Optimal Fiscal and Monetary Policy with Investment Spillovers and Endogenous Private Information

Online Supplement

Luca Colombo[*] Gianluca Femminis[†] Alessandro Pavan[‡]

July 28, 2023

Abstract

This document contains an extension to a family of economies in which the firms' managers, and hence the representative household, are risk averse; that is, they have diminishing marginal utility for the consumption of the final good. All numbered items in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text.

[*]Università Cattolica del Sacro Cuore (lucava.colombo@unicatt.it).
[†]Università Cattolica del Sacro Cuore (gianluca.femminis@unicatt.it).
[‡]Northwestern University and CEPR (alepavan@northwestern.edu).

S.1 Richer Economies with Risk-Averse Managers

Consider the following economy in which the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. Consistently with the rest of the pertinent literature, we assume that each manager is a member of a representative household, whose utility function is given by

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di,$$

where $R \geq 0$ is the coefficient of relative risk aversion in the consumption of the final good (the case $R = 0$ corresponds to what examined in Section 4 in the main text). The assumption that all managers are members of the same representative household is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the managers to fully insure against idiosyncratic consumption risk. The latter property, in turn, isolates the frictions (and associated inefficiencies) that originate in the presence of investment spillovers and endogenous dispersion of information at the time of technology adoption from the more familiar inefficiencies that originate in the lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i = l$ for all i), which is a consequence of the assumption that labor is homogenous and exchanged in a competitive market. Being a member of the representative household, each manager maximizes her firm's market valuation taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each manager maximizes

$$\mathbb{E}\left[C^{-R}\left(\frac{p_i y_i - W l_i}{P} + T\right)\middle| x_i; \pi_i^x\right] - kn_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consumption of the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open (but after firms make their technology and price decisions). The household faces a cash-in-advance constraint according to which the maximal expenditure on the purchase of the final good cannot exceed M, that is,

$$PY \leq M.$$

The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The government maximizes the ex-ante utility of the representative household, which is given by

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x),$$

by means of a monetary policy $M(\cdot)$ and a fiscal policy $T(\cdot)$, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in Section 4 in the main text (note, in particular, that prices are set under dispersed information about θ, that is, each p_i is based on x_i instead of θ). This richer economy is consistent with most of the assumptions typically made in the pertinent Macroeconomics literature.

S.1.1 Efficient Allocation

The following proposition characterizes the efficient allocation in this economy.

Proposition S.1. *(1) Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$ *and* $\bar{R} \equiv 1 - \frac{(v-1)(1+\varepsilon)}{(1+\varepsilon)v+\varepsilon\psi(1-v)}$. *Assume that* $\gamma^{\varphi} \geq 1+\beta$, $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$, *and* $0 \leq R \leq \bar{R}$. *For any precision of private information* π^x, *there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency requires that* $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. *The threshold* $\hat{x}(\pi^x)$, *along with the functions* $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$, *satisfy the following properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\right.$$

$$\left.\times\left(\frac{\gamma^{\varphi}-1}{\varphi\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^x)+1\right)}+\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^x)}\right)\middle|\hat{x}(\pi^x),\pi^x\right] = k,$$

$$\hat{N}(\theta;\pi^x) = 1-\Phi\left(\hat{x}(\pi^x)|\theta;\pi^x\right),$$

$$\hat{l}_0(\theta;\pi^x) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\times$$

$$\times\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^{x*})+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \quad \text{(S.1)}$$

and

$$\hat{l}_1(\theta;\pi^x) = \gamma^{\varphi}\hat{l}_0(\theta;\pi^x), \quad \text{(S.2)}$$

where $\Theta \equiv \exp(\theta)$.

(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}+\frac{v}{v-1}\frac{(\gamma^{\varphi}-1)}{\left((\gamma^{\varphi}-1)\hat{N}(\theta;\pi^{x*})+1\right)}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]+$$

$$+\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left[(\gamma^{\varphi}-1)\hat{N}(\theta;\pi^{x*})+1\right]^{\varepsilon}(\gamma^{\varphi}-1)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^x}\right] = \frac{d\mathcal{I}(\pi^{x*})}{d\pi^x}.$$

The restriction $0 \leq R \leq \bar{R}$ guarantees that the marginal utility of consuming the final good does not decrease 'too quickly' with C. Along with the other restrictions in the proposition, which are the same as in Lemma 1 in the main text, this property implies that the efficient investment strategy is monotone. When, instead, $R > \bar{R}$, a higher value of θ may entail a low enough marginal utility of consumption to induce the planner to ask some firms receiving a high signal to refrain from investing in the new technology. As we clarify below, our key results extend to this case, but the exposition is less transparent.

S.1.2 Equilibrium Allocation

Firms choose both their technology and the price for their intermediate goods under dispersed information about θ. Given these choices, they acquire labor l to meet their demands, after observing θ and the total investment N in the new technology. In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by the same conditions as in the main text. Likewise for the labor demands. Because labor is undifferentiated and the labor market is competitive, the supply of labor is then given by

$$\frac{W}{P}C^{-R} = l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P units, starting from a level of consumption equal to C. Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R} = \left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and labor demand, respectively, of each firm that invests in the new technology. The corresponding functions for the firms that retain the old technology are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$.[1]

The above equilibrium conditions are standard. The following definition identifies the components of the equilibrium allocation that are most relevant for our analysis.

Definition S.1. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then chooses its technology according to $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition S.2. The monetary policy $M^*(\cdot)$ along with the fiscal policy $T^*(\cdot)$ are **optimal** if they implement the efficient acquisition and usage of information as an equilibrium. That is, if they induce all firms to choose the efficient precision of information π^{x*}, follow the efficient rule $\hat{n}(x;\pi^{x*})$ to determine whether or not to upgrade their technology, and set prices according to rules $\hat{p}_0(x;\pi^{x*})$

[1] As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ, the measure of firms N adopting the new technology depends on the precision π^x of firms' information.

and $\hat{p}_1(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to $\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$ and result in firms employing labor according to the efficient schedules $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta;\pi^x)$ denote the optimal money supply in state θ. The following lemma characterizes the monetary policy $\hat{M}(\cdot;\pi^x)$.

Lemma S.1. *Suppose that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\cdot;\pi^x)$ that, together with some fiscal policy $\hat{T}(\cdot)$, implements the efficient use of information (for precision π^x) as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{\frac{1+\varepsilon}{1-R}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

for all θ, where m is an arbitrary positive constant. The monetary policy $\hat{M}(\cdot;\pi^x)$ induces all firms with the same technology to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ induces firms to disregard their private information about the fundamentals, and set prices based only on the selected technology. That prices do not respond to firms' information about θ is necessary to avoid allocative distortions in the induced employment and productions decisions. Relative prices must not vary with firms' signals about θ when the latter signals are imprecise. The monetary policy in Lemma S.1 is designed so that, even if firms could condition their prices on θ, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supplied in a way that replicates the same allocations sustained when the supply of money is constant and prices are flexible.

Lemma S.1, in turn, permits us to establish the following result.

Proposition S.2. *Irrespective of whether the economy satisfies the conditions in Proposition S.1, the fiscal policy*

$$T_0^*(r) = \frac{1}{v-1}r,$$

and

$$T_1^*(\theta,r) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{1}{v-1}r,$$

along with the monetary policy

$$M^*(\theta) = m\hat{l}_0(\theta;\pi^{x*})^{\frac{1+\varepsilon}{1-R}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

are optimal.

The monetary policy in the proposition (which belongs to the family in Lemma S.1, specialized to $\pi^x = \pi^{x*}$) neutralizes the effects of price rigidity by replicating the same allocations as under flexible prices. When paired with the fiscal policy in the proposition, it guarantees that, if firms were constrained to acquire information of precision π^{x*}, they would follow the efficient rule $\hat{n}(x;\pi^{x*})$ to choose which technology to operate and then set prices $\hat{p}_0(x;\pi^x)$ and $\hat{p}_1(x;\pi^x)$ that induce the efficient labor demands, and hence the efficient production of the intermediate and final goods. This is accomplished through a fiscal policy that, in addition to offsetting firms' market power with a familiar revenue subsidy $r/(v-1)$, realigns the private value of upgrading the technology with the social value through an additional subsidy to the innovating firms that operates as a Pigouvian correction. As in the baseline economy, the subsidy

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

makes each firm internalize the marginal effect of the investment in the new technology on the production of the final good, in each state θ. Once this realignment is established, the value that firms assign to acquiring information coincides with its social counterpart, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

S.2 Proofs

Proof of Proposition S.1. The proof is in two parts, each corresponding to the two claims in the proposition.

Part 1. Fix the precision of private information π^x and then drop it from all expressions to ease the notation. Let $n(x)$ denote the probability that a firm receiving signal x adopts the new technology, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the firms adopting the new technology and by those retaining the old one, respectively. The planner's problem can be written as

$$\begin{aligned}\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R}d\Omega(\theta) - k\int_\theta N(\theta)d\Omega(\theta) + \\ -\frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon}d\Omega(\theta) + \\ -\int_\theta \mathcal{Q}(\theta)\left(N(\theta)-\int_x n(x)\,\Phi(x|\theta)\right)d\Omega(\theta),\end{aligned}$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\,d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}}N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}, \tag{S.3}$$

with

$$y_1(\theta) = \gamma\Theta\left(1+\beta N(\theta)\right)^\alpha l_1(\theta)^\psi, \tag{S.4}$$

and

$$y_0(\theta) = \Theta(1+\beta N(\theta))^{\alpha} l_0(\theta)^{\psi}. \tag{S.5}$$

Using (S.3) and (S.4), the first-order condition of the planner's problem with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R}\left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}} \left(\gamma\Theta(1+\beta N(\theta))^{\alpha}\right)^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}-1}$$
$$- \left(l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right)^{\varepsilon} = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta)), \tag{S.6}$$

and using (S.3) and (S.4), we have that the above first-order condition reduces to

$$\psi C(\theta)^{\frac{1-vR}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta)L(\theta)^{\varepsilon}. \tag{S.7}$$

Following similar steps, the first-order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1-vR}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta)L(\theta)^{\varepsilon}. \tag{S.8}$$

Using (S.4) and (S.5), the ratio between (S.7) and (S.8) can be written as

$$\gamma^{\frac{v-1}{v}}\left(\frac{l_1(\theta)}{l_0(\theta)}\right)^{\psi\frac{v-1}{v}} = \frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{S.9}$$

Notice that (S.9) entails that, at the efficient allocation, the total labor demand, as defined in (S.6), is equal to

$$L(\theta) = l_0(\theta)\left[(\gamma^{\varphi}-1)N(\theta)+1\right]. \tag{S.10}$$

Using (S.4) and (S.5), we can also write aggregate consumption as

$$C(\theta) = \Theta(1+\beta N(\theta))^{\alpha}\left(\gamma^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}} N(\theta) + l_0(\theta)^{\psi\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}.$$

Using (S.9), we can rewrite the latter expression as

$$C(\theta) = \Theta(1+\beta N(\theta))^{\alpha} l_0(\theta)^{\psi}\left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{v}{v-1}}. \tag{S.11}$$

Next, use (S.9) and (S.11) to rewrite (S.8) as

$$\psi\left(\Theta(1+\beta N(\theta))^{\alpha}\right)^{\frac{1-vR}{v}} l_0(\theta)^{\psi\frac{1-vR}{v}}\left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{1-vR}{v-1}} \times$$
$$\times\left(\Theta(1+\beta N(\theta))^{\alpha}\right)^{\frac{v-1}{v}} l_0(\theta)^{\psi\frac{v-1}{v}} = l_0(\theta)L(\theta)^{\varepsilon},$$

which, using (S.10), can be expressed as

$$\psi\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}$$
$$= l_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\varepsilon}.$$

From the derivations above, we have that the efficient labor demands are given by

$$l_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \tag{S.12}$$

and by (S.9).

Note that $l_0(\theta)>0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1) for each θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0,1]$, we have that

$$\mathcal{Q}(\theta)\equiv C(\theta)^{-R}\frac{dC(\theta)}{dN\left(\theta\right)}-k-L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed, and varying the proportion of firms investing into the new technology and the amounts that each firm produces for given technology choice when N changes.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x)\equiv\int_{\theta}\mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right)=f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x)\overset{sgn}{=}\int_{\theta}\mathcal{Q}(\theta)f\left(\theta|x\right)d\theta=\mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all managers receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]>0$ adopt the new technology, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x]<0$ retain the old one.

Next, use (S.3) to observe that

$$C(\theta)^{-R}\tfrac{dC(\theta)}{dN(\theta)}=\tfrac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+$$

$$+C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta)+y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))\right],$$

and (S.4) and (S.5) to note that

$$y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta)+y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))$$

$$=\tfrac{\alpha\beta}{1+\beta N(\theta)}\left(y_1(\theta)^{\frac{v-1}{v}}N\left(\theta\right)+y_0(\theta)^{\frac{v-1}{v}}\left(1-N\left(\theta\right)\right)\right)=\tfrac{\alpha\beta}{1+\beta N(\theta)}C(\theta)^{\frac{v-1}{v}},$$

where the last equality uses again (S.3).

Finally, using (S.7) and (S.8), we have that

$$\psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=L(\theta)^{\varepsilon}\left(l_1(\theta)-l_0(\theta)\right).$$

We conclude that

$$\mathcal{Q}(\theta)=\left(\frac{v-\psi\,(v-1)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right]+C(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N\,(\theta)}-k.$$

Using (S.4), (S.5), (S.9), and (S.11), after some manipulations, we have that

$$\begin{aligned}C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}}-y_0(\theta)^{\frac{v-1}{v}}\right)=&\\ =\left(\left(\gamma^{\varphi}-1\right)N\,(\theta)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta N\,(\theta)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right).&\end{aligned}\tag{S.13}$$

Using (S.11), we also have that

$$C(\theta)^{1-R}=\left(\left(\gamma^{\varphi}-1\right)N\,(\theta)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\,(\theta)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}.$$

It follows that

$$\begin{aligned}\mathcal{Q}(\theta)=&\left(\left(\gamma^{\varphi}-1\right)N\,(\theta)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta\left(1+\beta N\,(\theta)\right)^{\alpha}\right)^{1-R}l_0(\theta)^{\psi(1-R)}\times\\ &\times\left(\frac{\gamma^{\varphi}-1}{\varphi[\left(\gamma^{\varphi}-1\right)N\,(\theta)+1]}+\frac{\alpha\beta}{1+\beta N\,(\theta)}\right)-k.\end{aligned}$$

Next, recall that the optimal labor demand for the firms retaining the old technology is given by (S.12). Replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\begin{aligned}\mathcal{Q}(\theta)=&\,\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\\ &\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right)-k.\end{aligned}$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. In turn, such a property implies that the first-best choice of N is either $N=0$ or $N=1$, for all θ. This observation, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ. These properties, in turn, imply that the optimal investment policy is monotone. For any $\hat{x}$, let

$$\bar{N}(\theta|\hat{x})\equiv 1-\Phi(\hat{x}|\theta)$$

denote the measure of firms investing in the new technology at θ when firms follow the monotone rule $n(x)=\mathbb{I}(x>\hat{x})$. Then let

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)\bar{N}(\theta|\hat{x})+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta\bar{N}(\theta|\hat{x})\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\bar{N}(\theta|\hat{x})+1\right)}{1+\beta\bar{N}(\theta|\hat{x})}\right)-k$$

denote the function $\mathcal{Q}(\theta)$ characterized above, specialized to $N(\theta)=\bar{N}(\theta|\hat{x})$.

Observe that, under the parameters' restrictions in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that

$$\lim_{\hat{x}\to-\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]<0<\lim_{\hat{x}\to+\infty}\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]=0$ admits exactly one solution. Letting $\hat{x}$ denote the solution to this equation, we have that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]<0$ for $x<\hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x]>0$ for $x>\hat{x}$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ such that the investment strategy $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(x\geq\hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition satisfy all the first-order conditions of the planner's problem. The threshold $\hat{x}(\pi^x)$ solves

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\right.$$
$$\left.\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)}{1+\beta\hat{N}\left(\theta;\pi^x\right)}\right)\middle|\,\hat{x}(\pi^x),\pi^x\right]=k,$$

with $\hat{N}\left(\theta;\pi^x\right)=1-\Phi(\hat{x}(\pi^x)|\theta;\pi^x)$.

Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}\left(x;\pi^x\right)$ is monotone), any solution to the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions (S.1) and (S.2) in the proposition and $\hat{n}\left(x;\pi^x\right)=\mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x]>0)$, where

$$\hat{\mathcal{Q}}(\theta;\pi^x) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi}+\frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta;\pi^x)+1\right)}{1+\beta\hat{N}(\theta;\pi^x)}\right)-k,$$

with $\hat{N}(\theta;\pi^x)=\int_\theta\hat{n}\left(x;\pi^x\right)d\Phi\left(x|\theta,\pi^x\right)$.

Part 2. For any precision of private information π^x, use Conditions (S.10) and (S.11) in part (1) to write ex-ante welfare as

$$\mathbb{E}\left[\mathcal{W}|\pi^x\right]=$$
$$=\frac{1}{1-R}\int_\theta\Theta^{1-R}\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^{\alpha(1-R)}\hat{l}_0\left(\theta;\pi^x\right)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{v}{v-1}(1-R)}d\Omega\left(\theta\right)+$$
$$-k\int_\theta\hat{N}\left(\theta;\pi^x\right)d\Omega\left(\theta\right)-\int_\theta\frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right]^{1+\varepsilon}d\Omega\left(\theta\right)-\mathcal{I}(\pi^x).$$

Using the envelope theorem, we have that the marginal effect of a variation in the precision of private information on welfare is given by

$$\frac{d\mathbb{E}\left[\mathcal{W}|\pi^x\right]}{d\pi^x} =$$

$$= \mathbb{E}\left[\hat{C}\left(\theta;\pi^x\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^x\right)}+\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+$$

$$-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+\mathbb{E}\left[\hat{l}_0\left(\theta;\pi^x\right)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]-\frac{d\mathcal{I}(\pi_x)}{d\pi_x}.$$

The result in part 2 then follows from the fact that, at the optimum, the above derivative must be equal to zero. Q.E.D.

Proof of Lemma S.1. We drop π^x from all formulas to ease the notation. Using (S.7) and (S.8), we have that

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}.$$

The Dixit and Stiglitz demand system implies that $y_i = C\left(P/p_i\right)^v$. Hence, the prices set by any two firms adopting the same technology coincide, implying that they are independent of the signal x. Let $\hat{p}_1$ be the (state-invariant) price set by the firms investing in the new technology, and $\hat{p}_0$ that set by firms retaining the old technology. Let $\hat{P}(\theta)$ denote the price of the final good when all firms follow the efficient policies. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_1\right)^{v-1}, \tag{S.14}$$

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_0\right)^{v-1}, \tag{S.15}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (S.9), implies that

$$\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0.$$

The price of the final good is then equal to

$$\hat{P}\left(\theta\right) = \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1}{1-v}}\hat{p}_0. \tag{S.16}$$

Combining the cash-in-advance constraint $M = PC$ with (S.15), we then have that

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\hat{P}\left(\theta\right)^{v+R-2}\hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi\hat{M}(\theta)^{1-R}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{v+R-2}{1-v}}\hat{p}_0^{R-1},$$

where we also used (S.16). Finally, using Condition (S.10), we obtain that

$$\hat{M}(\theta)^{1-R} = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}}\hat{p}_0^{1-R}.$$

It is immediate to verify that the same conclusion can be obtained starting from (S.14). Because $\hat{p}_0^{1-R}$ can be taken to be arbitrary, the result in the lemma obtains by setting $m^{1-R} = \frac{1}{\psi}\hat{p}_0^{1-R}$. Q.E.D.

Proof of Proposition S.2. The proof is in two parts and establishes a more general result than the one in the proposition. Part 1 fixes the precision of information and identifies a condition on the fiscal policy $T(\cdot)$ that guarantees that, when $T(\cdot)$ is paired with the monetary policy of Lemma S.1, and the economy satisfies the parameters' restrictions of Proposition S.1, firms have incentives to use information efficiently when the latter is exogenous. Part 2 identifies an additional restriction on the fiscal policy that, when combined with the condition in part 1, guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, agents have also incentives to acquire information efficiently. The arguments in parts 1 and 2 also allow us to establish that, irrespective of whether or not the economy satisfies the parameters' restrictions of Proposition S.1, when $M(\cdot)$ and $T(\cdot)$ are the specific policies of Proposition S.2, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same.

Part 1. We fix the precision of information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions when there is no risk of confusion.

Consider first the pricing decision of a firm that adopts the new technology. The firm sets p_1 to maximize

$$\mathbb{E}\left[C^{-R}\left(\frac{p_1 y_1 - W l_1}{P} + T_1\left(r_1\right)\right)\Bigg| x\right], \tag{S.17}$$

where $r_1 = p_1 y_1/P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^{v}, \tag{S.18}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma\Theta\left(1+\beta N\right)^{\alpha}}\right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.17) with respect to p_1 is given by

$$\mathbb{E}\left[C^{-R}\left(\left(1-v\right)CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\left(r_1\right)}{dr}\frac{d(p_1 y_1)}{dp_1}\right)\Bigg| x\right] = 0. \tag{S.19}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{S.20}$$

$$\frac{d\left(p_1 y_1\right)}{dp_1}=\left(1-v\right)CP^v p_1^{-v},$$

and (S.18), we have that (S.19) can be rewritten as

$$\mathbb{E}\left[C^{-R}\left(\left(1-v\right)\frac{y_1}{P}+\frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1}+\frac{dT_1\left(r_1\right)}{dr}\frac{\left(1-v\right)y_1}{P}\right)\middle| x\right]=0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}C^{-R}\frac{y_1 p_1}{P}+\frac{1}{\psi}C^{-R}\frac{W}{P}l_1+\frac{1-v}{v}C^{-R}\frac{dT_1\left(r_1\right)}{dr}\frac{y_1 p_1}{P}\middle| x\right]=0. \tag{S.21}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to determine which technology to use and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the technology they adopted but not on the signal x, as in the proof of Lemma S.1. Hereafter, we add 'hats' to all relevant variables to highlight that these are computed under the efficient policies. Observe that market clearing in the labor market requires that

$$\hat{C}^{-R}\frac{\hat{W}}{\hat{P}}=\hat{L}^{\varepsilon}, \tag{S.22}$$

and recall that, as established in the Proof of Proposition S.1,

$$\hat{L}=\hat{l}_0\left[\left(\gamma^{\varphi}-1\right)\hat{N}+1\right].$$

Also, consider that efficiency requires that

$$-\psi\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1=0.$$

Accordingly, using Condition (S.21), we have that each firm adopting the new technology finds it optimal to set the price $\hat{p}_1$ only if

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}C^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\middle| x\right]=0, \tag{S.23}$$

where $\hat{r}_1=\hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.18), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (S.23) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\middle| x\right]=0,$$

or, equivalently,

$$\mathbb{E}\left[\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\middle| x\right]=0.$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr=1/(v-1)$, the first-order condition of the firm's optimization problem with respect to its price is satisfied. Furthermore, one can verify that, under the proposed fiscal policy, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1=\hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r) = \frac{r}{v-1} + s = \frac{1}{v-1}\left(\frac{p_1 y_1}{P}\right) + s,$$

where s may depend on θ but is invariant in r, the firm's objective (S.17) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1 y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}} l_1 + s(\theta)\right)\middle| x\right].$$

Using (S.18) and (S.20), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\middle| x\right],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}} - \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\middle| x\right].$$

From the analysis above, we have that $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ when $p_1 = \hat{p}_1$. Furthermore, irrespective of x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\middle| x\right] = 0. \tag{S.24}$$

Using (S.24), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique stationary point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the firms retaining the old technology, we have that a fiscal policy that pays each firm retaining the old technology a policy equal to $T_0(r) = r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' technology choice. Hereafter, we reintroduce θ in the notation. When

$$T_0(r) = \frac{1}{v-1}r, \tag{S.25}$$

and

$$T_1(\theta, r) = s(\theta) + \frac{1}{v-1}r, \tag{S.26}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by innovating, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by retaining the old technology, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Let

$$\hat{\mathcal{R}}(\theta) \equiv \hat{C}(\theta)^{-R}\left(\hat{r}_1(\theta) - \hat{r}_0(\theta) - \frac{\hat{W}(\theta)}{\hat{P}(\theta)}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k,$$

where $\hat{r}_1(\theta)$ and $\hat{r}_0(\theta)$ are the firm's (real) revenues when the firm follows the efficient policies, after adopting the new technology and retaining the old one, respectively. Each firm receiving signal x finds it optimal to adopt the new technology if

$$\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0,$$

and retain the old technology if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$. Recall from (S.18) that the Dixit and Stiglitz demand system implies that $\hat{p}_f = \hat{P}(\theta)\,\hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\,\hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f = 0, 1$. Also, recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{C}(\theta)^{-R} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + \\ + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition S.1)

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon},$$

and

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon},$$

we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) \quad = \quad (1-\psi)\,\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) \; + \; \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) \; - \; k.$$

Next, use (S.18) to note that

$$\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}},$$

for $f = 0, 1$. It follows that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}s(\theta) - k. \tag{S.27}$$

Recall from the proof of Proposition S.1 that efficiency requires that each firm adopts the new technology if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and retains the old one if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient technology-adoption rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition S.1 (see Equations (S.13) and (S.12), respectively),

$$\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)= \\ =\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{1-R}\hat{l}_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right),$$

and

$$\hat{l}_0(\theta)=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}.$$

Using the last two expressions, we have that the first addendum in (S.27) can be rewritten as

$$\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}}-\hat{y}_0(\theta)^{\frac{v-1}{v}}\right)= \\ =\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right).$$

When the economy satisfies the conditions in Proposition S.1, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ in (S.27) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the parameters' restrictions in Proposition S.1, a subsidy $s(\theta)$ to the innovating firms satisfying conditions (a) and (b) below guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$:

(a) $$\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\text{ non-decreasing in }\theta;$$

(b) $$\mathbb{E}\left[\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\Big|\,\hat{x}\right]=\mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta)^{1-R}}{1+\beta\hat{N}\left(\theta\right)}\Bigg|\,\hat{x}\right].$$

The analysis above also reveals that, when the fiscal policy takes the form in (S.25) and (S.26) with

$$s(\theta)=\frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}\left(\theta\right)},$$

for all θ, and the monetary policy takes the form in Lemma S.1, then irrespective of whether or not the economy satisfies the conditions in Proposition S.1, each firm expecting all other firms to follow the efficient technology adoption rule $\hat{n}(x)$, and setting prices according to $\hat{p}_0$ and $\hat{p}_1$ (thus inducing the efficient employment decisions), finds it optimal to do the same.

Part 2. We now show that, when the economy satisfies the conditions in Proposition S.1, the fiscal policy in (S.25) and (S.26), when paired with the monetary policy

$$M^*(\theta)=m\hat{l}_0\left(\theta;\pi^{x*}\right)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

implement the efficient acquisition and usage of information if and only if the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) in part 1, is such that

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and follow the efficient technology and pricing rules. Consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the proposed fiscal and monetary policies, the firm finds it optimal to set a price equal to $\hat{p}_1^*$ after adopting the new technology and equal to $\hat{p}_0^*$ after retaining the old one, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.

Let

$$\hat{N}^*(\theta)\equiv\hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta)\equiv\hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta)\equiv\hat{l}_1(\theta;\pi^{x*}),$$

$$\hat{y}_1^*(\theta)\equiv\gamma\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_1^*(\theta)^{\psi},$$

$$\hat{y}_0^*(\theta)\equiv\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_0^*(\theta)^{\psi},$$

$$\hat{C}^*(\theta)=\hat{Y}^*(\theta)\equiv\left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta)+\hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}},$$

$$\hat{W}^*(\theta)\equiv\hat{W}(\theta;\pi^{x*}),$$

and

$$\hat{P}^*\left(\theta\right)\equiv\left(\hat{p}_1^{*1-v}\hat{N}^*\left(\theta\right)+\hat{p}_0^{*1-v}(1-\hat{N}^*\left(\theta\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal policy, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to

$$\bar{\Pi}_i(\pi_i^x)\equiv\sup_{\varsigma:\mathbb{R}\to[0,1]}\Pi_i(\varsigma;\pi_i^x),$$

where

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x)\equiv\;&\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]\\&-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]\\&+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x),\end{aligned}$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x) d\Phi(x|\theta, \pi_i^x)$ denoting the probability that firm i adopts the new technology when using the strategy $\varsigma : \mathbb{R} \to [0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$ under the new and the old technology, respectively. Using (S.18), we have that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}} \hat{y}_f^{*\frac{v-1}{v}}$ for $f = 0, 1$. Hence,

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\hat{y}_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x;\varsigma) + \hat{y}_0^{*\frac{v-1}{v}}\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Using

$$\hat{y}_1^* = \gamma\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_1^{*\psi}, \tag{S.28}$$

$$\hat{y}_0^* = \Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}, \tag{S.29}$$

and

$$\hat{l}_1^* = \gamma^{\varphi}\hat{l}_0^*, \tag{S.30}$$

we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] + \\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma) + \hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right] + \\ &- \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right] + \\ &+ \mathbb{E}\left[\hat{C}^{*-R}\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x},\end{aligned} \tag{S.31}$$

where $\partial\bar{n}(\pi_i^x;\varsigma)/\partial\pi_i^x$ is the marginal effect of varying π_i^x on the probability that the firm adopts the new technology at θ, holding fixed the rule ς.

Next, recall again that, for $f = 0, 1$,

$$\hat{r}_f^* \equiv \frac{\hat{p}_f^*\hat{y}_f^*}{\hat{P}^*} = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}.$$

Using (S.28) and (S.29), we have that

$$\hat{r}_1^* - \hat{r}_0^* = \hat{C}^{*\frac{1}{v}} \Theta^{\frac{v-1}{v}} \left(1 + \beta \hat{N}^*\right)^{\alpha \frac{v-1}{v}} \left(\gamma^{\frac{v-1}{v}} \hat{l}_1^{*\psi \frac{v-1}{v}} - \hat{l}_0^{*\psi \frac{v-1}{v}}\right).$$

Therefore, using (S.30) and the structure of the proposed fiscal policy, we have that

$$\hat{T}_1^* - \hat{T}_0^* = s + \frac{1}{v-1} \hat{C}^{*\frac{1}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \hat{l}_0^{*\psi \frac{v-1}{v}}.$$

Substituting this expression in (S.31), we obtain that

$$\frac{\partial \Pi_i(\varsigma; \pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1} \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \frac{\partial \bar{n}(\pi_i^x; \varsigma)}{\partial \pi_i^x} \hat{l}_0^{*\psi \frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R} \frac{\hat{W}^*}{\hat{P}^*} \left(\left(\gamma^{\varphi} - 1\right) \hat{l}_0^* \frac{\partial \bar{n}(\pi_i^x; \varsigma)}{\partial \pi_i^x}\right)\right] + \mathbb{E}\left[\hat{C}^{*-R} s \frac{\partial \bar{n}(\pi_i^x; \varsigma)}{\partial \pi_i^x}\right] - k \mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x; \varsigma)}{\partial \pi_i^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Next recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$, where $\hat{n}^*(x) \equiv \hat{n}(x; \pi^{x*})$ is the efficient technology choice for a firm receiving signal x after acquiring information of precision π^{x*}. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*; \pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1} \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \frac{\partial \hat{N}^*}{\partial \pi^x} \hat{l}_0^{*\psi \frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R} \frac{\hat{W}^*}{\hat{P}^*} \left(\left(\gamma^{\varphi} - 1\right) \hat{l}_0^* \frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right] + \mathbb{E}\left[\hat{C}^{*-R} s \frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k \mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] - \frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial \hat{N}^* / \partial \pi^x$ is the marginal change in the measure of firms adopting the new technology that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding the strategy $\hat{n}^*$ fixed. Note that, in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x; \varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*}; \hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the fiscal policy to induce efficiency in information acquisition (when paired with the monetary policy in the proposition), it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1} \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}} \left(\Theta \left(1 + \beta \hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi} - 1\right) \frac{\partial \hat{N}^*}{\partial \pi^x} \hat{l}_0^{*\psi \frac{v-1}{v}}\right] +$$
$$- \mathbb{E}\left[\hat{C}^{*-R} \frac{\hat{W}^*}{\hat{P}^*} \left(\left(\gamma^{\varphi} - 1\right) \hat{l}_0^* \frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right] +$$
$$+ \mathbb{E}\left[\hat{C}^{*-R} s \frac{\partial \hat{N}^*}{\partial \pi^x}\right] - k \mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right] = \frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(S.32)}$$

Next, use (S.22) and (S.30) to note that

$$\hat{C}^{*-R}\frac{\hat{W}^{*}}{\hat{P}^{*}}=\left(\hat{l}_{1}^{*}\hat{N}^{*}+\hat{l}_{0}^{*}\left(1-\hat{N}^{*}\right)\right)^{\varepsilon}=\hat{l}_{0}^{*\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^{*}+1\right)^{\varepsilon}.$$

Hence, using the fact that $\hat{C}^{*\frac{1-vR}{v}}=\hat{C}^{*1-R}\hat{C}^{*\frac{1-v}{v}}$, along with the fact that, as shown in the proof of Proposition S.1,

$$\hat{C}^{*}=\Theta\left(1+\beta\hat{N}^{*}\right)^{\alpha}\hat{l}_{0}^{*\psi}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^{*}+1\right)^{\frac{v}{v-1}},$$

we have that

$$\hat{C}^{*\frac{1-vR}{v}}=\hat{C}^{*1-R}\left(\Theta\left(1+\beta\hat{N}^{*}\right)^{\alpha}\right)^{\frac{1-v}{v}}\hat{l}_{0}^{*\psi\frac{1-v}{v}}\frac{1}{\left(\gamma^{\varphi}-1\right)\hat{N}^{*}+1}.$$

It follows that (S.32) is equivalent to

$$\mathbb{E}\left[\frac{v\left(\gamma^{\varphi}-1\right)\hat{C}^{*1-R}}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^{*}+1\right)}\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]+$$
$$-\mathbb{E}\left[\hat{l}_{0}^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^{*}+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^{x}}. \quad \text{(S.33)}$$

Recall that the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[\hat{C}^{*1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}^{*}}+\frac{v\left(\gamma^{\varphi}-1\right)}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^{*}+1\right)}\right)\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]$$
$$+\mathbb{E}\left[\hat{l}_{0}^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^{*}+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^{*}}{\partial\pi^{x}}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_{x}}. \quad \text{(S.34)}$$

Comparing (S.33) with (S.34), we have that, for the policy T to implement the efficient acquisition and usage of information (when paired with the monetary policy in the proposition, which, by virtue of Lemma S.1, is the only monetary policy that can induce efficiency in both information usage and information acquisition), the subsidy s to the innovating firms must satisfy the following condition

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right],$$

where we reintroduce the arguments of the various functions.

Finally, note that, independently of whether the economy satisfies the conditions in Proposition S.1, when the subsidy to the innovating firms is equal to

$$s(\theta)=\frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})}$$

in each state, then, as shown in part 1, the private value $\mathcal{R}$ that each firm assigns to adopting the new technology coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting all other firms to do the same. This establishes the claim in the proposition. Q.E.D.