Knowing your Lemon before you Dump It*

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Abstract

In many games of interest (e.g., trade, entry, leadership, warfare, and partnership environments), one player (the leader) covertly acquires information about the state of Nature before choosing whether to engage with another player (the follower). The friendliness of the follower's reaction depends on his beliefs about what motivated the leader's choice to engage. We provide necessary and sufficient conditions for the leader's value of acquiring more information to increase with the follower's expectations. We then derive the economic implications of this characterization, focusing on three closely related topics (expectation traps, disclosure, and cognitive styles), and drawing policy implications.

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1 Introduction

Many strategic situations of interest can be thought of as Stackelberg games in which one player, player L (the leader, "she") chooses between an "adverse-selection-sensitive" action and an "adverse-selection-insensitive" one. The reaction of the other player, player F (the follower, "he") to the adverse-selection-sensitive action depends on his beliefs about what motivated L's choice of action. For example, player L may represent a seller choosing between offering to trade with a buyer (the adverse-selection-sensitive action) and opting out of the negotiations, as in Akerlof's (1970) lemons model. More generally, player F may still act following the adverse-selection-insensitive action. For example, the latter action may represent the seller's decision to disclose hard information proving unambiguously what the seller knows about the value of the asset. In this case, the decision to disclose hard information is adverse-selection-insensitive because, once the state is revealed, the price offered by the buyer (the follower's reaction) is invariant to his beliefs about what motivated the seller's decision to disclose. The key assumption is that information that makes player L eager to engage with player F by choosing the adverse-selection-sensitive action (for example, by choosing not to disclose what she knows) makes player F react in an unfriendlier manner. Notable examples of such situations include, in addition to Akerlof's (1970) lemons model, many entry and partnership games that are central to the Industrial Organization, Finance, and Organization Economics literatures.

We enrich this classic model by allowing player L to covertly acquire information about the state of Nature before making her engagement decision. We are particularly interested in understanding how player L's information choice depends on player F's expectations (the relationship between the two naturally reflecting how strategic considerations shape the value of information in the class of games under consideration). We identify sufficient and/or necessary conditions for expectation conformity (EC) to emerge in these games, namely for player L to find it more valuable to acquire more information when player F expects her to do so. Besides being of independent interest, EC plays a major role in equilibrium analysis and has important policy implications. In particular, EC is often responsible for the multiplicity of equilibria. In its presence, interactions or markets may switch behavior abruptly. For instance, asset markets can tip from a pattern in which the assets receive little scrutiny and are frequently traded to one in which they are heavily scrutinized and traded infrequently, calling for policy interventions that disincentivize the acquisition of information and facilitate trade. EC also shapes the benefits of disclosure of hard information used to prove to others how much one has invested in learning the value of trading. Finally, EC plays a key role in the possibility that the players end up in an expectation trap, where they suffer from being expected to acquire more information.

Section 2 defines a broad class of generalized lemons environments, in which one of the players, player L, acquires information covertly and then decides whether or not to engage with another player (i.e., chooses between an adverse-selection-sensitive action, "trade," and an adverse-selection-insensitive one, "no trade"). As we show in the Supplement, a number of games can be reinterpreted within this framework. In the Supplement, we also discuss how the analysis changes in the anti-lemons case, i.e., in environments where the choice by a player to engage is good news for the other player (as in Spence (1973)'s signaling model), and present various examples of such interactions.

Section 3 introduces the notion of EC. To put flesh on the characterization, we compare information structures through the mean-preserving-spread (MPS) order, or the more refined rotations order. The MPS order says that the distribution over the posterior mean under a more informative structure is a mean-preserving spread of the corresponding distribution under a less informative structure, which is always the case when the former distribution is obtained through an experiment that Blackwell-dominates the one generating the latter distribution. The rotations order is a strengthening of the MPS order that obtains, for instance, under non-directed search, that is when player L's investment in information acquisition determines the probability of learning the state of Nature (equivalently, the value of the interaction with the other player). For more general environments, it is a property of the family of distributions over player F's posterior mean (we give examples with Uniform, Pareto and Exponential distributions).

The analysis delivers a sufficient (and, under further assumptions, necessary) condition for such games to satisfy EC. This condition says that the choice by player L of a Blackwell-more-informative experiment (a) aggravates the adverse selection problem, in a well-defined sense, which makes F's reaction less friendly to player L, and (b) that an unfriendlier reaction by F in turn raises L's incentive to acquire a more informative experiment; or that both conditions are simultaneously reversed. The condition for EC is easier to check than verifying directly that EC prevails. It obtains, for example, when, holding player F's reaction fixed, a more informative experiment reduces the probability of trade, both when such a probability is computed by player L, given her actual choice of experiment, and by player F, given the experiment that he expects player L to choose.

In the lemons game under non-directed search where the leader is a seller of an asset and the buyer a representative of a competitive market, as in Akerlof's model, EC holds when the gains from trade are large, but not for low gains. This is because large gains from trade induce the competitive buyer to offer a high price that the seller finds it optimal to accept when uninformed. The choice of a more informative experiment (which under non-direct search amounts to a higher probability of the seller learning the true value of the asset) then reduces the probability of trade by making the seller engage selectively when informed. Information thus unambiguously aggravates adverse selection, inducing the buyer to lower the price. This in turn raises the cost for the seller of parting with the asset when its value is high, raising the seller's value of acquiring more information. Hence, EC holds in this case. When, instead, the gains from trade are small, the price offered by the buyer is low, which makes the seller unwilling to trade based on her prior, i.e., when uninformed. Because the seller engages only when informed, the choice by the seller of a more informative experiment has no effect on the severity of the adverse selection problem and hence on the price offered by the buyer. EC thus does not obtain for low gains from trade.[1]

The paper then derives the economic implications of this characterization in Section 4, focusing on three closely-related topics: expectation traps, disclosure (of hard information), and cognitive styles. In generalized lemons games, under the key condition for EC mentioned above (namely, that more

[1] In the Supplement, we discuss how the results change in economies in which the lemons assumption is replaced by its anti-lemon counterpart (that is, states in which the leader is most eager to engage are those in which the follower's reaction is most favorable to the leader, as in Spence (1973) signaling model). The condition for EC is flipped. EC obtains when the choice of a more informative experiment induces the follower to respond in a *friendlier* manner and the marginal benefit of a more informative experiment *increases* (instead of decreases) with the friendliness of the follower's reaction; or both conditions are simultaneously reversed.

information reduces the probability of trade), the information-acquiring player is worse off in a high-information-intensity equilibrium than in a low-information-intensity one. This happens because, under the key condition for EC, information aggravates adverse selection. Consequently, the follower responds with an unfriendlier reaction when expecting the leader to choose a more informative experiment. Importantly, player L may be trapped into a high-information-intensity equilibrium even when information is free. In this case, the loss in player L's payoff originates entirely in the unfriendly response by player F and is unrelated to the cost of acquiring information. We then modify the game by assuming that the information-acquiring player can disclose evidence proving that she devoted external resources to the issue. For example, she can prove that she conducted an experiment resulting in a signal whose informativeness is no smaller than some threshold. Importantly, the hard information that the player discloses is about the experiment of her choice and not its realization. We show that the possibility to engage in this type of disclosure is mostly irrelevant. The intuition is related to the expectation-trap phenomenon: This type of disclosure serves to demonstrate that one is knowledgeable, which, under the key condition for EC, is not profitable. Along a similar vein, we show that it is optimal for the leader to choose a "cognitive style" whereby she poses as an "informational puppy dog," e.g., by convincing the other player that she is dumb or busy, or more generally that her cost to acquire information is high.

Section 5 contains policy analysis. It identifies conditions under which subsidies/taxes to trade are welfare enhancing as well as conditions under which the endogeneity of information calls for larger policy interventions. We show that, in the Akerlof's model, subsidies to trade are optimal when (a) the cost of public funds is small, (b) the choice of a more informative experiment aggravates the adverse selection problem, and (c) subsidies reduce the seller's investment in information acquisition. Furthermore, the endogeneity of the seller's information calls for a more generous program: relative to the case where information is exogenous, the optimal level of the subsidy is larger. This is because subsidies come with a *double dividend* under endogenous information: In addition to inducing player L to engage more often, they discourage player L from acquiring information, with the second effect further contributing to a reduction in the adverse selection problem and hence to an increase in trade.

Section 6 discusses the robustness of the key insights to the possibility for player L to choose the type of information to acquire: The results qualify in what sense the key conditions for EC extend to certain settings with flexible information acquisition. Section 7 concludes. Omitted proofs are in the Appendix at the end of the document. The Supplement studies how the results flip in the anti-lemon case, presents various examples of generalized lemons and anti-lemon problems, and discusses the connection to other covert investment games.

Related Literature. The paper is related to various strands of the literature. The first one is the literature on the lemons problem under alternative information structures. Kartik and Zhong (2024) consider a bilateral trading environment and characterize the payoffs that can be sustained in equilibrium under any possible information structure. The analysis parallels the one in Bergemann, Brooks and Morris (2015) but in a setting with possibly interdependent payoffs and general information structure whereby either player can be partially informed about the state (Bergemann, Brooks and Morris (2015) assume the buyer is always fully informed). Related are also Levin (2001), Kessler (2001), and Bar-Isaac et al. (2018). These papers, as Kartik and Zhong (2024), study how payoffs, the volume of trade, and the efficiency of bargaining outcomes vary with the information structure in variants of the Akerlof's model. In contrast, we study (a) how the acquisition of information is shaped by other players' expectations, (b) how the latter expectations depend on the information acquisition technology and the effect of information on the severity of the adverse selection problem, (c) how players may end up in an expectation trap, and (d) how policy interventions can alleviate the inefficiencies associated with the endogenous asymmetry of information.

Dang (2008), Lichtig and Weksler (2023), and Thereze (2024) also endogenize the information structure in the Akerlof's model. However, the focus of the analysis in these papers is different. Dang (2008) derives conditions under which no information is acquired in equilibrium as well as conditions under which the player acquiring information receives positive surplus despite not having bargaining power at the negotiation stage. Lichtig and Weksler (2023) consider a setting in which the seller can choose, at no cost, among a finite set of experiments (i.e., distributions over the posterior mean) and show that, when the distributions can be ranked according to the strict location-independent risk order (a strengthening of second-order stochastic dominance when distributions have the same mean), in equilibrium, the seller always selects the riskiest distribution. They also show the robustness of the conclusion to the possibility that trade is governed by a general (direct incentive compatible) mechanism instead of the familiar protocol whereby the competitive buyer makes a take-it-or-leave-it offer to the seller. Thereze (2024) considers a competitive adverse selection market in which the buyers' information also affects the sellers' costs (as in health markets), and investigates how the elasticity of the demand and the market equilibrium are affected by a change in the cost of information. In Thereze (2024), the buyers acquire information after seeing the prices asked by the sellers. In contrast, in our model, as in Dang (2008), and Lichtig and Weksler (2023), information acquisition takes place prior to observing the prices.[2]

A fairly vast literature studies information acquisition in bargaining games with private values. See for example Ravid (2020), and Ravid, Roesler, and Szentes (2022) and the references therein. The first paper considers a repeated bargaining setting with a rationally-inattentive buyer. The second paper investigates the properties of the equilibrium when the cost of the buyer's information vanishes in a one-shot ultimatum-bargaining game. Our paper, instead, considers games with interdependent payoffs (as in the lemons problem). It investigates how the information acquired in equilibrium is shaped by the effect of information on the severity of the adverse selection problem. It shows how EC is intrinsically related to the possibility of expectation traps whereby the information-acquiring player is worse off in a high-information-intensity equilibrium than in a low-information-intensity one, with these traps emerging even when information is free and the follower is a representative of a competitive market and hence obtains no surplus in equilibrium (as in Akerlof's original model).

Pavan and Tirole (2024) shares with the present paper the interest in how the possibility to disclose verifiable/hard information affects equilibrium outcomes in settings with interdependent payoffs. That paper focuses on the welfare effects of mandatory disclosure laws. The present paper, instead, focuses on the effects of information on the severity of the adverse selection problem and on policy interventions aimed at alleviating such a severity. Expectation conformity is also studied in Pavan and Tirole (2023). The analysis in that paper is not specific to settings with adverse selection and none of the results in

[2]See also Cremer and Khalil (1992), and Cremer, Khalil, and Rochet (1998) for earlier work on information acquisition in other contractual settings.

the present paper have counterparts in that paper.

Finally, the discussion of how governments can improve the efficiency of markets affected by adverse selection is related to Philippon and Skreta (2012) and Tirole (2012). The sellers' information in those papers is exogenous. In Section 5 of the present paper, we discuss how the governments' programs should be adjusted to account for the endogeneity of the sellers' information. In this respect, the paper is related also to Colombo, Femminis and Pavan (2024) and Pavan, Sundaresan, and Vives (2024). The first paper studies optimal fiscal and monetary policy in economies with investment complementarities and endogenous private information. The second paper studies how governments can influence the efficiency of financial market where traders acquire private information prior to submitting their limit orders. See also Koenig and Pothier (2022) for an analysis of how certain central banks interventions and macro-prudential regulations influence the collection of information by fund managers, with implications for the endogenous degree of adverse selection in the market and the risk of redemption runs and inefficient liquidity dry-ups caused by self-fulfilling fears of adverse selection.

2 Framework

2.1 Description

Consider the following game between two players, a "leader" (she) and a "follower" (he).

(a) Actions and timing

Player L (the "leader") first covertly acquires information about a relevant state of Nature. After updating her beliefs upon observing the realization of the selected information structure (equivalently, of the selected experiment), she chooses between two actions, $a = 0$ and $a = 1$. Player F (the "follower"), after observing player L's action a but not L's choice of an information structure and its realization, then chooses his reaction to the leader's action. Player F's reaction to $a = 0$ plays no role in the analysis and hence we do not formally describe it. His reaction to $a = 1$, instead, will be denoted by $r \in \mathbb{R}$. We assume a higher r stands for a friendlier response: player L's utility is increasing in r.

(b) Information

Prior to choosing a, player L acquires information about the state of Nature. The state of Nature, say the car's quality in the lemons model, is denoted by $\omega \in (-\infty, +\infty)$, and is commonly believed to be drawn from a distribution G with prior mean ω_0. We will assume that the two players' preferences are affine in ω, so they care only about the posterior mean m of the state. An experiment, indexed by $\rho \in \mathbb{R}_+$, will be taken to be the choice of a cumulative distribution function $G(\cdot;\rho)$ of the induced posterior mean m, satisfying the martingale property $\int_{-\infty}^{+\infty} m dG(m;\rho) = \omega_0$ for all ρ.[3] We will assume that the set of experiments (equivalently, of distributions, $G(\cdot;\rho)$) player L can choose from has the cardinality of the continuum, and then denote such a set by $[0,\bar{\rho})$, with $\bar{\rho} \in \mathbb{R}_+$. To ease the exposition, we also assume that the distributions are ordered in such a way that, for any $m \in \mathbb{R}$, the function $G(m;\cdot)$ is differentiable in ρ and then denote by $G_\rho(m;\rho)$ the partial derivative of $G(m;\rho)$ with respect to ρ. We will also assume that $G_\rho(\cdot;\rho)$ is integrable in m. These assumptions permit us to describe some

[3] Note that the support of $G(\cdot;\rho)$ can be a strict subset of $\mathbb{R}$.

of the key conditions in a concise form. None of the qualitative insights hinge on these differentiability assumptions. However, many of the relevant conditions are heavier when the derivatives are replaced with differentials across information structures.

For most of the results, we will also assume that the family of distributions $(G(\cdot;\rho))_{\rho\in\mathbb{R}_+}$ is consistent with the *mean-preserving-spread* (MPS) order.[4]

Assumption 1 (*MPS*). *Player L's set of feasible information structures is consistent with the MPS order if, for any ρ and $\rho' > \rho$, any $m^* \in \mathbb{R}$, $\int_{-\infty}^{m^*} G(m;\rho')dm \geq \int_{-\infty}^{m^*} G(m;\rho)dm$, with $\int_{-\infty}^{+\infty} G(m;\rho')dm = \int_{-\infty}^{+\infty} G(m;\rho)dm$.*

Consistently with what we have assumed above, when invoking Assumption 1, we will maintain that $G(m;\rho)$ is differentiable in ρ, for any $m \in \mathbb{R}$. Assumption 1 then boils down to the requirement that, for any $m^* \in \mathbb{R}$ and ρ, $\int_{-\infty}^{m^*} G_\rho(m;\rho)dm \geq 0$, with $\int_{-\infty}^{+\infty} G_\rho(m;\rho)dm = 0$. Some of the results below assume a strengthening of such an order whereby the spreads correspond to "rotations."

Definition 1 (rotations). The set of information structures Player L can choose from is consistent with the "rotation" order (equivalently, the distributions are "simple mean-preserving spreads" or experiments consistent with the "single-crossing property") if, for any ρ, there exists a rotation point m_ρ such that $G_\rho(m;\rho) \geq 0$ for $-\infty < m \leq m_\rho$ and $G_\rho(m;\rho) \leq 0$ for $m_\rho \leq m < +\infty$ (with some inequalities strict).

A simple mean-preserving spread is a mean-preserving spread, but the converse is not true. For example, a combination of two rotations need not be a rotation, unless they have the same rotation point. As is well known, however, any mean-preserving spread can be obtained through a sequence of simple mean-preserving spreads.
A family of distributions $G(\cdot;\rho)$ that are rotations is given by the following example.

Non-directed search. Assume that information collection follows the standard non-directed search technology. That is, L learns the true state with probability $\rho \in [0,1]$ and nothing with probability $1-\rho$. Then,

$$G(m;\rho) = \begin{cases} \rho G(m) & \text{for } m < \omega_0 \\ \rho G(m) + 1 - \rho & \text{for } m \geq \omega_0. \end{cases}$$

In this example, the rotation point is thus equal to the prior mean ω_0. Figure 1 below illustrates the idea for the special case in which G is uniform.

Other examples of rotations include a normally distributed state ω together with a signal that is normally distributed around the true state (ρ is then the precision of the signal), and the family of Pareto, Exponential, and Uniform distributions in Proposition 1 below. See Diamond and Stiglitz (1974) and Johnston and Myatt (2006) for a broader discussion of rotations and their properties.

Cost of information. Choosing information ρ costs $C(\rho)$ to player L. When invoking Assumption 1, we will assume that C is non-decreasing, differentiable, and weakly convex.

[4] Say that each distribution $G(\cdot;\rho)$ is obtained by observing the realization $z \in Z$ of some experiment $q^\rho : \Omega \to \Delta(Z)$, where Z is a Polish space of signal realizations. Then if higher ρ index distributions (over the posterior mean) generated by Blackwell-more-informative experiments, the family $(G(\cdot;\rho))_{\rho\in\mathbb{R}_+}$ must be consistent with the MPS order. The contrary, however, is not true. The MPS order is more permissive than the Blackwell order.

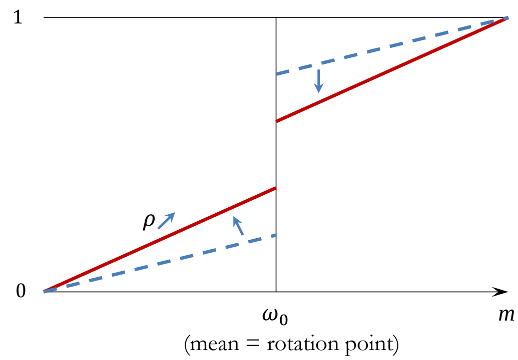

Figure 1: Cumulative distribution function $G(m;\rho)$ for non-directed search

(c) Preferences

Follower. Action $a = 1$ is "adverse-selection-sensitive," in the sense that player F's reaction to $a = 1$ depends on his beliefs about what information privately held by player L motivated L to engage. Consider a fictitious game in which L's information is exogenously fixed at $\rho^{\dagger}$. We assume that, for any $\rho^{\dagger}$, the equilibrium is unique and denote by $a^*(\cdot;\rho^{\dagger})$ and $r(\rho^{\dagger})$, respectively, L's engagement strategy and F's reaction to $a = 1$ in the unique equilibrium of the $\rho^{\dagger}$-game. The function $a^*(\cdot;\rho^{\dagger})$ specifies, for each posterior mean m, the probability $a^*(m;\rho^{\dagger}) \in [0,1]$ that player L engages when her posterior mean is m. In the game in which information is endogenous, we assume that, when F expects L to select information $\rho^{\dagger}$, he also expects L to engage according to $a^*(\cdot;\rho^{\dagger})$. We then denote by $\hat{G}(\cdot;\,\rho^{\dagger})$ the cumulative distribution function describing F's beliefs over L's posterior mean m, when expecting L to select information $\rho^{\dagger}$ and engaging according to $a^*(\cdot;\rho^{\dagger})$, after observing $a = 1$.[5] Given $\hat{G}(\cdot;\,\rho^{\dagger})$, F maximizes his expected payoff $\mathbb{E}_{\hat{G}(\cdot;\,\rho^{\dagger})}[u_F(1,\,r,\,m)]$ by means of an action $r \in \mathbb{R}$, where $u_F(1,\,r,\,m)$ is F's payoff when L engages (i.e., selects $a = 1$), F's reaction is r, and the posterior mean is m.[6]

By contrast, action $a = 0$ is "adverse-selection-insensitive." In some applications, such as Akerlof's lemons example below, action $a = 0$ involves no decision for the follower. More generally, we assume that the follower's reaction to $a = 0$ is independent of his beliefs about $\rho^{\dagger}$. This is the case, for instance, when $a = 0$ corresponds to the decision by player L to disclose hard information proving that the state (or L's posterior belief) is m, making F's conjecture about L's choice of information ρ irrelevant.[7] In the Supplement we discuss how the results may accommodate for the possibility that F's reaction to $a = 0$ also depends on F's beliefs about ρ and m, but with a lower sensitivity to these variables than F's reaction to $a = 1$; see the Supplement for an example of these situations.

Leader. Player L's payoff differential between $a = 1$ and $a = 0$ depends on the friendliness r of F's

[5] We are interested in situations in which, after choosing information $\rho^{\dagger}$,L engages with positive probability. In this case, when expecting information $\rho^{\dagger}$, player F, after observing $a = 1$, updates his beliefs $G(\cdot;\rho^{\dagger})$ about m using Bayes rule and the engagement strategy $a^*(\cdot;\rho^{\dagger})$. Also, in some of the applications of interest, it may be more natural to think of L as engaging after observing F's action r. Our results apply to some of these setting as well. For example, in the Akerlof's model where F stands for a competitive buyer, whether player L (the seller) observes the price offered by F before deciding to put the asset on sale, or puts the asset on sale anticipating the price offered by the competitive buyer is inconsequential because player F's reaction is predictable at the time player L engages.

[6] The assumption that F's payoff is affine in ω implies that $u_F(1,\,r,\,m)$ is also F's ex-post payoff when the state is $\omega = m$.

[7] See example (c) in the Supplement.

reaction and on player L's posterior mean m. Let $u_L(0,\, m)$ denote L's payoff when choosing $a = 0$. As just discussed, this payoff may depend on F's reaction. However, because the latter is invariant in F's expectations over L's choice of ρ, we can omit it to ease the notation and interpret $u_L(0, m)$ as L's payoff in state m given F's reaction to $a = 0$. Similarly let $u_L(1,\, r,\, m)$ denote L's payoff when choosing $a = 1$ and then denote by

$$\delta_L(r,\, m) \equiv u_L(1,\, r,\, m) - u_L(0,\, m)$$

L's payoff differential between $a = 1$ and $a = 0$, when F's reaction to $a = 1$ is r and L's posterior mean is m.

Assumption 2 (***leader's preferences***)**.** *Player L's payoff differential, δ_L, is Lipschitz continuous and twice continuously differentiable in each argument, strictly increasing in r, strictly decreasing in m, and such that the marginal impact of a friendlier reaction is weakly increasing in L's posterior mean: for any (r, m),*

$$\frac{\partial^2}{\partial r \partial m} \delta_L(r, m) \geq 0. \tag{1}$$

That δ_L is increasing in r reflects the assumption that a higher r represents a friendlier reaction, favoring $a = 1$. That δ_L is decreasing in m implies that a lower m favors $a = 1$. The strict monotonicity of δ_L in m in turn implies that, no matter the actual choice of information ρ, L optimally chooses to engage if and only if m falls below some cutoff $m^*(r)$ that depends on F's reaction r, with the cutoff $m^*(r)$ solving $\delta_L(r,\, m^*(r)) = 0$ and hence strictly increasing in r. Clearly, in any equilibrium in which L's actual information is ρ, the information $\rho^\dagger$ expected by F coincides with L's actual information ρ, and F's reaction is $r(\rho)$, where, as explained above, $r(\rho)$ is F's equilibrium reaction in a fictitious game in which L's information is exogenously fixed at ρ. Condition (1) in Assumption 2 says that L's marginal benefit of a friendlier reaction by F is larger in states in which L's payoff from engaging is lower. The condition will be used to determine whether information becomes more or less attractive to player L when player F behaves in a friendlier way (see the proof of Part (iii) of Proposition 1 below).

Let player F expect information $\rho^\dagger$ by player L. Out-of-equilibrium, $\rho^\dagger$ can differ from L's actual information ρ, because the choice of information is covert. However, suppose for a moment that information is exogenous and equal to $\rho^\dagger$. Because player F's payoff is quasilinear in ω, his reaction $r(\rho^\dagger)$ depends on the distribution $\hat{G}(\cdot;\, \rho^\dagger)$ describing his beliefs over L's posterior mean m only through the mean $\mathbb{E}_{\hat{G}(\cdot;\, \rho^\dagger)}[m]$ of $\hat{G}(\cdot;\, \rho^\dagger)$. Furthermore, as explained above, when L's information is exogenously fixed at $\rho^\dagger$, in equilibrium, player L's engagement strategy $a^*(\cdot; \rho^\dagger)$ takes the form of a cutoff rule, i.e., L optimally chooses $a = 1$ if and only if $m \leq m^*$, in which case $\mathbb{E}_{\hat{G}(\cdot;\, \rho^\dagger)}[m] = M^-(m^*;\, \rho^\dagger)$, where, for any $(m^*, \rho^\dagger)$,

$$M^-(m^*;\, \rho^\dagger) \equiv \mathbb{E}_{G(\cdot;\, \rho^\dagger)}[m | m \leq m^*] = m^* - \frac{\int_{-\infty}^{m^*} G(m; \rho^\dagger) dm}{G(m^*; \rho^\dagger)}$$

denotes the truncated mean of the distribution $G(\cdot;\, \rho^\dagger)$ of m, under information $\rho^\dagger$. An increase in M^- can then be viewed as a reduction of the adverse selection problem.

Assumption 3 (***lemons***)**.** The friendliness of player F's reaction increases with player L's investment in information if and only if more information alleviates the adverse selection problem:[8]

[8]Consistently with what anticipated above, to ease the exposition, we assume that $r(\cdot)$ and $M^-\big(m^*(r(\rho^\dagger));\, \cdot\big)$ are

$$\frac{dr(\rho^{\dagger})}{d\rho^{\dagger}} \overset{\text{sgn}}{=} \frac{\partial}{\partial \rho^{\dagger}} M^{-}\big(m^{*}(r(\rho^{\dagger}));\, \rho^{\dagger}\big). \tag{2}$$

As anticipated above, in the Supplement, we discuss how some of the results change in the anti-lemons case, i.e., when Assumption 3 is replaced with the following assumption, and present various examples of anti-lemons games.

Assumption 3′ (anti-lemons). The friendliness of player F's reaction increases with player $L's$investment in information if and only if more information reduces the truncated mean:

$$\frac{dr(\rho^{\dagger})}{d\rho^{\dagger}} \overset{\text{sgn}}{=} -\frac{\partial}{\partial \rho^{\dagger}} \mathrm{M}^{-}\big(\mathrm{m}^{*}(\mathrm{r}(\rho^{\dagger}));\, \rho^{\dagger}\big).$$

2.2 Examples

The Stackelberg game described above (and its key assumptions, 2 and 3) may look somewhat abstract. In this subsection, we show how Akerlof's lemons problem, augmented by the seller's endogenous covert information acquisition, maps into the general framework described above, and then briefly discuss other examples developed in the Supplement.

Akerlof's model. In Akerlof's (1970) model, player L is a seller of an asset (e.g., a used car). She can sell the good in the market ($a = 1$) or keep it for herself for own consumption ($a = 0$). Player F is a representative of a set of competitive buyers who choose a price r equal to the expected value of the good conditional on the good being put in the market. Suppose that the players' gross values for the good are m for the seller and $m + \Delta$ for the representative buyer, where Δ parametrizes the gains from trade, with $\Delta \in (0, \sup\{supp(G)\} - \omega_0\})$, where $supp(G)$ is the support of G.[9] Then, $r(\rho^{\dagger})$ is the price offered by the competitive buyer when the seller's information is exogenously fixed at $\rho^{\dagger}$ and is given by the solution to the following equation

$$r = \mathbb{E}_{G(\cdot;\,\rho^{\dagger})}\left[m + \Delta | m \leq r\right] = M^{-}(r; \rho^{\dagger}) + \Delta, \tag{3}$$

reflecting the fact that the cutoff $m^{*}(r)$ for L's equilibrium engagement strategy $a^{*}(\cdot; \rho^{\dagger})$ is equal to r. Consistently with what was explained above, we assume that the solution to (3) is unique, which is the case, for example, when $G(\cdot; \rho^{\dagger})$ is absolutely continuous with density $g(\cdot; \rho^{\dagger})$, and the inverse hazard rate $G(\cdot; \rho^{\dagger})/g(\cdot; \rho^{\dagger})$ of the distribution of m for information $\rho^{\dagger}$ is increasing in m.[10] Assumption 3 is

differentiable in $\rho^{\dagger}$ and denote by $\frac{\partial}{\partial \rho^{\dagger}} M^{-}\big(m^{*}(r(\rho^{\dagger}));\, \rho^{\dagger}\big)$ the partial derivative of $M^{-}(m^{*};\, \rho^{\dagger})$ with respect to $\rho^{\dagger}$, holding m^{*} fixed at $m^{*} = m^{*}(r(\rho^{\dagger}))$, where $m^{*}(r(\rho^{\dagger}))$ is the engagement threshold for L's equilibrium strategy $a^{*}(\cdot; \rho^{\dagger})$ in the fictitious game in which L's information is exogenously fixed at $\rho^{\dagger}$. These differentiability assumptions permit us to write Condition (2) in concise terms. The key property behind Assumption (3) is that, for any $\rho, \rho^{\dagger} \in \mathbb{R}_{+}$, $r(\rho) - r(\rho^{\dagger}) \overset{sgn}{=} M^{-}\big(m^{*}(r(\rho^{\dagger}));\, \rho\big) - M^{-}\big(m^{*}(r(\rho^{\dagger}));\, \rho^{\dagger}\big)$.

[9] When $\Delta \geq \sup\{supp(G)\} - \omega_0$, there is no adverse selection; the competitive buyer offers $\omega_0 + \Delta$ and the seller sells no matter her posterior mean. This case is not interesting.

[10] Then $\partial M^{-}(r; \rho^{\dagger})/\partial m^{*} \in (0, 1)$. See An (1998).

then satisfied. So is Assumption 2, given that, in this application, $\delta_L(r, m) = r - m$.
Turning to the case in which the seller's information is endogenous, we then have that L's optimal choice of ρ when L anticipates a reaction r by F is given by

$$\max_{\rho}\{G(r;\rho)r + \int_r^{\infty} mdG(m;\rho) - C(\rho)\}.$$

When C and G are differentiable in ρ and the above objective function for player L satisfies the appropriate quasi-concavity conditions (we will maintain these assumptions throughout the entire paper when referring to this example), the optimal level of ρ is then given by the following first-order condition[11]

$$-\int_r^{+\infty} G_\rho(m;\rho)dm = C'(\rho). \tag{4}$$

Other examples. The general model above also admits as a special case a different version of the Akerlof model in which the buyer, instead of being competitive, has full bargaining power. This version is the interdependent-value counterpart of the game considered in Ravid, Roesler, and Szentes (2022). In the Supplement, we show how a number of other games of interest fit into the framework introduced above. In the first example, a government engages in asset repurchases so as to jump-start a frozen market. In the second example, the good is divisible (a share in a project); the owner benefits from the synergies resulting from taking an associate in the project, but is hesitant about sharing the proceeds if she knows the project is highly profitable. In the third example, the seller may have hard information about the quality of the good and chooses whether to keep the evidence secret (which amounts to engaging in this example) or disclose it to the buyer (not engaging). The fourth example describes herding with interdependent payoffs; for example, by entering a market, a firm may encourage a rival to follow suit. The fifth example is a marriage game, in which covenants smooth the hardship of a subsequent divorce, but also signal bad prospects about the marriage. Some of these examples naturally feature a non-linear δ_L function which explains the generality introduced above.[12] We refer the reader to the Supplement for the details.

3 Expectation conformity

We now investigate how L's choice of information is influenced by F's expectations and how the latter in turn depend on whether L's information aggravates adverse selection. Adverse selection is here captured by the truncated mean $M^-(m^*;\, \rho^\dagger)$. Consistently with what discussed above, we will simplify the notation by assuming that $M^-(m^*;\, \rho^\dagger)$ is differentiable in ρ.

Definition 2 (impact of information on adverse selection). Starting from information $\rho^\dagger$, an increase in information by player L

- aggravates adverse selection if $\frac{\partial}{\partial \rho^\dagger} M^-(m^*(r(\rho^\dagger));\, \rho^\dagger) < 0$

[11] Note that the FOC for ρ can also be written as $\int_{-\infty}^{r} G_\rho(m;\rho)dm = C'(\rho)$. This is because $\int_{-\infty}^{+\infty} mdG(m;\rho)$ is invariant in ρ, implying that $\int_{-\infty}^{+\infty} G_\rho(m;\rho)dm = 0$.

[12] As explained below, a non-linear δ_L function also brings additional effects to the analysis, for example by making L's value for information depend, among other things, on the induced volatility of m.

- alleviates adverse selection if $\frac{\partial}{\partial \rho^\dagger} M^-(m^*(r(\rho^\dagger));\, \rho^\dagger) > 0$.

Simple computations show that, for any information $\rho^\dagger$ and truncation point m^*,

$$\frac{\partial}{\partial \rho^\dagger} M^-(m^*;\, \rho^\dagger) \overset{\text{sgn}}{=} A(m^*; \rho^\dagger) \tag{5}$$

where

$$A(m^*; \rho^\dagger) \equiv \left[m^* - M^-(m^*;\, \rho^\dagger)\right] G_\rho(m^*; \rho^\dagger) - \int_{-\infty}^{m^*} G_\rho(m; \rho^\dagger) dm. \tag{6}$$

The first term of A captures the direct effect of a change in the probability that player L engages on player F's expectation of the state. Because $m^* \geq M^-(m^*;\, \rho^\dagger)$, an increase in information alleviates adverse selection when it increases the chances that player L engages (i.e., when $G_\rho(m^*; \rho^\dagger) > 0$), whereas it aggravates it when it reduces the probability of such an event (i.e., when $G_\rho(m^*; \rho^\dagger) < 0$). The second term, of A, $\int_{-\infty}^{m^*} G_\rho(m; \rho^\dagger) dm$, in turn is related to the effect of information on the dispersion of L's posterior mean m. When more information induces more dispersion in the sense of second-order stochastic dominance (which is always the case when higher ρ index distributions $G(\cdot; \rho)$ generated by Blackwell-more-informative experiments), this second effect unambiguously contributes to an aggravation of the adverse selection problem. Hereafter, we will refer to

$$\mathrm{A}(\rho^\dagger) \equiv A(m^*(r(\rho^\dagger)); \rho^\dagger) \tag{7}$$

as the "*adverse-selection effect*" of an increase of information at $\rho^\dagger$. Note that, under Assumption 3, when $A(\rho^\dagger) > 0$ (alternatively, $A(\rho^\dagger) < 0$), starting from $\rho^\dagger$ a small increase in the informativeness of L's signal triggers a friendlier (alternatively, an unfriendlier) reaction by F.

Now recall that L's ex-ante payoff (gross of the cost) from choosing information ρ when expecting a reaction r to her decision to engage is equal to

$$\Pi(\rho; r) \quad \equiv \quad \sup_{a(\cdot)} \left\{ U_L(0) + \int_{-\infty}^{+\infty} a(m)\, \delta_L(r, m) dG(m; \rho) \right\}$$

where $U_L(0) \equiv \int_{-\infty}^{+\infty} u_L(0, m) dG(m)$ is L's ex-ante expected payoff when she never engages, and $a(m)$ represents the probability that L engages when her posterior mean is m.[13]

Then let

$$B(\rho; \rho^\dagger) \equiv -\frac{\partial^2 \Pi(\rho; r(\rho^\dagger))}{\partial \rho \partial r}$$

denote the effect of a reduction in the friendliness of F's reaction, starting from $r = r(\rho^\dagger)$, on L's marginal value of information, evaluated at ρ. Hereafter, we will refer to $B(\rho; \rho^\dagger)$ as the "*benefit of a friendlier reaction effect*".

Definition 3 (information incentive effect of unfriendly reactions). Given $(\rho, \rho^\dagger)$, a reduction in the friendliness of player F's reaction starting from $r = r(\rho^\dagger)$, raises (alternatively, lowers) player L's incentive to invest in information at ρ if $B(\rho; \rho^\dagger) > 0$ (alternatively, if $B(\rho; \rho^\dagger) < 0$).

[13]Note that, because $u_L(0, m)$ is affine in m, $\int_{-\infty}^{+\infty} u_L(0, m) dG(m; \rho) = \int_{-\infty}^{+\infty} u_L(0, m) dG(m)$ for any ρ, implying that $U_L(0)$ is invariant in ρ.

Using the envelope theorem along with the fact that, for any ρ, the optimal engagement strategy for L when F anticipates information $\rho^\dagger$, is to engage if and only if $m \leq m^*(r(\rho^\dagger))$, and integrating by parts, we have that

$$\begin{aligned} B(\rho;\rho^\dagger) &= -\frac{\partial \delta_L(r(\rho^\dagger), m^*(r(\rho^\dagger)))}{\partial r} G_\rho\Big(m^*(r(\rho^\dagger));\rho\Big) \\ &\quad + \int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial^2 \delta_L(r(\rho^\dagger), m)}{\partial r \partial m} G_\rho(m;\rho) dm. \end{aligned} \tag{8}$$

Because δ_L is increasing in r, the sign of the first term of $B(\rho;\rho^\dagger)$ is determined by whether an increase in information increases or reduces the chances that player L engages. Under Assumption 2, the marginal benefit of a friendlier reaction by player F is increasing in the posterior mean m. As a result, the second term of $B(\rho;\rho^\dagger)$ is always positive when a higher ρ indexes a mean-preserving spread of the induced posterior mean.

Next, let $V_L(\rho;\rho^\dagger) \equiv \Pi(\rho; r(\rho^\dagger))$ denote the maximal payoff that player L can obtain by choosing information ρ when player F expects information $\rho^\dagger$.

Definition 4 (expectation conformity). Expectation conformity (EC) holds at $(\rho, \rho^\dagger)$ if and only if

$$\frac{\partial^2 V_L(\rho;\rho^\dagger)}{\partial \rho \partial \rho^\dagger} > 0.$$

Suppose Assumption 1 holds (i.e., information structures are consistent with the MPS order and higher ρ index more informative experiments). EC then says that the marginal value to player L from choosing a more informative experiment starting from ρ is higher when player F, starting from $\rho^\dagger$, expects player L to choose a more informative experiment. When there is an interval $[\rho_1, \rho_2]$ such that the property holds for all $\rho, \rho^\dagger \in [\rho_1, \rho_2]$, the gross value to player L from moving from ρ_1 to ρ_2 is higher when player F expects her to choose ρ_2 than when he expects her to choose ρ_1: $V_L(\rho_2;\rho_2) - V_L(\rho_1;\rho_2) > V_L(\rho_2;\rho_1) - V_L(\rho_1;\rho_1)$. In this sense, EC captures a complementarity between actual and anticipated information choice. Below we relate this property to the determinacy of equilibria and a few other phenomena of interest.

Proposition 1 (*expectation conformity*). *Suppose that Assumptions 1, 2, and 3 hold.*
(i) EC holds at $(\rho, \rho^\dagger)$ if and only if the adverse selection effect and the benefit of a friendlier reaction effect are of opposite sign: $\mathrm{A}(\rho^\dagger)B(\rho;\rho^\dagger) < 0$.
(ii) Information always aggravates adverse selection at $\rho^\dagger$ (i.e., $\mathrm{A}(\rho^\dagger) < 0$) when the family of distributions $G(\cdot;\rho)$ from which m is drawn is Uniform, Pareto, or Exponential. For other distributions, a sufficient condition for information to aggravate adverse selection at $\rho^\dagger$ is that $G_\rho(m^*(r(\rho^\dagger));\rho^\dagger) < 0$.
(iii) Starting from $r(\rho^\dagger)$, a reduction in the friendliness of player F's reaction raises player L's incentive to invest in information at ρ (i.e., $B(\rho;\rho^\dagger) > 0$) if $G_\rho(m^*(r(\rho^\dagger));\rho) < 0$.
(iv) Therefore a sufficient condition for EC at $(\rho, \rho^\dagger)$ is that

$$\max\Big\{G_\rho(m^*(r(\rho^\dagger));\rho^\dagger), G_\rho(m^*(r(\rho^\dagger));\rho)\Big\} < 0. \tag{9}$$

(v) Suppose that, for any m^*, $M^-(m^*;\rho)$ is decreasing in ρ (which is the case for Uniform, Pareto, and Exponential distributions), implying that, for any $\rho^\dagger$, $\mathrm{A}(\rho^\dagger) < 0$. If $\partial^2\delta_L(r,m)/\partial r\partial m = 0$, as is the Akerlof's model described above, then $G_\rho(m^*(r(\rho^\dagger));\rho) < 0$ is a necessary and sufficient condition for EC at $(\rho,\rho^\dagger)$.[14] *When the distributions $G(\cdot;\rho)$ are rotations, in the sense of Definition 1, $G_\rho(m^*(r(\rho^\dagger));\rho) < 0$ if and only if $m^*(r(\rho^\dagger))$ is to the right of the rotation point m_ρ.*

Hence, EC holds at $(\rho,\rho^\dagger)$ when, fixing player F's reaction at $r(\rho^\dagger)$, an increase in the informativeness of player L's experiment decreases the probability that L engages, both when such an increase is evaluated from player L's perspective (i.e., starting from ρ) and when evaluated from player F's perspective (i.e., starting from $\rho^\dagger$)—formally, when Condition (9) holds. This is because, from F's perspective, that player L engages less often (formally, that $G_\rho(m^*(r(\rho^\dagger));\rho^\dagger) < 0$) implies an aggravation in the adverse selection problem, which induces player F to respond in an unfriendlier manner (part (ii) in the proposition). That player F responds in an unfriendlier manner, together with the fact that, in the eyes of player L, more information makes her engage less often (i.e., $G_\rho(m^*(r(\rho^\dagger));\rho) < 0$), implies a higher marginal value for player L to acquire more information starting from ρ (part (iii) in the proposition). Jointly, the above two properties (captured by Condition (9) in the proposition) thus imply that, when player F expects player L to acquire more information (starting from $\rho^\dagger$), the benefit for player L to acquire more information (starting from ρ) is higher. That is, EC holds at $(\rho,\rho^\dagger)$. Importantly, Condition (9) is sufficient for EC but not necessary. For example, when the family of distributions from which the posterior mean is drawn is Uniform, Pareto, or Exponential, more information always aggravates adverse selection, implying that EC holds at $(\rho,\rho^\dagger)$ if, in the eyes of player L, it reduces the probability of engagements (i.e., if $G_\rho(m^*(r(\rho^\dagger));\rho) < 0$), irrespectively of whether, in the eyes of player F, more information increases or decreases the probability of trade (i.e., irrespectively of the sign of $G_\rho(m^*(r(\rho^\dagger));\rho^\dagger)$). Furthermore, the sufficiency of Condition (9) hinges on the information structures being consistent with the MPS order. The result is thus perhaps less obvious than what it may look like.

EC holds at $(\rho,\rho^\dagger)$ also when $\mathrm{A}(\rho^\dagger) > 0$ and $B(\rho;\rho^\dagger) < 0$, that is, when the choice of a more informative signal by player L (starting from $\rho^\dagger$) induces player F to respond in a friendlier way because it alleviates adverse selection, and a friendlier reaction by player F (starting from $r(\rho^\dagger)$) raises player L's marginal value for information (starting from ρ).

Finally, the last part of the proposition establishes that, when information always aggravates adverse selection and L's payoff is separable in m and r, as in Akerlof's model, that more information reduces the probability of engagement starting from ρ (i.e., that $G_\rho(m^*(r(\rho^\dagger));\rho) < 0$) is not only sufficient for EC at $(\rho,\rho^\dagger)$, but also necessary.

As we document in the next section, EC is at the core of various economic phenomena. Before doing so, we first illustrate how EC naturally emerges in Akerlof's model under non-directed search.

[14]See also Examples (a), (c), and (d) in the Supplement for alternative games in which $\partial^2\delta_L(r,m)/\partial r\partial m = 0$.

3.1 Example: Akerlof's model under non-directed search

Under non-directed search, the rotation point is the prior mean. Proposition 1, when applied to the Akerlof's model of Subsection 2.2, thus says that EC holds at $(\rho, \rho^\dagger)$ whenever the engagement threshold $m^* = r(\rho^\dagger)$ is to the right of the prior mean, that is, when the price offered by the competitive buyer is sufficiently high. In other words, EC arises when the gains from trade (in the example parametrized by Δ) are large, and it never occurs when they are small.

To gather some intuition, recall that, in Akerlof's model, the seller puts the asset up for sale when her value for the asset is small (i.e., when the posterior mean is below a threshold m^* that coincides with the price $r(\rho^\dagger)$ offered by the buyer). Naturally, when the gains from trade Δ are large, the price offered by the buyer is also large, in which case $r(\rho^\dagger)$ exceeds the rotation point, which coincides with the prior mean ω_0 of the asset's value for the seller. Economically, what this implies is that the seller finds it optimal to enter the market both when she is uninformed and when she learns that her value for the asset, ω, is below the price $r(\rho^\dagger)$. Starting from such a situation, the expectation by the buyer of the seller acquiring more information reduces the quality of the asset perceived by the buyer after seeing that the asset is on sale. Faced with an exacerbated adverse selection problem, the buyer then reduces the price offered. But then it becomes even more important for the seller to learn the value of the asset, that is, to acquire more information starting from ρ. So EC naturally holds for $(\rho, \rho^\dagger)$ in this case.[15]

While the mechanism just described is fairly natural, it is important to appreciate that it need not always be in place. In fact, EC fails to obtain in this model when the gains from trade are positive but small. To see this, note that, when Δ is small, because of the adverse selection problem, the price offered by the buyer may well be lower than the ex-ante prior mean of the asset, meaning that $r(\rho^\dagger) < \omega_0$. Anticipating such a low price, the seller enters the market only if she receives information that reveals that $\omega \leq r(\rho^\dagger)$. The buyer then understands that the expected value of the asset, conditional on the seller putting it in on the market, is invariant in the seller's information: $M^-(r(\rho^\dagger); \rho^\dagger) = \int_{-\infty}^{r(\rho^\dagger)} \omega dG(\omega)/G(r(\rho^\dagger))$, which, fixing the price at $r(\rho^\dagger)$, is independent of the information that the buyer expects the seller to acquire. When this is this case, an increase in the information expected from the seller by the buyer does not affect the price offered by the buyer, and hence does not increase L's incentives to search. We thus have the following result:

Corollary 1 (***lemons under non-direct search***)**.** *In the Akerlof's model under non-directed search, EC holds at* $(\rho, \rho^\dagger)$ *if and only if the gains from trade* Δ *are sufficiently large (namely, if and only if the unique solution* $r(\rho^\dagger)$ *to* $r = M^-(r; \rho^\dagger) + \Delta$ *exceeds the prior mean* ω_0*).*

3.2 Gains from engagement

The example in the previous subsection suggests that EC is more likely to obtain when the gains from engagement for player L are large. The next result shows that this is true more generally.

Proposition 2 (***gains from engagement***)**.** *Suppose that Assumptions 1, 2 and 3 hold, and that information structures take the form of rotations, as in Definition 1. Further assume that player L's*

[15] Consistently with the result in Proposition 1, note that, when $r(\rho^\dagger) > \omega_0$, Condition (9) always holds (see Figure 1).

payoff differential from selecting $a = 1$ instead of $a = 0$ is $\delta_L(m,r) = \bar{\delta}_L(m,r) + \theta$, where $\bar{\delta}_L(m,r)$ is an arbitrary function satisfying Assumption 2, and $\theta \in \mathbb{R}$.[16] *For all $(\rho,\rho^\dagger)$, there exists $\theta^*(\rho,\rho^\dagger)$ such that, for all $\theta \geq \theta^*(\rho,\rho^\dagger)$, EC holds at $(\rho,\rho^\dagger)$: EC is more likely, the larger the leader's gains from engagement.*

Proof. See the Appendix.

Proposition 2 says that higher gains from engagement reinforce EC. On the other hand, holding player F's reaction fixed, larger gains from engagement reduce the marginal benefit of acquiring more information under the sufficient condition for EC identified in Proposition 1, namely that more information reduces the probability of trade, i.e., that $G_\rho(m^*(r(\rho^\dagger;\theta),\theta);\rho) \leq 0$:

$$\frac{\partial^2}{\partial\theta\partial\rho}\left[\int_{-\infty}^{m^*(r(\rho^\dagger;\theta),\theta)} [\bar{\delta}_L(r(\rho^\dagger;\theta),m) + \theta] dG(m;\rho)\right] = G_\rho(m^*(r(\rho^\dagger;\theta),\theta);\rho).$$

The reason for this last result is the following. Holding player F's reaction fixed, when more information reduces the probability of engagement, it is particularly costly when the gains from engagement are large. This property helps clarify that it is only because of the effects of information on the severity of the adverse selection problem that larger gains from engagement contribute to EC. They make player F respond to the anticipation of player L acquiring more information by reducing r more sharply, which in turn raises player L's value of information.

4 Expectation Traps, Disclosure of Hard Information, and Cognitive Styles

We now turn to three phenomena that are intrinsically related to EC in the type of situations described above, expectation traps, disclosure, and cognitive styles.

4.1 Expectation traps

Proposition 3 (***expectation traps***). *Suppose that Assumptions 2 and 3 hold and that ρ_1 and ρ_2 are both equilibrium levels, with $\rho_1 < \rho_2$. If, for any $\rho^\dagger \in [\rho_1, \rho_2]$, $\mathrm{A}(\rho^\dagger) < 0$ (which is the case, for example, when either the distributions are Uniform, Pareto, or Exponential, or when Assumption 1 holds and $G_\rho(m^*(r(\rho^\dagger)); \rho^\dagger) < 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$), then player L is better off in the low-information-intensity equilibrium ρ_1. Conversely, when for any $\rho^\dagger \in [\rho_1, \rho_2]$, $\mathrm{A}(\rho^\dagger) > 0$, player L is better off in the high-information-intensity equilibrium ρ_2.*

Expectation traps do not result just from the fact that, when $C(\rho)$ is increasing, in a high-information-intensive equilibrium, player L spends more resources in information acquisition. In fact, at the margin, player L's gain from a more informative structure is equal to the increase in the cost of information acquisition. Rather, expectation traps occur because player F, anticipating an exacerbated adverse selection problem when expecting player L to acquire more information, reacts in an unfriendlier way,

[16] For instance, in the examples in Section S.1 in the Supplement, an increase in θ corresponds to an increase in $d_L - c_L$ in example (b), a reduction in Δ in example (c), an increase in π^d and/or in $\pi^m - \pi^d$ in example (d), an increase in $\mathscr{L}_L - \ell_L$, or a reduction in c_L, in example (e), and an increase in K_0 in example (f).

which not only forces player L to acquire more information, vindicating player F's expectation, but hurts player L.

To illustrate, consider again the Akerlof's model under non-direct search of the previous section. The equilibrium levels of ρ and the corresponding prices $r(\rho)$ are given by the solutions to Conditions (3) and (4). For example, when G is Uniform over $[0,1]$, the cost of information is $C(\rho)=\rho^2/20$, and $\Delta=0.25$, there are two equilibria in which the price exceeds the prior mean $\omega_0=0.5$. In the first equilibrium $\rho_1\approx 0.48$ and $r(\rho_1)\approx 0.69$; in the second equilibrium, $\rho_2\approx 0.88$ and $r(\rho_2)\approx 0.58$. Because, for any $m^*>\omega_0$, $G(m^*;\rho^\dagger)$ is decreasing in $\rho^\dagger$, information always aggravates adverse selection at $\rho^\dagger$ when $r(\rho^\dagger)>\omega_0$. In this example, $r(\rho^\dagger)>\omega_0$ for all $\rho^\dagger\in[\rho_1,\rho_2]$, implying that $\mathrm{A}(\rho^\dagger)<0$ for all $\rho^\dagger\in[\rho_1,\rho_2]$. Hence, the conditions in the previous proposition apply. The seller is better off in the low-information-intensity equilibrium ρ_1 than in the high-information-intensity equilibrium ρ_2.

The result in the previous proposition contrasts with what one obtains in markets with private values and monopolistic screening. To see this, consider a setting in which player F is a seller maximizing expected profits $p-c(\omega)$ by means of a take-it-or-leave-it offer p, whereas player L is a buyer choosing how much information ρ to acquire about her gross value ω for the seller's product and whether or not to accept the seller's offer of trading at price p so as to maximize her net payoff $\omega-p-C(\rho)$. When the seller's cost c is invariant in ω, this model corresponds to the private-value setting of Ravid, Roesler, and Szentes (2022). In their setting, when information is free and the buyer can choose any mean-preserving contraction $G(\cdot;\rho)$ of the prior distribution G at no cost, there are multiple equilibria. All equilibria are Pareto ranked, with each player's payoff maximized in the equilibrium in which the buyer fully learns the state. When, instead, payoffs are interdependent and player F is a representative of a competitive market (as in the Akerlof example above), the result in Proposition 3 suggests that, when more information by the buyer aggravates the adverse selection problem and this leads the seller to ask for a higher price, then equilibria in which the buyer acquires more information are equilibria in which the buyer is necessarily worse off, no matter the cost of information.[17]

The result in Proposition 3 calls for government interventions aimed at discouraging the players from acquiring information. We discuss some of these interventions in Section 5. Here, instead, we want to emphasize that expectation traps are intrinsically related to EC. Recall that EC relates to the benefit of information in strategic settings. It does not depend on the cost of information. When the sufficient conditions for EC of Proposition 1 hold, one can identify cost functionals for which multiple equilibria arise.[18] The same conditions then imply that player L is worse-off in the more information-intense equilibria.

4.2 Disclosure and Cognitive Style

So far we have assumed that information acquisition is covert. Suppose that it is indeed covert, but that some form of disclosure prior to F's action is feasible. Namely, given her actual choice ρ, player L

[17]The payoff of player F (here in the role of a seller) is constant in the equilibrium signal ρ selected by the buyer when F has no bargaining power, i.e., when he is a representative of a competitive market.

[18]Namely, suppose that Assumptions 1, 2, and 3 hold, and that there exist ρ_1 and ρ_2, with $\rho_2>\rho_1$, such that $G_\rho(m^*(r(\rho^\dagger));\rho)<0$ for all $\rho^\dagger,\rho\in[\rho_1,\rho_2]$. There exist monotone cost functionals $C(\cdot)$ such that ρ_1 and ρ_2 are both equilibrium levels. Furthermore, under any such cost functionals, player L is better off in the low-information-intensive equilibrium ρ_1 than in the high-information-intensive equilibrium ρ_2.

can prove that her choice is above any level $\hat{\rho} \leq \rho$. The disclosed information is hard. For any $\hat{\rho} \in \mathbb{R}_+$, let the "$\hat{\rho}$-constrained game" be the no-disclosure game with modified cost function $\hat{C}(\rho; \hat{\rho}) = C(\rho)$ if $\rho \geq \hat{\rho}$ and $\hat{C}(\rho; \hat{\rho}) = +\infty$ if $\rho < \hat{\rho}$. Let $E(\hat{\rho})$ denote the set of equilibrium levels of ρ of the $\hat{\rho}$-constrained game and assume that $E(\hat{\rho})$ is non-empty for all $\hat{\rho} \in \mathbb{R}_+$. We say that the function $e(\cdot) : \mathbb{R}_+ \to \mathbb{R}_+$ is a selection if, for any $\hat{\rho} \in \mathbb{R}_+$, $e(\hat{\rho}) \in E(\hat{\rho})$.[19]

Definition 5 (monotone selections). The selection $e(\cdot)$ is *monotone* if for all $\hat{\rho}$ and $\hat{\rho}'$, with $\hat{\rho} < \hat{\rho}'$, $e(\hat{\rho}) \leq e(\hat{\rho}')$.

In words, the selection is monotone if, when L is constrained to choose among levels of ρ that exceed a certain threshold, in equilibrium, as the threshold increases, L selects a higher level. Note that, because $E(\hat{\rho}) \cap \{\rho \mid \rho \geq \hat{\rho}'\} \subseteq E(\hat{\rho}')$, it is always possible to construct monotone selections.

Definition 6 (regularity). Take any equilibrium of the primitive game with disclosure. The equilibrium is *regular* if the selection $e(\cdot)$ describing player L's choice of information following any possible disclosure $\hat{\rho} \in \mathbb{R}_+$ is monotone and $e(0)$ is an equilibrium of the no-disclosure game.

Clearly, in any pure-strategy equilibrium of the game with disclosure, L selects a unique ρ on path. In this case, regularity imposes restrictions on the off-path behavior of the two players. Namely, for any pure-strategy equilibrium of the game with disclosure in which player L's equilibrium investment is ρ^*, let $\hat{\rho}(\rho^*)$ denote the information L discloses on path. The equilibrium being regular implies, among other things, that, if L were to disclose any $\hat{\rho} < \hat{\rho}(\rho^*)$ (alternatively, any $\hat{\rho} > \hat{\rho}(\rho^*)$), in the continuation game, she would then select a ρ weakly below ρ^* (alternatively, weakly above ρ^*).

Let $\bar{\rho}$ be the highest level of ρ supported by a pure-strategy equilibrium of the game without disclosure. It is easy to see that, without the above refinement, the game with disclosure may admit pure-strategy equilibria supporting ρ^* strictly above $\bar{\rho}$. For example, suppose that information always aggravates adverse selection (i.e., $\mathrm{A}(\rho^\dagger) < 0$ for all $\rho^\dagger$, as in the case of Uniform, Pareto, or Exponential distributions). These equilibria can be sustained by a strategy for player L according to which, on path, L discloses $\hat{\rho}(\rho^*) = \rho^* > \bar{\rho}$. Off path, after disclosing any $\hat{\rho} < \rho^*$, L selects a ρ above ρ^* anticipating a low reaction by player F, supported by the expectation of the choice of an experiment by player L aggravating the adverse selection problem. In other words, without the refinement, there is not enough connection between the equilibrium information choices of the game with and without disclosure.

Proposition 4 (*disclosure*). *Assume that Assumptions 2 and 3 hold and that* $\mathrm{A}(\rho^\dagger) < 0$ *for all* $\rho^\dagger$*, implying that information always aggravates adverse selection.*

- *Any pure-strategy equilibrium choice of information* ρ *of the game in which disclosure is not feasible is also an equilibrium level in the disclosure game.*
- *Conversely, the largest and smallest levels of* ρ *sustained by pure-strategy regular equilibria of the disclosure game are also equilibrium levels in the game without disclosure.*

[19]Shishkin (2022) studies an evidence acquisition game. He shows that, when the probability of obtaining information is small, the Sender's optimal policy has a pass/fail structure and reveals only whether the quality is above or below a threshold. The game considered in this section differs in two respects: First, the acquired information is soft; second, the acquisition is either overt or "semi-overt" in that the intensity of information acquisition can be disclosed but not the actual information obtained.

Proof. See the Appendix.

Under Assumption 3, the choice of a more informative experiment by player L, by aggravating the adverse selection problem, reduces the friendliness of F's reaction. Player L then never gains from proving that her investment in information acquisition is large, when the disclosure of a higher investment is interpreted as a signal of a higher actual choice. When, in addition, the marginal benefit to player L of a more informative experiment decreases with the friendliness of F's reaction (which, by virtue of part (iii) of Proposition 1, is the case when more information reduces the probability that L engages, i.e., when, given $\rho^\dagger$, for any ρ, $G_\rho(m^*(r(\rho^\dagger)); \rho) < 0$), player L benefits from aligning her choice of information with player F's expectations (that is, EC holds) as in the game without disclosure.

In the same vein, one can consider the possibility of *transparency*, namely a commitment to reveal the exact amount of investment in information made. In this case, $\hat{\rho} = \rho$ for any ρ (overt information acquisition). Clearly, player L is better off committing ex ante to transparency than retaining the possibility to disclose information voluntary ex post (the case just studied). She is also better off under transparency than in the game with complete absence of any disclosure. More interestingly, when F's reaction is non-increasing in $\rho^\dagger$ (which is the case when information always aggravates adverse selection, i.e., when $\mathrm{A}(\rho^\dagger) < 0$ for all $\rho^\dagger$), under transparency, in equilibrium, player L may choose a level $\rho^* \leq \underline{\rho}$ that is lower than the lowest equilibrium level in the no-disclosure game. Similar conclusions obtain when player L cannot reveal her investment in information perfectly, but can prove that it is below some level $\hat{\rho}$ of her choice, for example by proving that she is unable to undertake more than a certain number of informative tests. In such situations, equilibria may exist in which player L proves that her investment is below the lowest equilibrium level of the no disclosure game.

Another focus of comparative statics concerns player L's *cognitive style*. We provide here only an informal account. Continue to assume that information aggravates adverse selection, but now suppose that the cost of information $C(\rho; \xi)$ depends on a parameter ξ, interpreted as ability. A higher-ability player L has a lower marginal cost of information: for any ξ, $C(0; \xi) = 0$ and $C_\rho(0; \xi) = 0$, whereas for any $\rho > 0$, $C_\rho(\rho; \xi) > 0$, $C_{\rho\rho}(\rho; \xi) > 0$, and $C_{\rho\xi}(\rho; \xi) < 0$. Under the conditions for EC of Proposition 1, as player L's ability increases, the equilibrium ρ also increases (in case of multiple equilibria, in the sense of monotone comparative statics, that is, the lowest and highest levels of the equilibrium set corresponding to ability ξ increase with ξ). Put it differently, player L's ability, while directly beneficial, indirectly hurts her as player F becomes more wary of adverse selection. This suggests that, if player L has side opportunities to signal her ability, she will want to adopt a dumbed-down profile.

Suppose indeed that player L can be bright (ξ_H) or dumb (ξ_L). A bright person can demonstrate that she is bright (and can always mimic a dumb one), but the reverse is impossible. The set of equilibrium levels of ρ is monotonically increasing in the posterior probability that $\xi = \xi_H$. Assume a monotone selection in this equilibrium set: Player F's action r is decreasing in the probability that she assigns to $\xi = \xi_H$ (a property automatically satisfied if the equilibrium is unique, for any possible belief). Then if one adds to this game a disclosure stage in which player L can disclose she is bright if this is indeed the case, the equilibrium is a pooling one, in which the bright player L does not disclose her brightness. Conversely, player L will disclose, if she can, that she is overloaded with work (assume that she cannot prove that she has a low workload), and therefore that her marginal cost of information is high. In either case, player L poses as an *"informational puppy dog"* (in the sense of Fudenberg and Tirole (1984)).

5 Policy Interventions

We now investigate how a benevolent government can improve over the laissez-faire equilibrium by subsidizing (alternatively, taxing) trade. For simplicity, we focus on the Akerlof's model of Subsection 2.2; in the Appendix, we extend the analysis (and generalize the results) to settings in which L's payoff differential is an arbitrary function $\delta_L(r, m)$ satisfying Assumption 2, and in which F is a representative of a competitive market with arbitrary payoff differential $\delta_F(r, m) \equiv u_F(1, r, m) - u_F(0, m)$.[20]

5.1 Optimality of subsidizing/taxing trade

Let s denote the subsidy (tax if $s < 0$) the government promises to pay to player L in case of trade.[21] For any r and s, let $m^*(r, s)$ denote the optimal engagement threshold for player L when player F's reaction is r and the subsidy is s. Recall that, in the Akerlof's model, $\delta_L(r, m) = r - m$, implying that $m^*(r, s) = r + s$. Let $\rho^*(s)$ and $r^*(s)$ denote, respectively, the leader's equilibrium investment in information and the follower's equilibrium response in the continuation game that starts after the government announces a subsidy equal of s. Throughout, we assume that, for any s, $\rho^*(s)$ and $r^*(s)$ are unique, Lipschitz continuous, and differentiable. Likewise, we assume that the the distributions $G(m; \rho)$ are differentiable and Lipschitz-continuous. In addition to facilitating the description of the relevant optimality conditions, these properties validate a certain envelope theorem that we use in the Appendix to establish some of the results.

For any s, total welfare is given by (up to scalars that are irrelevant for the analysis)

$$W(s) \equiv \int_{-\infty}^{m^*(r^*(s),s)} (\delta_L(r^*(s), m) + s)\, dG(m; \rho^*(s)) - C(\rho^*(s)) - (1 + \lambda)sG(m^*(r(s), s); \rho^*(s)),$$

where $\lambda \geq 0$ is the unit cost of public funds (linked to the deadweight loss of non-uniform taxation). The first two terms of $W(s)$ represent the leader's payoff, whereas the last term represents the cost of the program to the government. Hereafter, we assume that W is strictly quasi-concave.

Proposition 5 (***social value of subsidizing/taxing trade***)**.** Consider the Akerlof model of Subsection 2.2 and Suppose that Assumption 3 holds. A strictly positive subsidy is optimal when

$$\frac{\partial}{\partial m^*} M^-(r^*(0); \rho^*(0)) + \frac{\partial}{\partial \rho} M^-(r^*(0); \rho^*(0)) \frac{d\rho^*(0)}{ds} > \lambda,$$

whereas a tax on engagement is optimal when the above inequality is reversed.[22]

[20] The insights of this section extend to a broader class of problems in which F is not a representative of a competitive market and the welfare weights the government assigns to player$F$ is arbitrary. The conditions, however, are less transparent.

[21] More generally, both the decision to engage and that of not engage can be subject to taxes and subsidies. For example, the decision to hold on a car or a security can be taxed. Hence, in the analysis below, s should be interpreted as the differential in the subsidy/tax when player L engages relative to when she does not engage.

[22] As we show in the Appendix, for more general payoff functions δ_L for the leader, the result in the proposi-

Whether subsiding trade is preferable to taxing it depends on whether, fixing F's reaction at the laissez-faire level $r^*(0)$, subsidizing (alternatively, taxing) trade has a strong enough effect on the alleviation of the adverse selection problem to compensate for the cost of the program. When information is endogenous, there are two channels through which a subsidy alleviates (alternatively, aggravates) the adverse selection problem. The first one is through its effect on the leader's engagement, as captured by the threshold m^*, which, in the Akerlof's model is equal to $r+s$. The second one is through its effect on the leader's information, ρ. A higher subsidy always increases the engagement threshold $m^*(r,s) = r+s$. Because, for any ρ, M^- is increasing in m^*, the first effect always contributes to an alleviation of the adverse selection problem (i.e., to an increase in M^-). The second effect, instead, can be either positive or negative, depending on whether information aggravates or alleviates the adverse selection problem, and whether a positive subsidy increases or decreases the leader's investment in information ρ. From Proposition 1, we know that, when information structures are consistent with the MPS order (i.e., when Assumption 1 holds), information aggravates the adverse selection problem when it reduces the probability of trade, i.e., when $G_\rho(r^*(0); \rho^*(0)) < 0$.[23] Under this condition, a small subsidy induces the leader to invest less in information (i.e., $d\rho^*(0)/ds < 0$) when the comparative statics of the equilibrium have the same monotonicities as those of the best responses.[24]

5.2 Effects of endogeneity of seller's information on optimal policy

Let s^* denote the optimal policy when information is endogenous. Now suppose information is exogenous and equal to $\rho = \rho^*(s^*)$, where $\rho^*(s^*)$ is the equilibrium choice by L when information is endogenous and the policy is s^*. For any s, let $W^{\#}(s)$ denote welfare when the policy is s and information is exogenous and equal to $\rho^*(s^*)$. Assume that $W^{\#}(s)$ is strictly quasi-concave and then denote by s^{**} the level of the policy that maximizes $W^{\#}(s)$.

Proposition 6 (***double dividend of the subsidy***)**.** Consider the Akerlof's model of Subsection 2.2. Let s^* denote the optimal subsidy when information is endogenous. Assume that $G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$ is increasing in m, Assumption 1 holds, and $G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)) < 0$ (meaning that information aggravates adverse selection). Then, when information is exogenous and equal to $\rho^*(s^*)$, the optimal subsidy, s^{**}, satisfies $s^{**} < s^*$.

Recall that the property that $G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$ is increasing in m guarantees that Assumption 3 holds in the Akerlof's model (that is, the friendliness of the follower's reaction increases with ρ if and only if a larger ρ alleviates the adverse selection problem, i.e., it increases the truncated mean). In the context of the problem under consideration, this assumption guarantees that, starting from $\rho^*(s^*)$,

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tion becomes the following: there exists a positive scalar $K > 0$ such that a strictly positive scalar is optimal if $\frac{d}{ds}M^-(m^*(r^*(0), s); \rho^*(s))\big|_{s=0} > K$, whereas a strictly positive tax is optimal if the above inequality holds. As we discuss in the Supplement, the conclusion flips in the anti-lemon case (i.e., when Assumption 3 is replaced by Assumption 3'): in this case, there exists a negative scalar $K < 0$ such that a strictly positive subsidy is optimal if $\frac{d}{ds}M^-(m^*(r^*(0), s); \rho^*(s))\big|_{s=0} < K$, whereas a tax on engagement is optimal if $\frac{d}{ds}M^-(m^*(r^*(0), s); \rho^*(s))\big|_{s=0} > K$.

[23] Also recall that information always aggravates adverse selection, no matter whether it reduces or increases the probability of trade, when the distributions from which the mean m is drawn are Uniform, Pareto, or Exponential.

[24] Note that, holding the leader's choice of ρ fixed at $\rho^*(0)$, an increase in the subsidy, starting from $s = 0$, always increases the friendliness of the follower's reaction. Likewise, holding r fixed at $r^*(0)$, an increase in the subsidy, starting from $s = 0$, always reduces the leader's choice of ρ when $G_\rho(r^*(0); \rho^*(0)) < 0$.

a larger ρ increases $M^{-}(m^{*}(r^{*}(s^{*}), s^{*}); \rho^{*}(s^{*})))$. That information structures are consistent with the MPS order and $G_{\rho}(m^{*}(r^{*}(s^{*}), s^{*}); \rho^{*}(s^{*})) < 0$ in turn implies that, under the optimal subsidy s^{*}, information aggravates adverse selection (see Proposition 1). Because information aggravates adverse selection, starting from s^{*}, if the government were to cut the subsidy, it would trigger a larger reduction in the price offered by the buyer when information is endogenous than when it is exogenous. This is because, when information is endogenous, a smaller subsidy, by inducing the buyer to lower the price expecting the seller to engage less often, it also induces the seller to invest more in information acquisition which further aggravates adverse selection resulting in a sharper price reduction. As a result, the optimal subsidy is larger under endogenous information. Similar conclusions apply to Example (a) in the Supplement where we consider the design of government asset purchase problems. In the context of that example, the government directly controls the price at which the sellers can trade in their assets. Proposition 6 implies that, relative to the case in which information is exogenous, when information is endogenous, the government should run a more generous program, i.e., to offer a higher price.

The results above point to a general insight. When increasing trade is socially beneficial, information aggravates adverse selection, and a friendlier reaction by player F reduces the marginal benefit of acquiring more information for player L, the social value of subsidizing trade is higher when information is endogenous than when it is exogenous. This is because subsidizing trade comes with a double dividend: in addition to inducing player L to engage more often, it induces L to acquire less information which in turn alleviates adverse selection and further boosts welfare.

Table 1 in the Appendix summarizes some of the key insights, both in the lemons problem considered in this and the previous sections and in the anti-lemons case discussed in the Supplement.

6 Flexible Information Acquisition

The analysis in the previous sections assumes that the experiments player L has access to lead to distributions (over the posterior mean) consistent with the MPS order— Assumption 1. The key forces responsible for EC in Proposition 1, however, extend to situations in which player L can choose not only "how much" information to acquire but also "the nature" of the experiment, i.e., information is fully flexible. To see this, consider an arbitrary experiment $q : \Omega \to \Delta(Z)$ mapping states into probability distributions over a rich (Polish) space of signal realizations Z. Note that any such experiment, when combined with the prior G over Ω, leads to a distribution G^{q} of the posterior mean, m. Furthermore, when combined with the optimal engagement strategy (that is, the rule that, for any reaction r by player F, specifies to engage if and only $m \leq m^{*}(r)$), the experiment q leads to a stochastic choice rule $\sigma : \Omega \to [0, 1]$ specifying the probability that player L engages in each state ω.

Following the rational inattention literature, one can think of player L as choosing directly the rule $\sigma : \Omega \to [0, 1]$ subject to an appropriate specification of the cost functional $\mathcal{C}(\sigma)$, with the interpretation that, for any σ, $\mathcal{C}(\sigma)$ is the cost of the cheapest experiment $q : \Omega \to \Delta(Z)$ among those that permit L to implement the stochastic choice rule σ. A couple of cost functionals that have received special attention in the literature are those linked to "mutual information" and "maximal slope". Below we discuss both specifications and explain how our results are broadly consistent with these specifications.

Entropy cost. For any experiment q, let

$$I^q = \int_\Omega \int_Z \ln(q(z|\omega))q(\mathrm{d}z|\omega)dG(\omega) - \int_Z \ln\left(\int_\omega q(z|\omega)dG(\omega)\right)\int_\omega q(z|\omega)dG(\omega)$$

denote the mutual information between the random variables ω and z, where z is the random variable obtained by combining the prior G with the signal q. Now suppose that there exists a function $c : \mathbb{R}_+ \to \mathbb{R}_+$ such that, for any q, the cost of experiment q is given by $\mathbf{C}(q) = c(I^q)$. To facilitate the comparison with the analysis in the previous sections, assume ρ determines the easiness by which L can process information (equivalently, can reduce entropy). Specifically, assume that, for any $\rho \in \mathbb{R}_+$, player L's marginal cost of entropy reduction is $1/\rho$. To be able to process information at marginal cost $1/\rho$, player L must make an investment (e.g., train herself) whose cost is $C(\rho)$, with the function C satisfying the same assumptions as in the baseline model. The difference is that, once ρ is chosen, player L can now pick *any* experiment q of her choice, with each experiment costing her $c(I^q)/\rho$. For simplicity, one can then assume that c is the identity function (i.e., $c(I^q) = I^q$ for any q) so that the cost of each experiment q is given by the mutual information between its realizations z and the state ω (equivalently, by the reduction in entropy brought by the experiment), scaled by the (inverse) of L's choice of ρ.

Alternatively, one can let $\rho \in \mathbb{R}_+$ denote player L's "information capacity." Under this interpretation, L first purchases capacity ρ at cost $C(\rho)$, and then chooses the experiment that maximizes her expected payoff among those for which the mutual information between ω and the realization z of the selected experiment is no greater than ρ. The reason for allowing player L to choose both ρ and q is that, with flexible information, a change in the experiment expected by player F cannot, in general, be interpreted as player L acquiring "more/less" information. On the contrary, the anticipation of a larger choice of ρ can be interpreted, unambiguously, as player L "investing more in learning how to process information". This in turn facilitates the comparison with the analysis in the previous sections.

It is well known that, for any investment ρ and any anticipated reaction r by player F, the experiment $q^{\rho,r}$ that maximizes player L's expected payoff net of the above cost is binary, i.e., for any ω, it assigns positive probability only to two signal realizations. Without loss of generality, label these realizations by $z = 1$ and $z = 0$, and interpret $z = 1$ as a "recommendation to engage" and $z = 0$ as a "recommendation to not engage." Letting $q^{\rho,r}(1|\omega)$ denote the probability that signal $q^{\rho,r}$ recommends $z = 1$ when the state is ω and $q^{\rho,r}(1) \equiv \int_\omega q^{\rho,r}(1|\omega)dG(\omega)$ the total probability that player L engages under $q^{\rho,r}$, we have that, when $q^{\rho,r}(1) \in (0, 1)$, i.e., when q makes player L respond to the state, the optimal signal is given by the solution to the following functional equation (see, e.g., Woodford (2009) and Yang (2015)):[25]

$$\delta_L(r, \omega) = \frac{1}{\rho}\left[\ln\left(\frac{q^{\rho,r}(1|\omega)}{1 - q^{\rho,r}(1|\omega)}\right) - \ln\left(\frac{q^{\rho,r}(1)}{1 - q^{\rho,r}(1)}\right)\right].$$

That is, under the optimal signal, the change in the log-likelihood of player L engaging in state ω (relative to the prior) is proportional to player L's payoff differential between engaging and not engaging at state

[25] The formula is for when $1/\rho$ measures the marginal cost of entropy reduction. Conclusions similar to those reported below hold for the case where ρ determines the information capacity, i.e., the maximal level of entropy reduction, as in Sims (2003)'s original work on rational inattention (see also Mackowiak and Wiederholt (2009)).

ω, given the reaction r.

Max-slope cost. Next, consider the case in which the cost of inducing a stochastic choice rule $\sigma : \Omega \to [0,1]$ is given by $\mathcal{C}(\sigma) = c\left(\sup\left\{|\sigma'(\omega)|\right\}\right)$, where the function $c : \mathbb{R}_+ \cup \{+\infty\} \to \mathbb{R}_+ \cup \{+\infty\}$ is non-decreasing and satisfies $c(0) = 0$ and $c(k) < \infty$ for all $k \in \mathbb{R}_+$. Here $\sigma'(\omega)$ is the derivative of σ at ω. At any point of discontinuity of σ, $\sigma'(\omega) = +\infty$, whereas at any point ω at which σ is continuous but non-differentiable, $\sigma'(\omega)$ is the maximum between the left and the right derivative. Examples of this cost functional can be found in Robson (2001), Rayo and Becker (2007), Netzer (2009), and more recently Morris and Yang (2022).

Again, to facilitate the connection with the analysis in the previous sections, assume ρ is the maximal slope of player L's stochastic choice rule, selected at cost $C(\rho)$ with C satisfying the same properties as in the baseline model. Given ρ, player L then selects the experiment that maximizes her expected payoff, among those inducing a stochastic choice rule σ whose maximal slope is no greater than ρ. For any ρ and r, the optimal experiment can be taken to be binary and, when

$$\inf(supp\{G\}) \leq m^*(r) - \frac{1}{2\rho} < m^*(r) + \frac{1}{2\rho} \leq \sup(supp\{G\}),$$

for any ω, it recommends $z = 1$ (i.e., engagement) with probability $q^{\rho,r}(1|\omega)$ given by

$$q^{\rho,r}(1|\omega) = \begin{cases} 1 & \text{if} \quad \omega \leq m^*(r) - \frac{1}{2\rho} \\ \frac{1}{2} - \rho(\omega - m^*(r)) & \text{if} \quad m^*(r) - \frac{1}{2\rho} < \omega \leq m^*(r) + \frac{1}{2\rho} \\ 0 & \text{if} \quad \omega > m^*(r) + \frac{1}{2\rho} \end{cases}$$

where $m^*(r)$ is the same engagement cutoff as in the previous sections.

What distinguishes the two examples of flexible information acquisition above from the analysis in the previous sections is that, for any choice of ρ, there are multiple experiments that share the same cost $C(\rho)$ and that need not be rankable in the MPS order. After choosing ρ, player L chooses the experiment that maximizes her expected payoff, with the optimal choice depending on the anticipated reaction r by player F.

It is evident that, in each of the two cases of flexible information acquisition described above, under the optimal experiment $q^{\rho,r}$, when player L observes $z = 1$ (equivalently, when she engages), her posterior mean, which is given by

$$\mathbb{E}[\omega|z = 1; q^{\rho,r}] = \int \omega \frac{q^{\rho,r}(1|\omega)}{q^{\rho,r}(1)} dG(\omega),$$

is less than $m^*(r)$, and likewise, when she observes $z = 0$,

$$\mathbb{E}[\omega|z = 0; q^{\rho,r}] = \int \omega \frac{1 - q^{\rho,r}(1|\omega)}{1 - q^{\rho,r}(1)} dG(\omega)$$

is greater than $m^*(r)$.

For any choice $\rho^\dagger$ anticipated by F, any reaction r by player F, and any cutoff m^*, then let $M^-(m^*; \rho^\dagger, r)$ denote the expected value of m conditional on $m \leq m^*$, when player L chooses $\rho^\dagger$

and then selects the optimal experiment $q^{\rho^\dagger,r}$ anticipating a reaction r by player F. Then note that, for any $\rho^\dagger$ and r, when the cutoff is equal to $m^*(r)$,

$$M^-(m^*(r);\rho^\dagger,r) = \mathbb{E}[\omega|z=1;q^{\rho^\dagger,r}]$$

and $\partial M^-(m^*(r);\,\rho^\dagger,r)/\partial\rho^\dagger \overset{\text{sgn}}{=} A(m^*(r);\rho^\dagger,r)$, with

$$A(m^*(r);\rho^\dagger,r) \equiv \big[m^*(r) - M^-(m^*(r);\,\rho^\dagger,r)\big]G_\rho(m^*(r);\rho^\dagger,r) - \int_{-\infty}^{m^*(r)} G_\rho(m;\rho^\dagger,r)dm$$

where, for any m, $G(m;\rho^\dagger,r)$ denotes the probability that L's posterior mean is less than m under the experiment $q^{\rho^\dagger,r}$ and where $G_\rho(m;\rho^\dagger,r)$ denotes the partial derivative of such a probability with respect to ρ, evaluated at $\rho=\rho^\dagger$; such a derivative is computed accounting for the fact that, when ρ changes, the optimal experiment $q^{\rho,r}$ (which also depends on the expected reaction r) changes.

As in the baseline model, the sign of A determines whether a higher value of $\rho^\dagger$ anticipated by player F aggravates or alleviates the adverse selection problem. Consistently with the baseline model, we then continue to interpret $\mathrm{A}(\rho^\dagger) \equiv A(m^*(r(\rho^\dagger));\rho^\dagger,r(\rho^\dagger))$ as the "adverse selection effect." As in the baseline model, $r(\rho^\dagger)$ denotes the equilibrium reaction by player F in a fictitious setting in which player L's choice of ρ is exogenously fixed at $\rho^\dagger$. However, differently from the baseline model, in this fictitious setting, player L chooses the distribution $G(\cdot;\rho^\dagger,q)$ over her posterior mean m by selecting an experiment $q:\Omega\to\Delta(Z)$.[26] The equilibrium reaction $r(\rho^\dagger)$ is thus computed jointly with the equilibrium choice of experiment q and the equilibrium engagement strategy $a(\cdot)$.

Next, let

$$\Pi(\rho;r) \equiv U_L(0) + \int_{-\infty}^{m^*(r)} \delta_L(r,m)dG(m;\rho,r) = U_L(0) + \int_{-\infty}^{+\infty} \delta_L(r,\omega)q^{\rho,r}(1|\omega)dG(\omega)$$

denote the payoff, gross of the cost, that player L obtains by choosing ρ when expecting a reaction r by player F (with the expectation computed under the optimal experiment $q^{\rho,r}$) and then let

$$\begin{aligned} B(\rho;\rho^\dagger) \;&\equiv\; -\frac{\partial^2\Pi(\rho;r(\rho^\dagger))}{\partial\rho\partial r} = -\int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial\delta_L(r(\rho^\dagger),m)}{\partial r}dG_\rho(m;\rho,r(\rho^\dagger)) \\ &=\; -\frac{\partial\delta_L(r(\rho^\dagger),m^*(r(\rho^\dagger)))}{\partial r}G_\rho\Big(m^*(r(\rho^\dagger));\rho,r(\rho^\dagger)\Big) \\ &\quad\; + \int_{-\infty}^{m^*(r(\rho^\dagger))} \frac{\partial^2\delta_L(r(\rho^\dagger),m)}{\partial r\partial m}G_\rho(m;\rho,r(\rho^\dagger))dm. \end{aligned}$$

As in the baseline model, the function $B(\rho;\rho^\dagger)$ measures how a reduction in the friendliness of F's reaction around $r(\rho^\dagger)$ affects L's marginal benefit of expanding her investment in information processing, starting from ρ. Consistently with the baseline model, we will continue to refer to $B(\rho;\rho^\dagger)$ as the "benefit of friendlier reactions" effect.

[26]Recall that $\rho^\dagger$ only pins down the marginal cost of entropy reduction (alternatively, the maximal level of entropy reduction) or the maximal slope of the induced stochastic choice rule, leaving player L with flexibility over her choice of experiment $q:\Omega\to\Delta(Z)$.

The following proposition establishes the precise sense in which results analogous to those in Proposition 1 extend to a setting in which information is flexible and the cost of information is determined by either entropy reduction or the maximum slope of the induced stochastic choice rule.

Proposition 7 (***EC under flexible information acquisition***). *Suppose that Assumptions 2 and 3 hold and that ρ determines either the marginal cost of entropy reduction or the maximum-slope of the induced stochastic choice rule.*
(i) EC holds at $(\rho, \rho^\dagger)$ if and only if the adverse selection and the benefit of friendlier reactions effects are of opposite sign: $\mathrm{A}(\rho^\dagger)\mathrm{B}(\rho; \rho^\dagger) < 0$.
(ii) A sufficient condition for an increase in ρ to aggravate adverse selection at $\rho = \rho^\dagger$ (i.e., for $\mathrm{A}(\rho^\dagger) < 0$*) is that, $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$ at $\rho = \rho^\dagger$.*
(iii) A sufficient condition for a reduction in the friendliness of F's reaction at $r(\rho^\dagger)$ to raise L's marginal value of ρ (i.e., for $B(\rho; \rho^\dagger) > 0$) is that, in addition to $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ be increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, the total probability $q^{\rho,r(\rho^\dagger)}(1) \equiv \int q^{\rho,r(\rho^\dagger)}(1|\omega)dG(\omega)$ player L engages is non-increasing in ρ.
(iv) Therefore, a sufficient condition for EC to hold at $(\rho, \rho^\dagger)$ is that the conditions in parts (ii) and (iii) jointly hold.
(v) Suppose that $M^-(m^*(r(\rho^\dagger)); \rho, r(\rho^\dagger))$ is decreasing in ρ at $\rho = \rho^\dagger$, implying that $\mathrm{A}(\rho^\dagger) < 0$*, and that $\partial^2\delta_L(r, m)/\partial r\partial m = 0$. Then $q^{\rho,r(\rho^\dagger)}(1)$ non-increasing in ρ at $\rho = \rho^\dagger$ is necessary and sufficient for EC at $(\rho, \rho^\dagger)$.*

As in the baseline model, EC obtains when, and only when, the adverse-selection effect is of opposite sign than the benefit of friendlier reactions effect, i.e., when $\mathrm{A}(\rho^\dagger)B(\rho; \rho^\dagger) < 0$. The intuition is the same as in the baseline model.

When information is flexible, an increase in ρ (starting from $\rho^\dagger$) aggravates the severity of the adverse selection problem when it induces L to select an experiment that makes her engage with a higher probability at low states (namely for $\omega < m^*(r(\rho^\dagger))$) and with a lower probability at high states (namely for $\omega > m^*(r(\rho^\dagger))$), relative to the total probability $q^{\rho,r(\rho^\dagger)}(1)$ of engaging. This is because, in the eyes of player F, such changes make the engagement decision by player L a more informative signal of the state being less favorable to player F. When, in addition to the last property described, a higher ρ also reduces the overall probability $q^{\rho,r(\rho^\dagger)}(1)$ that player L engages, starting from the actual level ρ selected by player L, a reduction in the friendliness of player F's reaction (starting from $r(\rho^\dagger)$) increases L's marginal value of expanding ρ. The property that $q^{\rho,r(\rho^\dagger)}(1)$ decreases with ρ is equivalent to the property in the baseline model that a higher ρ reduces the probability of trade (i.e., $G_\rho\Big(m^*\big(r(\rho^\dagger)\big); \rho\Big) < 0$). This condition is both necessary and sufficient for EC when L's payoff is separable in r and ω (as in the Akerlof's model) and a higher ρ always aggravates the adverse selection problem in the eyes of player F. The results above are the analogs of those established in Proposition 1 for the case where ρ is a mean-preserving-spread index, thus establishing the robustness of the key insights to the flexible information structures considered in this section.

6.1 Akerlof with Entropy Cost

In this subsection, we specialize the analysis to the Akerlof model introduced above and assume that the cost of any signal $q : \Omega \to \Delta(Z)$ is given by I^q/ρ, that is, it is proportional to its entropy reduction. As explained above, $\rho \in [0, +\infty)$ parametrizes the seller's ability to process information. We first analyze the equilibrium of the game with exogenous ρ (inner game) and then endogenize ρ (outer game). After showing that the game with endogenous ρ typically admits multiple equilibria, we investigate whether the conditions for expectation conformity of Proposition 7 hold. For simplicity, we assume that the prior G from which ω is drawn is uniform over $[0, 1]$.

6.1.1 Inner game

We start by fixing the marginal cost ρ. As anticipated above, in solving for the seller's optimal signal, we can restrict attention to "action recommendations," that is, binary signals that recommend to engage (i.e., to sell) with probability $q(1|\omega)$ when the state is ω. Given any offer r by the buyer, the seller's payoff under any such a signal is equal to

$$\int_\omega (r - \omega) q(1|\omega) dG(\omega) + \omega_0 - \frac{I^q}{\rho}, \tag{10}$$

where

$$I^q = \int_\omega \phi(q(1|\omega)) dG(\omega) - \phi(q(1)),$$

with

$$q(1) \equiv \int_\omega q(1|\omega) dG(\omega)$$

and where ϕ is the function defined by $\phi(0) = \phi(1) \equiv 0$ and, for any $q \in (0, 1)$,

$$\phi(q) \equiv q \ln(q) + (1 - q) \ln(1 - q).$$

Let $\underline{r}(\rho)$ and $\overline{r}(\rho)$ be implicitly defined by, respectively, the following two equations

$$\int_\omega e^{-\rho\omega} dG(\omega) = e^{-\rho\underline{r}} \quad \text{and} \quad \int_\omega e^{\rho\omega} dG(\omega) = e^{\rho\overline{r}}$$

and note that $\omega_0 \in (\underline{r}(\rho), \overline{r}(\rho))$. Next, for any $r \in (\underline{r}(\rho), \overline{r}(\rho))$, let $\tilde{\omega}(r; \rho)$ be implicitly defined by

$$\tilde{\omega} = r + \frac{1}{\rho} \ln \left(\frac{\int_\omega \frac{1}{1+e^{\rho(\omega - \tilde{\omega})}} dG(\omega)}{1 - \int_\omega \frac{1}{1+e^{\rho(\omega - \tilde{\omega})}} dG(\omega)} \right). \tag{11}$$

Following Woodford (2008) and Yang (2015), one can show that, for any r, the seller's optimal signal when the marginal cost of entropy reduction is $1/\rho$ is such that, for all ω,

$$q^{\rho,r}(1|\omega) = \begin{cases} 0 & \text{if } r \leq \underline{r}(\rho) \\ \frac{1}{1+e^{\rho(\omega-\tilde{\omega}(r;\rho))}} & \text{if } r \in (\underline{r}(\rho), \overline{r}(\rho)) \\ 1 & \text{if } r \geq \overline{r}(\rho). \end{cases} \tag{12}$$

Intuitively, when $r \leq \underline{r}(\rho)$ (alternatively, $r \geq \overline{r}(\rho)$) the seller expects the realized value of ω to be greater (alternatively, smaller) than r with very high probability. She then finds it optimal not to acquire any information and refrain from selling (alternatively, sell) with probability one. It is only for intermediate values of r, namely for $r \in (\underline{r}(\rho), \overline{r}(\rho))$, that the seller acquires information. When this is the case, the optimal signal is a logistic function indexed by the position parameter $\tilde{\omega}(r;\rho)$. Figure 2 below illustrates the shape of the logistic function $q^{\rho,r}(1|\omega)$, when $r \in (\underline{r}(\rho), \overline{r}(\rho))$.

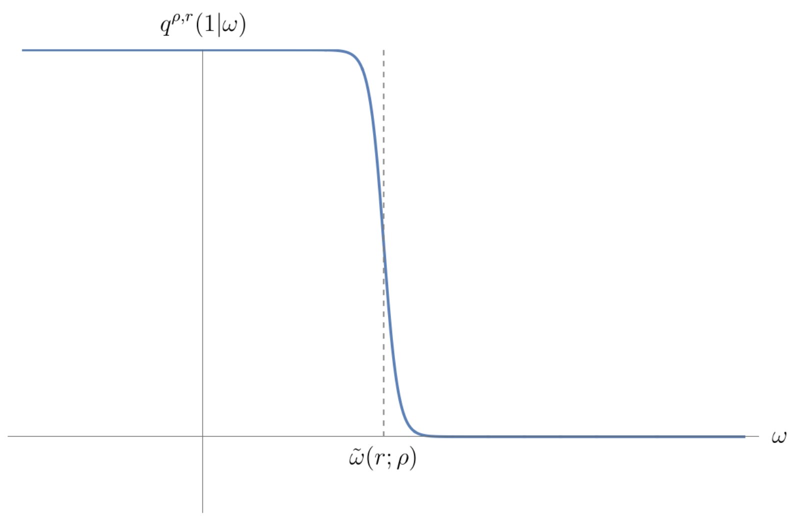

Figure 2: Shape of the optimal signal when the latter is interior.

An important implication of (12) is that, when the seller acquires information, the optimal signal depends on the buyer's price r *only* through the location parameter $\tilde{\omega}(r;\rho)$. We use this property below to conveniently characterize the interior equilibrium of the inner game.

Because the buyer is a representative of a competitive market, for any binary experiment q by the seller followed by an engagement strategy prescribing to sell if $z=1$ and not to sell if $z=0$, the price offered by the buyer is equal to

$$r = \mathbb{E}[\omega|z=1;q] + \Delta = \int_{\omega} \omega \frac{q(1|\omega)}{q(1)} dG(\omega) + \Delta$$

Using the characterization of the seller's optimal signal above, we thus have that when, in equilibrium, the seller acquires information, the equilibrium of the inner game is given by a pair $(\tilde{\omega}, r)$ that solves the following conditions

$$\begin{cases} \tilde{\omega} = r + \frac{1}{\rho} \ln \left(\frac{\int_\omega \frac{1}{1+e^{\rho(\omega-\tilde{\omega})}} dG(\omega)}{1-\int_\omega \frac{1}{1+e^{\rho(\omega-\tilde{\omega})}} dG(\omega)} \right) \\ r = \int_\omega \omega \frac{\frac{1}{1+e^{\rho(\omega-\tilde{\omega})}}}{\int_\omega \frac{1}{1+e^{\rho(\omega-\tilde{\omega})}} dG(\omega)} dG(\omega) + \Delta \\ r \in (\underline{r}(\rho), \overline{r}(\rho)) \end{cases} \tag{13}$$

The first equation represents how the seller's optimal signal depends on the (anticipated) price offered by the buyer, r. The second equation is the buyer's break-even condition, when expecting the seller to choose the logistic binary signal q indexed by the position parameter $\tilde{\omega}$ and engage if and only if $z = 1$. The last condition is a (necessary and sufficient) condition guaranteeing that the seller prefers to acquire some information to either holding the good or selling it without learning the state.

Figure 3 illustrates the solution to the above system of equations when $\omega \sim U[0, 1]$, $\rho = 8$, and $\Delta = 0.15$. The shaded light blue area are pairs $(\tilde{\omega}, r)$ for which $r \in (\underline{r}(\rho), \overline{r}(\rho))$. The solid blue curve represents the combination of $\tilde{\omega}$ and r solving the first equation in (13). This curve thus describes the seller's reaction (when the latter takes the form of an interior signal). The red solid line represents the combination of $\tilde{\omega}$ and r solving the second equation in (13), which is the buyer's break-even condition.

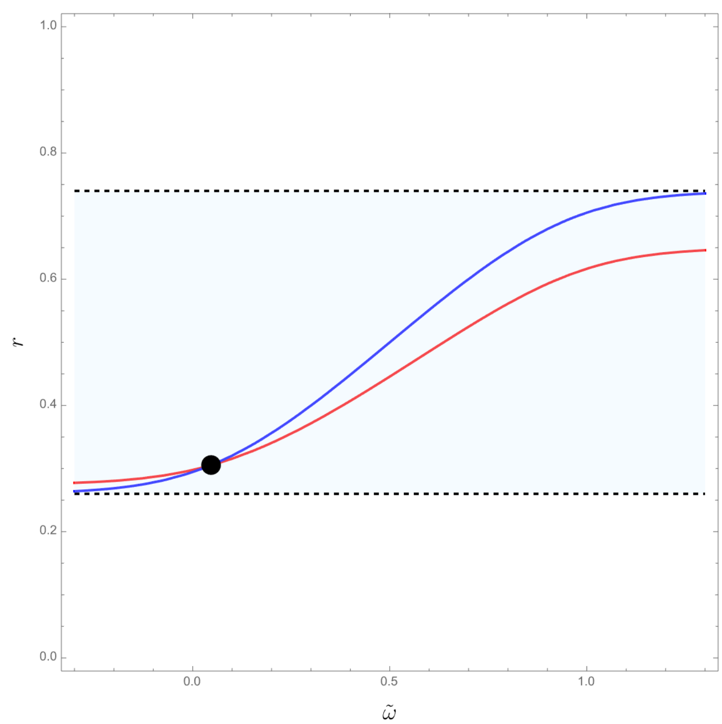

Figure 3: Solution of system (13) for $\omega \sim U[0, 1]$, $\rho = 8$ and $\Delta = 0.15$: $r^* \approx 0.305$, $\tilde{\omega}^* \approx 0.046$. The blue curve is the seller's reaction, the red curve the buyer's break-even condition.

Numerical simulations show that, when the gains from trade Δ are larger, the buyer's break-even curve is higher, implying a larger r^* and $\tilde{\omega}^*$. Moreover, in this example, when the seller's marginal cost of entropy reduction is larger (i.e., when ρ is smaller), the equilibrium price is larger: The buyer responds to the seller being a worse learner by offering a higher price.

The interaction between the buyer and the seller can also result in equilibria in which the seller does not acquire any information and then either never engages or always engages.

Consider first equilibria in which $r \geq \overline{r}(\rho)$. In this case, the seller always engages, that is, sells with certainty, implying that there is no adverse selection. The price offered by the buyer is then $r = \omega_0 + \Delta$. For such a price to be sustained in equilibrium, it must be that $\Delta \geq \overline{r}(\rho) - \omega_0$. If, instead, $\Delta < \overline{r}(\rho) - \omega_0$, there is no equilibrium in which the seller refrains from acquiring information and sells with certainty.

Next, consider the case in which $r \leq \underline{r}(\rho)$. In this case, the seller never engages. Whether such a behavior can be sustained in equilibrium depends on how the buyer responds in case the seller deviates and puts the asset on sale. Suppose that the buyer responds by offering a price $r = \Delta$, reflecting the belief that the seller acquired information and learned that the state is $\omega = 0$. As long as $\Delta \leq \underline{r}(\rho)$, it is an equilibrium for the seller not to acquire any information and never to engage. Note that, because the signal that recommends to sell only when the asset's value is the lowest (here $\omega = 0$) is costless, such an equilibrium survives most forward-induction refinements.

Figure 4 describes how the equilibrium price in the inner game changes with the (inverse of) the seller's marginal cost of entropy reduction, ρ. For low levels of ρ, there is no interior equilibrium with information acquisition, whereas the two corner equilibria with no information acquisition coexist. As ρ becomes larger (i.e., as the seller becomes a better learner), the corner equilibrium in which the seller trades with certainty disappears and is replaced by the interior equilibrium in which the seller acquires some information.

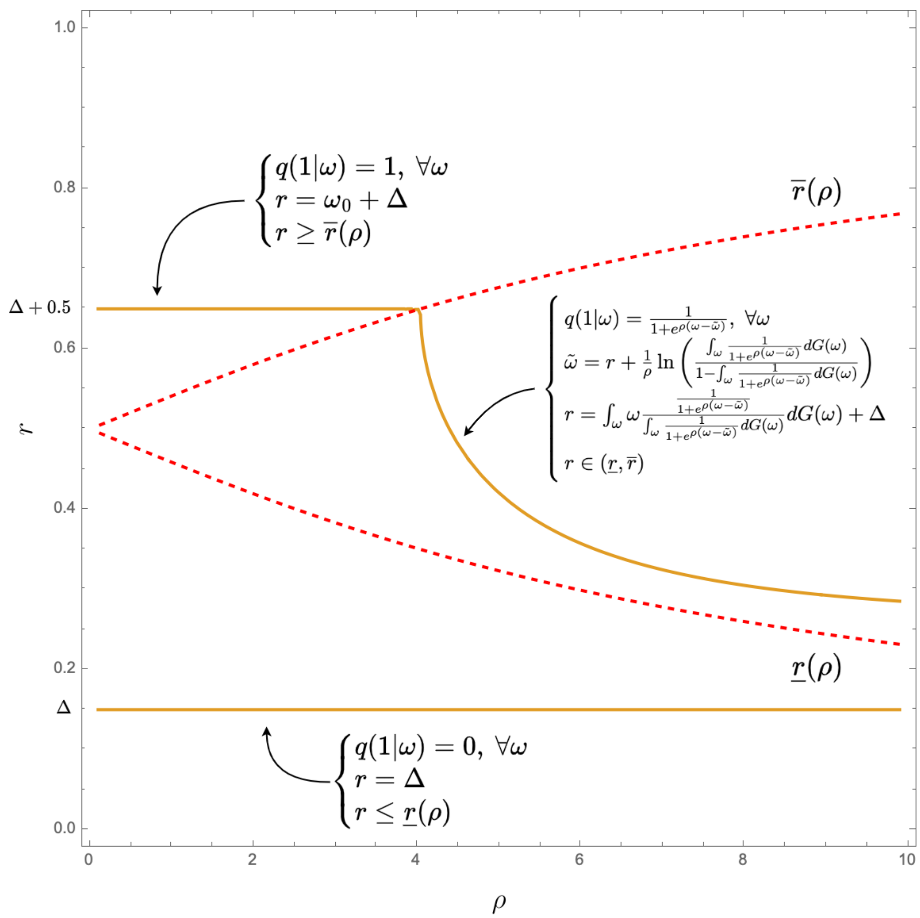

Figure 4: Equilibrium price correspondence (orange solid curve) when $\omega \sim U[0,1]$ and $\Delta = 0.15$, for different values of ρ. The dashed red curves correspond to the minimum and maximum prices compatible with interior solutions, $\underline{r}(\rho)$ and $\overline{r}(\rho)$.

6.1.2 Outer problem

We now turn to the outer game in which the seller first trains herself, i.e., invests in learning how to process information (formally captured by a choice of ρ), and then chooses the experiment of her choice. Recall that the seller's payoff when she covertly chooses ρ, the buyer offers r, and the seller selects an arbitrary (binary) signal q and then engages for $z = 1$ and does not engage for $z = 0$ is equal to

$$\Pi(r, q; \rho) \equiv \int_\omega (r - \omega) q(1|\omega) dG(\omega) + \omega_0 - \frac{I^q}{\rho} - C(\rho) \tag{14}$$

Then let $\Pi^*(r, \rho) \equiv \Pi(r, q^{\rho,r}; \rho)$ denote the seller's maximal payoff when she responds to the buyer offering a price of r with a choice of ρ, where $q^{\rho,r}$ is the optimal signal given (ρ, r), as characterized above. We use the Envelope Theorem to describe the seller's marginal value of investing in becoming a better learner (formally, in expanding ρ). Because $\Pi(r, q; \rho)$ is not Lipschitz continuous in ρ across all possible $(r, q; \rho)$, a little care is needed in establishing the result, which we provide in the following lemma.

Lemma 1. *For any r, there exists $\overline{\rho}(r) > 0$ such that, for any $\rho > \overline{\rho}(r)$, any q, $\Pi(r, q; \rho) \leq \omega_0$, whereas,* *for any* $\rho \leq \overline{\rho}(r)$*,* $\Pi^*(r,\rho)$ *is absolutely continuous in* ρ *with*

$$\frac{\partial \Pi^*(r,\rho)}{\partial \rho} = \frac{\partial \Pi(r, q^{\rho,r}; \rho)}{\partial \rho} = \frac{I^{q^{\rho,r}}}{\rho^2} - C'(\rho). \tag{15}$$

Using the result in the lemma, we thus have that, for any r for which the optimal choice of ρ is interior, the following optimality condition must hold

$$C'(\rho) = \frac{I^{q^{\rho,r}}}{\rho^2}.$$

This additional condition, when combined with the system of equations in (13) thus identifies the equilibrium in the full game with endogenous ρ. As an illustration, suppose that ω is drawn from a uniform distribution over $[0,1]$, $\Delta = 0.15$, and $C(\rho)$ is given by

$$C(\rho) = \begin{cases} \frac{a\rho^2}{2K} & \text{if} \quad \rho \leq 10 \\ +\infty & \quad \text{otherwise} \end{cases} \tag{16}$$

with $a \approx 1.46$ and $K = 1,000$. One can show that the full game admits a unique equilibrium with information acquisition and, in such an equilibrium, $(\rho^*, r^*) \approx (4.7, 0.45)$.

Such an interior equilibrium coexists with a corner equilibrium $(\rho_A, r_A) = (0, 0.65)$ in which the seller does not invest in learning how to process information ($\rho_A = 0$), acquires no information, and then sells with certainty at a price $r_A = \omega_0 + \Delta = 0.65$, and another corner equilibrium $(\rho_N, r_N) = (0, 0.15)$ in which the seller does not invest in learning how to process information ($\rho_N = 0$), acquires no information, and there is no trade (in such an equilibrium, if the seller were to deviate and put the asset on sale, the buyer would offer $r_N = \Delta = 0.15$, with such a price offer supported by the belief that the seller learnt that the state is $\omega = 0$). Because the buyer breaks even in each equilibrium, the three equilibria can be Pareto ranked, with the highest welfare attained in the equilibrium with full trade, and the lowest in the equilibrium with no trade—welfare in the equilibrium in which the seller invests in learning how to process information is in between the level of welfare in the two corner equilibria.

The game thus features a form of *expectation traps* between the two equilibria in which trade occurs with positive probability. When the buyer expects the seller to choose $\rho_A = 0$ and trade without acquiring information, she responds with a high price $r = r_A$ that gives a lot of surplus to the seller and induces the latter not to acquire any information. When, instead, the buyer expects the seller to invest ρ^* and then acquire information and trade selectively as a function of ω, she lowers her price to r^*, which forces the seller to acquire information and leaves her with a lower payoff.

6.1.3 Expectation conformity

We conclude by investigating whether expectation conformity holds under this model specification. We do so by using Proposition 7 above.

Recall that Proposition 7 assumes that Assumption 3 holds, which requires that, in the inner game, higher values of ρ are associated with higher equilibrium prices $r(\rho)$ if and only if higher values of ρ

lead to an increase in the truncated mean:

$$\left.\frac{dr(\rho)}{d\rho}\right|_{\rho=\rho^\dagger} \overset{sgn}{=} \frac{\partial}{\partial\rho} M^-(m^*(r(\rho^\dagger));\rho, r(\rho^\dagger))\Big|_{\rho=\rho^\dagger}$$
$$= \frac{\partial}{\partial\rho}\mathbb{E}[\omega|z=1; q^{\rho,r(\rho^\dagger)}]\Big|_{\rho=\rho^\dagger}$$
$$= \frac{\partial}{\partial\rho}\int_\omega \omega \frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)} dG(\omega)\Big|_{\rho=\rho^\dagger}.$$

Under flexible information, for any $\rho > 0$, the equilibrium price $r(\rho)$ is implicitly defined by the condition

$$r = \int_\omega \omega \frac{q^{\rho,r}(1|\omega)}{q^{\rho,r}(1)} dG(\omega) + \Delta. \tag{17}$$

Suppose again that ω is drawn from a uniform distribution over $[0,1]$ and that $\Delta = 0.15$. One can then show that Assumption 3 holds for example when $\rho^\dagger = 4.2, 4.3, 4.5, 5$. Starting from these levels of ρ, a local increase in ρ leads to the choice of experiments resulting in a smaller truncated mean and to a reduction in the equilibrium price in the inner game. We can then use Proposition 7 to verify whether expectation conformity holds. We have already established that $\mathrm{A}(\rho^\dagger) < 0$ for $\rho^\dagger = 4.2, 4.3, 4.5, 5$. Thus consider the function $B(\rho;\rho^\dagger)$. Fix some $\rho^\dagger$ and recall that

$$V^*(r,\rho) \equiv \int_\omega (r-\omega) q^{\rho,r}(1|\omega) dG(\omega) + \omega_0 - \frac{I^{q^{\rho,r}}}{\rho}$$

is the seller's gross payoff under the optimal signal $q^{\rho,r}$. Using the envelope theorem

$$\frac{\partial V^*(r,\rho)}{\partial\rho} = \frac{I^{q^{\rho,r}}}{\rho^2}.$$

Recall that $B(\rho^\dagger;\rho^\dagger)$ measures how the marginal value of information $\partial V^*(r,\rho)/\partial\rho$ changes with r around $r(\rho^\dagger)$, when $\rho = \rho^\dagger$. Numerical results indicated that, in this example, $B(\rho^\dagger;\rho^\dagger) > 0$ when $r(\rho^\dagger) > \omega_0 = 0.5$, whereas $B(\rho^\dagger;\rho^\dagger) < 0$ when $r(\rho^\dagger) < \omega_0$. In other words, when $r(\rho^\dagger) > \omega_0$, a local reduction in r around $r(\rho^\dagger)$ increases the marginal value of ρ around $\rho^\dagger$, whereas the opposite is true when $r(\rho^\dagger) < \omega_0$. These results indicate that the marginal value of information $\partial V^*(r,\rho)/\partial\rho$ is single-peaked at $r = \omega_0$, as illustrated in Figure 5.

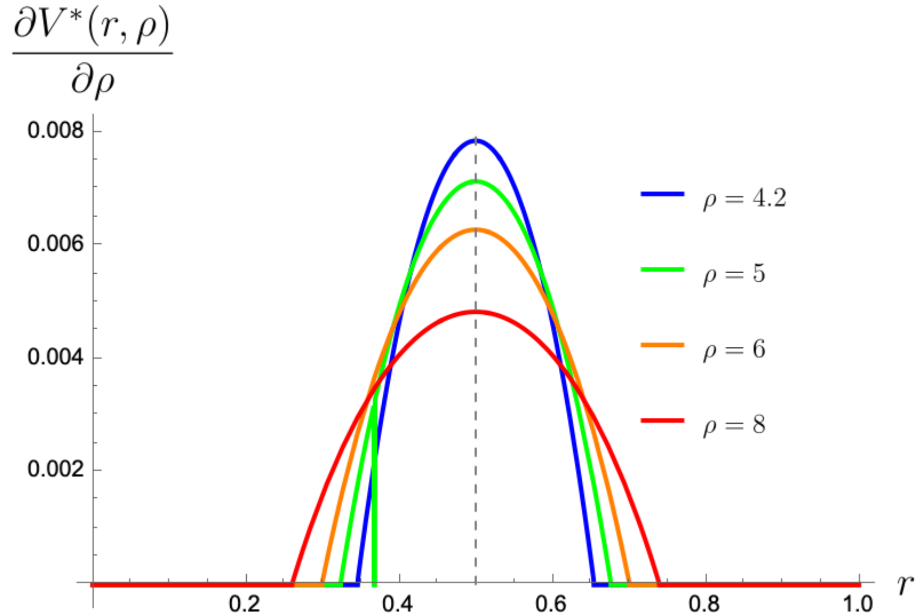

Figure 5: Effects of changes in r on $\partial V^*(r,\rho)/\partial\rho$ for different values of ρ, when $\omega \sim U[0,1]$.

Recall that Part (i) of Proposition 7 establishes that expectation conformity holds at $\rho = \rho^\dagger$ if $\mathrm{A}(\rho^\dagger)$ and $B(\rho^\dagger;\rho^\dagger)$ have opposite signs. In this example, this happens when $\rho = \rho^\dagger = 4.2$ and $\rho = \rho^\dagger = 4.3$ but not when $\rho = \rho^\dagger = 4.5$ and $\rho = \rho^\dagger = 5$.

Part (ii) of Proposition 7 in turn establishes that a sufficient condition for an increase in ρ to aggravate adverse selection at $\rho = \rho^\dagger$ (that is, for $\mathrm{A}(\rho^\dagger) < 0$) is that $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is increasing in ρ for $\omega < r(\rho^\dagger)$ and decreasing in ρ for $\omega > r(\rho^\dagger)$ at $\rho = \rho^\dagger$. These properties hold in the example under consideration for $\rho^\dagger = 4.2, 4.3, 4.5, 5$.

Finally, Part (iii) of Proposition 7 establishes that a sufficient condition for a reduction in r around $r(\rho^\dagger)$ to raise L's marginal value of $\rho = \rho^\dagger$ (i.e., for $B(\rho^\dagger;\rho^\dagger) > 0$) is that, in addition to $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ to be increasing in ρ at $r = \rho^\dagger$ for $\omega < r(\rho^\dagger)$ and decreasing in ρ at $\rho = \rho^\dagger$ for $\omega > r(\rho^\dagger)$, the total probability $q^{\rho,r(\rho^\dagger)}(1) \equiv \int q^{\rho,r(\rho^\dagger)}(1|\omega)dG(\omega)$ the seller puts the asset on sale is non-increasing in ρ at $\rho = \rho^\dagger$. The numerical simulations show that, in the example under consideration, the total probability $q^{\rho,r}(1)$ the seller puts the asset on sale is increasing in ρ when $r < \omega_0$ and decreasing in ρ when $r > \omega_0$. Part (iii) of Proposition 9 then implies that, when $\rho^\dagger$ is such that $r(\rho^\dagger) > \omega_0$, $B(\rho^\dagger;\rho^\dagger) > 0$. When, instead, $\rho^\dagger$ is such that $r(\rho^\dagger) < \omega_0$, because part (iii) of Proposition 9 provides only sufficient conditions for $B(\rho^\dagger;\rho^\dagger) > 0$, we cannot conclude from the above monotonicity that $B(\rho^\dagger;\rho^\dagger) < 0$ at $\rho = \rho^\dagger$. However, because in the Akerlof model $\partial^2\delta_L(r,m)/\partial r\partial m = 0$, and because $M^-(m^*(r(\rho^\dagger));\rho,r(\rho^\dagger))$ is decreasing in ρ at $\rho = \rho^\dagger = 4.5, 5$ (as shown above), part (v) in Proposition 7 implies that, when $\rho^\dagger = 4.5, 5$, because $\mathrm{A}(\rho^\dagger) < 0$ and $q^{\rho,r}(1)$ is increasing in ρ at $(\rho, r) = (\rho^\dagger, r(\rho^\dagger))$, expectation conformity does not obtain at $(\rho,\rho^\dagger) = (4.5, 4.5)$ and $(\rho,\rho^\dagger) = (5,5)$.

7 Conclusions

We investigate properties of generalized lemons (and anti-lemons) problems in which the information the player who engages possesses at the time of engagement is endogenous. We show how expectation conformity, i.e., the value to conform to the expectations held by other players, is affected by (a) the impact of information on the severity of the adverse selection problem, (b) the sensitivity of the marginal value of information to the friendliness of other players' reactions, and (c) the overall value of engagement, as captured by the size of the gains from trade. We then use the characterization to shed light on the connection between expectation conformity and the multiplicity of equilibria, the possibility of expectation traps (whereby the information acquiring player may be worse off in a high-information-intensity equilibrium than in a low-information-intensity one), and on the role of disclosure of hard information in such games (whereby players engage in activities that prove how well or poorly informed they are). We then use the model to investigate how a benevolent government can improve upon the laissez-faire equilibrium by subsidizing (alternatively, taxing) trade and identify conditions under which the endogeneity of information calls for more general programs due to the fact that such program, by disincentivizing the acquisition of information, they further boost trade and thus welfare. Finally, we show how the results can accommodate for flexible information acquisition.

There are many venues for future research. For example, in more applied work geared at understanding the role of endogenous information in financial trading, it would be interesting to study how public disclosures by benevolent governments impact the incentives for private information acquisition. In the context of stress testing, the announcement that a bank failed a test may induce a conservative response by potential asset buyers which may induce asset owners to collect more information, which in turn aggravates the severity of the adverse selection problem. To the best of our knowledge, this dimension has not been accounted for in the design of optimal stress tests. It would also be interesting to extend the analysis in the present paper by allowing both sides of the market to acquire information and investigate how strategic complementarity/substitutability in information acquisition is affected by the adverse selection problem.

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8 Appendix

Proof of Proposition 1 (i) By the chain rule and the definitions of the V_L and B functions, we have that

$$\frac{\partial^2 V_L(\rho;\rho^\dagger)}{\partial\rho\partial\rho^\dagger} = -B(\rho;\rho^\dagger)\frac{dr(\rho^\dagger)}{d\rho^\dagger}.$$

Assumption 3, together with Conditions (5), (6), and (7) imply that $dr(\rho^\dagger)/d\rho^\dagger$ is of the same sign as $\mathrm{A}(\rho^\dagger)$. EC thus holds at $(\rho,\rho^\dagger)$ if, and only if, $\mathrm{A}(\rho^\dagger)$ and $B(\rho;\rho^\dagger)$ are of opposite sign.

(ii) Using Condition (5), we have that, for any $m^* \in \mathbb{R}$ and $\rho^\dagger$, the sign of $\partial M^-(m^*;\rho^\dagger)/\partial\rho^\dagger$ is given by the sign of $A(m^*;\rho^\dagger)$, with $A(m^*;\rho^\dagger)$ as defined in (6). Because a higher ρ indexes a mean-preserving spread, the second term of (6) is always negative. Hence, starting from $\rho^\dagger$, information always aggravates adverse selection (that is, $\mathrm{A}(\rho^\dagger) < 0$) when the first term of (6) is also negative, which is the case when $G_\rho(m^*(r(\rho^\dagger));\rho^\dagger) < 0$. Note, however, that this condition is sufficient but not necessary for $\mathrm{A}(\rho^\dagger) < 0$. For a number of distributions, $\partial M^-(m^*(r(\rho^\dagger));\rho^\dagger)/\partial\rho^\dagger < 0$ regardless of the sign of $G_\rho(m^*(r(\rho^\dagger));\rho^\dagger)$. These distributions include the Uniform, Pareto, and Exponential distributions, as shown below.

- *Uniform distribution*: m is drawn uniformly from $[\underline{m}(\rho),\bar{m}(\rho)]$, with $\underline{m}(\rho)$ decreasing in ρ and satisfying $\underline{m}(\rho) \le \omega_0$ for all ρ, and $\bar{m}(\rho) = 2\omega_0 - \underline{m}(\rho)$ for all ρ (mean preservation). Then for any $m \in [\underline{m}(\rho),\bar{m}(\rho)]$, $G(m;\rho) = (m-\underline{m}(\rho))/[2(\omega_0-\underline{m}(\rho))]$. This family of distributions is thus consistent with the rotation order of Definition 1, with rotation point $m_\rho = \omega_0$ for all ρ. Furthermore, for any $m^* \in [\underline{m}(\rho),\bar{m}(\rho)]$,

$$M^-(m^*;\rho) = \frac{m^*+\underline{m}(\rho)}{2}$$

  which is decreasing in ρ.

- *Pareto distribution*: m is drawn from $[\underline{m}(\rho),+\infty)$ according to the survival function $1-G(m;\rho) = (\underline{m}(\rho)/m)^{\alpha(\rho)}$, with $\underline{m}(\rho)$ decreasing in ρ and $\alpha(\rho) = \omega_0/(\omega_0-\underline{m}(\rho))$ for all ρ.[27] This family of distributions too is consistent with the rotation order of Definition 1. For each ρ, the rotation point is $m_\rho = \underline{m}(\rho)\exp\left((\omega_0-\underline{m}(\rho))/\underline{m}(\rho)\right)$. Furthermore, for any $m^* > \underline{m}(\rho)$,

$$M^-(m^*;\rho) = \omega_0\frac{1-\left(\frac{\underline{m}(\rho)}{m^*}\right)^{\alpha(\rho)-1}}{1-\left(\frac{\underline{m}(\rho)}{m^*}\right)^{\alpha(\rho)}}$$

  which is decreasing in ρ.

- *Exponential distribution*: m is drawn from $[\underline{m}(\rho),+\infty)$ according to the survival function $1-$

[27] Note that the function $\alpha(\rho)$ is constructed so that, for any ρ, given $\underline{m}(\rho)$, $\mathbb{E}_{G(\cdot;\rho)}[m;\rho] = \frac{\alpha(\rho)\underline{m}(\rho)}{\alpha(\rho)-1} = \omega_0$ (mean preservation).

$G(m;\rho) = e^{-\lambda(\rho)(m-\underline{m}(\rho))}$, with $\underline{m}(\rho)$ decreasing in ρ and $\lambda(\rho) = 1/(\omega_0 - \underline{m}(\rho))$ for all ρ.[28] One can verify that an increase in ρ induces a rotation of $G(m;\rho)$ in the sense of Definition 1, with rotation point $m_\rho = \omega_0$ for all ρ. Furthermore, for any $m^* > \underline{m}(\rho)$,

$$M^-(m^*;\rho) = \omega_0 - \frac{(m^* - \underline{m}(\rho))\, e^{-\lambda(\rho)(m^*-\underline{m}(\rho))}}{1 - e^{-\lambda(\rho)(m^*-\underline{m}(\rho))}}$$

which is decreasing in ρ.

(iii) Recall that, starting from $r = r(\rho^\dagger)$, a reduction in the friendliness of F's reaction raises the incentive to acquire more information at ρ if and only if $B(\rho;\rho^\dagger) > 0$, with $B(\rho;\rho^\dagger)$ satisfying Condition (8). Note that the second term in the right-hand side of (8) is positive because, by Assumption 1, ρ is a mean-preserving-spread index and $\partial^2\delta_L/\partial r\partial m$ is positive (by Assumption 2) and constant in m (by the assumption that δ_L is affine in m). Because δ_L is increasing in r by Assumption 2, the first term in the right-hand-side of (8) is positive provided that $G_\rho\Big(m^*\big(r(\rho^\dagger)\big);\rho\Big) < 0$. Hence, starting from $r = r(\rho^\dagger)$, a reduction in the friendliness of F's reaction raises the incentive to acquire more information at ρ if $G_\rho\Big(m^*\big(r(\rho^\dagger)\big);\rho\Big) < 0$.

(iv) The result follow from parts (i)-(iii) in the proposition.

(v) The result follows from parts (i)-(iii) in the proposition, along with the fact that, in this case, the second term in the right-hand-side of (8) is zero. Because δ_L is increasing in r, we thus have that

$$B(\rho;\rho^\dagger) \overset{\text{sgn}}{=} -G_\rho\Big(m^*\big(r(\rho^\dagger)\big);\rho\Big).$$

Hence, $B(\rho;\rho^\dagger) > 0$ if and only if $G_\rho\Big(m^*\big(r(\rho^\dagger)\big);\rho\Big) < 0$. ■

Proof of Proposition 2. For any (r,θ), let $m^*(r;\theta)$ denote the optimal cut-off below which player L engages when the gains from engagement are parametrized by θ and F's reaction is r. For any $(\rho^\dagger,\theta)$, then let $r(\rho^\dagger;\theta)$ denote player F's response when player L's information is exogenously fixed at $\rho^\dagger$ and the gains from engagement are parametrized by θ. Observe that, under Assumption 2, given r, the engagement threshold $m^*(r;\theta)$, which is implicitly defined by the solution to $\bar{\delta}_L(r,m)+\theta = 0$, is strictly increasing in θ. Also observe that, under Assumption 3, given $\rho^\dagger$, $r(\rho^\dagger;\theta)$ is increasing in θ; this is because, fixing r and $\rho^\dagger$, a higher θ implies a higher engagement point $m^*(r;\theta)$, and hence a higher truncated mean $M^-(m^*(r;\theta);\rho^\dagger)$ which in turn implies a higher equilibrium response $r(\rho^\dagger;\theta)$ by virtue of Assumption 3. Because, for any θ, $m^*(r;\theta)$ is also increasing in r, we conclude that, for any $\rho^\dagger$, and any $\theta'' > \theta'$,

$$m^*(r(\rho^\dagger;\theta'');\theta'') \geq m^*(r(\rho^\dagger;\theta');\theta'). \tag{18}$$

Now take any $(\rho,\rho^\dagger,\theta')$ such that

$$\max\Big\{G_\rho(m^*(r(\rho^\dagger;\theta'),\theta');\rho^\dagger), G_\rho(m^*(r(\rho^\dagger;\theta'),\theta');\rho)\Big\} < 0. \tag{19}$$

Proposition 1 (part (iv)) implies that, when $\theta = \theta'$, EC holds at $(\rho,\rho^\dagger)$. That information structures are rotations in turn implies that $\max\{m_\rho, m_{\rho^\dagger}\} \leq m^*(r(\rho^\dagger;\theta'),\theta')$, which along with Condition (18), implies that $\max\{m_\rho, m_{\rho^\dagger}\} \leq m^*(r(\rho^\dagger;\theta''),\theta'')$ and hence that

$$\max\Big\{G_\rho(m^*(r(\rho^\dagger;\theta''),\theta'');\rho^\dagger), G_\rho(m^*(r(\rho^\dagger;\theta''),\theta'');\rho)\Big\} < 0. \tag{20}$$

[28] Again, the function $\lambda(\rho)$ is constructed so that, for any ρ, given $\underline{m}(\rho)$, $\mathbb{E}_{G(\cdot;\rho)}[m;\rho] = \underline{m}(\rho) + \frac{1}{\lambda(\rho)} = \omega_0$ (mean preservation).

Hence, when EC holds at (ρ,$\rho^\dagger$), it also holds at ($\rho, \rho^\dagger$). ■

Proof of Proposition 3 Under Assumptions 2 and 3, for any $\rho^\dagger \in [\rho_1, \rho_2]$, $dr(\rho^\dagger)/d\rho^\dagger \overset{\text{sgn}}{=} \mathrm{A}(\rho^\dagger)$. For any given r, player L's welfare is given by

$$\mathcal{V}(r) = \sup_{\rho} \left\{ U_L(0) + \int_{-\infty}^{m^*(r)} \delta_L(r, m) dG(m; \rho) - C(\rho) \right\}.$$

The envelope theorem, along with the property that $\delta_L(r, m)$ is increasing in r under Assumption 2, imply that $d\mathcal{V}(r)/dr > 0$. The result then follows from the fact that $r(\rho_2) < r(\rho_1)$ when $\mathrm{A}(\rho^\dagger) < 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$, whereas $r(\rho_2) > r(\rho_1)$ when $\mathrm{A}(\rho^\dagger) > 0$ for all $\rho^\dagger \in [\rho_1, \rho_2]$. ■

Proof of Proposition 4. (i) The logic is similar to the one behind Proposition 3. Consider a pure-strategy equilibrium of the game without disclosure in which player L selects ρ^*. To see that ρ^* can also be supported in a pure-strategy equilibrium of the game with disclosure, for any $\hat{\rho} \in \mathbb{R}_+$, let $e(\hat{\rho})$ denote the choice of information by L when disclosing $\hat{\rho}$. Consider the following strategy for L in the game with disclosure. For any $\hat{\rho} \leq \rho^*$, $e(\hat{\rho}) = \rho^*$, whereas for any $\hat{\rho} > \rho^*$, $e(\hat{\rho}) \geq \hat{\rho}$ (the precise value is not important). Under Assumption 3, that $\mathrm{A}(\rho^\dagger) < 0$ for all $\rho^\dagger$ implies that F's reaction $r(\rho^\dagger)$ is non-increasing in the choice $\rho^\dagger$ anticipated by player F. Hence, for any $\hat{\rho} > \rho^*$, F's reaction is $r(e(\hat{\rho})) \leq r^* \equiv r(\rho^*)$, whereas, for any $\hat{\rho} \leq \rho^*$, F's reaction is $r(e(\hat{\rho})) = r^*$. These properties imply that

$$\sup_{\{\rho,\hat{\rho}\}} \left\{ \int_{-\infty}^{m^*(r(e(\hat{\rho})))} \delta_L(r(e(\hat{\rho})), m) dG(m; \rho) - C(\rho) \right\} = \int_{-\infty}^{m^*(r^*)} \delta_L(r^*, m) dG(m; \rho^*) - C(\rho^*),$$

where the equality follow from the fact that ρ^* is an equilibrium of the no-disclosure game along with the fact that L's payoff is non-decreasing in F's reaction by Assumption 2.

(ii) Conversely, let ρ^* be an information choice supported by a regular equilibrium of the disclosure game (with associated disclosure $\hat{\rho}(\rho^*) \leq \rho^*$ and reaction $r^* \equiv r(\rho^*)$). Suppose that $\rho^* < \underline{\rho}$, where $\underline{\rho}$ is the lowest equilibrium level of the no-disclosure game. That the equilibrium supporting ρ^* is regular, along with the fact that $r(\cdot)$ is non-increasing in $\rho^\dagger$ (by virtue of the assumption that $\mathrm{A}(\rho^\dagger) < 0$ for all $\rho^\dagger$) implies that, for any $\hat{\rho} < \hat{\rho}(\rho^*)$, $e(\hat{\rho}) = \rho^*$ and hence $r(e(\hat{\rho})) = r^*$—for, otherwise, L has a profitable deviation—and that, for any $\hat{\rho} > \hat{\rho}(\rho^*)$, $e(\hat{\rho}) \geq \rho^*$ and hence $r(e(\hat{\rho})) \leq r^*$. Hence, given any actual choice ρ, the most profitable disclosure for player L induces a reaction r^*. This means that, under the reaction r^*, the payoff that L obtains by selecting ρ^* is weakly higher than the payoff that she obtains by selecting any other level ρ. Therefore, ρ^* can also be sustained in the no-disclosure game, a contradiction. Similar arguments imply that the highest level of ρ that can be sustained in any regular equilibrium of the disclosure game is $\bar{\rho}$, where $\bar{\rho}$ is the largest equilibrium level in the no-disclosure game. ■

Proof of Proposition 5. As explained in the main text, we consider here a more general settings in which L's payoff differential is an arbitrary function $\delta_L(r, m)$ satisfying Assumption 2, and in which F is a representative of a competitive market with arbitrary payoff differential $\delta_F(r, m) \equiv u_F(1, r, m) - u_F(0, m)$. Hence, $u_F(1, r, m)$ is F's payoff from responding with reaction r to L's choice of engaging, when the posterior mean (equivalently, the state) is m, whereas $u_F(0, m)$ is her payoff in case L does not engage. The payoff functions $\delta_L(r, m)$ and $\delta_F(r, m)$, as well as the distributions $G(m; \rho)$, are differentiable and Lipschitz-continuous. We also maintain that, for any s, the equilibrium choices $\rho^*(s)$ and $r^*(s)$ are unique, Lipschitz continuous, and differentiable, and the welfare function

$$W(s) \equiv \int_{-\infty}^{m^*(r^*(s),s)} (\delta_L(r^*(s), m) + s)\, dG(m; \rho^*(s)) - C(\rho^*(s)) - (1 + \lambda)sG(m^*(r(s), s); \rho^*(s)$$

is quasi-concave. Note that, under the more general payoff functions introduced above, for any r and s, the engagement threshold $m^*(r,s)$ is implicitly defined by the solution to $\delta_L(r,m^*)+s=0$ and the follower's reaction $r^*(s)$ satisfies

$$\int_{-\infty}^{m^*(r^*(s),s)} \delta_F(r^*(s),m)dG(m;\rho^*(s))=0.$$

Lemma 2. Suppose the environment satisfies the conditions above and Assumptions 2 and 3 hold. There exists a threshold $K>0$ such that a strictly positive subsidy is optimal if $\frac{d}{ds}M^-(m^*(r^*(0),s);\rho^*(s))\big|_{s=0}>K$, whereas a tax on engagement is optimal when the above inequality is reversed.[29]

Proof of Lemma 2. Using the envelope theorem, we have that[30]

$$W^{'}(s)=\int_{-\infty}^{m^*(r^*(s),s)}\left[\frac{\partial\delta_L(r^*(s),m)}{\partial r}\frac{dr^*(s)}{ds}+1\right]dG(m;\rho^*(s))$$

$$-\frac{d}{ds}\left[(1+\lambda)sG(m^*(r^*(s),s);\rho^*(s))\right].$$

The first line is simply the effect of a change in the subsidy on the leader's expected payoff (holding $\rho^*(s)$ and $m^*(r^*(s),s)$ fixed by usual envelope-theorem arguments). The second line is the (total) effect of a change in the subsidy on the cost of the program to the government.

Note that $W^{'}(s)$ can be expressed as

$$W^{'}(s)=\frac{dr^*(s)}{ds}\int_{-\infty}^{m^*(r^*(s),s)}\frac{\partial\delta_L(r^*(s),m)}{\partial r}dG(m;\rho^*(s))-s(1+\lambda)\frac{d}{ds}\left[G(m^*(r^*(s),s);\rho^*(s))\right]$$

$$-\lambda G(m^*(r^*(s),s);\rho^*(s)).$$

Because $W(s)$ is quasi-concave in s, the optimal s is thus strictly positive when

$$W^{'}(0)=\frac{dr^*(0)}{ds}\int_{-\infty}^{m^*(r^*(0),0)}\frac{\partial\delta_L(r^*(0),m)}{\partial r}dG(m;\rho^*(0))-\lambda G(m^*(r^*(0),0);\rho^*(0))>0$$

and strictly negative when the above inequality is reversed.

Because δ_L is affine in m, it can be expressed as $\delta_L(r,m)=a_L(r)m+b_L(r)$, for some functions $a_L(r)$ and $b_L(r)$. Assumption 2 then implies that, for any r and m, $a_L(r)<0$ and $a_L'(r)m+b_L'(r)>0$. This means that $W^{'}(0)>0$ when $dr^*(0)/ds>r^{\#}$, whereas $W'(0)<0$ when $dr^*(0)/ds<r^{\#}$, where

$$r^{\#}\equiv\frac{\lambda}{\frac{\partial}{\partial r}\delta_L(r^*(0),M^-(m^*(r^*(0),0);\rho^*(0))}$$

is a strictly positive constant that depends on the primitives of the problem. For example, in the Akerlof's model of Subsection 2.2, $\delta_L(r,m)=r-m$, in which case $r^{\#}=\lambda$.

[29] As the proof below shows, when Assumption 3 is replaced by Assumption A3' (i.e., in the anti-lemons case), the conclusion in the Lemma flips as follows: there exists a negative threshold $K<0$ such that a strictly positive subsidy is optimal if $\frac{d}{ds}M^-(m^*(r^*(0),s);\rho^*(s))\big|_{s=0}<K$, whereas a tax on engagement is optimal when the above inequality is reversed.

[30] Here we are using the fact that, given s and $r^*(s)$, $m^*(r^*(s),s)$ and $\rho^*(s)$ maximize the leader's payoff $\int_{-\infty}^{\hat{m}}(\delta_L(r^*(s),m)+s)\,dG(m;\rho)-C(\rho)$ over $(\hat{m},\rho)$.

Next, observe that, for any s, $\rho^*(s)$ and $r^*(s)$ jointly solve the following two conditions:

$$\int_{-\infty}^{m^*(r^*,s)} \delta_F(r^*, m)dG(m;\rho^*) = 0, \tag{21}$$

and

$$\rho^* = \arg\max_{\rho} \left\{ \int_{-\infty}^{m^*(r^*,s)} (\delta_L(r^*, m) + s)\, dG(m;\rho) - C(\rho) \right\}. \tag{22}$$

Because δ_F is affine in m, it can be expressed as $\delta_F(r, m) = a_F(r)m + b_F(r)$, for some functions $a_F(r)$ and $b_F(r)$, with $a_F(r) > 0$ when Assumption 3 holds (lemons), and $a_F(r) < 0$ when Assumption 3' holds (anti-lemons).[31]

Hence, for any s, $r^*(s)$ solves $\delta_F(r^*, M^-(m^*(r^*, s); \rho^*(s))) = 0$. Using the implicit-function theorem, we have that

$$\frac{dr^*(s)}{ds} = -\frac{\frac{d}{ds}\delta_F(r, M^-(m^*(r,s);\rho^*(s)))\big|_{r=r^*(s)}}{\frac{d}{dr}\delta_F(r, M^-(m^*(r,s);\rho^*(s)))\big|_{r=r^*(s)}}$$

where the denominator is negative, by assumption. We thus have that $dr^*(0)/ds > r^{\#}$ if

$$\frac{d}{ds}\delta_F(r^*(0), M^-(m^*(r^*(0), s); \rho^*(s)))\bigg|_{s=0} > \Lambda \equiv \lambda \left| \frac{\frac{d}{dr}\delta_F(r, M^-(m^*(r, 0); \rho^*(0)))\big|_{r=r^*(0)}}{\frac{d}{dr}\delta_L(r, M^-(m^*(r, 0); \rho^*(0)))\big|_{r=r^*(0)}} \right|$$

whereas $dr^*(0)/ds < r^{\#}$ if the above inequality is reversed. The condition says that, at the laissez-faire equilibrium, holding the follower's reaction fixed at $r^*(0)$, a small subsidy to engagement has a strong enough positive effect on the follower's payoff, accounting for the effect that the subsidy has on both the leader's engagement threshold and her choice of information.

Clearly,

$$\frac{d}{ds}\delta_F(r^*(0), M^-(m^*(r^*(0), s); \rho^*(s)))\bigg|_{s=0} = a_F(r^*(0))\, \frac{d}{ds}M^-(m^*(r^*(0), s); \rho^*(s))\bigg|_{s=0}.$$

The result in the lemma then follows by letting $K \equiv \Lambda/a_F(r^*(0))$ and noting that $K > 0$ when $a_F(r^*(0)) > 0$, i.e., when Assumption 3 (lemons) holds, whereas $K < 0$ when $a_F(r^*(0)) < 0$, i.e., when Assumption 3' (anti-lemons) holds. Q.E.D.

Now to see how the lemma, when applied to the Akerlof's model, implies the result in the proposition, observe that, in this case, $\Lambda = \lambda$ and $a_F(r^*(0)) = 1$, implying that $K = \lambda$, and that

$$\frac{d}{ds}M^-(m^*(r^*(0), s); \rho^*(s))\big|_{s=0} = \frac{\partial}{\partial m^*}M^-(r^*(0); \rho^*(0)) + \frac{\partial}{\partial \rho}M^-(r^*(0); \rho^*(0))\frac{d\rho^*(0)}{ds}.$$

■

Proof of Proposition 6. As in the proof of Proposition 5, we consider a more general settings in which

[31] To see this, note that, for any m^*, ρ, and r, $\int_{-\infty}^{m^*} \delta_F(r, m)dG(m;\rho) = G(m^*;\rho)\delta_F(r, M^-(m^*;\rho))$. Now fix s and drop it. The equilibrium r thus solves $a_F(r)M^-(m^*(r);\rho) + b_F(r) = 0$. Hence,

$$\frac{dr}{d\rho} = -\frac{a_F(r)\frac{\partial}{\partial\rho}M^-(m^*(r);\rho)}{\frac{\partial}{\partial r}\delta_F(r, M^-(m^*(r);\rho))}.$$

The denominator in the above expression is negative, by assumption. It follows that $dr/d\rho \overset{sgn}{=} a_F(r)\frac{\partial}{\partial\rho}M^-(m^*(r);\rho)$. Hence, $a_F(r) > 0$ when Assumption 3 holds, whereas $a_F(r) < 0$ when Assumption 3' holds.

L's payoff differential is $\delta_L(r,m)$, with δ_L satisfying Assumption 2, and in which F is a representative of a competitive market with payoff differential $\delta_F(r,m) \equiv u_F(1,\, r,\, m) - u_F(0,\, m)$, with $\delta_L(r,m)$, $\delta_F(r,m)$ and $G(m;\rho)$ differentiable and Lipschitz-continuous.

For any s, let $\hat{r}(s)$ denote the follower's equilibrium reaction when the subsidy is equal to s and information is exogenous and equal to $\rho^*(s^*)$, where s^* is the optimal policy when information is endogenous. Clearly, for $s = s^*$, $\hat{r}(s^*) = r^*(s^*)$, where $r^*(s^*)$ is the equilibrium reaction when information is endogenous. Recall that, for any r and s, the engagement threshold $m^*(r,s)$ is implicitly defined by the solution to $\delta_L(r,m) + s = 0$, and that, for any s, when $\rho = \rho^*(s^*)$, the follower's reaction $\hat{r}(s)$ satisfies

$$\int_{-\infty}^{m^*(\hat{r}(s),s)} \delta_F(\hat{r}(s),m)dG(m;\rho^*(s^*)) = 0.$$

For any (r,s), then let

$$\hat{W}(r,s) \equiv \int_{-\infty}^{m^*(r,s)} \left(\delta_L(r,m) + s\right) dG(m;\rho^*(s^*)) - C(\rho^*(s^*)) - (1+\lambda)sG(m^*(r,s);\rho^*(s^*))$$

denote the level of welfare that is attained when information is exogenous and equal to $\rho = \rho^*(s^*)$, the follower's reaction is r, the subsidy is s, and the leader engages if and only if $m < m^*(r,s)$. [32]

We start with the following lemma

Lemma 3 (***effect of endogeneity of information on optimal policy*****).** The endogeneity of the leader's information calls for larger policy interventions (i.e., $s^* > s^{**}$) if

$$\left(\frac{d\hat{r}(s^*)}{ds} - \frac{dr^*(s^*)}{ds}\right)\frac{\partial \hat{W}(r^*(s^*),s^*)}{\partial r} + (1+\lambda)s^*G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{d\rho^*(s^*)}{ds} < 0,$$

whereas the opposite is true (i.e., $s^* < s^{**}$) if the above inequality is reversed.

The lemma says that the endogeneity of the leader's information calls for larger policy interventions when (a) $\partial\hat{W}(r^*(s^*),s^*)/\partial r > 0$, meaning that the social value of increasing the follower's reaction beyond $r^*(s^*)$ is positive, accounting for the fact that a friendlier reaction induces more engagement which in turn comes with a larger cost to the government (due to the deadweight-loss of non-uniform taxation), (b) an increase in the subsidy, starting from s^*, triggers a larger response by the follower when information is endogenous than when it is exogenous, i.e., $dr^*(s^*)/ds > d\hat{r}(s^*)/ds$, and (c) the extra cost

$$(1+\lambda)s^*G_\rho(m^*(r(s^*),s^*);\rho^*(s^*))\frac{d\rho^*(s^*)}{ds} \tag{23}$$

that the government incurs to fund the program due to the endogeneity of information is small. Note that, when (i) $s^* > 0$, (ii) $G_\rho(m^*(r(s^*),s^*);\rho^*(s^*)) < 0$ (recall that, under the MPS order, i.e., when Assumption 1 holds, this is the key condition for EC in Proposition 1), and (iii) $d\rho^*(s^*)/ds < 0$, the term in (23) is positive: the government expects to pay s^* more often when the leader reduces her information in response to an increase in the subsidy. As a result, this last effect contributes to a lower level of the optimal policy when information is endogenous.

Proof of Lemma 3. Under the maintained assumptions that both W and $W^\#$ are quasi-concave, the optimal value of s^* solves $dW(s^*)/ds = 0$. That is, s^* solves

[32] Observe that $W^\#(s) = \hat{W}(\hat{r}(s),s)$.

$$\frac{dr^*(s^*)}{ds}\int_{-\infty}^{m^*(r^*(s^*),s^*)}\frac{\partial\delta_L(r^*(s^*),m)}{\partial r}dG(m;\rho^*(s^*))$$

$$-(1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*))\left[\frac{\partial m^*(r^*(s^*),s^*)}{\partial r}\frac{dr^*(s^*)}{ds}+\frac{\partial m^*(r^*(s^*),s^*)}{\partial s}\right] \tag{24}$$

$$-(1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*))-\lambda G(m^*(r^*(s^*),s^*);\rho^*(s^*))=0.$$

Next, use the envelope theorem along with the fact that $\hat{r}(s^*) = r^*(s^*)$ and, for any s $W^{\#}(s) = \hat{W}(\hat{r}(s), s)$, to observe that

$$\frac{dW^{\#}(s^*)}{ds}=\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial r}\frac{d\hat{r}(s^*)}{ds}+\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial s}$$

where

$$\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial r}=\int_{-\infty}^{m^*(r^*(s^*),s^*)}\frac{\partial\delta_L(r^*(s^*),m)}{\partial r}dG(m;\rho^*(s^*))$$

$$-(1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{\partial m^*(r^*(s^*),s^*)}{\partial r}$$

and

$$\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial s}=-(1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{\partial m^*(r^*(s^*),s^*)}{\partial s}-\lambda G(m^*(r^*(s^*),s^*);\rho^*(s^*)).$$

Using (24), we thus have that

$$\frac{dW^{\#}(s^*)}{ds}=\left(\frac{d\hat{r}(s^*)}{ds}-\frac{dr^*(s^*)}{ds}\right)\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial r}+(1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)).$$

Because $W^{\#}(s)$ is strictly quasi-concave and because s^{**} is obviously finite, we then have that $s^{**} < s^*$ if $dW^{\#}(s^*)/ds < 0$ and $s^{**} > s^*$ if the above inequality is reversed, which establishes the lemma. Q.E.D.

Next observe that the optimality of s^* when information is endogenous reveals that

$$\frac{dr^*(s^*)}{ds}\frac{\partial\hat{W}(r^*(s^*),s^*)}{\partial r}=(1+\lambda)s^*g(m^*(r^*(s^*),s^*);\rho^*(s^*))\frac{\partial m^*(r^*(s^*),s^*)}{\partial s}+\lambda G(m^*(r^*(s^*),s^*);\rho^*(s^*))$$

$$+(1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*)).$$

Hence, when (A) $s^* > 0$, (B) $dr^*(s^*)/ds > 0$, and (C)

$$\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*),s^*);\rho^*(s^*))>0,$$

necessarily $\partial\hat{W}(r^*(s^*),s^*)/\partial r > 0$. That is, under the welfare-maximizing policy s^*, welfare always increases with the friendliness of the follower's response when information reduces the probability of engagement (i.e., when $G_\rho(m^*(r(s^*),s^*);\rho^*(s^*)) < 0$) and the comparative statics of the equilibrium r and ρ have the same monotonicity as those of the best responses (i.e., r^* increases and ρ^* decreases with the subsidy).

Also note that

$$\frac{dr^*(s^*)}{ds} - \frac{d\hat{r}(s^*)}{ds} \overset{\text{sgn}}{=} \frac{\partial \delta_F(r^*(s^*), m)}{\partial m} \frac{\partial}{\partial \rho} M^-(m^*(r^*(s^*), s^*); \rho^*(s^*)) \frac{d\rho^*(s^*)}{ds},$$

where $\partial \delta_F(r^*(s^*), m)/\partial m$ is the sensitivity of the follower's payoff to the state (which is invariant in m under the maintained assumption that δ_F is affine in m). Hence, in the lemons case (i.e., when Assumption 3 holds, in which case $\partial \delta_F(r^*(s^*), m)/\partial m > 0$), an increase in the subsidy leads to a larger response by the follower under endogenous information when

$$\frac{\partial}{\partial \rho} M^-(m^*(r^*(s^*), s^*); \rho^*(s^*)) \frac{d\rho^*(s^*)}{ds} > 0 \tag{25}$$

and a smaller response when the inequality is reversed. That is, the follower responds more to an increase in the subsidy when information is endogenous than when it is exogenous when the increase in the subsidy leads to a reduction in information acquisition and, as a result of it, an alleviation of the adverse selection problem. The opposite conclusions holds in the anti-lemon case (i.e., under Assumption 3', in which case $\partial \delta_F(r^*(s^*), m)/\partial m < 0$).

Now to see how the above results, when applied to the Akerlof model of Subsection 2.2, imply the claim in the proposition, observe that, in this example,

$$\frac{d\hat{r}(s^*)}{ds} - \frac{dr(s^*)}{ds} = \frac{\frac{\partial}{\partial \rho} M^-(m^*(r^*(s^*), s^*); \rho^*(s^*)) \frac{d\rho^*(s^*)}{ds}}{\frac{\partial}{\partial m^*} M^-(m^*(r^*(s), s^*); \rho^*(s^*)) - 1}.$$

Using the fact that

$$\frac{\partial}{\partial \rho} M^-(m^*; \rho) = \frac{G_\rho(m^*; \rho)[m^* - M^-(m^*; \rho)] - \int_{-\infty}^{m^*} G_\rho(m; \rho) dm}{G(m^*; \rho)}$$

and

$$\frac{\partial}{\partial m^*} M^-(m^*; \rho) = \frac{g(m^*; \rho)[m^* - M^-(m^*; \rho)]}{G(m^*; \rho)},$$

along with the fact that $m^*(r^*(s^*), s^*) = r^*(s^*) + s^*$ and $r^*(s^*) = M^-(m^*(r^*(s^*), s^*); \rho^*(s^*)) + \Delta$, we have that

$$\tfrac{d\hat{r}(s^*)}{ds} - \tfrac{dr(s^*)}{ds} = \Big((s^* + \Delta) G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)) - \int_{-\infty}^{m^*(r^*(s^*), s^*)} G_\rho(m; \rho^*(s^*)) dm \Big) \frac{\frac{d\rho^*(s^*)}{ds}}{D},$$

where

$$D \equiv (s^* + \Delta) g(m^*(r^*(s^*), s^*); \rho^*(s^*)) - G(m^*(r^*(s^*), s^*); \rho^*(s^*)) < 0$$

when $G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$ is increasing in m. Hence, $d\hat{r}(s^*)/ds - dr(s^*)/ds < 0$ when

$$G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$$

is increasing in m, information structures are consistent with the MPS order, $G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)) < 0$, and $d\rho^*(s^*)/ds < 0$.

Furthermore, in this case,

$$\frac{\partial \hat{W}(r^*(s^*), s^*)}{\partial r} = G(m^*(r^*(s^*), s^*); \rho^*(s^*)) - (1 + \lambda) s^* g(m^*(r^*(s^*), s^*); \rho^*(s^*)).$$

Using the fact that

$$\frac{dW^{\#}(s^*)}{ds} = \left(\frac{d\hat{r}(s^*)}{ds} - \frac{dr^*(s^*)}{ds}\right)\frac{\partial \hat{W}(r^*(s^*), s^*)}{\partial r} + (1+\lambda)s^*\frac{d\rho^*(s^*)}{ds}G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*))$$

as established in the proof of Lemma 3, we then have that $dW^{\#}(s^*)/ds = \frac{d\rho(s^*)}{ds}\frac{J}{D}$, where

$$J \equiv (\Delta - \lambda s^*)\, G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*))G(m^*(r^*(s^*), s^*); \rho^*(s^*))$$

$$+\left(\int_{-\infty}^{m^*(r^*(s^*),s^*)} G_\rho(m; \rho^*(s^*))dm\right)\left[(1+\lambda)s^*g(m^*(r^*(s^*), s^*); \rho^*(s^*)) - G(m^*(r^*(s^*), s^*); \rho^*(s^*))\right].$$

Note that $J < 0$ when information structures are consistent with the MPS order, $G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$ is increasing in m, and $G_\rho(m^*(r^*(s^*), s^*); \rho^*(s^*)) < 0$.[33]

We conclude that, under the assumptions in the proposition, $dW^{\#}(s^*)/ds \overset{sgn}{=} d\rho(s^*)/ds$. To see that, under the assumptions in the proposition, $d\rho(s^*)/ds < 0$, note that

$$\frac{dr^*(s)}{ds} = -\frac{\frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{dm^*} + \frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{d\rho}\frac{d\rho^*(s)}{ds}}{\frac{\partial M^-(m^*(r^*(s),s);\rho^*(s))}{dm^*} - 1}.$$

Under the assumptions in the proposition,

$$\frac{\partial M^-(m^*(r^*(s), s); \rho^*(s))}{dm^*} - 1 = D \cdot G(m^*(r^*(s^*), s^*); \rho^*(s^*)) < 0$$

and $\partial M^-(m^*(r^*(s), s); \rho^*(s))/d\rho < 0$. Hence, $dr^*(s)/ds < 0$ if $d\rho(s^*)/ds > 0$. This cannot be consistent with the optimality of s^*. In fact, by cutting the subsidy, the planner would induce a friendlier reaction by the follower, permit the leader to economize on her investment in information, and save on the costs of public funds. The optimality of s^* thus implies that $d\rho(s^*)/ds < 0$. We conclude that $dW^{\#}(s^*)/ds < 0$. The strict quasi-concavity of $W^{\#}$ then implies that $s^{**} < s^*$. ■

Proof of Proposition 7. (i) The proof follows from the same arguments that establish part (i) of Proposition 1. (ii) Recall that

$$\frac{\partial}{\partial \rho}M^-\left(m^*(r(\rho^\dagger));\, \rho, r(\rho^\dagger)\right) = \frac{\partial}{\partial \rho}\left(\int \omega \frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)}dG(\omega)\right).$$

Both when the cost of information is given by entropy reduction and when it is given by maximum slope, $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is a decreasing function of ω. Hence, when $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ is increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, the collection of distributions $\left(F^{\rho,r(\rho^\dagger)}\right)_\rho$, indexed by ρ, with each cdf $F^{\rho,r(\rho^\dagger)}$ defined by the density

$$f^{\rho,r(\rho^\dagger)}(\omega) \equiv \frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)}g(\omega)$$

[33] Note that, under the optimal subsidy s^*, welfare is equal to $G(m^*(r^*(s^*), s^*); \rho^*(s^*))\,(\Delta - \lambda s^*) - C(\rho^*(s^*))$. Because welfare is non-negative under the laissez-faire equilibrium (i.e., when $s = 0$), it must be that $\Delta > \lambda s^*$. Also note that, when $\Delta > \lambda s^*$, $(1+\lambda)s^*g(m^*(r^*(s^*), s^*); \rho^*(s^*)) - G(m^*(r^*(s^*), s^*); \rho^*(s^*)) < D$ and hence the second line in J is negative when information structures are consistent with the MPS order and $G(m; \rho^*(s^*))/g(m; \rho^*(s^*))$ is increasing in m.

can be ranked according to FOSD, with $F^{\rho,r(\rho^\dagger)} \succ F^{\rho',r(\rho^\dagger)}$ for any $\rho < \rho'$. This means that $M^-(m^*(r(\rho^\dagger));\ \rho, r(\rho^\dagger))$ is decreasing in ρ at $\rho = \rho^\dagger$ which implies that $\mathrm{A}(\rho^\dagger) < 0$,implying that information aggravates adverse selection.

(iii) Note that

$$\frac{\partial \Pi(\rho; r(\rho^\dagger))}{\partial r} = q^{\rho,r(\rho^\dagger)}(1) \int \frac{\partial \delta_L(r(\rho^\dagger), \omega)}{\partial r} \frac{q^{\rho,r(\rho^\dagger)}(1|\omega)}{q^{\rho,r(\rho^\dagger)}(1)} dG(\omega). \tag{26}$$

Under Assumption 2, $\partial\delta_L(r(\rho^\dagger), \omega)/\partial r$ is increasing in ω. Hence, when

$$q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$$

is increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, the integral term in (26) is decreasing in ρ (the arguments are the same as in part (ii)). Hence, a sufficient condition for $B(\rho; \rho^\dagger) = -\partial^2\Pi(\rho; r(\rho^\dagger))/\partial\rho\partial r$ to be positive (equivalently, for a reduction in r around $r(\rho^\dagger)$ to raise the marginal value of expanding ρ) is that, in addition to $q^{\rho,r(\rho^\dagger)}(1|\omega)/q^{\rho,r(\rho^\dagger)}(1)$ to be increasing in ρ for $\omega < m^*(r(\rho^\dagger))$ and decreasing in ρ for $\omega > m^*(r(\rho^\dagger))$, $q^{\rho,r(\rho^\dagger)}(1)$ is non-increasing in ρ.

(iv) The proof is an immediate implication of parts (ii) and (iii).

(v) The proof follows from the fact that, in this case,

$$B(\rho; \rho^\dagger) = -\frac{\partial \delta_L\Big(r(\rho^\dagger), m^*(r(\rho^\dagger))\Big)}{\partial r} G_\rho\Big(m^*\big(r(\rho^\dagger)\big); \rho, r(\rho^\dagger)\big)\Big).$$

Because $\mathrm{A}(\rho^\dagger) < 0$, the result in part (i) implies that a necessary and sufficient condition for expectation conformity to hold at $(\rho, \rho^\dagger)$ is that $B(\rho; \rho^\dagger) > 0$ which is the case if and only if $G_\rho\Big(m^*\big(r(\rho^\dagger)\big); \rho, r(\rho^\dagger)\big)\Big) < 0$. Because, for any ρ,

$$G\Big(m^*\big(r(\rho^\dagger)\big); \rho, r(\rho^\dagger)\big)\Big) = \int q^{\rho,r(\rho^\dagger)}(1|\omega) dG(\omega) \equiv q^{\rho,r(\rho^\dagger)}(1)$$

the latter property is equivalent to $q^{\rho,r(\rho^\dagger)}(1)$ being non-increasing in ρ. ■

Proof of Lemma ???

Fix r and note that the seller's gross payoff $\int_\omega (r-\omega) q(1|\omega) dG(\omega) + \omega_0$ from trading with the buyer is bounded from above by

$$\int_\omega (r-\omega)\mathbb{I}[\omega \le r] dG(\omega) + \omega_0,$$

which is the seller's gross payoff under a signal that recommends to trade if and only if $\omega \le r$ (hereafter, we refer to such a signal as "fully-responsive"). Because C is increasing and convex, $\lim_{\rho\to\infty} C(\rho) = +\infty$. Now let $\overline{\rho}(r)$ be defined by

$$\int_\omega (r-\omega)\mathbb{I}[\omega \le r] dG(\omega) = C(\rho).$$

Clearly, for any $\rho > \overline{\rho}(r)$, any q,

$$\begin{aligned} \Pi(r, q; \rho) &= \int_\omega (r-\omega) q(1|\omega) dG(\omega) + \omega_0 - \tfrac{I^q}{\rho} - C(\rho) \\ &\le \int_\omega (r-\omega)\mathbb{I}[\omega \le r] dG(\omega) + \omega_0 - C(\rho) \\ &< \omega_0 \end{aligned}$$

which establishes that selecting ρ above $\overline{\rho}(r)$ is never optimal for the seller.

Next, let $\underline{\rho}(r)$ be the smallest value of ρ for which the inner problem admits an interior solution. For any $\rho < \underline{\rho}(r)$, the optimal signal $q^{\rho,r}$ entails no information acquisition and hence $I^{q^{\rho,r}} = 0$. This means that, for any $\rho < \underline{\rho}(r)$,

$$\Pi^*(r,\rho) \equiv \Pi(r, q^{\rho,r}; \rho) = \omega_0 - C(\rho)$$

when the optimal signal prescribes never to sell and

$$\Pi^*(r,\rho) \equiv \Pi(r, q^{\rho,r}; \rho) = r - C(\rho)$$

when it prescribes to sell for all possible ω. In either case, (15) holds.

Finally, consider $\rho \in [\underline{\rho}(r), \overline{\rho}(r)]$. Let $Q(r)$ denote the set of signals q for which $I^q \leq -\phi(G(r))$ and observe that this set includes the fully-responsive signal $q(1|\omega) = \mathbb{I}[\omega \leq r]$ for which the entropy cost is $I^q = -\phi(G(r)))$. Clearly, for any $\rho \in [\underline{\rho}(r), \overline{\rho}(r)]$, and any $q \notin Q(r)$,

$$\Pi(r, q; \rho) < \int_\omega (r - \omega)\mathbb{I}[\omega \leq r] dG(\omega) + \omega_0 - \frac{-\phi(G(r))}{\rho} - C(\rho),$$

where the right-hand side is the seller's payoff under the fully-responsive signal $q(1|\omega) = \mathbb{I}[\omega \leq r]$. For any $\rho \in [\underline{\rho}(r), \overline{\rho}(r)]$, the choice of q can thus be restricted to $Q(r)$. It is easy to see that, for any $\rho \in [\underline{\rho}(r), \overline{\rho}(r)]$, any $q \in Q(r)$, $\Pi(r, q; \rho)$ is differentiable in ρ with derivative uniformly bounded over $[\underline{\rho}(r), \overline{\rho}(r)] \times Q(r)$. That $\Pi^*(r,\rho)$ is absolutely continuous in ρ over $[\underline{\rho}(r), \overline{\rho}(r)]$ with derivative satisfying (15) then follows from Milgrom and Segal (2002). ■

	Lemons $\frac{dr}{d\rho^\dagger} \overset{sgn}{=} \frac{\partial}{\partial \rho^\dagger} M^-(m^*;\rho^\dagger)$	Anti-lemons $\frac{dr}{d\rho^\dagger} \overset{\text{sgn}}{=} -\frac{\partial}{\partial \rho^\dagger} \mathrm{M}^-(\mathrm{m}^*;\rho^\dagger)$
Does more information lead to an unfriendlier response? ($dr/d\rho^\dagger < 0$)	Yes if • MPS and $G_\rho < 0$ • or information always aggravates adverse selection (e.g., uniform, Pareto, exp.)	No if • MPS and $G_\rho < 0$ • or information always aggravates adverse selection (e.g., uniform, Pareto, exp.)
Does an unfriendlier response increase L's demand for information? $\left(-\frac{\partial^2 \Pi(\rho;r)}{\partial r \partial \rho} > 0\right)$	• Yes if MPS and $G_\rho < 0$ • No if $G_\rho \geq 0$ and $\frac{\partial^2 \delta_L}{\partial m \partial r} = 0$ (Akerlof's model +ex. a, c, d in Supplement)	• Yes if MPS and $G_\rho < 0$ • No if $G_\rho \geq 0$ and $\frac{\partial^2 \delta_L}{\partial m \partial r} = 0$ (ex. e, f, h in Supplement)
(Local) EC / expectation traps	• Yes if MPS and $G_\rho < 0$ • $G_\rho < 0$ is NSC if information always aggravates adverse selection and $\frac{\partial^2 \delta_L}{\partial m \partial r} = 0$	• Yes if information always aggravates adverse selection, $G_\rho > 0$, and $\frac{\partial^2 \delta_L}{\partial m \partial r} = 0$
Engagement channel of subsidy $\frac{\partial}{\partial m^*} M^-(m^*;\rho^\dagger) \frac{\partial m^*}{\partial s}$	• Benefits player L	• Hurts player L
Information channel of subsidy $\frac{\partial}{\partial \rho^\dagger} M^-(m^*;\rho^\dagger) \frac{d\rho^\dagger}{ds}$	• Benefits player L if MPS and $G_\rho < 0$	• Hurts player L if MPS and $G_\rho < 0$
Total effect of subsidy on welfare	• positive if engagement + information channels $> K > 0$	• negative if engagement + information channels $> K < 0$

Table 1: summary of a few results