Robust Procurement Design

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Motivation

- Procurement/regulation

  central to economics

  (a) public

  (b) private

- Difficulty in designing contracts

  - provider typically **privately informed** about cost

  - demand/value may not be known at contracting stage

Motivation

- Standard approaches
  - **Subjective Expected Utility (SEU)**
    - Buyer has conjecture/belief over

  (a) supplier's cost

  (b) demand/value for output

    - maximizes under conjecture

  - **Worst-case optimality**
    - buyer has no conjecture
    - maximizes under **worst-case scenario**

Motivation

- **Alternative robust approach**
  - buyer has conjecture/model but does not fully trust it
  - prepares for the worst (in case conjecture/model is wrong)
  - uses conjecture/model to select optimal mechanism among worst-case optimal ones
  - Lexicographic approach

    (a) political/hierarchical organizational constraints

    (b) attitude towards ambiguity

This Paper

- Uncertainty only over cost:
  - Baron-Myerson with **quantity floor**
- Uncertainty over both cost and demand
  - **upward quantity adjustment for high cost**
  - **downward quantity adjustment for intermediate costs**
- More general forms of cost uncertainty
  - **bridge between two different BM-schedules**
- Variations in uncertainty
  - cost uncertainty
  - demand uncertainty
- **Comparison of price vs quantity regulation**

Literature

- **Procurement under subjective expected utility**

  Baron-Myerson, (1982), Lewis and Sappington (1988), Armstrong, (1999)...

- **Alternative robustness criteria**
  - Garrett (2014)**: worst-case optimality**
  - Bergemann-Heumann-Morris (2024)**: competitive ratio**
  - Guo-Shmaya (2024)**: min-max regret**

- **Undominated mechanism design**
  - Mishra-Patil (2024)
  - Borgers-Li-Wang (2024)

- **Robust information design**
  - Dworczak-Pavan (2023)

- **Model mis-specification**
  - Cerreia-Vioglio-Hansen-Maccheroni-Marinacci (2022)

Plan

1. Introduction
2. **Model**
3. Baron-Myerson
4. Short List: Worst-Case Optimality
5. Robustly Optimal Quantity Mechanisms
6. More General Forms of Cost Uncertainty
7. Changes in Uncertainty
8. Price vs Quantity Regulation
9. Conclusions

Model

Model

- **Players**
  - Buyer (government/principal)
  - Seller (monopolist/agent)
- **Choices**
  - output $q \geq 0$
  - transfer $t \geq 0$

Model

- **Payoffs**
  - Social value of q:
    $$V^{\star}(\mathrm{q}) = \int_0^{\mathrm{q}} P^{\star}(s)ds$$
  - Cost: $\theta \mathrm{q}$

    θ drawn from ab. cont. $F^{\star}$ with $f^{\star}(\theta) > 0$ over $\Theta = [\underline{\theta}, \bar{\theta}]$
  - Ex-post welfare ($\alpha \in [0, 1]$)
    $$V^{\star}(\mathrm{q}) - \mathrm{t} + \alpha\,(\mathrm{t} - \theta \mathrm{q})$$
- **Asymmetric information**
  - θ: monopolist's private information

Model

- **Uncertainty/Model Mis-specification**
  - government does not fully trust conjecture/model $(V^\star, F^\star)$
  - concerned true demand and cost distribution may be $(V, F) \neq (V^\star, F^\star)$

Model

- **Admissible sets**
  - $\mathcal{V}$: set of possible consumer (gross) value fns
    - each $V \in \mathcal{V}$ strictly increasing, strictly concave, differentiable
  - $\mathcal{P}$: set of corresponding *inverse demand functions*
  - $P^\star \in \mathcal{P}$ and $V^\star \in \mathcal{V}$
  - lowest (inverse) demand function: $\underline{P}$
    - any $\mathrm{q} \geq 0$ and $P \in \mathcal{P}$
      $$P(\mathrm{q}) \geq \underline{P}(\mathrm{q})$$
    - $\underline{P}$: strictly decreasing, continuous, and s.t.
      $$\lim_{\mathrm{q} \to 0^+} \underline{P}(\mathrm{q}) > \overline{\theta}$$
  - $\mathcal{F} = \mathrm{CDF}(\Theta)$: set of cdfs supported on $\Theta = [\underline{\theta}, \bar{\theta}]$

Model

- Two equivalent interpretations
  - **robustness to model mis-specification**
    - $(V^\star, F^\star)$: government's model
    - $\mathcal{V} \times \mathcal{F}$: set of plausible models
  - **ambiguity**
    - state: $(\theta, V) \in \Theta \times \mathcal{V}$
    - belief $\mu \in \Delta(\Theta \times \mathcal{V})$ with marginals $\rho \in \Delta(\mathcal{V})$ and $F^\star \in \mathrm{CDF}(\Theta)$
    - $V^\star(\mathrm{q}) = \int V(\mathrm{q})\rho(\mathrm{d}V)$

Model

- **(Direct) mechanism** $M = (q, t)$
  - quantity schedule $q : \Theta \to \mathbb{R}_+$
  - transfer schedule $t : \Theta \to \mathbb{R}$
- $M = (q, t)$ IC and IR iff, for all θ, θ',

$$t(\theta) - \theta q(\theta) \geq t(\theta') - \theta q(\theta')$$

with

$$t(\theta) - \theta q(\theta) \geq 0$$

- $\mathcal{M}$: set of **IC** and **IR mechanisms**

Model

- Given IC and IR mechanism $M = (q, t)$, **ex-ante welfare** under (V, F)

$$W(M; V, F) := \int [V(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)]\, F(\mathrm{d}\theta)$$

where

$$u(\theta) := t(\theta) - \theta q(\theta)$$

is "**rent**" to type θ

Model

Definition

Given any IC and IR $M = (q, t)$, **welfare guarantee**

$$G(M) := \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F)$$

- **Short-list:**

$$\mathcal{M}^{\mathrm{SL}} := \{M \in \mathcal{M} : G(M) \geq G(M') \; \forall \; M' \in \mathcal{M}\}$$

(set of IC and IR mechanisms for which guarantee is maximal)

Model

- **Government's problem**
  - choose mechanism from $\mathcal{M}^{\mathrm{SL}}$ maximizing welfare under conjecture $(V^\star, F^\star)$

Definition

Mechanism $M \in \mathcal{M}^{\mathrm{SL}}$ **robustly optimal** iff, for every $M' \in \mathcal{M}^{\mathrm{SL}}$,

$$W(M; V^\star, F^\star) \geq W(M'; V^\star, F^\star)$$

- Robustly optimal mechanisms maximize ex-ante welfare under conjecture $(V^\star, F^\star)$ over all worst-case-optimal mechanisms

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Baron Myerson

Baron Myerson

Lemma

Mechanism $M = (q, t)$ IC and IR iff

1. q non-increasing
2. for all $\theta \in \Theta$,

$$u(\theta) = u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(z) dz$$

3. $u(\overline{\theta}) \geq 0$

Baron Myerson

Definition

F regular if abs continuous over $\mathbb{R}$ with density $f(\theta) > 0$ all $\theta \in \Theta$ and s.t. "virtual cost"

$$z(\theta) := \theta + (1 - \alpha)\frac{F(\theta)}{f(\theta)}$$

continuous and increasing over Θ.

Baron Myerson

Proposition

Assume $F^{\star}$ regular. SEU-optimal mechanism $M^{BM} = (q^{BM}, t^{BM})$ s.t., for all θ

$$q^{BM}(\theta) := P^{\star-1}(z^{\star}(\theta))$$

$$u^{BM}(\theta) := t^{BM}(\theta) - \theta q^{MB}(\theta) = \int_{\theta}^{\bar{\theta}} q^{BM}(z)dz$$

(BM-proof)

Baron Myerson

- FB-efficiency (under conjecture $(V^\star, F^\star)$):

$$q^{FB}(\theta) = P^{\star -1}(\theta)$$

- BM schedule (second-best efficiency)

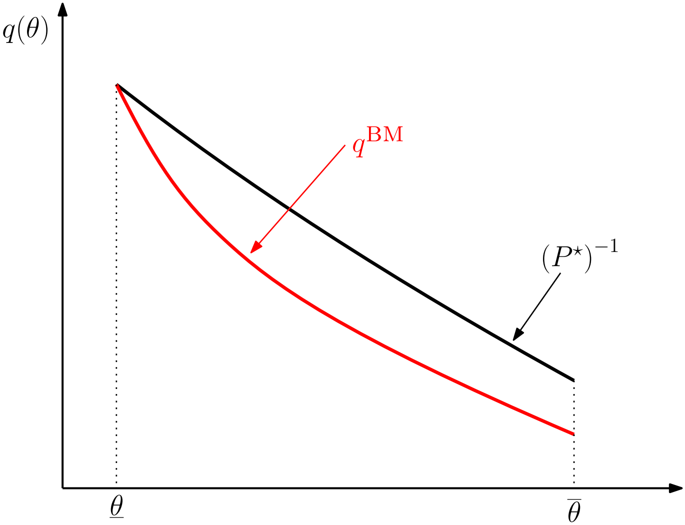
$$q^{BM}(\theta) = P^{\star -1}(z^\star(\theta)) :$$

  where

$$z^\star(\theta) := \theta + (1-\alpha)\frac{F^\star(\theta)}{f^\star(\theta)}$$

- Hence,
  - **no distortion "at top"**, i.e., for most efficient type, $\underline{\theta}$
  - **downward distortions for all** $\theta > \underline{\theta}$

Baron Myerson



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Short List

Worst-case optimality

Definition

Mechanism $M = (q, t)$ **worst-case optimal** iff, for any IC and IR $M' = (q', t') \in \mathcal{M}$,

$$G(M') := \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F) \leq \inf_{V \in \mathcal{V}, F \in \mathcal{F}} W(M; V, F) := G(M)$$

Worst-case optimality

- Let

$$\mathrm{q}_\ell := \arg\max_{\mathrm{q}} \left\{ \underline{V}(\mathrm{q}) - \bar{\theta}\mathrm{q} \right\}$$

(efficient output when $V = \underline{V}$ and $\theta = \bar{\theta}$)

Lemma

For any IC and IR mechanism $M = (q, t) \in \mathcal{M}$

$$G(M) = \inf_{\theta \in \Theta} \{ \underline{V}(q(\theta)) - \theta q(\theta) - (1 - \alpha) u(\theta) \}$$

with

$$u(\theta) := t(\theta) - \theta q(\theta)$$

Furthermore,

$$G(M) \leq G^* := \underline{V}(\mathrm{q}_\ell) - \bar{\theta}\mathrm{q}_\ell.$$

(Max-guarantee-proof)

Short List: Characterization

- Let

$$\mathcal{M}^{\mathrm{SL}} := \{M \in \mathcal{M} : G(M) \geq G(M') \ \forall \ M' \in \mathcal{M}\}$$

Lemma

$M = (q, t) \in \mathcal{M}^{\mathrm{SL}}$ iff (a) q non-increasing, (b) for any θ

$$u(\theta) := t(\theta) - \theta q(\theta) = \int_{\theta}^{\bar{\theta}} q(y)dy$$

and (c) for any θ

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q(y)dy \geq G^*$$

(Short List-proof)

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Robust Optimality

Robust Optimality: Full Program

- Recall def of **virtual cost** under conjecture $(V^\star, F^\star)$:

$$z^\star(\theta) := \theta + (1-\alpha)\frac{F^\star(\theta)}{f^\star(\theta)}$$

- Robustly optimal schedule q^{OPT} solves

$$\max_q \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta)q(\theta)\Big] F^\star(\mathrm{d}\theta)$$

$$\text{s.t.}$$

$$q \quad \text{non-increasing}$$

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q(y)dy \geq G^* \qquad \forall\ \theta \in \Theta$$

Robust Optimality: Relaxed Program

- Full program

$$\max_q \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \Big] F^\star(\mathrm{d}\theta)$$

$$\text{s.t.}$$

$$q \quad \text{non-increasing}$$

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha) \int_{\theta}^{\bar{\theta}} q(y) dy \geq G^* \qquad \forall\ \theta \in \Theta$$

- **Relaxed program**:

$$\max_q \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \Big] F^\star(\mathrm{d}\theta)$$

$$\text{s.t.}$$

$$q \quad \text{non-increasing}$$

$$q(\theta) \geq \mathrm{q}_\ell \qquad \forall\ \theta \in \Theta$$

$$q(\overline{\theta}) = \mathrm{q}_\ell$$

(Relaxation-proof)

Robust Optimality: **Baron-Myerson-with-quantity-floor**

- Let

$$q^{BM}(\theta) := \arg\max_{\mathrm{q}} \{V^\star(\mathrm{q}) - z^\star(\theta)\mathrm{q}\}$$

Definition

BM-with-qty-floor mechanism, $M^\star := (q^\star, t^\star)$, s.t.

$$q^\star(\theta) := \max\{q^{\mathrm{BM}}(\theta), \mathrm{q}_\ell\}$$

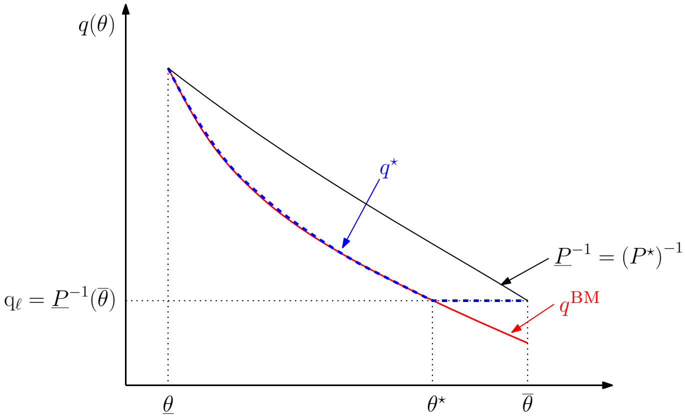
and

$$u^\star(\theta) := t^\star(\theta) - q^\star(\theta) = \int_{\theta}^{\bar{\theta}} q^\star(y)dy$$

Proposition

Suppose $F^\star$ regular and $V^\star = \underline{V}$. Then, BM-with-qty-floor robustly optimal

Robust Optimality: BM-with-qty-floor



Robust Optimality of BM-with-qty-floor: Proof

- When $V^\star = \underline{V}$ and $F^\star$ regular, $M^\star = (q^\star, u^\star)$ solves relaxed program
  - $q^\star(\theta)$ maximizes $V^\star(\mathrm{q}) - z^\star(\theta)\mathrm{q}$ under constraint $\mathrm{q} \geq \mathrm{q}_\ell$
  - $q^{\mathsf{BM}}(\overline{\theta}) < \mathrm{q}_\ell \Rightarrow q^\star(\bar{\theta}) = \mathrm{q}_\ell$
  - $z^\star$ non-decreasing $\Rightarrow M^\star$ IC and IR

- Ex-post welfare under $M^\star$ and $\underline{V}$

$$\underline{W}(\theta; q^\star) := \underline{V}(q^\star(\theta)) - \theta q^\star(\theta) - (1-\alpha)\int_\theta^{\overline{\theta}} q^\star(y)dy$$

  - $\underline{W}(\overline{\theta}; q^\star) = \underline{V}(\mathrm{q}_\ell) - \bar{\theta}\mathrm{q}_\ell := G^*$
  - If $\underline{W}(\cdot; q^\star)$ **non-increasing**, then $\underline{W}(\theta; q^\star) \geq G^*$ all θ

    $\Rightarrow M^\star = (q^\star, u^\star)$ **robustly optimal**

Monotonicity of $\underline{W}(\theta, q)$

- Recall

$$\underline{W}(\theta,q) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\overline{\theta}} q(y)dy$$

Lemma (Monotonicity)

Suppose $\underline{P}$ decreasing. For any non-increasing q and interval $I \subseteq \Theta$,

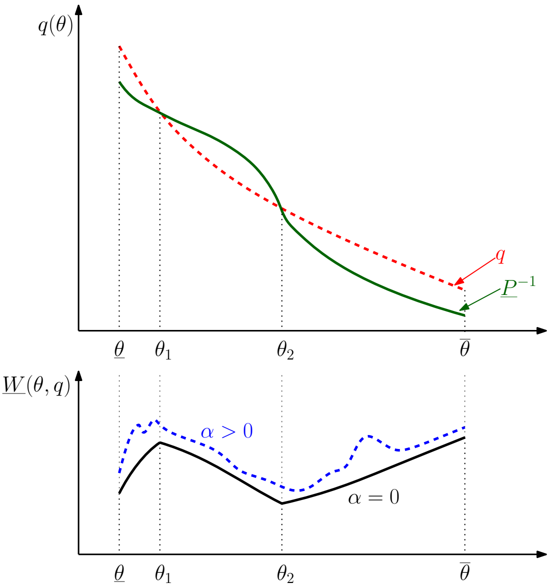
1. *If $0 < q(\theta) \leq \underline{P}^{-1}(\theta)$ for all $\theta \in I$, then $\underline{W}(\theta, q)$ non-increasing over I*

   (decreasing if $\alpha > 0$, or q decreasing and $q(\theta) < P^{-1}(\theta)$ all $\theta \in I$)

2. *If $q(\theta) > \underline{P}^{-1}(\theta)$ for all $\theta \in I$ and $\alpha = 0$, then $\underline{W}(\theta, q)$ non-decreasing over I*

   (increasing if q decreasing)

Monotonicity Lemma: Illustration



Robust Optimality of BM-with-qty-floor: Proof

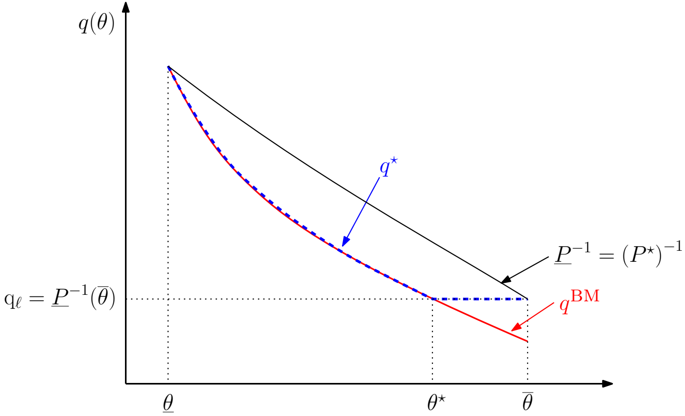
- Recall that

$$q^{\star}(\theta) := \max\{q^{\mathrm{BM}}(\theta), \mathrm{q}_{\ell}\}$$

- When $V^{\star} = \underline{V}$, for all $\theta \neq \underline{\theta}$,

$$q^{\star}(\theta) < \underline{P}^{-1}(\theta)$$

Robust Optimality of BM-with-qty-floor: Proof



Robust Optimality of BM-with-qty-floor: Proof

- Hence, when $V^\star = \underline{V}$, $\underline{W}(\cdot; q^\star)$ **non-increasing** in θ, with $\underline{W}(\bar{\theta}; q^\star) = G^*$

  $\Rightarrow \underline{W}(\theta; q^\star) \geq G^*$ all θ

  $\Rightarrow M^\star = (q^\star, u^\star)$ **robustly optimal**

Robust Optimality: BM-with-qty-floor

- When only uncertainty is over cost (more generally, $V^\star = \underline{V}$)
  - **efficiency at both bottom and top**
  - possibility cost being higher than conjectured

    $\rightarrow$ smaller distortions for high θ $\rightarrow$ more output from high θ
  - flat mechanism $q(\theta) = \mathrm{q}_\ell$ all θ
    - worst-case optimal
    - not robustly optimal

Robust Optimality: general case

- Suppose $F^\star$ regular and let

$$\theta^\star := \begin{cases} q^{\mathrm{BM}^{-1}}(\mathrm{q}_\ell) & \text{if} \quad q^{\mathrm{BM}}(\overline{\theta}) \le \mathrm{q}_\ell \\ \overline{\theta} & \text{if} \quad q^{\mathrm{BM}}(\overline{\theta}) > \mathrm{q}_\ell \end{cases}$$

and

$$\theta^m := \max\{\theta : \theta \in \arg\min_{\theta'} \underline{W}(\theta', q^\star)\}$$

($q^\star$ continuous and non-increasing $\Rightarrow$ θ^m well defined)

Proposition

Suppose $F^\star$ regular. Then
(1) BM-with-quantity-floor robustly optimal iff $\theta^m = \overline{\theta}$ and $q^{\mathrm{BM}}(\overline{\theta}) \le \mathrm{q}_\ell$

(2) If $\theta^m < \overline{\theta}$ or $\theta^m = \overline{\theta}$ and $q^{\mathrm{BM}}(\overline{\theta}) > \mathrm{q}_\ell$, then $\theta^m \le \theta^\star$ and

(a) $q^{\mathrm{OPT}}(\theta) = \mathrm{q}_\ell$ for all $\theta \in [\theta^\star, \overline{\theta}]$

(b) $q^{\mathrm{OPT}}(\theta) \le q^{\mathrm{BM}}(\theta)$ for almost all $\theta \le \theta^\star$

(inequality strict over positive-measure $I \subseteq [\underline{\theta}, \theta^\star]$)

Robust Optimality: general case

Proposition

Suppose $F^\star$ regular and $\alpha = 0$.

1. *Following conditions imply BM-with-qty-floor* **robustly optimal:**
   (a) $\underline{W}(\underline{\theta}, q^\star) \geq G^*$*;*
   (b) there exists $\hat{\theta} \in \Theta$ *s.t.* $q^\star(\theta) > \underline{P}^{-1}(\theta)$ *if* $\theta < \hat{\theta}$ *and* $q^\star(\theta) \leq \underline{P}^{-1}(\theta)$ *if* $\theta \geq \hat{\theta}$*.*

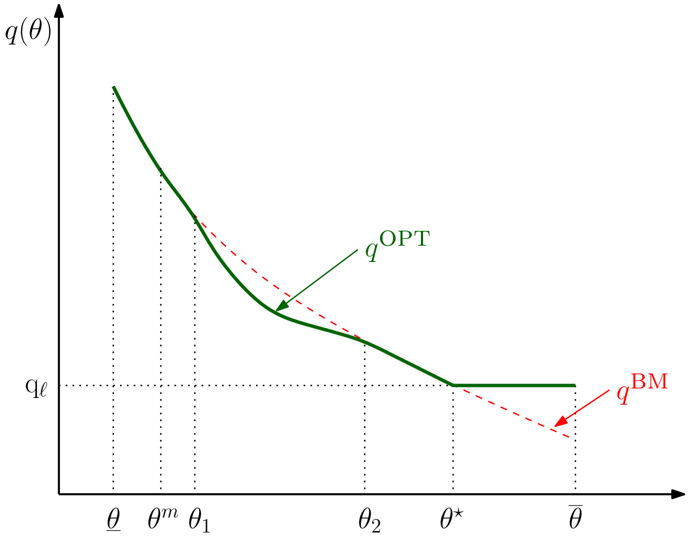
2. *If* $\theta^m \in (\underline{\theta}, \overline{\theta})$*, then* $q^{\mathrm{OPT}}(\theta) = q^{\mathrm{BM}}(\theta)$ *for almost all* $\theta \in (\underline{\theta}, \theta^m)$*.*

3. *Following conditions imply BM-with-qty-floor* **not robust***:*

   (a) $P^\star(q) - \underline{P}(q) > 1/f^\star(\overline{\theta})$ *for all* q*;*

   (b) $F^\star(\theta)/f^\star(\theta)$ *non-decreasing and continuous over* Θ*.*

Robust Optimality



Robust Optimality: general case

- Last two propositions leverage properties of

$$\underline{W}(\theta, q^\star) := \underline{V}(q^\star(\theta)) - \theta q^\star(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q^\star(y)dy$$

where

$$q^\star(\theta) := \max\{q^{\mathrm{BM}}(\theta), \mathrm{q}_\ell\}$$

together with **monotonicity lemma**

(General-case-proof)

Robust Optimality: general case

- **Upward quantity adjustments for high costs**
  - avoid inefficiencies motivated by rent extraction (for lower types)
- **Downward adjustments for intermediate costs**
  - avoid over-procurement in case demand lower than expected
- **Low costs**
  - welfare higher than G^* even when demand lower than conjectured
  - no adjustment necessary

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More General Cost Uncertainty

More General Cost Uncertainty

- Suppose there exist $\underline{F}, \overline{F} \in \text{CDF}(\Theta)$ s.t.

$$\mathcal{F} = \{F \in \text{CDF}(\Theta) : \underline{F}(\theta) \leq F(\theta) \leq \overline{F}(\theta), \ \forall \theta\}$$

  with $F^\star \in \mathcal{F}$

- So far, $\underline{F}(\theta) = \mathbb{I}(\theta \geq \bar{\theta})$ and $\bar{F}(\theta) = \mathbb{I}(\theta \geq \underline{\theta})$

More General Cost Uncertainty

- Let $\underline{M}^{\text{BM}} := (\underline{q}^{\text{BM}}, \underline{u}^{\text{BM}})$ be BM mechanism under $(\underline{F}, \underline{V})$.

Proposition

Suppose $\underline{F}$ regular. Then $M = (q, u) \in \mathcal{M}^{\text{SL}}$ only if $q(\theta) = \underline{q}^{\text{BM}}(\theta)$ almost all $\theta \in \Theta$.

- For any M, $G(M) \leq W(M : \underline{V}, \underline{F}) \leq W(\underline{M}^{\text{BM}} : \underline{V}, \underline{F})$
- Because $\underline{q}^{\text{BM}}(\theta) \leq \underline{P}^{-1}(\theta)$, $\underline{W}(\theta, \underline{M}^{\text{BM}})$ non-increasing in θ (monotonicity lemma)
- Hence, $W(\underline{M}^{\text{BM}} : \underline{V}, \underline{F}) \leq W(\underline{M}^{\text{BM}} : V, F)$, and

$$G^* = W(\underline{M}^{\text{BM}} : \underline{V}, \underline{F})$$

- Because

$$\underline{q}^{\text{BM}}(\theta) = \arg\max_{\mathrm{q}} \{\underline{V}(\mathrm{q}) - \underline{z}(\theta))\mathrm{q}\}$$

with

$$\underline{z}(\theta) := \theta + (1 - \alpha)\frac{\underline{F}(\theta)}{\underline{f}(\theta)}$$

$G(M) = G^*$ only if $q(\theta) = \underline{q}^{\text{BM}}(\theta)$ almost all θ.

More General Cost Uncertainty

Corollary

Suppose $F^\star$ regular and no uncertainty over cost (i.e., $\mathcal{F} = \{F^\star\}$). Unique robustly optimal mechanism is $\underline{M}^{\text{BM}} := (\underline{q}^{\text{BM}}, \underline{u}^{\text{BM}})$.

More General Cost Uncertainty

- Suppose there exist $\theta_s \in (\underline{\theta}, \overline{\theta})$ and $\delta_s \in [0,1]$ s.t. $\underline{F}$ (a) abs. cont over $(-\infty, \bar{\theta})$ with atom δ_s at $\bar{\theta}$, (b) $\underline{F}(\theta) = 0$ for all $\theta \leq \theta_s$, and $\underline{z}(\theta)$ increasing over $[\theta_s, \overline{\theta}]$ and continuous over $[\theta_s, \overline{\theta})$.

Proposition

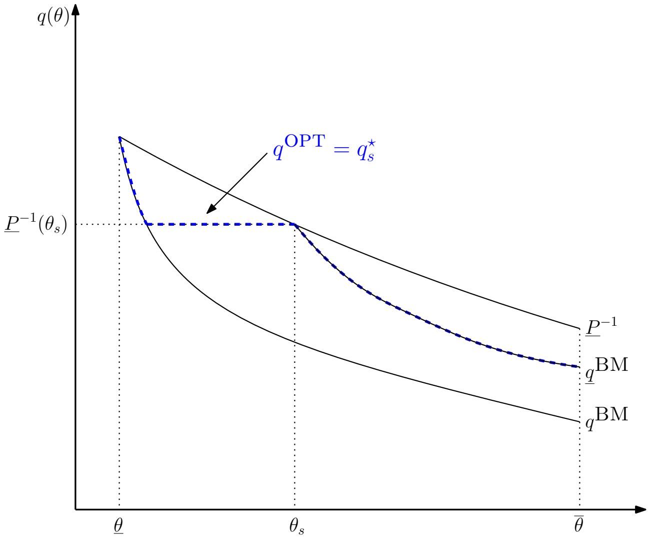
Suppose $F^\star$ regular, $V^\star = \underline{V}$, and $\underline{F}$ satisfies above properties. Then unique robustly optimal mechanism $M^{OPT} = (q^{OPT}, u^{OPT})$ s.t.

$$q^{OPT}(\theta) := \begin{cases} \max\{q^{\mathrm{BM}}(\theta), \underline{P}^{-1}(\theta_s)\} & \theta < \theta_s \\ \underline{q}^{\mathrm{BM}}(\theta) & \theta \geq \theta_s \end{cases}$$

- Optimal mechanism bridges two different schedules
  - q^{BM} with floor at $\underline{P}^{-1}(\theta_s)$ for low costs
  - $\underline{q}^{\mathrm{BM}}$ for high costs

- No distortion at top **and middle** (θ_s)

More General Cost Uncertainty



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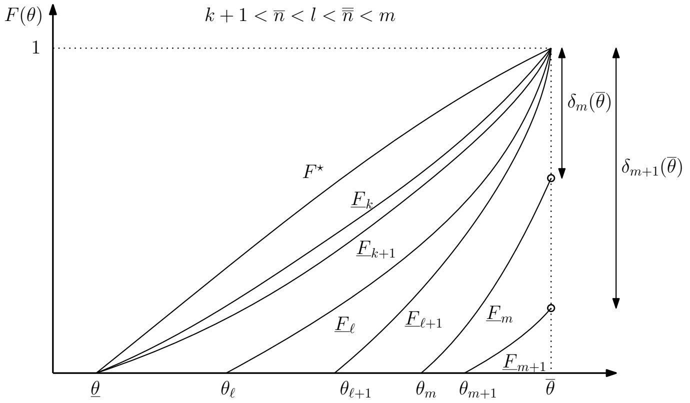
Change in Uncertainty

Change in Uncertainty

- Change in **cost uncertainty**, $\mathcal{F}$
- Change in **demand uncertainty,** $\mathcal{V}$ (equivalently, $\mathcal{P}$)

Change in Cost Uncertainty

- (Monotone) sequence $(\underline{F}_n)$ converging to $\mathbb{I}(\theta \geq \bar{\theta})$



Change in Cost Uncertainty

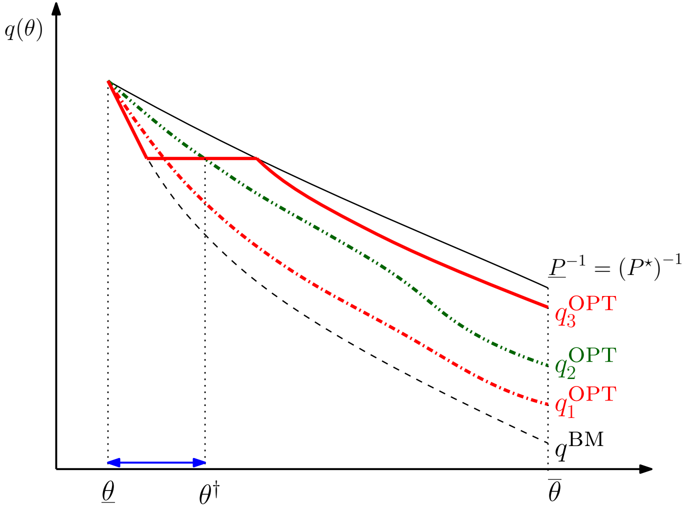
Proposition

Suppose $V^{\star} = \underline{V}$. *There exist monotone sequences* $(\underline{F}_n)$ *s.t., for every* $\theta \in (\underline{\theta}, \overline{\theta})$, *there exists* $n(\theta) \in \mathbb{N}$ *s.t.*

$$q_{n+1}^{OPT}(\theta) \left\{ \begin{array}{c} \geq \\ \leq \end{array} \right\} q_n^{OPT}(\theta) \quad \textit{for} \quad \left\{ \begin{array}{c} n \leq n(\theta) - 1 \\ n > n(\theta) \end{array} \right\}$$

Moreover, there exists $j, k \in \mathbb{N}$ *with* $j < k$, *such that* $q_j^{OPT}(\theta) > q_k^{OPT}(\theta)$

Change in Cost Uncertainty



Change in Demand Uncertainty

- Optimal mechanism depends on $\mathcal{P}$ only through lowest demand $\underline{P}$

- $\mathcal{P}_N$: new set of feasible demands with lowest element $\underline{P}_N$ s.t.

$$\underline{P}_N(\mathrm{q}) \geq \underline{P}(\mathrm{q}), \ \forall \mathrm{q}$$

  and $P^\star \in \mathcal{P}_N \cap \mathcal{P}$

- Let $(\mathrm{q}_\ell^N, \theta_N^\star, \theta_N^m)$ be analogs of $(\mathrm{q}_\ell, \theta^\star, \theta^m)$ for $\mathcal{P}_N$

Proposition

Suppose $\alpha = 0$. Then (a) $q_\ell^N \geq q_\ell$, (b) $\theta_N^\star \leq \theta^\star$, and (c) if $\theta_N^\star \geq \theta^m$, then $\theta_N^m \geq \theta^m$.

- As lowest demand increases
  - floor increases
  - region over which floor bids expands
  - region over which robustness calls for reduction in output vis-a-vis BM shrinks

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1. Introduction
2. Model
3. Baron-Myerson
4. Short List: Worst-Case Optimality
5. Robustly Optimal Quantity Mechanisms
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8. **Price vs Quantity Regulation**
9. Conclusions

Price vs Quantity Regulation

Price Mechanisms

- Let

$$\mathcal{D} := \{D : \mathbb{R}_+ \to \mathbb{R}_+ \text{ s.t. } \underline{D}(\mathrm{p}) \leq D(\mathrm{p}) \leq \overline{D}(\mathrm{p}) \ \forall \mathrm{p}\}.$$

  be set of direct demands associated with $\mathcal{P}$ (bijection between $\mathcal{P}$ and $\mathcal{D}$)

- Price mechanism $\tilde{M} = (p, t)$

$$p : \Theta \to \mathbb{R}_+$$

$$t : \Theta \times \mathcal{D} \to \mathbb{R}_+$$

Price Mechanisms: **ex-post robustness**

- Price mechanisms
  - adjustment in q to D
  - expose seller to uncertainty
- Robust (ex-post) IC and IR

Definition

Price mechanism $\tilde{M} = (p, t)$ **EPIC** and **EPIR** if, for all $\theta, \theta' \in \Theta$ and $D \in \mathcal{D}$,

$$t(\theta, D) - \theta D(p(\theta)) \geq t(\theta', D) - \theta D(p(\theta'))$$

$$\tilde{u}(\theta, D) := t(\theta, D) - \theta D(p(\theta)) \geq 0$$

- Welfare under IC and IR $\tilde{M} = (p, t)$ given (D, F):

$$\tilde{W}(\tilde{M}; D, F) := \int \{V(D(p(\theta))) - \theta D(p(\theta)) - (1 - \alpha)\tilde{u}(\theta, D)\}\, F(\mathrm{d}\theta)$$

Price Mechanisms: **guarantee**

Lemma

For any price mechanism $\tilde{M} \in \tilde{\mathcal{M}}$,

$$G(\tilde{M}) \leq G^*.$$

There exists price mechanism $\underline{\tilde{M}} \in \tilde{\mathcal{M}}$ *s.t.*

$$G(\underline{\tilde{M}}) = G^*.$$

Any $\tilde{M} \in \tilde{\mathcal{M}}$ *for which* $G(\tilde{M}) = G^*$ *is s.t.* $p(\overline{\theta}) = \overline{\theta}$ *and* $\tilde{u}(\overline{\theta}, \underline{D}) = 0$.

Price Mechanisms: **sub-optimality of mark-downs**

Lemma

Suppose $\tilde{M} \in \tilde{\mathcal{M}}$ *is s.t.* $p(\theta) < \theta$ *for some* θ*. There exists another price mechanism* $\tilde{M}^{\dagger} = (p^{\dagger}, t^{\dagger}) \in \tilde{\mathcal{M}}$ *with* $p^{\dagger}(\theta) \geq \theta$ *for all* θ *s.t.*

$$\tilde{W}(\tilde{M}^{\dagger}; D, F) \geq \tilde{W}(\tilde{M}; D, F)$$

for all $D \in \mathcal{D}$ *and* $F \in \mathcal{F}$*, with inequality strict if subset of* Θ *for which* $p(\theta) < \theta$ *has strict positive measure under* F*.*

Price Mechanisms: **characterization**

Lemma

Price mechanism $\tilde{M}^{\mathrm{OPT}} = (p^{OPT}, t^{OPT})$ *robustly optimal iff* p^{OPT} *solves*

$$\max_p \int_{\underline{\theta}}^{\bar{\theta}} \Big[V^\star(D^\star(p(\theta))) - z^\star(\theta)D^\star(p(\theta))\Big] F^\star(\mathrm{d}\theta)$$
$$\text{s.t.}$$
$$p \quad \textit{non-decreasing}$$
$$p(\bar{\theta}) = \bar{\theta}$$
$$p(\theta) \geq \theta \quad \textit{all } \theta \in \Theta$$

and $\tilde{u}^{OPT}(\theta, D) := t^{OPT}(\theta, D) - \theta D(p^{OPT}(\theta))$ *satisfies following properties:*

(a) for all $D \in \mathcal{D}$ *and* $\theta \in \Theta$, $\tilde{u}^{OPT}(\theta, D) = \tilde{u}^{OPT}(\bar{\theta}, D) + \int_\theta^{\bar{\theta}} D(p^{OPT}(y))dy$
(b) for all $D \in \mathcal{D}$,

$$0 \leq \tilde{u}^{OPT}(\overline{\theta}, D) \leq [V(D(\overline{\theta})) - \overline{\theta}D(\overline{\theta}) - G^*]/(1-\alpha)$$

(c) $\tilde{u}^{OPT}(\bar{\theta}, \underline{D}) = \tilde{u}^{OPT}(\bar{\theta}, D^\star) = 0$.

Price Mechanisms: Regular case

Proposition

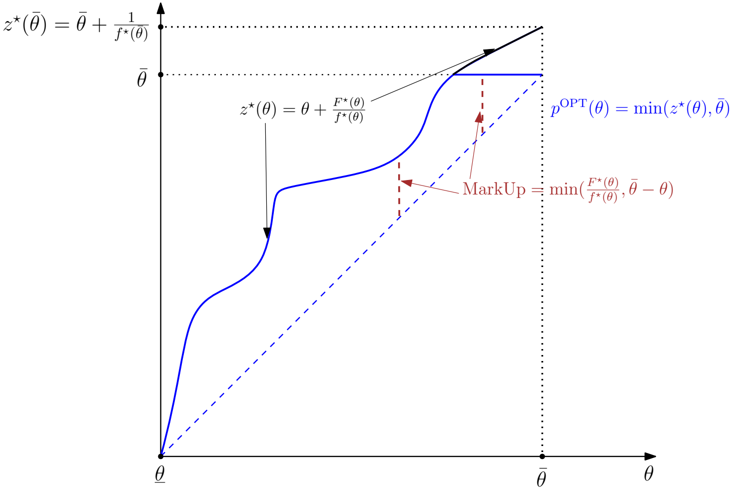
Suppose $F^\star$ regular. Unique robustly optimal price mechanism s.t. for all $\theta \in \Theta$,

$$p^{OPT}(\theta) = \min\{z^\star(\theta), \overline{\theta}\}$$

Corollary

Optimal price mechanism invariant in conjecture $D^\star$ and demand uncertainty $\mathcal{D}$. It consists in setting markup $F^\star(\theta)/f^\star(\theta)$, with price cap at $\overline{\theta}$.

Price Mechanisms



Price vs Quantity Regulation?

- M^{OPT} : robustly optimal **quantity** mechanism
- $\tilde{M}^{\text{OPT}}$: robustly optimal **price** mechanism

Definition

Price regulation superior to quantity regulation if

$$\tilde{W}(\tilde{M}^{\text{OPT}}; D^{\star}, F^{\star}) \geq W(M^{\text{OPT}}; V^{\star}, F^{\star})$$

Price vs Quantity Regulation?

Proposition

Assume $F^\star$ regular.

1. *If BM-with-qty-floor is robustly optimal, then quantity regulation superior to price regulation (strictly if $D^\star(\bar{\theta}) > \underline{D}(\bar{\theta})$)*

2. *If BM-with-qty-floor quantity is not robustly optimal and $D^\star(\bar{\theta}) = \underline{D}(\bar{\theta})$, price regulation strictly superior*

Corollary

Suppose $F^\star$ regular and no demand uncertainty. Price and quantity regulation equivalent.

Quantity regulation dominates

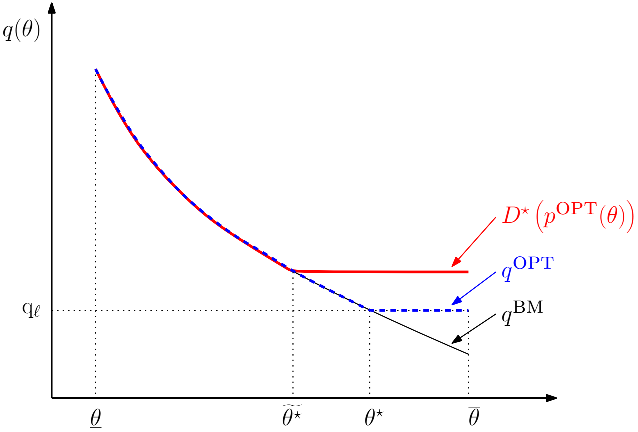
- Robustness under price mechanism $\Rightarrow$ **price cap** at $\overline{\theta}$
  - when $D^{\star}(\bar{\theta}) > \underline{D}(\bar{\theta})$

    $\Rightarrow$ **over-procurement (vis-a-vis qty regulation) for high costs**

- When BM-with-qty-floor mechanism robustly optimal

  $\Rightarrow$ **quantity regulation dominates**

Quantity regulation dominates



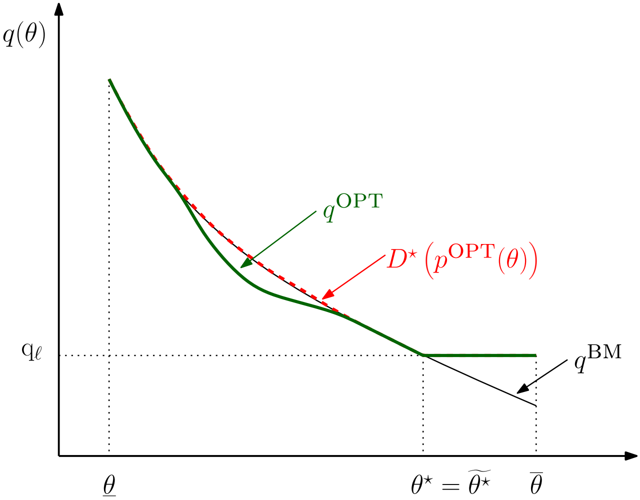
Price regulation dominates

- When BM-with-qty-floor mechanism **not** robustly optimal
    - robustness under quantity regulation

      $\Rightarrow$ **downward adjustments** for intermediate costs
    - adjustment necessary to avoid over-procurement when $D < D^\star$
    - adjustment not necessary under price regulation
    - if price cap at $\overline{\theta}$ does not induce over-procurement vis-a-vis qty regulation

      $\Rightarrow$ **price regulation strictly dominates**

Price regulation dominates



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Conclusions

Conclusions

- **Novel approach to robustness**
  - government has conjecture but does not fully trust it
  - first protects itself against worst-case
  - then uses conjecture to optimize over worst-case optimal mechanisms
- When only uncertainty is over cost
  - robustly optimal mechanism is BM-with-qty-floor
  - efficiency at top and bottom (of cost distribution)
- In general, robustness under quantity regulation
  - upward quantity adjustment for high cost
  - downward output adjustment for intermediate costs
- General cost uncertainty
  - robust qty mechanism bridges BM for F^* with BM for $\underline{F}$
  - efficiency at top and middle (of cost distribution)
- Price regulation superior when
  - price cap does not lead to over-procurement from high-cost firms
  - BM-with-qty-floor not robust

Most Important Slide

THANK YOU

BM-proof

- Familiar IC analysis $\rightarrow$ ex-ante welfare under conjecture $(V^{\star}, F^{\star})$

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[V^{\star}(q(\theta)) - z^{\star}(\theta)q(\theta) \right] dF^{\star}(\theta)$$

- $q^{BM}(\theta)$ maximizes virtual surplus $V^{\star}(\mathrm{q}) - z^{\star}(\theta)\mathrm{q}$ point-wise
- Regularity:

  $\Rightarrow q^{BM}$ decreasing

  $\Rightarrow (q^{BM}, u^{BM})$ IC and IR

Go back

Max-guarantee-proof

For any IC and IR mechanism $M = (q, u) \in \mathcal{M}$

$$W(M; V, F) := \int \{V(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\} F(\mathrm{d}\theta)$$
$$\geq \int \{\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\} F(\mathrm{d}\theta)$$
$$\geq \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Hence,

$$G(M) \geq \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Because $\underline{V} \in \mathcal{V}$ and, for each θ, Dirac distribution at θ is in $\mathcal{F}$

$$G(M) \leq \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Hence,

$$G(M) = \inf_{\theta\in\Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)\right]$$

Because $u(\bar{\theta}) \geq 0$ and

$$\underline{V}(q(\bar{\theta})) - \bar{\theta} q(\bar{\theta}) \leq \underline{V}(\mathrm{q}_\ell) - \bar{\theta}\mathrm{q}_\ell := G^*$$

$G(M) \leq G^*$

Go back

Short List-proof

- $M = (q, t)$ IC and IR:
  - q non-increasing
  - $u(\theta) := t(\theta) - \theta q(\theta) = u(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} q(y)dy$, with $u(\bar{\theta}) \geq 0$
- ONLY IF:
  - $M_L = (q_L, t_L)$ w. $q_L(\theta) = \mathrm{q}_\ell$ and $t_L(\theta) = \bar{\theta}\mathrm{q}_\ell$ all θ
    (i) IC and IR
    (ii) $u_L(\theta) := t_L(\theta) - \theta q_L(\theta) = (\bar{\theta} - \theta)\mathrm{q}_\ell$
    (iii) $\underline{W}(\theta, M_L) := \underline{V}(\mathrm{q}_\ell) - \theta \mathrm{q}_\ell - (1-\alpha)u_L(\theta) = G^* + \alpha(\bar{\theta} - \theta)\mathrm{q}_\ell$
    (iv) $G(M_L) = \inf_\theta\{\underline{W}(\theta, M_L)\} = G^*$
  - $M \in \mathcal{M}^{\mathrm{SL}}$ **only if** $G(M) = G^*$
  - Because $\underline{W}(\theta, M) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)u(\theta)$
    (1) $u(\bar{\theta}) = 0$
    (2) $\underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\bar{\theta}} q(y)dy \geq G^*$ all θ
- IF PART: immediate

Go back

Relaxation-Proof

- $M = (q, u) \in \mathcal{M}^{\mathrm{SL}}$ only if, for all θ,

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - (1-\alpha)\int_{\theta}^{\overline{\theta}} q(y)dy \geq G^*$$

- Because $G^* = \max_{\mathrm{q}}\{\underline{V}(\mathrm{q}) - \overline{\theta}\mathrm{q}\} = \underline{V}(\mathrm{q}_\ell) - \overline{\theta}\mathrm{q}_\ell$

  $\Rightarrow q(\overline{\theta}) = \mathrm{q}_\ell$

- q non-increasing

  $\Rightarrow q(\theta) \geq \mathrm{q}_\ell$ all θ

Go back

General-case-proof

- When $\alpha = 0$, $M^\star \equiv (q^\star, u^\star)$ robustly optimal if
  - $\underline{W}(\underline{\theta}, q^\star) \geq G^*$
  - $q^\star$ single -crosses $\underline{P}^{-1}$
- First condition equivalent to

$$\left(\underline{V}(q^\star(\underline{\theta})) - \underline{\theta} q^\star(\underline{\theta})\right) - \left(\underline{V}(\mathrm{q}_\ell) - \bar{\theta}\mathrm{q}_\ell\right) \geq \int_{\underline{\theta}}^{\bar{\theta}} q^\star(y) dy$$

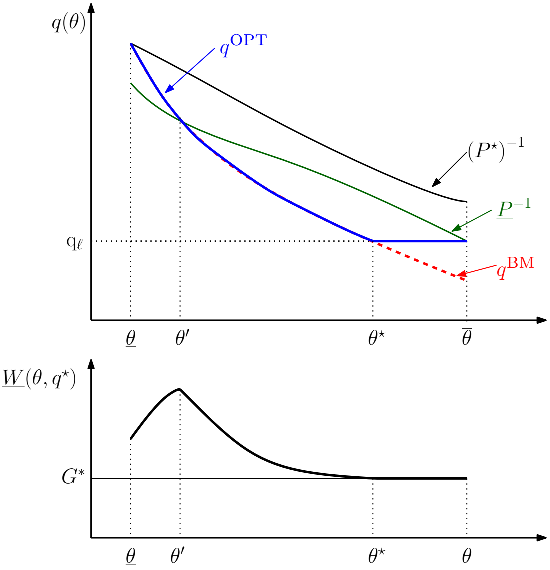
where $q^\star(\underline{\theta}) = \arg\max_{\mathrm{q}} \{V^\star(\mathrm{q}) - \underline{\theta}\mathrm{q}\}$

- When $V^\star = \underline{V}$,

$$\left(\underline{V}(q^\star(\underline{\theta})) - \underline{\theta} q^\star(\underline{\theta})\right) - \left(\underline{V}(\mathrm{q}_\ell) - \bar{\theta}\mathrm{q}_\ell\right) = \int_{\underline{\theta}}^{\bar{\theta}} q^{FB}(y) dy > \int_{\underline{\theta}}^{\bar{\theta}} q^\star(y) dy$$

- Hence inequality holds when $|| \; V^\star, \underline{V} \; ||$ small

Robust Optimality ($\alpha = 0$): BM-with-qty-floor



Optimality of "plateau" for high costs

Lemma

Suppose $F^\star$ regular. Assume $\theta^m \neq \bar{\theta}$ or $\theta^m = \bar{\theta}$ and $q^{\mathrm{BM}}(\bar{\theta}) > q_\ell$. Then $q^{\mathrm{OPT}}(\theta) = q_\ell$ for all $\theta \in [\theta^\star, \bar{\theta}]$

- **Robustness**: Minimizing quantity over $[\theta^\star, \bar{\theta}]$
  - increases $\underline{W}(\theta, q)$ for all $\theta \leq \theta^\star$ (by reducing rents)
  - possibly decreases $\underline{W}(\theta, q)$ for $\theta > \theta^\star$(by reducing $\underline{V}(\mathrm{q}) - \theta \mathrm{q}$)
  - however, $\underline{W}(\cdot, q^\star)$ decreasing over $[\theta^\star, \bar{\theta}]$ w. $\underline{W}(\bar{\theta}, q^\star) = G^*$
  - hence, it helps robustness
- **Virtual surplus maximization:** Minimizing quantity over $[\theta^\star, \bar{\theta}]$
  - increases VS because $V^\star(\mathrm{q}) - z^\star(\theta)\mathrm{q}$ maximized at $q^{\mathrm{BM}}(\theta) < \mathrm{q}_\ell$
- It follows that, under any $M^{\mathrm{OPT}} = (q^{\mathrm{OPT}}, u^{\mathrm{OPT}})$,

$$q^{\mathrm{OPT}}(\theta) = \mathrm{q}_\ell \qquad \forall \theta \in [\theta^\star, \bar{\theta}]$$

Suboptimality of upward adjustments for low costs

Lemma

Suppose $F^\star$ regular and assume $\theta^m \neq \overline{\theta}$ or $\theta^m = \overline{\theta}$ and $q^{\mathrm{BM}}(\overline{\theta}) > q_\ell$. Then $q^{\mathrm{OPT}}(\theta) \leq q^{\mathrm{BM}}(\theta)$ for almost all $\theta \in [\underline{\theta}, \theta^\star)$, with inequality strict over positive-measure $I \subseteq [\underline{\theta}, \theta^\star)$

Sub-optimality of upward adjustments for low costs

- Take any $M = (q, u) \in \mathcal{M}^{\mathrm{SL}}$ s.t.
  - $q(\theta) > q^{\star}(\theta) = q^{BM}(\theta)$ over positive-measure $I \subseteq [\underline{\theta}, \theta^{\star})$
  - $q(\theta) = \mathrm{q}_{\ell}$ for all $\theta \geq \theta^{\star}$
- Take $M' = (q', t')$ s.t.

$$q'(\theta) := \min\{q^{\star}(\theta), q(\theta)\}$$

and $u'(\theta) := t'(\theta) - \theta q'(\theta) = \int_{\theta}^{\bar{\theta}} q'(y) dy$

- Clearly,
  - M' is IC and IR
  - Higher VS under M' than M: q' closer to q^{BM}which maximizes VS
- $M' \in \mathcal{M}^{\mathrm{SL}}$?

Sub-optimality of upward adjustments for low costs

- $M' \in \mathcal{M}^{\mathrm{SL}} \Leftrightarrow$

$$\underline{W}(\theta, q') := \underline{V}(q'(\theta)) - \underline{\theta} q'(\theta) - \int_{\theta}^{\bar{\theta}} q'(y) dy \geq G^* \qquad \forall \theta$$

- Clearly so when
    - $q'(\theta) = q(\theta)$ — same TS (under $\underline{V}$), smaller rents
    - $\underline{P}^{-1}(\theta) \leq q'(\theta) < q(\theta)$ — higher TS (under $\underline{V}$), smaller rents

      [recall $\underline{P}^{-1}(\theta)$ maximizes $\underline{V}(\mathrm{q}) - \underline{\theta}\mathrm{q}$)]

Sub-optimality of upward adjustments for low costs

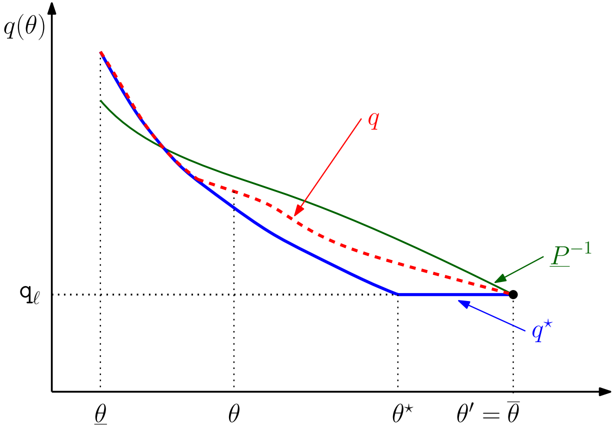
- When, instead, $q'(\theta) = q^\star(\theta) < \min\{q(\theta), \underline{P}^{-1}(\theta)\}$, **not clear** whether $\underline{W}(\theta, q') \geq G^*$
- Because $q^\star(\bar{\theta}) = \mathrm{q}_\ell = \underline{P}^{-1}(\bar{\theta})$, there exists $\theta' > \theta$ s.t.

$$q'(y) := \min\{q(y), q^\star(\theta)\} \leq \underline{P}^{-1}(y) \qquad \forall y \in [\theta, \theta']$$

$$q'(\theta') = \min\{\underline{P}^{-1}(\theta'), q(\theta')\}$$

- Monotonicity lemma $\Rightarrow \underline{W}(\cdot, q')$ non-increasing over $[\theta, \theta']$
- $\underline{W}(\theta', q') \geq G^*$
- Hence $\underline{W}(\theta, q') \geq G^*$, Q.E.D.

Sub-optimality of upward adjustments for low costs



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