# Investment Subsidies with Spillovers and Endogenous Private Information

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# This paper

- Firms face uncertainty about the returns to their investment in new technologies.
  - unknown fundamentals
  - spillovers
- Uncertainty is largely endogenous:
  - firms can acquire information about the fundamentals affecting profitability
- Combination of market power with firms' investment spillovers
  - inefficiency in information **acquisition** and **usage**.
- How should policy be designed to alleviate the inefficiencies?

# Main take-aways

- Exogenous information,
  - inefficiencies corrected with a constant subsidy to innovative firms
- Endogenous information
  - **Pigouvian corrections**
- Insights extend to richer economies with nominal rigidities

# Related Literature

- **Complementarities and investment spillovers**
  - Cooper and John (1988); Matsuyama (1991)
- **Corrective taxation in presence of externalities**
  - Sandmo (1975); Bovenberg and Goulder (1996); Barrage (2020)
- **(Pigouvian) subsidies in macro**
  - Heutel (2012); Grossman et al. (2013); Jeanne and Korinek (2019)
- **Policy with dispersed information**
  - Angeletos and Pavan (2009); Angeletos and La'O (2020); Angeletos, Iovino, and La'O (2020); Colombo, Femminis, and Pavan (2014); Llosa and Venkatesvaran (2022); Bergemann and Välimäki (2002)
- **Inefficiency in information acquisition**
  - Colombo, Femminis,Pavan (2014); Pavan (2017); Hebert and La'O (2020), Pavan, Sundaresan, Vives (2025)

# Plan

1. Introduction
2. **Model (price flexibility)**
3. Efficiency
4. Optimal Fiscal Policy
5. Richer Economies (sticky prices)
6. Conclusions

# Model

- Economy populated by
  - a (measure 1) continuum of agents
  - a (measure 1) continuum of monopolistically-competitive firms producing differentiated intermediate goods
  - a competitive retail sector producing the final good (using the intermediate goods as inputs)
  - a benevolent public authority controlling fiscal (and monetary) policy

# Model

- Each firm is run by a single entrepreneur (sole supplier of $n_i$)
  - chooses whether to upgrade technology for producing intermediate good $i \in [0, 1]$

$$y_i = \begin{cases} (1 + \Theta(1 + \beta N)) \, l_i^{\psi} & \text{if } n_i = 1 \text{ (new)} \\ l_i^{\psi} & \text{if } n_i = 0 \text{ (old)} \end{cases},$$

  - with $\beta \geq 0$ and $\psi \leq 1$.
  - $N = \int n_i di$: **aggregate investment** in new technology
  - $l_i$ undifferentiated labor
  - $\Theta$: aggregate fundamental
- Differential $y_i(n_i = 1) - y_i(n_i = 0)$ is increasing $\Theta$ and $N$
- Spillovers (within and across technologies)
- Cost of new technology: $k > 0$

# Model

- The final good is produced by a competitive retail sector using CES technology

$$Y = \left( \int y_i^{\frac{\nu-1}{\nu}} di \right)^{\frac{\nu}{\nu-1}},$$

- Profits of competitive retail sector

$$\Pi = PY - \int p_i y_i di,$$

- $P$: price of final good
- $p_i$: price of intermediate good of variety $i$

# Model (alternative production setup)

- Firms choose whether to produce the intermediate goods in
  - a traditional ($n_i = 0$)
  - "smart" (Industry 4.0) specification.($n_i = 1$)
- Each firm produces

$$y_i = l_i^{\psi},$$

  both using a smart ($n_i = 1$) or traditional ($n_i = 0$) specification
- The amount of the final good produced is equal to

$$Y = \left( \int_i \left( \left( 1 + \Theta \left( 1 + \beta N \right) n_i \right) y_i \right)^{\frac{v-1}{v}} di \right)^{\frac{v}{v-1}}.$$

# Model (alternative production setup)

- Smart specification increases production of final good
  - directly
  - enhancing the interoperability/productivity of other intermediate goods, including those supplied in the traditional specification
- Inputs produced under smart specification: charge a higher price

# Model

- Each entrepreneur is a member of a representative household whose utility is given by

$$U = C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x) di.$$

$\longrightarrow$ $C$ is consumption,.

$\longrightarrow$ $l^{1+\varepsilon}/(1+\varepsilon)$, $\varepsilon > 0$, denotes the disutility of labor

$\longrightarrow$ Homogenous labor exchanged in a competitive market

$\longrightarrow$ Each worker provides the same amount of labor

$\longrightarrow$ $I(\pi_i^x)$, disutility of acquiring information of precision $\pi_i^x$.

- Representative household collects profits and wages, and purchases final good under the **budget constraint**

$$C \leq \int \left( \frac{p_i y_i - W l_i}{P} \right) + T_i + \frac{W}{P} \int l_i di - \Sigma$$

$\longrightarrow$ $W$: nominal wage rate,

$\longrightarrow$ $T_i$; transfer to firm $i$ (in units of consumption of the final good), as a function of whether the firm invested or not, and of the firm's revenues $r = p_i y_i / P$

$\longrightarrow$ $\Sigma$; lump-sum tax (in units of consumption of the final good)

# Model

- Each entrepreneur maximizes firm's market valuation

$$\mathbb{E}\left[\frac{p_i y_i - W l_i}{P} + T_i - k n_i - \mathcal{I}(\pi_i^x)\right].$$

- Government's maximizes welfare

$$\mathcal{W} = \mathbb{E}\left[C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \mathcal{I}(\pi^x)\right]$$

- by designing fiscal policies

# Model - Timing

1. Government announces its policies
2. Nature draws $\theta = \ln(\Theta)$
3. Each entrepreneur chooses the precision $\pi_i^x$ of his private private signal $x_i = \theta + \xi_i$ (with $\xi_i \backsim N(0, 1/\pi_i^x)$)
4. Entrepreneurs simultaneously choose $n_i$ after observing $x_i$
5. $\theta$ and $N$ are publicly revealed
6. Entrepreneurs simultaneously set $p_i$
7. Retail sector chooses demand $y_i$ of each intermediate good, taking prices of intermediate goods $p_i$ and $P$ of the final good as given
8. Each entrepreneur hires $l_i$ on a competitive labor market to produce $y_i$ (taking $N$ and $\theta$ as given)
9. The representative household purchases final good, taking $P$ as given

# Equilibrium and Optimality

- Prices and employment depend on technology choice
  - old technology: $p_0(\theta, \pi^x)$ and $l_0(\theta, \pi^x)$
  - new technology: $p_1(\theta, \pi^x)$ and $l_1(\theta, \pi^x)$

**Definition**

Equilibrium is precision $\pi^x$ along with $n(x, \pi^x)$, $p_0(\theta, \pi^x)$ and $p_1(\theta, \pi^x)$ s.t., when each firm $j \neq i$, chooses $\pi^x$ and then invests according to $n(x, \pi^x)$ and sets prices $p_0(\theta, \pi^x)$ and $p_1(\theta, \pi^x)$, each entrepreneur $i$ maximizes her market valuation by doing the same

- Key features:

1. **endogeneity of private information**
2. **investment spillovers**

# Plan

1. Introduction
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3. **Efficiency**
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# Decentralized Efficiency: Efficient Information Use

- Suppose the precision of private information is fixed at $\pi^x$ for all $i$
- Efficient use of private information given by
  - $\hat{n}\left(x;\pi^x\right)$: investment
  - $\hat{l}_1\left(x,\theta;\pi^x\right)$, $\hat{l}_0\left(x,\theta;\pi^x\right)$: employment
- $\hat{N}\left(x;\pi^x\right)$: efficient aggregate investment
- $\hat{C}\left(x;\pi^x\right)$: efficient aggregate investment consumption of final good

# Efficient Information Use

Efficient upgrade policy: for any precision of private information $\pi^x$, there exists a constant $\hat{x}(\pi^x)$ such that $\hat{n}(x;\pi^x) = I(x \geq \hat{x}(\pi^x))$

- $\hat{y}_i = n_i\left(1+\Theta\left(1+\beta N\right)\right)\hat{l}_1^{\psi} + \left(1-n_i\right)\hat{l}_0^{\psi}$
- $\hat{Y} = \left(\int \hat{y}_i^{\frac{\nu-1}{\nu}} di\right)^{\frac{\nu}{\nu-1}}$

**Definition**

The economy is regular if $\nu < 1 + (1+\varepsilon)/\varepsilon\psi$.

**Lemma**

*Under regularity, for any $\pi^x$, there exists $\hat{x}(\pi^x)$ s.t. efficiency in technology adoption requires investing iff $x > \hat{x}(\pi^x)$.*

# Efficient Information Acquisition

**Efficient precision** $\pi^{x*}$ computed with the envelope theorem taking efficient rules $\hat{n}\left(x;\pi^{x*}\right)$, $\hat{l}_0\left(\theta;\pi^{x*}\right)$ and $\hat{l}_1\left(\theta;\pi^{x*}\right)$ fixed.

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# Optimal Fiscal Policy: Exogenous Information

**Definition**

The fiscal policy $\bar{T}$ is **optimal** if it induces all firms to invest according to efficient rule $\hat{n}(x;\pi^x)$ and set prices $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$ generating demands inducing efficient employment $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$.

# Optimal Fiscal Policy: Exogenous Information

$\mathcal{R}(\theta; \pi^x)$: private benefit of investing, net of subsidy and cost of acquiring information

## Lemma

*Optimal policy* $\bar{T}$ *pays to non-investing*

$$\bar{T}_0(r) = \frac{1}{v-1} r$$

*and to investing firms*

$$\bar{T}_1(r, \theta) = \frac{1}{v-1} r + s(\theta)$$

*with the* **additional subsidy** $s(\theta)$ *s.t.*

- $\mathbb{E}\left[\mathcal{R}(\theta; \pi^x) | x, \pi^x\right] < 0$ *when* $x < \hat{x}(\pi^x)$
- $\mathbb{E}\left[\mathcal{R}(\theta; \pi^x) | x, \pi^x\right] > 0$ *when* $x > \hat{x}(\pi^x)$

# Optimal Fiscal Policy: Exogenous Information

- The familiar revenue subsidy $r/(v-1)$ offsetting market power
- Additional subsidy $s(\theta)$ corrects for inefficiencies due to **spillovers**.

# Optimal Fiscal Policy: Exogenous Information

- $\mathcal{Q}(\theta;\pi^x)$: social benefit of investing
- Private benefit of investing

$$\mathcal{R}(\theta;\pi^x) = \mathcal{Q}(\theta;\pi^x) - \\ + \frac{\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\hat{C}(\theta;\pi^x)^{\frac{1}{\nu}}\hat{y}_1(\theta;\pi^x)^{\frac{\nu-1}{\nu}} + s(\theta)$$

where the **spillover externality**

$$\frac{\Theta\beta\hat{N}(\theta;\pi^x)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\hat{C}(\theta;\pi^x)^{\frac{1}{\nu}}\hat{y}_1(\theta;\pi^x)^{\frac{\nu-1}{\nu}}$$

is equal to the increase in production of final output when $N$ increases around the efficient level $\hat{N}(\theta;\pi^x)$, holding fixed all firms' investment and employment decisions

# Optimal Fiscal Policy: Exogenous Information

- Many subsidies $s(\theta)$ realign incentives
  - some are simple then others...

**Corollary**

*When information is exogenous, a constant subsidy to investing firms (over and above $r/(v-1)$) is optimal.*

# Optimal Fiscal Policy: Exogenous Information

- Constant (state invariant) subsidy is equal to the spillover externality

$$\bar{s}_{\pi^x} = \mathbb{E}\left[\frac{\Theta\beta\hat{N}\left(\theta;\pi^x\right)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\hat{C}\left(\theta;\pi^x\right)^{\frac{1}{\nu}}\hat{y}_1\left(\theta;\pi^x\right)^{\frac{\nu-1}{\nu}}\middle|\,\hat{x}(\pi^x),\pi^x\right]$$

  expected by the marginal investor with signal $\hat{x}(\pi^x)$.

- Marginal agent is indifferent since private benefit

$$\mathcal{R}(\theta;\pi^x) = \mathcal{Q}(\theta;\pi^x) - \\ + \frac{\Theta\beta\hat{N}\left(\theta;\pi^x\right)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^x)\right)}\hat{C}\left(\theta;\pi^x\right)^{\frac{1}{\nu}}\hat{y}_1\left(\theta;\pi^x\right)^{\frac{\nu-1}{\nu}} + \bar{s}_{\pi^x},$$

  is monotone in $\theta$, firms invest iff $x > \hat{x}(\pi^x)$,which is efficient.

# Optimal Fiscal Policy: Endogenous Information

**Definition**

The fiscal policy $T^*$ is **optimal** if it induces all firms to

1. choose the efficient precision $\pi^{x*}$
2. invest efficiently according to $\hat{n}(x; \pi^{x*})$
3. set prices $\hat{p}_1(\theta; \pi^{x*})$ and $\hat{p}_0(\theta; \pi^{x*})$ that induce efficient demands and hence efficient employment $\hat{l}_1(\theta; \pi^{x*})$ and $\hat{l}_0(\theta; \pi^{x*})$

# Optimal Fiscal Policy: Endogenous Information

$\partial \hat{N}\left(\theta; \pi^{x*}\right) / \partial \pi^{x}$: variation in firms investing at $\theta$ due to change in $\pi^{x}$ (holding fixed efficient investment rule $\hat{n}(x; \pi^{x*})$)

## Lemma

*Additional subsidy $s(\theta)$ to investing firms must satisfy*

$$\mathbb{E}\left[s(\theta)\right] = \mathbb{E}\left[\frac{\Theta\beta\hat{N}\left(\theta;\pi^{x}\right)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^{x})\right)}\hat{C}\left(\theta;\pi^{x}\right)^{\frac{1}{\nu}}\hat{y}_1\left(\theta;\pi^{x}\right)^{\frac{\nu-1}{\nu}}\middle|\,\hat{x}(\pi^{x}),\pi^{x}\right].$$

$$\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right] =$$

$$\mathbb{E}\left[\frac{\Theta\beta\hat{N}\left(\theta;\pi^{x}\right)}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^{x})\right)}\hat{C}\left(\theta;\pi^{x}\right)^{\frac{1}{\nu}}\hat{y}_1\left(\theta;\pi^{x}\right)^{\frac{\nu-1}{\nu}}\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\middle|\,\hat{x}(\pi^{x}),\pi^{x}\right].$$

- Covariance condition essential to realign incentives for acquisition

# Optimal Fiscal Policy: Endogenous Information

## Theorem

*Independently of whether economy is regular, fiscal policy*

$$T_0(r) = \frac{1}{\nu - 1} r,$$

$$T_1(r, \theta) = \frac{1}{\nu - 1} r + s(\theta).$$

*with state-contingent subsidy*

$$s(\theta) = \frac{\Theta \beta \hat{N}(\theta; \pi^{x*})}{1 + \Theta\left(1 + \beta \hat{N}(\theta; \pi^{x*})\right)} \hat{C}(\theta; \pi^{x*})^{\frac{1}{\nu}} \hat{y}_1(\theta; \pi^{x*})^{\frac{\nu-1}{\nu}}$$

*to investing firms induces efficiency in both acquisition and usage of information.*

# Optimal Fiscal Policy: Endogenous Information

- State-contingent subsidy

$$s(\theta) = \frac{\Theta\beta\hat{N}(\theta;\pi^{x*})}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^{x*})\right)}\hat{C}(\theta;\pi^{x*})^{\frac{1}{\nu}}\hat{y}_1(\theta;\pi^{x*})^{\frac{\nu-1}{\nu}}$$

  equals ex-post spillover externality, i.e., marginal change in final good production generated by a marginal change in $N$, at $N^*(\theta,\pi^{x*})$ holding firms' investment and employment fixed at the efficient level)

- $s(\theta)$: **Pigouvian correction**
- Known fact: Pigouvian subsidies work well under complete information
- Contribution: they also induce efficient info acquisition and usage (neither verifiable)

# Optimal Fiscal Policy: Endogenous Information

- Subsidy

$$s(\theta) = \frac{\Theta\beta\hat{N}(\theta;\pi^{x*})}{1+\Theta\left(1+\beta\hat{N}(\theta;\pi^{x*})\right)}\hat{C}(\theta;\pi^{x*})^{\frac{1}{\nu}}\hat{y}_1(\theta;\pi^{x*})^{\frac{\nu-1}{\nu}}$$

  requires knowing $\pi^{x*}$ and hence the cost of information $I(\pi^{x*})$

- More generally, government must be familiar with information technology

- Such difficulty bypassed by conditioning $s$ on distribution $\Lambda$ of
  - investing firms
  - employment decisions

# Optimal Fiscal Policy: Endogenous Information

- $C_N(\theta, \Lambda)$ marginal change in $C$ due to marginal change in $N$ holding investment and employment decisions fixed at level specified by $\Lambda$

**Theorem**

*Independently of whether economy is regular, efficiency in both info acquisition and usage is induced through fiscal policy*

$$T_0(r) = \frac{1}{\nu - 1} r, \quad T_1(r, \theta, \Lambda) = \frac{1}{\nu - 1} r + s(\theta, \Lambda)$$

*with*

$$s(\theta, \Lambda) = C_N(\theta, \Lambda).$$

# Optimal Fiscal Policy: Endogenous Information

- Subsidy equal to **ex-post spillover externality**, given
  - fundamental $\theta$
  - cross-section distribution of firms' investment and employment
- Planner leaves it to firms to figure out efficient allocation
  - no need to know information technology/cost

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# Richer Economies

Representative household's with function

$$U = \frac{C^{1-R}}{1-R} - kN - \frac{I^{1+\varepsilon}}{1+\varepsilon} - \int I(\pi_i^x)di$$

$R > 0$ in case agents are risk averse

- Sticky prices
  - $p_i$ set under imperfect information

# Richer Economies

- Money plays a role

- Money supply $M(\theta)$ used to meet a "**cash in advance**" constraint

$$PY \leq M$$

# Richer Economies

- Aggregate investment $N$ affects equally both investing and non-investing firms
- Production of intermediates is given by

$$y_i = \begin{cases} \gamma\Theta\left(1+\beta N\right)^{\alpha} I_i^{\psi} & \text{if } n_i = 1 \text{ (new)} \\ \Theta\left(1+\beta N\right)^{\alpha} I_i^{\psi} & \text{if } n_i = 0 \text{ (old)} \end{cases}$$

- with $\gamma > 1$ investment increases the amount of goods produced
- $\alpha \geq 0$ controls for the return to scale
- Differential $y_i(n_i = 1) - y_i(n_i = 0)$ is still increasing in $\Theta$ and $N$

# Richer Economies: Optimal fiscal *and monetary* policy

**Theorem**

*Above fiscal policies along with monetary policy that induces firms to disregard their private information when setting prices and only use it for investment purposes implements efficient acquisition and usage of information as a sticky-price equilibrium.*

# Richer Economies: Optimal fiscal *and monetary* policy

- Familiar monetary policy + novel **Pigouvian corrections** induce efficiency in
  - info acquisition
  - investment
  - employment
  - production and consumption of intermediate and final goods

# Conclusions

- Power of Pigouvian corrections extends to economies with endogenous and dispersed information
- Exogenous information + regularity
  - constant subsidy
- In general, subsidy to investing firms
  - state-dependent
  - contingent on distribution of investment and employment decisions

# Most Important Slide

Thank You!