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On the optimality of privacy in sequential contracting ☆

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Abstract

This paper studies the exchange of information between two principals who contract sequentially with the same agent, as in the case of a buyer who purchases from multiple sellers. We show that when (a) the upstream principal is not personally interested in the downstream level of trade, (b) the agent's valuations are positively correlated, and (c) preferences in the downstream relationship are separable, then it is optimal for the upstream principal to offer the agent full privacy. On the contrary, when any of these conditions is violated, there exist preferences for which disclosure is strictly optimal, even if the downstream principal does not pay for the information. We also examine the effects of disclosure on welfare and show that it does not necessarily reduce the agent's surplus in the two relationships and in some cases may even yield a Pareto improvement.

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Keywords: Contractual and informational externalities; Mechanism design; Optimal disclosure policies; Sequential common agency games; Exogenous and endogenous private information

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1. Introduction

In markets where different principals contract sequentially with the same agent, as when a buyer purchases from multiple sellers, the contracts offered by a downstream principal are often influenced by the decisions taken, as well as the information disclosed, in upstream relationships. [1]

A buyer's willingness to pay for a product may depend on its complementarity or substitutability with the products and services of upstream vendors. For example, the value of a new software application depends largely on its compatibility with the user's operating system, hardware, and other software applications. Furthermore, even in the absence of complementarities, the choice of a product, the request for a service, or simply the path followed in visiting a website may reveal valuable information about consumers' preferences and idiosyncratic characteristics. Knowing what products a consumer has purchased upstream thus allows a downstream seller to better tailor her contract offers and price discriminate. Personalized offers based on upstream transactions have indeed become common practice since the advent of online commerce and are present in a variety of markets including software, travel, and pharmaceutical products.

An upstream seller who expects her buyers to contract with a downstream one is thus likely to take advantage of her Stackelberg position by designing contract offers in a way that optimally controls for the influence they have on downstream contracting. There are two ways an upstream contract can affect a downstream one: directly, through the decisions it stipulates (*contractual externalities*), and indirectly, through the information it discloses (*informational externalities*).

In this paper we investigate how a principal should optimally control for both types of externalities, designing a menu of contract offers that screens the agent's type and strategically discloses information to a downstream principal.

We show that when (a) the upstream principal is not personally interested in the downstream level of trade, (b) the agent's valuations are positively correlated, and (c) preferences in the downstream relationship are separable so that the level of trade is independent of upstream decisions, then the optimal disclosure policy consists in offering the agent full privacy. This holds regardless of the price the downstream principal is willing to pay.

In fact, under conditions (a)–(c), a downstream seller is interested in getting information on upstream decisions only if this is indirectly informative about the buyer's exogenous type. The only benefits of disclosure then come from an *information-trade* effect, i.e. the possibility of making a profit by selling information to the downstream seller, and/or a *rent-shifting* effect, that is, the possibility of inducing the downstream seller to offer the buyer a personalized discount.

Suppose the buyer has either a low or a high valuation for the product of the downstream seller. If the latter believes the buyer's valuation is high (say, because marketing surveys indicate that the percentage of high-valuation buyers is significantly higher than that of low-valuation ones), the optimal price in the downstream relationship leaves no surplus to the buyer. In this case, the upstream seller may attempt to induce the downstream one to

[1] Hereafter, a principal is the party who designs the contract. We also adopt the convention of using masculine pronouns for the agent/buyer and feminine pronouns for the principals/sellers.

offer a discount by disclosing information that is correlated with the buyer's valuation. The rent-shifting effect then consists in making the buyer pay a higher price upstream for the increase in his expected utility downstream.

However, for disclosure to be valuable, the buyer must be given an incentive to reveal his type. When valuations are positively correlated, the extra rent a seller must leave to the buyer when she discloses information more than offsets both the rent-shifting and the information-trade effects, making full privacy optimal.

Conversely, we also prove that when any one of the above conditions is violated, there exist preferences for which disclosure is strictly optimal, even if the upstream principal is not allowed, or able, to sell information.

Firstly, consider direct externalities. When the upstream principal is personally interested in the downstream level of trade, as in the case of a vendor whose compensation depends on the sales (or market share) of another vendor, she may well accept to pay the incentive costs of disclosure (in terms of higher rents to the buyer) if this enables her to affect decisions downstream. With positive externalities, disclosure is optimal when it increases the downstream level of trade; with negative externalities, when it decreases it.

Next, we relax the assumption of positive correlation in the agents' valuations by considering the case of two horizontally differentiated sellers. When the single crossing condition is of opposite sign for upstream and downstream decisions, disclosure does not necessarily increase the rent that the upstream principal must leave to the buyer, it may actually reduce it. By increasing the downstream rent of those types who value the upstream product the least, disclosure creates *countervailing incentives* that can be used to minimize the informational rents required for information revelation. On the other hand, since disclosure is incentive-compatible only if trade in the upstream relationship is not certain, it is optimal only when the cost of not selling to all types is small as compared with the benefit of increasing the probability of a price discount downstream.

Finally, we consider environments where the agents' preferences are not separable, as in the case of a buyer whose willingness to pay for a product depends on its complementarity, or substitutability, with the products of an upstream seller (the case of contractual externalities). By introducing uncertainty about upstream decisions (for example, through lotteries, mixed strategies, or simply by selling only to a subset of types), a seller can create rents for her buyers in the downstream relationship. In this case, the optimal mechanism may also require a policy that discloses information correlated with the upstream level of trade. With *complements*, disclosure is motivated by the possibility of inducing the downstream seller to ask a lower price to consumers who have purchased upstream, whereas with *substitutes*, to those who did not.

For each of the environments described above, we also compare the equilibrium contracts when a principal cannot disclose information with the contracts that are offered in equilibrium when disclosure is permitted. Perhaps surprisingly, disclosure does not necessarily harm the agent, it may actually increase his surplus in the two relationships. This is consistent with a claim that is commonly made by vendors in their privacy policies, namely that consumers who agree to share information with the vendor's business partners may benefit from personalized discounts and tailor-made offers.

The effects of disclosure on total welfare—the sum of sellers' profits and consumer surplus—remain, however, ambiguous. On the one hand, by reducing the distortions due to the initial asymmetry of information, disclosure tends to increase efficiency in downstream contracting. On the other hand, disclosure may introduce new distortions in upstream decisions. This may be due to incentive compatibility or to the uncertainty about the level of upstream trade introduced with the intent of leading to an increase in consumer surplus downstream. [2]

None of these results is really specific to buyer–seller relationships. We expect the determinants of information disclosure discussed above to play an important role also in

Labor relationships: An employer who hires a worker typically receives letters of recommendation from previous employers describing the worker's characteristics (talent, fairness, relations with colleagues), but also the tasks performed in upstream relationships.

Insurance: Clients who purchase multiple policies are notified that relevant personal information (e.g. the number of accidents in the past few years and the type of risk borne by policy-holders) will be shared with partners.

Financial relationships: Venture capitalists often disclose information about a project's profitability, as well as personal characteristics of entrepreneurs, to other investors in order to convince them to join. Entrants in the credit card market get detailed information on potential customers from credit bureaus and other lenders.

Regulation and taxation of multinational firms: Foreign regulators usually operate on the basis of the information provided by domestic agencies. Information-sharing between domestic and foreign tax authorities is often considered to be largely strategic and is at the heart of political debates.

In what follows, first we briefly relate the paper to the pertinent literature. Section 2 then describes the sequential contracting game and illustrates how optimal policies can be obtained through a mechanism design approach. Section 3 derives the conditions for the optimality of full privacy. Sections 4 and 5 examine the determinants of the disclosure of exogenous and endogenous information. Section 6 concludes. Technical proofs are given in the appendix.

1.1. Related literature

This paper is related to several lines of research in contract theory, mechanism design, and industrial organization with asymmetric information.

Strategic information-sharing between firms has been examined in the literature on oligopolistic competition (see [26] for a survey), and in the financial intermediation literature [21,22]. In these papers, before competing, firms decide whether to share information with rivals. In our model, by contrast, upstream principals are initially uninformed; in fact, they learn by contracting with the agent and create new private information by taking deci-

[2] For disclosure to have positive effects on welfare and consumer surplus, it is important that buyers be able to trust that firms will keep their promises about their privacy policies, as we assume in this paper. One way vendors can increase consumers' confidence is by signing contracts with certification intermediaries such as Better Business Bureau, TRUSTe and WebTrust. By displaying the seal of these intermediaries, a vendor agrees to inform consumers of what personally identifiable information is collected, which organization collects it, how it is used, with whom it may be shared, and what choices are available to consumers regarding its collection, use and distribution. For a detailed discussion of the importance of trust in e-commerce, see the Federal Trade Commission report "Privacy Online: Fair Information Practices in the Electronic Marketplace" 2000.

sions that affect downstream principals. Sellers' disclosure policies have also been analyzed by Lizzeri [14] in a model where certification intermediaries have a technology to test the quality of the seller's product and commit on what to disclose to competitive buyers. Here, instead, we assume that the only way a principal can learn the agent's private information is through a screening mechanism.

A recent literature on consumers' privacy considers environments where sellers can use information on individual purchasing history to engage in product customization and price discrimination [1,9,31,32]. In this literature, however, the choice of the disclosure policy is not endogenous.

Informational linkages between markets have been studied in the literature on auctions followed by resale. Haile [12] examines the effects on revenue of bidders' incentives to signal information to the secondary market. Calzolari and Pavan [6] and Zheng [32] study optimal auctions and derive revenue-maximizing selling procedures and disclosure policies.

Sequential common agency models have also been examined in [2,4,16,25]. In this literature, principals offer their contracts sequentially, but decisions are taken only after the agents have received all proposals. On the contrary, we assume that the agent first contracts with an upstream principal, reveals his exogenous type, takes a payoff-relevant decision, and then enters into a new bilateral relationship with a second principal. This timing is more appropriate for examining the design of optimal disclosure policies.

Segal [27], and Segal and Whinston [28] provide a general and unifying framework for contracting with externalities. Martimort and Stole [18] consider direct externalities between principals in a simultaneous common agency game. Daughety and Reinganum [8] examine the role of informational externalities and confidentiality in a model where two plaintiffs sequentially file suit against the same defendant. Unlike these works, the current paper combines direct externalities with informational ones and shows how they are fashioned by an upstream principal through the design of an *optimal* disclosure policy.

2. The contracting environment

2.1. The model setup

2.1.1. Players

Since none of the results is truly specific to buyer–seller relationships, we find it convenient to describe the contracting environment as a common agency game where two principals, P_1 and P_2, contract sequentially with the same agent, A.[3]

2.1.2. Allocations and preferences

Each principal must select a *decision* $x_i \in X_i$ and a *transfer* $t_i \in T_i = \mathbb{R}$ from A to P_i. The vector $\mathbf{x} \equiv (x_1, x_2) \in \mathbf{X} \equiv X_1 \times X_2$ denotes a profile of decisions for the two principals. The agent's preferences are represented by the function $U_A = v_A(\mathbf{x}, \theta) - t_1 - t_2$,

[3] The model can also be read as one with a continuum of buyers with independent valuations, provided that there are no direct externalities among the buyers and that the sellers' payoffs are additive in the trades. See, for example [30].

and the two principals' preferences by $U_i = v_i(\mathbf{x}, \theta) + t_i$, for $i = 1, 2$. The variable $\theta \in \Theta$ denotes the agent's exogenous private information. We assume that X_i and Θ are finite sets with $X_i = \{0, 1\}$ and $\Theta \equiv \{\underline{\theta}, \overline{\theta}\}$. $x_i = 1$ denotes the decision to trade and $x_i = 0$ the "status quo," that is, $x_i = 0$ in the absence of a contract between A and P_i, with $v_i(0, 0, \theta) = 0$ for any $\theta \in \Theta$ and $i \in \{1, 2, A\}$. The two principals are assumed to share a common prior $\Pr(\overline{\theta}) = p = 1 - \Pr(\underline{\theta})$.

That Θ and X_i are finite sets simplifies the description of the stochastic mechanisms. As is shown in the appendix, Theorem 1 extends to environments where θ is continuously distributed over $[\underline{\theta}, \overline{\theta}]$ as well as to $X_1 = X_2 = \mathbb{R}_+$.

2.1.3. Contracts and privacy policies

Each principal offers the agent a mechanism (hereafter also referred to as a menu of contract offers). A mechanism $\phi_2 \in \Phi_2$ for P_2 consists of a message space $\mathcal{M}_2$ along with a mapping $\phi_2 : \mathcal{M}_2 \mapsto X_2 \times T_2$, where $x_2(m_2) \in X_2$ and $t_2(m_2) \in T_2$ denote respectively the decision and the transfer associated with message m_2.[4] For her part, P_1 offers a mechanism $\phi_1 \in \Phi_1$ that is characterized by a message space $\mathcal{M}_1$, a set of signals S that P_1 will disclose to P_2, and a mapping $\phi_1 : \mathcal{M}_1 \mapsto \Delta(X_1 \times S) \times T_1$. $\delta_1(m_1) \in \Delta(X_1 \times S)$ and $t_1(m_1) \in T_1$ stand for the joint lottery over $X_1 \times S$ and the expected transfer associated with message $m_1 \in \mathcal{M}_1$. With the standard abuse of notation in mechanism design, $\delta_1(x_1, s|m_1)$ will denote the conditional probability of x_1 and s, given m_1, and $\delta_1(x_1|m_1) = \sum_{s \in S} \delta_1(x_1, s|m_1)$ the associated probability of trade. The mechanism ϕ_1 embeds a *disclosure policy* $d : \mathcal{M}_1 \to \triangle(S)$: When the agent chooses message m_1, P_1 sends a signal s to P_2 with probability $d(s|m_1) = \sum_{x_1 \in X_1} \delta_1(x_1, s|m_1)$. We assume S is sufficiently rich to generate the desired posterior beliefs for P_2: as we show below, since Θ and X_1 are finite, it will suffice to treat S also as a finite set. Note that the mechanism ϕ_1 is (possibly) stochastic for two reasons: First, P_1 may want to create uncertainty about x_1 in order to influence the contracts offered by P_2; second, it may be in the interest of P_1 not to reveal to P_2 all the information disclosed in the upstream relationship. In other words, P_1 may find it optimal to disclose to P_2 only a noisy signal of (θ, x_1).[5] P_1 is not exogenously compelled to release any particular information, so she can select the disclosure policy she wants.

We assume each principal can commit perfectly to her mechanism, which also implies that P_1 can commit credibly to the disclosure policy of her choosing.[6] With this assumption we rule out two possible scenarios. In the first, P_1 discloses more information than allowed by ϕ_1. In the second, P_1 publicly announces a disclosure policy d but then secretly offers the agent a side contract with a different policy.

As is standard in common agency games, we also assume that neither principal can contract over the decisions of the other.

[4] In this environment, P_2 never benefits from offering a stochastic mechanism.

[5] Because of quasi-linearity, P_2 is never interested in learning t_1.

[6] If P_1 were obliged to disclose m_1, she might find it optimal to induce A to randomize over $\mathcal{M}_1$ (see [5,13] for dynamic contracting models with partial commitment).

Finally, we denote by $\tau(\phi_1)$ the price P_2 pays to observe the signals disclosed by ϕ_1. We want $\tau(\phi_1)$ to be the price for information and not for the distribution over X_1. To this end, we assume $\tau(\phi_1)$ is contracted after ϕ_1 has been executed, so that P_1 cannot threaten P_2 with changing her decision if she fails to pay τ. Instead of modelling a bargaining game between P_1 and P_2 explicitly, we consider a set of *rational* prices that can be the result of various bargaining procedures. Let $U_2\left(\phi_1\right)$ be the expected payoff for P_2 in the continuation game where she observes the signals disclosed by ϕ_1 and $U_2^{\mathrm{ND}}(\phi_1)$ in the continuation game in which she receives no information. Given ϕ_1, we define the set of rational prices as $T(\phi_1) = \{\tau : \tau = \gamma[U_2(\phi_1) - U_2^{\mathrm{ND}}(\phi_1)] \text{ for } \gamma \in [0, 1]\}$. The parameter γ captures the fraction of the value that P_2 attaches to the information disclosed by ϕ_1 that P_1 can appropriate through the price $\tau(\phi_1)$. Clearly, $\tau(\phi_1) = 0$ for any γ if ϕ_1 does not reveal any valuable information.

2.1.4. Timing: A sequential contracting game

- At $t = 0$, A privately learns θ.
- At $t = 1$, P_1 announces a public mechanism $\phi_1 \in \Phi_1$. If A rejects ϕ_1, the game ends and all players are left with their reservation payoffs, which are set to zero. If A accepts ϕ_1, he chooses a message m_1 and pays an expected transfer $t_1(m_1)$; a decision $x_1 \in X_1$ and a signal $s \in S$ are then selected with probability $\delta_1(x_1, s|m_1)$. The realization of the lottery $\delta_1(m_1)$ is observed jointly by A and P_1.
- At $t = 2$, P_2 pays $\tau(\phi_1)$, receives information from P_1 and offers a mechanism $\phi_2 \in \Phi_2$. If A rejects ϕ_2, the game is over. Otherwise, A reports a message m_2, which induces a decision $x_2(m_2)$ and a transfer $t_2(m_2)$.

Assuming that ϕ_1 is public is equivalent to assuming that P_2 can observe the mapping $\phi_1 : \mathcal{M}_1 \mapsto \Delta(X_1 \times S) \times T_1$ but not m_1 and x_1.

That the game ends after A rejects ϕ_1 is clearly not without loss of generality. However, note that in the game where A can contract with P_2 after rejecting ϕ_1, there exist equilibria where P_1 informs P_2 about the rejection such that all types obtain zero surplus with P_2 out-of-equilibrium. These equilibria also satisfy forward induction refinements such as the *intuitive criterion* of Cho and Kreps [7] and lead to the highest payoff for P_1. Rather than rely on refinements to determine A's outside option in the upstream relationship, we prefer, given the focus of the analysis, to assume it is exogenously fixed to zero.

2.2. Contract design

The game described above is a sequential version of the simultaneous common agency games with adverse selection examined in Martimort [15], Martimort and Stole [17,18], and Stole [29]. A strategy for P_1 is simply the choice of a mechanism $\phi_1 \in \Phi_1$. For P_2, a strategy is a mapping from Φ_1 and S onto the set of mechanisms Φ_2.[7] The agent's

[7] Although ϕ_2 depends on ϕ_1, the feasibility of the decisions contemplated in ϕ_2 does not depend on the particular decision x_1. This is a restriction. Calzolari and Pavan [6], for example, consider the design of optimal disclosure policies for an auctioneer who expects buyers to resell in a secondary market. As resale can take place only if a buyer has received the good in the primary market, the feasibility of an allocation in the secondary market depends on the decisions taken in the primary market, so that the above assumption is clearly violated in auctions followed by resale.

strategy specifies the reports to each principal as a function of the agent's information set, i.e. $m_1 = \phi_A^1(\theta, \phi_1)$, and $m_2 = \phi_A^2(\theta, \phi_1, m_1, x_1, t_1, s, \phi_2)$.

A strategy profile is a *perfect Bayesian equilibrium* if and only if: each principal selects a mechanism that is sequentially optimal given the strategies of the agent and the other principal; for each signal s on the equilibrium path, P_2 updates beliefs using Bayes' rule; and A sends only payoff-maximizing messages.

It is well known that in games where agents contract with multiple mechanism designers, the standard version of the revelation principle is not valid and the characterization of the entire set of common agency equilibria is problematic [10,17,23,24]. In this paper, however, we are interested only in the properties of the equilibrium contracts that lead to the highest payoff for the upstream principal. [8] It then suffices to search for mechanisms ϕ_1^* and $\{\phi_2^*(s)\}_{s\in S}$ with the following properties: [9]

(i) $\phi_1^* : \Theta \mapsto \Delta(X_1 \times S) \times T_1$ and $\phi_2^*(s) : \Theta \times X_1 \mapsto X_2 \times T_2$;
(ii) the agent finds it optimal to contract with both principals and truthfully report θ to P_1 and (θ, x_1) to P_2;
(iii) $\phi_2^*(s)$ is optimal for P_2—any other mechanism $\phi_2(s)$ that is individually rational and incentive-compatible for the agent leads to a lower payoff for P_2;
(iv) ϕ_1^* and $\{\phi_2^*(s)\}_{s\in S}$ are optimal for P_1—any other ϕ_1 and $\{\phi_2(s)\}_{s\in S}$ that dominate ϕ_1^* and $\{\phi_2^*(s)\}_{s\in S}$ necessarily violate either (ii) or (iii).

Conditions (i)–(iv) identify equilibrium allocations that yield the highest payoff for P_1 in an environment where both principals can induce the agent to follow their recommendations and where P_1 can also induce P_2 to offer the contracts that are most favorable to her when the latter is indifferent. [10]

If there exist mechanisms satisfying (i)–(iv), there also exists a sequential common agency equilibrium sustaining ϕ_1^* and $\{\phi_2^*(s)\}_{s\in S}$. That is, we can always complete the description of the equilibrium by specifying a reaction for P_2 to any possible ϕ_1 and a strategy for the agent $(\phi_A^{*1}, \phi_A^{*2})$ such that it is optimal for P_1 to offer ϕ_1^* and for P_2 to offer $\{\phi_2^*(s)\}_{s\in S}$.

The equilibrium described above can be characterized by backward induction. Consider first the mechanism design problem faced by P_2. For any extended type $\theta_2^{\mathrm{E}} = (\theta, x_1)$, let $v_A(x_2, \theta_2^{\mathrm{E}}) \equiv v_A(x_1, x_2, \theta)$ and $v_2(x_2, \theta_2^{\mathrm{E}}) \equiv v_2(x_1, x_2, \theta)$. Also, let

$$U_A^2(\theta_2^{\mathrm{E}}; s) \equiv v_A(x_2(\theta_2^{\mathrm{E}}; s), \theta_2^{\mathrm{E}}) - v_A(0, \theta_2^{\mathrm{E}}) - t_2(\theta_2^{\mathrm{E}}; s)$$

[8] For similar selection arguments in dynamic contracting with a single principal, see [13].

[9] In Pavan and Calzolari [23], we have shown that any equilibrium outcome of any *unrestricted* game in which principals can choose mechanisms with arbitrarily complex message spaces can also be sustained as an equilibrium outcome in the *restricted* game in which principals are constrained to offer direct mechanisms in which the message space is the agent's *extended type* and includes only payoff-relevant information. With quasi-linear utilities, upstream transfers play no role on downstream contracting and do not need to be included into Θ_2^{E}.

[10] The signal s can thus also be read as the recommendation that P_1 sends to P_2 about the mechanism to offer to the agent.

denote the downstream surplus A obtains with P_2 when he truthfully reports his extended type $\theta_2^{\mathrm{E}} = (\theta, x_1)$ and

$$U_A^2(\theta_2^{\mathrm{E}}, \widehat{\theta}_2^{\mathrm{E}}; s) \equiv v_A(x_2(\widehat{\theta}_2^{\mathrm{E}}; s), \theta_2^{\mathrm{E}}) - v_A(0, \theta_2^{\mathrm{E}}) - t_2(\widehat{\theta}_2^{\mathrm{E}}; s),$$

the corresponding payoff when he announces $\widehat{\theta}_2^{\mathrm{E}} \neq \theta_2^{\mathrm{E}}$.

Finally, let $S\left(d; \phi_1\right) \equiv \{s : d(s|\theta) > 0 \text{ for some } \theta \in \Theta\}$ represent the set of signals associated with the disclosure policy of mechanism ϕ_1. Assuming ϕ_1 induces A to truthfully reveal θ, for any signal $s \in S\left(d; \phi_1\right)$, P_2's posterior beliefs over Θ_2^{E} are given by [11]

$$\mu(\theta_2^{\mathrm{E}}; s) \equiv \Pr((\theta, x_1)|s) = \frac{\delta_1(x_1, s|\theta)\Pr(\theta)}{\sum_{\theta\in\Theta}\sum_{x_1\in X_1}\left[\delta_1(x_1, s|\theta)\right]\Pr(\theta)}.$$

An optimal mechanism for P_2 thus solves the following program:

$$\mathcal{P}_2(s): \begin{cases} \max \sum\limits_{\theta_2^{\mathrm{E}}\in\Theta_2^{\mathrm{E}}} [v_2(x_2(\theta_2^{\mathrm{E}}; s), \theta_2^{\mathrm{E}}) + t_2(\theta_2^{\mathrm{E}}; s)]\mu(\theta_2^{\mathrm{E}}; s) \\ \text{such that for any } \theta_2^{\mathrm{E}} \text{ and } \widehat{\theta}_2^{\mathrm{E}} \in \Theta_2^{\mathrm{E}}, \\ U_A^2(\theta_2^{\mathrm{E}}; s) \geqslant 0, \qquad\qquad (\mathrm{IR}_2) \\ U_A^2(\theta_2^{\mathrm{E}}; s) \geqslant U_A^2(\theta_2^{\mathrm{E}}, \widehat{\theta}_2^{\mathrm{E}}; s), \qquad (\mathrm{IC}_2) \end{cases}$$

where (IR_2) and (IC_2) are the individual rationality and incentive compatibility constraints. Note that we are implicitly assuming there is no way A can credibly disclose (x_1, t_1) to P_2, so that the latter has to provide incentives for truthful revelation.

Consider now the problem faced by P_1. At $t = 1$, P_1 designs a mechanism ϕ_1—with reaction $\{\phi_2(s)\}_{s\in S}$—that solves

$$\mathcal{P}_1: \begin{cases} \max \sum\limits_{\theta\in\Theta} \left\{ \sum\limits_{x_1\in X_1} \sum\limits_{s\in S} [v_1(x_1, x_2(\theta_2^{\mathrm{E}}; s), \theta)]\delta_1(x_1, s|\theta) + t_1(\theta) \right\} \\ \qquad \times \Pr(\theta) + \tau(\phi_1) \\ \text{s.t.} \\ U_A\left(\theta; \phi_1\right) \equiv \sum\limits_{x_1\in X_1} \sum\limits_{s\in S} \left[v_A\left(x_1, 0, \theta\right) + U_A^2(\theta_2^{\mathrm{E}}; s)\right] \\ \qquad \times \delta_1(x_1, s|\theta) - t_1(\theta) \geqslant 0 \quad \forall\theta \in \Theta, \qquad (\mathrm{IR}_1) \\ U_A\left(\theta; \phi_1\right) \geqslant \sum\limits_{x_1\in X_1} \sum\limits_{s\in S} \left[v_A\left(x_1, 0, \theta\right) + U_A^2(\theta_2^{\mathrm{E}}; s)\right] \\ \qquad \times \delta_1(x_1, s|\hat{\theta}) - t_1(\hat{\theta}) \quad \forall(\theta, \widehat{\theta}) \in \Theta^2, \qquad (\mathrm{IC}_1) \\ \phi_2(s) \text{ solves } \mathcal{P}_2(s) \text{ for any } s \in S\left(d; \phi_1\right). \qquad (\mathrm{SR}) \end{cases}$$

[11] To simplify the notation, we omit the dependence of μ on ϕ_1, when this does not create confusion.

In addition to standard individual rationality and incentive compatibility constraints, the (SR) constraint in $\mathcal{P}_1$ guarantees the optimality of P_2's reaction. Treating $\{\phi_2(s)\}_{s\in S}$ as a choice variable in $\mathcal{P}_1$ amounts to selecting the equilibrium which is most favorable to P_1.

Before analyzing the optimal contracts, we find it useful to formally define disclosure as well as contracts that optimally induce it.

Definition 1. The mechanism ϕ_1 discloses information if and only if it assigns positive measure to signals that lead to different posterior beliefs over Θ_2^{E} : Formally, there exist signals $s_l \in S\left(d;\phi_1\right)$ and $s_m \in S\left(d;\phi_1\right)$, with $s_l \neq s_m$, such that $\mu(\theta_2^{\mathrm{E}}; s_l) \neq \mu(\theta_2^{\mathrm{E}}; s_m)$ for some $\theta_2^{\mathrm{E}} \in \Theta_2^{\mathrm{E}}$.

Information disclosure is optimal for P_1 if and only if there exists a mechanism ϕ_1 that discloses information and solves $\mathcal{P}_1$, and there are no other solutions to $\mathcal{P}_1$ that do not disclose information.

3. On the optimality of privacy

In this section, we identify and discuss preferences that make full privacy the optimal policy for P_1. To save on notation, let $\Delta\theta \equiv \overline{\theta} - \underline{\theta} > 0$, $\Delta_\theta v_A(\mathbf{x},\theta) \equiv v_A(\mathbf{x},\overline{\theta}) - v_A(\mathbf{x},\underline{\theta})$, $\Delta_{x_1} v_A(\mathbf{x},\theta) \equiv v_A(1,x_2,\theta) - v_A(0,x_2,\theta)$, $\Delta_\theta[\Delta_{x_1} v_A(\mathbf{x},\theta)] \equiv \Delta_{x_1} v_A(\mathbf{x},\overline{\theta}) - \Delta_{x_1} v_A(\mathbf{x},\underline{\theta})$ and analogously for $\Delta_{x_2} v_A(\mathbf{x},\theta)$ and $\Delta_\theta[\Delta_{x_2} v_A(\mathbf{x},\theta)]$.

Player *i*'s preferences are additively *separable* if $v_i(\mathbf{x},\theta) = v_i^1(x_1,\theta) + v_i^2(x_2,\theta)$ with $v_i^1(0,\theta) = v_i^2(0,\theta) = 0$, and *independent* of x_j if $v_i(x_i,x_j,\theta) = v_i(x_i,\theta)$. The sign of the *single crossing condition* in player *i*'s preferences is the same for upstream and downstream decisions if, for any x_1 and x_2, $\operatorname{sign}\left\{\Delta_\theta[\Delta_{x_2} v_i(\mathbf{x},\theta)]\right\} = \operatorname{sign}\left\{\Delta_\theta[\Delta_{x_1} v_i(\mathbf{x},\theta)]\right\}$.

Theorem 1. *Assume the following*: (a) P_1*'s preferences are independent of* x_2; (b) *the sign of the single crossing condition in the agent's preferences is the same for upstream and downstream decisions*; (c) *the preferences of* P_2 *and A are additively separable. Then no disclosure is optimal for* P_1*, no matter what price* P_2 *is willing to pay to receive information.*

The formal proof is in the appendix. Here, let us simply sketch the intuition. Without loss, assume the sign of the single crossing condition is positive. When the agent's preferences are separable, this is equivalent to assuming that the valuations $v_A^1(1,\theta)$ and $v_A^2(1,\theta)$ are both increasing in θ. When P_2's preferences are also separable, the optimal mechanism for P_2 does not depend on x_1. It follows that under (a)–(c), the only benefit of influencing downstream decisions by disclosing information correlated with θ comes from a rent-shifting and/or an information-trade effect. The first consists in designing a policy that induces P_2 to leave the agent a rent and then set a higher price upstream. The second is the possibility of making money directly by selling information to the downstream principal.

Let $\phi_2^{\mathrm{ND}} \equiv (x_2^{\mathrm{ND}}(\theta), U_A^{2\mathrm{ND}}(\theta))$ denote the mechanism that P_2 offers if she receives no information from P_1. Under separability, this mechanism is not a function of ϕ_1, for the downstream surplus $W_2(x_2,\theta) \equiv v_2^2(x_2,\theta) + v_A^2(x_2,\theta)$ is independent of upstream decisions.

Now, suppose ϕ_1—with reaction ϕ_2—is optimal and discloses information. In this case, there exists another individually rational and incentive-compatible mechanism ϕ_1^{ND}—with reaction ϕ_2^{ND}—that does not release any information, that induces the same distribution over X_1, and is such that [12]

$$U_1(\phi_1) - U_1(\phi_1^{\mathrm{ND}}) = (1-\gamma) \sum_{\theta\in\Theta} \Big[\sum_{s\in S} U_A^2(\theta; s) d(s|\theta) - U_A^{2\mathrm{ND}}(\theta) \Big] \Pr(\theta)$$
$$+\gamma \sum_{\theta\in\Theta} \Big[\sum_{s\in S} W_2(x_2(\theta; s), \theta) d(s|\theta) - W_2(x_2^{\mathrm{ND}}(\theta), \theta) \Big] \Pr(\theta)$$
$$- \sum_{\theta\in\Theta} \Big[U_A\left(\theta; \phi_1\right) - U_A(\theta; \phi_1^{\mathrm{ND}}) \Big] \Pr(\theta) \leqslant 0. \tag{1}$$

When $\gamma = 0$, $\tau(\phi_1) = 0$ for any ϕ_1, the information-trade effect is absent, and hence the only benefit of disclosure comes from the rent-shifting effect, which corresponds to the first term in (1).

Conversely, when $\gamma = 1$, the rent-shifting effect is absent since any money that P_1 can extract from A for the increase in the informational rent she expects from P_2 must be deducted from the price $\tau(\phi_1)$. When this is the case, the only benefit of disclosure derives from the possibility of increasing efficiency in the downstream relationship, as indicated in the second term in (1).

Both the rent-shifting and the information-trade effects may well be positive. Disclosure, however, also affects the incentives for the agent to misrepresent his type to P_1 and hence the rent the latter must give A for truthful information, as indicated in the last term in (1). Under (b) and (c) this effect more than offsets the first two. It follows that in the absence of direct externalities (that is, when (a) also holds), the optimal policy for P_1 is to offer the agent full privacy.

To see this, note that ϕ_2 leaves no rent to $\underline{\theta}$ and a rent $U_A^2(\bar{\theta}; s) = \Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta)$ to $\bar{\theta}$ which is increasing in the posterior odds $\mu(\underline{\theta}; s)/\mu(\bar{\theta}; s)$ and hence in $d(s|\underline{\theta})/d(s|\bar{\theta})$. Furthermore, in any upstream mechanism that is optimal for P_1, $U_A(\underline{\theta}; \phi_1) = 0$ and $U_A(\bar{\theta}; \phi_1) = \Delta_\theta v_A^1(1, \theta)\delta_1(1|\underline{\theta}) + \sum_{s\in S} U_A^2(\bar{\theta}, s) d(s|\underline{\theta})$. Among all mechanisms that induce the same distribution over X_1 as ϕ_1 without disclosing information, consider a mechanism ϕ_1^{ND} such that $U_A(\underline{\theta}; \phi_1^{\mathrm{ND}}) = 0$ and $U_A(\bar{\theta}; \phi_1^{\mathrm{ND}}) = \Delta_\theta v_A^1(1, \theta)\delta_1(1|\underline{\theta}) + U_A^{2\mathrm{ND}}(\bar{\theta})$. It is easy to see that if ϕ_1 is individually rational and incentive-compatible, so is ϕ_1^{ND}. Furthermore,

$$\sum_{\theta\in\Theta} \Big[U_A\left(\theta; \phi_1\right) - U_A\left(\theta; \phi_1^{\mathrm{ND}}\right) \Big] \Pr(\theta) = p \Big[\sum_{s\in S} U_A^2(\bar{\theta}; s) d(s|\underline{\theta}) - U_A^{2\mathrm{ND}}(\bar{\theta}) \Big]. \tag{2}$$

[12] To compact notation, we omit the dependence of U_1 on ϕ_2.

Assume for a moment $\gamma = 0$ so that there is no information-trade effect. Then substituting (2) into (1) gives

$$U_1(\phi_1) - U_1(\phi_1^{\mathrm{ND}}) = p \sum_{s \in S} U_A^2(\bar{\theta}; s)[d(s|\bar{\theta}) - d(s|\underline{\theta})] \leqslant 0. \tag{3}$$

Indeed, suppose P_1 discloses only two signals, s_1 and s_2, and let $d(s_1|\bar{\theta}) = d(s_1|\underline{\theta}) + \varepsilon$ and $d(s_2|\bar{\theta}) = d(s_2|\underline{\theta}) - \varepsilon$ for some $\varepsilon > 0$. Since $U_A^2(\bar{\theta}; s)$ is increasing in $d(s|\underline{\theta})/d(s|\bar{\theta})$, $U_A^2(\bar{\theta}; s_1) \leqslant U_A^2(\bar{\theta}; s_2)$ and hence $U_1(\phi_1) - U_1(\phi_1^{\mathrm{ND}}) = p \left[U_A^2(\bar{\theta}; s_1) - U_A^2(\bar{\theta}; s_2) \right] \varepsilon \leqslant 0$. This result clearly extends to more general disclosure policies. The most favorable signals are always disclosed with a higher probability when A announces a low type. It follows that the additional surplus A obtains with P_2 when P_1 discloses information is more than offset by the increase in the rent P_1 must sacrifice to A to induce him to reveal information, making disclosure unprofitable for P_1.

Next consider the information-trade effect and assume $\gamma = 1$, in which case disclosure is motivated entirely by the possibility of increasing efficiency in the downstream relationship. Again substituting (2) into (1) and using the fact that the downstream decisions $x_2(\bar{\theta}; s)$ do not depend on s gives

$$\begin{aligned} U_1(\phi_1) - U_1(\phi_1^{\mathrm{ND}}) = (1-p)\Big[\sum_{s \in S} W_2(x_2(\underline{\theta}; s), \underline{\theta}) d(s|\underline{\theta}) - W_2(x_2^{\mathrm{ND}}(\underline{\theta}), \underline{\theta}) \Big] \\ - p \Big[\sum_{s \in S} U_A^2(\bar{\theta}; s) d(s|\underline{\theta}) - U_A^{2\mathrm{ND}}(\bar{\theta}) \Big]. \end{aligned}$$

Using $U_A^2(\bar{\theta}; s) = \Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta)$ and $U_A^{2\mathrm{ND}}(\bar{\theta}) = \Delta_\theta v_A^2(x_2^{\mathrm{ND}}(\underline{\theta}), \theta)$, the above further reduces to

$$\begin{aligned} \sum_{s \in S} \Big[(1-p) W_2(x_2(\underline{\theta}; s), \underline{\theta}) - p \Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta) \Big] d(s|\underline{\theta}) \\ - \Big[(1-p) W_2(x_2^{\mathrm{ND}}(\underline{\theta}), \underline{\theta}) - p \Delta_\theta v_A^2(x_2^{\mathrm{ND}}(\underline{\theta}), \theta) \Big] \end{aligned}$$

which is never positive since $x_2^{\mathrm{ND}}(\underline{\theta})$ maximizes $(1-p) W_2(x_2, \underline{\theta}) - p \Delta_\theta v_A^2(x_2, \theta)$. The explanation is simple. When $\gamma = 1$, the price $\tau(\phi_1) = U_2(\phi_1) - U_2(\phi_1^{\mathrm{ND}})$ allows P_1 to fully internalize the effect of disclosure on U_2. If P_1 could directly control $x_2(\underline{\theta})$, she would then optimally trade off efficiency and rent extraction by maximizing $(1-p) W_2(x_2, \underline{\theta}) - p \Delta_\theta v_A^2(x_2, \theta)$. But since this is exactly the same decision P_2 takes when her posterior beliefs are equal to the prior, the best P_1 can do is to commit not to disclose any information.

Finally, note that if disclosure is not profitable when $\gamma = 1$, it is clearly not profitable when $\gamma < 1$. We thus conclude that under (a)–(c), the optimal policy is always full privacy, irrespective of the price P_2 is willing to pay for information.

Theorem 1 does not depend on the discreteness of Θ, X_1 and X_2. As we show in the appendix, the theorem extends to environments where θ is continuously distributed over $[\underline{\theta}, \bar{\theta}]$ and $X_i = \mathbb{R}_+$ for $i = 1, 2$, under the usual additional assumptions for the continuous case, which guarantee that in the canonical single mechanism designer problem, the optimal policies $x_i(\theta)$ are deterministic with no bunching.

It is interesting to compare the result in Theorem 1 with Baron and Besanko [3]. They consider a dynamic single-principal single-agent relationship and show that when type is constant over time, the optimal long-term contract under full commitment consists in a sequence of static optimal contracts. Although the two results appear similar, they are actually quite different. In Baron and Besanko, there is a single principal who maximizes the intertemporal payoff $v_1(x_1, \theta) + v_2(x_2, \theta) + t(\theta)$, whereas in our setting the upstream principal maximizes only $v_1(x_1, \theta) + t(\theta) + \tau$, where $\tau = 0$ in the absence of disclosure. This implies that P_1 may well be happy to reduce the joint payoff for the two principals, if by so doing she can appropriate a larger part of the total surplus, as is illustrated in the next section. Also, even if P_1 were to maximize the principals' joint payoff, she would not necessarily offer the static optimal contracts. This would be the case if the preferences of the downstream principal were not only separable but also independent of x_1, as in Baron and Besanko. When instead they are only separable, the static optimal contracts—which coincide with the contracts that are offered in equilibrium when P_1 does not disclose information—fail to internalize the externality of x_1 on P_2.

The next result provides a converse to Theorem 1.

Theorem 2. *When any one of the conditions in Theorem* 1 *is violated, there exist preferences for which disclosure is strictly optimal for* P_1, *even if* P_2 *does not pay for information.*

In this sense, the conditions of Theorem 1 are not only sufficient but "almost necessary" to make privacy in sequential contracting optimal. The proof follows from the results of the next two sections, where we examine the determinants of the disclosure of exogenous and endogenous information separately. To prove that disclosure can be optimal whatever rational price P_2 is willing to pay, we consider the least favorable scenario where $\tau(\phi_1) = 0$ for any ϕ_1, in which case disclosure is free of charge.

4. Disclosure of exogenous information

To separate the effects associated with the disclosure of exogenous information (about θ) from those associated with the disclosure of endogenous information (about x_1), in this section, we again consider an environment where preferences in the downstream relationship are separable so that P_2 is interested in receiving information about x_1 only if this is indirectly informative about θ. In particular, assume the following holds.

Condition 1. The agent's preferences are separable: $v_A(x_1, x_2, \theta) = a(\theta)\, x_1 + b(\theta)\, x_2$; P_2's preferences are independent of θ and x_1: $v_2(x_1, x_2, \theta) = m_2 x_2$.

Assuming that the preferences of the downstream principal are not only separable but independent of θ and x_1 shortens the exposition without any significant effect on the results. [13]

[13] Adding an externality $q_2(\theta)\, x_1$ to P_2's preferences does not affect the downstream decisions. Also, letting m_2 depend on θ does not add much to the analysis since the virtual surplus for the P_2—A relationship already depends on θ through its effect on A's payoff.

In a buyer–seller relationship, $m_2 \leqslant 0$ can be interpreted as the marginal cost to the downstream seller. To make the analysis interesting, we then assume $m_2 + b(\theta) > 0$ for any θ, which guarantees that, under complete information, it is always efficient to trade downstream. We also assume that $\Delta b \equiv b(\bar{\theta}) - b(\underline{\theta}) > 0$. Under these conditions, the solution to $\mathcal{P}_2(s)$ assigns the same allocation to $\theta_2^{\mathrm{E}} = (\theta, 1)$ and $\theta_2^{\mathrm{E}} = (\theta, 0)$ and is equivalent to a take-it-or-leave-it offer at a price $t_2(s) = \bar{b}$ if $\Pr(\bar{\theta}|s) \geqslant (m_2 + \underline{b})/\left(m_2 + \bar{b}\right)$ and $t_2(s) = \underline{b}$ otherwise. As a consequence, P_1 needs to disclose only two signals, s_1 and s_2, such that $t_2(s_1) = \bar{b}$ and $t_2(s_2) = \underline{b}$.[14] This also implies that the optimal disclosure policy must satisfy

$$d(s_1|\bar{\theta}) \geqslant Hd(s_1|\underline{\theta}), \qquad (\mathrm{SR}_1)$$
$$d(s_2|\bar{\theta}) \leqslant Hd(s_2|\underline{\theta}), \qquad (\mathrm{SR}_2)$$

where $H \equiv (\frac{1-p}{p})(\frac{m_2+\underline{b}}{\Delta b})$. Given s_1, trade in the downstream relationship occurs only if $\theta = \bar{\theta}$ and the agent gets zero surplus; while, given s_2, trade occurs with both types and $\bar{\theta}$ enjoys a downstream rent equal to Δb.

When $H < 1$ [equivalently $p > (\underline{b} + m_2)/\left(\bar{b} + m_2\right)$], P_2 asks a high price in the event she receives no information from P_1. We call prior beliefs that satisfy this condition *unfavorable* to the agent. On the contrary, P_2's beliefs are *favorable* when $H \geqslant 1$. Also note that when $H < 1$, (SR_1) is implied by (SR_2) and no disclosure is formally equivalent to sending signal s_1, whereas the opposite is true with favorable beliefs in which case no disclosure corresponds to sending only signal s_2.

4.1. Direct externalities

Suppose now that P_1's payoff depends directly on x_2, as in the case of a seller whose compensation is based on his relative performance compared to another vendor. An alternative example examined in the literature [18] is one where P_1 and P_2 are two retailers purchasing from a common manufacturer. When the products of the two retailers are strategic substitutes, P_1 may find it optimal to disclose information about the manufacturer to influence the downstream retailer's decision to purchase additional units. To capture the possibility of direct externalities, assume the following holds.

Condition 2. P_1 is personally interested in $x_2 : v_1(x_1, x_2, \theta) = m_1 x_1 + e x_2$.

The term m_1 can be read as the marginal cost to P_1. We require that $m_1 + a(\theta) > 0$ for any θ so that it is always efficient to trade in the upstream relationship. We also assume that $\Delta a \equiv a(\bar{\theta}) - a(\underline{\theta}) > 0$: The sign of the single crossing condition is thus the same for x_1 and x_2, implying that disclosure is costly for P_1.

Depending on the environment, the externality e can be either positive or negative. It is probably negative in the examples above. However, it could be positive in the case of a telephone company that is considering switching to optical fiber and sharing the network of a downstream Internet or cable TV provider.

[14] For any mechanism ϕ_1 that discloses more than two signals, there exists another mechanism ϕ' that discloses at most two signals which is payoff-equivalent for all players.

Under Conditions (1) and (2), the surplus that A expects from the two relationships given ϕ_1 is $U_A(\overline{\theta};\phi_1) = \delta_1(1|\overline{\theta})\bar{a} + d(s_2|\overline{\theta})\Delta b - t_1(\overline{\theta})$ and $U_A(\underline{\theta};\phi_1) = \delta_1(1|\underline{\theta})\underline{a} - t_1(\underline{\theta})$. At the optimum ($\underline{\text{IR}}_1$) and ($\overline{\text{IC}}_1$) bind, which implies that $U_A(\underline{\theta};\phi_1) = 0$, $U_A(\overline{\theta};\phi_1) = \delta_1(1|\underline{\theta})\Delta a + d(s_2|\underline{\theta})\Delta b$ and

$$U_1(\phi_1) = p\delta_1(1|\overline{\theta})\,(m_1 + \bar{a}) + (1-p)\,\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + pe + (1-p)\,d(s_2|\underline{\theta})e - p[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b. \tag{4}$$

The optimal mechanism thus maximizes (4) subject to (SR_1), (SR_2) and

$$[\delta_1(1|\overline{\theta}) - \delta_1(1|\underline{\theta})]\Delta a \geqslant [d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b. \qquad (\underline{\text{IC}}_1)$$

Because trade in the downstream relationship occurs with certainty when $\theta = \overline{\theta}$ and with probability $d(s_2|\underline{\theta})$ when $\theta = \underline{\theta}$, the expected externality of x_2 on P_1 is $pe + (1-p)\,d(s_2|\underline{\theta})e$.

Since preferences in the downstream relationship are separable and there are no marginal effects of x_2 on $v_1(x_1, x_2, \theta) + v_A^1(x_1, \theta)$, the joint lottery $\delta_1(x_1, s|\theta)$ can be decomposed into a disclosure policy $d(s|\theta)$ and a trade policy $\delta_1(1|\theta)$, where $d(s|\theta)$ and $\delta_1(1|\theta)$ can be treated as independent distributions. This also implies that $\delta_1(1|\theta)$ can either be read as the probability of trade or as the quantity traded, with $\delta_1(1|\theta) \in [0, 1]$. [15]

Finally, note that constraint ($\underline{\text{IC}}_1$) is an "adjusted" monotonicity condition which reduces to the standard monotonicity condition $\delta_1(1|\overline{\theta}) \geqslant \delta_1(1|\underline{\theta})$ when no information is disclosed. On the contrary, when P_1 discloses information, monotonicity becomes strict for it requires $\delta_1(1|\underline{\theta}) < \delta_1(1|\overline{\theta})$. Indeed, suppose P_1 sells with certainty to both types. Then the low type, who does not expect any surplus in the downstream relationship, would always choose the contract with the lowest price. However, since disclosure requires that P_1 sends the most favorable signal s_2 with higher probability when A reports $\underline{\theta}$ than $\overline{\theta}$, the high type would also find it optimal to choose the low-type contract, making P_1's mechanism not incentive-compatible.

It follows that there are two possible costs associated with disclosure. The first is the extra rent $[d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b$ that P_1 must cede to $\overline{\theta}$, as discussed in the previous section. The second is the reduction in the level of trade with $\underline{\theta}$ required by ($\underline{\text{IC}}_1$). However, while it is always optimal for P_1 to trade with the high type, trading with the low type is profitable only if the "virtual surplus" $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$.

To see how P_1 optimally trades the possibility to influence x_2 off against the costs of disclosure, consider unfavorable beliefs. Since SR_2 is always binding at the optimum and $\delta_1^*(1|\overline{\theta}) = 1$, ($\underline{\text{IC}}_1$) can be rewritten as $\delta_1(1|\underline{\theta}) \leqslant 1 - (1-H)\frac{\Delta b}{\Delta a} d(s_2|\underline{\theta})$. Disclosure is then optimal for P_1 if and only if

$$(1-p)e \geqslant p(1-H)\Delta b + (1-p)(1-H)\frac{\Delta b}{\Delta a}\mathbb{I}\left[m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right],$$

[15] This is not true with non-separable preferences, because the joint distribution over X_1 and S is what determines the surplus that A and P_1 expect from downstream contracting.

where $\mathbb{I}[\cdot]$ is an indicator function taking value one if $[\cdot] > 0$ and zero otherwise. The left-hand side is the marginal externality associated with an increase in the downstream level of trade generated by an increase in $d(s_2|\underline{\theta})$. The right-hand side combines the cost of the increase in the rent for $\overline{\theta}$ with that of reducing the upstream level of trade with $\underline{\theta}$, which is relevant only when trading with the low type is profitable, that is when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a > 0$.

With favorable beliefs, things are symmetrically opposite. Disclosure is optimal only when P_1 has a strong incentive to reduce the downstream level of trade, as we show in the appendix.

Proposition 1. *With direct externalities, disclosure is motivated by the possibility of influencing the downstream level of trade. Suppose preferences are as in Conditions* (1) *and* (2). *When P_2's beliefs are unfavorable to the agent, disclosure is optimal if and only if there are large positive externalities. When they are favorable, disclosure is optimal for large negative externalities.*

Note that in either case, P_1 never fully informs P_2 about θ. Indeed, full disclosure is costly (in terms of rent for $\overline{\theta}$ and inefficient trade with $\underline{\theta}$) and is either unnecessary to induce the desired level of trade or else incentive-incompatible.

We now turn to the effects of disclosure on individual payoffs. We compare the optimal contracts with disclosure (formally derived in the proof of Proposition 1) with those that would be offered if P_1 were not able, or allowed, to disclose information. Because preferences are separable in the downstream relationship, these contracts simply consist in a take-it-or-leave-it offer at price $t_1 = \overline{a}$ if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$ and at price $t_1 = \underline{a}$ otherwise.

Corollary 1. *When P_2's beliefs are unfavorable, disclosure leads to a Pareto-improvement: P_1 and A are strictly better off, whereas P_2 is indifferent. When P_2's beliefs are favorable, disclosure makes A worse off, P_1 better off, and leaves P_2 indifferent. The effect of disclosure on total welfare is positive for large negative externalities and negative otherwise.*

P_2 is not affected by disclosure since the optimal mechanism ϕ_1^* makes her indifferent between asking the prices she would have asked in the absence of disclosure and the equilibrium ones. Together with the fact that P_2's preferences are independent of x_1 so that she is not personally affected by changes in upstream decisions, this implies that P_2 is just as well off as in the absence of disclosure.

Next, consider the effect of disclosure on the agent's payoff and recall that under the optimal contracts, $U_A(\underline{\theta}; \phi_1^*) = 0$ and $U_A(\overline{\theta}; \phi_1^*) = \delta_1^*(1|\underline{\theta})\Delta a + d^*(s_2|\underline{\theta})\Delta b$. First, assume unfavorable beliefs. If $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$, A is clearly better off, since in the absence of disclosure he gets no surplus with either principal. If instead $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$, then without disclosure, A gets $U_A(\overline{\theta}; \phi_1^{\text{ND}}) = \Delta a$ and $U_A(\underline{\theta}; \phi_1^{\text{ND}}) = 0$. As shown in the appendix (proof of Proposition 1), the optimal contracts with disclosure are such that $d^*(s_2|\underline{\theta}) = \min\{1; \Delta a/[(1-H)\Delta b]\}$ and $\delta_1^*(1|\underline{\theta}) = 1 - d^*(s_2|\underline{\theta})\,(1-H)\,\frac{\Delta b}{\Delta a}$, implying that A strictly benefits from disclosure. Indeed, even if disclosure comes at the expenses of a reduction of $\delta_1(1|\underline{\theta})$, this is more than compensated by the increase in the downstream rent.

The reason is that disclosure increases the surplus that $\overline{\theta}$ obtains by mimicking $\underline{\theta}$, but also the surplus that $\overline{\theta}$ obtains by truthfully reporting his type. In turn this allows P_1 to increase the rent she cedes to the high type without inducing the low type to mimic.

With favorable beliefs, things are different. In this case, P_1 induces P_2 to ask a higher price. Furthermore, when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$, P_1 reduces the level of trade with the low type to satisfy ($\underline{\text{IC}}_1$). As a consequence, A always suffers from disclosure. The effect on total welfare then depends on how strong the externality is. For moderate values, the negative effect on A prevails and welfare decreases with disclosure; for large negative externalities, the opposite is true.

4.2. Horizontal differentiation and countervailing incentives

We now turn to environments where the agent's valuations for x_1 and x_2 are negatively correlated, as when a buyer has horizontally differentiated preferences for the products of two sellers. Alternatively, A could be a retailer, or a marketing agent, with superior information than manufacturers about consumers' location in the space of characteristics differentiating the two brands.

Disclosure is now motivated by the rent-shifting effect, i.e. the possibility of appropriating the surplus A obtains in the downstream relationship. As was shown in the previous section, this is never possible when A's valuations are positively correlated, for in that case any increase in the agent's downstream surplus is more than offset by the increase in the rent that P_1 must cede to induce truthful revelation. But when the two products are horizontally differentiated, those types who can potentially benefit from the rent in the downstream relationship are those who attach less value to the product provided by the upstream principal. As a consequence, disclosure may create countervailing incentives that help P_1 extract more surplus from the agent. On the other hand, disclosure may come at the cost of an inefficient level of trade upstream, required by incentive compatibility.

To illustrate, assume preferences in the downstream relationship are described by Condition 1, and suppose the following also holds. [16]

Condition 3. P_1's preferences are independent of θ and x_2: $v_1(x_1, x_2, \theta) = m_1 x_1$; the single crossing condition in the agent's preferences has opposite signs for x_1 and x_2: $\Delta a < 0 < \Delta b$.

To make things interesting, we continue to assume that $m_1 + a(\theta) > 0$ for any θ so that it is always efficient to trade in the upstream relationship.

P_1's optimal mechanism maximizes

$$\begin{aligned} U_1(\phi_1) = & \; p\left[\delta_1(1|\overline{\theta})(m_1 + \overline{a}) + d(s_2|\overline{\theta})\Delta b - U_A(\overline{\theta}; \phi_1)\right] \\ & + (1-p)\left[\delta_1(1|\underline{\theta})(m_1 + \underline{a}) - U_A(\underline{\theta}; \phi_1)\right] \end{aligned}$$

[16] An example of horizontally differentiated preferences is $v_A(x_1, x_2, \theta) = (1-\theta)x_1 + \theta x_2$. See Mezzetti [19] for an analysis of countervailing incentives in (simultaneous) common agency games with horizontally differentiated preferences.

subject to $U_A(\overline{\theta};\phi_1)\geqslant 0$, $U_A(\underline{\theta};\phi_1)\geqslant 0$, ($SR_1$), ($SR_2$) and

$$U_A(\overline{\theta};\phi_1)\geqslant U_A(\underline{\theta};\phi_1)+d(s_2|\underline{\theta})\Delta b-\delta_1(1|\underline{\theta})|\Delta a|, \qquad (\overline{IC}_1)$$
$$U_A(\underline{\theta};\phi_1)\geqslant U_A(\overline{\theta};\phi_1)-d(s_2|\overline{\theta})\Delta b+\delta_1(1|\overline{\theta})|\Delta a|. \qquad (\underline{IC}_1)$$

Note that $\overline{\theta}$ continues to get Δb more than $\underline{\theta}$ when P_2 asks a low price, but now gets $|\Delta a|$ less than $\underline{\theta}$ from trading with P_1. As a consequence, it is not possible to determine which constraint binds a priori since this depends on which countervailing incentive prevails. Nevertheless, in any optimal mechanism, at least one (IR_1) and one (IC_1) constraint necessarily bind, and trade with the low type occurs with certainty, i.e. $\delta_1^*(1|\underline{\theta})=1$.

As for the optimal disclosure policy, when P_2's prior beliefs are favorable, no disclosure is always optimal, since having P_2 ask a low price increases the price $\overline{\theta}$ is willing to pay for the upstream product and reduces the rent for $\underline{\theta}$.

Consider next the case of unfavorable beliefs. In the absence of disclosure, the optimal mechanism consists in trading with either type at a price $t_1=\overline{a}$ if $m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\geqslant 0$ and only with the low type at a price $t_1=\underline{a}$ otherwise. When $m_1+\overline{a}-\frac{1-p}{p}|\Delta a|<0$, disclosure is always optimal. Indeed, by adopting a disclosure policy such that $d^*(s_2|\underline{\theta})=\min\{1,\frac{|\Delta a|}{\Delta b}\}$ and $d^*(s_2|\overline{\theta})=Hd(s_2|\underline{\theta})$, P_1 can fully appropriate the surplus $d^*(s_2|\overline{\theta})\Delta b$ that $\overline{\theta}$ expects from downstream contracting without increasing the rent for $\underline{\theta}$. What is more, disclosure allows P_1 to sell also to $\overline{\theta}$ with positive probability, once again without leaving any rent to the low type.

When $m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\geqslant 0$, things are more complicated because disclosure may require a reduction in the level of trade with $\overline{\theta}$, which is costly for P_1. Indeed, using (SR_2) and $\delta_1^*(1|\underline{\theta})=1$ and combining ($\overline{IC}_1$) with ($\underline{IC}_1$), gives $\delta_1(1|\overline{\theta})\leqslant 1-(1-H)\frac{\Delta b}{|\Delta a|}d(s_2|\underline{\theta})$ which is strictly less than one when P_1 discloses information, that is when $d(s_2|\underline{\theta})>0$.

The marginal effect of increasing $d(s_2|\underline{\theta})$ is then given by

$$pH\Delta b-p(1-H)\frac{\Delta b}{|\Delta a|}\Big(m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\Big)+(1-p)H\Delta b, \qquad (5)$$

where the first term is simply the benefit of increasing the probability of a downstream price discount for the high type (recall that $d(s_2|\overline{\theta})=Hd(s_2|\underline{\theta})$), the second term is the cost of reducing the level of trade with the high type, and the third term is the reduction in the rent for $\underline{\theta}$ generated by countervailing incentives.[17] Rewriting (5), we thus have that disclosure is optimal for P_1 if and only $m_1+\overline{a}-\frac{1-p}{p}|\Delta a|<\frac{H|\Delta a|}{p(1-H)}$.

Proposition 2. *When x_1 and x_2 are horizontally differentiated, disclosure is motivated by the possibility of exploiting countervailing incentives to appropriate surplus from downstream contracting. Suppose preferences are as in Conditions* (1) *and* (3). *Disclosure is optimal if and only if P_2's beliefs are unfavorable to the agent and the cost of reducing the level of trade with the high type is small.*

[17] As shown in appendix, at the optimum, $U_A(\underline{\theta};\phi_1^*)=\delta_1^*(1|\overline{\theta})\,|\Delta a|-d^*(s_2|\overline{\theta})\Delta b$.

Finally, consider the effect of disclosure on individual payoffs and welfare. P_2 is not affected by disclosure, since ϕ_1^* makes her indifferent between asking the prices she would have asked in the absence of disclosure and the equilibrium ones. As for the agent, when $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$, A gets the same payoff as when P_1 is not allowed to disclose information, since the increase in his rent with P_2 is entirely appropriated by P_1. But when $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geqslant 0$, disclosure reduces the rent of the low type from $|\Delta a|$ to $\delta_1^*(1|\overline{\theta})|\Delta a| - d^*(s_2|\overline{\theta})\Delta b$ without increasing that of the high type, thus making A strictly worse off. Indeed, by increasing the surplus of the high type, disclosure reduces the low type's incentive to mimic and thus allows P_1 to reduce the rent she must cede for truthful revelation.

In terms of welfare, when $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$, disclosure increases the level of trade in both relationships and thus boosts efficiency. When instead $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geqslant 0$, disclosure increases the level of trade in the downstream relationship but reduces it upstream, with a negative net effect on welfare.

Corollary 2. *Disclosure increases welfare if and only if* $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$. P_1 *strictly benefits from disclosure*, P_2 *is indifferent*, *and A is worse off if disclosure reduces the upstream level of trade*, *indifferent otherwise*.

5. Disclosure of endogenous information

In this Section, we consider environments where the agent's valuation in the downstream relationship depends on upstream decisions, as in the case of a buyer whose willingness to pay for a downstream product or service depends on complementarity, or substitutability, with the products and services purchased from an upstream vendor.

The reason why disclosure can be optimal when preferences are non-separable is that it permits P_1 to sustain a more profitable level of trade upstream without eliminating the rent the agent obtains in the downstream relationship. To illustrate, assume the following.

Condition 4. The agent's preferences are not separable: $v_A(x_1, x_2, \theta) = a(\theta)x_1 + bx_2 + gx_1x_2$. The two principals have preferences $v_i(x_1, x_2, \theta) = m_i x_i$ for $i = 1, 2$.

The two products are complements if $g > 0$ and substitutes if $g < 0$. That the downstream surplus does not depend on θ guarantees that disclosure is entirely about endogenous information. We also assume that trade continues to generate positive surplus in both relationships, that is $m_1 + a(\theta) \geqslant 0$ for any θ, $m_2 + b \geqslant 0$ and $m_2 + b + g > 0$.[18]

5.1. *Complements*

When preferences are as in Condition 4, the solution to $\mathcal{P}_2(s)$ assigns the same allocation to $\theta_2^{\mathrm{E}} = \left(x_1, \overline{\theta}\right)$ and $\theta_2^{\mathrm{E}} = \left(x_1, \underline{\theta}\right)$ and is equivalent to a take-it-or-leave-it offer at a price

[18] This also guarantees that P_2 is indeed interested in receiving information about x_1.

$t_2(s) \in \{b, b+g\}$. This implies that P_1 does not need to disclose more than two signals, s_1, and s_2, such that $t_2(s_1) = b+g$ and $t_2(s_2) = b$. Conditional on receiving information s_2, a low price is optimal for P_2 if and only if she assigns sufficiently low probability to A's having purchased the complementary product from P_1, that is, if and only if $\Pr(x_1 = 1|s_2)g \leqslant (m_2+b)\Pr(x_1 = 0|s_2)$, or equivalently

$$\delta_1(1, s_2)g \leqslant (m_2+b)\delta_1(0, s_2), \qquad (\mathrm{SR}_2)$$

where $\delta_1(x_1, s_2) = p\delta_1(x_1, s_2|\overline{\theta}) + (1-p)\,\delta_1(x_1, s_2|\underline{\theta})$.[19] The left-hand side is simply the cost of leaving the agent an informational rent when asking a low price $t_2 = b$, while the right-hand side is the cost of not trading when asking a high price $t_2 = b+g$.

Since A has no private information about his valuation for x_2, P_1 can appropriate the entire surplus $\delta_1(1, s_2)g$ that A expects from contracting with P_2. This also implies that the rent P_1 must cede to A is independent of the disclosure policy, and is the same as in the absence of downstream contracting, i.e. $U_A(\overline{\theta}; \phi_1) = \delta_1(1|\underline{\theta})\Delta a$ and $U_A(\underline{\theta}; \phi_1) = 0$. The optimal contracts then maximize

$$U_1 = p\delta_1(1|\overline{\theta})\,(m_1 + \bar{a}) + (1-p)\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + \delta_1(1, s_2)g$$

subject to (SR_2). Note that $(m_2+b)\delta_1(0, s_2)$ is an upper bound for the rent P_2 leaves to the agent. To maximize this upper bound, it is always optimal to send signal s_2 if trade does not occur, which implies that (SR_1) never binds and $\delta_1(0, s_2) = 1 - p\delta_1(1|\overline{\theta}) - (1-p)\delta_1(1|\underline{\theta})$. The cost of increasing the rent that P_2 leaves to the agent is thus the (virtual) surplus that P_1 forgoes by reducing the level of trade in the upstream relationship. It is then immediate that for $m_2 + b \leqslant m_1 + \underline{a} - \frac{p}{1-p}\Delta a$, it is optimal to sell to either type, in which case there is no disclosure.

However, when $m_2 + b > m_1 + \underline{a} - \frac{p}{1-p}\Delta a$, it is profitable for P_1 to sacrifice trade with the low type to induce P_2 to give the agent a price discount. The properties of the optimal mechanism then depend on the price that P_2 asks if P_1 sells only to $\overline{\theta}$. When the complementarity is small so that P_2 asks a low price, P_1 sells with certainty to the high type and with probability less than one to the low type and does not disclose any information.

When the complementarity is strong, so that P_2 is expected to ask a high price, P_1 has two options: sacrifice trade also with $\overline{\theta}$ and guarantee that P_2 will lower her price, or continue to trade with certainty with the high type and use the disclosure policy to induce P_2 to offer a price discount with probability positive, but less than one. When $m_1 + \bar{a} \leqslant m_2 + b$, P_1 finds it optimal to sacrifice trade. When instead $m_1 + \bar{a} > m_2 + b$, the optimal mechanism has the following structure:

$$\begin{array}{ccccccc} \overline{\theta} & \longrightarrow & x_1 = 1 & \longrightarrow & s_1 & \rightarrow & t_2 = b+g, \\ & & & \searrow & & & \\ \underline{\theta} & \longrightarrow & x_1 = 0 & \longrightarrow & s_2 & \rightarrow & t_2 = b. \end{array}$$

Signal s_1 can thus be interpreted as the decision to inform P_2 that trade occurred in the upstream relationship, s_2 as the decision to keep all information secret. The optimal policy

[19] The other constraint $\delta_1(1, s_1)g \geqslant (m_2+b)\delta_1(0, s_1)$ is omitted since it never binds at the optimum.

then consists in not disclosing any information if A decides not to purchase (which occurs if and only if $\theta = \underline{\theta}$) and informing P_2 with probability $\delta_1^*(1, s_2|\overline{\theta}) \in (0, 1)$ otherwise.[20]

Proposition 3. *Suppose preferences are as in Condition* (4) *and x_1 and x_2 are complements. Disclosure is motivated by the possibility of inducing P_2 to offer the agent a price discount without reducing the upstream level of trade. Disclosure is optimal when* (i) *the complementarity is sufficiently strong that excluding the low type is not sufficient to induce P_2 to ask a low price*; (ii) *the cost of reducing the level of trade with the high type is greater than the benefit of increasing the probability of a downstream price discount*, *whereas the opposite is true for the low type* (*i.e.* $m_1 + \bar{a} > m_2 + b > m_1 + \underline{a} - \frac{p}{1-p}\Delta a$).

As for the effects of disclosure on individual payoffs and welfare, when $m_1+\underline{a}-\frac{p}{1-p}\Delta a >$ 0 and $g < \Delta a(m_2+b)/(m_1+\underline{a}-m_2-b)$, P_1 would trade with either type with certainty if disclosure were not allowed. Clearly, in this case, disclosure benefits P_1 but harms A and P_2: by reducing trade with the low type, P_1 decreases the rent for $\overline{\theta}$ and the surplus that P_2 can extract from $\underline{\theta}$. Furthermore, since it is always efficient to trade in both relationships, disclosure is welfare-decreasing.

In all other cases, disclosure leads to a Pareto improvement, since it does not affect trade with the low type (hence the rent for $\overline{\theta}$) and it either increases trade with the high type or leaves it unchanged. P_2 clearly benefits from disclosure if it increases trade in the upstream relationship and is indifferent otherwise. Finally, since the optimal disclosure policy always induces P_2 to ask a low price when A does not purchase upstream, this guarantees that trade always occurs in the downstream relationship thus maximizing efficiency.

Corollary 3. *Disclosure harms P_2 and A and is welfare-decreasing if it reduces the upstream level of trade. Else*, *it leads to a Pareto improvement.*

5.2. Substitutes

Finally, consider a situation where the products of the two sellers are substitutes, in which case the agent obtains a positive surplus with P_2 only if he does *not* reduce his valuation by purchasing from P_1. To be consistent with the notation used so far, we continue to denote by s_1 the information that induces P_1 to ask a high price, so that $t(s_1) = b$ and $t(s_2) = b + g < b$. The optimal mechanism maximizes

$$U_1 = p\delta_1(1|\overline{\theta})\,(m_1+\bar{a}) + (1-p)\delta_1(1|\underline{\theta})\left(m_1+\underline{a}-\frac{p}{1-p}\Delta a\right) + \delta_1(0, s_2)\,|g|$$

subject to ($\underline{\text{IC}}_1$) and

$$|g|\,\delta_1(0, s_2) \leqslant (m_2+b+g)\delta_1(1, s_2). \qquad (\text{SR}_2)$$

[20] With a continuum of consumers, the optimal disclosure policy simply specifies the fraction of transactions that are disclosed to P_2.

Note that the upper bound for the rent that P_2 leaves to the agent is $(m_2 + b + g)\delta_1(1, s_2)$ so that it is always optimal to send signal s_2 when A purchases from P_1, which also implies that (SR_1) never binds and $\delta_1(1, s_2) = p\delta_1(1|\overline{\theta}) + (1 - p)\delta_1(1|\underline{\theta})$.

Since A now obtains a rent with P_2 only if he does buy upstream, the optimal contracts compare the surplus P_1 can appropriate by trading with either type with what she can get by not selling and making P_2 offer a lower price. Clearly, when $|g| \leqslant m_1 + \underline{a} - \frac{p}{1-p}\Delta a$, the rent A gets with P_2 is so small that it never pays to sacrifice upstream trade. On the contrary, when $|g| > m_1 + \underline{a} - \frac{p}{1-p}\Delta a$, P_1 finds it optimal not to sell to the low type. The optimal mechanism then depends on the price P_2 is expected to ask when P_1 sells only to the high type. When substitutability is small, selling only to $\overline{\theta}$ suffices to induce P_2 to ask a low price. In this case, the optimal mechanism is $\delta_1^*(1|\overline{\theta}) = 1 = \delta_1^*(0|\overline{\theta}) = 1$ if $|g| \leqslant m_1 + \bar{a}$ and $\delta_1^*(1|\overline{\theta}) \in (0, 1)$ and $\delta_1^*(0|\overline{\theta}) = 1$ otherwise. Indeed when $|g| > m_1 + \bar{a}$, P_1 finds it more profitable to sacrifice trade with the high type as well, so as to let the latter enjoy a downstream rent with positive probability. Trade with the high type is then stochastic, but again the optimal mechanism does not require disclosure.

Next consider the less favorable case in which P_2 is expected to ask a high price when P_1 sells only to $\overline{\theta}$. P_1 then needs to sell with positive probability also to $\underline{\theta}$ if she wants to reduce the downstream price. The value of selling to the low type must then be adjusted to take into account the increase in the probability of a downstream rent, as indicated in (SR_2). It follows that for $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g > 0$, selling to $\underline{\theta}$ with positive probability is optimal for P_1, in which case trade is stochastic and involves no disclosure. When this value is negative, however, it is more profitable to exclude the low type and induce P_2 to leave a rent with probability less than one by adopting a disclosure policy with the following structure:

$$\begin{array}{ccccccc} \overline{\theta} & \longrightarrow & x_1 = 1 & \longrightarrow & s_2 & \longrightarrow & t_2 = b + g < b, \\ & & & \nearrow & & & \\ \underline{\theta} & \longrightarrow & x_1 = 0 & \longrightarrow & s_1 & \longrightarrow & t_2 = b. \end{array}$$

As with complements, signal s_1 can be interpreted as the decision to inform P_2 that trade did not occur upstream and s_2 as the decision not to disclose any information.

Finally, note that for high levels of substitutability (i.e. $|g| \geqslant m_2 + b$) disclosure becomes irrelevant, since P_2 always asks a high price, whatever her beliefs about x_1.

Proposition 4. *Suppose preferences are as in Condition* (4) *and x_1 and x_2 are substitutes. Disclosure is motivated by the possibility of inducing P_2 to offer a price discount without increasing the level of trade in the upstream relationship. Disclosure is optimal when* (i) *selling only to the high type is not sufficient to induce P_2 to ask a low price*; (ii) *the cost of selling to the low type more than offsets the benefit of increasing the probability of a downstream price discount* (*i.e.* $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g < 0$).

Consider the effects of disclosure on payoffs. When disclosure is not allowed, P_1 has two options. She may trade with both types with positive probability or else she may exclude the low type by asking a price $t_1 = \overline{a}$ that induces P_2 to ask a high price $t_2 = b$. In this latter case, disclosure is clearly welfare-enhancing, since it does not affect trade upstream and increases it downstream. What is more, disclosure yields a Pareto improvement: P_1 is clearly better off since disclosure is strictly optimal; A is indifferent since he gets no surplus with either principal anyway; and P_2 is also unaffected, since the optimal mechanism makes her just indifferent between asking a high price with certainty—as in the absence of disclosure—and reducing the price conditional on receiving information s_2.

On the contrary, when the optimal mechanism in the absence of disclosure is such that P_1 sells also to $\underline{\theta}$ with positive probability so as to induce P_2 to lower her price, A strictly suffers from disclosure since it reduces the rent for $\overline{\theta}$. On the other hand, P_2 benefits from the reduction in upstream trade, since this increases the agent's willingness to pay downstream. The net effect on welfare then depends on whether it is efficient for P_1 to sell to the low type, that is on whether $m_1 + \underline{a} \gtrless |g|$.

Corollary 4. *When disclosure reduces the upstream level of trade, it damages A and benefits P_1 and P_2; its effect on welfare is positive if and only if it is inefficient to sell to the low type upstream. In all other cases, disclosure yields a Pareto improvement.*

6. Concluding remarks

We have considered the dynamic interaction between two principals who contract sequentially with the same agent. The focus is disclosure policies that control optimally for the exchange of information between the two bilateral relationships. We have shown that the optimal policy is keeping all information secret when: (a) the upstream principal is not personally interested in the level of trade downstream; (b) the agent's valuations are positively correlated so that the sign of the single crossing condition is the same for upstream and downstream decisions; and (c) preferences in the downstream relationship are additively separable, so that downstream decisions do not depend on the upstream level of trade.

When any of these conditions is violated, however, there exist preferences for which disclosure is strictly optimal, regardless of the price the downstream principal is willing to pay for information.

Finally, we have shown that the possibility of disclosing information does not necessarily harm the agent and in some cases even leads to Pareto improvements.

To bring out the various effects at work, we have examined the determinants of the disclosure of exogenous and endogenous information separately. Further, the results have been derived under the assumption that the upstream principal can commit perfectly to any privacy policy she chooses. The design of optimal policies in environments where disclosure may be driven by a combination of the different determinants discussed above is an interesting line for future research. Similarly, relaxing the assumption of full commitment may deliver new insights into the welfare effects of disclosure and the desirability of regulatory intervention in the area of privacy. We expect the main strategic effects that we have highlighted to prove useful also in the study of these more complex environments.

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Appendix

Proof of Theorem 1. Under conditions (a) and (c), the preferences for P_1, P_2 and A can be written as

$$v_1(x_1, x_2, \theta) = v_1(x_1, \theta), \quad v_2(x_1, x_2, \theta) = v_2^1(x_1, \theta) + v_2^2(x_2, \theta),$$
$$v_A(x_1, x_2, \theta) = v_A^1(x_1, \theta) + v_A^2(x_2, \theta)$$

with $v_1(0, \theta) = v_i^j(0, \theta) = 0$ for any $\theta \in \Theta$, $j = 1, 2$, and $i = 2, A$. To save on notation, we let $W_1(x_1, \theta) \equiv v_1(x_1, \theta) + v_A^1(x_1, \theta)$ and $W_2(x_2, \theta) \equiv v_2^2(x_2, \theta) + v_A^2(x_2, \theta)$.

The proof is by contradiction and is in four steps. Step 1 constructs the optimal mechanisms $\{\phi_2(s)\}_{s\in S}$. Step 2 identifies necessary conditions for ϕ_1 and $\{\phi_2(s)\}_{s\in S}$ to solve $\mathcal{P}_1$. Step 3 introduces an alternative mechanism ϕ_1^{ND}—with reaction ϕ_2^{ND}—that does not disclose information and induces the same upstream decisions as ϕ_1. Step 4 proves that if ϕ_1 and $\{\phi_2(s)\}_{s\in S}$ solve $\mathcal{P}_1$, so do $(\phi_1^{\text{ND}}, \phi_2^{\text{ND}})$, contradicting the assumption that disclosure is strictly optimal.

Step 1: Since preferences in the downstream relationship are separable, the mechanisms $\{\phi_2(s)\}_{s\in S}$ are independent of x_1 so that $x_2(\theta_2^{\text{E}}; s) = x_2(\widetilde{\theta}_2^{\text{E}}; s)$ and $U_A^2(\theta_2^{\text{E}}; s) = U_A^2(\widetilde{\theta}_2^{\text{E}}; s)$ for any $\theta_2^{\text{E}} = (\theta, x_1)$ and $\widetilde{\theta}_2^{\text{E}} = (\widetilde{\theta}, \widetilde{x}_1)$ such that $\theta = \widetilde{\theta}$. Indeed, for any mechanism $\phi_2(s)$ that depends on x_1, there exists another mechanism $\phi_2'(s)$ that is independent of x_1 and is payoff-equivalent for all players. This also implies that when P_2 does not receive information, her optimal mechanism does not depend of ϕ_1 and will be denoted by $\phi_2^{\text{ND}} = (x_2^{\text{ND}}(\theta), U_A^{2\text{ND}}(\theta))$. Finally, when $W_2(1, \theta) \leqslant 0$ for one of the two types, information disclosure is irrelevant since ϕ_2 does not depend on P_2's posterior beliefs. In what follows, we thus assume $W_2(1, \theta) > 0$ for all θ. The mechanisms ϕ_2 and ϕ_2^{ND} then satisfy

$$U_A^2(\underline{\theta}; s) = U_A^{2\text{ND}}(\underline{\theta}) = 0, \quad U_A^2(\overline{\theta}; s) = \Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta),$$
$$U_A^{2\text{ND}}(\overline{\theta}) = \Delta_\theta v_A^2(x_2^{\text{ND}}(\underline{\theta}), \theta), \tag{6}$$

where

$$x_2(\underline{\theta}; s) = \arg \max_{x_2 \in X_2} \left\{\mu(\underline{\theta}; s) W_2(x_2, \underline{\theta}) - \mu(\overline{\theta}; s)\Delta_\theta v_A^2(x_2, \theta)\right\},$$
$$x_2^{\text{ND}}(\underline{\theta}) = \arg \max_{x_2 \in X_2} \left\{(1 - p) W_2(x_2, \underline{\theta}) - p\Delta_\theta v_A^2(x_2, \theta)\right\},$$
$$x_2(\overline{\theta}; s) = x_2^{\text{ND}}(\overline{\theta}) = 1. \tag{7}$$

Step 2: Since $\tau(\phi_1) = \gamma\left[U_2(\phi_1) - U_2^{\text{ND}}(\phi_1)\right]$, for any individually rational and incentive-compatible mechanism ϕ_1—with reaction $\{\phi_2(s)\}_{s\in S}$

$$U_1(\phi_1) = \sum_{\theta\in\Theta} \left[W_1(1, \theta)\delta_1(1|\theta) + \sum_{s\in S} U_A^2(\theta; s) d(s|\theta) - U_A\left(\theta; \phi_1\right)\right]\Pr(\theta)$$
$$+\gamma\left[U_2(\phi_1) - U_2^{\text{ND}}(\phi_1)\right],$$

where

$$U_2(\phi_1) = \sum_{\theta\in\Theta}\Big[\sum_{s\in S}\Big(W_2(x_2(\theta;s),\theta) - U_A^2(\theta;s)\Big)d(s|\theta) + v_2^1(1,\theta)\,\delta_1(1|\theta)\Big]\Pr(\theta),$$

$$U_2^{\mathrm{ND}}(\phi_1) = \sum_{\theta\in\Theta}\Big[W_2(x_2^{\mathrm{ND}}(\theta),\theta) - U_A^{2\mathrm{ND}}(\theta) + v_2^1(1,\theta)\,\delta_1(1|\theta)\Big]\Pr(\theta).$$

If ϕ_1 and $\{\phi_2(s)\}_{s\in S}$ solve $\mathcal{P}_1$, then necessarily

$$U_A\left(\underline{\theta};\phi_1\right) = 0, \quad U_A\left(\overline{\theta};\phi_1\right) = \Delta_\theta v_A^1(1,\theta)\delta_1(1|\underline{\theta}) + \sum_{s\in S} U_A^2(\overline{\theta};s)d(s|\underline{\theta}) \tag{8}$$

and

$$\Delta_\theta v_A^1(1,\theta)\big[\delta_1(1|\overline{\theta}) - \delta_1(1|\underline{\theta})\big] + \sum_{s\in S} U_A^2(\overline{\theta};s)\big[d(s|\overline{\theta}) - d(s|\underline{\theta})\big] \geqslant 0. \tag{$\underline{\text{IC}}_1$}$$

Step 3: Consider now an alternative mechanism ϕ_1^{ND} that does not disclose information, that induces the same distribution over X_1 as ϕ_1 and is such that

$$U_A(\underline{\theta};\phi_1^{\mathrm{ND}}) = 0, \quad U_A(\overline{\theta};\phi_1^{\mathrm{ND}}) = \Delta_\theta v_A^1(1,\theta)\delta_1^{\mathrm{ND}}(1|\underline{\theta}) + U_A^{2\mathrm{ND}}(\overline{\theta}). \tag{9}$$

The mechanism ϕ_1^{ND}—with reaction ϕ_2^{ND}—is also individually rational and incentive-compatible and yields

$$U_1(\phi_1^{\mathrm{ND}}) = \sum_{\theta\in\Theta}\Big[W_1(1,\theta)\delta_1^{\mathrm{ND}}(1|\theta) + U_A^{2\mathrm{ND}}(\theta) - U_A(\theta;\phi_1^{\mathrm{ND}})\Big]\Pr(\theta).$$

It follows that

$$\begin{aligned} U_1(\phi_1) - U_1(\phi_1^{\mathrm{ND}}) = (1-\gamma) &\sum_{\theta\in\Theta}\Big[\sum_{s\in S} U_A^2(\theta;s)d(s|\theta) - U_A^{2\mathrm{ND}}(\theta)\Big]\Pr(\theta) \\ &+\gamma\sum_{\theta\in\Theta}\Big[\sum_{s\in S} W_2(x_2(\theta;s),\theta)d(s|\theta) - W_2(x_2^{\mathrm{ND}}(\theta),\theta)\Big]\Pr(\theta) \\ &-\sum_{\theta\in\Theta}\Big[U_A\left(\theta;\phi_1\right) - U_A\left(\theta;\phi_1^{\mathrm{ND}}\right)\Big]\Pr(\theta). \end{aligned} \tag{10}$$

Using (6), (8) and (9), (10) reduces to

$$
\begin{aligned}
&U_1(\phi_1) - U_1(\phi_1^{\mathrm{ND}}) \\
&\quad = (1-\gamma)\, p \sum_{s\in S} U_A^2(\overline{\theta}; s)\left[d(s|\overline{\theta}) - d(s|\underline{\theta})\right] \\
&\qquad + \gamma \sum_{s\in S}\left[(1-p) W_2(x_2(\underline{\theta}; s), \underline{\theta}) - p\Delta_\theta v_A^2(x_2(\underline{\theta}; s), \theta)\right] d(s|\underline{\theta}) \\
&\qquad - \gamma\left[(1-p) W_2(x_2^{\mathrm{ND}}(\underline{\theta}), \underline{\theta}) - p\Delta_\theta v_A^2(x_2^{\mathrm{ND}}(\underline{\theta}), \theta)\right]. \qquad (11)
\end{aligned}
$$

Step 4: First, consider the last two terms in (11). From (7), the difference between these two terms is never positive. Next, consider the first term in (11). $U_A^2(\overline{\theta}; s)$ is increasing in the posterior odds $\frac{\mu(\underline{\theta}; s)}{\mu(\overline{\theta}; s)}$ and hence in $\frac{d(s|\underline{\theta})}{d(s|\overline{\theta})}$. From standard representation theorems ([20]—Proposition 1), it then follows that $\sum_{s\in S} U_A^2(\overline{\theta}; s)[d(s|\overline{\theta}) - d(s|\underline{\theta})] \leqslant 0$.[21] We conclude that if ϕ_1 and $\{\phi_2(s)\}_{s\in S}$ solve $\mathcal{P}_1$, so do $(\phi_1^{\mathrm{ND}}, \phi_2^{\mathrm{ND}})$. □

Proof of Theorem 1 (*Continuum of types and decisions*). Assume θ is distributed over $\Theta \equiv [\underline{\theta}, \overline{\theta}]$ with absolutely continuous log-concave c.d.f. F and density f strictly positive over Θ. Furthermore, let $X_1 = X_2 = \mathbb{R}_+$ and suppose $v_A^i\,(x_i, \theta)$, $v_1\,(x_1, \theta)$ and $v_2^2\,(x_2, \theta)$ are thrice continuously differentiable and satisfy

$$
\begin{aligned}
&\frac{\partial^2 v_1\,(x_1, \theta)}{\partial x_1^2} < 0, \quad \frac{\partial^2 v_1\,(x_1, \theta)}{\partial x_1 \partial\theta} \geqslant 0, \quad \frac{\partial^2 v_2^2\,(x_2, \theta)}{\partial x_2^2} < 0, \\
&\frac{\partial^2 v_2^2\,(x_2, \theta)}{\partial\theta\partial x_2} \geqslant 0, \quad \frac{\partial v_A^i\,(x_i, \theta)}{\partial\theta} > 0, \quad \frac{\partial^2 v_A^i\,(x_i, \theta)}{\partial x_i^2} < 0, \\
&\frac{\partial^2 v_A^i\,(x_i, \theta)}{\partial x_i\partial\theta} \geqslant 0, \quad \frac{\partial^3 v_A^i\,(x_i, \theta)}{\partial\theta\partial x_i^2} \geqslant 0 \quad \text{and} \quad \frac{\partial^3 v_A^i\,(x_i, \theta)}{\partial\theta^2\partial x_i} \leqslant 0,
\end{aligned}
$$

for $i = 1, 2$. These conditions are standard in mechanism design with a continuum of types (see [11, Chapter 7]) and imply that the optimal mechanism for a single principal controlling both x_1 and x_2 is deterministic and is characterized by two schedules $x_1(\theta)$ and $x_2(\theta)$ with no bunching.

It suffices to prove the result for $\gamma = 1$; if disclosure is not optimal when $\gamma = 1$, it is clearly not optimal for any $\gamma < 1$. Letting $\Psi(s|\theta)$ and $\Gamma(x_1|\theta)$ denote the c.d.f. of the lotteries over $S \subseteq \mathbb{R}$ and X_1, we have that

$$
\begin{aligned}
U_2(\phi_1) = \int_\Theta \Bigg\{ &\int_S \left[W_2(x_2(\theta; s), \theta) - U_A^2(\theta; s)\right] \mathrm{d}\Psi(s|\theta) \\
&+ \int_{X_1} v_2^1\,(x_1, \theta)\ \mathrm{d}\Gamma(x_1|\theta)\Bigg\}\ \mathrm{d}F(\theta),
\end{aligned}
$$

[21] This also implies that if ($\underline{\mathrm{IC}}_1$) is satisfied by ϕ_1, then $\delta_1(1|\overline{\theta}) \geqslant \delta_1(1|\underline{\theta})$ and hence ($\underline{\mathrm{IC}}_1$) is satisfied also by ϕ_1^{ND}.

$$U_2^{\rm ND}(\phi_1) = \int_\Theta \left\{ W_2(x_2^{\rm ND}(\theta), \theta) - U_A^{2\rm ND}(\theta) + \int_{X_1} v_2^1(x_1, \theta)\, \mathrm{d}\Gamma(x_1|\theta) \right\} \mathrm{d}F(\theta).$$

P_1's expected payoff is thus

$$\begin{aligned} U_1(\phi_1) = & \int_\Theta \left\{ \int_{X_1} W_1(x_1, \theta)\, \mathrm{d}\Gamma(x_1|\theta) \right. \\ & \left. + \int_S U_A^2(\theta; s)\, \mathrm{d}\Psi(s|\theta) - U_A\left(\theta; \phi_1\right) \right\} \mathrm{d}F(\theta) + U_2(\phi_1) - U_2^{\rm ND}(\phi_1) \\ = & \int_\Theta \left\{ \int_{X_1} W_1(x_1, \theta)\, \mathrm{d}\Gamma(x_1|\theta) \right. \\ & \left. + \int_S W_2(x_2(\theta; s), \theta)\, \mathrm{d}\Psi(s|\theta) - U_A\left(\theta; \phi_1\right) \right\} \mathrm{d}F(\theta) \\ & - \int_\Theta \left\{ W_2(x_2^{\rm ND}(\theta), \theta) - U_A^{2\rm ND}(\theta) \right\} \mathrm{d}F(\theta), \end{aligned} \tag{12}$$

where

$$\begin{aligned} U_A\left(\theta; \phi_1\right) &= \int_{X_1} v_A^1(x_1, \theta)\, \mathrm{d}\Gamma(x_1|\theta) + \int_S U_A^2(\theta, s)\, \mathrm{d}\Psi(s|\theta) - t_1(\theta), \\ U_A^2(\theta; s) &= v_A^2(x_2(\theta; s), \theta) - t_2(\theta; s). \end{aligned}$$

Now suppose P_1 could control $x_2(\theta)$ and $t_2(\theta)$ directly. That is, consider a fictitious mechanism $\widetilde{\phi}_1 = (\widetilde{\Gamma}(x_1|\theta), \widetilde{\Psi}(s|\theta), \widetilde{x}_2(\theta; s), U_A(\theta; \widetilde{\phi}_1))$ in which A must report θ only at $t = 1$ and where the lotteries over X_2 are obtained by combining $\widetilde{\Psi}(s|\theta)$ with $\widetilde{x}_2(\theta; s)$. The mechanism $\widetilde{\phi}_1$ which maximizes (12) subject to standard individual rationality and incentive compatibility constraints is deterministic and is characterized by schedules $\widetilde{x}_1(\theta)$ and $\widetilde{x}_2(\theta)$ which maximize pointwise

$$W_1(x_1, \theta) - \frac{1-F(\theta)}{f(\theta)} \frac{\partial v_A^1(x_1, \theta)}{\partial \theta} \quad \text{and} \quad W_2(x_2, \theta) - \frac{1-F(\theta)}{f(\theta)} \frac{\partial v_A^2(x_2, \theta)}{\partial \theta}$$

together with a rent for the agent equal to

$$U_A(\theta; \widetilde{\phi}_1) = \int_{\underline{\theta}}^{\theta} \frac{\partial v_A^1(\widetilde{x}_1(z), z)}{\partial z} \mathrm{d}z + \int_{\underline{\theta}}^{\theta} \frac{\partial v_A^2(\widetilde{x}_2(z), z)}{\partial z} \mathrm{d}z.$$

Since in the absence of disclosure P_2 offers a mechanism such that $x_2^{\rm ND}(\theta)$ maximizes $W_2(x_2, \theta) - \frac{1-F(\theta)}{f(\theta)} \frac{\partial v_A^2(x_2, \theta)}{\partial \theta}$ and $U_A^{2\rm ND}(\theta) = \int_{\underline{\theta}}^{\theta} \frac{\partial v_A^2(x_2^{\rm ND}(z), z)}{\partial z} \mathrm{d}z$, it follows that P_1 can guarantee herself $U_1(\widetilde{\phi}_1)$ by offering a deterministic mechanism such that $x_1(\theta) = \widetilde{x}_1(\theta)$ and $t_1(\theta) = v_A^1(x_1(\theta), \theta) - \int_{\underline{\theta}}^{\theta} \frac{\partial v_A^1(x_1(z), z)}{\partial z} \mathrm{d}z$ and committing not to disclose any information. $\square$

Proof of Proposition 1. We prove that there exists a threshold $E(H)$ such that when $H < 1$ (respectively, $H \geqslant 1$) disclosure is optimal if and only if $e > E(H) > 0$ (respectively, $e \leqslant E(H) < 0$).

Consider the program for the optimal mechanism as in the main text. First, note that (SR_1) and (SR_2) cannot be both slack. If this were the case, P_1 could reduce $d(s_1|\bar{\theta})$ and increase $d(s_2|\bar{\theta})$, increasing her payoff and relaxing $(\underline{IC}_1)$. Second, using $d(s_1|\theta) = 1 - d(s_2|\theta)$, constraint (SR_1) can be rewritten as $d(s_2|\bar{\theta}) \leqslant Hd(s_2|\underline{\theta}) + 1 - H$. When $H < 1$, if (SR_2) is satisfied, so is (SR_1). When instead $H \geqslant 1$, (SR_1) implies (SR_2). Since at least one of these constraints must bind, it follows that for $H < 1$, (SR_2) is binding and (SR_1) is slack, whereas the opposite is true for $H \geqslant 1$.

Also note that by increasing $\delta_1(1|\bar{\theta})$, P_1 increases the objective function and relaxes $(\underline{IC}_1)$. Hence, at the optimum, trade occurs with certainty with $\bar{\theta}$.

Case 1: *Unfavorable beliefs* $(H < 1)$. Substituting $\delta_1^*(1|\bar{\theta}) = 1$ and $d(s_2|\bar{\theta}) = Hd(s_2|\underline{\theta})$, the program for ϕ_1^* reduces to

$$\mathcal{P}_1^{\text{Unf}}: \begin{cases} \max \; p\,(m_1 + \bar{a}) + (1-p)\,\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \dfrac{p}{1-p}\Delta a\right) \\ \qquad +pe + d(s_2|\underline{\theta})\left[(1-p)\,e - p\,(1-H)\,\Delta b\right] \\ \text{s.t.} \\ \left[1 - \delta_1(1|\underline{\theta})\right]\Delta a \geqslant d(s_2|\underline{\theta})\,(1-H)\,\Delta b. \qquad (\underline{IC}_1) \end{cases}$$

When $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$, it is always optimal not to trade with the low type, i.e. $\delta_1^*(1|\underline{\theta}) = 0$. If $(1-p)\,e \leqslant p\,(1-H)\,\Delta b$, the optimal disclosure policy is full privacy, that is $d^*(s_1|\theta) = 1$ for any θ. If instead $(1-p)\,e > p\,(1-H)\,\Delta b$, the optimal policy is $d^*(s_2|\underline{\theta}) = \min\{1;\, \Delta a/[(1-H)\Delta b]\}$ and $d^*(s_2|\bar{\theta}) = Hd^*(s_2|\underline{\theta})$.

Next, assume $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$. If $(1-p)\,e \leqslant p\,(1-H)\,\Delta b$, the optimal level of trade with $\underline{\theta}$ is $\delta_1^*(1|\underline{\theta}) = 1$ and no disclosure is optimal ($d^*(s_1|\theta) = 1$ for any θ). If instead $(1-p)\,e > p\,(1-H)\,\Delta b$, then $(\underline{IC}_1)$ binds. Substituting $\delta_1(1|\underline{\theta}) = 1 - d(s_2|\underline{\theta})\,(1-H)\,\frac{\Delta b}{\Delta a}$ from $(\underline{IC}_1)$ into the objective function in $\mathcal{P}_1^{\text{Unf}}$ gives

$$U_1 = p\,(m_1 + \bar{a} + e) + (1-p)\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + (1-p)d(s_2|\underline{\theta})\,(e - E)\,,$$

where

$$E = \frac{p}{1-p}\,(1-H)\,\Delta b + (1-H)\,\frac{\Delta b}{\Delta a}\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right).$$

Note that $E \geqslant p(1-H)\Delta b/(1-p)$ when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$. Hence, if $p(1-H)\Delta b/(1-p) < e \leqslant E$, $\delta_1^*(1|\underline{\theta}) = 1$ and $d^*(s_1|\theta) = 1$ for any θ. If instead $e > E$, the optimal contract maximizes $d(s_2|\underline{\theta})$ under the constraint $\delta_1(1|\underline{\theta}) \geqslant 0$. It follows that $d^*(s_2|\underline{\theta}) = \min\{1; \Delta a/[(1-H)\Delta b]\}$, $d^*(s_2|\bar{\theta}) = Hd^*(s_2|\underline{\theta})$ and $\delta_1^*(1|\underline{\theta}) = 1 - d^*(s_2|\underline{\theta})\,(1-H)\,\frac{\Delta b}{\Delta a}$.

We conclude that with unfavorable beliefs, disclosure is optimal if and only if

$$e > E(H) \equiv \frac{p}{1-p}(1-H)\Delta b + (1-H)\frac{\Delta b}{\Delta a}\mathbb{I}\left[m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right] > 0. \quad (13)$$

Case 2: *Favorable beliefs* ($H \geqslant 1$). Substituting $d(s_1|\bar{\theta}) = Hd(s_1|\underline{\theta})$ and $d(s_2|\theta) = 1 - d(s_1|\theta)$, the program for the optimal mechanism becomes

$$\mathcal{P}_1^{\text{Fav}}: \begin{cases} \max p\,(m_1 + \bar{a}) + (1-p)\,\delta_1(1|\underline{\theta})\left(m_1 + \underline{a} - \dfrac{p}{1-p}\Delta a\right) + \\ \qquad \times e - d(s_1|\underline{\theta})\left[(1-p)\,e - p\,(1-H)\,\Delta b\right] \\ \text{s.t.} \\ \left[1 - \delta_1(1|\underline{\theta})\right]\Delta a \geqslant (H-1)\,\Delta b d(s_1|\underline{\theta}). \qquad (\underline{\text{IC}}_1) \end{cases}$$

The proof follows the same steps as with unfavorable beliefs.

First, assume $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$ so that $\delta_1^*(1|\underline{\theta}) = 0$. When $(1-p)\,e \geqslant p\,(1-H)\,\Delta b$, the optimal policy is no disclosure: $d^*(s_1|\theta) = 0$ for any θ. When instead $(1-p)\,e < p\,(1-H)\,\Delta b$, U_1 is increasing in $d(s_1|\underline{\theta})$. The optimal policy is then $d^*(s_1|\underline{\theta}) = 1/H$ and $d^*(s_1|\bar{\theta}) = 1$ if $\frac{H\Delta a}{(H-1)\Delta b} \geqslant 1$ (the upper bound on $d^*(s_1|\underline{\theta})$ comes from SR_1), and $d^*(s_1|\underline{\theta}) = \frac{\Delta a}{(H-1)\Delta b}$ and $d^*(s_1|\bar{\theta}) = \frac{H\Delta a}{(H-1)\Delta b}$ otherwise (the upper bound on $d^*(s_1|\underline{\theta})$ comes from ($\underline{\text{IC}}_1$)).

Next, assume $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$. If $(1-p)\,e \geqslant p\,(1-H)\,\Delta b$, the optimal policy is $d^*(s_1|\theta) = 0$ for any θ, in which case $\delta_1^*(1|\underline{\theta}) = 1$. If on the contrary $(1-p)\,e < p\,(1-H)\,\Delta b$, then ($\underline{\text{IC}}_1$) binds. Substituting $\delta_1(1|\underline{\theta}) = 1 - (H-1)\,\frac{\Delta b}{\Delta a} d(s_1|\underline{\theta})$ from ($\underline{\text{IC}}_1$) into the objective function in $\mathcal{P}_1^{\text{Fav}}$ gives

$$U_1 = p\,(m_1 + \bar{a}) + (1-p)\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + e - (1-p) d(s_1|\underline{\theta})\,(e - E)\,.$$

where $E = E(H)$ is as in (13) but is now negative since $H > 1$. If $e > E$, then again $\delta_1^*(1|\underline{\theta}) = 1$ and $d^*(s_1|\theta) = 0$ for any θ. If instead $e \leqslant E$, the optimal mechanism is $d^*(s_1|\underline{\theta}) = 1/H$, $d^*(s_1|\bar{\theta}) = 1$ and $\delta_1^*(1|\underline{\theta}) = 1 - \frac{(H-1)\Delta b}{\Delta a H}$ if $\frac{(H-1)\Delta b}{\Delta a H} \leqslant 1$, and $d^*(s_1|\underline{\theta}) = \frac{\Delta a}{(H-1)\Delta b}$, $d^*(s_1|\bar{\theta}) = \frac{\Delta a H}{(H-1)\Delta b}$ and $\delta_1^*(1|\underline{\theta}) = 0$ otherwise.

We conclude that with favorable beliefs disclosure is optimal if and only if $e < E(H) < 0$. □

Proof of Corollary 1. To see that P_2 is not affected by disclosure, note that under the optimal contracts derived in the proof of Proposition 1, (SR_2) binds and (SR_1) is slack when $H < 1$, whereas the opposite is true when $H \geqslant 1$. This means that for $s = s_1$ (respectively, $s = s_2$ when $H \geqslant 1$), P_2 strictly prefers to ask the same price she would have asked in the absence of disclosure, whereas for $s = s_2$ (respectively, $s = s_1$) she is indifferent between asking $t_2 = \overline{b}$ and $t_2 = \underline{b}$. Together with the fact that U_2 is independent of x_1, this implies that P_2 is just as well off as in the absence of disclosure.

Next, consider the effect of disclosure on A and assume favorable beliefs (the case $H < 1$ is discussed in the main text). Without disclosure, $U_A(\underline{\theta}; \phi_1^{\text{ND}}) = 0$ and $U_A(\overline{\theta}; \phi_1^{\text{ND}}) = \Delta a + \Delta b$ if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$ and $U_A(\overline{\theta}; \phi_1^{\text{ND}}) = \Delta b$ otherwise. In contrast, with disclosure, $U_A(\underline{\theta}; \phi_1^*) = 0$ and $U_A(\overline{\theta}; \phi_1^*) = \delta_1^*(1|\underline{\theta})\Delta a + d^*(s_2|\underline{\theta})\Delta b$, where $d^*(s_2|\underline{\theta}) \in (0, 1)$ and $\delta_1^*(1|\underline{\theta}) > 0$ if and only if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$. It follows that $U_A(\overline{\theta}; \phi_1^*) < U_A(\overline{\theta}; \phi_1^{\text{ND}})$. While the negative effect of disclosure on U_A does not depend on e, the positive effect on U_1 increases without bound with $|e|$. It follows that for moderate values of $|e|$, disclosure is welfare-decreasing, whereas the opposite is true for large negative externalities. □

Proof of Proposition 2. Consider the program for the optimal mechanism as in the main text. First, note that it is always optimal to sell to the low type, i.e. $\delta_1^*(1|\underline{\theta}) = 1$. Second, note that when $H \geqslant 1$, (SR_1) binds and (SR_2) is slack, whereas the opposite is true when $H < 1$ (the argument is identical to that in the proof of Proposition 1).

Case 1: *Favorable beliefs* ($H \geqslant 1$): From (SR_1), $d(s_2|\overline{\theta}) = 1 - H + Hd(s_2|\underline{\theta})$. Suppose $d(s_2|\underline{\theta}) < 1$. Then reducing $d(s_1|\underline{\theta})$ to zero and increasing $U_A(\overline{\theta}; \phi_1)$ by $\Delta b d(s_1|\underline{\theta})$ increases U_1 without violating any of the constraints. Hence, necessarily $d^*(s_2|\underline{\theta}) = d^*(s_2|\overline{\theta}) = 1$, which implies that full privacy is always optimal with favorable beliefs. When $\Delta b \geqslant |\Delta a|$, the optimal contracts are such that $U_A(\overline{\theta}; \phi_1^*) = \Delta b - |\Delta a|$, $U_A(\underline{\theta}; \phi_1^*) = 0$ and $\delta_1^*(1|\overline{\theta}) = 1$. When instead $\Delta b < |\Delta a|$, $\delta_1^*(1|\overline{\theta}) = \frac{\Delta b}{|\Delta a|}$ and $U_A(\overline{\theta}; \phi_1^*) = U_A(\underline{\theta}; \phi_1^*) = 0$ if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \leqslant 0$, and $U_A(\overline{\theta}; \phi_1^*) = 0$, $U_A(\underline{\theta}; \phi_1^*) = |\Delta a| - \Delta b$ and $\delta_1^*(1|\overline{\theta}) = 1$ otherwise.

Case 2: *Unfavorable beliefs* ($H < 1$): First, observe that at the optimum ($\underline{IC}_1$) must be saturated. If this were not true, then necessarily $U_A(\underline{\theta}; \phi_1) = 0$ and $\delta_1(1|\overline{\theta}) = 1$, since otherwise P_1 could reduce $U_A(\underline{\theta}; \phi_1)$ and/or increase $\delta_1(1|\overline{\theta})$ enhancing her payoff. But then from ($\underline{IC}_1$) and ($\overline{IC}_1$), $0 \geqslant U_A(\overline{\theta}; \phi_1) - d(s_2|\overline{\theta})\Delta b + |\Delta a| \geqslant [d(s_2|\underline{\theta}) - d(s_2|\overline{\theta})]\Delta b$, which is consistent with (SR_2) only if $d(s_2|\underline{\theta}) = d(s_2|\overline{\theta}) = 0$, in which case $U_A(\overline{\theta}; \phi_1) = d(s_2|\overline{\theta})\Delta b - |\Delta a|$, implying that ($\underline{IC}_1$) is saturated.

Next, we establish that $U_A(\overline{\theta}; \phi_1^*) = 0$. Again, suppose this is not true. Then necessarily $U_A(\underline{\theta}; \phi_1) = 0$, since otherwise P_1 could reduce both rents by the same amount. Using the result that ($\underline{IC}_1$) necessarily binds, we have that $U_A(\overline{\theta}; \phi_1) = d(s_2|\overline{\theta})\Delta b - \delta_1(1|\overline{\theta})|\Delta a|$. Replacing $U_A(\overline{\theta}; \phi_1)$ and $U_A(\underline{\theta}; \phi_1)$ into U_1, gives $U_1 = p\{\delta_1(1|\overline{\theta})(m_1 + \overline{a}) + \delta_1(1|\overline{\theta}) \cdot |\Delta a|\} + (1 - p)\left\{m_1 + \underline{a}\right\}$ which is increasing in $\delta_1(1|\overline{\theta})$. But then $\delta_1(1|\overline{\theta}) = \min\{d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}; 1-(1-H)d(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}\}$, where the upper bound comes from ($\overline{IR}_1$) and ($\overline{IC}_1$) substituting $U_A(\overline{\theta}; \phi_1)$ and $U_A(\underline{\theta}; \phi_1)$ and using (SR_2). If $\Delta b \leqslant |\Delta a|$, $\min\{d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|};\ 1 - (1 - H)d(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}\} = d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}$ and hence $U_A(\overline{\theta}; \phi_1^*) = 0$. If instead $\Delta b > |\Delta a|$, then U_1 is maximized at $d(s_2|\underline{\theta}) = \frac{|\Delta a|}{\Delta b}$ and $\delta_1(1|\overline{\theta}) = H$ and again $U_A(\overline{\theta}; \phi_1^*) = 0$.

Substituting $U_A(\overline{\theta}; \phi_1^*) = 0$ and $U_A(\underline{\theta}; \phi_1^*) = \delta_1(1|\overline{\theta})|\Delta a| - d(s_2|\overline{\theta})\Delta b$ into U_1, and using (SR_2), the program for ϕ_1^* reduces to

$$\mathcal{P}_1^{HD}: \begin{cases} \max\limits_{\phi_1\in\Phi_1} \; p\delta_1(1|\overline{\theta})\big(m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\big)+(1-p)\,\big(m_1+\underline{a}\big) \\ \qquad\qquad +d(s_2|\underline{\theta})H\Delta b \\ \text{s.t.} \\ \delta_1(1|\overline{\theta})\geqslant d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}, \qquad\qquad (\underline{\text{IR}}_1) \\ \delta_1(1|\overline{\theta})\leqslant 1-d(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}(1-H). \qquad (\overline{\text{IC}}_1) \end{cases}$$

Note that ($\underline{\text{IR}}_1$) and ($\overline{\text{IC}}_1$) can be jointly satisfied if and only if $d(s_2|\underline{\theta})\leqslant\frac{|\Delta a|}{\Delta b}$.

If $m_1+\overline{a}-\frac{1-p}{p}|\Delta a|<0$, ($\underline{\text{IR}}_1$) binds. Replacing $\delta_1^*(1|\overline{\theta})=d(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}$ into the objective function in $\mathcal{P}_1^{HD}$ gives

$$U_1=d(s_2|\underline{\theta})H\Delta b\Big[1+\frac{p}{|\Delta a|}\left(m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\right)\Big]+(1-p)\,\big(m_1+\underline{a}\big),$$

which is increasing in $d(s_2|\underline{\theta})$ and maximized by setting $d^*(s_2|\underline{\theta})=\min\big\{1,\ \frac{|\Delta a|}{\Delta b}\big\}$. The optimal mechanism involves information disclosure and is such that $d^*(s_2|\underline{\theta})=\min\big\{1,\ \frac{|\Delta a|}{\Delta b}\big\}$, $d^*(s_2|\overline{\theta})=Hd^*(s_2|\underline{\theta})$, $\delta_1^*(1|\underline{\theta})=1$, and $\delta_1^*(1|\overline{\theta})=d^*(s_2|\underline{\theta})H\frac{\Delta b}{|\Delta a|}$.

If instead $m_1+\overline{a}-\frac{1-p}{p}\,|\Delta a|\geqslant 0$, then ($\overline{\text{IC}}_1$) binds, in which case P_1's payoff reduces to

$$U_1=\Big\{H\Delta b-p\big(m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\big)\frac{\Delta b}{|\Delta a|}\,(1-H)\,\Big\}d(s_2|\underline{\theta})$$
$$+p\big(m_1+\overline{a}-\frac{1-p}{p}|\Delta a|\big)+(1-p)(m_1+\underline{a}).$$

If $m_1+\overline{a}-\frac{1-p}{p}|\Delta a|<\frac{H|\Delta a|}{p(1-H)}$, U_1 is again increasing in $d(s_2|\underline{\theta})$. The optimal mechanism then discloses information and is such that $d^*(s_2|\underline{\theta})=\min\big\{1,\ \frac{|\Delta a|}{\Delta b}\big\}$, $d^*(s_2|\overline{\theta})=Hd^*(s_2|\underline{\theta})$, $\delta_1^*(1|\underline{\theta})=1$ and $\delta_1^*(1|\overline{\theta})=1-d^*(s_2|\underline{\theta})\frac{\Delta b}{|\Delta a|}(1-H)$. If instead $m_1+\overline{a}-\frac{1-p}{p}\,|\Delta a|\geqslant\frac{H|\Delta a|}{p(1-H)}$, then U_1 is decreasing in $d(s_2|\underline{\theta})$ and at the optimum $d^*(s_2|\underline{\theta})=d^*(s_2|\overline{\theta})=0$ and $\delta_1^*(1|\underline{\theta})=\delta_1^*(1|\overline{\theta})=1$.

We conclude that with unfavorable beliefs, disclosure is optimal if and only if $m_1+\overline{a}-\frac{1-p}{p}\,|\Delta a|<\frac{H|\Delta a|}{p(1-H)}$. □

Proof of Corollary 2. That P_2 is not affected by disclosure follows from the same arguments as in the proof of Corollary 1.

Consider the effect of disclosure on U_A. As shown in the proof of Proposition 2, $U_A(\overline{\theta}; \phi_1^*) = 0$ and $U_A(\underline{\theta}; \phi_1^*) = \delta_1^*(1|\overline{\theta})|\Delta a| - d^*(s_2|\overline{\theta})\Delta b$. In contrast, without disclosure, $U_A(\overline{\theta}; \phi_1^{\mathrm{ND}}) = 0$ and $U_A(\underline{\theta}; \phi_1^{\mathrm{ND}}) = |\Delta a|$ if $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| \geqslant 0$ and $U_A(\underline{\theta}; \phi_1^{\mathrm{ND}}) = 0$ otherwise.

When $m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < 0$, $U_A(\underline{\theta}; \phi_1^*) = 0$ and hence disclosure leads to a Pareto improvement (P_1 is strictly better off, A and P_2 are indifferent).

When instead $0 \leqslant m_1 + \overline{a} - \frac{1-p}{p}|\Delta a| < \frac{H|\Delta a|}{p(1-H)}$, $\delta_1^*(1|\overline{\theta}) = 1 - (1-H)\frac{\Delta b}{|\Delta a|}\min\left\{1, \frac{|\Delta a|}{\Delta b}\right\}$, $d^*(s_2|\overline{\theta}) = H\min\left\{1, \frac{|\Delta a|}{\Delta b}\right\}$ and $U_A(\underline{\theta}; \phi_1^*) = |\Delta a| - \Delta b \min\left\{1, \frac{|\Delta a|}{\Delta b}\right\}$, implying that A is strictly worse off. As for the effect of disclosure on total welfare,

$$U_1(\phi_1^*) - U_1(\phi_1^{\mathrm{ND}}) = \left\{H\Delta b - p\big(m_1 + \overline{a} - \frac{1-p}{p}|\Delta a|\big)\frac{\Delta b}{|\Delta a|}(1-H)\right\} \times \min\left\{1, \frac{|\Delta a|}{\Delta b}\right\}$$

and hence

$$\begin{aligned} W(\phi_1^*) - W(\phi_1^{\mathrm{ND}}) &= U_1(\phi_1^*) - U_1(\phi_1^{\mathrm{ND}}) + U_A(\underline{\theta}; \phi_1^*) - U_A(\underline{\theta}; \phi_1^{\mathrm{ND}}) \\ &= -\left\{(1-H)\Delta b + p\big(m_1 + \overline{a} - \frac{1-p}{p}|\Delta a|\big)\frac{\Delta b}{|\Delta a|}(1-H)\right\} \\ &\quad \times \min\left\{1, \frac{|\Delta a|}{\Delta b}\right\} < 0. \qquad \square \end{aligned}$$

Proof of Proposition 3. The optimal mechanism maximizes

$$\begin{aligned} U_1 = {} & p\left[\delta_1(1, s_1|\overline{\theta}) + \delta_1(1, s_2|\overline{\theta})\right](m_1 + \bar{a}) + (1-p)[\delta_1(1, s_1|\underline{\theta}) \\ & + \delta_1(1, s_2|\underline{\theta})]\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) \\ & + \left[p\delta_1(1, s_2|\overline{\theta}) + (1-p)\delta_1(1, s_2|\underline{\theta})\right]g \end{aligned}$$

subject to

$$\begin{aligned} & \delta_1(1, s_1|\overline{\theta}) + \delta_1(1, s_2|\overline{\theta}) \geqslant \delta_1(1, s_1|\underline{\theta}) + \delta_1(1, s_2|\underline{\theta}), \quad (\underline{\mathrm{IC}}_1) \\ & g\left[p\delta_1(1, s_1|\overline{\theta}) + (1-p)\,\delta_1(1, s_1|\underline{\theta})\right] \\ & \quad \geqslant (m_2 + b)\left[p\delta_1(0, s_1|\overline{\theta}) + (1-p)\,\delta_1(0, s_1|\underline{\theta})\right], \quad (\mathrm{SR}_1) \\ & g[p\delta_1(1, s_2|\overline{\theta}) + (1-p)\,\delta_1(1, s_2|\underline{\theta})] \\ & \quad \leqslant (m_2 + b)\left[p\delta_1(0, s_2|\overline{\theta}) + (1-p)\,\delta_1(0, s_2|\underline{\theta})\right]. \quad (\mathrm{SR}_2) \end{aligned}$$

At the optimum, (SR_1) never binds and $\delta_1^*(0, s_1|\theta) = 0$ for any θ. Indeed, reducing $\delta_1(0, s_1|\theta)$ and increasing $\delta_1(0, s_2|\theta)$ relaxes (SR_1) and (SR_2) without affecting ($\underline{\mathrm{IC}}_1$) and U_1. Constraint ($\underline{\mathrm{IC}}_1$) can also be ignored, since it is always satisfied at the optimum.

Next, note that the maximal surplus that P_1 can appropriate from P_2 by reducing the level of trade upstream and disclosing signal s_2 instead of s_1 is bounded from above by the right-hand side in (SR_2). On the other hand, the cost of creating a downstream rent is the surplus that P_1 forgoes when she does not trade, i.e.

$$p\delta_1(0, s_2|\overline{\theta})\,(m_1 + \bar{a}) + (1-p)\,\delta_1(0, s_2|\underline{\theta})\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right).$$

When $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant m_2 + b$,

$$p\delta_1(0, s_2|\overline{\theta})\,(m_1 + \bar{a}) + (1-p)\,\delta_1(0, s_2|\underline{\theta})\left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right)$$
$$> g[p\delta_1(1, s_2|\overline{\theta}) + (1-p)\,\delta_1(1, s_2|\underline{\theta})]$$

and hence the optimal mechanism is simply $\delta_1^*(1, s_1|\theta) = 1$ for any θ and does not require disclosure.

On the contrary, when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b$, at the optimum, $\delta_1^*(1, s_1|\underline{\theta}) = 0$. If this were not true, P_1 could transfer an ε probability from $\delta_1(1, s_1|\underline{\theta})$ to $\delta_1(0, s_2|\underline{\theta})$ and then increase $\delta_1(1, s_2|\underline{\theta})$ by $\frac{\varepsilon(m_2+b)}{g}$ reducing $\delta_1(1, s_1|\underline{\theta})$ by the same amount. This would increase her payoff, without affecting (SR_2). Hence $\delta_1^*(1, s_2|\underline{\theta}) = 1 - \delta_1^*(0, s_2|\underline{\theta})$. Furthermore, if $\delta_1^*(1, s_2|\underline{\theta}) > 0$, then necessarily $\delta_1^*(1, s_2|\overline{\theta}) = 1$. To see this, first suppose that $\delta_1^*(0, s_2|\overline{\theta}) > 0$. Since $m_1 + \bar{a} > m_1 + \underline{a} - \frac{p}{1-p}\Delta a$, P_1 could then transfer an ε probability from $\delta_1^*(0, s_2|\overline{\theta})$ to $\delta_1^*(1, s_2|\overline{\theta})$ and a $\frac{p}{1-p}\varepsilon$ probability from $\delta_1^*(1, s_2|\underline{\theta})$ to $\delta_1^*(0, s_2|\underline{\theta})$ increasing U_1 without any effect on (SR_2). Hence, if $\delta_1^*(1, s_2|\underline{\theta}) > 0$, then necessarily $\delta_1^*(0, s_2|\overline{\theta}) = 0$. Next, suppose that $\delta_1^*(1, s_1|\overline{\theta}) > 0$. P_1 could then transfer an ε probability from $\delta_1^*(1, s_2|\underline{\theta})$ to $\delta_1^*(0, s_2|\underline{\theta})$ and then reduce $\delta_1^*(1, s_1|\overline{\theta})$ by $\frac{1-p}{p}\varepsilon(1 + \frac{m_2+b}{g})$ and increase $\delta_1^*(1, s_2|\overline{\theta})$ by the same amount. Once again, since $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b$, this would increase U_1, without affecting (SR_2). We conclude that if $\delta_1^*(1, s_2|\underline{\theta}) > 0$, then necessarily $\delta_1^*(1, s_2|\overline{\theta}) = 1$.

First, consider the case in which $-g < m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b$. Since $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + g > 0$, U_1 is increasing in $\delta_1(1, s_2|\underline{\theta})$ and hence (SR_2) binds at the optimum. When $gp \leqslant (m_2 + b)\,(1-p)$, the optimal mechanism is $\delta_1^*(1, s_2|\overline{\theta}) = 1$, $\delta_1^*(1, s_2|\underline{\theta}) = \frac{(1-p)(m_2+b)-pg}{(1-p)[m_2+b+g]}$ and $\delta_1^*(0, s_2|\underline{\theta}) = 1 - \delta_1^*(1, s_2|\underline{\theta})$. On the contrary, when $pg > (1-p)(m_2 + b)$, necessarily $\delta_1^*(0, s_2|\underline{\theta}) = 1$ and $\delta_1^*(1, s_2|\overline{\theta}) \in (0, 1)$. The optimal mechanism then depends on the comparison between $m_1 + \bar{a}$ and $m_2 + b$. If $m_1 + \bar{a} > m_2 + b$, then $\delta_1^*(0, s_2|\overline{\theta}) = 0$. To see this, note that by reducing $\delta_1^*(0, s_2|\overline{\theta})$ and $\delta_1^*(1, s_2|\overline{\theta})$ respectively by ε and $\frac{\varepsilon(m_2+b)}{g}$ and increasing $\delta_1^*(1, s_1|\overline{\theta})$ by $\varepsilon\left[\frac{(m_2+b)}{g} + 1\right]$, P_1 increases U_1 without any effect on (SR_2). It follows that for $m_1 + \bar{a} > m_2 + b$, $\delta_1^*(1, s_2|\overline{\theta}) = \frac{(1-p)(m_2+b)}{pg} = 1 - \delta_1^*(1, s_1|\overline{\theta})$, whereas for $m_1 + \bar{a} \leqslant m_2 + b$, $\delta_1^*(1, s_1|\overline{\theta}) = 0$ and $\delta_1^*(1, s_2|\overline{\theta}) = \frac{m_2+b}{p[m_2+b+g]} = 1 - \delta_1^*(0, s_2|\overline{\theta})$.

Finally, consider $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \leqslant -g$. In this case, $\delta_1^*(0, s_2|\underline{\theta}) = 1$ is always optimal since U_1 is decreasing in $\delta_1(1, s_2|\underline{\theta})$. Following the same steps as in the previous case, when $pg \leqslant (1-p)(m_2+b)$, $\delta_1^*(1, s_2|\bar{\theta}) = 1$. When instead $pg > (1-p)(m_2+b)$,

$$\delta_1^*(1, s_2|\bar{\theta}) = \begin{cases} \frac{(1-p)(m_2+b)}{pg} = 1 - \delta_1^*(1, s_1|\bar{\theta}) & \text{if } m_1 + \bar{a} > m_2 + b, \\ \frac{m_2+b}{p[m_2+b+g]} = 1 - \delta_1^*(0, s_2|\bar{\theta}) & \text{otherwise.} \end{cases}$$

We conclude that disclosure is optimal if and only if (i) $g > [(1-p)(m_2+b)]/p$, i.e. when the complementarity is sufficiently strong that excluding the low type is not sufficient to induce P_2 to ask a low price and (ii) $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b < m_1 + \bar{a}$, that is, when the cost of reducing trade with the high type is higher than the benefit of increasing the downstream rent, whereas the opposite is true with the low type. □

Proof of Corollary 3. Step 1 derives the optimal mechanism ϕ_1^{ND} when P_1 is not allowed to disclose information and (i) $g > [(1-p)(m_2+b)]/p$ and (ii) $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < m_2 + b < m_1 + \bar{a}$, in which case disclosure would have been optimal for P_1. Step 2 compares payoffs in this mechanism with those in the optimal mechanism derived in the proof of Proposition 3.

Step 1: Among all mechanisms that induce P_2 to set a high price $t_2 = b + g$, the one that maximizes U_1 is $\delta_1(1|\bar{\theta}) = \delta_1(1|\underline{\theta}) = 1$ if $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$, and $\delta_1(1|\bar{\theta}) = 1$ and $\delta_1(1|\underline{\theta}) = 0$ otherwise, yielding a payoff $U_1^{b+g} = \max\{m_1 + \underline{a};\, p(m_1 + \bar{a})\}$. In contrast, among all mechanisms that induce P_2 to set a low price $t_2 = b$, the one that maximizes U_1 solves

$$\mathcal{P}_1^{\mathrm{ND}}: \begin{cases} \max \quad p\delta_1(1|\bar{\theta})(m_1 + \bar{a} + g) + (1-p)\,\delta_1(1|\underline{\theta}) \\ \qquad \times \left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a + g\right) \\ \text{s.t.} \\ \delta_1(1|\bar{\theta}) \geqslant \delta_1(1|\underline{\theta}), & (\underline{\mathrm{IC}}_1) \\ g\left[p\delta_1(1|\bar{\theta}) + (1-p)\,\delta_1(1|\underline{\theta})\right] \\ \quad \leqslant (m_2+b)\left[p\left(1 - \delta_1(1|\bar{\theta})\right) + (1-p)\left(1 - \delta_1(1|\underline{\theta})\right)\right]. & (\mathrm{SR}) \end{cases}$$

Following the same arguments as in the proof of Proposition (3), under (i) and (ii), the solution to $\mathcal{P}_1^{\mathrm{ND}}$ is $\delta_1(1|\bar{\theta}) = \frac{m_2+b}{p(m_2+b+g)}$ and $\delta_1(1|\underline{\theta}) = 0$ and yields a payoff $U_1^b = \frac{(m_2+b)(m_1+\bar{a}+g)}{m_2+b+g}$.

The optimal contract ϕ_1^{ND} is obtained comparing U_1^{b+g} with U_1^b. When $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$, $U_1^{b+g} \geqslant U_1^b$ if and only if $g \geqslant \Delta a(m_2+b)/\left[m_1 + \underline{a} - m_2 - b\right]$, whereas for $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < 0$, $U_1^{b+g} \geqslant U_1^b$ if and only if $g \geqslant \frac{(1-p)(m_2+b)(m_1+\bar{a})}{p(m_1+\bar{a}) - m_2 - b}$.

Step 2: Since $U_A(\bar{\theta}; \phi_1^{\mathrm{ND}}) = \delta_1(1|\underline{\theta})\Delta a$, $U_A(\bar{\theta}; \phi_1^*) = [\delta_1(1, s_1|\underline{\theta}) + \delta_1(1, s_2|\underline{\theta})]\Delta a$ and $U_A(\underline{\theta}; \phi_1^{\mathrm{ND}}) = U_A(\underline{\theta}; \phi_1^*) = 0$, disclosure damages A if and only if it reduces the upstream level of trade with the low type. From Step 1, this occurs when $m_1 + \underline{a} - \frac{p}{1-p}\Delta a \geqslant 0$ and $g \geqslant \Delta a(m_2+b)/[m_1 + \underline{a} - m_2 - b]$. In this case, disclosure also harms P_2 since it decreases the value $\underline{\theta}$ attaches to downstream contracting. Furthermore, since it is efficient to trade in both relationships, disclosure is welfare-decreasing. In all other cases, disclosure yields a Pareto improvement, since it does not affect trade with $\underline{\theta}$ and it either increases trade with $\bar{\theta}$, or else it leaves it unchanged. □

Proof of Proposition 4. The optimal contracts maximize

$$U_1 = p\big[\delta_1(1, s_1|\overline{\theta}) + \delta_1(1, s_2|\overline{\theta})\big](m_1 + \bar{a}) + (1-p)\big[\delta_1(1, s_1|\underline{\theta}) + \delta_1(1, s_2|\underline{\theta})\big] \times \left(m_1 + \underline{a} - \frac{p}{1-p}\Delta a\right) + \left[p\delta_1(0, s_2|\overline{\theta}) + (1-p)\delta_1(0, s_2|\underline{\theta})\right]|g|$$

subject to

$$\delta_1(1, s_1|\overline{\theta}) + \delta_1(1, s_2|\overline{\theta}) \geqslant \delta_1(1, s_1|\underline{\theta}) + \delta_1(1, s_2|\underline{\theta}), \qquad (\underline{\mathrm{IC}}_1)$$

$$|g|\left[p\delta_1(0, s_1|\overline{\theta}) + (1-p)\,\delta_1(0, s_1|\underline{\theta})\right] \geqslant (m_2 + b + g)\left[p\delta_1(1, s_1|\overline{\theta}) + (1-p)\,\delta_1(1, s_1|\underline{\theta})\right], \quad (\mathrm{SR}_1)$$

$$|g|\left[p\delta_1(0, s_2|\overline{\theta}) + (1-p)\,\delta_1(0, s_2|\underline{\theta})\right] \leqslant (m_2 + b + g)\left[p\delta_1(1, s_2|\overline{\theta}) + (1-p)\,\delta_1(1, s_2|\underline{\theta})\right]. \quad (\mathrm{SR}_2)$$

At the optimum, $\delta_1^*(1, s_1|\theta) = 0$ for any θ. Indeed, by reducing $\delta_1^*(1, s_1|\theta)$ and increasing $\delta_1^*(1, s_2|\theta)$, P_1 relaxes (SR_1) and (SR_2) with no effect on ($\underline{\mathrm{IC}}_1$) and U_1. It follows that constraint (SR_1) can be neglected. Constraint ($\underline{\mathrm{IC}}_1$) will also be ignored since it never binds. Also note that $\delta_1^*(0, s_1|\overline{\theta}) = 0$, since otherwise P_1 could reduce $\delta_1^*(0, s_1|\overline{\theta})$ and increase $\delta_1^*(1, s_2|\overline{\theta})$ relaxing (SR_2) and increasing U_1.

If $|g| \leqslant m_1 + \underline{a} - \frac{p}{1-p}\Delta a$, the optimal mechanism is simply $\delta_1^*(1, s_2|\overline{\theta}) = \delta_1^*(1, s_2|\underline{\theta}) = 1$.

If, instead, $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < |g| < m_1 + \bar{a}$, the unconstrained solution is $\delta_1^*(1, s_2|\overline{\theta}) = \delta_1^*(0, s_2|\underline{\theta}) = 1$ and satisfies (SR_2) if and only if $|g| \leqslant p(m_2+b)$. If, however, $p(m_2+b) < |g| \leqslant m_2 + b$, then ($\mathrm{SR}_2$) binds and $\delta_1^*(0, s_2|\underline{\theta}) < 1$. The optimal mechanism then depends on the sign of $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g$. When it is positive, $\delta_1^*(0, s_1|\underline{\theta}) = 0$; indeed, reducing $\delta_1^*(0, s_1|\underline{\theta})$ by $(1 + \frac{m_2+b+g}{|g|})\varepsilon$ and increasing $\delta_1^*(1, s_2|\underline{\theta})$ and $\delta_1^*(0, s_2|\underline{\theta})$, respectively, by ε and $\frac{m_2+b+g}{|g|}\varepsilon$ increases U_1 without any effect on (SR_2). The optimal mechanism is then $\delta_1^*(1, s_2|\overline{\theta}) = 1$, $\delta_1^*(1, s_2|\underline{\theta}) = \frac{|g| - p(m_2+b)}{(1-p)(m_2+b)}$ and $\delta_1^*(0, s_2|\underline{\theta}) = 1 - \delta_1^*(1, s_2|\underline{\theta})$. When instead $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g < 0$, by the same argument, $\delta_1^*(1, s_2|\underline{\theta}) = 0$, in which case the optimal mechanism is $\delta_1^*(1, s_2|\overline{\theta}) = 1$, $\delta_1^*(0, s_2|\underline{\theta}) = \frac{p(m_2+b+g)}{(1-p)|g|}$ and $\delta_1^*(0, s_1|\underline{\theta}) = 1 - \delta_1^*(0, s_2|\underline{\theta})$.

Finally, if $|g| > m_1 + \bar{a}$, then (SR_2) always binds, since the unconstrained solution is $\delta_1^*(0, s_2|\overline{\theta}) = \delta_1^*(0, s_2|\underline{\theta}) = 1$. If $\delta_1^*(0, s_2|\overline{\theta}) > 0$, then necessarily $\delta_1^*(0, s_2|\underline{\theta}) = 1$. Otherwise, P_1 could transfer an ε probability from $\delta_1^*(0, s_2|\overline{\theta})$ to $\delta_1^*(1, s_2|\overline{\theta})$ and $\frac{p}{1-p}\varepsilon$ probability from either $\delta_1^*(1, s_2|\underline{\theta})$ or $\delta_1^*(0, s_1|\underline{\theta})$ to $\delta_1^*(0, s_2|\underline{\theta})$ increasing U_1 without violating (SR_2). It follows that for $|g| \leqslant p(m_2 + b)$, $\delta_1^*(0, s_2|\underline{\theta}) = 1$, $\delta_1^*(1, s_2|\overline{\theta}) = |g|/[p(m_2 + b)]$ and $\delta_1^*(0, s_2|\overline{\theta}) = 1 - \delta_1^*(1, s_2|\overline{\theta})$, whereas for $|g| > p(m_2 + b)$, $\delta_1^*(1, s_2|\overline{\theta}) = 1$, in which case the solution coincides with that for $m_1 + \underline{a} - \frac{p}{1-p}\Delta a < |g| < m_1 + \bar{a}$.

We conclude that disclosure is optimal if and only if (i) $p(m_2 + b) < |g| < m_2 + b$ and (ii) $m_1 + \underline{a} - \frac{p}{1-p}\Delta a + m_2 + b + g < 0$. □

Proof of Corollary 4. Step 1 derives the optimal mechanism when P_1 cannot disclose information and (i) $p(m_2+b) < |g| < m_2+b$ and (ii) $m_1+\underline{a}-\frac{p}{1-p}\Delta a+m_2+b+g<0$, in which case disclosure would have been optimal for P_1. Step 2 compares payoffs in this mechanism with those in the optimal mechanism derived in the proof of Proposition (4).

Step 1: When (ii) holds, necessarily $m_1+\underline{a}-\frac{p}{1-p}\Delta a<0$, since $m_2+b+g>0$. This implies that among all mechanisms that induce P_2 to ask a high price, the one that maximizes U_1 is $\delta_1(1|\overline{\theta})=1$ and $\delta_1(0|\underline{\theta})=1$ and yields a payoff $U_1^b=p(m_1+\overline{a})$. In contrast, among all mechanisms that induce P_2 to ask a low price, the one that maximizes U_1 solves

$$\mathcal{P}_1^{\mathrm{ND}}:\begin{cases}\max\ \ p\left\{\delta_1(1|\overline{\theta})\,(m_1+\bar{a})+\delta_1(0|\overline{\theta})\,|g|\right\}+(1-p)\left\{\delta_1(1|\underline{\theta})\right.\\ \qquad\left.\times\left(m_1+\underline{a}-\dfrac{p}{1-p}\Delta a\right)+\delta_1(0|\underline{\theta})\,|g|\right\}\\ \text{s.t.}\\ \delta_1(1|\overline{\theta})\geqslant\delta_1(1|\underline{\theta}), \qquad\qquad (\underline{\mathrm{IC}})\\ |g|\left[p\delta_1(0|\overline{\theta})+(1-p)\,\delta_1(0|\underline{\theta})\right]\\ \qquad\leqslant(m_2+b+g)\left[p\delta_1(1|\overline{\theta})+(1-p)\,\delta_1(1|\underline{\theta})\right]. \quad (\mathrm{SR})\end{cases}$$

Using $\delta_1(0|\theta)=1-\delta_1(1|\theta)$, constraint (SR) reduces to $\delta_1(1|\underline{\theta})\geqslant\frac{|g|}{(1-p)(m_2+b)}-\frac{p}{1-p}\delta_1(1|\overline{\theta})$, which clearly binds since $m_1+\underline{a}-\frac{p}{1-p}\Delta a<0$. Substituting $\delta_1(1|\underline{\theta})$ into the objective function, we have that U_1 is increasing in $\delta_1(1|\overline{\theta})$ and hence the solution to $\mathcal{P}_1^{\mathrm{ND}}$ is $\delta_1(1|\overline{\theta})=1$ and $\delta_1(1|\underline{\theta})=\frac{|g|-p(m_2+b)}{(1-p)(m_2+b)}$. Comparing the payoff for P_1 in this mechanism with the payoff in the mechanism that induces a high downstream price, we have that the optimal mechanism is

$$\delta_1(1|\overline{\theta})=1,$$
$$\delta_1(1|\underline{\theta})=\begin{cases}\frac{|g|-p(m_2+b)}{(1-p)(m_2+b)} & \text{if } m_1+\underline{a}-\frac{p}{1-p}\Delta a\leqslant\frac{|g|(m_2+b+g)}{p(m_2+b)-|g|},\\ 0 & \text{otherwise.}\end{cases}$$

Step 2: If $m_1+\underline{a}-\frac{p}{1-p}\Delta a>\frac{|g|(m_2+b+g)}{p(m_2+b)-|g|}$, disclosure leads to a Pareto improvement: A and P_2 are indifferent, P_1 is strictly better off. If instead $m_1+\underline{a}-\frac{p}{1-p}\Delta a\leqslant\frac{|g|(m_2+b+g)}{p(m_2+b)-|g|}$, disclosure reduces the level of trade upstream and leaves it unchanged downstream: A is worse off, P_1 and P_2 better off. Disclosure is welfare-increasing if and only if it is inefficient to sell to $\underline{\theta}$ upstream, i.e. if and only if $|g|\geqslant m_1+\underline{a}$. □

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