Investment Subsidies with Spillovers and Endogenous Private Information

Online Supplement

Luca Colombo* Gianluca Femminis† Alessandro Pavan‡

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Abstract

This document contains two sections. Section S.1 establishes that the results for the version of the model in Subsection 2.2 in the main text (where firms produce intermediate goods in a "smart" or "traditional" specification) are equivalent to those for the version of the model in Subsection 2.1 analyzed in the rest of the paper. Section S.2 contains an extension to a family of economies in which the firms' managers, and hence the representative household, are risk averse with a diminishing marginal utility for the consumption of the final good. All numbered items in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text.

*Università Cattolica del Sacro Cuore (lucava.colombo@unicatt.it).
†Università Cattolica del Sacro Cuore (gianluca.femminis@unicatt.it).
‡Northwestern University and CEPR (alepavan@northwestern.edu).

S.1 Equivalence between Specifications in Subsections 2.1 and 2.2 in main text

Lemma 1. To see that Lemma 1 in the main text holds under the production function of Subsection 2.2 in the main text, note that, in each state θ, the amount of the final good produced is equal to (we dropped the dependence of the various functions on π_x to ease the notation)

$$Y(\theta) = \Theta\left(1+\beta N(\theta)\right)^{\alpha}\left(N(\theta)\left(\gamma y_1(\theta)\right)^{\frac{v-1}{v}} + (1-N(\theta))\, y_0(\theta)^{\frac{v-1}{v}}\right)^{\frac{v}{v-1}}. \tag{S.1}$$

Because $C(\theta) = Y(\theta)$, using Condition (4) in the main text, we have that the consumption of the final good in each state θ is equal to

$$C(\theta) = \left(N(\theta)\left(\gamma\Theta\left(1+\beta N(\theta)\right)^{\alpha} l_1(\theta)^{\psi}\right)^{\frac{v-1}{v}} + (1-N(\theta))\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha} l_0(\theta)^{\psi}\right)^{\frac{v-1}{v}}\right)^{\frac{v}{v-1}}, \tag{S.2}$$

which coincides with the expression under the specification of Subsection 2.1 – see Conditions (A.1), (A.2), and (A.3) in the proof of Lemma 1 in the main text. It is then easy to see that all the remaining steps in the proof of Lemma 1 in the main text characterizing the policies $l_1(\theta)$, $l_0(\theta)$, $N(\theta)$, and $n(x)$ that maximize welfare apply also to the specification in Subsection 2.2.

Lemma 2. We start by characterizing the equilibrium price of the final good under the production specification in Subsection 2.2. Recall that the final good is produced in a competitive market in which profits are equal to

$$\Pi = PY - \int p_i y_i di,$$

where Y is given by Condition (S.1) above. Note that, for each intermediate input i, the price y_i naturally depends on whether the good is provided in its smart or traditional specification. Letting p_1 denote the price for the goods provided in the smart specification and p_0 the price for the goods provided in the traditional specification, we have that the first-order conditions for the maximization of Π yield

$$p_1 = P\left(\frac{y_1}{Y}\right)^{-\frac{1}{v}}\left(\gamma\Theta\left(1+\beta N\right)^{\alpha}\right)^{\frac{v-1}{v}} \qquad p_0 = P\left(\frac{y_0}{Y}\right)^{-\frac{1}{v}}\left(\Theta\left(1+\beta N\right)^{\alpha}\right)^{\frac{v-1}{v}}, \tag{S.3}$$

where we dropped the arguments of all the functions to ease the notation. The demands for the intermediate goods supplied in their smart specification are then given by

$$y_1 = \gamma^{v-1} \left(\frac{p_1}{p_0} \right)^{-v} y_0.$$

Using Conditions (S.1) and (S.3) above, we thus have that the amount of the final good produced in each state θ is equal to

$$Y = \Theta \left(1 + \beta N\right)^{\alpha} \left(N p_1^{1-v} \gamma^{v-1} + (1-N) p_0^{1-v}\right)^{\frac{v}{v-1}} \frac{y_0}{p_0^{-v}},$$

which in turn implies that the price of the final good is equal to

$$P = \frac{\left(N \left(p_1/\gamma\right)^{1-v} + (1-N) p_0^{1-v}\right)^{\frac{1}{1-v}}}{\Theta \left(1 + \beta N\right)^{\alpha}}.$$

This condition is the analog of Condition (A.21) in the main text. Notice, however, that under this specification an increase in the productivity θ of the final good reduces the price of the latter P relative to that of the inputs p_0 and p_1. Note also that, because smart inputs (that is, goods provided in the smart specification) are more productive than traditional ones, their relative price in terms of the final good is larger (by a factor of $1/\gamma$) than that of the traditional inputs. To verify that the optimal policies under the production specification of Subsection 2.2 coincides with those under the specification of Subsection 2,1, we show that the extra profit (net of the subsidy) $\mathcal{R}(\theta)$ that each firm makes by choosing the smart specification takes the same form as in the proof of Lemma 2 in the main text.

Given W and P, each firm providing its input in the smart specification chooses p_1 to maximize[1]

$$\frac{p_1 y_1 - W l_1}{P} + T_1 \left(\frac{p_1 y_1}{P} \right), \tag{S.4}$$

where

$$y_1 = \left(\gamma \Theta \left(1 + \beta N\right)^{\alpha}\right)^{v-1} C \left(\frac{p_1}{P} \right)^{-v}, \tag{S.5}$$

and $l_1 = y_1^{1/\psi}$. After some algebra, the first-order condition of the above maximization problem for p_1 yields

$$\frac{1-v}{v} \frac{y_1 p_1}{P} + \frac{1}{\psi} \frac{W}{P} l_1 + \frac{1-v}{v} \frac{dT_1 \left(p_1 y_1/P\right)}{dr} \frac{y_1 p_1}{P} = 0, \tag{S.6}$$

which is the same as (A.19) in the main text.

Next, use (S.2) and (S.5), along with the fact that efficiency requires that $\hat{l}_1 = \gamma^{\varphi} \hat{l}_0$ (as shown in

[1] We drop π^x and θ from all the formulas to ease the notation.

the proof of Lemma 1 in the main text which, as argued above, is valid also for the specification in Subsection 2.2) and that $\hat{y}_i = \hat{l}_i^{\psi}$, to verify that, in any equilibrium implementing the efficient allocation, firms must set prices equal to

$$\hat{p}_1 = \Theta\left(1+\beta\hat{N}\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\gamma^{\left(1+\frac{\varphi}{1-v}\right)}\hat{P}, \tag{S.7}$$

and

$$\hat{p}_0 = \Theta\left(1+\beta\hat{N}\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\hat{P}, \tag{S.8}$$

with

$$\hat{P} = \frac{\left(\hat{N}\hat{p}_1^{1-v}\gamma^{v-1}+\left(1-\hat{N}\right)\hat{p}_0^{1-v}\right)^{\frac{1}{1-v}}}{\Theta\left(1+\beta\hat{N}\right)^{\alpha}}. \tag{S.9}$$

Equilibrium in the labor market requires that $\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}$ where $\hat{L} = \hat{l}_1\hat{N}+\hat{l}_0(1-\hat{N})$. Furthermore, efficiency requires that

$$-\psi\hat{C}^{\frac{1}{v}}\left(\gamma\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{\frac{v-1}{v}}\hat{l}_1^{\psi\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1 = 0. \tag{S.10}$$

This condition is the analog of Condition (A.4) in the main text.
Condition (S.6) then implies that T implements the efficient allocation only if

$$T_1\left(r\right) = \frac{1}{v-1}r+s, \tag{S.11}$$

and

$$T_0\left(r\right) = \frac{1}{v-1}r \tag{S.12}$$

exactly as under the production specification of Subsection 2.1 (see the proof of Lemma 2 in the main text).
Using again (S.5) above, we have that

$$\frac{y_1 p_1}{P} = y_1^{\frac{v-1}{v}}Y^{\frac{1}{v}}\left(\gamma\Theta\left(1+\beta N\right)^{\alpha}\right)^{\frac{v-1}{v}}.$$

Hence, when the labor market clears, the extra profit (net of the subsidy) from producing inputs in their smart specification (relative to the profits of producing them in their traditional specification) is equal to

$$\mathcal{R} = \left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\frac{v-1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)+s\left(\theta\right)-k.$$

Using Condition (S.1) above along with the fact that $y_i = l_i^{\psi}$, the above expression can be rewritten as

$$\begin{aligned}\mathcal{R} &= \left(\frac{v-\psi(v-1)}{v-1}\right)\left(\hat{N}(\theta)\gamma^{\frac{v-1}{v}}\hat{l}_1(\theta)^{\psi\frac{v-1}{v}} + \left(1-\hat{N}(\theta)\right)\hat{l}_0(\theta)^{\psi\frac{v-1}{v}}\right)^{\frac{1}{v}} \cdot \\ &\cdot\theta\left(1+\beta\hat{N}\right)^{\alpha}\left(\gamma\hat{l}_1(\theta)^{\psi\frac{v-1}{v}} - \hat{l}_0(\theta)^{\psi\frac{v-1}{v}}\right) + s(\theta) - k.\end{aligned} \quad \text{(S.13)}$$

Finally, use Condition (2) and (3) in the main text to observe that the formula for $\mathcal{R}$ at the end of the proof of Lemma 2 in the main text coincides with the one in (S.13). Hence Lemma 2 continues to hold under the specification of Subsection 2.2.

That Lemmas 1 and 2 hold under the specification of Subsection 2.2 implies that all the other results in Sections 3 and 4 in the main text for the specification of Subsection 2.1 hold verbatim also for the specification of Subsection 2.2.

Corollary 1. The result follows from the same argument as in the proof in the main text.

Lemma 3. Part 1 follows from the results in Lemma 1. As for Part 2, we show that, when production is efficient, revenues under the specification of Subsection 2.2 coincide with those in Subsection 2.1.

Using (S.2) and (S.8), and recalling that $\hat{y}_0 = \hat{l}_0^{\psi}$, we obtain that

$$\frac{\hat{p}_0}{\hat{P}}\hat{y}_0 = \Theta\left(1+\beta\hat{N}\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\hat{l}_0^{\psi}. \quad \text{(S.14)}$$

Recalling that $\hat{l}_1 = \gamma^{\varphi}\hat{l}_0$, $\hat{y}_1 = \gamma^{\varphi\psi}\hat{l}_0^{\psi}$, and $1 + \frac{\varphi}{1-v} + \varphi\psi = \varphi$, and using (S.7), we also have that

$$\frac{\hat{p}_1}{\hat{P}}\hat{y}_1 = \Theta\left(1+\beta\hat{N}\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\gamma^{\varphi}\hat{l}_0^{\psi}. \quad \text{(S.15)}$$

To see that the revenues in (S.14) and (S.15) coincide with those under the specification in Subsection 2.1, it suffices to use (A.1)-(A.3) to rewrite (A. 27) in the main text.

Propositions 1 and 2. The results follow from the same arguments as in the main text.

Lemma 4. The proof mimicks that in the main text for the specification of Subsection 2.1. Efficiency requires that (refer to Conditions (A.6) and (A.7))

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1}{v}}\left(\gamma\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}}\hat{l}_1(\theta)^{\psi\frac{v-1}{v}},$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1}{v}}\left(\Theta\left(1+\beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}}\hat{l}_0(\theta)^{\psi\frac{v-1}{v}},$$

with $\hat{L}(\theta)$ defined by (A.5). The Dixit and Stiglitz demand system implies that

$$y_1 = Y\left(\frac{p_1}{P}\right)^{-v}\left(\gamma\Theta\left(1+\beta N\right)^{\alpha}\right)^{v-1},$$

$$y_0 = Y\left(\frac{p_0}{P}\right)^{-v}\left(\Theta\left(1+\beta N\right)^{\alpha}\right)^{v-1}.$$

Hence, for efficiency to obtain, the prices set by any two firms producing inputs with the same specification (smart vs. traditional) must coincide. Recalling that $\hat{y}_1(\theta) = \hat{l}_1(\theta)^{\psi}$ and $\hat{y}_0(\theta) = \hat{l}_0(\theta)^{\psi}$, the prices inducing the efficient allocation must thus satisfy

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)\left(\frac{p_1(\theta)}{P(\theta)}\right)^{1-v}\left(\gamma\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{v-1} \tag{S.16}$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)\left(\frac{p_0(\theta)}{P(\theta)}\right)^{1-v}\left(\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{v-1}. \tag{S.17}$$

Recalling that $\hat{l}_1(\theta) = \gamma^{\varphi}\hat{l}_0(\theta)$, we obtain that

$$\frac{\hat{p}_1}{\hat{p}_0} = \gamma^{1+\frac{\varphi}{1-v}}.$$

Using (S.9) the price of the final good is then equal to

$$\hat{P}(\theta) = \frac{\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{1}{1-v}}\hat{p}_0}{\Theta\left(1+\beta\hat{N}(\theta)\right)^{\alpha}}. \tag{S.18}$$

Combining (S.17) with the cash-in-advance constraint $M = PC$, we have that, in each state θ,

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)\hat{P}(\theta)^{v-2}\hat{p}_0^{1-v}\left(\Theta\left(1+\beta\hat{N}(\theta)\right)^{\alpha}\right)^{v-1}.$$

Using (S.18) to express $\hat{P}(\theta)$ as a function of $\hat{N}(\theta)$ and $\hat{p}_0$, and rewriting $\hat{L}(\theta)$ using (A.5), we obtain that, in each state θ, the money supply must be given by

$$\hat{M}(\theta) = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}}\hat{p}_0\left(\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right).$$

Because $\hat{p}_0$ can be taken to be arbitrary, by setting $m = \frac{1}{\psi}\hat{p}_0$, we obtain that

$$\hat{M}(\theta) = m\hat{l}_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}}\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}.$$

The only difference with the specification in Subsection 2.1 is that here the money stock depends directly upon $\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}$ since the price of any intermediate good in terms of the final good is affected by aggregate productivity and by the external effect.

Proposition 3. The proof is in two parts and parallels the one in the main text.
Part 1. Consider first the pricing decision of a firm producing the good in its smart specification. The firm sets p_1 to maximize

$$\mathbb{E}\left[\frac{p_1 y_1 - W l_1}{P} + T_1\left(r_1\right)\middle| x\right], \tag{S.19}$$

where $r_1 = p_1 y_1/P$, taking C, W, and P as given. Using (S.5) and $l_1 = (y_1)^{\frac{1}{\psi}}$, the first-order condition for the maximization of (S.19) with respect to p_1 yields

$$\mathbb{E}\left[\frac{1}{P}\frac{d(p_1 y_1)}{dp_1} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\left(r_1\right)}{dr}\frac{d(p_1 y_1)}{dp_1}\middle| x\right] = 0. \tag{S.20}$$

From (S.5) and $l_1 = (y_1)^{\frac{1}{\psi}}$, we also obtain $dl_1/dp_1 = -vl_1/\psi p_1$ and

$$\frac{d\left(p_1 y_1\right)}{dp_1} = \left(1-v\right)\left(\gamma\Theta\left(1+\beta N\right)^{\alpha}\right)^{v-1}CP^{v}p_1^{-v} = \left(1-v\right)\frac{y_1}{P}.$$

Condition (S.20) can thus be rewritten as

$$\mathbb{E}\left[\left(1-v\right)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT_1\left(r_1\right)}{dr}\frac{\left(1-v\right)y_1}{P}\middle| x\right] = 0.$$

Finally, multiplying all the addenda by p_1/v, we obtain that

$$\mathbb{E}\left[\frac{1-v}{v}\frac{y_1 p_1}{P} + \frac{1}{\psi}\frac{W}{P}l_1 + \frac{1-v}{v}\frac{dT_1\left(r_1\right)}{dr}\frac{y_1 p_1}{P}\middle| x\right] = 0. \tag{S.21}$$

Suppose now that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to choose which specification of the product to provide and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend on the signals x only through the effect that the latter has on the firms' decision of which specification of the input to supply, as in the proof of Lemma 4.

Consistent with the notation used above, we add "hats" to all relevant variables to highlight that these are computed under the efficient rules.

Observe that market clearing in the labor market requires that $\hat{W}/\hat{P} = \hat{L}^{\varepsilon}$, and recall that $\hat{L} = \hat{l}_0 \left[(\gamma^{\varphi} - 1) \hat{N} + 1 \right]$, as shown in the proof of Lemma 1 in the main text. Also, observe that – as established in the proof of Lemma 4 in this supplement – efficiency requires that $\hat{l}_1 \hat{L}^{\varepsilon} = \psi \hat{C}^{\frac{1}{v}} (\gamma \Theta (1 + \beta N)^{\alpha})^{\frac{v-1}{v}} \hat{l}_1^{\psi \frac{v-1}{v}}$. Using (S.5), we obtain that

$$\hat{C}^{\frac{1}{v}} (\gamma \Theta (1 + \beta N)^{\alpha})^{\frac{v-1}{v}} \frac{\hat{P}}{\hat{p}_i} = \hat{y}_1.$$

Recalling also that $\hat{y}_1 = \hat{l}_1^{\psi}$ and using Condition (S.21), we have that each firm choosing the smart specification finds it optimal to set the price $\hat{p}_1$ that sustains the efficient allocation only if

$$\mathbb{E} \left[\frac{1-v}{v} \frac{\hat{y}_1 \hat{p}_1}{\hat{P}} + \frac{\hat{y}_1 \hat{p}_1}{\hat{P}} + \frac{1-v}{v} \frac{dT_1 (\hat{r}_1)}{dr} \frac{\hat{y}_1 \hat{p}_1}{\hat{P}} \middle| x \right] = 0, \tag{S.22}$$

or, equivalently,

$$\mathbb{E} \left[\hat{r}_1 \left(\frac{1}{v} + \frac{1-v}{v} \frac{dT_1 (\hat{r}_1)}{dr} \right) \middle| x \right] = 0,$$

where $\hat{r}_1 = \hat{p}_1 \hat{y}_1 / \hat{P}$.

It follows that, to induce the firm to set the efficient price $\hat{p}_1$ irrespective of his signal x, the fiscal policy must satisfy $dT_1 (r_1) / dr = 1/(v-1)$ for all r_1. Furthermore, as in the main text, one can verify that, when $dT_1 (r_1) / dr = 1/(v-1)$ for all r_1, the firm's payoff is quasi-concave in p_1, which implies that setting the price $p_1 = \hat{p}_1$ is indeed optimal for all x. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient rules and

$$T_1(r) = \frac{r}{v-1} + s = \frac{1}{v-1} \left(\frac{p_1 y_1}{P} \right) + s,$$

where s is invariant in r, the firm's objective is equal to

$$\mathbb{E} \left[\frac{v}{v-1} \frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}} l_1 + s \middle| x \right].$$

Using (S.5) and the fact that $dl_1/dp_1 = -vl_1/\psi p_1$, the first derivative of the firm's objective with respect to p_1 is equal to

$$\mathbb{E} \left[-v \frac{y_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}} \frac{v}{\psi} \frac{l_1}{p_1} \middle| x \right],$$

whereas the second derivative is equal to

$$\mathbb{E}\left[\frac{1}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\middle| x\right].$$

From the analysis above, when $p_1=\hat{p}_1$, $y_1=\hat{y}_1$ and $l_1=\hat{l}_1$ in each state θ. Furthermore, irrespective of x, the derivative of the firm's objective function with respect to p_1, evaluated at $p_1=\hat{p}_1$, is

$$\mathbb{E}\left[-v\frac{\hat{y}_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\middle| x\right]=0. \tag{S.23}$$

Using (S.23), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1=\hat{p}_1$, is negative. Because the firm's objective function has a unique critical point at $p_1=\hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the firms choosing the traditional specification, we have that any fiscal policy that induces efficiency in information usage must pay to each firm selecting the traditional specification a transfer equal to $T_0(r_0)$ such that $dT_0(r_0)/dr=1/(v-1)$, and that any such policy indeed induces these firms to set a price equal to $\hat{p}_0$ irrespective of the signals x.

Part 2. Observe that, under any monetary and fiscal policy that implement the efficient allocation, the real revenues, i.e. the revenues expressed in terms of the consumption of the final good, must be the same as under flexible prices. This follows from the fact that the equilibrium in the market for intermediate goods implies that $\hat{y}_1=\left(\gamma\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{v-1}\hat{C}\left(\hat{P}/\hat{p}_1\right)^{v}$ and $\hat{y}_0=\left(\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{v-1}\hat{C}\left(\hat{P}/\hat{p}_0\right)^{v}$, which means that $\hat{p}_f/\hat{P}$ for $f=0,1$ – and hence $\hat{r}_f=(\hat{p}_f\hat{y}_f)/\hat{P}$ – are uniquely pinned down by the efficient allocation. Because, as in the version of the model in the main text, the transfers to the firms are in terms of real revenues, and because real wages are also uniquely pinned down by the efficient allocation (as $\hat{W}/\hat{P}=\hat{L}^{\varepsilon}$), the value of choosing the smart specification and of acquiring information must coincide with their counterparts under flexible prices. In turn, this implies that the subsidy to the firms choosing the smart specification $s(\theta)$ must satisfy the same conditions as in Lemma 2 when information is exogenous, and those in Lemma 3 when information is endogenous. Finally, that the conclusions in Propositions 1 and 2 hold follows directly from the same arguments as in the proofs of these propositions.

S.2 Richer Economies with Risk-Averse Managers

Consider the following economy in which the production function is the same as in Subsection 2.1. in the main text, but the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. Consistently with the rest of the pertinent literature, we assume that each manager is a member of a representative household, whose utility function is given by

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di,$$

where $R \geq 0$ is the coefficient of relative risk aversion in the consumption of the final good (the case $R = 0$ corresponds to what examined in Section 4 in the main text). The assumption that all managers are members of the same representative household is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the managers to fully insure against idiosyncratic consumption risk. The latter property, in turn, isolates the frictions (and associated inefficiencies) that originate in the interaction between (a) investment spillovers and (b) endogenous private information at the time of the investment decisions from the more familiar inefficiencies that originate in the lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i = l$ for all i), which is a consequence of the assumption that labor is homogenous and exchanged in a competitive market. Being a member of the representative household, each manager maximizes her firm's market valuation taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each manager maximizes

$$\mathbb{E}\left[C^{-R}\left(\frac{p_i y_i - W l_i}{P} + T\right)\middle| x_i; \pi_i^x\right] - kn_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consumption of the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open (but after firms make their investment and pricing decisions). The household faces a cash-in-advance constraint according to which the maximal expenditure on the purchase of the final good cannot exceed M, that is, $PY \leq M$. The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The government maximizes the ex-ante utility of the representative household, which is given by

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x),$$

by means of a monetary policy $M(\cdot)$ and a fiscal policy $T(\cdot)$, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in Section 4 in the main text (note, in particular, that prices are set under dispersed information about θ, that is, each p_i is based on x_i instead of θ). This richer economy is consistent with most of the assumptions typically made in the pertinent literature.

S.2.1 Efficient Allocation

The following proposition characterizes the efficient allocation in this economy.

Proposition S.1. *(1) Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$ *and* $\bar{R} \equiv 1 - \frac{(v-1)(1+\varepsilon)}{(1+\varepsilon)v+\varepsilon\psi(1-v)}$. *Assume that* $\gamma^{\varphi} \geq 1+\beta$, $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$, *and* $0 \leq R \leq \bar{R}$. *For any precision of private information* π^x, *there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency requires that* $\hat{n}\left(x;\pi^x\right) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. *The threshold* $\hat{x}(\pi^x)$, *along with the functions* $\hat{N}\left(\theta;\pi^x\right)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$, *satisfy the following properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \right.$$
$$\left. \times\left(\frac{\gamma^{\varphi}-1}{\varphi\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)} + \frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^x\right)}\right)\middle|\hat{x}(\pi^x),\pi^x\right] = k,$$

$$\hat{N}\left(\theta;\pi^x\right) = 1 - \Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \times$$
$$\times\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \quad \text{(S.24)}$$

and

$$\hat{l}_1(\theta;\pi^x) = \gamma^{\varphi}\hat{l}_0(\theta;\pi^x), \tag{S.25}$$

where $\Theta \equiv \exp(\theta)$.

(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}+\frac{v}{v-1}\frac{\left(\gamma^{\varphi}-1\right)}{\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]+$$
$$+\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left[\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right]^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^{x}}.$$

The restriction $0 \leq R \leq \bar{R}$ guarantees that the marginal utility of consuming the final good does not decrease 'too quickly' with C. Along with the other restrictions in the proposition, which are the same as in Lemma 1 in the main text, this property implies that the efficient investment strategy is monotone. When, instead, $R > \bar{R}$, a higher value of θ may entail a low enough marginal utility of consumption to induce the planner to ask some firms receiving a high signal to refrain from investing. As we clarify below, our key results extend to this case, but the exposition is less transparent.

S.2.2 Equilibrium Allocation

Firms make their investment decisions and set the price for their intermediate goods under dispersed information about θ. Given these choices, they acquire labor l to meet their demands, after observing θ and aggregate investment N. In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by the same conditions as in the main text. Likewise for the labor demands. Because labor is undifferentiated and the labor market is competitive, the supply of labor is then given by

$$\frac{W}{P}C^{-R}=l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P units, starting from a level of consumption equal to C. Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R}=\left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1\left(x;\pi^x\right)$ and $l_1\left(x,\theta;\pi^x\right)$ denote the equilibrium price and labor demand, respectively, of each investing firm. The corresponding functions for the firms that do not invest are $p_0\left(x;\pi^x\right)$ and $l_0\left(x,\theta;\pi^x\right)$.[2]

[2]As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ,

The above equilibrium conditions are standard. The following definition identifies the components of the equilibrium allocation that are most relevant for our analysis.

Definition S.1. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then invests according to $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition S.2. The monetary policy $M^*(\cdot)$ along with the fiscal policy $T^*(\cdot)$ are **optimal** if they implement the efficient acquisition and usage of information as an equilibrium. That is, if they induce all firms to choose the efficient precision of information π^{x*}, follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and set prices according to rules $\hat{p}_0(x;\pi^{x*})$ and $\hat{p}_1(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to $\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$ and result in firms employing labor according to the efficient schedules $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta;\pi^x)$ denote the optimal money supply in state θ. The following lemma characterizes the monetary policy $\hat{M}(\cdot;\pi^x)$.

Lemma S.1. *Suppose that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\cdot;\pi^x)$ that, together with some fiscal policy $\hat{T}(\cdot)$, implements the efficient use of information (for precision π^x) as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{\frac{1+\varepsilon}{1-R}} \left((\gamma^{\varphi} - 1)\, \hat{N}\,(\theta;\pi^x) + 1 \right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

for all θ, where m is an arbitrary positive constant. The monetary policy $\hat{M}(\cdot;\pi^x)$ induces all firms that make the same investment decision to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ induces firms to disregard their private information about the fundamentals, and set prices based only on their investment decision. That prices do not respond to firms' information about θ, given the firms' investments, is necessary to avoid allocative distortions in the induced employment

the measure of investing firms N depends on the precision π^x of firms' information.

and productions decisions. Relative prices must not vary with firms' signals about θ when the latter signals are imprecise. The monetary policy in Lemma S.1 is designed so that, even if firms could condition their prices on θ, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supplied in a way that replicates the same allocations sustained when the supply of money is constant and prices are flexible.

Lemma S.1, in turn, permits us to establish the following result.

Proposition S.2. *Irrespective of whether the economy satisfies the conditions in Proposition S.1, the fiscal policy*

$$T_0^*(r) = \frac{1}{v-1}r,$$

and

$$T_1^*(\theta, r) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{1}{v-1}r,$$

along with the monetary policy

$$M^*(\theta) = m\hat{l}_0(\theta;\pi^{x*})^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^{x*})+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

are optimal.

The monetary policy in the proposition (which belongs to the family in Lemma S.1, specialized to $\pi^x = \pi^{x*}$) neutralizes the effects of price rigidity by replicating the same allocations as under flexible prices. When paired with the fiscal policy in the proposition, it guarantees that, if firms were constrained to acquire information of precision π^{x*}, they would follow the efficient rule $\hat{n}(x;\pi^{x*})$ to make their investment decisions and then set prices $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$ that induce the efficient labor demands, and hence the efficient production of the intermediate and final goods. This is accomplished through a fiscal policy that, in addition to offsetting firms' market power with a familiar revenue subsidy $r/(v-1)$, realigns the private value of investing with the social value through an additional subsidy to the investing firms that operates as a Pigouvian correction. As in the baseline economy, the subsidy

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

makes each firm internalize the marginal effect of investment on the production of the final good, in each state θ. Once this realignment is established, the value that firms assign to acquiring information coincides with its social counterpart, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

Appendix

Proof of Proposition S.1. The proof is in two parts, each corresponding to the two claims in the proposition.

Part 1. Fix the precision of private information π^x and then drop it from all expressions to ease the notation. Let $n(x)$ denote the probability that a firm receiving signal x invests, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the investing firms and by those deciding not to invest, respectively. The planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R} d\Omega(\theta) - k \int_\theta N(\theta) d\Omega(\theta) + \\ - \frac{1}{1+\varepsilon} \int_\theta \left[l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) + \\ - \int_\theta \mathcal{Q}(\theta) \left(N(\theta) - \int_x n(x)\, d\Phi(x|\theta) \right) d\Omega(\theta),$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}} (1 - N(\theta)) \right)^{\frac{v}{v-1}}, \tag{S.26}$$

with

$$y_1(\theta) = \gamma \Theta (1 + \beta N(\theta))^\alpha l_1(\theta)^\psi, \tag{S.27}$$

and

$$y_0(\theta) = \Theta (1 + \beta N(\theta))^\alpha l_0(\theta)^\psi. \tag{S.28}$$

Using (S.26) and (S.27), the first-order condition of the planner's problem with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R} \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}} (1 - N(\theta)) \right)^{\frac{1}{v-1}} \left(\gamma \Theta (1 + \beta N(\theta))^\alpha \right)^{\frac{v-1}{v}} l_1(\theta)^{\psi \frac{v-1}{v} - 1} \\ - \left(l_1(\theta)N(\theta) + l_0(\theta)(1 - N(\theta)) \right)^\varepsilon = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1 - N(\theta)), \tag{S.29}$$

and using (S.26), (S.27), and (S.28), we have that the above first-order condition reduces to

$$\psi C(\theta)^{\frac{1-vR}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta) L(\theta)^{\varepsilon}. \tag{S.30}$$

Following similar steps, the first-order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1-vR}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon}. \tag{S.31}$$

Using (S.27) and (S.28), the ratio between (S.30) and (S.31) can be written as

$$\gamma^{\frac{v-1}{v}} \left(\frac{l_1(\theta)}{l_0(\theta)}\right)^{\psi \frac{v-1}{v}} = \frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{S.32}$$

Notice that (S.32) entails that, at the efficient allocation, the total labor demand, as defined in (S.29), is equal to

$$L(\theta) = l_0(\theta)\left[(\gamma^{\varphi} - 1) N(\theta) + 1\right]. \tag{S.33}$$

Using (S.27) and (S.28), we can also write aggregate consumption as

$$C(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha} \left(\gamma^{\frac{v-1}{v}} l_1(\theta)^{\psi \frac{v-1}{v}} N(\theta) + l_0(\theta)^{\psi \frac{v-1}{v}} (1 - N(\theta))\right)^{\frac{v}{v-1}}.$$

Using (S.32), and the fact that

$$\frac{v-1}{v}(1 + \varphi\psi) = \varphi, \tag{S.34}$$

we can rewrite the latter expression as

$$C(\theta) = \Theta\left(1 + \beta N(\theta)\right)^{\alpha} l_0(\theta)^{\psi} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\frac{v}{v-1}}. \tag{S.35}$$

Next, use (S.32) and (S.28) to rewrite (S.31) as

$$\psi\left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{1-vR}{v}} l_0(\theta)^{\psi \frac{1-vR}{v}} \left((\gamma^{\varphi} - 1) N(\theta) + 1\right)^{\frac{1-vR}{v-1}} \times$$
$$\times \left(\Theta\left(1 + \beta N(\theta)\right)^{\alpha}\right)^{\frac{v-1}{v}} l_0(\theta)^{\psi \frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon},$$

which, using (S.33), can be expressed as

$$\psi\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}$$
$$= l_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\varepsilon}.$$

From the derivations above, we have that the efficient labor demands are given by

$$l_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \quad \text{(S.36)}$$

and by (S.32).

Note that $l_0(\theta)>0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique critical point in (l_0, l_1) for each θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0,1]$, we have that

$$\mathcal{Q}(\theta)\equiv C(\theta)^{-R}\frac{dC(\theta)}{dN\left(\theta\right)}-k-L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed, and varying the proportion of investing firms and the amounts that each firm produces (for given investment decision) when N changes.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x)\equiv\int_{\theta}\mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right)=f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x)\overset{sgn}{=}\int_{\theta}\mathcal{Q}(\theta)f\left(\theta|x\right)d\theta=\mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all firms receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]>0$ invest, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]<0$ refrain from investing.

Next, use (S.26) to observe that

$$C(\theta)^{-R}\tfrac{dC(\theta)}{dN(\theta)}=\tfrac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+$$

$$+C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta)+y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))\right],$$

and (S.27) and (S.28) to note that

$$y_1(\theta)^{-\frac{1}{v}}\frac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta)+y_0(\theta)^{-\frac{1}{v}}\frac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))$$

$$=\tfrac{\alpha\beta}{1+\beta N(\theta)}\left(y_1(\theta)^{\frac{v-1}{v}}N\left(\theta\right)+y_0(\theta)^{\frac{v-1}{v}}\left(1-N\left(\theta\right)\right)\right)=\tfrac{\alpha\beta}{1+\beta N(\theta)}C(\theta)^{\frac{v-1}{v}},$$

where the last equality uses again (S.26).

Finally, using (S.30) and (S.31), we have that

$$\psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

We conclude that

$$\mathcal{Q}(\theta)=\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+C(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N\left(\theta\right)}-k.$$

Using (S.27), (S.28), (S.32), (S.34), and (S.35), after some manipulations, we have that

$$C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=$$
$$=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right). \quad \text{(S.37)}$$

Using (S.35), we also have that

$$C(\theta)^{1-R}=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}.$$

It follows that

$$\mathcal{Q}(\theta)=\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi[\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1]}+\frac{\alpha\beta}{1+\beta N\left(\theta\right)}\right)-k.$$

Next, recall that the optimal labor demand for the non-investing firms is given by (S.36). Replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\mathcal{Q}(\theta)=\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right)-k.$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. In turn, such a property implies that the first-best choice of N is either $N = 0$ or $N = 1$, for all θ. This observation, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ. These properties, in turn, imply that the optimal investment policy is monotone. For any $\hat{x}$, let $\bar{N}(\theta|\hat{x}) \equiv 1 - \Phi(\hat{x}|\theta)$ denote the measure of investing firms at θ when firms follow the monotone rule $n(x) = \mathbb{I}(x > \hat{x})$. Then let

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left((\gamma^{\varphi} - 1)\,\bar{N}(\theta|\hat{x}) + 1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \times$$
$$\times \left(1 + \beta\bar{N}(\theta|\hat{x})\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\frac{\gamma^{\varphi}-1}{\varphi} + \frac{\alpha\beta\left((\gamma^{\varphi}-1)\,\bar{N}(\theta|\hat{x}) + 1\right)}{1+\beta\bar{N}(\theta|\hat{x})}\right) - k$$

denote the function $\mathcal{Q}(\theta)$ characterized above, specialized to $N(\theta) = \bar{N}(\theta|\hat{x})$.

Observe that, under the parameters' restrictions in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that $\lim_{\hat{x}\to-\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] < 0 < \lim_{\hat{x}\to+\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$. Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] = 0$ admits exactly one solution. Letting $\hat{x}$ denote the solution to this equation, we have that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] < 0$ for $x < \hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] > 0$ for $x > \hat{x}$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ such that the investment strategy $\hat{n}\,(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition satisfy all the first-order conditions of the planner's problem. The threshold $\hat{x}(\pi^x)$ solves

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^x) + 1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \left(1+\beta\hat{N}\,(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times \right.$$
$$\left. \times \left(\frac{\gamma^{\varphi}-1}{\varphi} + \frac{\alpha\beta\left((\gamma^{\varphi}-1)\,\hat{N}\,(\theta;\pi^x)+1\right)}{1+\beta\hat{N}\,(\theta;\pi^x)}\right) \right| \hat{x}(\pi^x), \pi^x \Bigg] = k,$$

with $\hat{N}\,(\theta;\pi^x) = 1 - \Phi(\hat{x}(\pi^x)|\theta;\pi^x)$.

Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}\,(x;\pi^x)$ is monotone), any solution to the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions

(S.24) and (S.25) in the proposition and $\hat{n}(x;\pi^x) = \mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x] > 0)$, where

$$\hat{\mathcal{Q}}(\theta;\pi^x) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left((\gamma^\varphi - 1)\,\hat{N}(\theta;\pi^x) + 1 \right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1} \times$$
$$\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\frac{\gamma^\varphi - 1}{\varphi} + \frac{\alpha\beta\left((\gamma^\varphi-1)\,\hat{N}(\theta;\pi^x)+1\right)}{1+\beta\hat{N}(\theta;\pi^x)} \right) - k,$$

with $\hat{N}(\theta;\pi^x) = \int_\theta \hat{n}(x;\pi^x)\, d\Phi(x|\theta,\pi^x)$.

Part 2. For any precision of private information π^x, use Conditions (S.33) and (S.35) in part (1) to write ex-ante welfare as

$$\begin{aligned}
&\mathbb{E}\left[\mathcal{W}|\pi^x\right] = \\
&= \frac{1}{1-R}\int_\theta \Theta^{1-R}\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha(1-R)} \hat{l}_0(\theta;\pi^x)^{\psi(1-R)} \left((\gamma^\varphi-1)\,\hat{N}(\theta;\pi^x)+1\right)^{\frac{v}{v-1}(1-R)} d\Omega(\theta) + \\
&\quad - k\int_\theta \hat{N}(\theta;\pi^x)\, d\Omega(\theta) - \int_\theta \frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[(\gamma^\varphi-1)\,\hat{N}(\theta;\pi^x)+1\right]^{1+\varepsilon} d\Omega(\theta) - \mathcal{I}(\pi^x).
\end{aligned}$$

Using the envelope theorem, we have that the marginal effect of a variation in the precision of private information on welfare is given by

$$\begin{aligned}
&\frac{d\mathbb{E}\left[\mathcal{W}|\pi^x\right]}{d\pi^x} = \\
&= \mathbb{E}\left[\hat{C}(\theta;\pi^x)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)} + \frac{v(\gamma^\varphi-1)}{(v-1)\left((\gamma^\varphi-1)\,\hat{N}(\theta;\pi^x)+1\right)}\right)\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\right] + \\
&-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\right] + \mathbb{E}\left[\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}\left((\gamma^\varphi-1)\,\hat{N}(\theta;\pi^x)+1\right)^\varepsilon(\gamma^\varphi-1)\frac{\partial\hat{N}(\theta;\pi^x)}{\partial\pi^x}\right] - \frac{d\mathcal{I}(\pi_x)}{d\pi_x}.
\end{aligned}$$

The result in part 2 then follows from the fact that, at the optimum, the above derivative must be equal to zero. Q.E.D.

Proof of Lemma S.1. We drop π^x from all formulas to ease the notation. Recall that

$$\hat{l}_1(\theta)\hat{L}(\theta)^\varepsilon = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^\varepsilon = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}.$$

The Dixit and Stiglitz demand system implies that $y_i = C\,(P/p_i)^v$. Hence, the prices set by any two investing firms coincide, implying that they are independent of the signal x. Let $\hat{p}_1$

be the (state-invariant) price set by the investing firms, and $\hat{p}_0$ that set by the non-investing firms. Let $\hat{P}(\theta)$ denote the price of the final good when all firms follow the efficient policies. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}(\theta)/\hat{p}_1\right)^{v-1}, \tag{S.38}$$

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}(\theta)/\hat{p}_0\right)^{v-1}, \tag{S.39}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (S.32), implies that $\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0$. The price of the final good is then equal to

$$\hat{P}(\theta) = \left((\gamma^{\varphi} - 1)\hat{N}(\theta) + 1\right)^{\frac{1}{1-v}}\hat{p}_0. \tag{S.40}$$

Combining the cash-in-advance constraint $M = PC$ with (S.39), we then have that

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\hat{P}(\theta)^{v+R-2}\hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\left((\gamma^{\varphi} - 1)\hat{N}(\theta) + 1\right)^{\frac{v+R-2}{1-v}}\hat{p}_0^{R-1},$$

where we also used (S.40). Finally, using Condition (S.33), we obtain that

$$\hat{M}(\theta)^{1-R} = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left((\gamma^{\varphi} - 1)\hat{N}(\theta) + 1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}}\hat{p}_0^{1-R}.$$

It is immediate to verify that the same conclusion can be obtained starting from (S.38). Because $\hat{p}_0^{1-R}$ can be taken to be arbitrary, the result in the lemma obtains by setting $m^{1-R} = \hat{p}_0^{1-R}/\psi$. Q.E.D.

Proof of Proposition S.2. The proof is in two parts and establishes a more general result than the one in the proposition. Part 1 fixes the precision of information and identifies a condition on the fiscal policy $T(\cdot)$ that guarantees that, when $T(\cdot)$ is paired with the monetary policy of Lemma S.1, and the economy satisfies the parameters' restrictions of Proposition S.1, firms have incentives to use information efficiently when the latter is exogenous. Part 2 identifies an additional restriction on the fiscal policy that, when combined with the condition in part 1, guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, agents have also incentives to acquire information efficiently. The arguments in parts 1 and 2 also allow us to establish that, irrespective of whether or not the economy satisfies the parameters' restrictions of Proposition S.1, when $M(\cdot)$ and $T(\cdot)$ are the specific policies of Proposition S.2, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same.

Part 1. We fix the precision of information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions when there is no risk of confusion.
Consider first the pricing decision of an investing firm. The firm sets p_1 to maximize

$$\mathbb{E}\left[C^{-R}\left(\frac{p_1 y_1 - W l_1}{P} + T_1(r_1)\right)\middle| x\right], \tag{S.41}$$

where $r_1 = p_1 y_1 / P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^v, \tag{S.42}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta(1+\beta N)^{\alpha}}\right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.41) with respect to p_1 is given by

$$\mathbb{E}\left[C^{-R}\left((1-v)CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1(r_1)}{dr}\frac{d(p_1 y_1)}{dp_1}\right)\middle| x\right] = 0. \tag{S.43}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{S.44}$$

$$\frac{d(p_1 y_1)}{dp_1} = (1-v)CP^v p_1^{-v},$$

and (S.42), we have that (S.43) can be rewritten as

$$\mathbb{E}\left[C^{-R}\left((1-v)\frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT_1(r_1)}{dr}\frac{(1-v)y_1}{P}\right)\middle| x\right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}C^{-R}\frac{y_1 p_1}{P} + \frac{1}{\psi}C^{-R}\frac{W}{P}l_1 + \frac{1-v}{v}C^{-R}\frac{dT_1(r_1)}{dr}\frac{y_1 p_1}{P}\middle| x\right] = 0. \tag{S.45}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to determine whether or not to invest, and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the investment decision but not on the signal x, as in the proof of Lemma S.1. Hereafter, we add 'hats' to all relevant variables to highlight that these are computed under the efficient policies.

Observe that market clearing in the labor market requires that

$$\hat{C}^{-R}\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}, \tag{S.46}$$

and recall that, as established in the Proof of Proposition S.1, $\hat{L} = \hat{l}_0\left[\left(\gamma^{\varphi}-1\right)\hat{N}+1\right]$.Also, consider that efficiency requires that $-\psi\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon}\hat{l}_1 = 0$. Accordingly, using Condition (S.45), we have that each investing firm finds it optimal to set the price $\hat{p}_1$ only if

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}} + \frac{1-v}{v}C^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\middle| x\right] = 0, \tag{S.47}$$

where $\hat{r}_1 = \hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.42), we have that $\hat{y}_1^{-\frac{1}{v}} = \hat{C}^{-\frac{1}{v}}\hat{p}_1/\hat{P}$, which allows us to rewrite Condition (S.47) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}} + \frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\middle| x\right] = 0,$$

or, equivalently,

$$\mathbb{E}\left[\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v} + \frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\middle| x\right] = 0.$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr = 1/(v-1)$, the first-order condition of the firm's optimization problem with respect to its price is satisfied. Furthermore, under the proposed fiscal policy, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1 = \hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r) = \frac{r}{v-1} + s = \frac{1}{v-1}\left(\frac{p_1 y_1}{P}\right) + s,$$

where s may depend on θ but is invariant in r, the firm's objective (S.41) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}l_1 + s\left(\theta\right)\right)\middle| x\right].$$

Using (S.42) and (S.44), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\middle| x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\middle| x\right].$$

From the analysis above, we have that $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ when $p_1 = \hat{p}_1$. Furthermore, irrespective of x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\middle| x\right]=0. \tag{S.48}$$

Using (S.48), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique critical point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the non-investing firms, we have that a fiscal policy that pays to each non-investing firm a transfer equal to $T_0(r) = r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' investment choice. Hereafter, we reintroduce θ in the notation. When

$$T_0\left(r\right)=\frac{1}{v-1}r, \tag{S.49}$$

and

$$T_1\left(\theta,r\right)=s(\theta)+\frac{1}{v-1}r, \tag{S.50}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by investing, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by not investing, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Let

$$\hat{\mathcal{R}}(\theta)\equiv\hat{C}\left(\theta\right)^{-R}\left(\hat{r}_1(\theta)-\hat{r}_0(\theta)-\frac{\hat{W}(\theta)}{\hat{P}\left(\theta\right)}\left(\hat{l}_1(\theta)-\hat{l}_0(\theta)\right)+T_1\left(\theta,\hat{r}_1(\theta)\right)-T_0\left(\hat{r}_0(\theta)\right)\right)-k,$$

where $\hat{r}_1(\theta)$ and $\hat{r}_0(\theta)$ are the firm's (real) revenues when the firm follows the efficient policies, respectively, after investing and not investing. Each firm receiving signal x finds it optimal to invest if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] > 0$, and not to invest if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] < 0$. Recall from (S.42) that the Dixit and Stiglitz demand system implies that $\hat{p}_f = \hat{P}(\theta)\,\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f = 0, 1$. Also, recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{C}(\theta)^{-R} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + \\ + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition S.1) $\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon}$,and $\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon}$,we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) = (1-\psi)\,\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Next, use (S.42) to note that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f = 0, 1$. It follows that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}s(\theta) - k. \tag{S.51}$$

Recall from the proof of Proposition S.1 that efficiency requires that each firm invests if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and does not invest if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient investment rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition S.1 (see Equations (S.37) and (S.36), respectively),

$$\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)=$$

$$=\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{1-R}\hat{l}_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right),$$

and

$$\hat{l}_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}.$$

Using the last two expressions, we have that the first addendum in (S.51) can be rewritten as

$$\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)=$$

$$=\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right).$$

When the economy satisfies the conditions in Proposition S.1, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ in (S.51) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the parameters' restrictions in Proposition S.1, a subsidy $s(\theta)$ to the investing firms satisfying conditions (a) and (b) below guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$:

(a) $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ non-decreasing in θ;

(b)

$$\mathbb{E}\left[\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\middle|\,\hat{x}\right]=\mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta)^{1-R}}{1+\beta\hat{N}\left(\theta\right)}\middle|\,\hat{x}\right].$$

The analysis above also reveals that, when the fiscal policy takes the form in (S.49) and (S.50) with $s(\theta)=\alpha\beta\hat{C}(\theta)/\left(1+\beta\hat{N}\left(\theta\right)\right)$, for all θ, and the monetary policy takes the form in Lemma S.1, then irrespective of whether or not the economy satisfies the conditions in Proposition S.1, each firm expecting all other firms to follow the efficient investment rule $\hat{n}(x)$, and setting prices according to $\hat{p}_0$ and $\hat{p}_1$ (thus inducing the efficient employment decisions), finds it optimal to do the same.

Part 2. We now show that, when the economy satisfies the conditions in Proposition S.1, the fiscal policy in (S.49) and (S.50), when paired with the monetary policy

$$M^*(\theta) = m\hat{l}_0\left(\theta;\pi^{x*}\right)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

implement the efficient acquisition and usage of information if and only if the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) in part 1, is such that

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right] = \mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and follow the efficient investment and pricing rules. Consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the proposed fiscal and monetary policies, the firm finds it optimal to set a price equal to $\hat{p}_1^*$ after investing and equal to $\hat{p}_0^*$ if it does not invest, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.

Let

$$\hat{N}^*(\theta) \equiv \hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta) \equiv \hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta) \equiv \hat{l}_1(\theta;\pi^{x*}),$$

$$\hat{y}_1^*(\theta) \equiv \gamma\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_1^*(\theta)^{\psi},$$

$$\hat{y}_0^*(\theta) \equiv \Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_0^*(\theta)^{\psi},$$

$$\hat{C}^*(\theta) = \hat{Y}^*(\theta) \equiv \left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta)+\hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}},$$

$$\hat{W}^*(\theta) \equiv \hat{W}(\theta;\pi^{x*}),$$

and

$$\hat{P}^*\left(\theta\right) \equiv \left(\hat{p}_1^{*1-v}\hat{N}^*\left(\theta\right)+\hat{p}_0^{*1-v}(1-\hat{N}^*\left(\theta\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal policy, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to $\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma;\pi_i^x)$, where

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) \equiv{}& \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x),\end{aligned}$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i invests when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$, respectively in case it invests and in case it does not invest.

Using (S.42), we have that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$ for $f=0,1$. Hence,

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) ={}& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\hat{y}_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x;\varsigma)+\hat{y}_0^{*\frac{v-1}{v}}\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Using

$$\hat{y}_1^* = \gamma\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_1^{*\psi}, \tag{S.52}$$

$$\hat{y}_0^* = \Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}, \tag{S.53}$$

and

$$\hat{l}_1^* = \gamma^{\varphi}\hat{l}_0^*, \tag{S.54}$$

we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) ={}& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] + \\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left((\gamma^{\varphi}-1)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*-\hat{T}_0^*\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}, \quad \text{(S.55)}$$

where $\partial\bar{n}(\pi_i^x;\varsigma)/\partial\pi_i^x$ is the marginal effect of varying π_i^x on the probability that the firm invests at θ, holding fixed the rule ς.

Next, recall again that, for $f=0,1$,

$$\hat{r}_f^* \equiv \frac{\hat{p}_f^*\hat{y}_f^*}{\hat{P}^*} = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}.$$

Using (S.52) and (S.53), we have that

$$\hat{r}_1^*-\hat{r}_0^* = \hat{C}^{*\frac{1}{v}}\Theta^{\frac{v-1}{v}}\left(1+\beta\hat{N}^*\right)^{\alpha\frac{v-1}{v}}\left(\gamma^{\frac{v-1}{v}}\hat{l}_1^{*\psi\frac{v-1}{v}}-\hat{l}_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (S.54) and the structure of the proposed fiscal policy, we have that

$$\hat{T}_1^*-\hat{T}_0^* = s+\frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\,\hat{l}_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (S.55), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right]+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}.$$

Next recall that, when $\pi_i^x=\pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma=\hat{n}^*$, where $\hat{n}^*(x)\equiv\hat{n}(x;\pi^{x*})$ is the efficient investment choice for a firm receiving signal x

after acquiring information of precision π^{x*}. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial \hat{N}^*/\partial \pi^x$ is the marginal change in the measure of investing firms that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding the strategy $\hat{n}^*$ fixed. Note that, in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x;\varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*};\hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the fiscal policy to induce efficiency in information acquisition (when paired with the monetary policy in the proposition), it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]=\frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(S.56)}$$

Next, use (S.46) and (S.54) to note that

$$\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*} = \left(\hat{l}_1^*\hat{N}^* + \hat{l}_0^*\left(1-\hat{N}^*\right)\right)^{\varepsilon} = \hat{l}_0^{*\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}.$$

Hence, using the fact that $\hat{C}^{*\frac{1-vR}{v}} = \hat{C}^{*1-R}\hat{C}^{*\frac{1-v}{v}}$, along with the fact that, as shown in the proof of Proposition S.1,

$$\hat{C}^* = \Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\frac{v}{v-1}},$$

we have that

$$\hat{C}^{*\frac{1-vR}{v}} = \hat{C}^{*1-R}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{1-v}{v}} \hat{l}_0^{*\psi\frac{1-v}{v}} \frac{1}{\left(\gamma^{\varphi}-1\right)\hat{N}^*+1}.$$

It follows that (S.56) is equivalent to

$$\begin{aligned}\mathbb{E}\left[\frac{v\left(\gamma^{\varphi}-1\right)\hat{C}^{*1-R}}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+ \\ -\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+ \\ +\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}.\end{aligned} \tag{S.57}$$

Recall that the efficient precision of private information π^{x*} solves

$$\begin{aligned}\mathbb{E}\left[\hat{C}^{*1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}^*}+\frac{v\left(\gamma^{\varphi}-1\right)}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right. \\ \left.+\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}.\end{aligned} \tag{S.58}$$

Comparing (S.57) with (S.58), we have that, for the policy T to implement the efficient acquisition and usage of information (when paired with the monetary policy in the proposition, which, by virtue of Lemma S.1, is the only monetary policy that can induce efficiency in both information usage and information acquisition), the subsidy s to the investing firms must satisfy the following condition

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right],$$

where we reintroduce the arguments of the various functions.

Finally, note that, independently of whether the economy satisfies the conditions in Proposition S.1, when the subsidy to the investing firms is equal to

$$s(\theta)=\frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

in each state, then, as shown in part 1, the private value $\mathcal{R}$ that each firm assigns to investing coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting all other firms to do the same. This establishes the claim in the proposition. Q.E.D.