# (In)efficiency in Information Acquisition and Aggregation through Prices

# Online Appendix

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**Abstract**

This document contains results for a market in which competition is restricted to be in market orders (i.e., flat, price-invariant schedules). The numbering of the conditions in this supplement is independent from the one in the main text and any cross-referencing in the supplement is meant with respect to the numbering of the conditions in the supplement, not the one in the main text.

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# 1 Competition in Market Orders

Below we show that, when traders are restricted to market orders, there is no inefficiency in either the collection or usage of information. That traders collect and use information efficiently when restricted to submitting market orders does not imply that welfare is higher under this microstructure than when traders compete in schedules. This is because the latter permit the traders to adjust their asset holding based on the information held by other traders and also respond to shocks to the supply (namely, to $u$). In particular, under the optimal policies in the main text inducing efficiency in both information acquisition and trading, welfare is strictly higher when traders compete in schedules than when they are restricted to market orders. This is because these policies implement the efficient allocations, and the latter are obtained by maximizing welfare over a set of allocations that includes those that can be sustained with market orders.

## 1.1 Efficiency in usage

To start with, suppose that $y_i = y$ for all $i$, with $y$ exogenous. In any symmetric equilibrium in which the price is affine in $(\theta, u, \eta)$, each trader's market order is an affine function of her private signal. That is, $x_i = as_i + b$, for some scalars $(a, b)$ that depend on the exogenous parameters of the model. Aggregate demand is then equal to $\tilde{x} = \int x_i di = a(\theta + f(y)\eta) + b$. Combining the above expression with the inverse aggregate supply function $p = \alpha - u + \beta\tilde{x}$, we then have that the equilibrium price must satisfy

$$p = \alpha - u + \beta b + \beta a(\theta + f(y)\eta). \tag{1}$$

For each $s_i$, the equilibrium market order $x_i = as_i + b$ must maximize trader $i$'s expected profits

$$\Pi_i \quad = \quad \mathbb{E}\left[(\theta - p)\, x_i - \lambda \frac{x_i^2}{2} | s_i; y_i\right] - \mathcal{C}(y_i),$$

where $x_i = a_i s_i + b$. Following steps similar to those in the baseline model, we have that, for any $s_i$, the derivative of $\Pi_i$ with respect to $x_i$, evaluated at $x_i = a_i s_i + b$, must be equal to zero, which yields $\mathbb{E}[\theta|s_i] - \alpha - \beta b - \beta a \mathbb{E}[\theta + f(y)\eta|s_i] = \lambda(as_i + b)$.[1]

We conclude that the equilibrium value of $b$, which we denote by $b^*$, is equal to $b^* = -\alpha/(\beta + \lambda)$. To obtain the equilibrium value of $a$, which we denote by $a^*$, we replace $\mathbb{E}[\theta|s_i] = \frac{\tau_\epsilon}{\tau_\epsilon + \tau_\theta} s_i$ and

$$\mathbb{E}[\eta|s_i] \frac{f(y)\frac{1}{\tau_\eta}\tau_\theta\tau_\epsilon}{\tau_\epsilon + \tau_\theta} s_i$$

into the above first-order condition from which we obtain that

$$a^* \quad = \quad \frac{\tau_\epsilon}{\lambda(\tau_\epsilon + \tau_\theta) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}}.$$

Next, we can derive the expression for the welfare losses. When the market orders are affine with coefficients $a$ and $b$, $x_i - \tilde{x} = a(s_i - \theta - f(y)\eta)$, from which we obtain that $\mathbb{E}[(x_i - \tilde{x})^2] = \mathbb{E}[a^2 f(y)^2 e_i^2] = a^2/y\tau_e$, as in the model in the main text. Observe that the first-best action is $x^o = (\theta - \alpha + u)/(\beta + \lambda)$.

[1] Note that $\mathbb{E}[u|s_i] = 0$.

One can then show that, for any $(a, b)$, the welfare losses are equal to

$$WL = \frac{\beta+\lambda}{2}\mathbb{E}[(\tilde{x} - x^o)^2] + \frac{\lambda}{2}\mathbb{E}[(x_i - \tilde{x})^2] =$$

$$\frac{1}{2(\beta+\lambda)^2}\left[\frac{(\beta a+\lambda a-1)^2}{\tau_\theta} + \frac{(\beta+\lambda)^2 a^2}{y\tau_\eta} + \frac{1}{\tau_u} + b^2(\beta+\lambda)^2 + \alpha^2 + 2\alpha b(\beta+\lambda)\right].$$

For any $a$, the value of $b$ that minimizes the welfare losses is thus given by the solution to

$$\frac{\partial WL}{\partial b} = b + \frac{\alpha}{\beta+\lambda} = 0.$$

We conclude that the optimal value of $b$ is the equilibrium one: $b^T = b^* = -\alpha/(\beta+\lambda)$. Replacing the above value of $b^T$ into the expression for the welfare losses, we have that the latter can be expressed as a function of $a$ as follows

$$\tilde{WL}(a; y) = \frac{1}{2}\left(\frac{(\beta a + \lambda a - 1)^2}{(\beta+\lambda)\tau_\theta} + \frac{(\beta+\lambda)a^2}{y\tau_\eta} + \frac{1}{(\beta+\lambda)\tau_u} + \frac{\lambda a^2}{y\tau_e}\right).$$

Differentiating $\tilde{WL}(a; y)$ with respect to $a$ and setting the derivative equal to zero, we have that the socially-optimal value of $a$, which we denote by $a^T$, must satisfy

$$\frac{\partial}{\partial a}\tilde{WL}(a^T; y) = \frac{(\beta a^T + \lambda a^A - 1)}{\tau_\theta} + \frac{(\beta+\lambda)a^T}{y\tau_\eta} + \frac{\lambda a^T}{y\tau_e} = 0,$$

from which we obtain that

$$a^T = \frac{\tau_\epsilon}{\lambda\tau_\epsilon + \beta\tau_\epsilon + \lambda\tau_\theta + \frac{\beta\tau_\epsilon\tau_\theta}{y\tau_\eta}} = a^*.$$

We thus conclude that there is no inefficiency in the usage of information when traders are restricted to market orders.

### 1.2 Efficiency in information acquisition

We now characterize the equilibrium acquisition of private information and show that it is also efficient. When each trader $j \neq i$ chooses $y_j = y$ and then submits the equilibrium affine market order $x_j = as_j + b$ for quality of information $y$, and trader $i$ instead acquires information of quality $y_i$ and then, after observing $s_i$, submits the market order $x_i$, his expected payoff is equal to

$$\Pi_i = \mathbb{E}\left[(\theta - p)\, x_i - \lambda\frac{x_i^2}{2}|s_i, y_i\right] - \mathcal{C}(y_i),$$

where $p = \alpha - u + \beta\tilde{x}$, with $\tilde{x} = a(\theta + f(y)\eta) + b$, with

$$a = \frac{\tau_\epsilon}{\lambda\,(\tau_\epsilon + \tau_\theta) + \beta\tau_\epsilon + \beta\frac{\tau_\theta\tau_\epsilon}{y\tau_\eta}} \tag{2}$$

and $b = -\alpha/(\beta+\lambda)$, as shown above. For any $(s_i, y_i)$, the optimal market order for trader $i$ is given

by the solution to the first-order condition with respect to $x_i$ which yields $x_i = a_i s_i + b$, with

$$a_i = \frac{y_i \tau_e \tau_\eta (1 - \beta a(y)) - \beta a(y) \frac{\sqrt{y_i}}{\sqrt{y}} \tau_\theta \tau_e}{\lambda \left( y_i \tau_e \tau_\eta + \tau_\theta (\tau_e + \tau_\eta) \right)} \tag{3}$$

and $b = -\alpha/(\beta + \lambda)$. That is, for any $(y, y_i)$, trader $i$'s expected profits when all other traders acquire information of quality $y$ and then submit the equilibrium market orders for quality of information $y$, and trader $i$ instead acquires information of quality $y_i$ and then submits the market order that maximizes his payoff (the one described above) is given by

$$\begin{aligned} \Pi_i(y, y_i) &= \mathbb{E}\left[ \left( \theta - \alpha + u - \beta \left( a\theta + a f(y) \eta + b \right) \right) (a_i s_i + b) - \lambda \frac{(a_i s_i + b)^2}{2} ; y, y_i \right] - \mathcal{C}(y_i) \\ &= \frac{a_i - \beta a a_i}{\tau_\theta} - \frac{\beta a a_i}{\sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda a_i^2}{2} \left( \frac{1}{\tau_\theta} + \frac{1}{y_i \tau_\eta} + \frac{1}{y_i \tau_e} \right) - \mathcal{C}(y_i) - \alpha b + (1 - \beta) b^2 \end{aligned}$$

where we used the fact that $s_i = \theta + f(y_i)(\eta + e_i)$.

For any $(y, y_i)$, let $a^*(y)$ and $a_i^*(y, y_i)$ be the value in (2) and (3), respectively. Using the Envelope Theorem, we then have that

$$\frac{\partial}{\partial y_i} \Pi_i(y, y_i) = \frac{1}{2} \frac{\beta a^*(y) a_i^*(y, y_i)}{y_i \sqrt{y}\sqrt{y_i}\tau_\eta} - \frac{\lambda \left( a_i^*(y, y_i) \right)^2}{2} \left( -\frac{1}{y_i^2 \tau_\eta} - \frac{1}{y_i^2 \tau_e} \right) - \mathcal{C}'(y_i).$$

When $y$ is equal to the equilibrium level, which we denote by $y^*$, it must be that $\frac{\partial}{\partial y_i} \Pi_i(y^*, y^*) = 0$, which, using the fact that $a_i^*(y^*, y^*) = a^*(y^*)$, yields[2]

$$\mathcal{C}'(y^*) = \frac{1}{2} \left( \frac{(\beta + \lambda) \left( a(y^*) \right)^2}{(y^*)^2 \tau_\eta} + \frac{\lambda \left( a(y^*) \right)^2}{(y^*)^2 \tau_e} \right).$$

Following steps similar to those in the proof of Proposition 5 in the main text, one can show that, when $\mathcal{C}'(0)$ is small and $\mathcal{C}$ is sufficiently convex, the above equation admits one and only one solution $y^*$, and individual payoffs $\Pi_i(y^*, \cdot)$ are quasi-concave in $y_i$, implying that an equilibrium in the full game exists and is unique within the family of equilibria in affine strategies.

Next, we characterize the socially optimal value of $y$. Because for any $y$, the socially-optimal usage of information coincides with the equilibrium, as shown above, using the Envelope Theorem, we have that the socially optimal value of $y$, which we denote by $y^T$ is given by the condition[3]

$$\frac{\partial}{\partial y} WL(a(y^T); y^T) = \frac{1}{2} \left( -\frac{(\beta + \lambda) \left( a^*(y^T) \right)^2}{(y^T)^2 \tau_\eta} - \frac{\lambda \left( a^*(y^T) \right)^2}{(y^T)^2 \tau_e} \right) + \mathcal{C}'(y^T) = 0.$$

We conclude that $y^T$ is given by the solution to the following condition

[2] The notation $\frac{\partial}{\partial y_i} \Pi_i(y^*, y^*)$ stands for the partial derivative of $\Pi_i(y, y_i)$ with respect to $y_i$ evaluated at $y_i = y = y^*$.

[3] The notation $\frac{\partial}{\partial y} WL(a(y^T); y^T)$ stands for the partial derivative of $WL(a; y)$ with respect to $y$ evaluated at $y = y^T$ and $a = a(y^T)$.

$$\mathcal{C}'(y^T) = \frac{1}{2}\left(\frac{(\beta+\lambda)\left(a^*(y^T)\right)^2}{(y^T)^2\,\tau_\eta} + \frac{\lambda\left(a^*(y^T)\right)^2}{(y^T)^2\,\tau_e}\right).$$

It is immediate to see that $y^T = y^*$, implying that the equilibrium of the laissez-faire economy is efficient. Q.E.D.