Matching Auctions

Alessandro Pavan
Northwestern University

Daniel Fershtman
Tel Aviv University

Motivation

- Mediated matching central to "**sharing economy**"

- Most matching markets intrinsically **dynamic — re-matching**

  - shocks to profitability of existing matching allocations

  - gradual resolution of uncertainty about attractiveness

  - preference for variety

- Re-matching, while pervasive, largely ignored by matching theory

This paper

- Dynamic matching
  - mediated (many-to-many) interactions
  - evolving private information
  - payments
  - capacity constraints
- Applications
  - scientific outsourcing (Science Exchange)
  - lobbying
  - sponsored search
  - internet display advertising
  - lending (Prospect, LendingClub)
  - B2B
  - health-care (MEDIGO)
  - organized events (meetings.com)
- Matching auctions
- Dynamics under profit vv welfare maximization

Plan

- **Model**
- Matching auctions
- Truthful bidding
- Profit maximization
- Distortions
- Endogenous processes
- Conclusions

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- Profit-maximizing platform mediates interactions between 2 sides, A, B

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- Agents: $N_A = \{1, ..., n_A\}$ and $N_B = \{1, ..., n_B\}$, $n_A, n_B \in \mathbb{N}$
- Period-t match between agents $(i, j) \in N_A \times N_B$ yields gross payoffs

$$v_{ijt}^A = \theta_i^A \cdot \varepsilon_{ijt}^A \quad \text{and} \quad v_{ijt}^B = \theta_j^B \cdot \varepsilon_{ijt}^B$$

θ_i^k: "vertical" type

ε_{ijt}^k: "horizontal" type (time-varying match-specific)

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$$v_{it}^A = (v_{i1t}^A, v_{i2t}^A, ..., v_{in_Bt}^A)$$

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- Agent i's payoff ($i \in N_A$):

$$U_i^A = \sum_{t=0}^{\infty} \delta^t \sum_{j \in N_B} v_{ijt}^A \cdot x_{ijt} - \sum_{t=0}^{\infty} \delta^t p_{it}^A$$

with $x_{ijt} = 1$ if (i, j)-match active, $x_{ijt} = 0$ otherwise.

Model

- Platform's profits:

$$\sum_{t=0}^{\infty} \delta^t \left(\sum_{i \in N_A} p_{it}^A + \sum_{j \in N_B} p_{jt}^B - \sum_{i \in N_A} \sum_{j \in N_B} c_{ijt} \cdot x_{ijt} \right)$$

Model

- In each period $t \geq 1$, each agent $l \in N^k$ from each side $k = A, B$ can be matched to at most m_l^k agents from side $-k$.
  - *one-to-one matching:* $m_l^k = 1$ all $l = 1, ..., n^k$, $k = A, B$

  - *many-to-many mathcing with no binding capacity constraints*:
  $m_l^k \geq n^{-k}$, all $l = 1, ..., n^k$, $k = A, B$

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- In each period $t \geq 1$, platform can match up to M pairs of agents

  - space, time, services constraint

  - platform can delete previously formed matches and create new ones. Total number of existing matches cannot exceed M in all periods.

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- Agents observe θ_i^k prior to joining, but learn (ε_{ijt}^k) over time

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Matching auctions

- At $t = 0$ (i.e., upon joining the platform), each agent $l \in N_k$ purchases *membership status* $\theta_l^k \in \Theta_l^k$ at price $p_l^k(\theta)$

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- Full transparency - bids, payments, membership, matches all public.

Payments (PST + BV)

- Fixing weights β, *weighted surplus*:

$$w_t \equiv \sum_{i \in N_A} \sum_{j \in N_B} S_{ijt} \cdot \chi_{ijt}$$

- $w_t^{-i,A}$ = weighted surplus in absence of agent $i \in N_A$ (same as W_t, but with $S_{ijs}^A = 0$, all $j \in N_B$).
- Period-t payments, $t \geq 1$:

$$\psi_{it}^A = \sum_{j \in N_B} b_{ijt}^A \cdot \chi_{ijt} - \frac{w_t - w_t^{-i,A}}{\beta_i^A(\theta_i^A)}$$

- (Horizontal) match quality under rule χ:

$$D_l^A(\theta) \equiv \mathbb{E}^{\lambda[\chi]|\theta} \left[\sum_{t=1}^{\infty} \delta^t \sum_{j \in N_B} \varepsilon_{ijt}^A \chi_{ijt} \right]$$

- Period-0 membership fees:

$$\psi_{i0}^A = \theta_i^A D_i^A(\theta) - \int_{\underline{\theta}_i^A}^{\theta_i^A} D_i^A(\theta_{-i}^A, y) dy - \mathbb{E}^{\lambda[\chi]|\theta_0} \left[\sum_{t=1}^{\infty} \delta^t \psi_{it}^A \right] - L_i^A$$

Payments

- Payments similar to GSPA for sponsored search but adjusted for
  - dynamic externalities
  - costs of information rents (captured by β)
  - matches need not maximize true surplus

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Truthful bidding

Definition

Strategy profile $\sigma = (\sigma_l^k)_{l \in N^k}^{k=A,B}$ *truthful* if each agent
- selects membership status corresponding to true vertical type
- at each $t \geq 1$, bids given by $b_{ijt}^k = v_{ijt}^k = \theta_l^k \cdot \varepsilon_{ijt}^k$, all $(i,j) \in N^A \times N^B$, $k = A, B$, irrespective of membership status selected at $t = 0$ and of past bids.

Truthful equilibrium is an equilibrium in which strategy profile is truthful.

Truthful bidding

Theorem

Any matching auction in which L_l^k *large enough admits an equilibrium in which all agents participate in each period and follow truthful strategies. Furthermore, such truthful equilibria are periodic ex-post (agents' strategies are sequentially rational, regardless of beliefs about other agents' past and current types).*

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Profit maximization

Theorem

Let

$$\beta_l^{k,P}(\theta_l^k) \equiv 1 - \frac{1 - F_l^k(\theta_l^k)}{f_l^k(\theta_l^k)\theta_l^k}, \text{ all } l \in N^k, \ k = A, B. \tag{1}$$

Suppose $D_l^k(\theta^{-l,k}, \underline{\theta}_l^k; \beta^P) \geq 0$, *all* $l \in N^k$, $k = A, B$, *and all* $\theta^{-l,k}$. *Matching auctions with weights* β^P *and payments s.t.* $L_l^k = 0$, *all* $l \in N^k$, $k = A, B$, *maximize platform's profits across all possible mechanisms.*

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Welfare maximization

Theorem

Let $\beta_l^{k,W}(\theta_l^k) = 1$*, all* θ_l^k*,* $l \in N^k$*,* $k = A, B$*.*
(i) Matching auctions with weights β^W *and payments with* L_l^k *large enough, all* $l \in N^k$*,* $k = A, B$*, maximize ex-ante welfare over all possible mechanisms.*
(ii) Suppose $D_l^k(\theta^{-l,k}, \underline{\theta}_l^k; \beta^W) \geq 0$*, all* $l \in N^k$*,* $k = A, B$*, and all* θ_{-l}^k*.* *Matching auctions with payment s.t.* $L_l^k = 0$*, all* $l \in N^k$*,* $k = A, B$*, admit ex-post periodic equilibria in which agents participate and follow truthful strategies at all histories. Furthermore, such auctions maximize the platform's profits over all mechanisms implementing welfare-maximizing matches and inducing the agents to join platform in period zero.*

Distortions

Theorem

Assume horizontal types ε **non-negative**

(1) If none of capacity constraints binds

$$\chi_{ijt}^{P} = 1 \;\Rightarrow\; \chi_{ijt}^{W} = 1$$

(2) If only platform's capacity constraint potentially binding

$$\sum_{(i,j)\in N^A\times N^B} \chi_{ijt}^{W} \geq \sum_{(i,j)\in N^A\times N^B} \chi_{ijt}^{P}$$

(3) If some of individual capacity constraints potentially binding,

$$\sum_{(i,j)\in N^A\times N^B} \chi_{ijt}^{P} > 0 \;\Rightarrow\; \sum_{(i,j)\in N^A\times N^B} \chi_{ijt}^{W} > 0.$$

(*) Above conclusions can be reversed with negative horizontal types (upward distortions)

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Endogenous Processes

- **Endogenous processes:**
  - when $x_{ijt-1} = 0$, $\varepsilon^k_{ijt} = \varepsilon^k_{ijt-1}$ a.s.
  - when $x_{ijt-1} = 1$, kernel G_{ijt} depends on $\sum_{s=1}^{t-1} x_{ijs}$
  - costs c_{ijt} may also depend on $\sum_{s=1}^{t-1} x_{ijs}$
  - example 1: experimentation in Gaussian world $(\varepsilon^k_{ijt} = \mathbb{E}[\omega^k_{ij} | (z^k_{ijs})_s])$
  - example 2: preference for variety

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- ε drawn independently across agents and from θ, given x

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- Auctions similar to those above but where at each t agents adjust membership status to $\theta_{lt}^k \in \Theta_l^k$ and scores given by following indexes

$$S_{ijt} \equiv \sup_{\tau} \frac{\mathbb{E}^{\lambda_{ij}|\theta_0,\theta_t,b_t,x^{t-1}} \left[\sum_{s=t}^{\tau} \delta^{s-t} \left(\beta_i^A(\theta_{i0}^A) \cdot b_{ijt}^A + \beta_j^B(\theta_{j0}^B) \cdot b_{ijt}^B - c_{ijs}(x^{s-1}\right)\right]}{\mathbb{E}^{\lambda_{ij}|\theta_0,\theta_t,b_t,x^{t-1}} \left[\sum_{s=t}^{\tau} \delta^{s-t}\right]}$$

where
τ: stopping time

$\lambda_{ij}|\theta_0, \theta_t, b_t, x^{t-1}$: process over bids under truthful bidding, when $\varepsilon_{ijt}^k = \frac{b_{ijt}^k}{\theta_{it}^k}$

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- Same qualitative conclusions as for exogenous processes

Conclusions

- Mediated (dynamic) matching
  - agents learn about attractiveness of partners over time
  - shocks to profitability of matching allocations
- Matching auctions
  - similar in spirit to GSPA for sponsored search BUT
  (i) richer externalities
  (i) costs of info rents
- Ongoing work:
  - searching for arms/partners

Thank You!