Information Management and Pricing in Platform Markets
Supplementary Material

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This document contains additional results for the manuscript "*Information Management and Pricing in Platform Markets*." All numbered items (i.e., sections, subsections, lemmas, conditions, propositions, and equations) in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text. Please refer to the main text for notation and definitions.

Section S1 contains results for the monopoly case. It formally shows that the same Conditions (M) and (Q) in the main text that guarantee existence and uniqueness of a continuation equilibrium in the duopoly case also guarantee existence and uniqueness of a continuation equilibrium in the monopoly case, as claimed (but not proved) in Section 4.1 in the main text. In the same section, we also discuss platform design and information management in monopolistic markets. Section S2 identifies conditions guaranteeing that the equilibrium allocations (prices and participation decisions) in the baseline-model remain equilibrium allocations in an enriched model in which agents can opt out of the market, or multihome. Section S3 contains a dynamic extension in which platforms revise their prices at the same frequency at which agents revise their beliefs about the distribution of preferences, and hence their participation decisions. Section S4 contains an extension to a market with within-side network effects. Finally, Section S5 contains a description of a flexible family of economies with Gaussian information satisfying the conditions in Section 5.1 in the main text.

S1 Monopoly

S1.1 Existence and Uniqueness of Continuation Equilibria in the Monopoly Case

In this section, we formally establish the result claimed in the main text that, in the monopoly case, the continuation equilibrium is unique when network effects are small.

Lemma S1. *Assume Conditions (M) and (Q) in the main text hold. Then, for any vector of prices $p = (p_1^B, p_2^B)$ set by the monopolist, there exists one and only one solution to the system of equations given by*

$$s_i + \frac{\hat{v}_i}{2} + \gamma_i[1 - M_{ji}(\hat{v}_j \mid \hat{v}_i)] = p_i^B \qquad i, j = 1, 2, j \neq i. \tag{S1}$$

In turn, this implies that, in each equilibrium of the game, given any vector of prices set by the platform, all agents from side i whose v_i exceeds $\hat{v}_i$ join the platform, whereas all agents whose v_i is less than $\hat{v}_i$ refrain from joining, where $(\hat{v}_1, \hat{v}_2)$ is the unique solution to (S1).

Proof of Lemma S1. The proof parallels the one for Lemma 1 in the main text. To ease the reading, we re-write Conditions (M) and (Q) here:

Condition (M): for any $i, j = 1, 2,\ j \neq i$, any $v_1, v_2 \in \mathbb{R}$,

$$1 - 2\gamma_i \frac{\partial M_{ji}(v_j|v_i)}{\partial v_i} > 0;$$

Condition (Q): for any $i, j = 1, 2,\ j \neq i$, any $v_1, v_2 \in \mathbb{R}$,

$$\gamma_1\gamma_2 < \frac{\left[\frac{1}{2} - \gamma_1 \frac{\partial M_{21}(v_2|v_1)}{\partial v_1}\right]\left[\frac{1}{2} - \gamma_2 \frac{\partial M_{12}(v_1|v_2)}{\partial v_2}\right]}{\frac{\partial M_{12}(v_1|v_2)}{\partial v_1}\frac{\partial M_{21}(v_2|v_1)}{\partial v_2}}.$$

First, observe that, when Condition (M) holds, for any $\hat{v}_j \in \mathbb{R}$, the gross payoff $s_i + \frac{v_i}{2} + \gamma_i\left[1 - M_{ji}(\hat{v}_j|v_i)\right]$ that an agent from side $i = 1, 2$ with valuation $v_{il}^B = s_i + \frac{v_i}{2}$ obtains from joining platform B when he expects agents from side $j \neq i$ to join the platform if and only if $v_j > \hat{v}_j$ is strictly increasing in v_i. From the implicit function theorem, this in turn means that, given the prices $p = (p_1^B, p_2^B)$, for any $\hat{v}_j \in \mathbb{R}$, there exists one and only one solution $\zeta_i(\hat{v}_j)$ to the equation $s_i + \frac{\hat{v}_i}{2} = p_i^B - \gamma_i\left[1 - M_{ji}(\hat{v}_j|\hat{v}_i)\right]$ with $\zeta_i(\hat{v}_j)$ satisfying

$$\zeta_i'(\hat{v}_j) = \frac{\gamma_i \frac{\partial M_{ji}(\hat{v}_j|\zeta_i(\hat{v}_j))}{\partial v_j}}{1/2 - \gamma_i \frac{\partial M_{ji}(\hat{v}_j|\zeta_i(\hat{v}_j))}{\partial v_i}} \tag{S2}$$

Note that the denominator in (S2) is strictly positive under Condition (M).

Next, let

$$z_j(v_j) \equiv s_i + \frac{v_j}{2} + \gamma_j\left[1 - M_{ij}(\zeta_i(v_j)|v_j)\right]$$

denote the gross payoff that an agent from side $j = 1, 2$ derives from joining the platform when he expects all agents from side $i \neq j$ to join if and only if $v_i \geq \zeta_i(v_j)$. The function $z_j(v_j)$ is differentiable with derivative equal to

$$z_j'(v_j) = \frac{1}{2} - \gamma_j \left\{ \frac{\partial M_{ij}\left(\zeta_i(v_j)|v_j\right)}{\partial v_i} \zeta_i'(v_j) + \frac{\partial M_{ij}\left(\zeta_i(v_j)|v_j\right)}{\partial v_j} \right\}$$
$$= \frac{1}{2} - \gamma_j \frac{\partial M_{ij}\left(\zeta_i(v_j)|v_j\right)}{\partial v_j} - \left(\frac{1}{\frac{1}{2} - \gamma_i \frac{\partial M_{ji}(v_j|\zeta_i(v_j))}{\partial v_i}} \right) \gamma_i \gamma_j \frac{\partial M_{ij}\left(\zeta_i(v_j)|v_j\right)}{\partial v_i} \frac{\partial M_{ji}\left(v_j|\zeta_i(v_j)\right)}{\partial v_j}$$

Together, Conditions (M) and (Q) imply that the function $z_j(v_j)$ is strictly increasing. Because $\lim_{v_j \to -\infty} z_j(v_j) = -\infty$ and $\lim_{v_j \to +\infty} z_j(v_j) = +\infty$, we then have a solution to the equation $z_j(\hat{v}_j) = p_j$ exists and is unique. This in turn implies existence and uniqueness of a solution to the system of equations given by (S1). That, in each equilibrium of the game, given any vector of prices, all agents from side i whose v_i exceeds $\hat{v}_i$ join the platform, whereas all agents whose v_i is less than $\hat{v}_i$ refrain from joining then follows from the fact that the thresholds $(\hat{v}_1, \hat{v}_2)$ defined by the unique solution to (S1) coincide with the thresholds defined by the procedure of iterated deletion of interim strictly dominated strategies. Q.E.D.

S1.2 Platform Design and Information Management in Monopolistic Markets

Suppose a single platform, platform B, serves both sides of the market and assume information is Gaussian as in Section 5 in the main text. Adapting the analysis in Section 4.1 in the main text to the possibility that agents may be imperfectly informed about their own stand-alone valuations (in which case participation decisions depend on *estimated* stand-alone valuations), we then have that the profit-maximizing prices, along with the participation thresholds they induce, must satisfy the following optimality conditions for $i, j = 1, 2,\ j \neq i$,

$$p_i^B = \frac{1 - \Phi\left(\sqrt{\beta_i^v} v_i\right)}{2\sqrt{\beta_i^v}\phi\left(\sqrt{\beta_i^v} v_i\right)} - \gamma_j \sqrt{1 + \Omega^2} \frac{\phi\left(\sqrt{1 + \Omega^2}\sqrt{\beta_i^v} v_i - \Omega\sqrt{\beta_j^v} v_j\right)}{\phi\left(\sqrt{\beta_i^v} v_i\right)} \left[1 - \Phi\left(\sqrt{\beta_j^v} v_j\right)\right]$$
$$+ \gamma_i \Omega \frac{\phi\left(\sqrt{1 + \Omega^2}\sqrt{\beta_j^v} v_j - \Omega\sqrt{\beta_i^v} v_i\right)}{\phi\left(\sqrt{\beta_i^v} v_i\right)} \left[1 - \Phi\left(\sqrt{\beta_i^v} v_i\right)\right]$$

along with

$$s_i + \frac{v_i}{2} + \gamma_i \left[1 - \Phi\left(\sqrt{1 + \Omega^2}\sqrt{\beta_j^v} v_j - \Omega\sqrt{\beta_i^v} v_i\right)\right] = p_i^B \qquad i, j = 1, 2,\ j \neq i.$$

Note that the formulas above specialize the ones in Proposition 3 in the main text to the Gaussian environment under examination. We now compare the platform's incentives to align preferences across sides, and/or to help agents predict participation decisions on the opposite side, to their counterparts in the duopoly case.

Proposition S1. *In general, monopoly profits can either increase or decrease with a higher alignment in preferences across the two sides (a higher ρ_v) and/or with information campaigns that help agents predict participation decisions on the opposite side (higher $|\Omega|$). When the monopolist serves exactly half of the market on each side, (i.e., $\hat{v}_i = 0$, $i = 1, 2$, as in the duopoly case), such policies have only second-order effects in the monopoly case, while they have first-order effects in the duopoly case. Suppose the market is symmetric across sides (i.e., $\beta_i^v = \beta^v$, $\gamma_i = \gamma > 0$, and $\hat{v}_i = \hat{v}$, $i = 1, 2$). When the monopolist serves less than half the market on each side, profits increase with the alignment in preferences across sides. They increase with information campaigns that help agents predict participation decisions on the opposite side when preferences are aligned ($\rho_v > 0$), whereas they decrease with such policies when preferences are misaligned ($\rho_v < 0$). In contrast, when the monopolist serves more than half the market on each side, more alignment is detrimental to profits; furthermore, information campaigns that help agents predict participation decisions on the opposite side decrease profits when preferences are aligned, whereas they increase profits when preferences are misaligned. Importantly, the marginal effects of the above policies (platform design and information management) on the monopolist's profits are always smaller than in the duopoly case.*

Proof of Proposition S1. Observe that

$$\hat{\Pi}^B = \sum_{i=1,2,} p_i^B \left[1 - \Phi\left(\sqrt{\beta_i^v}\hat{v}_i\right)\right],$$

with p_i^B as defined above. Holding β_i^v fixed, $i = 1, 2$, and using the envelope theorem, we thus have that

$$\frac{\partial \hat{\Pi}^B}{\partial \Omega} = \sum_{i,j=1,2,\ j\neq i} -\gamma_i \phi\left(\sqrt{1+\Omega^2}\sqrt{\beta_j^v}\hat{v}_j - \Omega\sqrt{\beta_i^v}\hat{v}_i\right)\left(\frac{\Omega}{\sqrt{1+\Omega^2}}\sqrt{\beta_j^v}\hat{v}_j - \sqrt{\beta_i^v}\hat{v}_i\right)\left[1 - \Phi\left(\sqrt{\beta_i^v}\hat{v}_i\right)\right].$$

Hence, in general, monopoly profits can either increase or decrease with Ω. Because Ω is increasing in ρ_v, this means that monopoly profits can either increase or decrease with a higher alignment in preferences across the two sides (a higher ρ_v) and/or with information campaigns that help agents predict participation decisions on the opposite side (higher $|\Omega|$).

Now suppose the monopolist serves exactly half of the market on each side (i.e., $v_1 = v_2 = 0$, as in the duopoly case). Using the envelope formula above, it is easy to see that $\partial\hat{\Pi}^B/\partial\Omega = 0$. Hence, in this case, marginal variations in platform design and/or in information management have only second-order effects on the monopolist profits.

Next, suppose the market is perfectly symmetric across the two sides (i.e., $\beta_i^v = \beta^v$, $\gamma_i = \gamma > 0$, and $\hat{v}_i = \hat{v}$, $i = 1, 2$). It is easy to see that

$$sign\left(\partial\hat{\Pi}^B/\partial\Omega\right) = sign(\hat{v}).$$

Hence, when the monopolist serves less than half the market on each side (i.e., $\hat{v} > 0$), profits increase with the alignment in preferences across sides , for a higher ρ_v implies a higher Ω. They increase with information campaigns that help agents predict participation decisions on the opposite side (i.e., with $|\Omega|$) when preferences are aligned, for, in this case, a higher $|\Omega|$ implies a higher Ω; instead, profits decrease with such policies when preferences are misaligned, for, in this case, a higher $|\Omega|$ implies a lower Ω.

It is immediate to see that the above conclusions are reversed when the monopolist serves more than half of the market on each side (i.e., $\hat{v} < 0$).

Finally, consider the last statement in the proposition. Note that, irrespective of whether the monopolist serves less or more than half of the market on each side (i.e., of whether $v > 0$),

$$\begin{gathered}\frac{\partial \hat{\Pi}^B}{\partial \Omega} = (\gamma_1 + \gamma_2)\phi\left(\sqrt{1+\Omega^2}\sqrt{\beta^v}v - \Omega\sqrt{\beta^v}v\right)\sqrt{\beta^v}v\left(1 - \frac{\Omega}{\sqrt{1+\Omega^2}}\right)\left[1 - \Phi\left(\sqrt{\beta^v}v\right)\right] \\ < \frac{1}{2}(\gamma_1 + \gamma_2)\left(1 - \frac{\Omega}{\sqrt{1+\Omega^2}}\right) = \frac{\partial \Pi^*}{\partial \Omega}\end{gathered}$$

where $\partial \Pi^*/\partial \Omega$ is the marginal effect of a variation in Ω on duopoly profits (in the case of a market that is symmetric across sides), as one can derive from Condition (20) in the main text. Note that the inequality follows from the fact that, for any $x \geq 0$, the function

$$z(x) \equiv \phi\left(\sqrt{1+\Omega^2}x - \Omega x\right) x \left[1 - \Phi\left(x\right)\right]$$

takes value less than 1/2. The above inequality implies that the marginal effects of the above policies (platform design and information management) on the monopolist's profits are always smaller than in the duopoly case, as claimed in the proposition. Q.E.D.

S2. Opts-out and Multihoming

In this section, we show that, under additional assumptions, the equilibrium prices characterized in the baseline model (along with the participation decisions they induce) continue to remain equilibrium outcomes in a richer environment in which agents can (a) "opt out" of the market, or (b) "multihome" by joining both platforms.

We start by considering robustness to partial participation. The reason why, in general, the equilibrium in the game with compulsory participation need not be an equilibrium in the game in which agents can opt out of the market is the following. First, when the platforms set the same prices as in the equilibrium of the game with compulsory participation, some agents may experience a negative payoff under the cut-off strategy profile of the game with compulsory participation and hence prefer to opt out. Because the equilibrium prices in the game with compulsory participation are invariant in the intensity of the individual stand-alone valuations (formally, in s_1 and s_2), on-path full participation in the game with voluntary participation can always be guaranteed by assuming that the marginal agents' equilibrium payoffs are positive, which amounts to assuming that s_1 and s_2 are sufficiently high. When this is the case, given the equilibrium prices, no agent finds it optimal to opt out, given that, under the cut-off strategy profile any agent's payoff is at least as high as that of the marginal agents.

The above condition, however, does not suffice. In fact, platforms may have an incentive to raise one of their prices above the equilibrium levels of the game with compulsory participation if they expect that, by inducing some agents to opt out, their demand will fall less than that of the rival platform, relative to the case in which participation is compulsory. For this to be the case, it must be that the intensity of the network effects is sufficiently strong to prevail over the direct effect coming from the stand-alone valuations. The proof below shows that this is never the case when s_1 and s_2 are sufficiently large.

Consider arbitrary values $\hat{s}_1$ and $\hat{s}_2$ and, from now on, assume that $s_i > \hat{s}_i$ for $i = 1, 2$. For any vector of prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, any $i = 1, 2$, let

$$v_i^{A+}(p) \equiv p_i^B - p_i^A + \gamma_i,\ v_i^{A-}(p) \equiv \min\left\{2\hat{s}_i - 2p_i^A; p_i^B - p_i^A - \gamma_i\right\}$$

$$v_i^{B-}(p) \equiv p_i^B - p_i^A - \gamma_i,\ v_i^{B+}(p) \equiv \max\left\{2p_i^B - 2\hat{s}_i; p_i^B - p_i^A + \gamma_i\right\}$$

Note that, given the prices p, irrespective of what other agents do, any agent from side i with stand-alone differential $v_i > v_i^{A+}(p)$ prefers joining platform B to joining platform A, whereas any agent with stand-alone differential $v_i < v_i^{A-}(p)$ prefers joining platform A to either joining platform B or opting out of the market. Likewise, irrespective of what other agents do, any agent from side i with stand-alone differential $v_i < v_i^{B-}(p)$ prefers joining platform A to joining platform B, whereas any agent from side i with stand-alone differential $v_i > v_i^{B+}(p)$ prefers joining platform B to either joining platform A or opting out.

Now, for any $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, $i = 1, 2$, let

$$\bar{\Pi}_i^A(p) = \begin{cases} p_i^A Q_i^A(v_i^{A+}(p)) & \text{if } p_i^A \geq 0 \\ p_i^A Q_i^A(v_i^{A-}(p)) & \text{if } p_i^A < 0 \end{cases}$$

and

$$\bar{\Pi}_i^B(p) = \begin{cases} p_i^B \left[Q_i^B(v_i^{B-}(p))\right] & \text{if } p_i^B \geq 0 \\ p_i^B \left[Q_i^B(v_i^{B+}(p))\right] & \text{if } p_i^B < 0. \end{cases}$$

Note that $\bar{\Pi}_i^k(p)$ are platform k's profits when all agents follow the rationalizable strategy most advantageous to platform k.

We assume that the following condition holds, which guarantees that deviations to arbitrarily large prices are never profitable.

Condition (S-P). *For any vector of equilibrium prices $\bar{p} = (\bar{p}_1^A, \bar{p}_2^A, \bar{p}_1^B, \bar{p}_2^B)$ in the game in which participation to one of the two platforms is compulsory there exist strictly positive scalars $R_i^k(\bar{p}) \in \mathbb{R}_{++}$, $i = 1, 2$, $k = A, B$, with $R_i^k(\bar{p}) > |\bar{p}_i^k|$ such that the following is true for $k = A, B$:*

$$\sum_{i=1,2} \bar{\Pi}_i^k(p_1^k, p_2^k, \bar{p}_1^{-k}, \bar{p}_2^{-k}) \leq \Pi^k(\bar{p}) \text{ for all } (p_1^k, p_2^k) \text{ s.t. } |p_i^k| > R_i^k(\bar{p}), \text{ for some } i = 1, 2.$$

where $\Pi^k(\bar{p})$ are the equilibrium profits in the game in which participation to one of the two platforms is compulsory.

The condition says that asking for (or offering) an extreme price on one, or both, sides of the market leads to profits lower than in the equilibrium in the game in which participation to one of the two platforms is compulsory, either when revenues are inflated, and/or losses are reduced, according to the majorization associated with the definition of the $\bar{\Pi}_i^k$ functions above. Recall that, in the game in which participation to one of the two platforms is compulsory, for any vector of prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, the participation thresholds $\hat{v}_i(p)$ are given by

$$\hat{v}_i = p_i^B - p_i^A + \gamma_i \left[2M_{ji}\left(\hat{v}_j \mid \hat{v}_i\right) - 1\right] \qquad i,j = 1,2, j \neq i$$

so the majorizations are small when the network effects are small. We then have the following result:

Proposition S2. *Suppose Condition (S-P) holds and assume that the game in which the agents can "opt out" by not joining any platform admits an equilibrium. Let $\bar{p} = (\bar{p}_1^A, \bar{p}_2^A, \bar{p}_1^B, \bar{p}_2^B)$ be equilibrium prices in the game in which participation to one of the two platforms is compulsory. There exist finite scalars $(\underline{s}_i(\bar{p}))_{i=1,2}$ such that, for any $(s_i)_{i=1,2}$ with $s_i > \underline{s}_i(\bar{p})$, $i = 1,2$, the following is true: the game in which all agents can opt out of the market admits an equilibrium in which (a) platforms offer the same prices $\bar{p} = (\bar{p}_1^A, \bar{p}_2^A, \bar{p}_1^B, \bar{p}_2^B)$ and all agents make the same participation decisions as in the game in which participation to one of the two platforms is compulsory.*

Proof of Proposition S2. Let $\bar{p} = (\bar{p}_1^A, \bar{p}_2^A, \bar{p}_1^B, \bar{p}_2^B)$ be equilibrium prices in the game in which participation to one of the two platforms is compulsory. Observe that $\bar{p}$ is independent of s_1 and s_2. Likewise, observe that, for any vector of prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, the participation thresholds $\hat{v}_i(p)$ in the game in which participation is compulsory are independent of s_1 and s_2, $i = 1,2$.

Next, observe that, given any vector of prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, when all agents from side $j \neq i$, $i,j = 1,2$, follow a cut-off strategy with cut-off $\hat{v}_j(p)$ — meaning that all agents from side j with stand-alone differential $v_j < \hat{v}_j(p)$ join platform A, whereas all agents with $v_j > \hat{v}_j(p)$ join platform B — the payoff that each agent $l \in [0,1]$ from side i with stand-alone differential v_{il} obtains by joining the platform for which his utility is the highest is at least as high as the payoff

$$r_i(p; s_i) \equiv s_i - \frac{1}{2}\hat{v}_i(p) + \gamma_i M_{ji}(\hat{v}_j(p)|\hat{v}_i(p)) - p_i^A$$

that the marginal agent from side i with stand-alone differential equal to $\hat{v}_i(p)$ obtains by joining platform A (this follows directly from Condition (M) in the main text). That $r_i(p; s_i)$ is continuous in $(p; s_i)$,[1] and strictly increasing in s_i, with $lim_{s_i \to +\infty} r_i(p; s_i) = +\infty$, in turn implies that there exist finite scalars $(\underline{s}_i(\bar{p}))_{i=1,2}$ with $\underline{s}_i(\bar{p}) > \hat{s}_i$, $i = 1,2$, such that, for any $(s_i)_{i=1,2}$ with $s_i > \underline{s}_i(\bar{p})$, $i = 1,2$, $r_i(p; s_i) \geq 0$ for any p such that $|p_i^k| \leq R_i^k(\bar{p})$, $i = 1,2$, $k = A,B$.

Next pick any $(s_i)_{i=1,2}$ with $s_i > \underline{s}_i(\bar{p})$, $i = 1,2$, and consider the game in which agents can opt out of the market by not joining any platform. Observe that, by assumption, such a game admits an

[1]The functions $\hat{v}_i(\cdot)$ are continuous in p, $i = 1,2$, and the functions $M_{ji}(v_j|v_i)$ are differentiable, and hence continuous, in each argument.

equilibrium. This in turn means that, for any vector of prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, one can construct a strategy profile for the agents such that each agent's strategy is a best response to the other agents' strategies in the game that starts with the observation of the prices p. It is also easy to see that, for any vector of prices p such that $|p_i^k| \leq R_i^k(\bar{p})$, $i = 1, 2$, $k = A, B$, the strategy profile in which each agent $l \in [0, 1]$ from each side $i = 1, 2$ joins platform A for $v_i \leq \hat{v}_i(p)$ and joins platform B for $v_i > \hat{v}_i(p)$, is a continuation equilibrium, exactly as in the game with compulsory participation.

Now consider the following strategy profile for the agents in the game in which all agents can opt out of the market. For any p such that $r_i(p; s_i) \geq 0$, $i = 1, 2$, all agents follow the same cut-off strategy as in the game with compulsory participation. For any p such that, instead, $r_i(p; s_i) < 0$, for some $i = 1, 2$, take any collection of strategies such that each agent's strategy is a best response to the other agents' strategies, given the prices p (again, that at least one such strategy profile exists follows from the fact that the game in which agents can opt out is assumed to have at least one equilibrium).

We now show that, when the agents follow the above specified strategy profile, each platform finds it optimal to offer the same prices $(\bar{p}_1^k, \bar{p}_2^k)$ it would have offered in the game with compulsory participation, when it expects the rival platform to also offer the prices $(\bar{p}_1^{-k}, \bar{p}_2^{-k})$, $k, -k = A, B$, $-k \neq k$.

To see this, consider the problem faced by platform A (the problem faced by platform B is symmetric and hence omitted). Suppose that platform B offers the equilibrium prices $(\bar{p}_1^B, \bar{p}_2^B)$. Clearly any deviation by platform A to a pair of prices (p_1^A, p_2^A) such that, given $p = (p_1^A, p_2^A, \bar{p}_1^B, \bar{p}_2^B)$, $r_i(p; s_i) \geq 0$, $i = 1, 2$, is unprofitable, for it leads to the same participation decisions (and hence the same profits) as in the game in which participation to one of the two platforms is compulsory. Thus consider a deviation to a pair of prices (p_1^A, p_2^A) such that, given $p = (p_1^A, p_2^A, \bar{p}_1^B, \bar{p}_2^B)$, $r_i(p; s_i) < 0$ for some $i \in \{1, 2\}$. Observe that this implies that $|p_j^A| > R_j^A(\bar{p})$ for some $j \in \{1, 2\}$ possibly different than i. Condition (S-P) then implies that such deviation is unprofitable for platform A, irrespective of which particular continuation equilibrium the agents play following the observation of $p = (p_1^A, p_2^A, \bar{p}_1^B, \bar{p}_2^B)$.

Applying the same arguments to platform B yields the result. Q.E.D.

Next, consider the possibility that agents multihome by choosing to join both platforms. We assume that, by doing so, each agent $l \in [0, 1]$ from each side $i = 1, 2$ obtains a gross payoff equal to $(2 - \kappa_i)s_i + \gamma_i(q_j^A + \mu_j^B)$, where μ_j^B is the measure of agents from side $j \neq i$ who join platform B without joining platform A (to avoid double counting), and where $\kappa_i \in \mathbb{R}$ is a scalar that parametrizes the degree of subadditivity (for $\kappa_i > 0$) or superadditivity (for $\kappa_i < 0$) between the two stand-alone valuations.[2] We then have the following result:

Proposition S3. *Consider the variant of the game in which agents from each side of the market can multihome, as described above. For any vector of prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$ such that*

[2]Note that $(2 - \kappa_i)s_i + \gamma_i(q_j^A + \mu_j^B) = v_i^A + v_i^B - \kappa_i s_i + \gamma_i(q_j^A + \mu_j^B)$.

$p_i^A + p_i^B \geq \gamma_i + 2(1-\kappa_i)s_i$, $i = 1, 2$, *there exists a continuation equilibrium in which each agent from each side singlehomes. Conversely, such a continuation equilibrium fails to exist for any vector of prices for which* $p_i^A + p_i^B < \gamma_i + 2(1-\kappa_i)s_i$, *for some* $i \in \{1, 2\}$.

Proof of Proposition S3. Recall that each agent $l \in [0, 1]$ from each side $i = 1, 2$ prefers joining platform A to joining platform B if and only if

$$v_{il} + \gamma_i \mathbb{E}\left[q_j^B - q_j^A \mid v_{il}\right] \leq p_i^B - p_i^A. \tag{S4}$$

The same agent prefers joining platform A to multihoming if and only if

$$\frac{1}{2}v_{il} + (1-\kappa_i)s_i + \gamma_i \mathbb{E}\left[\mu_j^B \mid v_{il}\right] - p_i^B \leq 0. \tag{S5}$$

Note that Condition (S5) is implied by Condition (S4) if and only if

$$2(1-\kappa_i)s_i + 2\gamma_i \mathbb{E}\left[\mu_j^B \mid v_{il}\right] - \gamma_i \mathbb{E}\left[q_j^B - q_j^A \mid v_{il}\right] \leq p_i^A + p_i^B. \tag{S6}$$

In any continuation equilibrium where all agents singlehome, $q_j^B = \mu_j^B = 1 - q_j^A$. In this case, the inequality in (S6) becomes equivalent to $\gamma_i + 2(1-\kappa_i)s_i \leq p_i^A + p_i^B$. The same conclusion applies to those agents that prefer platform B to platform A. From the results in the main text, we know that the game where multihoming is not possible always admits a continuation equilibrium. We then conclude that, when $p_i^A + p_i^B \geq \gamma_i + 2(1-\kappa_i)s_i$ such a continuation equilibrium is also a continuation equilibrium in the game where agents can multihome.

Conversely, when $p_i^A + p_i^B < \gamma_i + 2(1-\kappa_i)s_i$, there exists no continuation equilibrium where all agents singlehome, for, if such equilibrium existed, then it would satisfy $q_j^B = \mu_j^B = 1 - q_j^A$. Inverting the inequalities above, we would then have that some agent from side $i \in \{1, 2\}$ would necessarily prefer to multihome. Q.E.D.

The condition in the proposition guarantees that *any* agent who expects all other agents to singlehome (according to the same threshold rule as in the game in which multihoming is not possible) prefers to join his most preferred platform to multihoming. As the proposition makes clear, the condition is also necessary, in the sense that, when it is violated, then in any continuation equilibrium some agents necessarily multihome. The following corollary is then an immediate implication of the above result:

Corollary S1. *Suppose that platforms cannot set negative prices. Let* $\bar{p} = (\bar{p}_1^A, \bar{p}_2^A, \bar{p}_1^B, \bar{p}_2^B)$ *be equilibrium prices in the game in which multihoming is not possible, and assume that* $\bar{p}_i^k \geq \gamma_i + 2(1-\kappa_i)s_i$, $i = 1, 2$, $k = A, B$. *In the game in which multihoming is possible, there exists an equilibrium in which platforms offer the same prices* $\bar{p}$ *and all agents make the same participation decisions as in the game in which agents can only singlehome.*

The result in Corollary (S1) appears consistent with the finding in Armstrong and Wright (2007) that strong product differentiation on both sides of the market implies that agents have no incentive to multihome when prices are restricted to be non-negative (As argued in that paper, and in other contexts as well, the assumption that prices must be non-negative can be justified by the fact that negative prices can create moral hazard and adverse selection problems).

Together, the results in Proposition (S1) and Corollary (S1) imply that, when (a) the stand-alone valuations of the marginal agents are neither too high nor too low (intermediate s_i), (b) the two platforms are sufficiently differentiated on both sides of the market (so that the equilibrium prices are sufficiently high) and (c) prices are restricted to be positive, then the equilibrium prices and participation decisions in the baseline game are also equilibrium allocations in the more general game where agents can multihome and opt out of the market.

S3. Dynamics under Flexible Prices

In this section, we study duopoly pricing in the same two-period dynamic economy examined in Section 5.3 in the main text, but assuming the platforms change prices at the same frequency at which agents revise their beliefs about the distribution of stand-alone valuations in the cross-section of the population. We continue to denote the period-1 prices by p_i^k, and then denote the period-2 prices in each state θ by $p_i^k(\theta)$, $k = A, B$, $i = 1, 2$.

Because there are no switching costs, the period-1 prices naturally coincide with those in the static benchmark in the main text. Under these prices, the period-1 participation rates expected by the two platforms are $Q_i^A = \mathbb{E}_\theta[\Lambda_i^\theta(\hat{v}_i)]$ and $Q_i^B = 1 - \mathbb{E}_\theta[\Lambda_i^\theta(\hat{v}_i)]$, where $(\hat{v}_1, \hat{v}_2)$ are the unique solutions to the system of equations given by

$$\hat{v}_i - 2\gamma_i M_{ji}(\hat{v}_j \mid \hat{v}_i) + \gamma_i = p_i^B - p_i^A \qquad i, j = 1, 2,\ j \neq i$$

and where the participation rates reflect the assumption that the two platforms share a common prior, as in Section 5.3 in the main text. [Recall that Λ_i^θ is the true cumulative distribution of stand-alone differentials on side i in state θ, with density λ_i^θ].

Next, consider the choice of the period-2 prices. The observation of the period-1 participation rates reveals to all agents and to the platforms the true state θ. Paralleling the analysis in the static benchmark in the main text, but observing that, under complete information, $M_{ji}(v_j \mid v_i) = \Lambda_j^\theta(v_j)$ all $v_i, v_j \in \mathbb{R}$, we then have that the period-2 equilibrium prices, $p_i^A(\theta)$, and the associated participation rates, $q_i^A(\theta)$, must satisfy

$$p_i^A(\theta) = \frac{\Lambda_i^\theta(\hat{v}_i^\theta)}{\lambda_i^\theta(\hat{v}_i^\theta)} - 2\gamma_j \Lambda_j^\theta(\hat{v}_j^\theta), \qquad q_i^A(\theta) = \Lambda_i^\theta(\hat{v}_i^\theta), \qquad i, j = 1, 2,\ j \neq i,$$

$$p_i^B(\theta) = \frac{1 - \Lambda_i^\theta(\hat{v}_i^\theta)}{\lambda_i^\theta(\hat{v}_i^\theta)} - 2\gamma_j \left(1 - \Lambda_j^\theta(\hat{v}_j^\theta)\right), \qquad q_i^B(\theta) = 1 - \Lambda_i^\theta(\hat{v}_i^\theta), \qquad i, j = 1, 2,\ j \neq i,$$

with the equilibrium thresholds satisfying

$$\hat{v}_i^\theta = p_i^B(\theta) - p_i^A(\theta) - \gamma_i \left(q_j^B(\theta) - q_j^A(\theta)\right), i, j = 1, 2, j \neq i.$$

Clearly, the period-2 differential in the prices set by the platforms, $p_i^B(\theta) - p_i^A(\theta)$, now varies with the state, reflecting the property that different states feature a different degree of relative appreciation for the two platforms' products. Our primary interest is in how the period-1 prices compare to the average period-2 prices. In what follows, we focus on the case of symmetric competition.

Definition S1. *We say that competition is ex-ante symmetric in period two if, based on the period-1 prior, (a) both platforms expect to set equal prices and enjoy equal participation rates on each side in period two (that is,* $\mathbb{E}[p_i^A(\theta)] = \mathbb{E}[p_i^B(\theta)]$ *and* $\mathbb{E}[q_i^A(\theta)] = \mathbb{E}[q_i^B(\theta)] = 1/2$, $i = 1, 2$*).*

When beliefs are consistent with a common prior, as assumed here, the definition amounts to assuming that the prior distribution over the actual cross-sectional distributions of stand-alone valuations is symmetric. Note that this condition is satisfied in the Gaussian model.

It is easy to verify that, when period-2 competition is ex-ante symmetric, the expected period-2 prices must satisfy the following conditions

$$\mathbb{E}[p_i^A(\theta)] = \mathbb{E}\left[\frac{\Lambda_i^\theta(\hat{v}_i^\theta)}{\lambda_i^\theta(\hat{v}_i^\theta)}\right] - \gamma_j = \mathbb{E}\left[\frac{1 - \Lambda_i^\theta(\hat{v}_i^\theta)}{\lambda_i^\theta(\hat{v}_i^\theta)}\right] - \gamma_j = \mathbb{E}[p_i^B(\theta)] \qquad i, j = 1, 2,\ j \neq i.$$

Comparing the expected period-2 prices to their period-1 counterparts (see Corollary 1 in the main text), we have that

$$\mathbb{E}[p_i^k(\theta)] - p_i^k = \left(\mathbb{E}\left[\frac{\Lambda_i^\theta(\hat{v}_i^\theta)}{\lambda_i^\theta(\hat{v}_i^\theta)}\right] - \frac{1}{2\mathbb{E}\left[\lambda_i^\theta(0)\right]}\right) + \gamma_i \left(\frac{\frac{\partial M_{ji}(0|0)}{\partial v_i}}{\mathbb{E}[\lambda_i^\theta(0)]}\right) + \gamma_j \left(\frac{\frac{\partial M_{ij}(0|0)}{\partial v_i}}{\mathbb{E}[\lambda_i^\theta(0)]} - 1\right) \tag{S7}$$

where we used the fact that, under a common prior, $\psi_i(0) = \mathbb{E}\left[\lambda_i^\theta(0)\right]$.

There are three effects that contribute to the difference between the prices that, on average, the platforms set in period two under complete information and the prices that they set in period one under dispersed information. The first effect is the change in the (inverse) semi-elasticity of the stand-alone demand functions. This effect is captured by the first round bracket in (S7) and is present also in the absence of network effects. This effect originates from the fact that the platforms learn the exact distribution of the stand-alone valuations in the second period.

The second effect is due to the fact that, under dispersed information, the beliefs of the marginal agent on each side i differ from the platform's beliefs. As discussed in the main text, this effect is only present in the first period, when information is dispersed, and is captured by the second round bracket in (S7).

The third effect is due to the fact that the adjustment to the side-j's price necessary to maintain the side-j's participation constant when the side-i's participation changes depends on whether or not information is dispersed. This effect is captured by the last round bracket in (S7).

As discussed in the main text, the sign of the second effect is negative when preferences are aligned, thus contributing to higher prices in the first than in the second period. The sign of the other two terms is in general ambiguous. However, there are interesting markets in which all three effects contribute to lower prices in the second period, as we show next.

Definition S2. *When competition is symmetric, information softens competition on side i if*

$$\mathbb{E}\left[\frac{\Lambda_i^\theta(\hat{v}_i^\theta)}{\lambda_i^\theta(\hat{v}_i^\theta)}\right] \geq \frac{1}{2\mathbb{E}\left[\lambda_i^\theta(0)\right]}$$

whereas it strengthens it when the inequality is reversed.

Definition S3. *The period-1 marginal agents' beliefs are more concentrated than the platforms' beliefs around the mean if for all $i, j = 1, 2,\ j \neq i$,*

$$\frac{\partial M_{ij}\left(0 \mid 0\right)}{\partial v_i} > \mathbb{E}\left[\lambda_i^\theta(0)\right].$$

Definition S3 says that the density of agents from side i who are expected to be indifferent between the two platforms' products by an agent from side j who is himself indifferent, $\partial M_{ij}\left(0 \mid 0\right)/\partial v_i$, is larger than the density expected by the platforms ex-ante. When competition is symmetric in period one, $\mathbb{E}[v_{il}|0] = 0$. In this case, the condition says that agents who are indifferent have beliefs that are more concentrated around the mean than the platforms, a property that appears natural when the prior is symmetric. In fact, the property always holds under a Gaussian information structure. More generally the property holds under symmetric competition when platforms' beliefs are a simple mean-preserving spread (SMPS) of the marginal agents' beliefs.[3]

We then have the following result (the proof follows directly from the arguments above):

Proposition S4. *Suppose that competition is symmetric in period one (as defined in the main text) and is ex-ante symmetric in period two (in the sense of Definition (S1)). Further assume that the marginal agents' period-1 beliefs are more concentrated than the platforms' beliefs around the mean (in the sense of Definition (S3)). The expected period-2 prices on side i are larger than their period-1 counterparts if (i) information softens competition (in the sense of Definition (S2)) and (ii) either preferences are misaligned or γ_i/γ_j is small. Conversely, the expected period-2 prices on side i are lower than their period-1 counterparts if (i) information strengthens competition and (ii) preferences are aligned and γ_i/γ_j is large.*

The dynamics of expected prices thus combine the dynamics of the inverse semi-elasticities of the stand-alone valuations with the dynamics of the slopes of the inverse residual demands that originate

[3]See Diamond and Stiglitz (1974). Under SMPS, $M_{ij}\left(v|0\right)$ crosses $\mathbb{E}\left[\Lambda_i^\theta(v)\right]$ only once and from below. Given that, under symmetric competition, $M_{ij}\left(0|0\right) = 1/2 = \mathbb{E}\left[\Lambda_i^\theta(0)\right]$, the slope of $M_{ij}\left(v|0\right)$ at zero, $\partial M_{ij}\left(0|0\right)/\partial v_i$, must be higher than the slope of $\mathbb{E}\left[\Lambda_i^\theta(v)\right]$ at $v = 0$ which is given by $\psi_i\left(0\right) = \mathbb{E}\left[\lambda_i^\theta(0)\right]$.

from the ability of the two sides to predict the participation decisions on the other side. While, in general, prices may either increase or decrease over time, the proposition identifies special cases in which the dynamics of the averages prices can be signed.

S4. Within-Side Network Effects

Consider a market in which agents on one or both sides care not only about the number of agents joining the platform from the opposite side but also about the number of agents joining from their own side. For example, advertisers may compete with other advertisers for viewers' attention; when this is the case, the payoff that an advertiser derives from joining a platform decreases with the number of advertisers who also join, which amounts to negative within-side network effects. When, instead, the platform is a firm producing a new smart-phone operating system, end-users may benefit not only from a high adoption rate by developers from the opposite side of the market, but also from other end-users from the same side adopting the same technology, which amounts to positive within-side network effects.

Such possibilities can be captured by assuming that individual payoffs are given by

$$U_{il}^k = v_{il}^k + \gamma_i q_j^k + \xi_i q_i^k - p_i^k, \qquad i, j = 1, 2, j \neq i, k = A, B,$$

where the new parameter $\xi_i \in \mathbb{R}$ controls for the intensity of the within-side network effects on side $i = 1, 2$, and where all other terms are as in the baseline model.

Now let $M_{ii}(\hat{v}|v)$ denote the measure of agents from side $i = 1, 2$ with differential in stand-alone valuations smaller than or equal to $\hat{v}$, as expected by an agent from side i with differential in stand-alone valuations equal to v and then let $M_i(v) \equiv M_{ii}(v|v)$.

Below, we show that, under conditions analogous to Conditions (M) and (Q) in the baseline model, for any vector of prices there exist a unique continuation equilibrium in threshold strategies. Such continuation equilibrium is the unique continuation equilibrium when the within-side network effects are non-negative on both sides, but not necessarily when they are negative on one or both sides. To be consistent with the analysis in the rest of the paper, in the discussion below, we assume that, in case there are multiple continuation equilibria, the selected one is the one in threshold strategies.

Condition (M-C). *For any $i, j = 1, 2$, $j \neq i$, any $v_1, v_2, v_{il} \in \mathbb{R}$,*

$$1 - 2\gamma_i \frac{\partial M_{ji}(v_j|v_{il})}{\partial v_{il}} - 2\xi_i \frac{\partial M_i(v_i|v_{il})}{\partial v_{il}} > 0.$$

Condition (Q-C). *For any $i, j = 1, 2$, $j \neq i$, any $v_j \in \mathbb{R}$,*

$$v_i - 2\gamma_i M_{ji}(v_j|v_i) - 2\xi_i M_i(v_i)$$

is strictly increasing in v_i. Furthermore, for any $v_1, v_2 \in \mathbb{R}$,

$$\gamma_1\gamma_2 < \frac{\left[\frac{1}{2} - \gamma_1 \frac{\partial M_{21}(v_2|v_1)}{\partial v_1} - \xi_1 \frac{dM_1(v_1)}{dv}\right]\left[\frac{1}{2} - \gamma_2 \frac{\partial M_{12}(v_1|v_2)}{\partial v_2} - \xi_2 \frac{dM_2(v_2)}{dv}\right]}{\frac{\partial M_{12}(v_1|v_2)}{\partial v_1} \frac{\partial M_{21}(v_2|v_1)}{\partial v_2}}.$$

Paralleling the analysis in the baseline model, we then have the following result:

Proposition S5 *Suppose Conditions (M-C) and (Q-C) hold, and competition is symmetric, as defined in the baseline model. Then there exist scalars $\psi_i(0)$, $i = 1, 2$, such that beliefs must satisfy the following conditions: (a) $Q_i^k(0) = 1/2$, and (b) $|dQ_i^k(0)/dv_i| = \psi_i(0)$, $k = A, B$, $i = 1, 2$. Furthermore, equilibrium prices are given by*

$$p_i^k = \frac{1}{2\psi_i(0)} - \gamma_j \left[\frac{\frac{\partial M_{ij}(0|0)}{\partial v_i}}{\psi_i(0)} \right] - \gamma_i \left[\frac{\frac{\partial M_{ji}(0|0)}{\partial v_i}}{\psi_i(0)} \right] - \xi_i \left[\frac{\frac{dM_i(0)}{dv}}{\psi_i(0)} \right]$$

$k = A, B$, $i = 1, 2$.

Proof of Proposition S5. The proof is in three steps. Step 1 shows that, under conditions (M-C) and (Q-C), there exists a unique continuation equilibrium in threshold strategies, for all prices. Step 2 then shows that, when platforms expect the continuation equilibrium to be in threshold strategies (which, as explained above, is always the case when $\xi_i \geq 0$, $i = 1, 2$, for, in this case, the continuation equilibrium is unique), then the equilibrium prices must satisfy optimality conditions similar to those in the main text, but augmented by a new term. Finally, Step 3 shows how the aforementioned optimality conditions yield the formulas in the proposition when competition is symmetric.

Step 1. We start with the following lemma.

Lemma S2. *Suppose Conditions (M-C) and (Q-C) hold. Then, for any vector of prices, there exist a unique continuation equilibrium in threshold strategies.*

Proof of Lemma S2. The proof parallels the one in the baseline model without within-side network effects. Given the platforms' prices, each agent $l \in [0, 1]$ from each side $i = 1, 2$, chooses platform B if

$$v_{il} + \gamma_i \mathbb{E}[q_j^B - q_j^A | v_{il}] + \xi_i \mathbb{E}[q_i^B - q_i^A | v_{il}] > p_i^B - p_i^A$$

and platform A if the above inequality is reversed.

When Condition (Q-C) holds, for any $\hat{v}_j \in \mathbb{R}$, the gross payoff differential

$$\hat{v}_i + \gamma_i + \xi_i - 2\gamma_i M_{ji}(\hat{v}_j|\hat{v}_i) - 2\xi_i M_i(\hat{v}_i)$$

that an agent from side $i = 1, 2$ with differential in stand-alone valuations equal to $\hat{v}_i$ obtains from joining platform B relative to joining platform A, when he expects (a) all agents from side $j \neq i$ to join platform B when $v_j > \hat{v}_j$ and platform A when $v_j < \hat{v}_j$ and (b) all agents from his own side i to join platform B when $v_i \geq \hat{v}_i$ and platform A when $v_i < \hat{v}_i$ is strictly increasing in $\hat{v}_i$. From the intermediate and implicit function theorems, this means that, given the prices $p = (p_1^A, p_2^A, p_1^B, p_2^B)$, for any $\hat{v}_j \in \mathbb{R}$, there exists a unique solution $\hat{v}_i = \varrho_i(\hat{v}_j)$ to the equation

$$\hat{v}_i + \gamma_i + \xi_i - 2\gamma_i M_{ji}(\hat{v}_j|\hat{v}_i) - 2\xi_i M_i(\hat{v}_i) = p_i^B - p_i^A, \tag{S8}$$

with $\varrho_i(\hat{v}_j)$ satisfying

$$\varrho_i'(\hat{v}_j) = \frac{2\gamma_i \frac{\partial M_{ji}(\hat{v}_j|\varrho_i(\hat{v}_j))}{\partial v_j}}{1 - 2\gamma_i \frac{\partial M_{ji}(\hat{v}_j|\varrho_i(\hat{v}_j))}{\partial v_i} - 2\xi_i \frac{dM_i(\varrho_i(\hat{v}_j))}{dv}}. \tag{S9}$$

Note that the denominator in (S9) is strictly positive under Condition (M-C). Next observe that, under Condition (M-C), any agent from side i with stand-alone differential $v_i < \varrho_i(\hat{v}_j)$ strictly prefers joining platform A to joining platform B if he expects (a) all agents from side $j \neq i$ to follow a threshold strategy with cut-off equal to $\hat{v}_j$ and (b) all agents from his own side i to follow a threshold strategy with cut-off equal to $\varrho_i(\hat{v}_j)$. Likewise, any agent from side i with the same expectations as above but with stand-alone differential $v_i > \varrho_i(\hat{v}_j)$ prefers joining platform B to joining platform A.

Next, let

$$L_j(\hat{v}_j) = \hat{v}_j + \gamma_j + \xi_j - 2\gamma_j M_{ij}\left(\varrho_i(\hat{v}_j)|\hat{v}_j\right) - 2\xi_j M_j\left(\hat{v}_j\right)$$

denote the gross payoff differential that an agent from side $j = 1, 2$ derives from joining platform B relative to joining platform A when he expects (a) all agents from side $i \neq j$ to join platform B when $v_i \geq \varrho_i(\hat{v}_j)$ and platform A when $v_i < \varrho_i(\hat{v}_j)$ and (b) all agents from his own side j to join platform B when $v_j \geq \hat{v}_j$ and platform A when $v_j < \hat{v}_j$. The function $L_j(\hat{v}_j)$ is differentiable with derivative equal to

$$\begin{aligned} L_j'(\hat{v}_j) &= 1 - 2\gamma_j \left\{ \frac{\partial M_{ij}\left(\varrho_i(v_j)|v_j\right)}{\partial v_i} \varrho_i'(\hat{v}_j) + \frac{\partial M_{ij}\left(\varrho_i(\hat{v}_j)|\hat{v}_j\right)}{\partial v_j} \right\} - 2\xi_j \frac{dM_j\left(\hat{v}_j\right)}{dv} \\ &= 1 - \frac{4\gamma_i\gamma_j \frac{\partial M_{ji}(\hat{v}_j|\varrho_i(\hat{v}_j))}{\partial v_j} \frac{\partial M_{ij}(\varrho_i(v_j)|v_j)}{\partial v_i}}{1 - 2\gamma_i \frac{\partial M_{ji}(\hat{v}_j|\varrho_i(\hat{v}_j))}{\partial v_i} - 2\xi_i \frac{dM_i(\varrho_i(\hat{v}_j))}{dv}} - 2\gamma_j \frac{\partial M_{ij}\left(\varrho_i(\hat{v}_j)|\hat{v}_j\right)}{\partial v_j} - 2\xi_j \frac{dM_j\left(\hat{v}_j\right)}{dv}. \end{aligned}$$

Together, Conditions (M-C) and (Q-C) imply that the function $L_j(\hat{v}_j)$ is strictly increasing. Because $\lim_{v_j \to -\infty} L_j(\hat{v}_j) = -\infty$ and $\lim_{v_j \to +\infty} L_j(\hat{v}_j) = +\infty$, we then have a solution to the equation $L_j(\hat{v}_j) = p_j^B - p_j^A$ exists and is unique. This in turn implies that there exists one and only one solution to the system of equations given by (S8).

Step 2. Now assume agents play threshold strategies (as explained above, this is always the case when $\xi_i \geq 0$, $i = 1, 2$). In this case, the demands expected by the two platforms continue to be given by $Q_i^A(\hat{v}_i)$ and $Q_i^B(\hat{v}_i)$ but with the thresholds now solving the indifference conditions

$$\hat{v}_i + \gamma_i + \xi_i - 2\gamma_i M_{ji}\left(\hat{v}_j|\hat{v}_i\right) - 2\xi_i M_i\left(\hat{v}_i\right) = p_i^B - p_i^A. \tag{S10}$$

Paralleling the analysis in the baseline model, now suppose that platform B aims at getting on board Q_1 agents from side one and Q_2 agents from side two. Given its beliefs, the platform has to set prices equal to

$$p_i^B = p_i^A + V_i^B(Q_i) + \gamma_i + \xi_i - 2\gamma_i M_{ji}\left(V_j^B(Q_j) \mid V_i^B(Q_i)\right) - 2\xi_i M_i\left(V_i^B(Q_i)\right).$$

This means that the slopes of the inverse (residual) demand curves are now given by

$$\frac{\partial p_i^B}{\partial Q_i} = \frac{dV_i^B(Q_i)}{dQ_i} - 2\gamma_i \frac{\partial M_{ji}\left(V_j^B(Q_j) \mid V_i^B(Q_i)\right)}{\partial v_i} \frac{dV_i^B(Q_i)}{dQ_i} - 2\xi_i \frac{dM_i\left(V_i^B(Q_i)\right)}{dv} \frac{dV_i^B(Q_i)}{dQ_i}.$$

Note that the same conditions that guarantee existence and uniqueness of monotone continuation equilibria also guarantee that the above demand curves slope downwards, even in the presence of within-side network effects.

Expressing each platform's profits as a function of the participation thresholds, and then fixing the prices offered by the rival firm and differentiating profits with respect to the participation thresholds, we have that the profit-maximizing prices, expressed as a function of the participation thresholds they induce, must satisfy the following condition

$$\begin{aligned} p_i^k \quad = \quad & \frac{Q_i^k(\hat{v}_i)}{|dQ_i^k(\hat{v}_i)/dv_i|} - 2\gamma_i \frac{\partial M_{ji}(\hat{v}_j \mid \hat{v}_i)}{\partial v_i} \frac{Q_i^k(\hat{v}_i)}{|dQ_i^k(\hat{v}_i)/dv_i|} \quad -2\xi_i \frac{dM_i(\hat{v}_i)}{dv} Q_i^k(\hat{v}_i) \\ & -2\gamma_j \frac{\partial M_{ij}(\hat{v}_i \mid \hat{v}_j)}{\partial v_i} \frac{Q_j^k(\hat{v}_j)}{|dQ_i^k(\hat{v}_i)/dv_i|}, \end{aligned} \tag{S11}$$

$i, j = 1, 2$, $j \neq i$, $k = A, B$, where the participation thresholds $\hat{v}_1$ and $\hat{v}_2$ are implicitly defined by the system of equations given by (S10), $i = 1, 2$.

Step 3. The final step consists in observing that, when competition is symmetric, the participation thresholds are given by $\hat{v}_i = 0$, $i = 1, 2$. Furthermore, because the two platforms expect the same participation rates, $\Psi_i^k(0) = 1/2$, $k = A, B$. From (S11), we then have that the equilibrium prices must satisfy

$$\begin{aligned} p_i^A &= \frac{1}{\psi_i^A(0)} \left\{ \frac{1}{2} - \gamma_i \frac{\partial M_{ji}(0 \mid 0)}{\partial v_i} - \gamma_j \frac{\partial M_{ij}(0 \mid 0)}{\partial v_i} - \xi_i \frac{\partial M_i(0)}{\partial v} \right\} \\ p_i^B &= \frac{1}{\psi_i^B(0)} \left\{ \frac{1}{2} - \gamma_i \frac{\partial M_{ji}(0 \mid 0)}{\partial v_i} - \gamma_j \frac{\partial M_{ij}(0 \mid 0)}{\partial v_i} - \xi_i \frac{\partial M_i(0)}{\partial v} \right\} \end{aligned}$$

Clearly, the terms in curly brackets cannot be equal to zero, for otherwise the platforms' prices would be equal to zero and platforms would have a profitable deviations. For the platforms' prices to coincide, it must then be that $\psi_i^B(0) = \psi_i^B(0) = \psi_i(0)$, $i = 1, 2$. The above results imply that, when competition is symmetric, the equilibrium prices must satisfy the formulas in the proposition. Q.E.D.

The pricing formulas in Proposition S5 are qualitatively similar to the one in the baseline model. The only difference is the last term, which captures the impact on the side-i price of a marginal variation in the side-i within-side network effects originating from a higher participation by side i. Interestingly, the impact of this effect on the side-i price combines the direct effect of a higher side-i participation with the variation in the beliefs of the side-i marginal agent about his own side's participation. To see this notice that

$$\frac{dM_i(\hat{v}_i)}{dv} = \left. \frac{\partial M_i(\hat{v}_i|v_i)}{\partial \hat{v}} \right|_{v_i = \hat{v}_i} + \left. \frac{\partial M_i(\hat{v}_i|v_i)}{\partial v_i} \right|_{v_i = \hat{v}_i} \tag{S12}$$

The first term on the right hand side of (S12) is the *direct effect* of increasing the side-i participation holding the beliefs of the side-i marginal agent fixed. The second term is the *indirect effect* of varying the beliefs of the side-i marginal agent for given participation threshold**,** and is negative when preferences are aligned within side. Clearly, this second effect is present only under dispersed information. In fact, under complete information,

$$\frac{dM_i\left(\hat{v}_i\right)}{dv} = \lambda_i^{\theta}(\hat{v}_i)$$

in which case the equilibrium prices become

$$p_i^k = \frac{1}{2\lambda_i^{\theta}(0)} - \gamma_j - \xi_i, \qquad i,j = 1,2,\ j \neq i,\ k = A,B.$$

Under complete information, whether within-side network effects contribute to higher or lower equilibrium prices then depends only on the sign of the within-side network effects.

Under dispersed information, instead, whether within-side network effects contribute to higher or lower equilibrium prices thus depends not only on the sign of the within-side network effects ξ_i (as under complete information), but also on whether the direct or the indirect effect dominates in the marginal agents' beliefs about the distribution of valuations on their own side. In particular, under dispersed information, within-side network effects contribute to higher equilibrium prices when either (a) the within-side network externalities are negative (i.e., $\xi_i < 0$) and the direct effects dominate the indirect effects in the marginal agents' beliefs about the distribution of valuations on their own side (i.e., $dM_i\left(0\right)/dv > 0$) or (b) the within-side network externalities are positive (i.e., $\xi_i > 0$), preferences are aligned within sides (i.e., $\partial M_i\left(\hat{v}_i|v_i\right)/\partial v_i < 0$), and the indirect effects dominate the direct effects in the marginal agents' beliefs (i.e., $dM_i\left(0\right)/dv < 0$). On the contrary, within-side network effects contribute to lower equilibrium prices when either (c) the within-side network externalities are positive (i.e., $\xi_i > 0$) and the direct effects in the marginal agents' beliefs dominate the indirect effects (i.e., $dM_i\left(0\right)/dv > 0$), or (d) the within-side network externalities are negative (i.e., $\xi_i < 0$), preferences are aligned within side (i.e., $\partial M_i\left(\hat{v}_i|v_i\right)/\partial v_i < 0$) and the indirect effects dominate the direct effects in the marginal agents' beliefs (i.e., $dM_i\left(0\right)/dv < 0$).

To see why this is the case, suppose that valuations are drawn from a common prior and that information is Gaussian, as in the previous section. In this case, $M_i(v) = M_{ii}\left(v|v\right)$ is naturally increasing in v. Positive within-side network effects then contribute to flatter inverse demands which in turn contribute to lower equilibrium prices.[4] As the discussion above clarifies, this conclusion extends to more general information structures insofar as $M_i(v)$ remains increasing in v (meaning that those agents who are most enthusiastic about a platform's product are also those who expect most agents from their own side to have an appreciation lower than their own). When this property fails to hold, platforms may, instead, raise their equilibrium prices (relative to the benchmark model), despite within-side network effects being positive on both sides.

We conclude by considering the effect of platform design and information policies on profits, welfare, and consumer surplus in the presence of within-side network effects. To this purpose, consider again the Gaussian specification introduced above. As we show in Corollary S2 below,

[4]The opposite is true when within-side network effects are negative, as in the case of congestion.

equilibrium prices are then given by

$$p_i^{**} = \frac{1}{2\sqrt{\beta_i^v}\phi(0)} - \gamma_j\sqrt{1+\Omega^2} + \gamma_i\Omega - \xi_i\left[\sqrt{1+\Omega_i^2} - \Omega_i\right].$$

where $\Omega_i \equiv \rho_v^i/\sqrt{1-(\rho_v^i)^2}$ captures the ability of the side-i agents to predict participation decisions by other agents from their own side, with $\rho_i^v \equiv cov(v_i, v_i')/var(v_i)$ denoting the coefficient of linear correlation between the valuations of any pair of agents from side i.[5] Equilibrium profits and equilibrium welfare can then be expressed as

$$\Pi^{**} \quad = \quad \Pi^* - \tfrac{1}{2}\xi_1\left[\sqrt{1+\Omega_1^2} - \Omega_1\right] - \tfrac{1}{2}\xi_2\left[\sqrt{1+\Omega_2^2} - \Omega_2\right]$$

and

$$W^{**} = W^* + 2\xi_1 \Pr(v_1 \geq 0, v_1' \geq 0) + 2\xi_2 \Pr(v_2 \geq 0, v_2' \geq 0),$$

where Π^* and W^* are, respectively, equilibrium profits and equilibrium welfare in the absence of within-side network effects. We then have the following result:

Corollary S2. *Suppose information is Gaussian, as in Section 5 in the main text. When within-side network effects are positive, equilibrium prices are lower than in the benchmark without within-side network effects. Furthermore, policies that align preferences within sides (formally captured by increases in Ω_i for given Ω and β_i^v, $i = 1, 2$) increase profits and welfare but reduce consumer surplus. The opposite conclusions hold when within-side network effects are negative.*

Proof of Corollary S2. When information is Gaussian, an agent with estimated stand alone differential equal to v_i expects any other agent from his own side to have an estimated stand alone differential v_i' normally distributed with mean

$$\mathbb{E}[v_i'|v_i] = \frac{cov(v_i', v_i)}{var(v_i)}v_i = \rho_v^i v_i$$

and variance

$$var(v_i'|v_i) = var(v_i')(1-(\rho_v^i)^2) = \frac{1-(\rho_i^v)^2}{\beta_i^v}$$

where

$$\rho_v^i \equiv \frac{cov(v_i, v_i')}{\sqrt{var(v_i)var(v_i')}} = \frac{cov(v_i, v_i')}{var(v_i)} = \frac{\beta_i^\eta}{\beta_i^\eta + \beta_i^\theta}$$

with $(\beta_i^\theta)^{-1} = \delta_i^2 + (1-\delta_i)^2 + 2\delta_i(1-\delta_i)\rho_\omega$.

Letting

$$\Omega_i \equiv \frac{\rho_v^i}{\sqrt{1-(\rho_v^i)^2}}$$

[5] Formally speaking, the ability of the side-i agents to predict participation decisions by agents from their own side is captured by $|\Omega_i|$. However, because$\Omega_i > 0$, we do not need to distinguish between Ω_i and $|\Omega_i|$.

we then have that

$$M_i(v) = Pr(v_i' \le v | v_i = v) = \Phi\left(\frac{\sqrt{\beta_i^v}(1-\rho_v^i)v}{\sqrt{1-(\rho_v^i)^2}}\right) = \Phi\left(\sqrt{\beta_i^v}v\left[\sqrt{1+\Omega_i^2}-\Omega_i\right]\right)$$

and hence

$$\frac{dM_i(0)}{dv} = \sqrt{\beta_i^v}\phi(0)\left[\sqrt{1+\Omega_i^2}-\Omega_i\right].$$

Replacing the latter into the formula for the equilibrium prices and using the fact that $\psi_i(0) = \sqrt{\beta_i^v}\phi(0)$, $\partial M_{ji}(0 \mid 0)/\partial v_i = -\Omega\sqrt{\beta_i^v}\phi(0)$, and $\partial M_{ij}(0 \mid 0)/\partial v_i = \sqrt{1+\Omega^2}\sqrt{\beta_i^v}\phi(0)$, we have that the equilibrium prices are given by

$$p_i^{**} = \frac{1}{2\sqrt{\beta_i^v}\phi(0)} - \gamma_j\sqrt{1+\Omega^2} + \gamma_i\Omega - \xi_i\left[\sqrt{1+\Omega_i^2}-\Omega_i\right].$$

Each platform equilibrium profits are then equal to

$$\Pi^{**} = \Pi^* + \tfrac{1}{2}\xi_1\left[\Omega_1 - \sqrt{1+\Omega_1^2}\right] + \tfrac{1}{2}\xi_2\left[\Omega_2 - \sqrt{1+\Omega_2^2}\right]$$

where

$$\Pi^* = \frac{1}{4\phi(0)}\left[\frac{1}{\sqrt{\beta_1^v}} + \frac{1}{\sqrt{\beta_2^v}}\right] + \frac{1}{2}(\gamma_1+\gamma_2)\left[\Omega - \sqrt{1+\Omega^2}\right]$$

are the equilibrium profits in the benchmark without within-side network effects.

Likewise, steps similar to those that lead to the formula for equilibrium welfare in the absence of within-side network effects imply that equilibrium welfare in the presence of within-side network effects is equal to

$$W^{**} = W^* + 2\xi_1\Pr(v_1 \ge 0, v_1' \ge 0) + 2\xi_2\Pr(v_2 \ge 0, v_2' \ge 0)$$

where

$$W^* = \sum_{i=1,2}\mathbb{E}\left[V_{il}^A\mathbb{I}(v_{il} \le 0) + V_{il}^B\mathbb{I}(v_{il} > 0)\right]$$

$$+2(\gamma_1+\gamma_2)\Pr(v_1 \ge 0, v_2 \ge 0)$$

is equilibrium welfare in the absence of within-side network effects and

$$\Pr(v_i \ge 0, v_i' \ge 0) = \frac{1}{4} + \phi^2(0)\arcsin(\rho_v^i), \qquad i = 1, 2.$$

Consider now the marginal effect on profits, total welfare, and consumer surplus of the same policies discussed in the baseline model.

The effect of the policies discussed in the Corollary on profits, welfare, and consumer surplus can then be read from the above formulas along with the formulas that relate Ω and Ω_i to the primitive parameters. Q.E.D.

The effect of policies aligning preferences within sides is thus similar to the effect of the policies examined in the previous section. The only difference is that preferences are always aligned within sides.

What is perhaps more interesting is the effect of policies that affect both the agents' ability to predict participation decisions from agents on the opposite side as well as from agents on their own side. Consider, for example, the promotion of forums, blogs, and other information policies that shift the agents' attention towards platform dimensions of interest primarily to agents from the opposite side, possibly at the expenses of dimensions of interest primarily to agents from their own side. An example of such policy is the promotion of blogs that attract agents from both sides as opposed to specialized blogs that target agents only from one side.

When within-side network effects are positive and preferences are aligned across sides ($\xi_i > 0$, $i = 1, 2$, and $\rho_v > 0$) such policies increase the agents' ability to predict participation decisions from the opposite side but reduce their ability to predict participation decisions from agents on their own side (formally, Ω increases but Ω_i decreases). In this case, within-side network externalities reduce the positive effect of such policies on profits and welfare as well as the negative effect of such policies on consumer surplus.

When, instead, within-side network effects are positive but preferences are misaligned across sides ($\xi_i > 0$, $i = 1, 2$, and $\rho_v < 0$), such policies always increase the agents' ability to predict participation decisions by agents on their own side, but have ambiguous effects on the agents' ability to predict participation decisions from the opposite side.

S6. Gaussian economies

The following family of Gaussian economies is an example of a class of economies satisfying the various conditions in Section 5.1 in the main text.

The aggregate state $\theta = (\theta_1, \theta_2)$ is drawn from a bi-variate Gaussian distribution with zero mean and variance-covariance matrix

$$\begin{bmatrix} \left(\beta_1^{\theta}\right)^{-1} & \frac{\rho_\theta}{\sqrt{\beta_1^{\theta}\beta_2^{\theta}}} \\ \frac{\rho_\theta}{\sqrt{\beta_1^{\theta}\beta_2^{\theta}}} & \left(\beta_2^{\theta}\right) \end{bmatrix}.$$

Each individual true stand-alone differential is given by $V_{il} = k_i[\theta_i + \varepsilon_{il}]$, whereas each agent's information is summarized in the uni-dimensional statistics $x_{il} = \delta_i\theta_i + (1-\delta_i)\theta_j + \eta_{il}$, where δ_1 and δ_2 are positive scalars, and where each pair $(\varepsilon_{il}, \eta_{il})$ is drawn independently from θ and independently across all agents from a bi-variate Gaussian distribution with mean $(0, 0)$ and variance-covariance matrix

$$\begin{bmatrix} (\beta_i^{\varepsilon})^{-1} & \frac{\rho_i}{\sqrt{\beta_i^{\varepsilon}\cdot\beta_i^{\eta}}} \\ \frac{\rho_i}{\sqrt{\beta_i^{\varepsilon}\cdot\beta_i^{\eta}}} & (\beta_i^{\eta})^{-1} \end{bmatrix}$$

with the parameter $\rho_i \geq 0$ denoting the coefficient of linear correlation between ε_{il} and η_{il}.

Next observe that, in this economy, each individual estimated stand-alone differential is equal to $v_{il} \equiv \mathbb{E}\left[V_{il} \mid x_{il}\right] = \kappa_i x_{il}$, with

$$\kappa_i \equiv \frac{cov[V_{il}, x_{il}]}{var[x_{il}]} = k_i \frac{\delta_i \left(\beta_i^\theta\right)^{-1} + (1-\delta_i)\frac{\rho_\theta}{\sqrt{\beta_1^\theta \beta_2^\theta}} + \frac{\rho_i}{\sqrt{\beta_i^\eta \beta_i^\varepsilon}}}{\delta_i^2 \left(\beta_i^\theta\right)^{-1} + (1-\delta_i)^2 \left(\beta_j^\theta\right)^{-1} + (\beta_i^\eta)^{-1} + 2\delta_i(1-\delta_i)\frac{\rho_\theta}{\sqrt{\beta_1^\theta \beta_2^\theta}}} \tag{1}$$

To see how β_i^v and Ω depend on the primitive parameters, then let

$$\omega_i \equiv \delta_i \theta_i + (1-\delta_i)\theta_j,$$

$i = 1, 2$, and observe that, ex-ante, the pair $\omega \equiv (\omega_1, \omega_2)$ is drawn from a bivariate Normal distribution with mean $(0, 0)$ and variance-covariance matrix

$$\Sigma_\omega = \begin{bmatrix} (\beta_1^\omega)^{-1} & \frac{\rho_\omega}{\sqrt{\beta_1^\omega \beta_2^\omega}} \\ \frac{\rho_\omega}{\sqrt{\beta_1^\omega \beta_2^\omega}} & (\beta_2^\omega)^{-1} \end{bmatrix}$$

where

$$(\beta_i^\omega)^{-1} = \delta_i^2 \left(\beta_i^\theta\right)^{-1} + (1-\delta_i)^2 \left(\beta_j^\theta\right)^{-1} + 2\delta_i(1-\delta_i)\frac{\rho_\theta}{\sqrt{\beta_1^\theta \beta_2^\theta}} \tag{2}$$

and

$$\frac{\rho_\omega}{\sqrt{\beta_1^\omega \beta_2^\omega}} = cov(\omega_1, \omega_2) = \delta_i(1-\delta_j)\left(\beta_i^\theta\right)^{-1} + (1-\delta_i)\delta_j\left(\beta_j^\theta\right)^{-1} + \left[\delta_i\delta_j + (1-\delta_i)(1-\delta_j)\right]\frac{\rho_\theta}{\sqrt{\beta_1^\theta \beta_2^\theta}}. \tag{3}$$

From an ex-ante perspective, the distribution of estimated stand-alone differentials on each side $i = 1, 2$ is thus Normal with mean 0 and variance-covariance matrix

$$\begin{bmatrix} (\beta_1^v)^{-1} & \frac{\rho_v}{\sqrt{\beta_1^v \beta_2^v}} \\ \frac{\rho_v}{\sqrt{\beta_1^v \beta_2^v}} & (\beta_2^v)^{-1} \end{bmatrix}$$

where

$$(\beta_i^v)^{-1} = \kappa_i^2\left[(\beta_i^\omega)^{-1} + (\beta_i^\eta)^{-1}\right]$$

and

$$\rho_v = \frac{cov(x_1, x_2)}{\sqrt{var(x_1)var(x_2)}} = \frac{\frac{\rho_\omega}{\sqrt{\beta_1^\omega \beta_2^\omega}}}{\sqrt{\left((\beta_1^\omega)^{-1} + (\beta_1^\eta)^{-1}\right)\left((\beta_2^\omega)^{-1} + (\beta_2^\eta)^{-1}\right)}}$$

The coefficient of mutual forecastibility can then be expressed as a function of the primitive parameters as follows:

$$\Omega \equiv \frac{\rho_v}{\sqrt{1-\rho_v^2}} = \frac{\frac{\rho_\omega}{\sqrt{\beta_1^\omega \beta_2^\omega}}}{\sqrt{\left((\beta_1^\omega)^{-1} + (\beta_1^\eta)^{-1}\right)\left((\beta_2^\omega)^{-1} + (\beta_2^\eta)^{-1}\right) - \left(\frac{\rho_\omega}{\sqrt{\beta_1^\omega \beta_2^\omega}}\right)^2}} \tag{4}$$

Finally, observe that the normalization in the main text is obtained by letting the various parameters be such that $\kappa_i = 1$.

The above specification has the advantage of being tractable, while at the same time rich enough to capture a variety of situations.

The pure common-value case where all agents from side i have identical stand-alone valuations for the two platforms but different information about the stand-alone differential is captured as the limit in which $\beta_i^\varepsilon \to \infty$, in which case, almost surely, $V_{il} = k_i\theta_i$ all $l \in [0,1]$. The parameter β_i^θ is then a measure of differentiation between the two platforms, as perceived by side i. Letting $\beta_1^\theta = \beta_2^\theta$ and $\rho_\theta = 1$ while allowing $\beta_1^\eta \neq \beta_2^\eta$ then permits us to capture situations where the quality differential between the two platforms is the same on both sides, but where one side may have superior information than the other. Letting $k_i = 0$ in turn permits one to capture situations where agents on side i do not care about the intrinsic quality differential between the two platforms but nonetheless possess information about the distribution of preferences on the opposite side (as in the case of advertisers who choose which media platform to place ads on entirely on the basis of their expectation of the platform's ability to attract readers and viewers from the opposite side).

More generally, allowing the correlation coefficient ρ_θ to be different from one permits one to capture situations where the quality differential between the two platforms differs across the two sides (including situations where it is potentially negatively correlated), as well as situations where one side may be able to perfectly predict the behavior of each agent from that side but not the behavior of agents from the opposite side (which corresponds to the limit in which $\beta_i^\eta \to \infty$).

The model can also capture situations in which different users from the same side have different preferences for the two platforms. This amounts to letting the variance of ε_{il} be strictly positive or, equivalently, $\beta_i^\varepsilon < \infty$. Depending on the degree of correlation ρ_i between ε_{il} and η_{il}, agents may then possess more or less accurate information about their own stand-alone valuations. For example, the case where each agent perfectly knows his own valuations but is imperfectly informed about the valuations of other agents (from either side) is captured as the limit in which $\delta_i \to 1$ and $\rho_i \to 1$. The extreme case of independent private values then corresponds to the limit in which $\beta_i^\theta \to \infty$ and $\beta_i^\varepsilon < \infty$.

Finally, the scalars δ_i control for the "nature" of the agents' information, that is, the extent to which their information correlates with the information possessed by the agents from the opposite side, for given distribution of true stand-alone valuations.

References

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