under equilibrium use. The results in Proposition 5 compare the equilibrium precision of private information to the precision of information that maximizes welfare *when the planner can dictate to the agents how to use their available information*. This comparison is of interest for it tells us in which direction the policy maker would like to correct inefficiencies in the acquisition of private information when the policy maker can also correct inefficiencies in the use of information (see, *e.g.* Angeletos and Pavan, 2009 for how fiscal policy can restore efficiency in the use of information).

For certain problems, though, it is important to compare the precision of private information acquired in equilibrium to the precision that the planner would like the agents to acquire when the *planner is unable to change the way society uses the information it collects*. This is akin to investigating how welfare, under the equilibrium use of information, changes with the precision of private information. As it will become clear in the next section, addressing this question is also instrumental to understanding how the social value of public information (that is, the comparative statics of equilibrium welfare with respect to the precision of public information) is affected by the endogeneity of private information. The remainder of this subsection is thus devoted to the analysis of this question.

Recall that, for any precision of private and public information (π_x, π_z), the equilibrium use of information consists in each agent following the rule $k(\cdot;\pi_x,\pi_z)$ that solves the functional equation (1). It is standard practice to show that, in the case of a Gaussian information structure, the unique solution to this functional equation is the linear rule[11]

$$k(x,y;\pi_x,\pi_z) = \kappa_0 + \kappa_1(1-\gamma)x + \kappa_1\gamma\frac{\pi_\theta\mu + (\pi_z-\pi_\theta)y}{\pi_z}, \tag{6}$$

where[12]

$$\gamma = \gamma(\pi_x,\pi_z) \equiv \frac{\pi_z}{\pi_z+(1-\alpha)\pi_x}. \tag{7}$$

The marginal social benefit of inducing the agents to acquire more precise private information is then given by the result in the following proposition.

Proposition 6. *Suppose that the planner cannot control the way the agents use their available information. For any precision of private and public information* (π_x, π_z), *the (gross) marginal*

11. It is also standard practice to abuse terminology and refer to the rule in (6) as linear, as opposed to affine.

12. Throughout, the notation γ is meant to be a shortcut for the function $\gamma(\pi_x,\pi_z)$. To ease the exposition, we find it convenient to use this shortcut whenever highlighting the specific point where the function is evaluated is not important. A similar notation will be used below for other functions, as it will become clear in due course.

social benefit of inducing the agents to acquire more precise private information is given by

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\kappa_1^2(1-\gamma)^2}{\pi_x^2}+\frac{|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)}{\pi_z+(1-\alpha)\pi_x}\left|\frac{\partial\gamma}{\partial\pi_x}\right| \tag{8}$$
$$+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_z}\frac{\kappa_1^*-\kappa_1}{\kappa_1}\left|\frac{\partial\gamma}{\partial\pi_x}\right|,$$

where $\gamma=\gamma(\pi_x,\pi_z)$ *measures the sensitivity of equilibrium actions to private information, relative to public, and is determined by the expression in (7).*

The first term in (8) is the direct marginal benefit of a reduction in the cross-sectional dispersion of individual actions around the mean action that obtains holding fixed the equilibrium rule $k(\cdot;\pi_x,\pi_z)$. As we show in the Appendix, the second term combines the marginal effect of changing the rule $k(\cdot;\pi_x,\pi_z)$ by inducing the agents to rely more on their private information and less on public information (observe that $\partial\gamma/\partial\pi_x<0$) on (a) the dispersion $Var[k-K]$ of individual actions and (b) the volatility of the aggregate action around its complete-information counterpart, $Var[K-\kappa]$. Finally, the last term, which is relevant only in economies that are inefficient under complete information, captures the welfare effects of changing the way the "error" in aggregate activity due to the incompleteness of information covaries with the inefficiency of the complete-information allocation. Clearly, these last two terms are absent in economies in which the equilibrium use of information is efficient ($\kappa=\kappa^*$ and $\alpha=\alpha^*$) or, alternatively, when the planner can dictate to the agents how to use their information.

Comparing the marginal benefit that each individual agent assigns in equilibrium to an increase in the precision of her private information to the marginal benefit that the planner assigns to the same increase then yields the following result.

Proposition 7. *Let* π_x^{**} *denote the precision of private information that maximizes welfare under the equilibrium use of information (i.e. when the planner cannot control the way the agents use their available information). The same conclusions as in Corollary 3 hold for the comparison between the precision of private information acquired in equilibrium,* $\hat{\pi}_x$*, and* π_x^{**}.

Clearly, in economies that are efficient in their equilibrium use of information ($\kappa=\kappa^*$ and $\alpha=\alpha^*$), the precision of private information that maximizes welfare coincides with the level π_x^* that maximizes welfare when the planner can dictate to the agents how to use their available information (*i.e.* $\pi_x^{**}=\pi_x^*$). As discussed above, in this case, inefficiencies in the equilibrium acquisition of information originate entirely in the discrepancy between the private and the social value of reducing the cross-sectional dispersion of individual actions.

Next, consider economies that are efficient under complete information and in which there are no externalities from the dispersion of individual actions ($\kappa=\kappa^*$ and $u_{\sigma\sigma}=0$). Recall that, in these economies, inefficiencies in the acquisition of private information originate in the discrepancy between the equilibrium and the socially optimal degrees of coordination, with such a discrepancy affecting the sensitivity of individual actions to private information, relative to public. In this case, the precision of private information π_x^{**} that the planner would like the agents to acquire when he cannot control the way information is used in equilibrium can be higher or lower than the precision π_x^* that maximizes welfare under the efficient use of information. Nonetheless, what remains true is that

$$sign(\hat{\pi}_x-\pi_x^{**})=sign(\hat{\pi}_x-\pi_x^*).$$

The same conclusions as in part (ii) of Corollary 3 then apply. When agents overvalue aligning their actions with the actions of others, relative to the planner (that is, when $\alpha > \alpha^*$), they rely too much on their public sources of information when committing their actions. In this case, the precision of private information acquired in equilibrium is inefficiently low. The opposite conclusion holds when they undervalue aligning their actions with the actions of others, *i.e.* when $\alpha < \alpha^*$.

Finally, consider economies in which there are no externalities from the dispersion of individual actions and in which the equilibrium degree of coordination coincides with the socially optimal one ($u_{\sigma\sigma} = 0$ and $\alpha = \alpha^*$). Recall that, in such economies, inefficiencies in the acquisition of information continue to originate in the inefficiency in the use of information, but the latter now comes from the inefficiency of the complete-information actions, as opposed to the discrepancy between α and α^*. In these economies, the wedge between the social and the private benefit of more precise private information, evaluated at the equilibrium level $\hat{\pi}_x$, is given by

$$\frac{|u_{kk} + 2u_{kK} + u_{KK}|\kappa_1^2}{\pi_z} \frac{\kappa_1^* - \kappa_1}{\kappa_1} \left| \frac{\partial \gamma}{\partial \pi_x} \right|.$$

This wedge is positive (implying that the precision of private information acquired in equilibrium is inefficiently low) if $\kappa_1 < \kappa_1^*$ and negative if $\kappa_1 > \kappa_1^*$. The intuition for this result is the following. Economies in which $\kappa_1 < \kappa_1^*$ are economies in which agents respond too little to variations in the fundamentals. In these economies, the "error" $K - \kappa$ due to the incompleteness of information covaries positively with the inefficiency of the complete-information allocation. More precise private information, by bringing the aggregate activity closer to the complete-information level, then increases efficiency. This first-order effect is, however, not internalized by the agents, which explains why the planner would like them to collect more precise private information than they actually do in equilibrium. The opposite is true for economies in which $\kappa_1 > \kappa_1^*$. In this case, the 'error' $K - \kappa$ due to incomplete information covaries negatively with the inefficiency of the complete-information allocation (*i.e.* $K - \kappa$ tends to be positive when the equilibrium activity under complete information κ falls short of the efficient level κ^*, and negative otherwise). In this case, "ignorance is a blessing", for it partially corrects the inefficiency of the complete-information actions. As a result, the planner would like the agents to acquire less private information than they do in equilibrium.

Remark. The economies considered in Corollary 3 and Proposition 7 are benchmark cases in which the source of the inefficiency in the acquisition of private information can be isolated and in which the inefficiency can be unambiguously signed. More generally, one can show that there exists a function $\Lambda(\kappa_1^* - \kappa_1, \alpha - \alpha^*; \pi_z)$ that is increasing in the differences $(\kappa_1^* - \kappa_1, \alpha - \alpha^*)$ and equal to zero at $(0,0)$, such that, for any $(\kappa_1^*, \kappa_1, \alpha, \alpha^*)$, the equilibrium precision of private information falls short of the efficient level, *i.e.* $\hat{\pi}_x < \pi_x^{**}$, if and only if the externality from the cross-sectional dispersion of individual actions is small, that is, if and only if $u_{\sigma\sigma} < \Lambda(\kappa_1^* - \kappa_1, \alpha - \alpha^*; \pi_z)$.

3.4. *The social value of public information when private information is endogenous*

We now turn to the welfare effects of variations in the precision of public information. The key novelty with respect to previous work in this literature is that the analysis below takes into account how variations in the precision of public information affect the agents' incentives to acquire private information.

Intuitively, relative to the case where the precision of private information is exogenous, an increase in the precision of public information, by inducing the agents to cut on their acquisition of private information, results in a stronger increase in the correlation of the agents' posteriors and in a smaller increase (or even a decrease) in the accuracy of their posteriors. Whether the net effect on welfare is larger than when neglecting the endogeneity of private information in turn depends on whether the precision of private information acquired in equilibrium is inefficiently low or high, as highlighted in the next proposition. Let $w(\pi_x,\pi_z)$ denote welfare under the equilibrium use of information, that is, the ex ante expected payoff of each agent, net of the cost of information acquisition, computed under the equilibrium rule $k(\cdot;\pi_x,\pi_z)$.

Proposition 8. *The crowding-out effects of public information on private information reduce the social value of public information if and only if the precision of the private information acquired in equilibrium is inefficiently low:*[13]

$$\frac{dw\left(\hat{\pi}_x,\pi_z\right)}{d\pi_z}<\frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z}\Leftrightarrow\hat{\pi}_x<\hat{\pi}_x^{**}.$$

The result follows directly from the fact that, when taking into account the crowding-out effects of public information on private information, the total change in welfare due to a variation in the precision of public information is given by

$$\frac{dw\left(\hat{\pi}_x,\pi_z\right)}{d\pi_z}=\frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z}+\frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_x}\frac{\partial\hat{\pi}_x}{\partial\pi_z},$$

along with the fact that $\partial w\left(\hat{\pi}_x,\pi_z\right)/\partial\pi_x>0$ if and only if $\hat{\pi}_x<\hat{\pi}_x^{**}$.

The following result is then an immediate implication of the above proposition along with the results in Corollary 3 and Proposition 7.

Corollary 4. *The crowding-out effects of public information on private information reduce the social value of public information in economies in which $\kappa_1\leq\kappa_1^*$, $\alpha\geq\alpha^*$, and $u_{\sigma\sigma}\leq 0$ (strictly when at least one of the inequalities is strict). They increase the social value of public information in economies in which $\kappa_1\geq\kappa_1^*$, $\alpha\leq\alpha^*$, and $u_{\sigma\sigma}\geq 0$ (strictly when at least one of the inequalities is strict).*

In certain cases, the acquisition of private information is particularly important, because not only it affects the "magnitude" of the social value of public information, it may also affect its sign. However, as the next proposition shows, this is never the case in economies in which the precision of private information acquired in equilibrium is inefficiently low and in which the equilibrium degree of coordination is not too high relative to the socially optimal level.

Proposition 9. *Take any economy in which the precision of private information acquired in equilibrium is inefficiently low (i.e. $\hat{\pi}_x\leq\pi_x^{**}$). There exists a critical threshold $\Delta>0$ (which depends only on the payoff structure) such that, irrespective of the equilibrium degree of substitutability between public and private information ($\partial\hat{\pi}_x/\partial\pi_z$), welfare always increases with the precision of public information if $\alpha-\alpha^*<\Delta$, i.e. if the discrepancy between the equilibrium and the socially optimal degrees of coordination is not too large.*

13. The expression $dw(\hat{\pi}_x,\pi_z)/d\pi_z$ denotes the total derivative of w with respect to π_z, taking into account that the equilibrium precision of private information $\hat{\pi}_x$ depends on π_z. In contrast, $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_z$ denotes the partial derivative of w with respect to π_z, holding π_x fixed. Both derivatives are evaluated at $\pi_x=\hat{\pi}_x$.

Note that, when $\alpha-\alpha^{*}<\Delta$, both private and public information can contribute either positively or negatively to welfare; that is, $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_z$ and $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_x$ can be of either sign. What the bound on the discrepancy $\alpha-\alpha^{*}$ guarantees is that, when agents underinvest in their collection of private information (*i.e.* when $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_x>0$), then public information has a positive direct effect on welfare (*i.e.* $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_z\geq 0$) that is always strong enough to prevail over the crowding-out effects of public information on private information.

The following corollary is then an implication of the previous proposition.

Corollary 5. *In economies in which $\alpha-\alpha^{*}<\Delta$, the crowding-out effects of public information on private information may either increase or decrease the social value of public information. However, they can never turn the social value of public information negative when it is positive in the absence of such effects. That is,*

$$\frac{dw\left(\hat{\pi}_x,\pi_z\right)}{d\pi_z}\gtreqless\frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z};\ \textit{however,}\ \frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z}\geq 0\Rightarrow\frac{dw\left(\hat{\pi}_x,\pi_z\right)}{d\pi_z}\geq 0.$$

The first part of the statement follows directly from the fact that, in these economies, agents may either overinvest or underinvest in the acquisition of private information. The crowding-out effects of public information on private information can thus either strengthen or weaken the direct effect that more precise public information exerts on welfare. The second part follows by contradiction. Suppose that the direct effect of an increase in the precision of public information on welfare is positive and yet that the total net effect that one obtains when considering the crowding-out effects of public information on private information is negative. That is, suppose that

$$\frac{dw\left(\hat{\pi}_x,\pi_z\right)}{d\pi_z}<0\leq\frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z}.$$

For this to be possible, it must be that, at the equilibrium level $\hat{\pi}_x$, agents underinvest in the acquisition of private information, *i.e.* $\partial w\left(\hat{\pi}_x,\pi_z\right)/\partial\pi_x\geq 0$. However, because $\alpha-\alpha^{*}<\Delta$, the result in Proposition 9 implies that $dw(\hat{\pi}_x,\pi_z)/d\pi_z\geq 0$, a contradiction.

The most interesting implication of such crowding-out effects obtains by considering economies in which $0<\alpha-\alpha^{*}<\Delta$. It is well known that, when $\alpha-\alpha^{*}>0$, the direct effect of more precise public information on welfare may be negative (*i.e.* there are situations in which $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_z<0$). The reason is that the excessive value that the agents assign to aligning their actions with the actions of others induces them to rely too much on public sources of information and too little on private ones. More precise public information, by further increasing the sensitivity of the equilibrium actions to public sources and further decreasing their sensitivity to private ones, may thus contribute negatively to welfare (see, among others, Morris and Shin, 2002 and Angeletos and Pavan, 2007). Interestingly, these effects are reinforced when private information is endogenous for, in this case, an increase in the precision of public information also comes with a reduction in the precision of private information, which contributes to an even higher sensitivity of equilibrium actions to public sources relative to private. This notwithstanding, the crowding-out effects that public information exerts on the acquisition of private information may turn the social value of public information from negative to positive. The reason is that the precision of private information acquired in equilibrium may be excessively high. By inducing the agents to cut on their acquisition of private information, more precise public information thus comes with a benefit that is neglected in the previous literature. As we show below, this new benefit can be strong enough to turn the social value of public information positive for the same economies in which it is found to be negative when neglecting the crowding-out effects on the acquisition of private information.

3.4.1. Beauty contests. As an illustration of the above results, consider the celebrated beauty-contest model of Morris and Shin (2002) in which payoffs are given by

$$u(k_i, K, \sigma_k, \theta) = -(1-r)(k_i-\theta)^2 - r\left(L_i - \bar{L}\right) = -(1-r)(k_i-\theta)^2 - r(k_i-K)^2 + r\sigma_k^2,$$

where $r \in (0,1)$ is a scalar that parametrizes the intensity of the coordination motive, $L_i = \int_{[0,1]}[k_h - k_i]^2 dh$ is the dispersion of other agents' actions around agent i's action, and $\bar{L} = \int_{[0,1]} L_i di = 2\sigma_k^2$ is a positive externality that comes from the dispersion of other agents' actions around the mean action. As it is by now well understood, these are economies in which the equilibrium degree of coordination is inefficiently high, *i.e.* $\alpha > \alpha^*$, implying that the direct effect of more precise public information on welfare $\partial w\left(\hat{\pi}_x, \pi_z\right)/\partial \pi_z$ can be negative, as documented in the literature. As for the effect of more precise private information, the latter is always positive when it is exogenous. When, instead, the precision of private information is determined endogenously, then one can show that the precision acquired in equilibrium can be either excessively low or excessively high, depending on the precision of public information. Formally, there exists a threshold $\pi_z' > 0$ such that the precision of private information acquired in equilibrium is inefficiently high if and only if $\pi_z < \pi_z'$ (see the proof of Proposition 10 below). When this is the case, the crowding-out effects of public information on private information increase the social value of public information. In particular, these new effects can be sufficiently strong to completely overturn the partial effect identified in the literature, making the social value of public information positive under the same parameter conditions that would have predicted it to be negative ignoring the endogeneity of private information.

At a first glance, this result may seem counterintuitive. It is known that, in these economies, welfare increases with the accuracy $\pi_x + \pi_z$ of the agents' information while it decreases with its commonality $\delta = \pi_z/[\pi_z + \pi_x]$, due to the inefficiently high level of coordination.[14] Given that the endogeneity of private information reduces the positive effects of accuracy and increases the negative effects of commonality, how is it possible that the crowding-out effects of public information on private information turn the social value of public information from negative to positive? The answer lies in the inefficiently high level of private information acquired in equilibrium. By inducing agents to cut on their collection of private information, an increase in the precision of public information can boost welfare when the costs saved in the acquisition of private information more than compensate for the increase in non-fundamental volatility relative to dispersion.

At this point, one may wonder whether the opposite is also true: can the endogeneity of private information turn the sign of the social value of public information from positive to negative? As shown in Proposition 8, for this to be possible, it must be that the precision of private information acquired in equilibrium is inefficiently low, *i.e.* $\pi_x < \pi_x^{**}$. However, because in these economies the discrepancy between the equilibrium and the socially-optimal degrees of coordination $\alpha - \alpha^*$ is positive but never larger than the critical value Δ of Proposition (9), the social value of public information always remains positive when it is positive ignoring the endogeneity of private information.

We summarize the above observations in the following proposition.

Proposition 10. *Consider the class of "beauty-contest" economies described above. The crowding-out effects of public information on private information may either increase or decrease the social value of public information. However, while such effects may turn the sign of the social*

14. See Corollary 8 in Angeletos and Pavan (2007).

value of public information from negative to positive, they can never turn it from positive to negative.

4. MONETARY ECONOMIES WITH PRICE-SETTING COMPLEMENTARITIES

4.1. *The economy*

We show how the insights from the Gaussian-quadratic setup extend to a fully microfounded model with price rigidities and dispersed information, along the lines of those considered in the recent literature (see, among others, Hellwig, 2005; Adam, 2007; Lorenzoni, 2010).

The economy is populated by a (measure-one) continuum of agents, indexed by $j \in [0,1]$. Each agent j is both a consumer of all goods and the sole producer of good j, which is produced with labour as the only input. Denote by N_j the agent's supply of labour and by

$$C_j = \left(\int_{[0,1]} c_{jh}^{\frac{\nu-1}{\nu}} dh \right)^{\frac{\nu}{\nu-1}}$$

the usual Dixit-Stiglitz aggregator of the agent's consumption of the various goods, with $\nu > 1$ parametrizing the elasticity of substitution among the goods, and with c_{jh} denoting the agent's consumption of good $h \in [0,1]$. The agent's preferences (over N_j and C_j or, equivalently, over N_j and the bundle $(c_{jh})_{h\in[0,1]}$ of individual goods) take the familiar isoelastic form

$$U(C_j, N_j) \equiv \frac{C_j^{1-R}}{1-R} - N_j, \tag{9}$$

where $R > 0$ parametrizes the curvature of the utility function with respect to the consumption aggregator C_j. Note that, while the above specification assumes a linear disutility of labour, all the results extend to the case of a strictly convex disutility of labour.[15]

Each agent j chooses the bundle $(c_{jh})_{h\in[0,1]}$ so as to maximize her utility subject to the budget constraint

$$\int_{[0,1]} P_h c_{jh} dh \leq P_j Y_j, \tag{10}$$

where P_h denotes the price of good $h \in [0,1]$ and where Y_j denotes the aggregate demand for good j ($P_j Y_j$ is thus agent j's disposable income). Given the aggregate demand Y_j for good j, agent j's supply of labour is then determined by the technological constraint

$$Y_j = \Theta N_j^{1/\omega}, \tag{11}$$

where $\omega > 1$ parametrizes the return to scale, and Θ is a log-Normally distributed technology shock (that is $\theta \equiv \ln(\Theta)$ is Normally distributed, with mean zero and variance π_θ^{-1}).

As is standard in the literature, we assume a cash-in-advance constraint, which implies that the following identity must hold

$$\int_{[0,1]} P_h Y_h dh = M,$$

15. To see this, suppose that the disutility of labour was equal to N^β, for $\beta > 1$. It then suffices to replace ω with $\omega' = \omega\beta$ in the payoff formula (12) below and then conduct the entire analysis verbatim as in the case where $\beta = 1$.

where M is the aggregate money supply. Note that here we follow Hellwig (2005) in assuming that the money supply is fixed.[16] Such a rule appears plausible if the time horizon is sufficiently short but need not be optimal. Any serious analysis of optimal monetary policy requires a richer environment, which is beyond the scope of this article.[17] Likewise, we do not consider here optimal fiscal policy. We just notice that, in this economy where private information is endogenous, fiscal policies that implement the efficient price rule (for example, through state-contingent subsidies and lump sum taxes) need not implement the efficient acquisition of private information (see Appendix B for an example).[18]

As in the Gaussian-quadratic model, we assume that all agents share a common prior over θ. Before setting prices, each producer–consumer decides how much private information to acquire about θ. This is modelled as in the Gaussian-quadratic model by assuming that each agent chooses the precision of her private signal π_{x_j}, for given precision of public information $\pi_z = \pi_\theta + \pi_y$. Each agent j then receives an idiosyncratic and a public signal about θ, x_j and y respectively, where the properties of these signals are as in the Gaussian-quadratic model. Given (x_j, y), agent j then sets a price P_j for her product and commits to supply any quantity that will be demanded at this price by adjusting her labour supply according to the technological constraint (11). As is standard in the literature, consumption decisions take place once θ is realized and publicly observed.

4.2. *Equilibrium*

4.2.1. Price rule. Given the log-Normality of the TFP shock and of the information structure, it is natural to focus on equilibria in which prices are log-Normally distributed. In the Appendix, we show that in any such equilibrium each producer–consumer's payoff can be expressed as follows

$$u(p_j, \bar{p}, \sigma_p^2, \theta) \equiv \frac{1}{1-R}\left(\exp\left\{-(v-1)p_j + (v-2)\bar{p} - (v-1)(v-2)\frac{\sigma_p^2}{2} + m\right\}\right)^{1-R} \quad (12)$$
$$- \exp\left\{-v\omega p_j + \omega(v-1)\bar{p} - \omega(v-1)^2\frac{\sigma_p^2}{2} + \omega(m-\theta)\right\},$$

16. Our setup is similar to that in Hellwig (2005), except for the fact that each consumer j is the sole producer of good j, which simplifies the analysis. Another difference is that Hellwig (2005) assumes that agents are uncertain about a monetary policy shock whereas here we assume that they are uncertain about a productivity shock.

17. In this environment where the only decisions that must be made under incomplete information are nominal, there trivially exists a monetary rule that, along with an appropriately chosen production subsidy, implements the first-best allocation, for any precision of private information π_x. Under such a rule—obtained by conditioning money supply on θ or, equivalently, on aggregate prices—no agent acquires any information. Assuming that such a rule is in place is obviously inappropriate in this setting. One way one could accommodate optimal monetary rules without trivializing the acquisition of private information is by introducing a second decision that must be made under incomplete information, say an investment decision or an early employment decision.

18. These results may appear in contrast to those in Llosa and Venkateswaran (2013) and to those in Angeletos and La'O (2013). Llosa and Venkateswaran (2013) show that a simple state-contingent subsidy that implements the efficient price rule also induces the agents to acquire the efficient amount of private information. That result, however, hinges on the assumption that $R=0$, which corresponds to the case where all households are risk neutral. In a similar vein, Angeletos and La'O (2013) find that policies that correct inefficiencies in the use of information also correct inefficiencies in the acquisition of information. However, their result hinges on the assumption of perfect insurance in consumption, in which case the degree of risk aversion is irrelevant.

where $p_j \equiv \ln(P_j)$, $\bar{p} \equiv \int_{[0,1]} p_h dh$, $\sigma_p \equiv \int_{[0,1]} (p_h - \bar{p})^2 dh$, and $m \equiv \ln(M)$. The expression in (12) is obtained by combining standard properties of the demand functions in the familiar monopolistic-competition model with isoelastic preferences and a log-Normal structure of all relevant variables. Note that the first term in (12) is the utility of consumption while the second term is the disutility of labour, both expressed in terms of the cross-sectional distribution of the log-prices.

Let $\mathbb{E}_j[\cdot]$, $Var_j[\cdot]$, and $Cov_j[\cdot,\cdot]$ denote shortcuts for, respectively, the conditional expectation, variance, and covariance operators, given the agent's information (x_j, y) and given the quality of information (π_{x_j}, π_z). In the symmetric equilibrium in which all producers–consumers acquire information of quality π_x, each consumer–producer j then optimally sets her price p_j according to the following rule (see Appendix B for details on the derivation)[19]

$$p_j = \mathbb{E}_j[(1-\alpha)\kappa + \alpha\bar{p}] - (v-1)\alpha\frac{\sigma_p^2}{2} - \frac{(1-\alpha)(R-1)^2(v-2)^2}{R+\omega-1}\frac{Var_j[\bar{p}]}{2} \tag{13}$$
$$+ \frac{1-\alpha}{R+\omega-1}\frac{\omega^2(v-1)^2 Var_j[\bar{p}]}{2} + \frac{\omega^2 Var_j[\theta]}{2} - \omega^2(v-1)Cov_j[\bar{p},\theta],$$

where (a) $\kappa \equiv \kappa_0 + \kappa_1\theta$ denotes the complete-information equilibrium log-price, with

$$\kappa_0 \equiv \frac{1}{R+\omega-1}\ln\left(\frac{v\omega}{v-1}\right) + m \text{ and } \kappa_1 \equiv -\frac{\omega}{R+\omega-1},$$

and where (b)

$$\alpha \equiv \frac{\omega(v-1)+(R-1)(v-2)}{v\omega+(v-1)(R-1)}$$

denotes the slope of best responses with respect to the average log-price. In the following, we restrict attention to the interesting case in which $\alpha > 0$, which requires the parameters of the model to satisfy[20]

$$v > \max\left\{1, 1 + \frac{R-1}{R+\omega-1}\right\}. \tag{14}$$

Note that the right-hand side of (14) is increasing in the curvature parameter R and approaches 2 as R goes to infinity, which implies a price-to-cost ratio $v/(v-1)$ lower than 200%. Hence, the restriction appears plausible even under extreme parameter configurations.

The structure of best responses in (13) highlights certain similarities, but also important differences, with the Gaussian-quadratic model. The first term in (13) parallels the structure of best responses in the Gaussian-quadratic model. The other terms are novel and worth discussing.

First, each agent's best response (here the log-price) now decreases with the cross-sectional dispersion of individual prices. This is due to the fact that a higher cross-sectional price dispersion lowers the Dixit–Stiglitz price index

$$P \equiv \left(\int_{[0,1]} P_h^{1-v} dh\right)^{\frac{1}{1-v}} = \exp\left(\bar{p} - (v-1)\frac{\sigma_p^2}{2}\right),$$

via the substitution effect in the aggregate consumption bundles (consumers substitute goods with high prices for those with lower prices). This effect unambiguously contributes to lower

19. As it will become clear from the discussion below, the terms for $Var_j[\bar{p}]$ in (13) are not combined in order to ease the interpretation of the different addenda of the formula.

20. Note that this condition guarantees that the numerator of α is positive. That the denominator is also positive follows from the fact that $v\omega+(v-1)(R-1) = v\omega+1-v+R(v-1) > 0$, given that $v, \omega > 1$. Also note that $\alpha < 1$.

individual prices, due to the complementarity in pricing decisions, as indicated in the second term of (13).

Second, note that a higher volatility in the average price level (formally, a higher $Var_j[\bar{p}]$) implies higher volatility in individual revenues and hence in individual consumption. This effect unambiguously contributes to a higher expected marginal utility of consumption, which in turn induces the producer to lower her price, as reflected in the third term of (13).

Third, note that a higher volatility of the aggregate price level, by contributing to a higher volatility of the producer's own sales, induces a higher marginal disutility of labour. This effect induces the producer to raise her own price in order to cut on production and hence reduce individual labour supply. This effect is stronger the more convex the marginal productivity of labour (see (B.6) in Appendix B). Similarly, a higher volatility in the fundamental (equivalently, in the productivity of labour) induces a higher marginal disutility of labour, which again calls for a higher price (see again Condition (B.6) in Appendix B). Finally, a positive covariance between the average price and the productivity shock mitigates the former two effects, by reducing their impact on the producer's marginal disutility of labour, which induces the producer to expand production by lowering her price. Combined, these last three effects give rise to the fourth term in (13).

Because the agents' expectations of both the fundamental shock θ and of the common noise shock ε in their information are linear in their signals, it is natural to conjecture a linear symmetric equilibrium in which each producer–consumer sets her price according to the linear rule

$$p_j = p(x_j, z; \pi_x, \pi_z) \equiv \lambda_0 + \lambda_1 x_j + \lambda_2 z, \tag{15}$$

where $z \equiv \mathbb{E}[\theta|y]$. In Appendix B, we verify that such an equilibrium exists, is unique (within the family of equilibria in which prices are log-linear), and is such that the sensitivity of the price rule to private and public information is given by

$$\lambda_1 = \kappa_1(1-\gamma) \text{ and } \lambda_2 = \kappa_1 \gamma,$$

where

$$\gamma = \gamma(\pi_x, \pi_z) \equiv \frac{\pi_z}{\pi_z + (1-\alpha)\pi_x}.$$

Interestingly, note that these are exactly the same formulas as in the Gaussian-quadratic model (recall the characterization in (6)). The key difference with respect to the Gaussian-quadratic model is the fixed term λ_0, which represents the ex ante unconditional expectation of each household's equilibrium price (that is, $\lambda_0 = \mathbb{E}[p] = \mathbb{E}[\bar{p}]$) and which is equal to

$$\lambda_0 = \kappa_0 + \frac{\omega[\omega v^2 - (v-1)^2] + (1-R)(v-1)[R(v-1)-1]}{2(R+\omega-1)} \frac{\lambda_1^2}{\pi_x} + \frac{\omega^2(1+\lambda_1)^2 - (1-R)^2 \lambda_1^2}{2(R+\omega-1)} \frac{1}{\pi_z}. \tag{16}$$

The first term in (16) coincides with the one in the Gaussian-quadratic model. The other terms reflect the novel effects discussed above in connection to the structure of the individual best responses (13). They combine the direct effects of price dispersion on the ex ante average price with the effects of the precision of public and private information on the agent's ability to estimate (a) the volatility of the aggregate price level $\bar{p}$, (b) the volatility of the productivity shock θ, and (c) the covariance between $\bar{p}$ and θ. As explained above, such estimates, along with the expected cross-sectional price dispersion, translate into the expected marginal utility of consumption and the expected marginal disutility of labour that, combined, determine the ex ante level of each agent's equilibrium price.

Finally, note that $\lambda_1^2/\pi_x = \sigma_p^2$ coincides with the cross-sectional dispersion of prices under the equilibrium rule. One can then interpret the term

$$\frac{\partial \mathbb{E}[\bar{p}]}{\partial \sigma_p^2} \equiv \frac{\omega[\omega v^2 - (v-1)^2] + (1-R)(v-1)[R(v-1)-1]}{2(R+\omega-1)} \tag{17}$$

in (16) as the net impact of price-dispersion on ex ante expected prices. As explained above, such net impact combines the direct effect of price dispersion on best responses with the various indirect effects via the agents' ability to forecast $\bar{p}$ and θ. As we show below, this effect plays an important role in determining the direction in which the economy fails to acquire the efficient amount of private information.

4.2.2. Information acquisition. We turn to the acquisition of private information. As in the Gaussian-quadratic model, the (gross) marginal private benefit of more precise private information can be computed using the envelope theorem. In the symmetric equilibrium, this marginal benefit coincides with the marginal effect of a higher precision π_x on each agent's ex ante payoff, holding fixed the agents' equilibrium pricing rule, as given by (15). As we show in Appendix B, the equilibrium private benefit of more precise private information can then be shown to be equal to

$$\frac{u_{kk}}{2}\frac{\partial}{\partial \pi_x} Var[p-\bar{p} \mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z], \tag{18}$$

where (a)

$$u_{kk} = u_{kk}(\pi_x,\pi_z) \equiv \mathbb{E}\left[\frac{\partial^2 u(p,\bar{p},\sigma_p^2,\theta)}{\partial p^2} \mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right] < 0$$

denotes the average curvature of individual payoffs around own prices, when all agents follow the linear rule $p(\cdot;\pi_x,\pi_z)$ given by (15), and (b) $\frac{\partial}{\partial \pi_x} Var[p-\bar{p} \mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ denotes the marginal reduction in the dispersion of individual prices around the mean price that obtains when one changes the precision of private information, holding fixed the equilibrium price rule $p(\cdot;\pi_x,\pi_z)$.

Interestingly, note that the equilibrium benefit of more accurate private information is the same as in the Gaussian-quadratic model, except for the fact that the curvature of each agent's payoff around her own action (here the log-price p) is not constant and hence must be computed under the equilibrium price rule. Naturally, the curvature of the agent's expected payoff around her own price also depends on the quality of the agent's information. Apart from this change, the private benefit of more precise private information continues to be given by the marginal reduction in the dispersion of the agent's action around the mean action (holding fixed the equilibrium rule), scaled by the importance that the individual assigns to such a reduction, which continues to be given by the expected curvature of the individual's payoff around her own action, evaluated under the equilibrium rule.

We conclude that all key positive properties of the Gaussian-quadratic model extend to this fully microfounded application. In particular, given that the dispersion of individual prices around the mean price continues to be given by $\kappa_1^2(1-\gamma)^2/\pi_x^2$, we have that the precision of private information acquired in equilibrium continues to be given by

$$\hat{\pi}_x = \sqrt{\frac{\left|u_{kk}(\hat{\pi}_x, \pi_z)\right| \kappa_1^2}{2C'\left(\hat{\pi}_x\right)}} - \frac{\pi_z}{1-\alpha}, \tag{19}$$

which is exactly the same expression as in Proposition 2 in the Gaussian-quadratic model, except for the dependence of u_{kk} on π_x and π_z.

4.3. *Efficiency*

4.3.1. Price rule. The notion of efficiency we consider here is the same as in the Gaussian-quadratic model and coincides with the notion of decentralized efficiency favoured in the literature. We look for a pricing strategy that, when adopted by all producers–consumers, maximizes the ex ante utility of a representative household.[21] More precisely, we are interested in a linear rule

$$p = p^*(x, z; \pi_x, \pi_z) \equiv \lambda_0^* + \lambda_1^* x + \lambda_2^* z \tag{20}$$

that maximizes the ex ante expectation of $u(p, \bar{p}, \sigma_p^2, \theta)$. Restricting attention to linear rules is justified by two considerations. First, as shown above, the equilibrium rules are linear. Comparing the equilibrium rule with the linear rule that maximizes welfare thus permits us to identify primitive conditions that are responsible for inefficiencies in the acquisition of private information. Second, because the fundamental shocks and the agents' information are log-Normally distributed, we conjecture (but did not prove) that the price rule that maximizes welfare among all possible rules is indeed linear.

As we show in Appendix B, the efficient rule is given by

$$\lambda_1^* = \kappa_1^*(1-\gamma^*),\ \lambda_2^* = \kappa_1^* \gamma^*$$

and

$$\lambda_0^* = \kappa_0^* + \frac{\omega[\omega v^2 - (v-1)^2] + (1-R)(v-1)[R(v-1)-1]}{2(R+\omega-1)} \frac{\lambda_1^{*2}}{\pi_x} + \frac{\omega^2\left(1+\lambda_1^*\right)^2 - (1-R)^2 \lambda_1^{*2}}{2(R+\omega-1)} \frac{1}{\pi_z}, \tag{21}$$

where $\kappa^* \equiv \kappa_0^* + \kappa_1^* \theta$ is the complete-information welfare-maximizing allocation (differing from the first-best allocation only by the usual monopolistic-competition mark-up) with

$$\kappa_1^* = \kappa_1 = -\frac{\omega}{R+\omega-1} \text{ and } \kappa_0^* = \frac{1}{R+\omega-1} \ln(\omega) + m,$$

and where

$$\gamma^* = \gamma^*(\pi_x, \pi_z) \equiv \frac{\pi_z}{\pi_z + (1-\alpha^*)\pi_x},$$

21. Note that society could do even better by committing not to set prices until the productivity shock θ is realized, thus replicating the complete-information allocation. This alternative notion of efficiency is clearly inappropriate to identify the sources of inefficiency in the collection of private information that emerge when prices have to be set under dispersed information.

with

$$\alpha^* \equiv 1 - \frac{R+\omega-1}{v(v\omega+1-v)+R(v-1)^2}.$$

The parameter α^* continues to represent the slope of the best responses of individual prices to aggregate prices that makes the price rule efficient, as in the Gaussian-quadratic model (see Condition (B.26) in Appendix B). In the Appendix, we also show that $\alpha^* > \alpha$ when $\alpha > 0$. This reflects the fact that a price setter expecting her competitors to raise their prices in response to, say, a drop in productivity has an incentive to raise her own price less than her competitors so as to gain market share. Such a motive is obviously not warranted from a social stand-point.[22] That $\alpha < \alpha^*$ in turn implies that the sensitivity of the equilibrium price rule to public information is inefficiently low, while its sensitivity to private information is inefficiently high, suggesting the possibility that the precision of private information acquired in equilibrium is inefficiently high, a possibility that we discuss in the next subsection. Importantly, note that the inefficiency of the equilibrium price rule comes not only from the fact that the sensitivities to private and public information differ from the socially optimal levels, but also from the fact that the ex ante average price $\mathbb{E}[p] = \mathbb{E}[\bar{p}] = \lambda_0$ differs from the socially optimal one, λ_0^*, both because $\alpha \neq \alpha^*$ and because the average price under complete information, κ_0, differs from the welfare-maximizing level κ_0^*.

4.3.2. Information acquisition. We now use the above characterization to derive the precision of private information π_x^* that maximizes welfare when the planner can control the way in which the producers set their prices (that is, when he can control the agents' use of information).

Paralleling the analysis in the Gaussian-quadratic model, and using the fact that prices are log-Normally distributed under the efficient rule, we show in Appendix B that the social benefit of more precise private information under the efficient rule is given by

$$\frac{u_{kk}^* + u_{\sigma\sigma}^*}{2} \frac{\partial}{\partial \pi_x} Var[p-\bar{p} \mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z], \tag{22}$$

where

$$u_{kk}^* = u_{kk}^*(\pi_x,\pi_z) \equiv \mathbb{E}\left[\frac{\partial^2 u(p,\bar{p},\sigma_p^2,\theta)}{\partial p^2} \mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right] < 0$$

denotes the average curvature of a representative household's utility around her own price, and where

$$u_{\sigma\sigma}^* = u_{\sigma\sigma}^*(\pi_x,\pi_z) \equiv 2\mathbb{E}\left[\frac{\partial u(p,\bar{p},\sigma_p^2,\theta)}{\partial \sigma_p^2} \mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right] > 0$$

denotes the average externality from the dispersion of prices in the cross section of the population, under the efficient price rule $p^*(\cdot;\pi_x,\pi_z)$, as given by (20).[23] To understand why $u_{\sigma\sigma}^*$ is positive, note that, because of the Dixit–Stiglitz substitution effect, consumers respond to a higher price dispersion by increasing their expenditure on the cheapest goods. An increase in price dispersion thus has three effects on each agent's payoff. First, it reduces, on average, the agent's sales and

22. That, in these economies, $\alpha^* > \alpha$ is consistent with what noticed, for example, in Hellwig (2005).

23. Note that the reason why we scale the average externality from the cross-sectional dispersion of prices by 2 is to facilitate the comparison with the Gaussian-quadratic model. There we assumed that payoffs depend linearly on σ_k^2 with coefficient $u_{\sigma\sigma}/2$ and then denoted by $u_{\sigma\sigma}$ the second derivative of u with respect to the standard deviation $\sigma_k = \sqrt{\sigma_k^2}$. This is equivalent to saying that the derivative of u with respect to σ_k^2 is equal to $u_{\sigma\sigma}/2$.

hence her labour supply. This effect contributes positively to the agent's payoff. Second, by reducing sales, it lowers income, which contributes negatively to the agent's payoff. Finally, for given income, an increase in price dispersion increases the agent's ability to procure goods at low prices, which contributes positively to payoffs. As we show in the Appendix, the total effect is always positive, thus making payoffs increase with price dispersion.

The above result about the (gross) marginal social value of more precise private information parallels the one in the Gaussian-quadratic model. When all agents follow the efficient price rule, the social benefit of more precise private information is simply the marginal reduction in the cross-sectional dispersion of prices around the mean price that obtains by holding fixed the price rule (20) by usual envelope arguments, scaled by the social benefit of such a reduction. As in the Gaussian-quadratic model, the social benefit of a lower cross-sectional price dispersion combines the private benefit (given by the expected curvature u^*_{kk} of individual payoffs around own prices) with the expected external effect $u^*_{\sigma\sigma}$ that price dispersion exerts on individual payoffs. Contrary to the Gaussian-quadratic model, however, and in analogy with what was discussed above for the equilibrium, both u^*_{kk} and $u^*_{\sigma\sigma}$ must be computed by averaging across states, and are thus a function of the quality of information.

4.4. *Inefficiency of the equilibrium acquisition of information*

One can use the above results to compare the precision of information acquired in equilibrium, $\hat{\pi}_x$, with the precision π^*_x that maximizes welfare when the planner can control the price rule. Using (18) and (22), we have that the precision of private information acquired in equilibrium is inefficiently low if and only if

$$\frac{|u_{kk}(\hat{\pi}_x,\pi_z)|}{2}\left(\frac{\kappa_1(1-\alpha)}{\pi_z+(1-\alpha)\hat{\pi}_x}\right)^2<\frac{|u^*_{kk}(\hat{\pi}_x,\pi_z)+u^*_{\sigma\sigma}(\hat{\pi}_x,\pi_z)|}{2}\left(\frac{\kappa^*_1(1-\alpha^*)}{\pi_z+(1-\alpha^*)\hat{\pi}_x}\right)^2. \tag{23}$$

Note that (23) is the analogue of Condition (5) in Proposition 5 in the Gaussian-quadratic model.

The above condition can be related to the primitives of the model. However, we do not pursue this here for brevity.[24] Instead, we show that, as in the Gaussian-quadratic model, one can also use the above results to address the question of whether the planner would like the agents to acquire more or less precise private information when she is *unable* to change the price rule used in equilibrium (note that this is akin to the analysis in Subsection 3.3.2 in the Gaussian-quadratic model). In other words, the above results also permit us to examine how welfare, under the equilibrium price rule, changes with the precision of private information. To this purpose, let

$$u_K=u_K(\pi_x,\pi_z)\equiv\mathbb{E}\left[\frac{\partial u(p,\bar{p},\sigma_p^2,\theta)}{\partial\bar{p}}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]<0$$

and

$$u_{\sigma\sigma}=u_{\sigma\sigma}(\pi_x,\pi_z)\equiv 2\mathbb{E}\left[\frac{\partial u(p,\bar{p},\sigma_p^2,\theta)}{\partial\sigma_p^2}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]>0.$$

Note that u_K and $u_{\sigma\sigma}$ represent, respectively, the marginal effect on payoffs of a higher average price and of a higher cross-sectional price dispersion, when all agents set prices according to

24. We find it more interesting to relate the primitive parameters of the model to the inefficiency in the equilibrium acquisition of private information in the case in which the planner is unable to induce the agents to follow the efficient price rule—this is what we do in Proposition 11.

the equilibrium rule $p(\cdot;\pi_x,\pi_z)$, as given in (15). That u_K is negative reflects the fact that higher ex ante prices contribute negatively to welfare by depressing consumption. That $u_{\sigma\sigma}$ is positive follows from arguments similar to those discussed above for $u^*_{\sigma\sigma}$ in the case of the efficient price rule.

As we show in Appendix B, the (gross) marginal social benefit of inducing the agents to acquire more precise private information is equal to

$$\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\left(-\frac{\kappa_1^2(1-\gamma)^2}{\pi_x^2}\right) \tag{24}$$
$$+2\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\frac{\kappa_1^2(\alpha^*-\alpha)}{\pi_z+(1-\alpha)\pi_x}\left|\frac{\partial\gamma}{\partial\pi_x}\right|,$$

where u_{kk}, $u_{\sigma\sigma}$, u_K, γ, and $\partial\gamma/\partial\pi_x$ are all functions of (π_x,π_z), as in the definitions above.

The result in (24) highlights certain similarities with Condition (8) in the Gaussian-quadratic model, but also a few interesting differences (recall that $\kappa_1=\kappa_1^*$ in this application). As in the Gaussian-quadratic model, the first line in (24) captures the effects on expected payoffs of a reduction in the cross-sectional price dispersion that obtains as a result of a higher precision of private information, holding fixed the sensitivity of the equilibrium prices to private and public information. The new effect, relative to the Gaussian-quadratic model, is captured by the term

$$u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}. \tag{25}$$

To understand this term note that, holding fixed the sensitivity of the equilibrium price rule to private and public information, a variation in the precision of private information, by triggering a variation in price dispersion, also triggers a variation in the ex ante expected price, which is given by λ_0 (see Condition (16) above and observe that $\sigma_p^2=\kappa_1^2(1-\gamma)^2/\pi_x$). The marginal impact of such a variation on payoffs is then given by the term in (25).

The second line in (24) captures the net effect of inducing a variation in the relative sensitivity of the equilibrium price rule to private and public information, as captured by γ. As in the Gaussian-quadratic model, the value of changing such a sensitivity comes from a discrepancy between the equilibrium degree of coordination, α, and the socially optimal degree of coordination, α^*. The key novelty relative to the Gaussian-quadratic model is the fact that a variation in the relative sensitivity of prices to private and public information, by triggering a variation in price dispersion, now also triggers a variation in the ex ante expected prices. The impact of this novel effect on welfare is once again captured by the new term $u_K\partial\mathbb{E}[\bar{p}]/\partial\sigma_p^2$.

One way to appreciate how these new effects ultimately affect the weights that the planner assigns to variations in (a) the cross-sectional dispersion of individual prices, (b) the volatility of the average prices, and (c) the ex ante level of the average prices is by looking at the problem from a different angle. As we show in Appendix B, the (gross) marginal social benefit of more precise private information can also be expressed as follows

$$\frac{\left|u^*_{kk}+u^*_{\sigma\sigma}\right|}{2(\nu-1)(1-\alpha)}\frac{\kappa_1^2(1-\gamma)^2}{\pi_x^2}+\frac{\left|u^*_{kk}+u^*_{\sigma\sigma}\right|}{(\nu-1)(1-\alpha)}\frac{\kappa_1^2\left(\alpha-\alpha^*\right)}{\pi_z+(1-\alpha)\pi_x}\left|\frac{\partial\gamma}{\partial\pi_x}\right|, \tag{26}$$

where u^*_{kk}, $u^*_{\sigma\sigma}$, γ, and $\partial\gamma/\partial\pi_x$ are all functions of (π_x,π_z), as in the definitions above. Note that this expression closely parallels (8) in the Gaussian-quadratic model (recall that here $\kappa_1=\kappa_1^*$), except for one important qualification. The weight that the planner assigns to a reduction in the cross-sectional price dispersion is here equal to the weight she would have assigned had she been able to dictate the price rule $(u^*_{kk}+u^*_{\sigma\sigma})$, adjusted by the term $1/((v-1)(1-\alpha))$. This adjustment, which is unnecessary in the Gaussian-quadratic model, is what permits the planner to take into account that variations in the cross-sectional price dispersion under the equilibrium price rule also impact the ex ante level of the average prices, as discussed above.

The results in (24), or equivalently in (26), can in turn be used to identify primitive conditions under which the precision of private information acquired in equilibrium is inefficiently high or low. We discuss some of these implications in the result below.

Proposition 11. *Suppose that the planner cannot change the way the agents set their prices in equilibrium. Let*

$$R^* \equiv \frac{1}{2}+\sqrt{\frac{1}{4}+\frac{\omega v(1+v(\omega-1))}{(v-1)^2}}>1. \tag{27}$$

When utility is sufficiently concave in consumption, that is, when $R>R^*$, irrespective of the cost of information acquisition, the economy always overinvests in the acquisition of private information (meaning that ex ante welfare would be higher if the agents were to acquire less precise private information). When, instead, $R<R^*$, whether the economy over- or underinvests in the acquisition of private information depends on the cost of information acquisition.

To gain some intuition for why the direction of the inefficiency in the acquisition of private information depends on the curvature of the utility function over consumption, consider the *net* marginal social benefit of having the agents acquire more precise private information, evaluated at the equilibrium level $\hat{\pi}_x$. Using (24) and the fact that the equilibrium precision of private information satisfies

$$\frac{u_{kk}}{2}\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right)=C'(\hat{\pi}_x),$$

with $u_{kk}=u_{kk}(\hat{\pi}_x,\pi_z)$ and $\gamma=\gamma(\hat{\pi}_x,\pi_z)$, we have that the *net* marginal social benefit, evaluated at $\pi_x=\hat{\pi}_x$, is given by

$$\frac{u_{\sigma\sigma}}{2}\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right)+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right) \tag{28}$$
$$+2\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\frac{\kappa_1^2(\alpha^*-\alpha)}{\pi_z+(1-\alpha)\hat{\pi}_x}\left|\frac{\partial\gamma}{\partial\pi_x}\right|,$$

where u_{kk}, $u_{\sigma\sigma}$, u_K, γ and $\partial\gamma/\partial\pi_x$ are all evaluated at $(\hat{\pi}_x,\pi_z)$.

The economy thus underinvests in the acquisition of private information when the expression in (28) is positive, and overinvests when this expression is negative.

Now recall that this economy features an excessively low degree of coordination, that is $\alpha<\alpha^*$. As a result, under the equilibrium rule, prices respond too little to public information and too much to private information, relative to what is efficient. An increase in the precision of private information, by further increasing the sensitivity of equilibrium prices to private information relative to public may thus contribute to a welfare loss. This effect, which is captured by the second line in (28), is not internalized in equilibrium and contributes unambiguously to overinvestment in the acquisition of private information.[25]

Next, consider the first term in (28). This term captures the fact that, for given price rule, an increase in the precision of private information leads to a reduction in price dispersion, as captured by the term $-\left(\kappa_1^2(1-\gamma)^2\right)/\hat{\pi}_x^2$. This effect contributes to a welfare loss, given that in this economy a higher price dispersion has a positive direct effect on welfare, as captured by the term $u_{\sigma\sigma}>0$. As a result, the first term in (28) is also negative and thus contributes to overinvestment in information acquisition, irrespective of the curvature of individual payoffs in consumption.

The only term in (28) whose sign depends on R is thus

$$u_K \frac{\partial \mathbb{E}[\bar{p}]}{\partial \sigma_p^2}\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right) \tag{29}$$

in the first line of (28). As discussed above, this term captures the effects on welfare of a variation in the average price level, triggered by a variation in price dispersion. Recall that the term u_K is always negative, reflecting the fact that higher ex ante prices contribute negatively to consumption. On the other hand, the sign of $\partial\mathbb{E}[\bar{p}]/\partial\sigma_p^2$ depends on R. To see this, consider each agent's incentives to lower her own price in response to a reduction in price dispersion. When prices are less disperse, a reduction in the agent's own price translates into a stronger boost in sales, relative to the case where price dispersion is high. In turn, this means a stronger increase in both consumption and expected labour supply. When R is high, the marginal utility from increasing consumption is small relative to the extra disutility of labour, thus inducing the agent to raise her price in response to a reduction in price dispersion. As all agents do the same, a reduction in price dispersion then leads to an increase in the average price, contributing negatively to welfare. In this case, this effect reinforces the other two effects discussed above and the economy unambiguously overinvests in information acquisition.

In contrast, when R is small, ex ante prices may actually decline with a reduction in price dispersion, reflecting the incentives that each individual has in cutting her own price when expecting a high marginal utility of consumption relative to the marginal disutility of labour. In this case, whether the economy over- or underinvests in information acquisition then depends on how strong this last effect is relative to the other two effects discussed above, which in turn depends on the amount of private information acquired in equilibrium and hence on the cost of information acquisition.

Also note that the critical value R^* for the curvature of the utility function above which the economy always overinvests in information acquisition is decreasing in v and increasing in ω. To understand these comparative statics, recall that a higher v implies a higher substitutability across goods. This reinforces the effect of price dispersion on the ex ante level of the average prices discussed above. In fact, a higher substitutability across goods implies a sharper increase in sales in response to a reduction in prices. For each agent to find it optimal to cut her own

25. Note that a higher price dispersion σ_p^2 has three effects on welfare, captured by the three terms in the round bracket in the second line of (28). First, there is an unambiguously negative effect via the term u_{kk}, reflecting the fact that each household, when acting in the role of a producer, suffers from the dispersion of her own price around the average price. Second, there is a positive direct effect via the term $u_{\sigma\sigma}$, reflecting the fact that each household, when acting as a consumer, benefits from the dispersion of prices in the cross-section of the population. Third, there is the effect of price dispersion on the ex ante level of the average price, as captured by the term $u_K\partial\mathbb{E}[\bar{p}]/\partial\sigma_p^2$. As explained below, this term can be of either sign, depending on the curvature R of utility in consumption. As we show in the Appendix (Condition (B.34)), the sum of these three terms is, however, always negative, irrespective of R.

price in response to a reduction in price dispersion it then becomes essential that marginal utility drops less fast with consumption than in the case where sales are less sensitive to price cuts. The threshold R^* for the curvature of individual payoffs above which agents always overinvest in information acquisition thus naturally decreases with the substitutability parameter v.

Next, consider an increase in ω and recall that a higher ω means lower returns-to-scale in production. An increase in ω thus implies a higher value of information, since lower returns-to-scale imply that the amount of labour that each agent has to supply to compensate for low productivity becomes more significant. This enhances the value that each agent assigns to learning θ. Importantly, the higher value that the individual assigns to her private information is shared by the planner, implying that the instances in which the economy overinvests in information acquisition become more rare. As a result, the threshold R^* above which the economy overinvests in information acquisition naturally increases with ω.

Finally, to illustrate the possibility of underivestment in information acquisition, consider an economy in which the elasticity of substitution across goods is $v=3$, a value that implies a realistic price-to-cost ratio of 150%. Further recall that—as highlighted in Section (4.1)—our economy is isomorphic to one in which the marginal disutility of labour is increasing, and assume that the production technology exhibits constant returns to scale (a benchmark consistent, for example, with the estimates in Basu and Fernald, 1997). Lastly, assume a Frisch elasticity of 0.5, which is consistent with most estimates (see, *e.g.* Chetty *et al.*, 2011). Now recall that, in this economy, the parameter ω corresponds to the product of (a) the parameter for the returns-to-scale technology and (b) the inverse of the Frisch elasticity of labour supply (see footnote 14).

Replacing $v=3$ and $\omega=2$ into the formula for R^* in Condition (27), we obtain a critical value $R^*=3$. The result in the proposition then implies that, no matter the cost of information acquisition, the economy overinvests in information acquisition if $R>3$, a value that appears consistent, for example, with the estimate in Hall (1988). If, instead $R<3$, as some more recent estimates seem to indicate,[26] then the result in the proposition indicates that whether this economy underinvests or overinvests in information acquisition depends on the costs of information acquisition. However, using Condition (B.38) in Appendix B, one can verify that, for the same values of ω and v considered above and for values of R for example equal to zero or to one, for the economy to underinvest in the information acquisition, the marginal cost of acquiring private information must be so low that the equilibrium precision of private information is 7–10 times higher than that of public information, which seems unreasonable. Clearly, we are not suggesting that one should feel comfortable using the above specification for serious quantitative analysis. We leave it to future work to further enrich the environment to arrive to satisfactory quantitative results.

5. CONCLUDING REMARKS

We investigate the sources of inefficiency in the acquisition of private information and relate them to the sources of inefficiency in the equilibrium use of information. We then use such a characterization to show how the social value of public information is affected by the endogenous response in the acquisition of private information.

The analysis is conducted within the family of Gaussian-quadratic economies extensively studied in the literature. Many of the insights from the Gaussian-quadratic model appear to help

26. For example, Gourinchas and Parker (2002) find values of R that vary from 0.3 to 2.3 depending on educational background, whereas Vissing-Jørgensen (2002) finds values of R that range from 2.5 to 3.3 for stockholders and close to 1 for bondholders. Other papers such as Bansal and Yaron (2004), Guvenen (2006) and Barro (2009) suggest that the intertemporal elasticity of substitution is greater than one, which corresponds to $R<1$ in our model.

also outside such a model, as suggested by the fully microfounded business cycle application considered in the second part of the article.

In future work, it would be interesting to extend the analysis to richer information structures, such as those recently considered in the rational inattention literature, as well as those studied in Myatt and Wallace (2012), in which the "publicity" of any given source of information is determined endogenously by the attention allocated by the agents.

It would also be interesting to extend the analysis to other microfounded applications, such as the business-cycle specifications recently examined in Angeletos *et al.* (2011), and in Paciello and Wiederholt (2013), as well as the economies with information spillovers between real and financial activity examined in Angeletos *et al.* (2011).

APPENDIX A: PROOFS FOR THE GAUSSIAN-QUADRATIC MODEL

Proof of Proposition 1. Let $U_i(\pi_x;\hat{\pi}_x)$ denote agent i's ex ante expected payoff when she acquires information of precision π_x while any other agent acquires information of precision $\hat{\pi}_x$. Note that, to ease the exposition, hereafter we are suppressing the dependence of U_i on the precision of public information π_z, when there is no risk of confusion. Because (i) the agent expects her information to be used optimally once acquired and (ii) because the equilibrium use of information when all agents acquire information of precision $\hat{\pi}_x$ consists in all agents following the unique rule $k(\cdot;\hat{\pi}_x,\pi_z)$ that solves the functional equation (1), we have that the marginal benefit of more precise private information evaluated at $\pi_x=\hat{\pi}_x$ is simply the direct effect that more precise private information has on the agent's ex ante expected payoff, holding fixed the rule $k(\cdot;\hat{\pi}_x,\pi_z)$ by usual envelope reasoning.

Next observe that, when all agents (including agent i) follow the rule $k(\cdot;\hat{\pi}_x,\pi_z)$, agent i's ex ante expected payoff when she acquires information of precision π_x is given by

$$U_i(\pi_x;\hat{\pi}_x)=\mathbb{E}[u(K,K,\sigma_k,\theta)]+\frac{u_{kk}}{2}Var[k_i-K\mid\pi_x,\pi_z]-C(\pi_x), \tag{A.1}$$

where the first term in the right-hand side of (A.1) is the ex ante expectation of the payoff that the agent would obtain if her action coincided with the average action in each state, while the second term is the ex ante dispersion of the agent's action around the mean action.

Now note that, when all agents follow the unique strategy $k(\cdot;\hat{\pi}_x,\pi_z)$ that solves the functional equation (1) then both K and σ_k depend only on (θ,y) and $\mathbb{E}[u(K,K,\sigma_k,\theta)]$ is independent of the choice of π_x. The result then follows from the above observations. Q.E.D.

Proof of Proposition 2. The result follows from observing that, for any (π_x,π_z), the unique strategy $k(\cdot;\pi_x,\pi_z)$ that solves the functional equation (1) is the linear strategy

$$k(x,y;\pi_x,\pi_z)=\kappa_0+\kappa_1(1-\gamma)x+\kappa_1\gamma\left(\frac{\pi_\theta\mu+(\pi_z-\pi_\theta)y}{\pi_z}\right),$$

where

$$\gamma=\gamma(\pi_x,\pi_z)\equiv\frac{\pi_z}{\pi_z+(1-\alpha)\pi_x}. \tag{A.2}$$

Fixing the precision of private information at the equilibrium level $\pi_x=\hat{\pi}_x$, holding the rule $k(\cdot;\hat{\pi}_x,\pi_z)$ fixed (which also means holding γ fixed), and noting that $k-K=\kappa_1(1-\gamma)\xi$, we then have that

$$\frac{\partial}{\partial\pi_x}Var[k-K\mid k(\cdot;\hat{\pi}_x,\pi_z),\hat{\pi}_x,\pi_z]=-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2},$$

with $\gamma=\gamma(\hat{\pi}_x,\pi_z)$. In the unique symmetric equilibrium, each agent equates the marginal benefit of more precise private information to its marginal cost. Combining the result in Proposition 1 with the characterization above, we then have that, in any symmetric equilibrium, the precision of private information acquired in equilibrium must satisfy the first-order condition

$$\frac{|u_{kk}|}{2}\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}=C'(\hat{\pi}_x), \tag{A.3}$$

with $\gamma=\gamma(\hat{\pi}_x,\pi_z)$. Substituting (A.2) into (A.3) we arrive at the formula in the proposition. Finally, that the symmetric equilibrium is unique follows from the convexity of the cost function. Q.E.D.

Proof of Propositions 3 and 4. From Angeletos and Pavan (2007), we know that, for any given precision of private and public information (π_x, π_z), efficiency in the use of information requires that all agents follow the linear rule

$$k^*(x,y;\pi_x,\pi_z) = \kappa_0^* + \kappa_1^*\left(1-\gamma^*\right)x + \kappa_1^*\gamma^*\left(\frac{\pi_\theta\mu + (\pi_z - \pi_\theta)y}{\pi_z}\right), \tag{A.4}$$

where

$$\gamma^* = \gamma^*(\pi_x,\pi_z) \equiv \frac{\pi_z}{\pi_z + (1-\alpha^*)\pi_x} \tag{A.5}$$

and

$$\alpha^* \equiv 1 - \frac{u_{kk} + 2u_{kK} + u_{KK}}{u_{kk} + u_{\sigma\sigma}}.$$

We also know that, for any given precisions (π_x, π_z), welfare under the efficient rule $k^*(\cdot;\pi_x,\pi_z)$ is given by

$$w^*(\pi_x,\pi_z) \equiv \mathbb{E}[u(\kappa^*,\kappa^*,0,\theta)] - \mathcal{L}^*(\pi_x,\pi_z) - C(\pi_x),$$

where $u(\kappa^*,\kappa^*,0,\theta)$ denotes welfare under the first-best allocation and where

$$\mathcal{L}^*(\pi_x,\pi_z) \equiv \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} Var[K - \kappa^* \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z] + \frac{|u_{kk} + u_{\sigma\sigma}|}{2} Var[k - K \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$$

are the welfare losses due to incomplete information. The two terms $Var[K-\kappa^* \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ and $Var[k-K \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ denote, respectively, the ex ante dispersion of the aggregate action around the first-best allocation and the ex ante dispersion of individual actions in the cross-section of the population. Both dispersions are computed under the efficient rule $k^*(\cdot;\pi_x,\pi_z)$.

Now note that, holding fixed the rule $k^*(\cdot;\pi_x,\pi_z)$, the ex ante dispersion of the aggregate action around the first-best allocation is independent of π_x. Envelope arguments similar to those used in the proof of Proposition 1 then permit us to establish that the social marginal benefit of more precise private information under the efficient rule $k^*(\cdot;\pi_x,\pi_z)$ is given by (4) in Proposition 3.

Next, use (A.4) to verify that

$$\frac{\partial}{\partial \pi_x} Var[k - K \mid k^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z] = -\frac{\kappa_1^{*2}(1-\gamma^*)^2}{\pi_x^2}, \tag{A.6}$$

where $\gamma^* = \gamma^*(\pi_x,\pi_z)$ is given by (A.5). The result in Proposition 4 then follows from replacing $\gamma^* = \gamma^*(\pi_x,\pi_z)$ into (A.6) and equating the marginal cost of private information to its social marginal benefit. The uniqueness of π_x^* follows from the quasi-concavity of the welfare function $w^*(\pi_x,\pi_z)$ in π_x. Q.E.D.

Proof of Proposition 5. The result follows from comparing the (gross) marginal private benefit of more precise private information, evaluated at the equilibrium level $\hat{\pi}_x$, as given by

$$\frac{|u_{kk}|}{2}\frac{\kappa_1^2(1-\gamma(\hat{\pi},\pi_z))^2}{\hat{\pi}_x^2} = \frac{|u_{kk}|}{2}\left(\frac{\kappa_1(1-\alpha)}{\pi_z + (1-\alpha)\hat{\pi}_x}\right)^2$$

with the (gross) marginal social benefit, evaluated at the same equilibrium level $\pi_x = \hat{\pi}_x$, as given by

$$\frac{|u_{kk} + u_{\sigma\sigma}|}{2}\frac{\kappa_1^{*2}(1-\gamma^*(\hat{\pi},\pi_z))^2}{\hat{\pi}_x^2} = \frac{|u_{kk} + u_{\sigma\sigma}|}{2}\left(\frac{\kappa_1^*(1-\alpha^*)}{\pi_z + (1-\alpha^*)\hat{\pi}_x}\right)^2.$$

Q.E.D.

Proof of Proposition 6. From Angeletos and Pavan (2007), we know that welfare under the equilibrium use of information is given by

$$w(\pi_x,\pi_z) = \mathbb{E}[u(\kappa,\kappa,0,\theta)] - \mathcal{L}(\pi_x,\pi_z) - C(\pi_x),$$

where $\mathbb{E}[u(\kappa,\kappa,0,\theta)]$ is expected welfare under the complete-information equilibrium allocation κ and where

$$\mathcal{L}(\pi_x,\pi_z) \equiv \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2}\frac{\kappa_1^2\gamma^2}{\pi_z} + \frac{|u_{kk} + u_{\sigma\sigma}|}{2}\frac{\kappa_1^2(1-\gamma)^2}{\pi_x} + |u_{kk} + 2u_{kK} + u_{KK}|\frac{\kappa_1^* - \kappa_1}{\kappa_1}\frac{\kappa_1^2\gamma}{\pi_z}$$

are the welfare losses due to the incompleteness of information, with $\gamma = \gamma(\pi_x,\pi_z)$ given by (A.2). The first two terms in $\mathcal{L}(\pi_x,\pi_z)$ are the analogs of the two terms in $\mathcal{L}^*(\pi_x,\pi_z)$ in the proof of Propositions 3 and 4: they represent, respectively, the welfare losses due to the dispersion $Var[K-\kappa \mid k(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ of the aggregate activity around its complete-information counterpart and the welfare losses due to the cross-sectional dispersion $Var[k-K \mid k(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ of individual actions (note that both dispersions are computed under the equilibrium rule $k(\cdot;\pi_x,\pi_z)$). The last term is a first-order effect that is present only in economies that are inefficient under complete information (*i.e.* for which $\kappa \neq \kappa^*$). This term captures how the "error" $K-\kappa$ in aggregate activity due to incomplete information covaries with the inefficiency $\kappa-\kappa^*$ of the complete-information equilibrium allocation.

Using the above results, we then have that the (gross) marginal social benefit of an increase in the precision of private information is given by

$$\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{\kappa_1^2(1-\gamma)^2}{\pi_x^2} \tag{A.7}$$
$$-\gamma|u_{kk}+2u_{kK}+u_{KK}|\frac{\kappa_1^2}{\pi_z}\frac{\partial\gamma}{\partial\pi_x}+(1-\gamma)|u_{kk}+u_{\sigma\sigma}|\frac{\kappa_1^2}{\pi_x}\frac{\partial\gamma}{\partial\pi_x}$$
$$-|u_{kk}+2u_{kK}+u_{KK}|\frac{\kappa_1^*-\kappa_1}{\kappa_1}\frac{\kappa_1^2}{\pi_z}\frac{\partial\gamma}{\partial\pi_x},$$

where $\gamma=\gamma(\pi_x,\pi_z)$ is given by (A.2).

Substituting $|u_{kk}+2u_{kK}+u_{KK}|=(1-\alpha^*)|u_{kk}+u_{\sigma\sigma}|$, we can rewrite the sum of the second and third addendum of (A.7) as

$$\left[\frac{1-\gamma}{\pi_x}-\frac{(1-\alpha^*)\gamma}{\pi_z}\right]|u_{kk}+u_{\sigma\sigma}|\kappa_1^2\frac{\partial\gamma}{\partial\pi_x}. \tag{A.8}$$

Using the fact that $(1-\gamma^*)/\pi_x=(1-\alpha^*)\gamma^*/\pi_z$, we then have that (A.8) is equivalent to

$$\left(\gamma^*-\gamma\right)\left[\frac{1}{\pi_x}+\frac{1-\alpha^*}{\pi_z}\right]|u_{kk}+u_{\sigma\sigma}|\kappa_1^2\frac{\partial\gamma}{\partial\pi_x}. \tag{A.9}$$

Replacing the second and third term in (A.7) with (A.9) and using the definitions of $\gamma=\gamma(\pi_x,\pi_z)$ and $\gamma^*=\gamma^*(\pi_x,\pi_z)$ in (A.2) and (A.5), respectively, we then arrive at (8). Q.E.D.

Proof of Proposition 9. Replacing the formula for $\gamma=\gamma(\pi_x,\pi_z)$ into the formula for the the (gross) marginal social benefit of more precise private information (8) and using the fact that the marginal cost of more precise private information at the equilibrium level $\hat{\pi}_x$ is

$$C'(\hat{\pi}_x)=\frac{|u_{kk}|}{2}\frac{\kappa_1^2(1-\gamma(\hat{\pi}_x,\pi_z))^2}{\hat{\pi}_x^2}=\frac{|u_{kk}|}{2}\frac{\kappa_1^2(1-\alpha)^2}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2},$$

we have that the net effect of an increase in π_x on equilibrium welfare, evaluated at the equilibrium level $\hat{\pi}_x$, is given by (the derivation in (A.10) uses the formula for $\gamma=\gamma(\pi_x,\pi_z)$):

$$\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial\pi_x}=\left(\frac{|u_{kk}+u_{\sigma\sigma}|}{2}-\frac{|u_{kk}|}{2}\right)\frac{\kappa_1^2(1-\alpha)^2}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2} \tag{A.10}$$
$$+\frac{|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)(1-\alpha)\pi_z}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^3}$$
$$+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2(1-\alpha)}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2}\frac{\kappa_1^*-\kappa_1}{\kappa_1}.$$

Likewise, after some algebra, and using the fact that $|u_{kk}+2u_{kK}+u_{KK}|=(1-\alpha^*)|u_{kk}+u_{\sigma\sigma}|$, we have that the direct effect of an increase in the precision of public information π_z on equilibrium welfare is given by

$$\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial\pi_z}=\frac{|u_{kk}+2u_{kK}+u_{KK}|}{2}\frac{\kappa_1^{*2}}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2} \tag{A.11}$$
$$-\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\kappa_1^2\left(\alpha-\alpha^*\right)(1-\alpha)\frac{2\hat{\pi}_x}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^3}$$
$$-\frac{|u_{kk}+2u_{kK}+u_{KK}|}{2}\frac{(\kappa_1^*-\kappa_1)^2}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2}.$$

Combining (A.10) with (A.11) and using the result in Corollary 1 that $\partial\hat{\pi}_x/\partial\pi_z \geq -1/(1-\alpha)$, we have that, when $\partial w\left(\hat{\pi}_x,\pi_z\right)/\partial\pi_x > 0$,

$$\begin{aligned}\frac{dw\left(\hat{\pi}_x,\pi_z\right)}{d\pi_z} &= \frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z} + \frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_x}\frac{\partial\hat{\pi}_x}{\partial\pi_z} \\ &\geq \frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_z} - \frac{\partial w\left(\hat{\pi}_x,\pi_z\right)}{\partial\pi_x}\frac{1}{1-\alpha} \\ &= \frac{\kappa_1^2}{2\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2}\left[-|u_{kk}+u_{\sigma\sigma}|\left(\alpha-\alpha^*\right)+|u_{kk}|(1-\alpha)\right] \\ &= \frac{\kappa_1^2}{2\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2}\left[-\left(|u_{kk}+u_{\sigma\sigma}|+|u_{kk}|\right)\left(\alpha-\alpha^*\right)+|u_{kk}|\frac{|u_{kk}+2u_{kK}+u_{KK}|}{|u_{kk}+u_{\sigma\sigma}|}\right],\end{aligned}$$

which is always positive for

$$\alpha-\alpha^* < \Delta \equiv \frac{|u_{kk}+2u_{kK}+u_{KK}|\cdot|u_{kk}|}{|u_{kk}+u_{\sigma\sigma}|\cdot\left(|u_{kk}+u_{\sigma\sigma}|+|u_{kk}|\right)}.$$

Q.E.D.

Proof of Proposition 10. This payoff specification is nested in our model with $u_{kk}=-2$, $u_{kK}=2r$, $u_{KK}=-2r$, $u_{k\theta}=2(1-r)$, $u_{K\theta}=0$, $u_{\sigma\sigma}=2r$. It follows that $\alpha\equiv u_{kK}/|u_{kk}|=r$, $\kappa_1\equiv -u_{k\theta}/(u_{kk}+u_{kK})=\kappa_1^*\equiv -(u_{K\theta}+u_{k\theta})/(u_{kk}+2u_{kK}+u_{KK})=1$, and $\alpha^*\equiv 1-(u_{kk}+2u_{kK}+u_{KK})/(u_{kk}+u_{\sigma\sigma})=0$.

Using (A.10), we then have that the net effect on welfare of an increase in the precision of private information, evaluated at the equilibrium level, is given by

$$\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial\pi_x} = \frac{(1-r)^2 r[\pi_z-(1-r)\hat{\pi}_x]}{\left[\pi_z+(1-r)\hat{\pi}_x\right]^3}. \tag{A.12}$$

As one can easily see from (A.12), the precision of private information acquired in equilibrium can be either inefficiently low or inefficiently high, depending on the precision of public information π_z. Formally, there exists a threshold $\pi_z'>0$ such that $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_x<0$ if and only if $\pi_z<\pi_z'$.[27]

Next, use (A.11) to see that the direct effect on welfare of an increase in the precision of public information is given by

$$\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial\pi_z} = \frac{(1-r)[\pi_z-(2r-1)(1-r)\hat{\pi}_x]}{\left[\pi_z+(1-r)\hat{\pi}_x\right]^3}, \tag{A.13}$$

which is positive if and only if

$$\pi_z > (2r-1)(1-r)\hat{\pi}_x, \tag{A.14}$$

as shown in Morris and Shin (2002). Using the negative dependence of $\hat{\pi}_x$ on π_z, we can then show that there exists a threshold $\pi_z''<\pi_z'$ such that the inequality in (A.14) holds if and only if $\pi_z>\pi_z''$.

One can then easily see how the crowding-out effects of public information on private information may affect the social value of public information. As shown in Proposition 8, when the precision of private information acquired in equilibrium is inefficiently high (in these economies, this occurs when $\pi_z<\pi_z'$), such crowding-out effects increase the social value of public information. These new effects can be sufficiently strong to overturn the partial effect identified in the literature, making the social value of public information positive under the same conditions that would have predicted it to be negative by ignoring such crowding-out effects.

To see this more clearly, suppose that the cost of acquiring private information is given by the following isoelastic cost function $C(\pi_x)=(\pi_x^{1+\eta})/(1+\eta)$, where $\eta\in[0,\infty)$ is the elasticity of the marginal cost. Using (3), we can then verify that the equilibrium degree of substitutability between public and private information (equivalently, the crowding-out effect of public information on private information) is given by

$$\frac{\partial\hat{\pi}_x}{\partial\pi_z} = -\frac{2\hat{\pi}_x}{\eta\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]+2(1-\alpha)\hat{\pi}_x}, \tag{A.15}$$

with $-1/(1-\alpha)\leq\partial\hat{\pi}_x/\partial\pi_z\leq 0$, as predicted by Corollary 1. Combining (A.12) with (A.13) and using (A.15), we then have that the total effect on welfare of an increase in the precision of public information (controlling for the crowding-out

27. Observe from (3) that $\lim_{\pi_z\to 0}\hat{\pi}_x(\pi_z)=\bar{\pi}_x>0$, with $\bar{\pi}_x$ implicitly given by $\bar{\pi}_x^2 C'(\bar{\pi}_x)=|u_{kk}|\kappa_1^2/2$, while $\lim_{\pi_z\to\infty}\hat{\pi}_x(\pi_z)=0$. Along with the fact that $\hat{\pi}_x$ is strictly decreasing in π_z, these properties give the result.

effect of public information on private information) is given by

$$\frac{dw(\hat{\pi}_x,\pi_z)}{d\pi_z}=\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial \pi_z}+\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial \pi_x}\frac{\partial \hat{\pi}_x}{\partial \pi_z}$$

$$=\frac{(1-r)}{\left[\pi_z+(1-r)\hat{\pi}_x\right]^3}\left\{\pi_z-(2r-1)(1-r)\hat{\pi}_x-\frac{r(1-r)\left[\pi_z-(1-r)\hat{\pi}_x\right]2\hat{\pi}_x}{\eta\left[\pi_z+(1-r)\hat{\pi}_x\right]+2(1-r)\hat{\pi}_x}\right\}.$$

The above expression is positive if and only if

$$\eta\pi_z^2+2(1-r)^2(1+\eta)\hat{\pi}_x\pi_z-(\eta(2r-1)-2(1-r))(1-r)^2\hat{\pi}_x^2\geq 0,$$

or, equivalently, if and only if

$$\pi_z\geq\left[2r-1-\frac{2(1-r)}{\eta}\right](1-r)\hat{\pi}_x.$$

Given that

$$\left[2r-1-\frac{2(1-r)}{\eta}\right](1-r)\hat{\pi}_x<(2r-1)(1-r)\hat{\pi}_x,$$

this means that there exists a third threshold $\pi_z'''<\pi_z''$ such that the social value of public information is positive, when private information is endogenous, if and only if $\pi_z>\pi_z'''$. It is then easy to see that, for any $\pi_z\in(\pi_z''',\pi_z'')$, the social value of public information turns from negative to positive when one takes into account the crowding-out effect of public information on private information. Q.E.D.

APPENDIX B: DERIVATION OF RESULTS FOR MONETARY ECONOMY APPLICATION

Reduced-form payoffs: We start by showing how the economy can be represented as a game in which the reduced-form payoff for each producer–consumer depends only on the cross-sectional distribution of log-prices. To this purpose, let

$$P\equiv\left(\int_{[0,1]}P_h^{1-\nu}dh\right)^{\frac{1}{1-\nu}} \tag{B.1}$$

denote the usual Dixit–Stiglitz price index. Solving the game backwards starting from the agents' consumption choices, it is standard to show that each agent j's demand for each good $h\in[0,1]$ satisfies

$$c_{jh}=\left(\frac{P}{P_h}\right)^{\nu}C_j, \tag{B.2}$$

where C_j is the Dixit–Stiglitz aggregator of the agent's consumption, as defined in the main text. Aggregating the individual demand functions (B.2) for each good h over all agents yields the aggregate demand for each good h

$$Y_h=\left(\frac{P}{P_h}\right)^{\nu}Y, \tag{B.3}$$

where

$$Y\equiv\int_{[0,1]}C_jdj.$$

Substituting the individual demand for each good h, as given by (B.2), into each agent's budget constraint (10), and using the definition of the aggregate price index in (B.1), we have that each agent's budget constraint can be rewritten as

$$C_j=\frac{P_j}{P}Y_j. \tag{B.4}$$

Combining (B.4) with (B.3) then permits us to express C_j as follows

$$C_j=\left(\frac{P}{P_j}\right)^{\nu-1}Y. \tag{B.5}$$

Solving (11) for N_j and using (B.3) then allows us to express the agent's individual labour supply as a function of her own price, the aggregate price index, and the technology shock Θ as follows

$$N_j=Y_j^{\omega}\Theta^{-\omega}=\left(\frac{P}{P_j}\right)^{\nu\omega}Y^{\omega}\Theta^{-\omega}. \tag{B.6}$$

Finally, substituting (B.5) and (B.6) into the agent's utility function (9) and rewriting the money supply equation as $PY=M$ permits us to express each agent's payoff as a function of her own price, the Dixit–Stiglitz price index P, and the

TFP shock Θ:

$$\mathcal{U}(P_j,P,\Theta)\equiv\frac{\left(P_j^{1-v}P^{v-2}M\right)^{1-R}}{1-R}-P_j^{-v\omega}P^{\omega(v-1)}M^{\omega}\Theta^{-\omega}. \tag{B.7}$$

Using (B.1), we can show that, in any equilibrium in which prices are log-Normally distributed, the Dixit–Stiglitz price index can be rewritten as follows

$$P=\left(\int_{[0,1]}\exp\{p_h(1-v)\}dh\right)^{\frac{1}{1-v}}=(\mathbb{E}[\exp\{p_h(1-v)\}])^{\frac{1}{1-v}}=\exp\left\{\bar{p}-(v-1)\frac{\sigma_p^2}{2}\right\}.$$

Replacing P into (B.7) and using the definitions of p_j, θ, and m then yields the expression in (12).

Equilibrium price rule: Let

$$\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)\equiv\frac{1}{1-R}\left(\exp\left\{(1-v)p+(v-2)\bar{p}-(v-1)(v-2)\frac{\sigma_p^2}{2}+m\right\}\right)^{1-R}$$

and

$$\mathcal{N}(p,\bar{p},\sigma_p^2,\theta)\equiv\exp\left\{-v\omega p+\omega(v-1)\bar{p}-\omega(v-1)^2\frac{\sigma_p^2}{2}+\omega m-\omega\theta\right\}$$

denote the utility of consumption and the disutility of labour, respectively, for an agent with log-price p, when the aggregate log-price is $\bar{p}$, the dispersion of log-prices is σ_p^2, and the log-productivity shock is θ.

Using (12), we have that each agent's payoff can be rewritten concisely as

$$u(p,\bar{p},\sigma_p^2,\theta)\equiv\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)-\mathcal{N}(p,\bar{p},\sigma_p^2,\theta). \tag{B.8}$$

Denoting by $\mathbb{E}_j[\cdot]=\mathbb{E}[\cdot|x_j,y;\pi_x,\pi_z]$ the expected value operator given the agent's information (x_j,y) and the quality of information (π_x,π_z), and using (B.8), we then have that the optimal choice of p_j is given by the first-order condition

$$\frac{\partial}{\partial p_j}\mathbb{E}_j\left[u(p_j,\bar{p},\sigma_p^2,\theta)\right]=0. \tag{B.9}$$

Using the notation above, we can rewrite (B.9) as

$$\mathbb{E}_j\left[\mathcal{V}(p_j,\bar{p},\sigma_p^2,\theta)\right]=\frac{v\omega}{(v-1)(1-R)}\mathbb{E}_j\left[\mathcal{N}(p_j,\bar{p},\sigma_p^2,\theta)\right]. \tag{B.10}$$

Assuming that $\bar{p}$ is Normally distributed and that σ_p^2 is a constant (both properties will be shown to hold in equilibrium), taking logs, and rearranging, we can rewrite the optimality condition (B.10) as (13).

Next note that, when all agents other than j follow the linear price rule in (15), (a) the average price is given by

$$\bar{p}=\lambda_0+\lambda_1\theta+\lambda_2 z, \tag{B.11}$$

which is indeed Normally distributed with a constant variance, and (b) the dispersion of prices in the cross section of the population is given by

$$\sigma_p^2=\frac{\lambda_1^2}{\pi_x}, \tag{B.12}$$

which is also constant, as conjectured. Replacing (B.11) and (B.12) into agent j's best response (13), noting that $Var_j[\theta]=1/(\pi_z+\pi_x)$, $Var_j[\bar{p}]=\lambda_1^2/(\pi_z+\pi_x)$, and $Cov_j[\bar{p},\theta]=\lambda_1/(\pi_z+\pi_x)$, and solving for the fixed point, we obtain that, in the unique log-linear symmetric equilibrium,

$$\lambda_1=\lambda_1(\pi_x,\pi_z)\equiv\kappa_1\frac{(1-\alpha)\pi_x}{\pi_z+(1-\alpha)\pi_x}=\kappa_1(1-\gamma), \tag{B.13}$$

$$\lambda_2=\lambda_2(\pi_x,\pi_z)\equiv\kappa_1\frac{\pi_z}{\pi_z+(1-\alpha)\pi_x}=\kappa_1\gamma, \tag{B.14}$$

with

$$\gamma=\gamma(\pi_x,\pi_z)\equiv\frac{\pi_z}{\pi_z+(1-\alpha)\pi_x},$$

and

$$\lambda_0=\lambda_0(\pi_x,\pi_z)\equiv\kappa_0-(v-1)\frac{\omega(v-1)+(R-1)(v-2)}{2(R+\omega-1)}\frac{\lambda_1^2}{\pi_x} \tag{B.15}$$

$$+\frac{\omega^2[(v-1)\lambda_1-1]^2-(1-R)^2(v-2)^2\lambda_1^2}{2(R+\omega-1)(\pi_x+\pi_z)}.$$

Notice that the second line of (B.15) can be rewritten as

$$\frac{\left[\omega^2 v^2-(1-R)^2(v-1)^2\right]\lambda_1^2}{2(R+\omega-1)(\pi_x+\pi_z)}+\frac{\omega^2(1+\lambda_1)^2-(1-R)^2\lambda_1^2}{2(R+\omega-1)(\pi_x+\pi_z)}+\frac{(1-R)^2(v-1)\lambda_1^2-\omega^2 v\lambda_1(1+\lambda_1)}{(R+\omega-1)(\pi_x+\pi_z)}.$$

Observing that

$$\frac{1}{\pi_x+\pi_z}=\frac{1}{\pi_x}-\frac{\pi_z}{\pi_x(\pi_x+\pi_z)}=\frac{1}{\pi_z}-\frac{\pi_x}{\pi_z(\pi_x+\pi_z)},$$

we then have that the second line of (B.15) can be rewritten as

$$\frac{1}{2(R+\omega-1)}\left\{\frac{\left[\omega^2 v^2-(1-R)^2(v-1)^2\right]\lambda_1^2}{\pi_x}+\frac{\omega^2(1+\lambda_1)^2-(1-R)^2\lambda_1^2}{\pi_z}\right\}$$

$$+\frac{1}{2(R+\omega-1)}\left\{\frac{(1-R)^2\lambda_1^2}{\pi_z+\pi_x}\frac{[(v-1)\pi_z+\pi_x]^2}{\pi_z\pi_x}-\frac{\omega^2}{\pi_z+\pi_x}\frac{[v\lambda_1\pi_z+(1+\lambda_1)\pi_x]^2}{\pi_z\pi_x}\right\}.$$

Using the formulas for λ_1, κ_1, γ, and α, we then have that

$$\frac{(1-R)^2\lambda_1^2}{\pi_z+\pi_x}\frac{[(v-1)\pi_z+\pi_x]^2}{\pi_z\pi_x}-\frac{\omega^2}{\pi_z+\pi_x}\frac{[v\lambda_1\pi_z+(1+\lambda_1)\pi_x]^2}{\pi_z\pi_x}=0,$$

which permits us to rewrite (B.15) as

$$\lambda_0=\kappa_0-(v-1)\frac{\omega(v-1)+(R-1)(v-2)}{2(R+\omega-1)}\frac{\lambda_1^2}{\pi_x}+\frac{\omega^2 v^2-(1-R)^2(v-1)^2}{2(R+\omega-1)}\frac{\lambda_1^2}{\pi_x}+\frac{\omega^2(1+\lambda_1)^2-(1-R)^2\lambda_1^2}{2(R+\omega-1)}\frac{1}{\pi_z}. \quad \text{(B.16)}$$

Combining the second and third terms in (B.16), we obtain (16) in the main text.

Equilibrium acquisition of private information: Fix the precision of public information π_z, and let $\hat{U}_j(\pi_{x_j},\pi_x)$ denote agent j's ex ante expected payoff when the precision of her information is π_{x_j}, the precision of all other agents information is π_x and all agents' (including agent j) follow the equilibrium price rule, as given by (15) (the dependence of $\hat{U}_j$ on π_z is dropped to ease the exposition). Using (B.8), and the fact that, when s is Normally distributed, $\mathbb{E}[\exp\{s\}]=\exp\left\{\mathbb{E}[s]+\frac{1}{2}Var(s)\right\}$, we then have that

$$\frac{\partial\hat{U}_j(\pi_x,\pi_x)}{\partial\pi_{x_j}}=\mathbb{E}[\mathcal{V};\pi_x,\pi_z]\frac{(1-R)^2(1-v)^2}{2}\left(-\frac{\lambda_1^2}{\pi_x^2}\right)-\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\frac{(v\omega)^2}{2}\left(-\frac{\lambda_1^2}{\pi_x^2}\right)-C'(\pi_x), \quad \text{(B.17)}$$

where $\mathbb{E}[\mathcal{V};\pi_x,\pi_z]$ and $\mathbb{E}[\mathcal{N};\pi_x,\pi_z]$ are shortcuts for $\mathbb{E}\left[\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)\right]$ and $\mathbb{E}\left[\mathcal{N}(p,\bar{p},\sigma_p^2,\theta)\right]$. Importantly, these ex ante expectations are computed under the price rule (15). That is, they are computed using the fact that (a) $\bar{p}=\lambda_0+\lambda_1\theta+\lambda_2 z$, (b) $\sigma_p^2=\lambda_1^2/\pi_x$, and (c) $p=\lambda_0+\lambda_1 x+\lambda_2 z$, with the coefficients $(\lambda_0,\lambda_1,\lambda_2)$ given by (B.16), (B.13), and (B.14), respectively, with x drawn from a Normal distribution with mean zero and variance $1/\pi_\theta+1/\pi_x$, and with z drawn from a Normal distribution with mean zero and variance $(\pi_z-\pi_\theta)/\pi_z\pi_\theta$.

Applying the law of iterated expectations to (B.10), we have that

$$\mathbb{E}[\mathcal{V};\pi_x,\pi_z]=\frac{v\omega}{(v-1)(1-R)}\mathbb{E}[\mathcal{N};\pi_x,\pi_z]. \quad \text{(B.18)}$$

Replacing (B.18) into (B.17) and recalling that $\lambda_1=\kappa_1(1-\gamma)$, we then have that

$$\frac{\partial\hat{U}_j(\pi_x,\pi_x)}{\partial\pi_{x_j}}=\{-v\omega[v\omega+(R-1)(v-1)]\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\}\left(-\frac{[\kappa_1(1-\gamma)]^2}{2\pi_x^2}\right)-C'(\pi_x), \quad \text{(B.19)}$$

with $\gamma=\gamma(\pi_x,\pi_z)$. To interpret this condition, first use the envelope theorem to note that $\partial\hat{U}_j(\pi_x,\pi_x)/d\pi_{x_j}$ is agent j's net marginal private benefit of acquiring more precise private information when all agents (including agent j) acquire information of quality π_x and then use their information according to (15). Next, use (B.8) and (B.10) to note that, when all agents follow the linear rule (15),

$$\begin{aligned} u_{kk}=u_{kk}(\pi_x,\pi_z)&\equiv\mathbb{E}\left[\frac{\partial^2 u(p,\bar{p},\sigma_p^2,\theta)}{\partial p^2}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]\\ &=(1-R)^2(1-v)^2\mathbb{E}[\mathcal{V};\pi_x,\pi_z]-(v\omega)^2\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\\ &=-v\omega[v\omega+(R-1)(v-1)]\mathbb{E}[\mathcal{N};\pi_x,\pi_z]<0. \end{aligned}$$

We thus have that the first curly bracket in (B.19) is simply the expected curvature of a representative agent's payoff around her own price when all agents follow the linear rule (15) and when all agents have private information of quality π_x.

Next, note that the second curly bracket in (B.19) is simply the marginal reduction in the dispersion of each agent's own price around the mean price that obtains by increasing the precision of the agent's private information holding fixed the equilibrium price rule (which means holding fixed $\gamma=\gamma(\pi_x,\pi_z)$). In a symmetric equilibrium, this reduction coincides with the marginal reduction of each agent's price around the cross-sectional average price, that is,

$$-\frac{[\kappa_1(1-\gamma)]^2}{\pi_x^2}=\frac{\partial}{\partial\pi_x}Var[p-\bar{p}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z].$$

We conclude that, in any symmetric equilibrium, the gross private benefit of more accurate private information is given by (18) in the main text.

Efficient price rule: The efficient (linear) price rule maximizes

$$\mathbb{E}[u(p,\bar{p},\sigma_p^2,\theta)]=\mathbb{E}[\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)]-\mathbb{E}[\mathcal{N}(p,\bar{p},\sigma_p^2,\theta)].$$

Using again the property that, when s is Normally distributed, $\mathbb{E}[\exp\{s\}]=\exp\left\{\mathbb{E}[s]+\frac{1}{2}Var[s]\right\}$, and noting that, under any linear rule, $p=\lambda_0+\lambda_1x+\lambda_2z=\bar{p}+\lambda_1(x-\theta)$ so that $\sigma_p^2=\lambda_1^2/\pi_x$, we have that

$$\begin{aligned}&\mathbb{E}[u(p,\bar{p},\sigma_p^2,\theta)]=\\ &\frac{1}{1-R}\exp\left\{\begin{array}{c}-(1-R)\mathbb{E}[\bar{p}]+\frac{1}{2}(1-R)^2Var[\bar{p}]\\ +\left[(1-R)^2(v-1)^2-(1-R)(v-1)(v-2)\right]\frac{\sigma_p^2}{2}+(1-R)m\end{array}\right\}\\ &-\exp\left\{-\omega\mathbb{E}[\bar{p}+\theta]+\omega^2\frac{1}{2}Var[\bar{p}+\theta]+\left[\omega^2v^2-\omega(v-1)^2\right]\frac{\sigma_p^2}{2}+\omega m\right\}.\end{aligned}\tag{B.20}$$

Next, observe that, under any linear rule,

$$\mathbb{E}[\bar{p}]=\mathbb{E}[\bar{p}+\theta]=\lambda_0,$$

$$Var[\bar{p}]=\frac{1}{\pi_\theta}\left(\lambda_1+\frac{\pi_y}{\pi_z}\lambda_2\right)^2+\frac{\pi_y}{\pi_z^2}\lambda_2^2,$$

$$Var[\bar{p}+\theta]=\frac{1}{\pi_\theta}\left(1+\lambda_1+\frac{\pi_y}{\pi_z}\lambda_2\right)^2+\frac{\pi_y}{\pi_z^2}\lambda_2^2.$$

Using these expressions and writing $\mathbb{E}[\mathcal{V};\pi_x,\pi_z]$ and $\mathbb{E}[\mathcal{N};\pi_x,\pi_z]$ again as shortcuts for $\mathbb{E}\left[\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)\right]$ and $\mathbb{E}\left[\mathcal{N}(p,\bar{p},\sigma_p^2,\theta)\right]$, we can write the first-order conditions of (B.20) with respect to λ_0, λ_1 and λ_2, respectively, as

$$\mathbb{E}[\mathcal{V};\pi_x,\pi_z]=\frac{\omega}{1-R}\mathbb{E}[\mathcal{N};\pi_x,\pi_z],\tag{B.21}$$

$$\begin{aligned}&\left\{(1-R)^2\frac{1}{\pi_\theta}\left(\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)+\left[(1-R)^2(v-1)^2-(1-R)(v-1)(v-2)\right]\frac{\lambda_1^*}{\pi_x}\right\}\mathbb{E}[\mathcal{V};\pi_x,\pi_z]\\ &=\left\{\omega^2\frac{1}{\pi_\theta}\left(1+\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)+\left[\omega^2v^2-\omega(v-1)^2\right]\frac{\lambda_1^*}{\pi_x}\right\}\mathbb{E}[\mathcal{N};\pi_x,\pi_z],\end{aligned}\tag{B.22}$$

and

$$\begin{aligned}&\left\{(1-R)^2\frac{\pi_y}{\pi_\theta\pi_z}\left(\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)+(1-R)^2\frac{\pi_y}{\pi_z^2}\lambda_2^*\right\}\mathbb{E}[\mathcal{V};\pi_x,\pi_z]\\ &=\left\{\omega^2\frac{\pi_y}{\pi_\theta\pi_z}\left(1+\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)+\omega^2\frac{\pi_y}{\pi_z^2}\lambda_2^*\right\}\mathbb{E}[\mathcal{N};\pi_x,\pi_z].\end{aligned}\tag{B.23}$$

Combining (B.21) with (B.22) and (B.23), we can then show that

$$\lambda_1^*=\lambda_1^*(\pi_x,\pi_z)\equiv\kappa_1^*\frac{\pi_x(1-\alpha^*)}{\pi_z+\pi_x(1-\alpha^*)}\text{ and }\lambda_2^*=\lambda_2^*(\pi_x,\pi_z)\equiv\kappa_1^*\frac{\pi_z}{\pi_z+\pi_x(1-\alpha^*)}=\kappa_1^*-\lambda_1^*,$$

where

$$1-\alpha^*=\frac{R+\omega-1}{v(v\omega+1-v)+R(v-1)^2}$$

and where $\kappa_1^*(=\kappa_1)=-\omega/(R+\omega-1)$. Substituting the above expressions into (B.21) yields

$$\lambda_0^* = \frac{1}{R+\omega-1}\ln(\omega)+m-(v-1)\frac{\omega(v-1)+(R-1)(v-2)}{2(R+\omega-1)}\frac{\lambda_1^{*2}}{\pi_x}$$
$$+\frac{\omega^2v^2-(1-R)^2(v-1)^2}{2(R+\omega-1)}\frac{\lambda_1^{*2}}{\pi_x}+\frac{\omega^2}{2(R+\omega-1)}\left(\frac{\left(1+\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)^2}{\pi_\theta}+\frac{\pi_y}{\pi_z^2}\lambda_2^{*2}\right)$$
$$-\frac{(1-R)^2}{2(R+\omega-1)}\left(\frac{\left(\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)^2}{\pi_\theta}+\frac{\pi_y}{\pi_z^2}\lambda_2^{*2}\right). \tag{B.24}$$

Using the fact that $\lambda_1^*+\lambda_2^*=\kappa_1^*$ in turn permits us to write

$$\frac{\left(1+\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)^2}{\pi_\theta}+\frac{\pi_y}{\pi_z^2}\lambda_2^{*2}=\frac{\left(1+\lambda_1^*\right)^2}{\pi_z}+\frac{\pi_y}{\pi_\theta\pi_z}\left(1+\kappa_1^*\right)^2,$$

and

$$\frac{\left(\lambda_1^*+\frac{\pi_y}{\pi_z}\lambda_2^*\right)^2}{\pi_\theta}+\frac{\pi_y}{\pi_z^2}\lambda_2^{*2}=\frac{\lambda_1^{*2}}{\pi_z}+\frac{\pi_y}{\pi_\theta\pi_z}\kappa_1^{*2}.$$

Using the definition for κ_1^*, we can then express (B.24) as

$$\lambda_0^*=\kappa_0^*-(v-1)\frac{\omega(v-1)+(R-1)(v-2)}{2(R+\omega-1)}\frac{\lambda_1^{*2}}{\pi_x}$$
$$+\frac{\omega^2v^2-(1-R)^2(v-1)^2}{2(R+\omega-1)}\frac{\lambda_1^{*2}}{\pi_x}+\frac{\omega^2\left(1+\lambda_1^*\right)^2-(1-R)^2\lambda_1^{*2}}{2(R+\omega-1)}\frac{1}{\pi_z}. \tag{B.25}$$

Combining the second and third terms in (B.25) then permits us to obtain the expression in (21) in the main text.

Finally, note that, after some tedious algebra, one can show that under the efficient price rule

$$p_j^*=\mathbb{E}_j[(1-\alpha^*)\kappa^*+\alpha^*\bar{p}^*]-\left(1-\alpha^*\right)\left[(v-1)\frac{\omega(v-1)+(R-1)(v-2)}{2(R+\omega-1)}\frac{\lambda_1^{*2}}{\pi_x}\right] \tag{B.26}$$
$$+\left(1-\alpha^*\right)\left[\frac{\omega^2v^2-(1-R)^2(v-1)^2}{2(R+\omega-1)}\frac{\lambda_1^{*2}}{\pi_x}+\frac{\omega^2\left(1+\lambda_1^*\right)^2-(1-R)^2\lambda_1^{*2}}{2(R+\omega-1)}\frac{1}{\pi_z}\right],$$

from which it is easy to see that α^* continues to measure the socially optimal degree of coordination, that is, the slope of the individual best responses to the aggregate price level, under the efficient price rule.

Comparison between α^* and α: Note that

$$\alpha^*-\alpha=\frac{(R+\omega-1)(v-1)[v\omega+1-v+R(v-2)]}{\left[v(v\omega+1-v)+R(v-1)^2\right][v\omega+1-v+R(v-1)]}.$$

It is immediate to see that all the terms in the denominator and those in the first two brackets of the numerator of the above expression are positive for all $v,\omega>1$ and $R\geq 0$, so that

$$sign\left(\alpha^*-\alpha\right)=sign[v\omega+1-v+R(v-2)].$$

Recall that $\alpha>0$ requires that $\omega(v-1)+(R-1)(v-2)>0$, which can be rewritten as $v\omega+R(v-2)+2-\omega-v>0$. Since $\omega>1$, this implies that $v\omega+1-v+R(v-2)>0$. Hence, $\alpha>0$ implies that $\alpha^*>\alpha$.

Efficient acquisition of information: Using (B.8), we have that, for any precision of private and public information, welfare under the efficient price rule (20) is given by

$$w^*(\pi_x,\pi_z)=\mathbb{E}\left[\mathcal{V}^*;\pi_x,\pi_z\right]-\mathbb{E}\left[\mathcal{N}^*;\pi_x,\pi_z\right]-C(\pi_x),$$

where $\mathbb{E}[\mathcal{V}^*;\pi_x,\pi_z]$ and $\mathbb{E}[\mathcal{N}^*;\pi_x,\pi_z]$ are shortcuts for $\mathbb{E}\left[\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)\right]$ and $\mathbb{E}\left[\mathcal{N}(p,\bar{p},\sigma_p^2,\theta)\right]$, and where each of these expectations is computed assuming that prices are determined according to the efficient price rule (20) for quality of information (π_x,π_z).

Using again the envelope theorem, we then have that the net marginal benefit of more precise private information is given by the direct effect on welfare of a marginal increase in π_x, holding fixed the efficient price rule. That is,

$$\frac{\partial w^*(\pi_x,\pi_z)}{\partial \pi_x} = \frac{1}{2}\Big[(1-R)(\nu-1)^2+(1-\nu)(\nu-2)\Big](1-R)\mathbb{E}\big[\mathcal{V}^*;\pi_x,\pi_z\big]\left(-\frac{\lambda_1^{*2}}{\pi_x^2}\right) \tag{B.27}$$

$$-\frac{1}{2}\Big[(\nu\omega)^2-\omega(\nu-1)^2\Big]\mathbb{E}\big[\mathcal{N}^*;\pi_x,\pi_z\big]\left(-\frac{\lambda_1^{*2}}{\pi_x^2}\right)-C'(\pi_x),$$

where

$$\lambda_1^* = \lambda_1^*(\pi_x,\pi_z) \equiv \kappa_1^* \frac{\pi_x(1-\alpha^*)}{\pi_z+\pi_x(1-\alpha^*)}.$$

Now recall that, under the efficient price rule $\mathbb{E}[\mathcal{V}^*;\pi_x,\pi_z]=\frac{\omega}{1-R}\mathbb{E}[\mathcal{N}^*;\pi_x,\pi_z]$. Replacing this expression into (B.27), we then have that

$$\frac{\partial w^*(\pi_x,\pi_z)}{\partial \pi_x} = \frac{1}{2}\omega\Big[(1-R)(\nu-1)^2-(1-\nu)-\omega\nu^2\Big]\mathbb{E}\big[\mathcal{N}^*;\pi_x,\pi_z\big]\left(-\frac{\lambda_1^{*2}}{\pi_x^2}\right)-C'(\pi_x). \tag{B.28}$$

Next, observe that, under the efficient price rule, the expected curvature of each agent's payoff around her own price is given by

$$u_{kk}^* = u_{kk}^*(\pi_x,\pi_z) \equiv \mathbb{E}\left[\frac{\partial^2 u(p,\bar{p},\sigma_p^2,\theta)}{\partial p^2} \mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]$$

$$=(1-R)^2(1-\nu)^2\mathbb{E}\big[\mathcal{V}^*;\pi_x,\pi_z\big]-(\nu\omega)^2\mathbb{E}\big[\mathcal{N}^*;\pi_x,\pi_z\big]$$

$$=-\omega\Big[\nu^2\omega+(R-1)(1-\nu)^2\Big]\mathbb{E}\big[\mathcal{N}^*;\pi_x,\pi_z\big]<0.$$

Then, use again (B.8) and $\mathbb{E}[\mathcal{V}^*;\pi_x,\pi_z]=\frac{\omega}{1-R}\mathbb{E}[\mathcal{N}^*;\pi_x,\pi_z]$ to note that, under the efficient price rule,

$$u_{\sigma\sigma}^* = u_{\sigma\sigma}^*(\pi_x,\pi_z) \equiv 2\mathbb{E}\left[\frac{\partial u(p,\bar{p},\sigma_p^2,\theta)}{\partial \sigma_p^2} \mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]$$

$$=(1-\nu)(\nu-2)(1-R)\mathbb{E}\big[\mathcal{V}^*;\pi_x,\pi_z\big]+\omega(1-\nu)^2\mathbb{E}\big[\mathcal{N}^*;\pi_x,\pi_z\big]$$

$$=(\nu-1)\omega\mathbb{E}\big[\mathcal{N}^*;\pi_x,\pi_z\big]>0.$$

Using the definitions of u_{kk}^* and $u_{\sigma\sigma}^*$ and the fact that $\lambda_1^*=\kappa_1^*(1-\gamma^*)$, we can then rewrite the net marginal benefit of more precise private information (B.28) as[28]

$$\frac{\partial w^*(\pi_x,\pi_z)}{\partial \pi_x} = \frac{u_{kk}^*+u_{\sigma\sigma}^*}{2}\left(-\frac{\kappa_1^{*2}(1-\gamma^*)^2}{\pi_x^2}\right)-C'(\pi_x)$$

$$=\frac{u_{kk}^*+u_{\sigma\sigma}^*}{2}\frac{\partial}{\partial \pi_x}Var[p-\bar{p}\mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]-C'(\pi_x),$$

where $\frac{\partial}{\partial \pi_x}Var[p-\bar{p}\mid p^*(\cdot;\pi_x,\pi_z),\pi_x,\pi_z]$ denotes the reduction in the dispersion of prices around the mean price that obtains when all agents acquire information of higher precision, holding fixed the efficient price rule $p^*(\cdot;\pi_x,\pi_z)$, as given by (20).

Policies implementing the efficient price rule need not implement the efficient acquisition of information: To see this, suppose that the planner were able to implement the efficient price rule by using a system of Pigouvian taxes that impose high penalties whenever the price set by each producer–consumer is different from what prescribed by the efficient rule $p^*(\cdot;\pi_x^*,\pi_z)$ and else are equal to zero. Such taxes require that the policy maker be able to observe the individual signals (x_j,y), which is clearly unrealistic, but nonetheless helps us illustrating the point in the simplest possible way.

Towards a contradiction, suppose that such policies also implement the efficient acquisition of private information π_x^*. As above, drop the dependence on π_z to ease the notation and denote by $\hat{U}_j(\pi_{x_j},\pi_x^*)$ agent j's ex ante expected payoff when (a) the precision of agent j's information is π_{x_j}, (b) the precision of all other agents' information is π_x^*, and (c) all agents (including j) follow the efficient price rule (20). Because the Pigouvian taxes make it optimal for the agent to

28. Observe that $u_{kk}^*+u_{\sigma\sigma}^*<0$ for all (π_x,π_z).

follow the efficient price rule $p^*(\cdot;\pi_x^*,\pi_z)$ irrespective of her choice of π_{x_j}, from envelope arguments similar to those discussed above, we have that the net private marginal benefit of varying π_{x_j} around π_x^* is given by

$$\frac{\partial \hat{U}_j(\pi_x^*,\pi_x^*)}{\partial \pi_{x_j}}=\mathbb{E}\big[\mathcal{V}^*;\pi_x^*,\pi_z\big]\frac{(1-R)^2(1-v)^2}{2}\left(-\frac{\lambda_1^{*2}}{\pi_x^{*2}}\right)-\mathbb{E}\big[\mathcal{N}^*;\pi_x^*,\pi_z\big]\frac{(v\omega)^2}{2}\left(-\frac{\lambda_1^{*2}}{\pi_x^{*2}}\right)-C'(\pi_x^*), \tag{B.29}$$

where $\mathbb{E}\big[\mathcal{V}^*;\pi_x^*,\pi_z\big]$ and $\mathbb{E}\big[\mathcal{N}^*;\pi_x^*,\pi_z\big]$ are shortcuts for $\mathbb{E}\Big[\mathcal{V}(p,\bar{p},\sigma_p^2,\theta)\Big]$ and $\mathbb{E}\Big[\mathcal{N}(p,\bar{p},\sigma_p^2,\theta)\Big]$, with each of these expectations computed assuming that prices are determined according to the efficient price rule (20) for quality of information (π_x^*,π_z), and where

$$\lambda_1^*=\lambda_1^*(\pi_x^*,\pi_z)=\kappa_1^*\frac{\pi_x^*(1-\alpha^*)}{\pi_z+\pi_x^*(1-\alpha^*)}.$$

Now recall that, under the efficient price rule, $\mathbb{E}\big[\mathcal{V}^*;\pi_x^*,\pi_z\big]=\frac{\omega}{1-R}\mathbb{E}\big[\mathcal{N}^*;\pi_x^*,\pi_z\big]$. Replacing this condition into (B.29) and using the definition of $u_{kk}^*(\cdot)$, $u_{\sigma\sigma}^*(\cdot)$, and $\gamma^*(\cdot)$ given above, we have that

$$\begin{aligned}\frac{\partial \hat{U}_j(\pi_x^*,\pi_x^*)}{\partial \pi_{x_j}}&=\frac{u_{kk}^*(\pi_x^*,\pi_z)}{2}\left(-\frac{(\lambda_1^*(\pi_x^*,\pi_z))^2}{\pi_x^{*2}}\right)-C'(\pi_x^*)\\&=\frac{\partial w^*(\pi_x^*,\pi_z)}{\partial \pi_x}+\frac{u_{\sigma\sigma}^*(\pi_x^*,\pi_z)}{2}\frac{\kappa_1^{*2}(1-\gamma^*(\pi_x^*,\pi_z))^2}{\pi_x^{*2}}\\&=\frac{u_{\sigma\sigma}^*(\pi_x^*,\pi_z)}{2}\frac{\kappa_1^{*2}(1-\gamma^*(\pi_x^*,\pi_z))^2}{\pi_x^{*2}}\\&=\frac{(v-1)\omega}{2}\mathbb{E}\big[\mathcal{N}^*;\pi_x^*,\pi_z\big]\left(\frac{\lambda_1^*(\pi_x^*,\pi_z)}{\pi_x^*}\right)^2>0,\end{aligned}$$

where the last two equalities use the definition of π_x^*. Hence, under such Pigouvian taxes, the agent has an incentive to deviate and acquire more precise private information than what is socially efficient. As mentioned above, this example is chosen for its simplicity. Similar conclusions hold for more realistic policies such as subsidies to production that do not require the Government to be able to observe individual signals.

Comparison between $\hat{\pi}_x$ and π_x^*: The (gross) marginal social benefit of more precise private information (evaluated at the equilibrium level $\pi_x=\hat{\pi}_x$) when the planner can control the price rule is given by

$$\frac{u_{kk}^*(\hat{\pi}_x,\pi_z)+u_{\sigma\sigma}^*(\hat{\pi}_x,\pi_z)}{2}\left(-\frac{\big[\kappa_1^*(1-\gamma^*(\hat{\pi}_x,\pi_x))\big]^2}{\hat{\pi}_x^2}\right),$$

whereas the (gross) marginal private benefit is

$$\frac{u_{kk}(\hat{\pi}_x,\pi_z)}{2}\left(-\frac{\big[\kappa_1(1-\gamma(\hat{\pi}_x,\pi_z))\big]^2}{\hat{\pi}_x^2}\right).$$

We thus have that the precision of private information $\hat{\pi}_x$ acquired in equilibrium is inefficiently low relative to π_x^* if and only if

$$\frac{\big|u_{kk}(\hat{\pi}_x,\pi_z)\big|}{2}\frac{\kappa_1^2(1-\gamma(\hat{\pi}_x,\pi_z))^2}{\hat{\pi}_x^2}<\frac{\big|u_{kk}^*(\hat{\pi}_x,\pi_z)+u_{\sigma\sigma}^*(\hat{\pi}_x)\big|}{2}\frac{\kappa_1^{*2}(1-\gamma^*(\hat{\pi}_x,\pi_z))^2}{\hat{\pi}_x^2}. \tag{B.30}$$

Replacing the formulas for $\gamma(\cdot)$ and $\gamma^*(\cdot)$ into (B.30) and simplifying we arrive at the conclusion in (23).

Social benefit of private information under the equilibrium price rule: Note that welfare, under the equilibrium price rule, is given by

$$w(\pi_x,\pi_z)=\mathbb{E}[\mathcal{V};\pi_x,\pi_z]-\mathbb{E}[\mathcal{N};\pi_x,\pi_z]-C(\pi_x),$$

where $\mathbb{E}[\mathcal{V};\pi_x,\pi_z]-\mathbb{E}[\mathcal{N};\pi_x,\pi_z]$ is given by (B.20). Differentiating $w(\pi_x,\pi_z)$ with respect to π_x, we have that

$$\begin{aligned}\frac{\partial w(\pi_x,\pi_z)}{\partial \pi_x}=&(1-R)\mathbb{E}[\mathcal{V};\pi_x,\pi_z]\left(-\frac{\partial \mathbb{E}[\bar{p}]}{\partial \pi_x}+\frac{(1-R)}{2}\frac{\partial Var[\bar{p}]}{\partial \pi_x}+\frac{(1-R)(1-v)^2-(v-1)(v-2)}{2}\frac{\partial \sigma_p^2}{\partial \pi_x}\right)\\&-\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\left(-\omega\frac{\partial \mathbb{E}[\bar{p}]}{\partial \pi_x}+\frac{\omega^2}{2}\frac{\partial Var[\bar{p}+\theta]}{\partial \pi_x}+\frac{\omega^2 v^2-\omega(v-1)^2}{2}\frac{\partial \sigma_p^2}{\partial \pi_x}\right)-C'(\pi_x).\end{aligned}$$

Since $\lambda_1+\lambda_2=\kappa_1$, we have that $Var[\bar{p}]=\lambda_1^2/\pi_z+(\pi_z-\pi_\theta)\kappa_1^2/\pi_\theta\pi_z$ and $Var[\bar{p}+\theta]=(1+\lambda_1)^2/\pi_z+(\pi_z-\pi_\theta)(1+\kappa_1)^2/\pi_\theta\pi_z$ with $\lambda_1=\lambda_1(\pi_{x,}\pi_z)$ as defined above. Using the fact that, under the equilibrium price rule, $\mathbb{E}[\mathcal{V};\pi_x,\pi_z]=\frac{v\omega}{(v-1)(1-R)}\mathbb{E}[\mathcal{N};\pi_x,\pi_z]$, we obtain that

$$\frac{\partial w(\pi_x,\pi_z)}{\partial \pi_x}=\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\left(-\frac{\omega}{v-1}\frac{\partial \mathbb{E}[\bar{p}]}{\partial \pi_x}+\frac{\omega[1-v((1-R)(1-v)+\omega v)]}{2}\frac{\partial \sigma_p^2}{\partial \pi_x}\right)$$
$$+\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\left(-\frac{v\omega(1-R)}{v-1}\lambda_1+\omega^2(1+\lambda_1)\right)\frac{\kappa_1}{\pi_z}\frac{\partial \gamma}{\partial \pi_x}-C'(\pi_x).$$

Now, note that $\partial\mathbb{E}[\bar{p}]/\partial\pi_x=\partial\lambda_0/\partial\pi_x$. Using (16), and recalling that in equilibrium the dispersion of individual prices is given by $\sigma_p^2=\lambda_1^2/\pi_x$, we then have that

$$\frac{\partial \mathbb{E}[\bar{p}]}{\partial \pi_x}=\frac{\partial \mathbb{E}[\bar{p}]}{\partial \sigma_p^2}\frac{\partial \sigma_p^2}{\partial \pi_x}-\frac{\omega^2(1+\lambda_1)-(1-R)^2\lambda_1}{R+\omega-1}\frac{\kappa_1}{\pi_z}\frac{\partial \gamma}{\partial \pi_x},$$

where $\partial\mathbb{E}[\bar{p}]/\partial\sigma_p^2$ is given by (17). Hence,

$$\frac{\partial w(\pi_x,\pi_z)}{\partial \pi_x}=\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\left\{-\frac{\omega}{v-1}\frac{\partial \mathbb{E}[\bar{p}]}{\partial \sigma_p^2}+\frac{\omega[1-v((1-R)(1-v)+\omega v)]}{2}\right\}\frac{\partial \sigma_p^2}{\partial \pi_x}$$
$$+\mathbb{E}[\mathcal{N};\pi_x,\pi_z]\left\{\frac{\omega}{v-1}\frac{\omega^2(1+\lambda_1)-(1-R)^2\lambda_1}{R+\omega-1}-\frac{v\omega(1-R)}{v-1}\lambda_1+\omega^2(1+\lambda_1)\right\}\frac{\kappa_1}{\pi_z}\frac{\partial \gamma}{\partial \pi_x}$$
$$-C'(\pi_x).$$

The term in the curly bracket in the second line of the above expression can be rewritten as

$$\left[\frac{\omega}{v-1}\frac{\omega^2-(1-R)^2}{R+\omega-1}-\frac{v\omega(1-R)}{v-1}+\omega^2\right](\lambda_1-\kappa_1) \tag{B.31}$$
$$+\frac{\omega}{v-1}\frac{\omega^2(1+\kappa_1)-(1-R)^2\kappa_1}{R+\omega-1}-\frac{v\omega(1-R)\kappa_1}{v-1}+\omega^2(1+\kappa_1).$$

Using the expressions for λ_1 and κ_1, we can rewrite (B.31) as

$$-\left[\frac{\omega}{v-1}\frac{\omega^2-(1-R)^2}{R+\omega-1}-\frac{v\omega(1-R)}{v-1}+\omega^2\right]\frac{\kappa_1\pi_z}{\pi_z+\pi_x(1-\alpha)}$$
$$=-\frac{\omega}{v-1}[v\omega+(R-1)(v-1)]\frac{\kappa_1\pi_z}{\pi_z+\pi_x(1-\alpha)}.$$

Exploiting this result and recalling that

$$1-\alpha^*=\frac{R+\omega-1}{v(v\omega+1-v)+R(v-1)^2}>0,$$
$$u_K=u_K(\pi_x,\pi_z)\equiv\mathbb{E}\left[\frac{\partial u(p,\bar{p},\sigma_p^2,\theta)}{\partial \bar{p}}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]=-\frac{\omega}{v-1}\mathbb{E}[\mathcal{N};\pi_x,\pi_z]<0,$$
$$u_{kk}=u_{kk}(\pi_x,\pi_z)\equiv\mathbb{E}\left[\frac{\partial u^2(p,\bar{p},\sigma_p^2,\theta)}{\partial p^2}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]=-v\omega[v\omega+(R-1)(v-1)]\mathbb{E}[\mathcal{N};\pi_x,\pi_z]<0,$$

and

$$u_{\sigma\sigma}=u_{\sigma\sigma}(\pi_x,\pi_z)\equiv 2\mathbb{E}\left[\frac{\partial u(p,\bar{p},\sigma_p^2,\theta)}{\partial \sigma_p^2}\mid p(\cdot;\pi_x,\pi_z),\pi_x,\pi_z\right]=\omega\mathbb{E}[\mathcal{N};\pi_x,\pi_z]>0,$$

we then obtain that

$$\frac{\partial w(\pi_x,\pi_z)}{\partial \pi_x}=\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial \mathbb{E}[\bar{p}]}{\partial \sigma_p^2}\right)\frac{\partial \sigma_p^2}{\partial \pi_x} \tag{B.32}$$
$$+2\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial \mathbb{E}[\bar{p}]}{\partial \sigma_p^2}\right)\frac{\kappa_1^2}{\pi_z+\pi_x(1-\alpha)}\left(1-\alpha^*\right)\frac{\partial \gamma}{\partial \pi_x}-C'(\pi_x).$$

Next, using the fact that $\sigma_p^2=\frac{\kappa_1^2(1-\gamma)^2}{\pi_x}$, we obtain that

$$\frac{\partial \sigma_p^2}{\partial \pi_x}=-\frac{\kappa_1^2(1-\gamma)^2}{\pi_x^2}-\frac{2\kappa_1^2(1-\gamma)}{\pi_x}\frac{\partial \gamma}{\partial \pi_x}. \tag{B.33}$$

Substituting (B.33) into (B.32) we finally arrive at (24) in the main text.

Derivation of Equation (26). Using the definition of α, we have that the expression for $\partial\mathbb{E}[\bar{p}]/\partial\sigma_p^2$ in (17) can be conveniently rewritten as

$$\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}=\frac{1}{2}\left(\frac{(R-1)(1-v)+\omega v+1-v}{1-\alpha}+v-1\right).$$

Using the expressions for u_K, u_{kk}, and $u_{\sigma\sigma}$, we have that

$$\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}$$
$$=\frac{\omega}{2(1-\alpha)(v-1)}[-v(R+\omega-1)(v-1)+(R-1)(v-1)-(\omega v+1-v)]\mathbb{E}[\mathcal{N};\pi_x,\pi_z].$$

Finally, using the formulas for u^*_{kk} and $u^*_{\sigma\sigma}$, evaluated at (π_x,π_z), we have that

$$\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}=\frac{1}{2}\frac{u^*_{kk}+u^*_{\sigma\sigma}}{(1-\alpha)(v-1)}<0. \tag{B.34}$$

Replacing the latter into (24), we arrive at (26).

Proof of Proposition 11. Recall that the social benefit of more precise private information under the equilibrium price rule is given by (24). Evaluating the expression in (24) at the precision $\hat{\pi}_x$ acquired in equilibrium, we then have that the net marginal social benefit of inducing the agents to acquire more precise information, evaluated at $\pi_x=\hat{\pi}_x$, is

$$\frac{\partial w(\hat{\pi}_x,\pi_z)}{\partial\pi_x}=\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right)$$
$$+2\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\frac{\kappa_1^2(\alpha^*-\alpha)}{\pi_z+(1-\alpha)\hat{\pi}_x}\left|\frac{\partial\gamma}{\partial\pi_x}\right|-C'(\hat{\pi}_x),$$

where u_{kk}, $u_{\sigma\sigma}$, u_K, γ, and $\partial\gamma/\partial\pi_x$ are all evaluated at $(\hat{\pi}_x,\pi_z)$.

Now recall that the equilibrium $\hat{\pi}_x$ satisfies

$$\frac{u_{kk}}{2}\frac{\partial}{\partial\pi_x}Var[p-\bar{p};\,|\,p(\cdot;\hat{\pi}_x,\pi_z),\hat{\pi}_x,\pi_z]=\frac{u_{kk}}{2}\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right)=C'(\hat{\pi}_x).$$

Hence, $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_x$ is positive if and only if

$$\left(\frac{u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\left(-\frac{\kappa_1^2(1-\gamma)^2}{\hat{\pi}_x^2}\right) \tag{B.35}$$
$$+2\left(\frac{u_{kk}+u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}\right)\frac{\kappa_1^2(\alpha^*-\alpha)}{\pi_z+(1-\alpha)\hat{\pi}_x}\left|\frac{\partial\gamma}{d\pi_x}\right|>0.$$

Using $u_{\sigma\sigma}=\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]$ and

$$u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}=\left(\frac{-\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]}{v-1}\right)\left\{\frac{v^2\omega^2-(v-1)^2(R-1)^2-(v-1)[\omega(v-1)+(R-1)(v-2)]}{2(R+\omega-1)}\right\},$$

we have that

$$\frac{u_{\sigma\sigma}}{2}+u_K\frac{\partial\mathbb{E}[\bar{p}]}{\partial\sigma_p^2}$$
$$=\frac{\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]}{2}-\frac{\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]}{2}\left\{\frac{v^2\omega^2-(v-1)^2(R-1)^2-(v-1)[\omega(v-1)+(R-1)(v-2)]}{(v-1)(R+\omega-1)}\right\}$$
$$=\frac{\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]}{2}\left\{1-\frac{v^2\omega^2-(v-1)^2(R-1)^2-(v-1)[\omega(v-1)+(R-1)(v-2)]}{(v-1)(R+\omega-1)}\right\}$$
$$=\frac{\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]}{2}\left\{\frac{(v-1)^2R(R-1)-\omega v[1+v(\omega-1)]}{(v-1)(R+\omega-1)}\right\}.$$

Replacing the above expression into (B.35) and using

$$\frac{u_{kk}}{2}=-v[v\omega+(R-1)(v-1)]\frac{\omega\mathbb{E}\left[\mathcal{N};\hat{\pi}_x,\pi_z\right]}{2},$$

$$1-\gamma=\frac{(1-\alpha)\hat{\pi}_x}{\pi_z+(1-\alpha)\hat{\pi}_x},$$

$$\left|\frac{\partial\gamma(\hat{\pi}_x,\pi_z)}{\partial\pi_x}\right|=\frac{\pi_z(1-\alpha)}{\left[\pi_z+(1-\alpha)\hat{\pi}_x\right]^2},$$

we then have that the precision of information acquired in equilibrium is inefficiently low (that is, $\partial w(\hat{\pi}_x,\pi_z)/\partial\pi_x$ is positive) if and only if

$$-\frac{(1-\alpha)}{2}\left\{(v-1)^2R(R-1)-\omega v[1+v(\omega-1)]\right\} \tag{B.36}$$

$$-[v\omega+(R-1)(v-1)]\left[v(v\omega+1-v)+R(v-1)^2\right]\frac{(\alpha^*-\alpha)\pi_z}{\pi_z+(1-\alpha)\hat{\pi}_x}>0.$$

Recalling that

$$1-\alpha=\frac{R+\omega-1}{v\omega+(R-1)(v-1)},$$

and

$$\alpha^*-\alpha=\frac{(R+\omega-1)(v-1)[v\omega+1-v+R(v-2)]}{\left[v(v\omega+1-v)+R(v-1)^2\right][v\omega+1-v+R(v-1)]},$$

we then have that Condition (B.36) can be rewritten as

$$\frac{\pi_z+(1-\alpha)\hat{\pi}_x}{\pi_z}\left\{\omega v[1+v(\omega-1)]-(v-1)^2R(R-1)\right\} \tag{B.37}$$

$$-2(v-1)[v\omega+(R-1)(v-1)][v\omega+1-v+R(v-2)]>0.$$

Observe that the term in the second line of (B.37) is negative for any $v,\omega>1$ and $R\geq 0$. It follows that a sufficient condition for the inequality to be reversed is that $\omega v[1+v(\omega-1)]<(v-1)^2R(R-1)$, which is equivalent to $R>R^*$, where R^* is as in (27) in the proposition. It follows that, for $R>R^*$, the economy overinvests in information acquisition.

Next, use (B.37) to see that, when $R<R^*$, under-acquisition of private information obtains if

$$\frac{\hat{\pi}_x}{\pi_z}>\frac{v\omega+(R-1)(v-1)}{(R+\omega-1)\left[v\omega(v\omega+1-v)-(v-1)^2R(R-1)\right]} \tag{B.38}$$

$$\cdot\left\{(2v-3)\left[v\omega(v\omega+1-v)+(v-1)^2R(R-1)+2(v-1)v\omega R\right]-2(v-1)^2[v\omega+(R-1)(v-1)]\right\},$$

whereas over-acquisition obtains if the inequality in (B.38) is reversed. Using (19), we then have that, when $R<R^*$, whether the economy over- or underinvests in the acquisition of private information depends on the cost of information acquisition. Q.E.D.

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