Investment Subsidies with Spillovers and Endogenous Private Information: Why Pigou Got it All Right*

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Abstract

How should firms be incentivized to invest efficiently (e.g., in AI or the supply of smart inputs) when such investments come with spillovers and their profitability depends on uncertain aggregate economic conditions? We show that firms can be encouraged to collect information and then use it in society's best interest through a subsidy that resembles a Pigouvian correction but accounts for the non-verifiability of firms' acquisition and usage of information.

Keywords: endogenous information, investment spillovers, optimal fiscal policy, Pigouvian corrections

JEL classification: D21, D62, D83, E60, E62

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1 Introduction

New technologies typically come with spillovers that influence firms' investment decisions. In fact, one of the key challenges that most firms face these days is whether to switch to new AI-based technologies or keep operating under traditional ones. The new technologies are typically superior. However, the profitability of adopting them is often uncertain. Importantly, this uncertainty is endogenous as firms can collect information about relevant economic fundamentals affecting the profitability of the investment decisions. In such contexts, how should a benevolent government use subsidies to encourage firms to collect information and use it in society's best interest? This question is at the center of an active debate, as many countries are devoting significant resources to incentivize firms to familiarize themselves with new technologies, develop "smart" inputs, provide critical infrastructure, and, more broadly, invest in sectors, products, and production processes of strategic importance.

In this paper, we consider a stylized but flexible framework that permits us to capture some of the key trade-offs that firms face in a broad class of investment problems with spillovers and endogenous private information. We show that firms can be induced to invest efficiently by combining familiar revenue subsidies correcting for firms' market power with additional subsidies to the investing firms designed to induce firms to invest when, and only when, investment is socially efficient, not ex-post but given firms' limited information about the relevant fundamentals.

We also show that, when information is dispersed but exogenous and the efficient investment decisions are monotone in fundamentals, these additional subsidies are simple, namely, they need not condition on the aggregate economic conditions and/or other firms' decisions. When, instead, firms must also be incentivized to collect information in society's best interest, it is essential to condition the subsidies on aggregate fundamentals. Furthermore, if the cost of information is unknown to the government, the optimal subsidy must also condition on the cross-sectional distribution of firms' investment decisions. Such richer subsidies operate as a Pigouvian correction, realigning the private value of investment to its social counterpart, by inducing firms to internalize the externality associated with the spillovers generated by their investment decisions. Importantly, these Pigouvian-like policies also realign the private value of acquiring more precise information to its social counterpart, accounting for the fact that neither the acquisition nor the usage of information is verifiable. The property that, when information is complete and firms' activities are verifiable, Pigouvian subsidies/taxes correct externalities and induce efficient allocations is known. The paper's contribution is in showing that a specific version of these policies also creates the right incentives for information acquisition and its subsequent utilization when neither of the two activities is verifiable.

In our model, the externality originates in investment spillovers. We expect Pigouvian policies similar to those discussed in the paper to induce efficiency in information acquisition and usage also in the presence of other externalities such as those associated with pollution and/or the adoption of "green" technologies.

Related literature. Government interventions under endogenous private information have been studied by a few recent papers in macroeconomics. Angeletos and La'O (2020) study optimal fiscal and monetary policy over the business cycle with dispersed information, in a setup in which inefficiencies in the acquisition and usage of information originate in sticky prices and monopolistic competition. Under the familiar constant elasticity of substitution (CES) structure, the paper shows that efficiency in the usage of information can be obtained with an appropriate combination of fiscal policy (a proportional subsidy on revenues correcting for firms' market power) and monetary policy (with the latter replicating the flexible price allocation by inducing firms to disregard their private information when setting prices). The same policy mix induces firms to collect private information in society's best interest. Our contribution is in introducing investment spillovers, showing how the latter interact with the acquisition of private information in a stylized but standard general-equilibrium model, and investigating how the interaction shapes the design of optimal subsidies. We find that some of the policies that induce efficiency in the usage of information need not induce efficiency in the acquisition of information. In our model, prices are flexible and there is no role for monetary policy.[1] Inefficiencies in the collection and usage of information originate in the combination of market power with investment spillovers. The former can be corrected with the same revenue subsidies as in Angeletos and La'O (2020) and the rest of the pertinent macro literature. The latter, instead, require an additional subsidy to the investing firms that can be taken to be state-invariant (a constant) when information is exogenous but must co-move with the state when information is endogenous. We also show that the optimal subsidy must condition on the cross-section distribution of investment when the cost of information is unknown to the policymaker.

Angeletos and Sastry (2025) consider a macro model with rationally-inattentive agents and a perfectly competitive economy with complete markets. They study how efficiency in actions and in attention depend on the attention cost and in particular on whether such a cost depends on the choices of others. In our model, the welfare theorems do not apply because of the missing market for the externality created by the investment spillovers. Consequently, the

[1] We study sticky prices and optimal monetary policy in the online Supplement. There, we show that fiscal policies similar to those in the main text remain optimal when paired with a monetary rule that induces firms to disregard their endogenous private information when setting prices and only use it for technology adoption.

laissez-faire allocations are inefficient even when the cost of information is invariant in other agents' actions.

Hébert and La'O (2023) study the inefficiency of information acquisition in a linear-quadratic-Gaussian economy focusing on a stylized beauty context. The quadratic payoff specification in that paper has the property that welfare coincides with the potential of the game. Under this structure, the acquisition and usage of information are efficient when the cost of information is invariant in other agents' information choices and agents do not learn directly from the actions of others.

Colombo, Femminis and Pavan (2014) study the inefficiency of information acquisition in a class of linear-quadratic-Gaussian economies that features a rich set of possible externalities, including those originating in investment spillovers. They show that, in the absence of externalities from the dispersion of individual actions, when the laissez-faire economy is efficient in the usage of information, it is also efficient in the acquisition of information. That paper does not investigate policy interventions. The connection between the efficiency in the usage and in the acquisition of information is for the laissez-faire economy, and does not necessarily hold under the policies that induce efficiency in the usage of information. In the present paper, there are no externalities from dispersion, but the laissez-faire equilibrium is inefficient in the utilization of information because of the investment spillovers. In this economy, we find that the simplest policies that correct for the inefficiency in the usage of information fail to induce efficiency in the acquisition of information.

In all the papers cited above the decisions made under dispersed information are continuous. In the present paper, instead, these decisions are binary (firms must choose whether to operate under an existing technology or switch to a new one; operating under a mix of the two is not reasonable for the type of technologies we have in mind). Binary choices provide the government with more flexibility on how to induce efficiency in the usage of information when the latter is exogenous. As in global games of regime change (see, e.g., Morris and Shin (2004), Angeletos, Hellwig and Pavan (2006), Inostrosa and Pavan (2025), and the references therein), it suffices to target the marginal agent (the one who is indifferent between investing and not investing) and make him coincide with the one under the efficient allocation. When, instead, information is endogenous, the optimal subsidy must co-move with the state in a way that makes the private value of information also coincide with its social counterpart. The Pigouvian policies identified in the present paper (in which the subsidy parallels the state-dependent externality in production) have this property. Other simpler state-invariant policies that only target the marginal agent do not induce the right co-movement and hence fail to realign the private to the social value of information. More generally, the present paper provides a complete characterization of the properties that the subsidies need to satisfy when investment decisions are binary. Extending the analysis of this characterization to economies with a continuum of actions represents an interesting line for future work.[2]

Hébert and La'O (2023) and Angeletos and Sastry (2025) study the interaction between the acquisition of private information and the aggregation of the latter through prices and other macroeconomic statistics (see also Angeletos, Iovino, and La'O (2020)). These papers, however, abstract from direct payoff externalities such as those originating in investment spillovers and instead focus on the learning externalities that emerge when agents' ability to learn (e.g., from prices) is influenced by the choices of others. The present paper abstracts from these learning externalities and instead focus on those originating in investment spillovers.

In the microeconomic literature, various authors study how to incentivize efficient information acquisition in context such as procurement, auctions, and voting. A general result is that Vickrey-Clarke-Groves (VCG) transfers can be used to realign individual payoffs with total welfare, inducing agents to acquire information in society's best interest (see, e.g., Bergemann and Välimäki (2002)). In the present paper, VCG transfers do not work because agents are atomistic – the contribution of each agent to total welfare is zero. The Pigouvian policies identified in the present paper induce efficiency by realigning the agents' marginal incentives instead of their payoffs. They induce efficiency in both the acquisition and usage of information by making the agents pay, at the margin, for the variation in the production of the final good that would obtain had the agents invested differently.

In this respect, our analysis is related to the literature on corrective taxation in the presence of externalities, as pioneered by Pigou (1920)—see also Baumol (1972). This is a broad literature that is too vast to summarize here. See Sandmo (1975) for one of the earlier applications to environmental economics, and Barrage (2020) for recent developments within the same literature. See also Bovenberg and Goulder (1996) for one of the early general-equilibrium analyses of Pigouvian policies, and Romer (1986), Barro (1990), and more recently Chan et al. (2009), Grossman et al. (2013), Heutel (2012), and Jeanne and Korinek (2019) for the growth and business-cycle implications of these policies. Finally, see Cooper and John (1988), and Matsuyama (1991) for the role of externalities originating in investment spillovers and how they can be corrected with appropriate policy interventions. Our contribution is in endogenizing private information about relevant economic fundamentals affecting the profitability of the investment decisions and showing how appropriate subsidies resembling Pigouvian in-

[2]The papers with continuous choice cited above show that revenue subsidies correcting for market power suffice to induce efficiency in the acquisition of information. A complete characterization of the (necessary and sufficient) conditions that optimal fiscal policy must satisfy in economies with richer payoff interdependencies (e.g., investment spillovers) is still missing, at least to the best of our knowledge.

terventions can correct for inefficiencies in both the acquisition and usage of information.

To isolate the novel effects, we abstract from the familiar learning externalities that arise when firms learn from the behavior of other firms, as in the literature on observational and social learning pioneered by Banerjee (1992) and Bikhchandani et al (1992)—see also Wolitzky (2018) for a recent contribution in which firms learn from the outcomes instead of the decisions of their predecessors. Learning externalities also arise when prices aggregate information.

In addition to the papers by Angeletos, Iovino, and La'O (2020), Hébert and La'O (2023), and Angeletos and Sastry (2025) cited above, see Grossman and Stiglitz (1980) for one of the early pioneering contributions, Angeletos and Werning (2006) for information aggregation preceding financial crises, and Pavan, Sundaresan, and Vives (2025) for the design of taxes in markets in which traders compete in schedules and private information is endogenous. None of these papers investigates how to correct the inefficiencies (in information acquisition and usage) that arise in the presence of investment spillovers when private information is endogenous, which is the focus of the present paper. The closest work to ours in this literature is Lemoine (2024) who studies climate change policies in the presence of pollution externalities when financial markets aggregate private information. We share with this paper the focus on how to correct for direct payoff-relevant externalities under dispersed information. Contrary to it, however, we do not consider information aggregation and instead endogenize the acquisition of private information.

Outline. The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 contains the characterization of the optimal subsidies. Section 4 concludes. All proofs omitted in the main text are in the Appendix. In the online Supplement, we show that similar results obtain in economies in which (a) the spillovers affect also the non-investing firms or are in the production of the final goods instead of the intermediate ones, (b) consumers' preferences are non-linear over the consumption of the final good, and (c) firms set prices under imperfect information (nominal rigidities) under an optimal monetary policy that induces firms to disregard their endogenous private information when setting prices and only use it for investment purposes.

2 The Model

The economy is populated by (a) a measure-1 continuum of firms, each producing a differentiated intermediate good, (b) a competitive retail sector producing a final good using the intermediate goods as inputs, (c) a measure-1 continuum of agents who are consumers, entrepreneurs, and suppliers of homogeneous labor, and (d) a benevolent government designing optimal subsidies.

Each firm is run by an entrepreneur who must decide whether or not to invest. Indexing firms by $i \in [0,1]$, we denote by $n_i = 1$ the decision by firm i to "invest," and by $n_i = 0$ the decision to "not invest". The investment decision may have different interpretations depending on the context of interest. Hereafter, we interpret the decision to invest as the choice to adopt a new superior technology. More broadly, however, the decision to invest may represent a wide range of economic activities for which spillovers play an important role. Investing costs $k > 0$, with the cost expressed in terms of the entrepreneur's disutility of labor.

Let

$$N = \int n_i di$$

denote the aggregate investment, y_i the amount of the intermediate good produced by firm i, l_i the amount of labor employed by firm i, and $\Theta \in \mathbb{R}_+$ "aggregate economic fundamentals" affecting the profitability of the investment decisions. The amount of the intermediate good produced by each firm i is given by

$$y_i = \begin{cases} (1 + \Theta (1 + \beta N))\, l_i^{\psi} & \text{if } n_i = 1 \\ l_i^{\psi} & \text{if } n_i = 0 \end{cases}, \tag{1}$$

with $\beta > 0$ and $\psi \leq 1$. The parameters β and ψ control for the intensity of the production spillovers and for the returns to scale of labor, respectively.

Importantly, firms do not observe Θ at the time they make their investment decisions. Let $\theta \equiv \log \Theta$. Both the firms and the government commonly believe that θ is drawn from a Normal distribution with mean 0 and precision π_θ. Before investing, each firm chooses the precision π_i^x of a private signal $x_i = \theta + \xi_i$ about θ, with ξ_i drawn from a Normal distribution with mean zero and precision π_i^x, independently from θ and independently across i. The cost of acquiring information of precision π_i^x is equal to $\mathcal{I}(\pi_i^x)$, with $\mathcal{I}$ continuously differentiable and such that $\mathcal{I}'(0) = 0$, $\mathcal{I}'(\pi_i^x) > 0$ and $\mathcal{I}''(\pi_i^x) \geq 0$ for all $\pi_i^x > 0$.[3] Such a cost can also be interpreted as disutility of effort.

Let Y denote the amount of the final good. The latter is produced in a competitive retail sector according to the CES technology

$$Y = \left(\int y_i^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}}, \tag{2}$$

with $v > 1$ denoting the elasticity of substitution between intermediate goods. The price of

[3]We denote such a cost with $\mathcal{I}$ (which is meant to be mnemonic for information cost) instead of C to avoid confusion with the consumption of the final good.

the final good is P and the profits of the competitive retail sector are given by

$$\Pi = PY - \int p_i y_i di, \tag{3}$$

where p_i is the price of the intermediate good paid to firm i. In the absence of nominal rigidities, P is indeterminate. We thus normalize P to 1 and drop it from the analysis below (see the online Supplement for an extension to an economy with price rigidities in which the price of the final good P is determined by an optimal monetary policy).

We assume that each entrepreneur is a member of a representative household whose utility is given by

$$U = C - kN - \frac{L^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x)di, \tag{4}$$

where $L^{1+\varepsilon}/(1+\varepsilon)$ denotes the disutility of labor, with $\varepsilon > 0$. [4]

The representative household collects profits and wages from all firms, and uses them to purchase the final good. Its budget constraint is given by

$$C \leq \int \left((1+\tau)p_i y_i - Wl_i + s_i n_i\right) di + WL - \Upsilon, \tag{5}$$

where W is the wage rate, τ is a standard revenue-subsidy designed to offset the firms' market power, s_i is an additional transfer to firm i paid in case the firm invested in the new technology (i.e., when $n_i = 1$), and Υ is a lump-sum tax paid to the government. All these variables are expressed in units of consumption of the final good.[5]

The government budget constraint is satisfied in each state; that is, given the subsidies paid to the firms the government asks the representative consumer to pay a lump-sum transfer equal to

$$\Upsilon = \int (\tau p_i y_i + s_i n_i)di. \tag{6}$$

Being a member of the representative household, each entrepreneur maximizes his firm's market valuation, taking into account that the profits the firm generates are used for the purchase

[4] That entrepreneurs are members of a representative household is not important when preferences over the consumption of the final good are linear, as assumed here. When, instead, they are concave over C, as in the online Supplement, our assumption guarantees insurance in consumption risk, which is standard in the pertinent literature — see, e.g., Angeletos and La'O (2020).

[5] One can also consider other policies in which the transfers to the firms are a function of employment, profits, or a combination of these and other verifiable variables. Following the pertinent literature, we focus on revenue-based subsidies $\tau p_i y_i$, augmented by an additional subsidy s_i that is paid if the firm invested, that is, if $n_i = 1$.

of the final good. This means that each entrepreneur maximizes

$$\mathbb{E}\left[(1+\tau)p_i y_i - W l_i + (s_i - k)n_i - \mathcal{I}(\pi_i^x)\right]. \tag{7}$$

Using the fact that (a) the government's budget is balanced, (b) the total labor demand must equal the total labor supply ($\int l_i di$=L), (c) all entrepreneurs choose the same precision of private information in equilibrium, (d) firms' total revenues coincide with the total expenditure on the final good, and (e) the total consumption of the final good C coincides with its production Y, we have that the government's objective can be expressed as

$$\mathcal{W} = \mathbb{E}\left[C - kN - \frac{L^{1+\varepsilon}}{1+\varepsilon} - \mathcal{I}(\pi^x)\right]. \tag{8}$$

The government thus designs its fiscal policy to maximize aggregate consumption net of investment, labor, and information costs.

The sequence of events is the following:

1. **Investment phase**

   (1.a) the government announces its transfer policy;

   (1.b) each entrepreneur chooses the precision π_i^x of his private information;

   (1.c) Nature draws θ and each entrepreneur receives a private signal x_i about θ;

   (1.d) entrepreneurs simultaneously choose n_i.

2. **Production and consumption phase**

   After θ and N are publicly revealed:

   (2.a) entrepreneurs simultaneously set prices p_i;

   (2.b) the competitive retail sector chooses how much of each intermediate good to purchase, taking the prices of the intermediate goods $\{p_i\}_{i\in[0,1]}$ as given; given the demand y_i for his intermediate good, entrepreneur i hires l_i units of labor on a competitive labor market to produce y_i; the representative household purchases the final good, after collecting all firms' profits and after being paid for the labor supplied.

Definition 1. A (symmetric) **equilibrium** for the economy described above consists of a precision π^x of private information, an investment strategy $n(x;\pi^x)$, price and employment functions $(p_1(\theta;\pi^x), l_1(\theta;\pi^x))$ and $(p_0(\theta;\pi^x), l_0(\theta;\pi^x))$, for the investing and non-investing firms, respectively, a total labor supply $L(\theta;\pi^x)$ and a final-good consumption demand $C(\theta;\pi_x)$ for the representative consumer, intermediate-good demands $y_1(\theta;\pi^x)$ and $y_0(\theta;\pi^x)$ by the final-good representative producer, a wage function $W(\theta;\pi^x)$, and a total investment function $N(\theta;\pi_x)=\int n(x;\pi_x)d\Phi(x|\theta;\pi_x)$ such that the following conditions hold:

1. given θ, $N(\theta;\pi_x)$, and $W(\theta;\pi^x)$, each firm i maximizes its market valuation (as given in (7)) by setting a price equal to $p_1(\theta;\pi^x)$ after investing and equal to $p_0(\theta;\pi^x)$ after non investing, taking the wage rate $W(\theta;\pi^x)$ as given, and accounting for how the demand y_i for its product by the final-good producer varies with its price p_i when all investing firms set prices according to $p_1(\theta;\pi^x)$, all non-investing firms set prices according to $p_0(\theta;\pi^x)$, and, given y_i, the labor the firm needs to employ to produce y_i is given by the production function in (1);

2. given the prices of the intermediate goods, the final-good producer purchases the intermediate goods to maximize its profits (as given in (3)) taking as given the price $P=1$ of the final good and the wage rate $W(\theta;\pi^x)$, and accounting for the fact that the production function for the final good is given by (2);

3. the demand $C(\theta;\pi_x)$ for the final good and the labor supply $L(\theta;\pi^x)$ maximize the representative household's utility (as given in (4)) given the budget constraint in (5);

4. the labor market clears at wage $W(\theta;\pi^x)$, meaning that the total labor demand

$$N(\theta;\pi_x)l_1\left(\theta;\pi^x\right)+(1-N(\theta;\pi_x))l_0\left(\theta;\pi^x\right)$$

   equals the labor supply $L(\theta;\pi^x)$;

5. in the investment phase, each entrepreneur $i\in[0,1]$ maximizes its market valuation by choosing a precision of information equal to π^x and investing according to $n(x;\pi^x)$ when, in the production and consumption phase, for each θ, the measure of investing firms is $N(\theta;\pi_x)$, the wage rate is $W(\theta;\pi^x)$, and all firms set prices, employ labor, and produce output according to $(p_1(\theta;\pi^x),l_1(\theta;\pi^x),y_1(\theta;\pi^x))$ after investing and according to $(p_0(\theta;\pi^x),l_0(\theta;\pi^x),y_0(\theta;\pi^x))$ after non investing.

Comments. That firms are differentiated monopolists, the production of the intermediate goods is Cobb-Douglas, and the technology for producing the final good is iso-elastic, are standard assumptions in the macroeconomics literature. Assuming the same structure facilitates the comparison with previous work and permits us to illustrate the novel effects originating in the interaction between (a) the investment spillovers and (b) the endogeneity of firms' private information, which is the contribution of the paper.

The functional forms in (1) and (2) favor the characterization of the efficient and equilibrium allocations in closed form but are not essential to the main results. What matters is that, for any choice of labor l, the output differential $\Theta\left(1+\beta N\right)l^{\psi}$ between the investing and the non-investing firms is increasing in N and Θ. Likewise, that the spillovers affect only the investing firms is not important (in the online Supplement, we consider an alternative specification whereby the spillovers affect also the non-investing firms; e.g., investments in AI that may come with the development of knowledge, auxiliary products and services such as AI-based software useful to all firms, including those operating under traditional technologies).

Also note that, whereas the economy described above is static, the following dynamic interpretation is fully consistent with the model. There are two periods. The first period is identical to the static economy described above. The second period is also identical except for the fact that the cost of adopting the new technology is zero. As a result, those firms that did not adopt the new technology early in the first period, do so late in the second period, when the adoption cost drops (for simplicity, to zero). Provided that profits and preferences separate over the two periods, the analysis in the first period is strategically equivalent to the one for the static economy described above.[6]

In the online Supplement we also show that all results apply to an alternative production economy in which firms choose whether to produce the intermediate goods in a traditional or a "smart" (Industry 4.0) specification. In this alternative economy, the amount of intermediate good produced by each firm is equal to

$$y_i = l_i^{\psi}, \tag{9}$$

irrespective of whether the firm supplies its good in a smart ($n_i = 1$) or traditional ($n_i = 0$) specification. The amount of the final good produced, instead, is equal to

$$Y = \left(\int_i ((1+\Theta\left(1+\beta N\right)n_i)y_i)^{\frac{v-1}{v}}\,di\right)^{\frac{v}{v-1}}. \tag{10}$$

Consistently with what discussed in the pertinent literature, the decision to provide an intermediate good in a smart specification increases the amount of the final good produced, both

[6] When, instead, firms' early investment decisions have long-lasting effects (e.g., in the presence of learning-by-doing), a result similar to that in Proposition 2 below, establishing the optimality of Pigouvian corrections, continues to hold but with the static externality replaced by its dynamic counterpart, accounting for the effects that early decisions have on continuation profits.

directly and by enhancing the interoperability/productivity of other intermediate goods (see, e.g., Bai et al., 2020). In this alternative economy, the spillover is thus in the production of the final good rather than in the production of the intermediate goods.

More generally, whereas the closed-form characterization of the efficient allocation in Lemma 1 and of the optimal subsidies in Lemmas 2 and 3 below uses the functional forms in (1) and (2), the optimality of Pigouvian corrections in Proposition 2 extends to arbitrary production functions and arbitrary information technologies (see also the discussion following Proposition 2).[7] Also note that, as anticipated above, the assumption that U is linear in C is not important for the results. In the online Supplement, we show that results analogous to those below hold in economies in which U is iso-elastic in C.

Finally, observe that the government announces its policies before the fundamental variable θ is drawn. There is no signaling by the government to the private sector. The role of these policies is to induce firms to acquire and use information in society's best interest by eliminating wedges between private and social returns to investment due to the spillovers. Once the firms' decisions have been made, there is no value for the government in redistributing resources from one firm to the other, even after learning the fundamental variable θ. There is thus no time-inconsistency problem in the government's choice of policies.

3 Constrained Efficiency and Optimal Subsidies

Subsection 3.1 characterizes constrained efficiency, whereas Subsection 3.2 characterizes subsidies that implement the constrained efficient allocations in equilibrium.

3.1 Constrained Efficiency

We assume that the government cannot transfer information across agents. This restriction is standard in the literature (see, among others, Vives (1988), Angeletos and Pavan (2007), Colombo, Femminis and Pavan (2014), Angeletos, Iovino and La'O (2016), Angeletos and La'O (2020), and Llosa and Venkateswaran (2022)).

The constrained efficient allocation has three parts: the precision of private information, π^{x*}, a rule specifying whether or not firms should invest based on their private information x, and a rule describing how much labor each firm should employ as a function of θ and x (equivalently, θ and its investment). These three parts are chosen jointly to maximize *ex-ante*

[7]Likewise, results similar to those in Proposition 2 hold in economies in which investment comes with an intensive instead of an extensive margin, i.e., n is a continuous choice. In this economy, the subsidy is proportional to investment and equal to the one for the investing firms in Proposition 2, i.e., to $s^{\#}(\theta,\Lambda)\,n$.

welfare, $\mathcal{W}$, as given in (8). Lemma 1 focuses on efficient investment decisions. The rule describing the efficient employment of labor is in the proof of Lemma 1, whereas the formula for the efficient precision of private information π^{x*} is in the proof of Lemma 3. The reason for relegating these parts to the Appendix is that they are useful for comparative statics but not essential to the arguments establishing the key results.

Definition 2. The economy is regular if the following condition holds: $v < 1 + (1+\varepsilon)/\varepsilon\psi$.

The parameters' restrictions in the regularity definition guarantee that the social value of investing (net of its cost) is increasing in the fundamental θ and in the mass N of investing firms. This restriction is fairly standard. It plays a role similar to the one played by the assumption that substitution effects are stronger than income effects in other settings. The monotonicities in θ and N, in turn, imply that the efficient rule for investment is monotone in the firms' private information.

Lemma 1. *Suppose that the economy is regular. For any precision of private information π^x, there exists a threshold $\hat{x}(\pi^x)$ such that efficiency in investment decisions requires that each firm with signal $x > \hat{x}(\pi^x)$ invests, whereas each firm with signal $x < \hat{x}(\pi^x)$ does not.*

Proof. See the Appendix.

That the efficient investment rule is monotone in signals is not essential for our results but facilitates the exposition. In particular, it permits us to fully characterize necessary and sufficient conditions for a subsidy policy to implement the efficient allocation, both when information is exogenous (Lemma 2), and when it is endogenous (Lemma 3). The results in Propositions 1 and 2 below, establishing that Pigouvian corrections eliminate any discrepancy between private and social objectives (and hence induce efficiency in both information acquisition and usage, despite the fact that neither of the two activities is verifiable), apply also to economies in which the constrained-efficient allocation is not monotone.

3.2 Optimal Subsidies

Following the pertinent literature, we assume that the revenue subsidy is given by $\tau = 1/(v-1)$. As is well know, such a subsidy offsets the firms' market power. Our focus is on the additional subsidy s to the investing firms.[8]

[8]In the proof of Lemma 2, we establish that any revenue-based transfer scheme paying a total transfer $T_1(py)$ to the investing firms and $T_0(py)$ to the non-investing firms as a function of their revenues must take the form $T_0(py) = \tau py$ and $T_1(py) = \tau py + s$, with $\tau = 1/(v-1)$ and s invariant in py to implement the efficient allocations.

We first characterize (jointly necessary and sufficient) conditions that any such subsidy must satisfy when the precision of private information π^x is exogenous. Next, we characterize additional conditions that any optimal subsidy must satisfy when information is endogenous. The comparison between the two sets of conditions permits us to illustrate the general point that policies that are optimal under exogenous information need not be optimal when information is endogenous. We also show that simple subsidies to the investing firms that are invariant in the fundamentals θ suffice to induce efficiency in the usage of information but fail to induce efficiency in the acquisition of information. The latter requires that the subsidies co-move with the marginal effect of more precise private information on the measure of investing firms, which in turn requires conditioning the subsidies on the fundamentals θ. At the end of the section, we discuss how a government that does not know the cost of information (alternatively, the experiments that firms can conduct to learn the fundamentals) can induce efficiency in both information acquisition and usage with subsidies that condition on both the fundamentals and the cross section of firms' employment and investment decisions.

3.2.1 Exogenous Information

Suppose that the precision of private information is exogenous and equal to π^x. Let $\hat{n}(x;\pi^x)$ denote the rule describing the efficient investment decisions, and $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ the rules describing the efficient labor employment, for the investing and the non-investing firms, respectively. Let $\hat{y}_1(\theta;\pi^x)$ and $\hat{y}_0(\theta;\pi^x)$ denote the efficient production of the intermediate goods for each of the two types of firms. Finally, let $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$ denote the prices for the investing and the non-investing firms, respectively, that induce demands equal to $\hat{y}_1(\theta;\pi^x)$ and $\hat{y}_0(\theta;\pi^x)$ and hence employment equal to the efficient levels $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$.

Definition 3. Assume that the precision of private information is exogenous and equal to π^x. The subsidy s is **optimal** if it implements the efficient usage of information as an equilibrium; that is, if it induces all firms to invest according to the efficient rule $\hat{n}(x;\pi^x)$ and set prices according to the rules $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$.

Let $\hat{C}(\theta;\pi^x)$ and $\hat{N}(\theta;\pi^x)$ denote, respectively, the amount of the final good consumed and the measure of investing firms in state θ, when the precision of private information is π^x and all firms make all decisions efficiently.

The following lemma provides a complete characterization of the subsidies that, when information is exogenous, implement the efficient use of information.

Lemma 2. *Assume that the precision of private information is exogenous and equal to* π^x

and that the economy is regular. Let

$$\mathcal{R}(\theta;\pi^x) \equiv \; \tfrac{v-\psi(v-1)}{v-1} \; \hat{C}\left(\theta;\pi^x\right)^{\frac{1}{v}} \left(\hat{y}_1(\theta;\pi^x)^{\frac{v-1}{v}} - \hat{y}_0(\theta;\pi^x)^{\frac{v-1}{v}}\right) + s\left(\theta;\pi^x\right) - k. \tag{11}$$

be the private benefit of investing, net of its cost. The state-contingent subsidy $s\left(\theta;\pi^x\right)$ to the investing firms is optimal if and only if $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right] < 0$ when $x < \hat{x}(\pi^x)$, and $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right] > 0$ when $x > \hat{x}(\pi^x)$, where $\hat{x}(\pi^x)$ is the signal threshold for the efficient investment decision as defined in Lemma 1.

Proof. See the Appendix.

Naturally, because the new technology is superior, the investing firms expect higher revenues, and hence a higher subsidy $py/(v-1)$. However, this standard subsidy alone is not sufficient to induce firms to invest efficiently. This is because firms do not internalize that, by investing, they increase other firms' output. The additional subsidy $s\left(\theta;\pi^x\right)$ to the investing firms must correct for such an externality. In the proof of the lemma in the Appendix, we show that the private benefit of investing $\mathcal{R}(\theta;\pi^x)$ can be written as

$$\mathcal{R}(\theta;\pi^x) = \mathcal{Q}(\theta;\pi^x) - \frac{\Theta\beta\hat{N}\left(\theta;\pi^x\right)}{1+\Theta\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)}\hat{C}\left(\theta;\pi^x\right)^{\frac{1}{v}}\hat{y}_1\left(\theta;\pi^x\right)^{\frac{v-1}{v}} + s\left(\theta;\pi^x\right),$$

where $\mathcal{Q}(\theta;\pi^x)$ is the social benefit, and

$$\frac{\Theta\beta\hat{N}\left(\theta;\pi^x\right)}{1+\Theta\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)}\hat{C}\left(\theta;\pi^x\right)^{\frac{1}{v}}\hat{y}_1\left(\theta;\pi^x\right)^{\frac{v-1}{v}}$$

is the marginal externality created by the investment spillover. The externality coincides with the increase in the production of the final good that obtains if, hypothetically, one increases the total mass N of investing firms by a small amount $\varepsilon > 0$ around the efficient level $\hat{N}\left(\theta;\pi^x\right)$, holding fixed all firms' technology and employment. The subsidy $s\left(\theta;\pi^x\right)$ must thus be designed to compensate for the fact that firms do not internalize such an externality. Many subsidies $s\left(\theta;\pi^x\right)$ accomplish this objective. In fact, because efficiency requires that firms invest when $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right] > 0$ and refrain from investing when $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right] < 0$, any subsidy that targets the "marginal firm" (the one that is indifferent between investing and not investing) by aligning the sign of the expected private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ to the sign of the expected social benefit $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right]$ does the job. When the economy is regular, $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right]$ turns from negative to positive at $x = \hat{x}(\pi^x)$. Hence, any subsidy that makes the expected private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ turn from negative to positive at

$x = \hat{x}(\pi^x)$ induces all firms to invest efficiently. A subsidy that is particularly simple is one that is constant, i.e., invariant in θ.

Corollary 1. *Assume that the precision of private information is exogenous and equal to π^x and that the economy is regular. A state-invariant subsidy equal to*

$$s(\theta;\pi^x) = \bar{s}_{\pi^x} \equiv \mathbb{E}\left[\frac{\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\hat{C}(\theta;\pi^x)^{\frac{1}{v}}\hat{y}_1(\theta;\pi^x)^{\frac{v-1}{v}}\middle|\hat{x}(\pi^x),\pi^x\right], \tag{12}$$

for all θ, is optimal.

Proof. See the Appendix.

The constant subsidy $\bar{s}_{\pi^x}$ to the investing firms is thus the externality expected by the marginal investor with signal equal to the efficient threshold $\hat{x}(\pi^x)$. The advantage of such a simple policy is that it does not require the government to track the fundamental variable θ. When the government promises to pay to the investing firms a constant subsidy equal to $\bar{s}_{\pi^x}$, a firm with signal equal to $\hat{x}(\pi^x)$ that expects all other firms to invest efficiently and then set prices according to the rules $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$ that induce the efficient demands $\hat{y}_1(\theta;\pi^x)$ and $\hat{y}_0(\theta;\pi^x)$ (and hence the efficient employment decisions $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$), is indifferent between investing and not investing. Because

$$\mathcal{Q}(\theta;\pi^x) - \frac{\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\hat{C}(\theta;\pi^x)^{\frac{1}{v}}\hat{y}_1(\theta;\pi^x)(\theta)^{\frac{v-1}{v}}$$

is monotone in θ, under the same expectations, any firm with signal above $\hat{x}(\pi^x)$ finds it optimal to invest, whereas any firm with signal below $\hat{x}(\pi^x)$ finds it optimal not to invest. This means that the constant subsidy $\bar{s}_{\pi^x}$ to the investing firms, along with the revenue subsidy $py/(v-1)$ that offsets market power, aligns the sign of the private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ to its social counterpart $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right]$, and hence implements the efficient allocation.

3.2.2 Endogenous Information

We now turn to the case in which firms' information is endogenous. Let π^{x*} denote the precision of the firms' private information that maximizes welfare (its characterization is in the proof of Lemma 3). In the presence of endogenous information, optimality is defined as follows.

Definition 4. The subsidy s^* is **optimal** if it implements the efficient acquisition and usage of information as an equilibrium. That is, it induces all firms to (1) choose the efficient precision of private information π^{x*}, (2) follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and (3) set prices $\hat{p}_1(\theta;\pi^{x*})$ and $\hat{p}_0(\theta;\pi^{x*})$ that induce demands for the intermediate products equal to $\hat{y}_1(\theta;\pi^{x*})$ and $\hat{y}_0(\theta;\pi^{x*})$, and hence efficient employment $\hat{l}_1(\theta;\pi^{x*})$ and $\hat{l}_0(\theta;\pi^{x*})$.

Let $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$ denote the marginal variation in the measure of firms investing at θ that obtains when one varies π^x infinitesimally at $\pi^x=\pi^{x*}$, holding fixed the rule for efficient investment $\hat{n}(x;\pi^{x*})$.

Lemma 3. *Assume that information is endogenous and that the economy is regular. A subsidy $s\left(\theta;\pi^{x*}\right)$ is optimal if and only if, in addition to the condition in Lemma 2 applied to $\pi^x=\pi^{x*}$, it satisfies the following condition*

$$\mathbb{E}\left[s\left(\theta;\pi^{x*}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\mathbb{E}\left[\frac{\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)}{1+\Theta\left(1+\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}\hat{y}_1\left(\theta;\pi^{x*}\right)^{\frac{v-1}{v}}\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]. \tag{13}$$

Proof. See the Appendix.

The lemma provides a complete characterization of the subsidies that induce efficiency in both information acquisition and information usage. Relative to the case in which information is exogenous (with precision π^{x*}), the subsidy to the investing firms must satisfy an additional restriction on the co-movement between (a) the subsidy $s\left(\theta;\pi^{x*}\right)$ and (b) the marginal change $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$ in the measure of investing firms due to the increase in the precision of information (equivalently, the marginal effect of more precise private information on aggregate investment under the efficient investment rule). The restriction is necessary to align the private benefit of acquiring more information with its social counterpart. When the economy is regular, the externality

$$\frac{\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)}{1+\Theta\left(1+\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}\hat{y}_1\left(\theta;\pi^{x*}\right)^{\frac{v-1}{v}}$$

increases with the state θ. The marginal variation $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$ in the measure of investing firms due to more precise private information is also monotone in θ (it is negative for $\theta<\hat{x}(\pi^x)$ and positive for $\theta>\hat{x}(\pi^x)$). The subsidy $s\left(\theta;\pi^{x*}\right)$ must thus change with the state θ, so that the co-movement between $s\left(\theta;\pi^{x*}\right)$ and $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$ is the same as that between the externality and $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$. That both the private and the social values of information are related to these co-movements reflects the fact that information permits the agents to reduce mistakes in their decisions. Observe that $\hat{N}\left(\theta;\pi^{x*}\right)$ coincides with the probability that each firm invests at state θ. Hence, the government must pay a larger subsidy in states in which information results in a larger increase $\partial \hat{N}(\theta; \pi^{x*}) / \partial \pi^x$ in the probability of investment, with the co-movement between the two paralleling exactly the one between $\partial \hat{N}(\theta; \pi^{x*}) / \partial \pi^x$ and the externality so as to align the private value of information to its social counterpart.

As a result of the additional restriction, policies that are optimal under exogenous information need not be optimal when information is endogenous. For example, the simple policy of Corollary 1, specialized to $\pi^x = \pi^{x*}$, under which the government pays a constant subsidy $\bar{s}_{\pi^{x*}}$ to the investing firms in addition to the revenue subsidy $py/(v-1)$, fails to induce efficiency in information acquisition. Hence, it is not optimal when information is endogenous. This is because a constant subsidy equal to the externality expected by the marginal investor with signal $\hat{x}(\pi^{x*})$ does not induce the right co-movement between the subsidy $s(\theta; \pi^{x*})$ and the (state-dependent) marginal effect of more precise private information on aggregate investment, $\partial \hat{N}(\theta; \pi^{x*}) / \partial \pi^x$, which is necessary to realign the private benefit of information acquisition to its social counterpart. Conversely, a policy that pays, in each state θ, a subsidy to the investing firms equal to the state-contingent externality from the investment spillover satisfies the co-movement condition in (13), and hence induces efficiency in both information acquisition and information usage.

Proposition 1. *Irrespective of whether the economy is regular, a state-contingent subsidy to the investing firms equal to*

$$s(\theta; \pi^{x*}) = \frac{\Theta \beta \hat{N}(\theta; \pi^{x*})}{1 + \Theta \left(1 + \beta \hat{N}(\theta; \pi^{x*})\right)} \hat{C}(\theta; \pi^{x*})^{\frac{1}{v}} \hat{y}_1(\theta; \pi^{x*})^{\frac{v-1}{v}} \tag{14}$$

is optimal.

Proof. Suppose that all other firms (i) acquire information of precision π^{x*}, (ii) invest when, and only when, it is socially efficient to do so (i.e., invest when $\mathbb{E}[\mathcal{Q}(\theta; \pi^{x*}) | x, \pi^{x*}] > 0$ and not invest when $\mathbb{E}[\mathcal{Q}(\theta; \pi^{x*}) | x, \pi^{x*}] < 0$), and (iii) set prices $\hat{p}_1(\theta; \pi^{x*})$ and $\hat{p}_0(\theta; \pi^{x*})$ that induce efficient employment and production decisions. Then, in each state θ, irrespective of the precision π^x of its private information, each firm finds it optimal to set a price equal to $\hat{p}_1(\theta; \pi^{x*})$ when investing, and equal to $\hat{p}_0(\theta; \pi^{x*})$ when not investing. Furthermore, the subsidy in the proposition guarantees that the private value of investing coincides with the social value in each state (see the proof of Lemma 2 in the Appendix for the formal arguments). These properties hold irrespective of whether the precision π^x selected by the firm coincides with the efficient level π^{x*}. They also hold irrespective of whether the economy satisfies the regularity conditions, the sole role of which is to guarantee that, when $\pi^x = \pi^{x*}$, the social benefit $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right]$ of investing expected by a firm with signal x turns from negative to positive at $x=\hat{x}(\pi^{x*})$. The same properties also imply that the value the firm assigns to acquiring information coincides with the social value. The above results thus imply that acquiring information of precision π^{x*} and then using the information efficiently (both when it comes to choosing whether or not to invest and when setting the prices) is individually optimal for each firm expecting all other firms to do the same. Q.E.D.

As anticipated above, the state-contingent subsidy in (14) operates as a Pigouvian correction that induces each firm to internalize the effect of its investment on the production of the final good when all other firms acquire and use information efficiently. To see this, let Λ denote the cross-sectional distribution of firms' investment and employment decisions (n_i, l_i). Let $C_N(\theta,\Lambda)$ denote the marginal change in the production of the final good that obtains when, holding θ and Λ fixed, one changes N in all firms' production function by a small $\varepsilon>0$, starting from $N=N_\Lambda$, where N_Λ is aggregate investment under the distribution Λ. Next, let $\hat{\Lambda}(\theta;\pi^{x*})$ denote the cross-sectional distribution of firms' investment and employment decisions (n_i, l_i) under the efficient allocation. Then one has that

$$C_N\left(\theta,\hat{\Lambda}(\theta;\pi^{x*})\right)=\frac{\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)}{1+\Theta\left(1+\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}\hat{y}_1\left(\theta;\pi^{x*}\right)^{\frac{v-1}{v}}.$$

That is, the state-dependent subsidy in (14) coincides with the marginal change in the production of the final good that obtains as a result of a marginal change in N, evaluated at $N=\hat{N}\left(\theta;\pi^{x*}\right)$, holding all firms' investment and employment decisions fixed at the efficient level. Such a policy is thus reminiscent of familiar Pigouvian corrections for complete-information economies. Importantly, these corrections also induce firms to collect and use information in society's best interest when firms' decisions (i.e., how much they invest in information acquisition and how they use their information) are not verifiable.

The Pigouvian policy of Proposition 1 is not the unique one implementing the efficient allocation. Other state-contingent policies do the job. One of the limitations of many of these policies (including the one in Proposition 1) is that they require the government to know the type of information the firms can collect (equivalently, the cost of different information structures). This knowledge is necessary to compute $\hat{C}\left(\theta;\pi^{x*}\right)$ and $\hat{N}\left(\theta;\pi^{x*}\right)$, and hence the state-contingent subsidy $s(\theta;\pi^{x*})$ in (14), but may not be available in some economies of interest. When this is the case, efficiency in both information acquisition and usage can be induced by making the subsidy to the investing firms equal to the *ex-post externality* of investment on the production of the final good. Such a subsidy induces efficiency (in both the acquisition and usage of information), irrespective of the signals the firms can use to learn the fundamentals and their costs.

Proposition 2. *Suppose the government does not know the type of information the firms can collect (equivalently, the cost of different information structures). Efficiency in both information acquisition and usage can be induced through a subsidy to the investing firms equal to*

$$s^{\#}(\theta, \Lambda) = C_N(\theta, \Lambda),$$

where Λ is the ex-post cross-sectional distribution of firms' investment and employment decisions (n_i, l_i), and where $C_N(\theta, \Lambda)$ is the marginal change in the production of the final good that obtains as a result of a marginal change in N holding all firms' investment and employment decisions fixed at the level specified by Λ.

Proof. Under the assumption that the type of information that firms can collect is unknown to the government, suppose that all other firms (i) acquire information efficiently (with information acquisition taking the form of a private signal $q : \Theta \rightarrow \Delta(\mathcal{S})$ mapping θ into a distribution over a Polish space $\mathcal{S}$ of signal realizations that, without loss of generality can be taken to coincide with $[0, 1]$), (ii) use information efficiently to make their investment decisions, and (iii) in each state θ, given aggregate investment N, set prices so as to induce the efficient employment (and hence production) decisions. Then, each firm has enough knowledge about the economy to compute the efficient allocation, and has incentives to follow the same efficient policies as any other firm. In fact, the revenue subsidy $py/(v-1)$ guarantees that each firm, no matter its investment decision, after learning θ, has the right incentives to set the price for its intermediate good at a level that induces the efficient demand for its product, and hence the efficient employment decisions (see the proof of Lemma 2 in the Appendix where the result is established without using the specific properties of the firms' information structure). Furthermore, when for each (θ, Λ), $s^{\#}(\theta, \Lambda) = C_N(\theta, \Lambda)$, the marginal value that each firm assigns to investing coincides with the government's value (see the proof of Lemma 2 in the Appendix). The above properties imply that the private value of information acquisition coincides with the social one, no matter the cost of each experiment q. Hence, all firms have the right incentives to acquire and use information efficiently when expecting all other firms to do the same. Q.E.D.

The result in Proposition 2 illustrates the power of the Pigouvian logic. When the policy maker announces that investing firms will receive a subsidy equal to the *ex-post* (marginal) externality $C_N(\theta, \Lambda)$ that each firm's investment exerts on the production of the final good, it re-aligns firms' (marginal) incentives with their social counterpart, not just at the *interim* stage but also *ex-post*. The government can then delegate to firms the computation of the efficient allocation, while guaranteeing that, in equilibrium, they acquire and use information efficiently.[9]

One can also show that the power of the Pigouvian logic extends to economies in which firms are heterogeneous in their cost of acquiring information and/or in their investment cost. It also extends to economies in which investment features an intensive instead of an extensive margin, i.e., firms decide how much to invest, with the latter decision taking a continuum of possible values. This is because there are no discrepancies between private and social marginal costs. As a result, the subsidy in Proposition 2, by aligning each firm's private benefit to investment with its social counterpart induces efficiency in both information acquisition and usage, irrespective of whether investment is a discrete or a continuous choice and of any heterogeneity across firms.

Propositions 1 and 2 complement each other. Proposition 1 shows that, when the government knows the cost of different information structures, efficiency in both information acquisition and usage can be induced with a fiscal policy that conditions the subsidy s to the investing firms solely on the fundamental state θ — no further contingencies are necessary. Proposition 2, instead, shows that, when the cost is unknown to the government, efficiency in information acquisition and usage requires expanding the contingencies in the optimal subsidy by conditioning on the cross-sectional distribution of investment and employment decisions.

As anticipated in the Introduction, the policies of Propositions 1 and 2 resemble VCG transfers, but with the correction operating at the margin instead of the levels. While the VCG transfers eliminate the wedge between the private and the social objectives by making firms' profits (net of the transfers) proportional to their contribution to total welfare, the policies in Propositions 1 and 2 eliminate the wedge between the marginal private and social benefit of varying the firms' decisions, as envisioned by Pigou.

4 Conclusions

We investigate optimal fiscal policy in economies in which firms face endogenous uncertainty about aggregate economic conditions affecting the profitability of their investment decisions (e.g., in AI-based technologies, or in smart intermediate products), and where the output they produce is affected by investment spillovers. We show that firms can be incentivized to acquire

[9] In equilibrium, each firm i's employment decision depends only on the firm's own technology choice n_i, the fundamental θ, and aggregate investment N. Hence, the subsidy paid, in equilibrium, to the investing firms depends on (θ, Λ) only through (θ, N).

information efficiently and then use it in society's best interest through a fiscal policy that, in addition to correcting for firms' market power, provides the investing firms with a subsidy that makes them internalize the effects of their investments on the production of intermediate and final goods. This result shows how the power of Pigouvian corrections extends to economies in which neither the collection nor the usage of information is verifiable. We expect results similar to those discussed in the present paper to obtain in economies in which externalities originate in pollution, and/or spillovers from investments in human capital.

Our analysis can be extended in several directions. To isolate the novel effects from the familiar learning externalities that are present when late adopters learn from early ones and/or where financial markets imperfectly aggregate private information, we consider a static general-equilibrium economy in which all the relevant production decisions occur simultaneously and there is no information aggregation. In future work, it would be interesting to extend the analysis to combine the externalities from investment spillovers discussed in the present paper with the familiar learning ones, as, e.g., in Dasgupta (2007), but in a setting with endogenous private information. It would also be interesting to enrich the model to allow for partial information aggregation in financial markets and study how inefficiencies in investment and production decisions interact with those in the trading of financial assets (see Angeletos, Lorenzoni, and Pavan (2023), and Pavan, Sundaresan and Vives (2024) for models with some of these ingredients, but without spillovers). Finally, it would be interesting to extend the analysis to economies in which firms expand the set of available products over time and strategically choose when to replace existing products with new ones, thus contributing to the understanding of how governments can increase the efficiency of the innovation diffusion process.

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Appendix

Proof of Lemma 1. Fix π^x and drop it from all expressions to ease the notation. Efficiency requires that any two firms making the same investment decision employ the same amount of labor. Letting $n(x)$ denote the probability that a firm receiving signal x invests, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the investing and the non-investing firms respectively when the fundamental variable is θ, we have that the planner's problem can be written as

$$\begin{aligned}\max_{n(\cdot),l_1(\cdot),l_0(\cdot)} \int_\theta C(\theta)d\Omega\,(\theta) - k\int_\theta N(\theta)d\Omega\,(\theta) + \\ -\frac{1}{1+\varepsilon}\int_\theta \left(l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))\right)^{1+\varepsilon} d\Omega\,(\theta) + \\ -\int_\theta \mathcal{Q}(\theta)\left(N\,(\theta)-\int_x n\,(x)\,d\Phi\,(x|\theta)\right)d\Omega\,(\theta)\,,\end{aligned}$$

where $\Omega\,(\theta)$ is the cumulative distribution function of θ (with density $\omega\,(\theta)$), $\Phi\,(x|\theta)$ is the cumulative distribution function of x given θ (with density $\phi\,(x|\theta)$), $\mathcal{Q}(\theta)$ is the multiplier associated with the constraint $N\,(\theta)=\int_x n\,(x)\,d\Phi\,(x|\theta)$, and

$$C(\theta) = \left(y_1\,(\theta)^{\frac{v-1}{v}}\,N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}\,, \tag{A.1}$$

with

$$y_1(\theta) = (1 + \Theta(1 + \beta N(\theta)))\, l_1(\theta)^{\psi}, \tag{A.2}$$

and

$$y_0(\theta) = l_0(\theta)^{\psi}. \tag{A.3}$$

Denoting for convenience

$$A(\theta, N) \equiv 1 + \Theta(1 + \beta N), \tag{A.4}$$

the first-order condition with respect to $l_1(\theta)$ is thus equal to

$$\psi \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}} (1 - N(\theta)) \right)^{\frac{1}{v-1}} A(\theta, N(\theta))^{\frac{v-1}{v}} l_1(\theta)^{\psi \frac{v-1}{v} - 1} + \\ - (l_1(\theta) N(\theta) + l_0(\theta)(1 - N(\theta)))^{\varepsilon} = 0. \tag{A.5}$$

Letting

$$L(\theta) \equiv l_1(\theta) N(\theta) + l_0(\theta)(1 - N(\theta)), \tag{A.6}$$

and using (A.1) and (A.2), we have that the first order condition for $l_1(\theta)$ above can be expressed as

$$\psi C(\theta)^{\frac{1}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta) L(\theta)^{\varepsilon}. \tag{A.7}$$

Following similar steps, the first-order condition for $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon}. \tag{A.8}$$

Let

$$\varphi \equiv (v - 1) / (v - \psi (v - 1)).$$

Jointly, the first-order conditions (A.7) and (A.8) – together with (A.2) and (A.3) – yield

$$l_1(\theta) = A(\theta, N(\theta))^{\varphi} l_0(\theta). \tag{A.9}$$

Using (A.9) – together with (A.1), (A.6), and (A.8) – we then obtain

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon-\psi}} \left((A(\theta, N(\theta))^{\varphi} - 1) N(\theta) + 1 \right)^{\frac{1+\varepsilon-v\varepsilon}{(v-1)(1+\varepsilon-\psi)}}. \tag{A.10}$$

Notice that (A.9) implies that, at the efficient allocation, the total labor demand, as defined in (A.6), is equal to

$$L(\theta) = l_0(\theta) \left((A(\theta, N(\theta))^{\varphi} - 1) N(\theta) + 1 \right). \tag{A.11}$$

The above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1) for any θ.
Differentiating the government's objective with respect to $N(\theta)$, we have that

$$\begin{aligned}\mathcal{Q}(\theta) = \tfrac{v}{v-1} C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}} \right) + C(\theta)^{\frac{1}{v}} y_1(\theta)^{-\frac{1}{v}} \Theta \beta N(\theta) \, l_1(\theta)^{\psi} \\ -k - L(\theta)^{\varepsilon} \left(l_1(\theta) - l_0(\theta) \right).\end{aligned} \tag{A.12}$$

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_{\theta} \mathcal{Q}(\theta) \phi(x|\theta) \, \omega(\theta) \, d\theta.$$

Using the fact that $\phi(x|\theta) \, \omega(\theta) = f(\theta|x) \, g(x)$, where $f(\theta|x)$ is the conditional density of θ given x, and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_{\theta} \mathcal{Q}(\theta) f(\theta|x) \, d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that $n(x) = 1$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and $n(x) = 0$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$.
Use (A.7) and (A.8) to observe that

$$L(\theta)^{\varepsilon} \left(l_1(\theta) - l_0(\theta) \right) = \psi C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}} \right).$$

Replacing the above expression into (A.12), we have that

$$\mathcal{Q}(\theta) = \left(\frac{v - \psi(v-1)}{v-1} \right) C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}} \right) + C(\theta)^{\frac{1}{v}} y_1(\theta)^{-\frac{1}{v}} \Theta \beta N(\theta) \, l_1(\theta)^{\psi} - k. \tag{A.13}$$

Using (A.1), (A.2), (A.3), and (A.9), after some manipulations, we have that

$$C(\theta)^{\frac{1}{v}} \left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}} \right) = \left(\left(A(\theta, N(\theta))^{\varphi} - 1 \right) N(\theta) + 1 \right)^{\frac{1}{v-1}} l_0(\theta)^{\psi} \left(A(\theta, N(\theta))^{\varphi} - 1 \right),$$

and

$$C(\theta)^{\frac{1}{v}} y_1(\theta)^{-\frac{1}{v}} \Theta \beta N(\theta) \, l_1(\theta)^{\psi} = \left(\left(A(\theta, N(\theta))^{\varphi} - 1 \right) N(\theta) + 1 \right)^{\frac{1}{v-1}} l_0(\theta)^{\psi} A(\theta, N(\theta))^{\varphi} \frac{\Theta \beta N(\theta)}{A(\theta, N(\theta))}.$$

Replacing the above expressions into (A.13) and using (A.10), we have that

$$\mathcal{Q}(\theta) = \psi^{\frac{\psi}{1+\varepsilon-\psi}} \left(\left(A\left(\theta, N\left(\theta\right)\right)^{\varphi} - 1 \right) N(\theta) + 1 \right)^{\frac{1+\varepsilon+\varepsilon\psi(1-v)}{(v-1)(1+\varepsilon-\psi)}} \times$$
$$\times \left(\frac{A\left(\theta, N\left(\theta\right)\right)^{\varphi} - 1}{\varphi} + \frac{\Theta\beta N(\theta)}{A\left(\theta, N\left(\theta\right)\right)} A\left(\theta, N\left(\theta\right)\right)^{\varphi} \right) - k. \quad \text{(A.14)}$$

When the parameters satisfy the regularity conditions, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). Because $\mathcal{Q}$ is the derivative of welfare with respect to N, the property that, for any θ, $\mathcal{Q}$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the firms (and hence by the planner) at the time the investment decisions are made. Such a property in turn implies that the first-best choice of N is either $N = 0$ or $N = 1$, for all θ. This last property, along with the fact that $\mathcal{Q}$ is increasing in θ for any N, implies that the first-best level of N is increasing in θ. This property, in turn, implies that the efficient strategy $\hat{n}\left(x\right)$ is monotone. For any θ and $\hat{x}$, then let $\bar{\mathcal{Q}}(\theta|\hat{x})$ denote the function defined in (A.14) when $N(\theta) = 1 - \Phi(\hat{x}|\theta)$, that is, when firms invest if and only if $x > \hat{x}$. Under the regularity conditions, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that $\lim_{\hat{x}\to-\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] < 0 < \lim_{\hat{x}\to+\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$. Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] = 0$ admits one and only one solution. Let $\hat{x}$ denote the solution to this equation. Then note that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] < 0$ for $x < \hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] > 0$ for $x > \hat{x}$. We conclude that, under the assumptions in the lemma, there exists a threshold $\hat{x}$ such that the investment rule $\hat{n}\left(x\right) = \mathbb{I}(x \geq \hat{x})$, along with the employment functions $\hat{l}_1(\theta)$ and $\hat{l}_0(\theta)$ satisfying the first-order conditions above, constitute a solution to the planner's problem. Q.E.D.

Proof of Lemma 2. As in the proof of Lemma 1, we drop π^x from all formulas to ease the notation. We also drop θ and use $r = py$ to denote a firm's revenues. First, we show that a transfer policy paying a total transfer $T_1(r)$ to the investing firms and $T_0(r)$ to the non-investing firms implements the efficient allocations if and only if $T_1(r) = \tau r + s$ and $T_0(r) = \tau r$ with $\tau = 1/(v-1)$ and with s possibly dependent on θ but invariant in r. To see this, first observe that the demand for each firm setting a price p for its product is given by

$$y = Cp^{-v}. \quad \text{(A.15)}$$

Next observe that, given y_1, each investing firm must employ labor

$$l_1 = \left(\frac{y_1}{1 + \Theta\left(1 + \beta N\right)} \right)^{1/\psi} \quad \text{(A.16)}$$

to produce y_1, whereas, given y_0, each non-investing firm must employ labor

$$l_0 = y_0^{1/\psi} \tag{A.17}$$

to produce y_0. The problem of each-investing firm then consists in choosing p_1 to maximize

$$p_1 y_1 - W l_1 + T_1(p_1 y_1), \tag{A.18}$$

taking W as given, accounting for the fact that the demand for its product is given by (A.15), with C exogenous to the firm's problem, and that l_1 is given by (A.16). The first-order condition with respect to p_1 is thus given by

$$\frac{d\left(p_1 y_1\right)}{dp_1} - W\frac{dl_1}{dp_1} + \frac{dT_1(p_1 y_1)}{dr}\frac{d\left(p_1 y_1\right)}{dp_1} = 0. \tag{A.19}$$

Using (A.15) and (A.16), we have that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{A.20}$$

and

$$\frac{d\left(p_1 y_1\right)}{dp_1} = \left(1 - v\right) C p_1^{-v}. \tag{A.21}$$

Replacing (A.20) and (A.21) into (A.19), using (A.15), and rearranging terms, we obtain that

$$\frac{1-v}{v} p_1 y_1 + \frac{1}{\psi} W l_1 + \frac{1-v}{v}\frac{dT_1(p_1 y_1)}{dr} p_1 y_1 = 0. \tag{A.22}$$

Next use (1) and (A.15), along with (A.9) and

$$\hat{A} \equiv A\left(\theta, \hat{N}\left(\theta\right)\right) = 1 + \Theta\left(1 + \beta \hat{N}\left(\theta\right)\right),$$

to observe that, in any equilibrium implementing the efficient allocation, firms must set prices equal to (hereafter we use "hats" to denote variables under the rules inducing the efficient allocation)

$$\hat{p}_1 = \hat{A}^{\frac{\varphi}{1-v}}\left(\left(\hat{A}^{\varphi} - 1\right)\hat{N} + 1\right)^{\frac{1}{v-1}}, \tag{A.23}$$

and

$$\hat{p}_0 = \left(\left(\hat{A}^{\varphi} - 1\right)\hat{N} + 1\right)^{\frac{1}{v-1}} \tag{A.24}$$

Furthermore, equilibrium in the labor market requires that $\hat{L}^{\varepsilon} = \hat{W}$, with $\hat{L} = \hat{N}\hat{l}_1 + (1-\hat{N})\hat{l}_0$.

Using (A.7), efficiency entails that

$$\hat{W}\hat{l}_1 = \psi \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}}, \tag{A.25}$$

which, taking advantage of (A.15), can be rewritten as

$$\hat{W}\hat{l}_1 = \psi \hat{p}_1 \hat{y}_1. \tag{A.26}$$

Condition (A.22) then implies that T_1 induces the investing firms to set the price $\hat{p}_1$ only if

$$\frac{1}{v} = \frac{v-1}{v} \frac{dT_1(p_1 y_1)}{dr}.$$

Because $\hat{p}_1 \hat{y}_1$ is state dependent, we have that T_1 must be affine and satisfy

$$T_1(r) = \frac{1}{v-1} r + s, \tag{A.27}$$

with s invariant in r. Furthermore, one can verify that, when the transfer to the firm is the one in (A.27), the firm's objective is quasi-concave in its price, which implies that setting a price equal to $\hat{p}_1$ is indeed optimal for each investing firm.
Similar arguments imply that the transfer $T_0(r)$ to the non-investing firms induces them to set $p_0 = \hat{p}_0$ if and only if

$$T_0(r) = \frac{1}{v-1} r. \tag{A.28}$$

Next, consider the properties that the subsidy s to the investing firms must satisfy to induce them to invest effciiently. As s is possibly dependent on θ, we reintroduce θ whenever relevant. The extra profit from investing can be written as

$$\mathcal{R}(\theta) \equiv \left(\frac{v - \psi(v-1)}{v-1} \right) \hat{C}(\theta)^{\frac{1}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) + s(\theta) - k, \tag{A.29}$$

where we made use of (A.25) and of the corresponding conditions for non-investing firms. Hence, each firm finds it optimal to invest if $\mathbb{E}[\mathcal{R}(\theta)|x] > 0$ and to not invest if $\mathbb{E}[\mathcal{R}(\theta)|x] < 0$. Now use the proof of Lemma 1 to note that efficiency requires that each firm invests if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and does not invest if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$, where $\mathcal{Q}(\theta)$ can be conveniently rewritten as

$$\mathcal{Q}(\theta) = \left(\tfrac{v - \psi(v-1)}{v-1} \right) \hat{C}(\theta)^{\frac{1}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) + \hat{C}(\theta)^{\frac{1}{v}} \hat{y}_1(\theta)^{\frac{v-1}{v}} \tfrac{\Theta \beta \hat{N}(\theta)}{A\left(\theta, \hat{N}(\theta)\right)} - k.$$

When the economy satisfies the regularity conditions, $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ turns from negative to positive at $x = \hat{x}$. Hence, for a subsidy $s(\theta)$ to induce efficiency in investment decisions it is both necessary and sufficient that $\mathbb{E}[\mathcal{R}(\theta)|x]$ turns from negative to positive at $x = \hat{x}$. Q.E.D.

Proof of Corollary 1. Use the derivations in the proof of Lemma 2 to observe that

$$\mathcal{R}(\theta) = \mathcal{Q}(\theta) - \frac{\Theta\beta\hat{N}(\theta)}{A\left(\theta, \hat{N}(\theta)\right)}\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} + s(\theta),$$

where $A(\theta, N(\theta))$ is the function defined in (A.4). Next observe that the function

$$\mathcal{Q}(\theta) - \frac{\Theta\beta\hat{N}(\theta)}{A\left(\theta, \hat{N}(\theta)\right)}\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}}$$

is non-decreasing in θ under the regularity conditions of Definition 2. We thus have that, when $s(\theta) = \bar{s}_{\pi^x}$ for all θ, $\mathbb{E}[\mathcal{R}(\theta)|x]$ turns from negative to positive at $x = \hat{x}$, implying that the subsidy s satisfies all the conditions in Lemma 2 and hence it is optimal. Q.E.D.

Proof of Lemma 3. The proof is in two parts. Part 1 characterizes the efficient precision of information π^{x*}. Part 2 uses the characterization in Part 1 to establish the claim in the lemma.

Part 1. Using the derivations in the proof of Lemma 1, we have that, for any π^x, irrespective of whether the economy is regular, *ex-ante* welfare under the efficient allocation is equal to

$$\mathcal{W} = \int_\theta \hat{l}_0(\theta;\pi^x)^{\psi}\left(\left(A\left(\theta, \hat{N}(\theta;\pi^x)\right)^{\varphi} - 1\right)\hat{N}(\theta;\pi^x) + 1\right)^{\frac{v}{v-1}} d\Omega(\theta) +$$
$$-k\int_\theta \hat{N}(\theta;\pi^x)\, d\Omega(\theta) - \int_\theta \frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left(\left(A\left(\theta, \hat{N}(\theta;\pi^x)\right)^{\varphi} - 1\right)\hat{N}(\theta;\pi^x) + 1\right)^{1+\varepsilon} d\Omega(\theta) - \mathcal{I}(\pi^x),$$

where $A(\theta, N(\theta))$ is the function defined in (A.4). Using the envelope theorem, we then have that π^{x*} solves

$$\mathbb{E}\left[\frac{v}{v-1}\hat{l}_0\left(\theta;\pi^{x*}\right)^{\psi\frac{v-1}{v}}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}\left(A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1+\right.\right.$$
$$\left.\left.+\ \varphi A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]+$$
$$-\ \mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\times\right.$$
$$\left.\times\left(A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1+\varphi A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \tag{A.30}$$

The above condition identifies the efficient precision of private information π^{x*}.

Part 2. Suppose that all firms other than i acquire information of precision π^{x*} and consider firm i's problem. Under the policy in the lemma, in each state θ, the price that maximizes firm i's profit coincides with the one that induces the efficient allocation for precision π^{x*}, irrespective of firm i's choice of π_i^x. This price is equal to $\hat{p}_1^*$ if the firm invests and $\hat{p}_0^*$ if the firm does not invest, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the functions in (A.23) and (A.24), respectively, evaluated at $\pi^x=\pi^{x*}$. Note that we use the combination between "^" and "*" to denote variables under the efficient allocation for precision π^{x*} (this notation applies not only to $\hat{p}_1^*$ and $\hat{p}_0^*$ but to all expressions below).

Dropping θ from the argument of each function to ease the notation, we have that firm i's value function is equal to $\bar{\Pi}_i(\pi_i^x)\equiv\sup_{\varsigma:\mathbb{R}\to[0,1]}\Pi_i(\pi_i^x;\varsigma)$, where

$$\Pi_i(\pi_i^x;\varsigma)\equiv\mathbb{E}\left[\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right]-\mathbb{E}\left[\hat{W}^*\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]+$$
$$+\,\mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x;\varsigma)\equiv\int\varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i invests when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*=\tau\hat{r}_1^*+s$ and $\hat{T}_0^*=\tau\hat{r}_0^*$ denoting the transfers received when generating revenues $\hat{r}_1^*=\hat{p}_1^*\hat{y}_1^*$ and $\hat{r}_0^*=\hat{p}_0^*\hat{y}_0^*$, after investing and not investing, respectively.

Substituting

$$\hat{r}_f^*=\hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}, \tag{A.31}$$

$f=0,1$, into $\Pi_i(\varsigma;\pi_i^x)$ and using Conditions (A.9) and (1), as well as the fact that $1+\varphi\psi=$

$\frac{v}{v-1}\varphi$, we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\left(\hat{A}^{*\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+\\ &-\mathbb{E}\left[\hat{W}^{*}\left(\left(\hat{A}^{*\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*}\right]+\\ &+\mathbb{E}\left[\hat{T}_1^{*}\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^{*}\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly,

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\hat{A}^{*\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+\\ &-\mathbb{E}\left[\hat{W}^{*}\left(\hat{A}^{*\varphi}-1\right)\hat{l}_0^{*}\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]+\\ &+\mathbb{E}\left[\left(\hat{T}_1^{*}-\hat{T}_0^{*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}.\end{aligned}\tag{A.32}$$

Using (1), (A.9), and (A.31), we have that

$$\hat{T}_1^{*}-\hat{T}_0^{*}=s+\frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(\hat{A}^{*\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}.$$

Replacing this expression into (A.32), we obtain that

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\hat{A}^{*\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+\\ &-\mathbb{E}\left[\hat{W}^{*}\left(\hat{A}^{*\varphi}-1\right)\hat{l}_0^{*}\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]+\mathbb{E}\left[s\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}.\end{aligned}\tag{A.33}$$

Recall that, when $\pi_i^x=\pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma=\hat{n}^*$. Using the envelope theorem, we thus have that

$$\begin{aligned}\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial\Pi_i(\hat{n}^*;\pi^{x*})}{\partial\pi_i^x} =& \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}}\left(\hat{A}^{*\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+\\ &-\mathbb{E}\left[\hat{W}^{*}\left(\hat{A}^{*\varphi}-1\right)\hat{l}_0^{*}\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+\mathbb{E}\left[s\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x},\end{aligned}$$

where $\partial\hat{N}^*/\partial\pi^x$ is the marginal change in the measure of investing firms that obtains when one changes π^x at $\pi^x=\pi^{x*}$, holding $\hat{n}^*$ fixed. For the proposed policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. This requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}\left(A\left(\theta,\hat{N}(\theta;\pi^{x*})\right)^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\hat{l}_0(\theta;\pi^{x*})^{\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(A\left(\theta,\hat{N}(\theta;\pi^{x*})\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\left(A\left(\theta,\hat{N}(\theta;\pi^{x*})\right)^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]+$$
$$+\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^{x}}, \quad \text{(A.34)}$$

where we use the fact that $\hat{W}^* = \hat{L}^{*\epsilon}$, and reintroduce all the arguments of the various functions to make the result consistent with the claim in the main text.
The difference between Conditions (A.30) and (A.34) net of the the subsidy $s(\theta)$ is given by

$$\mathbb{E}\left[\left(\frac{v}{v-1}\hat{l}_0\left(\theta;\pi^{x*}\right)^{\psi\frac{v-1}{v}}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}-\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\right)\times\right.$$
$$\left.\times\,\varphi A\left(\theta,\hat{N}\left(\theta;\pi^{x*}\right)\right)^{\varphi-1}\Theta\beta\hat{N}\left(\theta;\pi^{x*}\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]. \quad \text{(A.35)}$$

Taking advantage of (A.3), (A.8), and (A.11), we have that

$$\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(A\left(\theta,\hat{N}(\theta;\pi^{x*})\right)^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}=\psi\hat{C}(\theta;\pi^{x*})^{\frac{1}{v}}\hat{l}_0(\theta;\pi^{x*})^{\psi\frac{v-1}{v}},$$

so that (A.35) becomes

$$\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{\psi\frac{v-1}{v}}\hat{C}\left(\theta;\pi^{x*}\right)^{\frac{1}{v}}A\left(\theta,\hat{N}(\theta;\pi^{x*})\right)^{\varphi-1}\Theta\beta\hat{N}(\theta;\pi^{x*})\right], \quad \text{(A.36)}$$

which, using (A.2), (A.4), and the fact that $1+\varphi\psi=\frac{v}{v-1}\varphi$, immediately gives Condition (13) in Lemma 3. Q.E.D.