Robust Procurement: Bayesian Design under Worst-Case Approval Constraints

Debasis Mishra, Sanket Patil, Alessandro Pavan

Motivation

- Contract design
  - central to many economic problems
    - procurement
    - regulation
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  - **uncertainty**
    - cost technology
    - product value/demand
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  - **asymmetric information**
  - **uncertainty**
    - cost technology
    - product value/demand
    - ...
  - **need of supervisory approval**
    - regulatory authorities
    - upstream management
    - ...

This Paper

- **Bayesian design under conservative approval constraints**

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- **(Max-min) supervisor/approver**
  - does not trust designer's confidence in the model
    - limited expertise
    - legal requirements/impediments
    - value-at-risk obligations
    - ...

  asks for **satisfactory welfare guarantee**

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  asks for **satisfactory welfare guarantee**

- **(Bayesian) designer**
  - same objective as approver
  - **uses model to max over contracts with satisfactory guarantee**

Alternative Interpretation (of robustness constraint)

- **Designer's own attitude toward uncertainty**
- Ambiguity-averse designer has *model* but does not fully trust it
  - first prepares for worst (in case model is mis-specified)
  - then uses model to select among mechanisms delivering satisfactory guarantee
- **Two interpretations are equivalent**

Literature

- **Bayesian (subjective expected utility) design**
  - ...
  - procurement: Baron-Myerson (1982)...
- **Robust model/belief-free (procurement/regulation) design**
  - Garrett (2014): **worst-case optimality**
  - Bergemann-Heumann-Morris (2025): **competitive ratio**
  - Guo-Shmaya (2024): **min-max regret**

  (Difference: Designer uses **model/conjecture** to select over robust mechanisms)
- **Un-dominated design**
  Mishra-Patil (2025), Borgers-Li-Wang (2025)
- **Robust information design**
  Dworczak-Pavan (2023), Cheng-Klibanoff-Mukerji-Renou (2024)
- **Robust control, model mis-specification, and other robustness approaches**
  Bergemann and Morris (2005), Bergemann and Schlag (2008), Lopomo-Rigotti-Shannon (2021), Cerreia-Vioglio-Hansen-Maccheroni-Marinacci (2025), Kambhampati (2025)...

Plan

1. Introduction
2. **Designer's problem**
3. Robustly Optimal Mechanisms
4. Downstream Markets and Price Regulation
5. Quantity vs Price Regulation
6. Conclusions

Designer's Problem

Environment

- **Players**
  - Designer (buyer)
  - Agent (supplier)
  - Supervisor (upstream authority)
- **Contracting variables**
  - output $\mathrm{q} \geq 0$ (intensity of trade)
  - transfer t

Environment

- **Payoffs**
  - Value of q to designer and supervisor:

$$V(\mathrm{q}) = \int_0^{\mathrm{q}} P(s)ds$$

  - Seller's cost $\theta\mathrm{q}$
  - Players' outside option payoff: 0
  - Designer/supervisor's payoff:

$$V(\mathrm{q}) - \mathrm{t}$$

  - Seller's payoff:

$$\mathrm{t} - \theta\mathrm{q}$$

- **Asymmetric information**
  - θ: seller's private information

Environment

- **Designer's model** $(V^\star, F^\star)$
  - Value function: $V^\star(\mathrm{q}) = \int_0^{\mathrm{q}} P^\star(s)ds$
  - Cost technology: θ drawn from abs. cont $F^\star$ with $f^\star(\theta) > 0$ over $\Theta = [\underline{\theta}, \overline{\theta}]$ and "**virtual cost**"
$$z^\star(\theta) := \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$$
    cont. and increasing (**regularity**)

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    cont. and increasing (**regularity**)

- **Supervisor's quest for robustness**
  - Supervisor does not share designer's confidence in model $(V^\star, F^\star)$
  - concerned true model may be $(V, F) \neq (V^\star, F^\star)$
  - approves only mechanisms with highest guarantee

    (Supplement: more permissive constraints)

Environment

- **Admissible set (of models):** $\mathcal{A} = \mathcal{V} \times \mathcal{F}$
  - $\mathcal{V}$: set of possible (gross) value fns
    - each $V \in \mathcal{V}$ abs continuous, increasing, concave
    - $\mathcal{P}$: set of corresponding *inverse demand functions*, with

$$V(\mathrm{q}) = \int_0^{\mathrm{q}} P(s)ds \qquad \forall \mathrm{q}$$

    - $\mathcal{D}$: set of corresponding *direct demand functions*, $D = P^{-1}$
    - $P^\star \in \mathcal{P}$, $D^\star \in \mathcal{D}$, $V^\star \in \mathcal{V}$
    - Lowest (inverse) demand function: $\underline{P}$

      any $\mathrm{q} \geq 0$ and $P \in \mathcal{P}$

$$P(\mathrm{q}) \geq \underline{P}(\mathrm{q})$$

      $\underline{P}$: strictly decreasing, continuous, and s.t.

$$\lim_{\mathrm{q} \to 0^+} \underline{P}(\mathrm{q}) > \overline{\theta}$$

  - $\mathcal{F} = \mathrm{CDF}(\Theta)$: all cdfs with support contained in $\Theta = [\underline{\theta}, \overline{\theta}]$

    (Supplement: more general $\mathcal{F}$, w. lowest element $\underline{F}$)

Environment

- $\mathcal{M}$: set of IC and IR mechanisms
- $W(M; V, F)$: buyer's welfare under M, given (V, F)

Definition

Welfare guarantee for $M \in \mathcal{M}$,

$$G(M) := \inf_{(V,F)\in\mathcal{A}} W(M; V, F).$$

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Short-list (of approved mechanisms)

$$\mathcal{M}^{\mathrm{SL}} \equiv \arg\max_{M\in\mathcal{M}} G(M)$$

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(mechanisms with largest guarantee)

- (Supplement: $\mathcal{M}^{\mathrm{SL}} \equiv \{M \in \mathcal{M} : G(M) \geq \gamma G^*\})$, $\gamma \in [0, 1]$

Designer's Problem

- Choose mechanism that maximizes expected welfare under conjectured model $(V^\star, F^\star)$ over SL of worst-case-optimal mechanisms

Designer's Problem

- Choose mechanism that maximizes expected welfare under conjectured model (V^*, F^*) over SL of worst-case-optimal mechanisms

Definition

Mechanism M **robustly optimal** if

$$M \in \arg \max_{M' \in \mathcal{M}^{\mathrm{SL}}} W(M'; V^*, F^*)$$

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Robustly Optimal Mechanisms

Procurement Mechanisms

- **(Direct) procurement mechanism** $M = (q, t)$
  - quantity schedule $q : \Theta \to \mathbb{R}_+$
  - transfer schedule $t : \Theta \to \mathbb{R}$
- **"Rent"** to type θ:

$$u(\theta) := t(\theta) - \theta q(\theta)$$

- Equivalent representation: $M = (q, u)$
- Given $M = (q, u)$, **expected welfare** under (V, F)

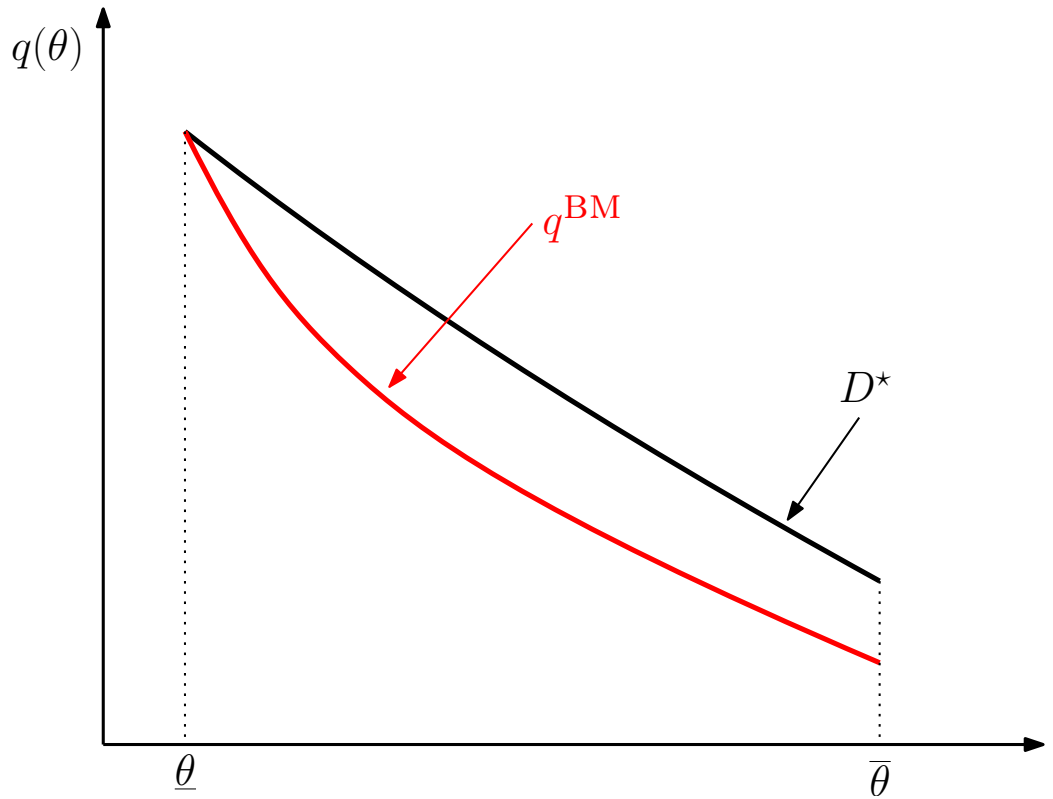
$$W(M; V, F) := \int [V(q(\theta)) - \theta q(\theta) - u(\theta)] \, F(\mathrm{d}\theta)$$

SEU/Bayesian Benchmark: Baron Myerson

- Suppose supervisor trusts designer's model

Proposition

SEU/Bayesian optimum: $q^{BM}(\theta) := D^\star(z^\star(\theta))$, where $z^\star(\theta) := \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$ is "virtual cost".



(BM-Proposition)

Short List: Worst-case Optimality

- Let

$$\mathrm{q}_\ell := \arg\max_{\mathrm{q}} \left\{ \underline{V}(\mathrm{q}) - \overline{\theta}\mathrm{q} \right\} = \underline{D}(\overline{\theta})$$

$$G^* := \underline{V}(\mathrm{q}_\ell) - \overline{\theta}\mathrm{q}_\ell$$

Lemma

For any IC and IR mechanism $M = (q, u) \in \mathcal{M}$

$$G(M) = \inf_{\theta \in \Theta} \left\{ \underline{V}(q(\theta)) - \theta q(\theta) - \int_\theta^{\overline{\theta}} q(y)dy - u(\bar{\theta}) \right\}.$$

Furthermore,

$$G(M) \leq G^*$$

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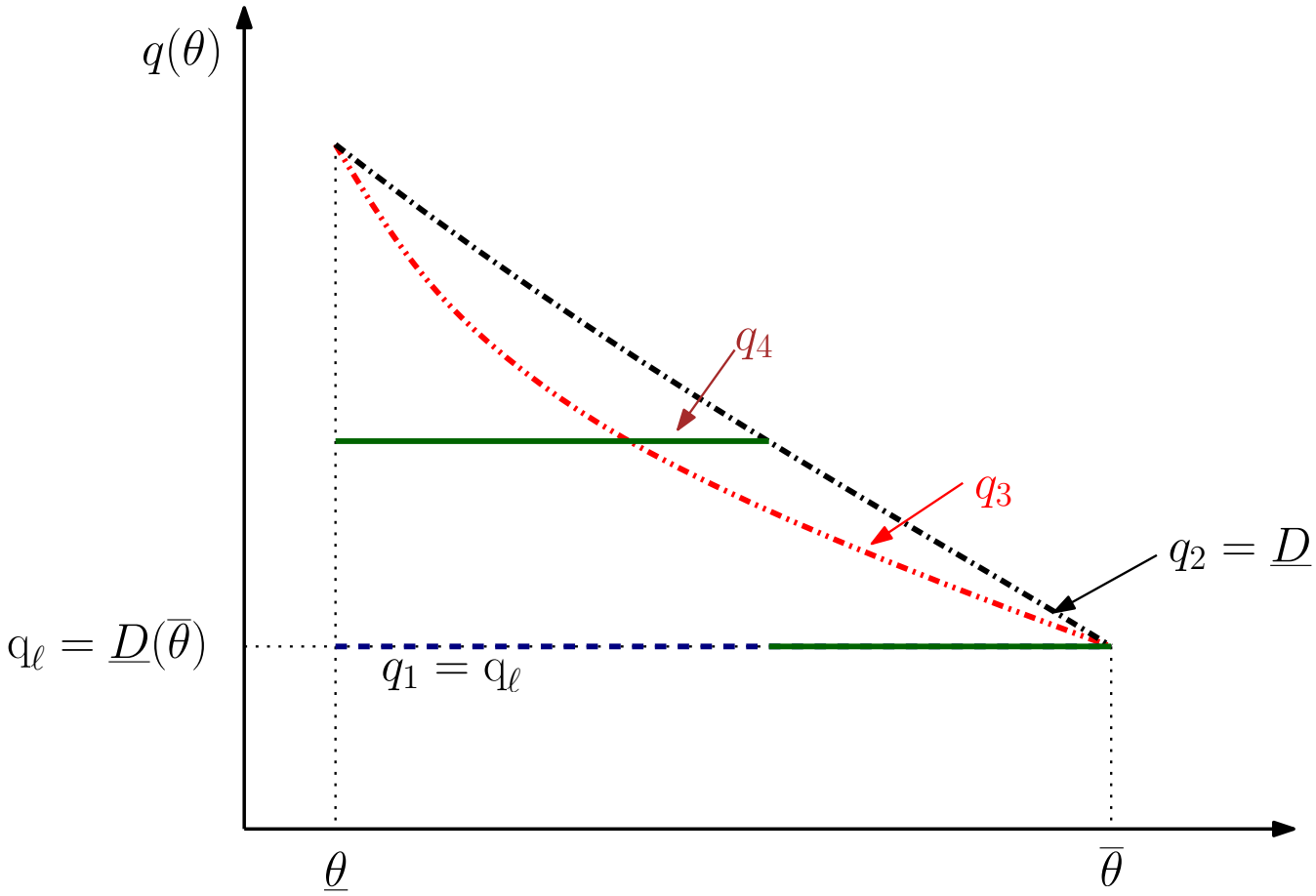
- Worst case always under lowest demand **but not necessarily highest cost**
- "Flat" mechanism $q(\theta) = \mathrm{q}_\ell$, $t(\theta) = \overline{\theta}\mathrm{q}_\ell$, all θ, delivers maximal guarantee G^*

(Max-guarantee-proof)

Short List: Illustration

- Continuum of worst-case optimal mechanisms



Short List: Characterization

Lemma

$M = (q, u) \in \mathcal{M}^{\mathrm{SL}}$ *iff*

1. q *non-increasing,*
2. $u(\theta) = \int_{\theta}^{\overline{\theta}} q(y)dy$ *all* θ
3. *for all* θ,

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y)dy \geq G^*$$

Short List: Majorizations

Lemma

Take any non-increasing q s.t. $q(\overline{\theta}) = \mathrm{q}_\ell$. Following are equivalent

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_\theta^{\overline{\theta}} q(y)dy \geq G^* \qquad \forall \theta \in \Theta$$

$$\int_\theta^{\overline{\theta}} q(y)dy \leq \int_\theta^{\overline{\theta}} \underline{D}(y)dy - \underbrace{\int_\theta^{\underline{P}(q(\theta))} (\underline{D}(y) - q(\theta))\, dy}_{DWL(\theta, q(\theta)) \geq 0} \qquad \forall \theta \in \Theta$$

$$\int_\theta^{\overline{\theta}} q(y)dy \leq \int_\theta^{\overline{\theta}} \underline{D}(y)dy \qquad \forall \theta \in \Theta \qquad \textit{and} \qquad \underline{W}(\underline{\theta}, q) \geq G^*$$

Robust Optimality

- **Robustly optimal schedule** q^{OPT} solves

$$\max_{q} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \Big] F^\star(\mathrm{d}\theta)$$

$$\text{s.t.}$$

$$q \quad \text{non-increasing}$$

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y) dy \geq G^* \qquad \forall\ \theta \in \Theta$$

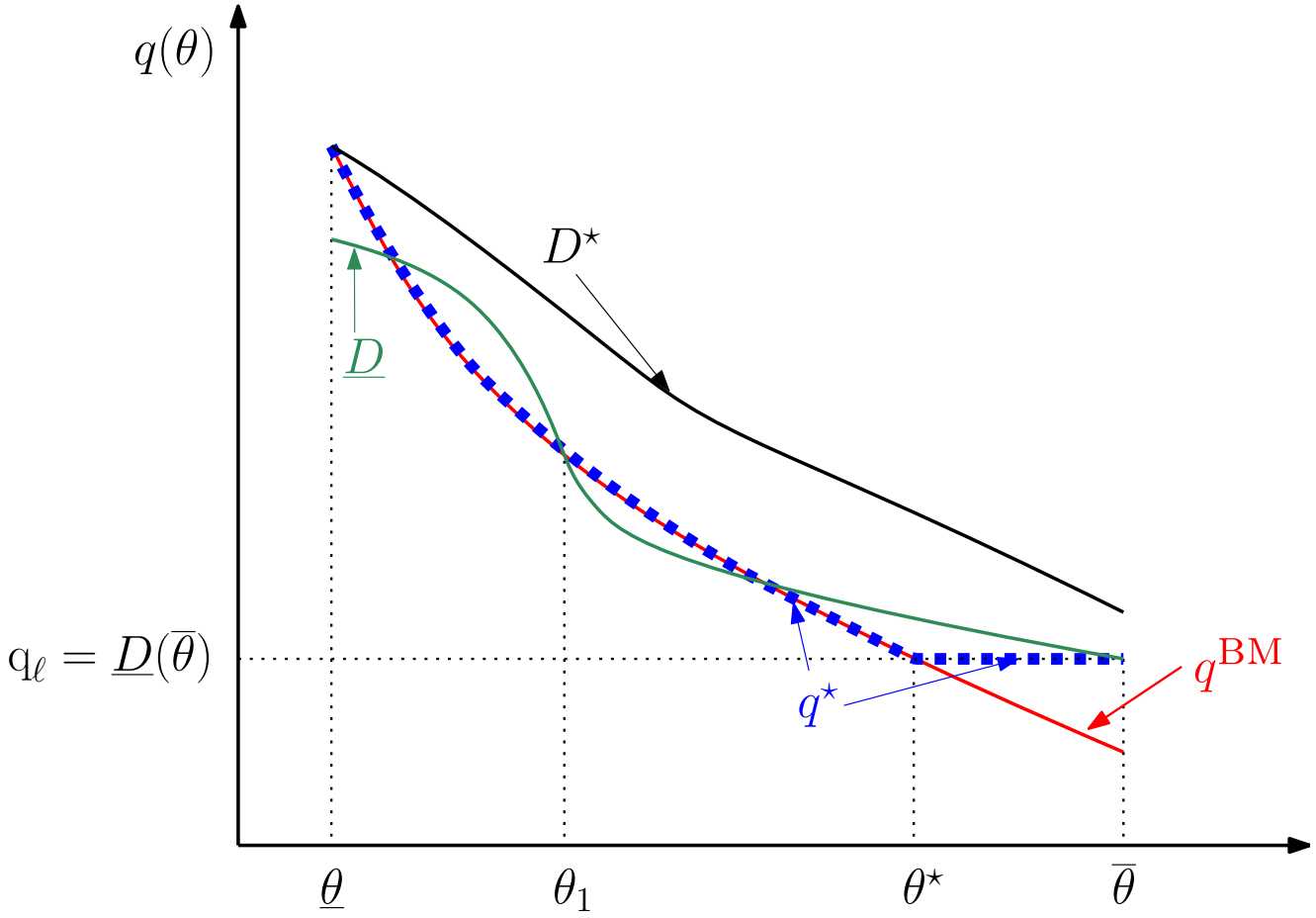
where $z^\star(\theta) := \theta + \frac{F^\star(\theta)}{f^\star(\theta)}$ is **virtual cost** under conjectured model $(V^\star, F^\star)$.

Baron-Myerson-with-quantity-floor

Definition

BM-with-quantity-floor:

$$q^\star(\theta) := \max\{q^{\mathrm{BM}}(\theta), \mathrm{q}_\ell\}$$



Robust Optimality of Baron-Myerson-with-quantity-floor

Proposition

BM-with-quantity-floor $M^\star = (q^\star, u^\star)$ *robustly optimal iff* $\underline{W}(\underline{\theta}, q^\star) \geq G^*$ *and*

$$\int_{\theta}^{\overline{\theta}} q^\star(y)dy \leq \int_{\theta}^{\overline{\theta}} \underline{D}(y)dy \qquad \forall\ \theta \in \Theta$$

Furthermore, when $M^\star = (q^\star, u^\star)$ *robustly optimal, any* $M^{\mathrm{OPT}} = (q^{\mathrm{OPT}}, u^{\mathrm{OPT}})$ *s.t.*

$$q^{\mathrm{OPT}}(\theta) = q^\star(\theta) \qquad \forall\ \theta > \underline{\theta}$$

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Corollary

Suppose uncertainty only over cost (more generally, $D^\star = \underline{D}$*)*

- *BM-with-quantity-floor robustly optimal*
- *efficiency at both bottom and top*

Robust Optimality of Baron-Myerson-with-quantity-floor

- Full program:

$$\max_{q} \int_{\underline{\theta}}^{\overline{\theta}} \left[V^{\star}(q(\theta)) - z^{\star}(\theta) q(\theta) \right] F^{\star}(\mathrm{d}\theta)$$

$$q \quad \text{non-increasing}$$

$$\underline{W}(\theta; q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y) dy \geq G^{*} \qquad \forall \theta \in \Theta$$

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- **Relaxed program:**

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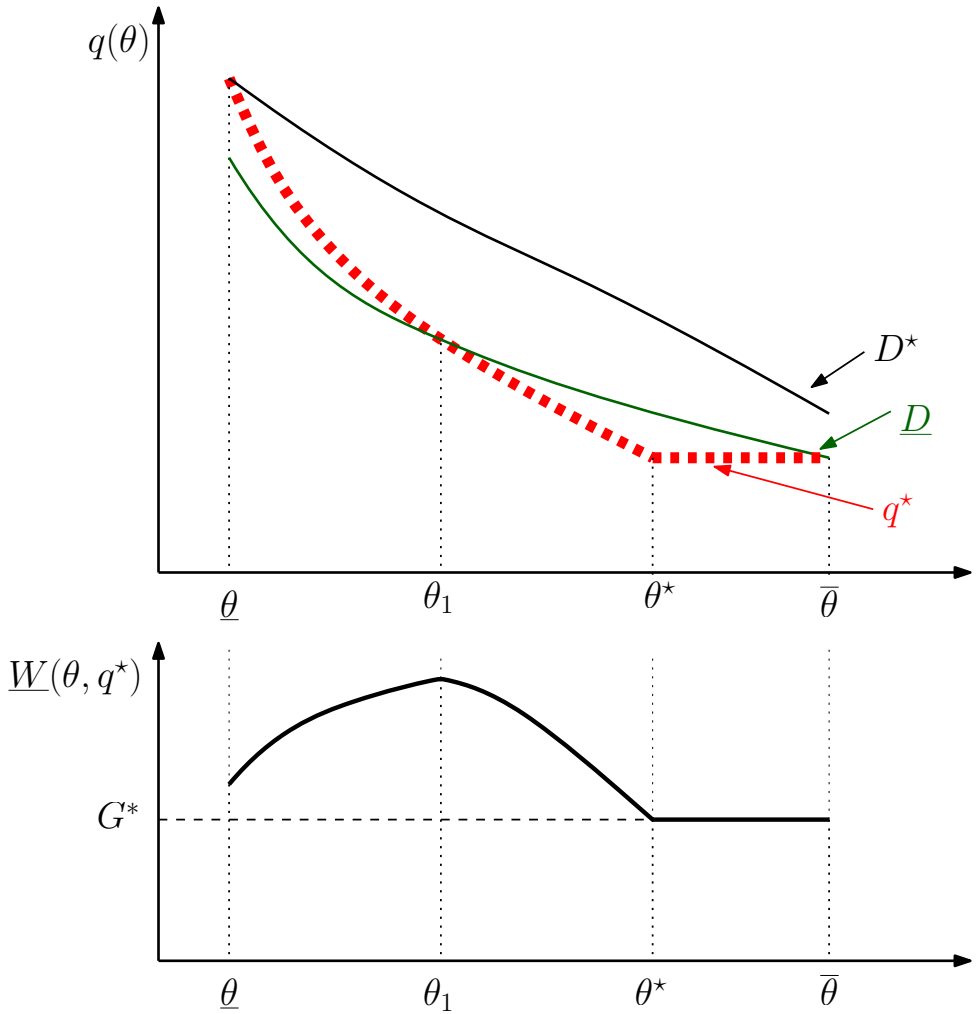
- BM-with-quantity-floor schedule $q^{\star}$ solves relaxed program
- Majorization lemma: conditions in Prop $\Rightarrow \underline{W}(\theta; q^{\star}) \geq G^{*}$ all θ

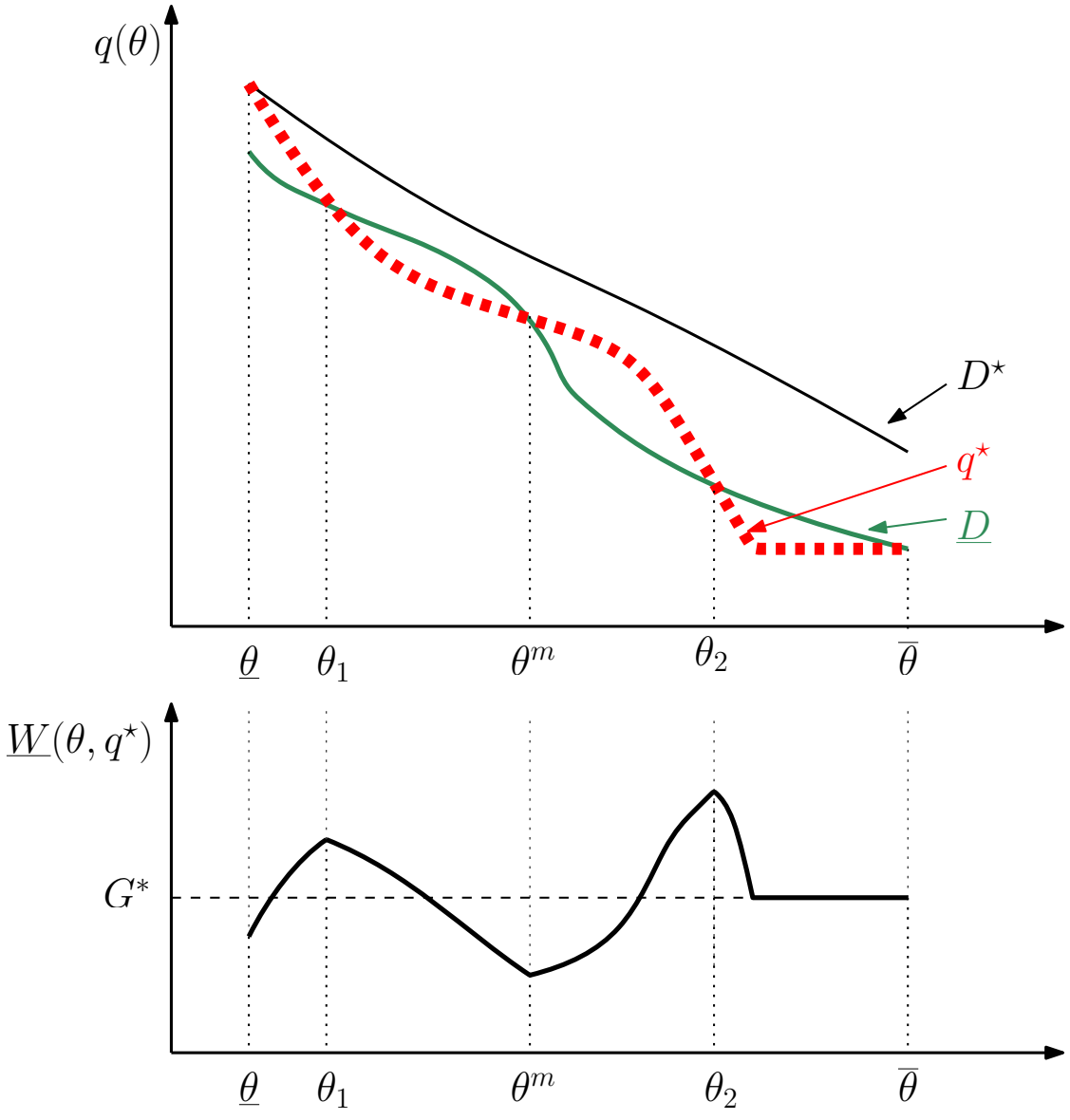
Robust Optimality of BM-with-quantity-floor

Suppose $D^\star \neq \underline{D}$, q^{BM} single-crosses $\underline{D}$ from above, and $\underline{W}(\underline{\theta}, q^\star) \geq G^*$. Then BM-with-quantity-floor robustly optimal

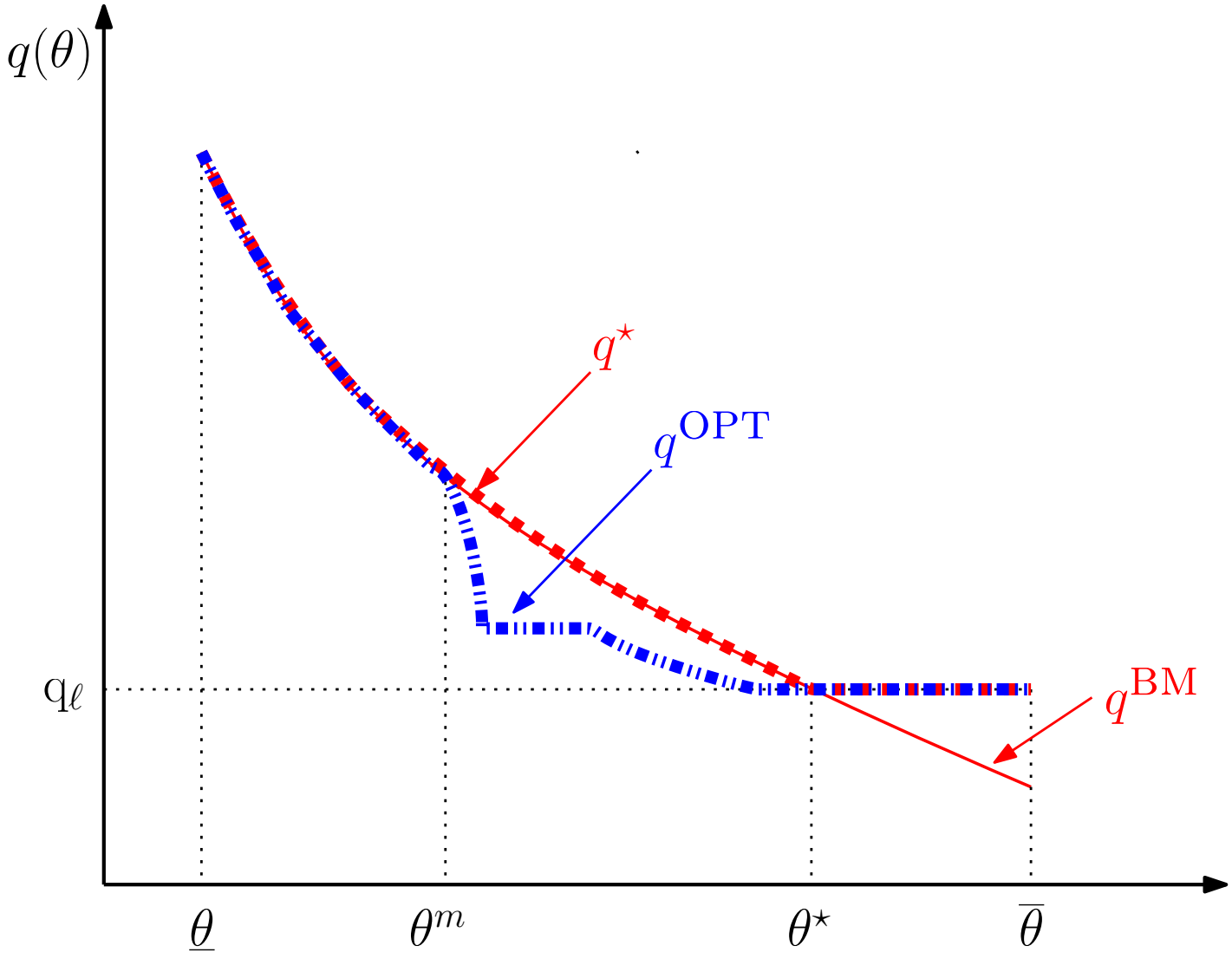


(Monotonicity Lemma)

Non-robustness of BM-with-quantity-floor



General Case



Robust Optimality: General Case

- **Reduced procurement for intermediate costs**
  - Avoid over-procurement and excessive rents when $D < D^*$
- **Increased procurement for high costs**
  - Avoid inefficiencies motivated by rent extraction (for lower types)
- **Low costs**
  - Welfare higher than G^* even when $D < D^*$
  - No adjustments necessary

(Proposition: General Case)

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Downstream Markets and Price regulation

Price Regulation

- Suppose supplier sells to **downstream mkt**
- Idea: use **price regulation** for **demand discovery**
- (Direct) price mechanism $\widetilde{M} = (p, t)$
  - **price schedule** $p : \Theta \to \mathbb{R}_+$
  - transfer schedule $t : \Theta \times \mathcal{D} \to \mathbb{R}$
  - conditioning t on realized demand D
    - irrelevant under qty regulation
    - protects seller from uncertainty under price regulation

(EPIC and EPIR)

Price Regulation: Welfare Guarantee

Lemma

For any price mechanism $\widetilde{M} \in \widetilde{\mathcal{M}}$,

$$G(\widetilde{M}) \leq G^*.$$

There exists price mechanism $\underline{\widetilde{M}} \in \widetilde{\mathcal{M}}$ *s.t.*

$$G(\underline{\widetilde{M}}) = G^*.$$

Any $\widetilde{M} \in \widetilde{\mathcal{M}}^{\mathrm{SL}}$ *(i.e., for which* $G(\widetilde{M}) = G^*$*) s.t.*

$$p(\overline{\theta}) = \overline{\theta}$$

and $\tilde{u}(\overline{\theta}, \underline{D}) = 0$.

- Maximal guarantee same as with quantity mechanisms
- Price cap at $\overline{\theta}$

Robust Optimality: Baron-Myerson-with-price-cap

- Designer's price regulation problem

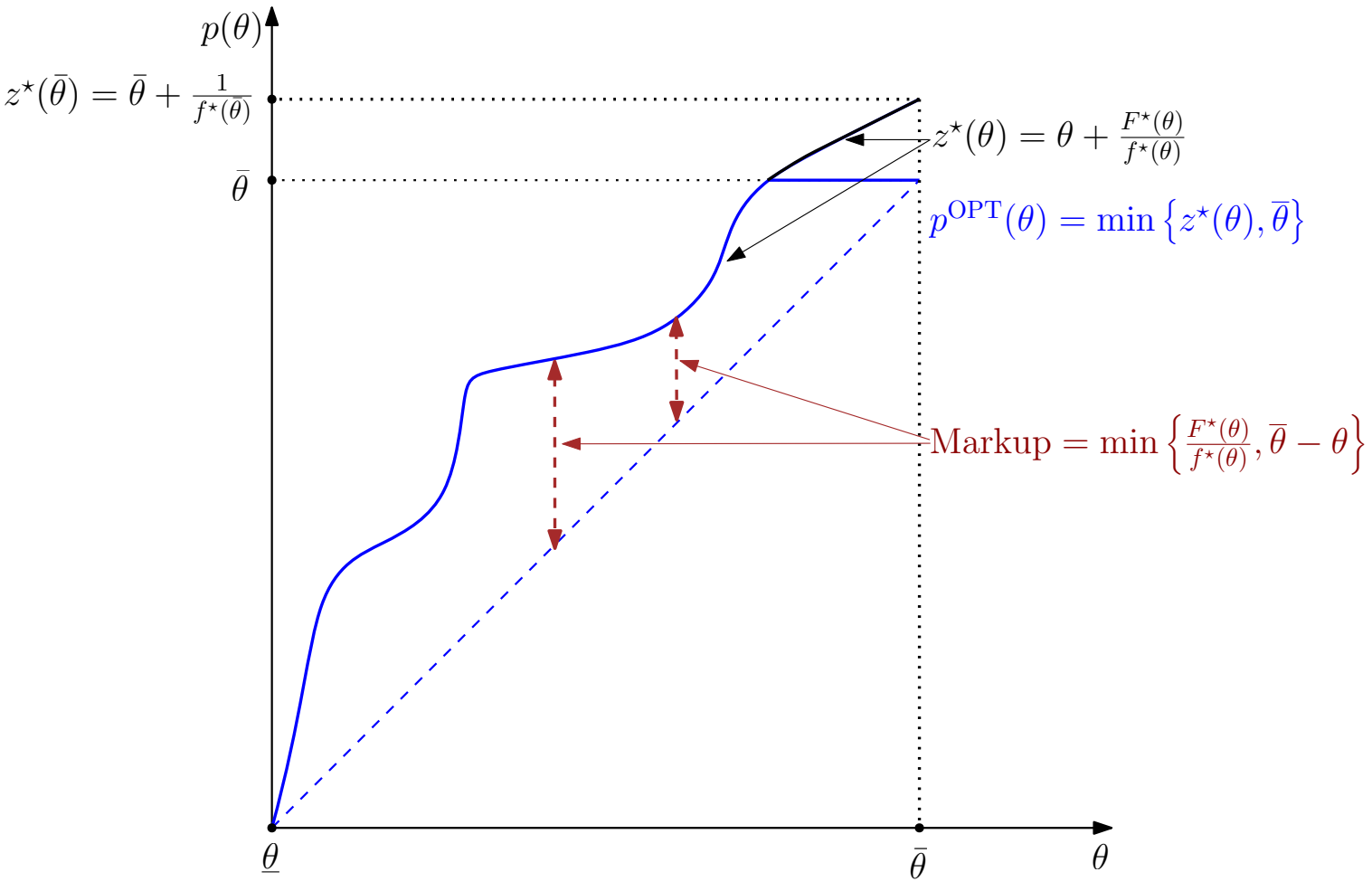
$$\max_{\tilde{M} \in \widetilde{M}^{\mathsf{SL}}} W(\tilde{M}; D^*, F^*)$$

Proposition

In any robustly optimal price mechanism $\widetilde{M}^{\mathsf{OPT}} = (p^{\mathsf{OPT}}, \tilde{u}^{\mathsf{OPT}})$

$$p^{OPT}(\theta) = \min\{z^\star(\theta), \overline{\theta}\} \qquad \forall \theta \in \Theta$$

Robust Optimality: Baron-Myerson-with-price-cap



(Proof Sketch)

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Quantity vs Price Regulation

Quantity vs Price Regulation?

- When is qty regulation superior to price regulation?
  - Weitzman 78: depends
  - Bayesian design: price wins
  - Worst-case optimality: tie
  - Robust optimality?

- M^{OPT} : robustly optimal **quantity** mechanism

- $\widetilde{M}^{\text{OPT}}$: robustly optimal **price** mechanism

Definition

Price regulation dominates if

$$\widetilde{W}(\widetilde{M}^{\text{OPT}}; D^\star, F^\star) \geq W(M^{\text{OPT}}; V^\star, F^\star)$$

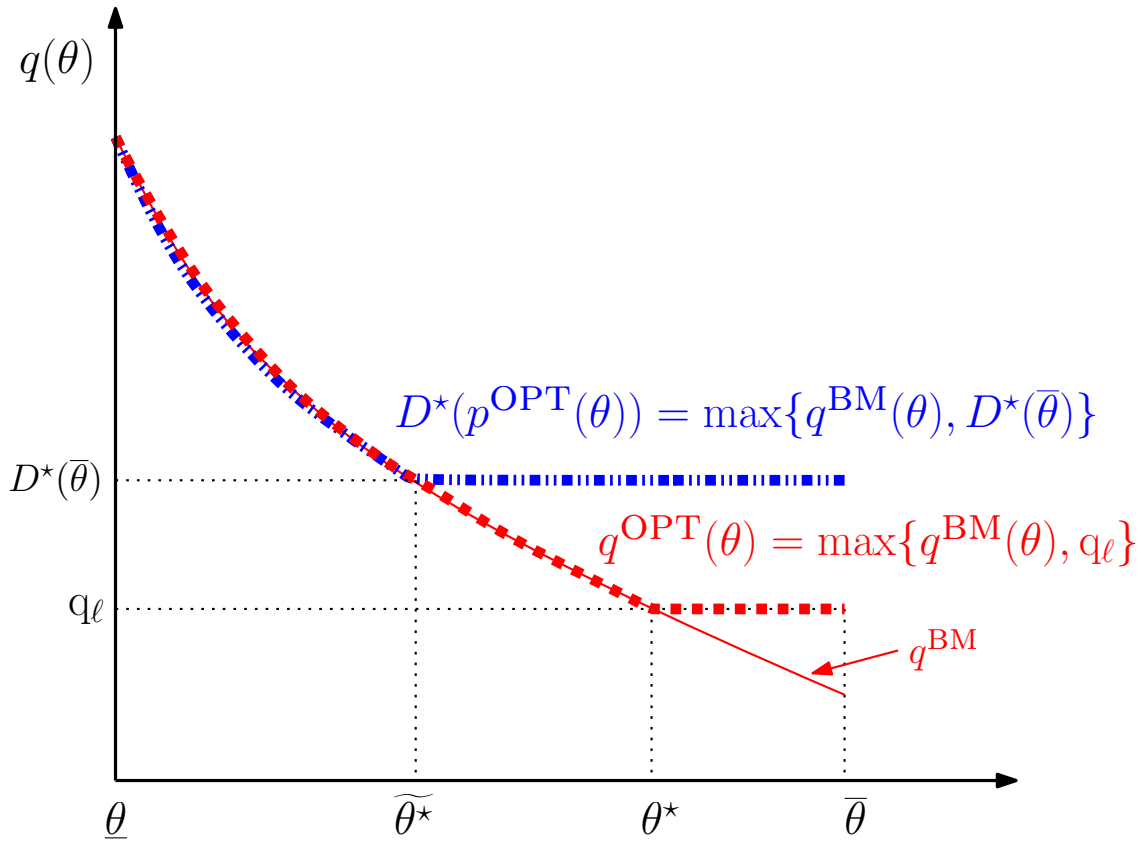
Quantity regulation dominates if

$$\widetilde{W}(\widetilde{M}^{\text{OPT}}; D^\star, F^\star) \leq W(M^{\text{OPT}}; V^\star, F^\star)$$

Quantity Regulation Dominates

Proposition

If BM-with-quantity-floor is robustly optimal, then quantity regulation dominates (strictly if $D^\star(\overline{\theta}) > \underline{D}(\overline{\theta})$).

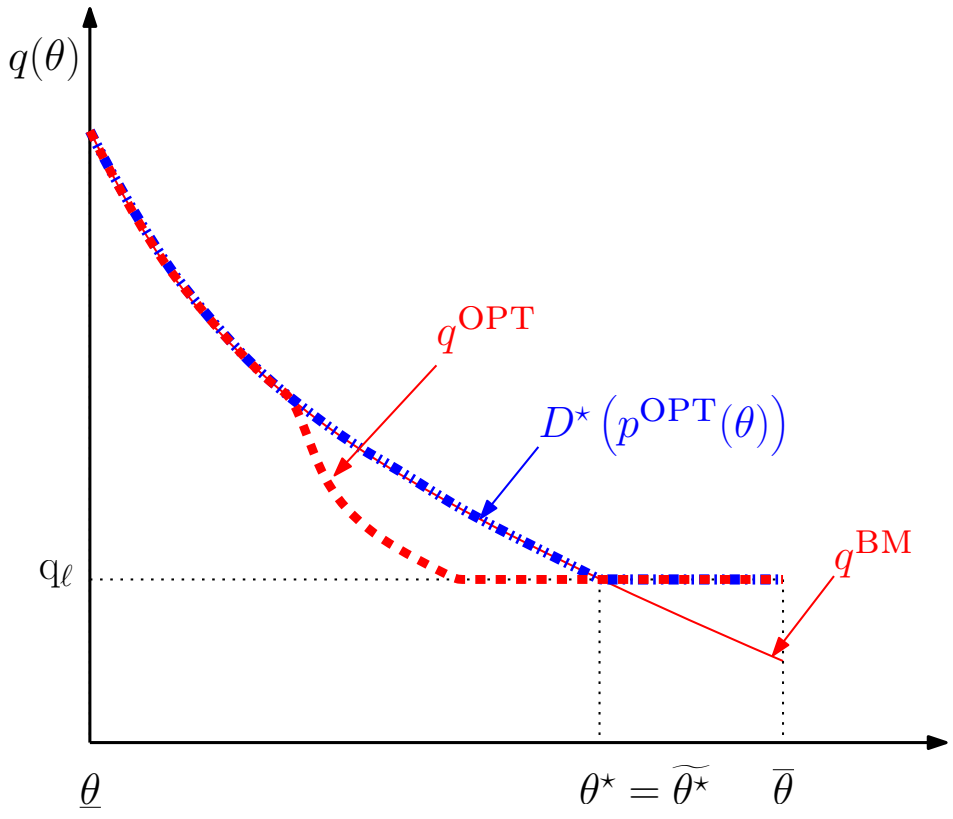


(Reasoning)

Price Regulation Dominates

Proposition

If BM-with-quantity-floor **not** *robustly optimal and* $D^\star(\overline{\theta}) = \underline{D}(\overline{\theta})$*, price regulation strictly dominates.*



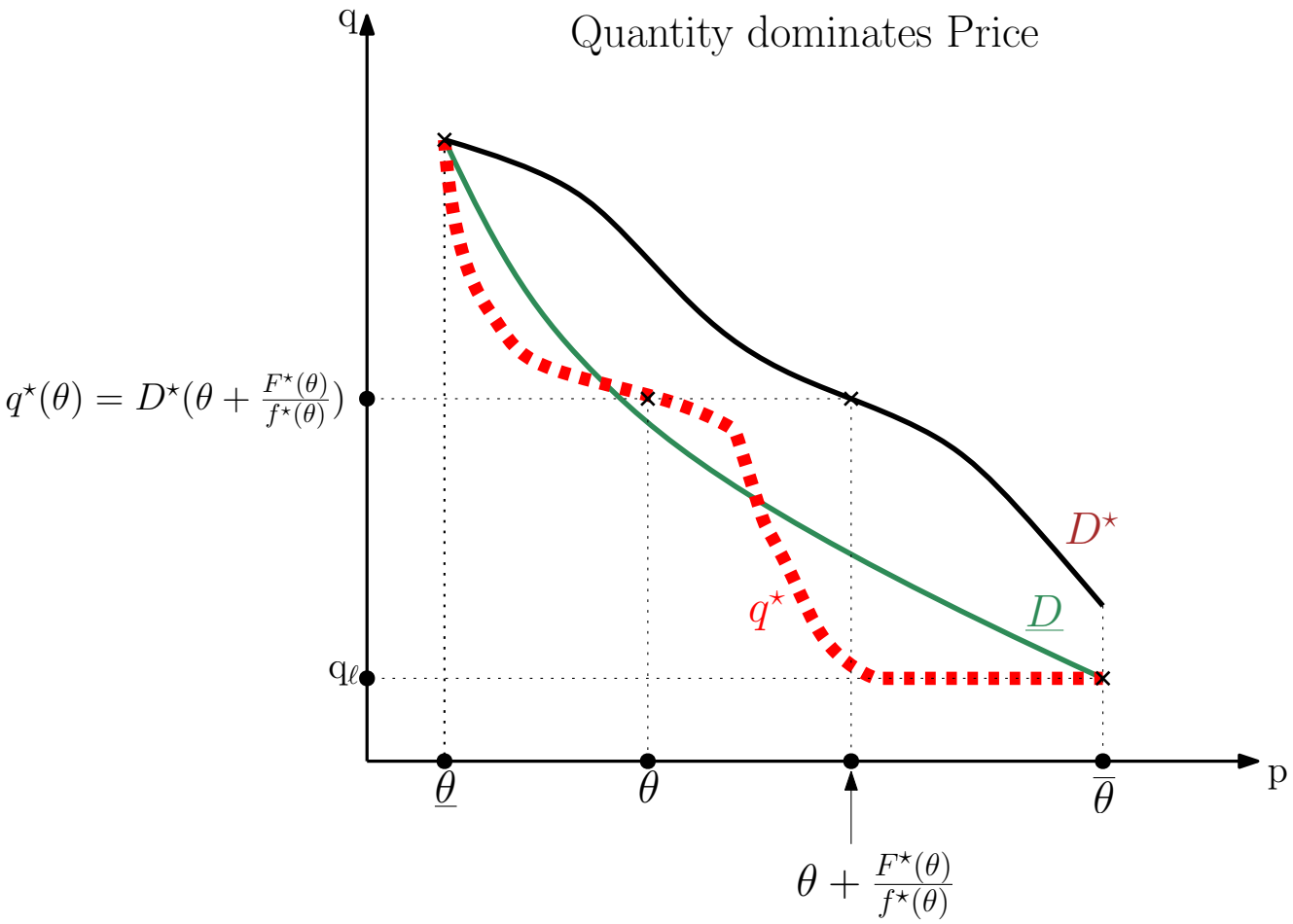
(Reasoning)

Quantity vs Price Regulation: Primitives

- **Qty regulation dominates** when
  - markups $F^\star(p)/f^\star(p)$ large relative to $D^* - \underline{D}$
- **Price regulation dominates** when
  - markups $F^\star(p)/f^\star(p)$ small relative to $D^* - \underline{D}$

(Proposition: Qty vs Price Regulation)

Quantity Regulation Dominates



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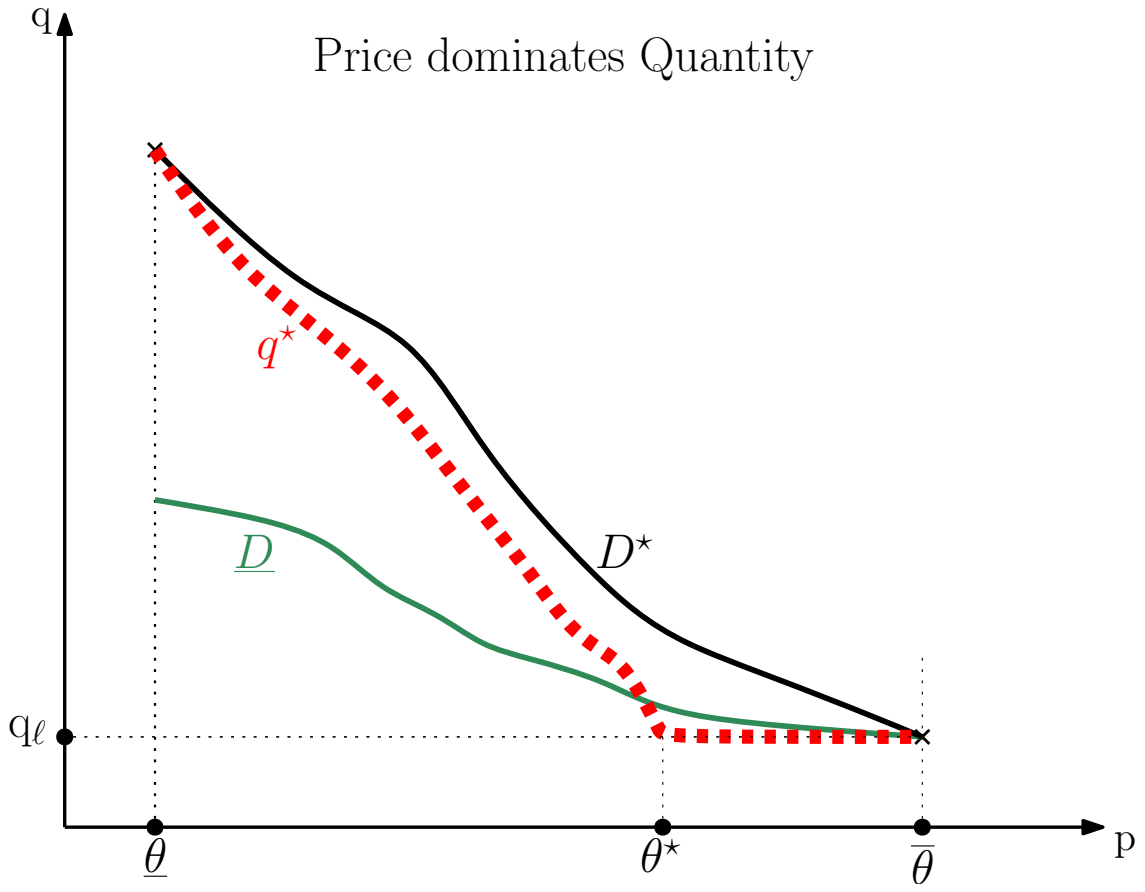

General Cost Uncertainty

(General Cost Uncertainty)

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Conclusions

Conclusions

- Novel approach to robustness where designer
  - has model/conjecture
  - but needs approval from conservative supervisor
- Designer uses model to select over mechanisms delivering satisfactory guarantee
- When uncertainty only over costs
  - robustly optimal mechanism: **BM-with-quantity-floor**
    - efficiency at top and bottom (of cost distribution)
- When uncertainty is also over demand/value for good/service
  - reduced procurement from intermediate-cost sellers
  - increased procurement from high-cost sellers
- Quantity regulation superior to price regulation
  - large mark-ups
  - small demand uncertainty

Most Important Slide

THANK YOU

Baron Myerson: Proof

Lemma

Mechanism $M = (q, u)$ IC and IR iff

1. q non-increasing
2. for all $\theta \in \Theta$,

$$u(\theta) = u(\overline{\theta}) + \int_{\theta}^{\overline{\theta}} q(z)dz$$

3. $u(\overline{\theta}) \geq 0$

Baron Myerson: Proof

- Expected payoff of any IC and IR M under model $(V^\star, F^\star)$:

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[V^\star(q(\theta)) - z^\star(\theta) q(\theta) \right] F^\star(\mathrm{d}\theta) - u(\overline{\theta})$$

- $q^{BM}(\theta)$ maximizes virtual surplus $V^\star(\mathrm{q}) - z^\star(\theta)\mathrm{q}$, all θ

- Regularity:

  $\Rightarrow$ q^{BM} decreasing

  $\Rightarrow$ q^{BM} (along with $u^{BM}(\overline{\theta}) = 0$) **SEU-optimal**

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Max-guarantee-proof

For any IC and IR mechanism $M = (q, u)$

$$W(M; V, F) := \int \{V(q(\theta)) - \theta q(\theta) - u(\theta)\} F(\mathrm{d}\theta)$$
$$\geq \int \{\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\} F(\mathrm{d}\theta)$$
$$\geq \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Hence,

$$G(M) \geq \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Because $\underline{V} \in \mathcal{V}$ and, for each θ, Dirac distribution at θ is in $\mathcal{F}$

$$G(M) \leq \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Hence,

$$G(M) = \inf_{\theta \in \Theta} \left[\underline{V}(q(\theta)) - \theta q(\theta) - u(\theta)\right]$$

Because $u(\overline{\theta}) \geq 0$ and

$$\underline{V}(q(\overline{\theta})) - \overline{\theta} q(\overline{\theta}) \leq \underline{V}(\mathrm{q}_\ell) - \overline{\theta} \mathrm{q}_\ell := G^*$$

$G(M) \leq G^*$

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Monotonicity Lemma

- Recall

$$\underline{W}(\theta, q) := \underline{V}(q(\theta)) - \theta q(\theta) - \int_{\theta}^{\overline{\theta}} q(y)dy$$

Lemma (Monotonicity)

For any non-increasing q and interval $I \subseteq \Theta$,

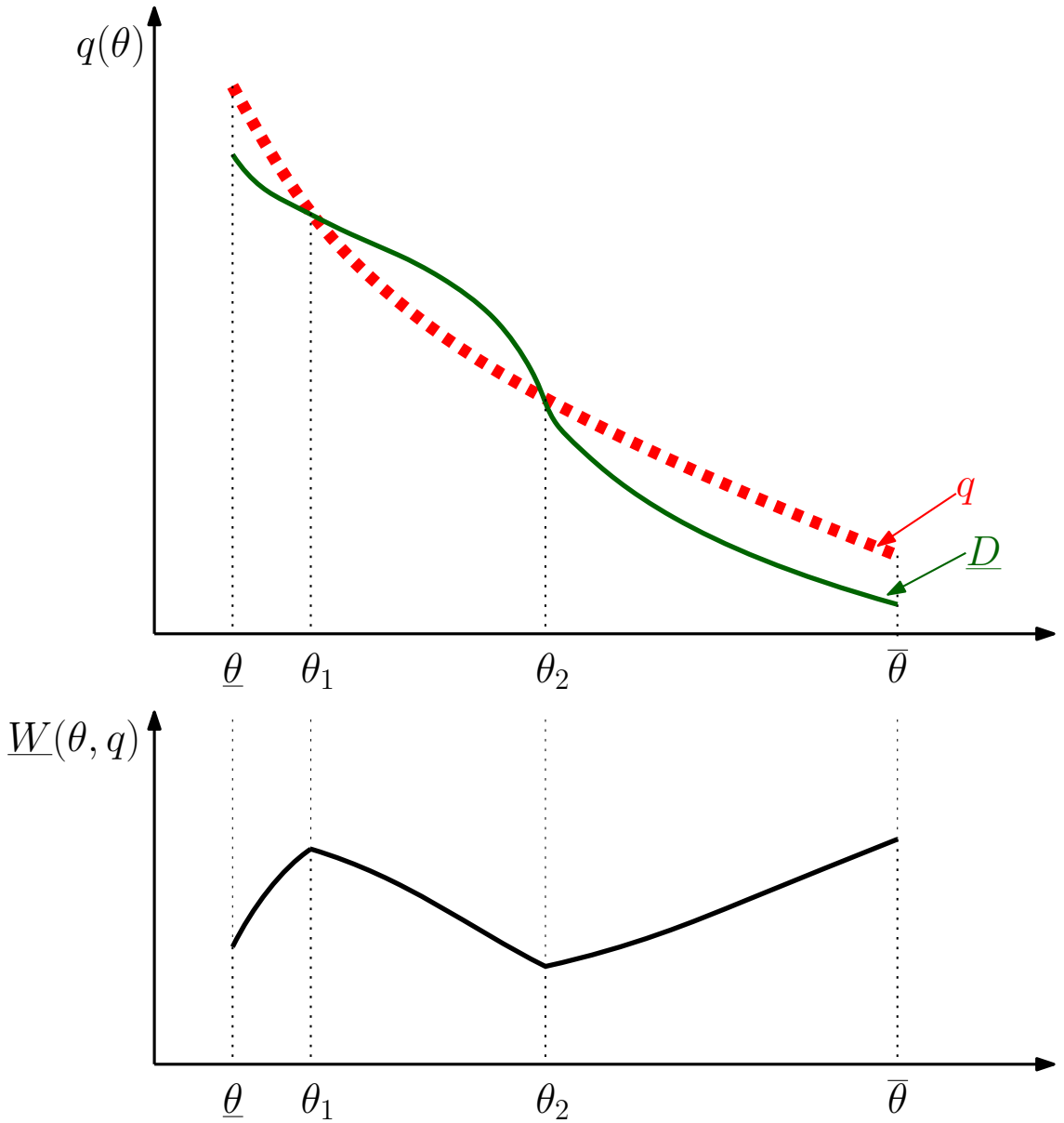
1. *If $0 < q(\theta) \leq \underline{D}(\theta)$ for all $\theta \in I$, then $\underline{W}(\cdot, q)$ non-increasing over I*

   (decreasing if q decreasing and $q(\theta) < \underline{D}(\theta)$ all $\theta \in I$)

2. *If $q(\theta) > \underline{D}(\theta)$ for all $\theta \in I$, then $\underline{W}(\cdot, q)$ non-decreasing over I*

   (increasing if q decreasing)

Monotonicity Lemma: Illustration



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Proposition: General Case

Let

$$\theta^m := \max\{\arg\min_{\theta\in\Theta} \underline{W}(\theta, q^\star)\}$$

$$\theta^\star := \begin{cases} (q^{\mathrm{BM})^{-1}}(\mathrm{q}_\ell) & \text{if} \quad q^{\mathrm{BM}}(\overline{\theta}) < \mathrm{q}_\ell \\ \overline{\theta} & \text{otherwise} \end{cases}$$

Proposition

When BM-with-quantity-floor **not** *robustly optimal,*

(a) $q^{\mathrm{OPT}}(\theta) = q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\underline{\theta}, \theta^m)$

(b) $q^{\mathrm{OPT}}(\theta) \leq q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^\star)$

(c) $q^{\mathrm{OPT}}(\theta) = \mathrm{q}_\ell$ *for all* $\theta \in [\theta^\star, \overline{\theta}]$

Furthermore, if $\theta^\star < \overline{\theta}$, *then* $q^{\mathrm{OPT}}(\theta) < q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^\star)$. *If, instead,* $\theta^\star = \overline{\theta}$, *there exists* $\theta^{\star\star} \in [\theta^m, \theta^\star]$ *such that* $q^{\mathrm{OPT}}(\theta) < q^{\mathrm{BM}}(\theta)$ *for all* $\theta \in (\theta^m, \theta^{\star\star})$ *and* $q^{\mathrm{OPT}}(\theta) = \underline{D}(\theta)$ *for all* $\theta \in [\theta^{\star\star}, \overline{\theta}]$.

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Price Mechanisms: EPIC and EPIR

Definition

Price mechanism $\widetilde{M} = (p, t)$ **EPIC** and **EPIR** if, for all $\theta, \theta' \in \Theta$ and $D \in \mathcal{D}$,

$$t(\theta, D) - \theta D(p(\theta)) \geq t(\theta', D) - \theta D(p(\theta'))$$

$$\tilde{u}(\theta, D) := t(\theta, D) - \theta D(p(\theta)) \geq 0$$

- $\widetilde{\mathcal{M}}$: set of EPIC and EPIR price mechanisms $\widetilde{M} := (p, \tilde{u})$

Price Mechanisms: EPIC and EPIR

Lemma

$\widetilde{M} = (p, t)$ is EPIC and EPIR iff p is non-decreasing, and for every $\theta \in \Theta$ and $D \in \mathcal{D}$,

$$\tilde{u}(\theta, D) = \tilde{u}(\overline{\theta}, D) + \int_{\theta}^{\overline{\theta}} D(p(y))dy,$$

with $\tilde{u}(\overline{\theta}, D) \geq 0$.

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Robust Optimality of Baron-Myerson-with-price-cap

- Welfare under $\widetilde{M} = (p, \tilde{u}) \in \widetilde{\mathcal{M}}$ given (D, F):

$$\widetilde{W}(\widetilde{M}; D, F) := \int \{V(D(p(\theta))) - \theta D(p(\theta)) - \tilde{u}(\theta, D)\} F(\mathrm{d}\theta)$$

Robust Optimality of Baron-Myerson-with-price-cap

$$\max_{p,\{\tilde{u}(\overline{\theta},D)\geq 0\}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(D^\star(p(\theta))) - z^\star(\theta)D^\star(p(\theta))\Big] F^\star(\mathrm{d}\theta) - \tilde{u}(\overline{\theta}, D^\star)$$

$$\text{s.t.}$$

$$p \quad \text{non-decreasing}$$

$$\int_0^{D(p(\theta))} D^{-1}(y)dy - \theta D(p(\theta)) - \int_{\theta}^{\overline{\theta}} D(p(y))dy - \tilde{u}(\overline{\theta}, D) \geq G^* \quad \forall\ \theta \in \Theta \ \ \forall\ D \in \mathcal{D}$$

$$p(\overline{\theta}) = \overline{\theta}$$

Robust Optimality of Baron-Myerson-with-price-cap

- Relaxed program:

$$\max_{p,\{\tilde{u}(\overline{\theta},D)\geq 0\}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[V^\star(D^\star(p(\theta))) - z^\star(\theta)D^\star(p(\theta))\Big] F^\star(\mathrm{d}\theta) - \tilde{u}(\overline{\theta}, D^\star)$$

$$\text{s.t.}$$

$$p \quad \text{non-decreasing}$$

$$\int_0^{D(\overline{\theta})} D^{-1}(y)dy - \overline{\theta}D(\overline{\theta}) - \tilde{u}(\overline{\theta}, D) \geq G^* \qquad \forall\ D \in \mathcal{D}$$

$$p(\overline{\theta}) = \overline{\theta}$$

- $p^{\mathrm{OPT}}(\theta) = \min\{z^\star(\theta), \overline{\theta}\}$, along with $\{\tilde{u}^{\mathrm{OPT}}(\overline{\theta}, D)\}$ chosen to satisfy remaining robustness constraints (blue inequalities), solves relaxed program

- Because $p^{\mathrm{OPT}}(\theta) \geq \theta$ all θ, remaining robustness constraint satisfied

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Quantity Regulation Dominates

- Robustness under price mechanism $\Rightarrow$ **price cap** at $\overline{\theta}$
- When $D^{\star}(\overline{\theta}) > \underline{D}(\overline{\theta})$

  $\Rightarrow$ **over-procurement (vis-a-vis quantity mechanism) for high costs**
- When BM-with-quantity-floor mechanism robustly optimal

  $\Rightarrow$ **quantity regulation dominates**

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Price Regulation Dominates

- When BM-with-quantity-floor mechanism **not** robustly optimal

  $\Rightarrow$ **downward qty adjustments** for intermediate costs (under D^* vis-a-vis q^{BM})

  - adjustments necessary to avoid over-procurement when $D < D^\star$
  - under price regulation, demand adjusts itself
  - adjustment do not occur when $D = D^*$

- If price cap at $\overline{\theta}$ does not induce over-procurement vis-a-vis q_ℓ

  $\Rightarrow$ **price regulation strictly dominates**

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Qty vs Price: Primitives

Let $\underline{\delta} \equiv \inf\{D^*(\theta) - \underline{D}(\theta) : \theta \in \Theta\}$, $\bar{\delta} \equiv \sup\{D^*(\theta) - \underline{D}(\theta) : \theta \in \Theta\}$, $\underline{f} \equiv \inf\{f^\star(\theta) : \theta \in \Theta\}$ and $\bar{f} \equiv \sup\{f^\star(\theta) : \theta \in \Theta\}$.

Proposition

Suppose $D^\star$ is bi-Lipschitz continuous over $[0, P^\star(0)]$ with constants $0 \leq k \leq K$.

- *Quantity regulation dominates if, for all $\theta \in \Theta$,*

$$\frac{F^\star(\theta)}{f^\star(\theta)} \geq \frac{D^*(\theta) - \underline{D}(\theta)}{k},$$

  (strictly if $D^*(\bar{\theta}) - \underline{D}(\bar{\theta}) > 0$)

- *When, instead, for all $\theta \in \Theta$,*

$$\frac{F^\star(\theta)}{f^\star(\theta)} \leq \frac{D^*(\theta) - \underline{D}(\theta)}{K\left(1 + \frac{\bar{\delta}}{\underline{\delta}}\sqrt{\frac{2\bar{f}}{\underline{f}}}\right)},$$

and $\underline{\delta} > 0$, price regulation strictly dominates.

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General Cost Uncertainty

- Suppose there exist $\underline{F} \in \text{CDF}(\Theta)$ s.t.

$$\mathcal{F} = \{F \in \text{CDF}(\Theta) : \underline{F}(\theta) \leq F(\theta), \quad \forall\ \theta\}$$

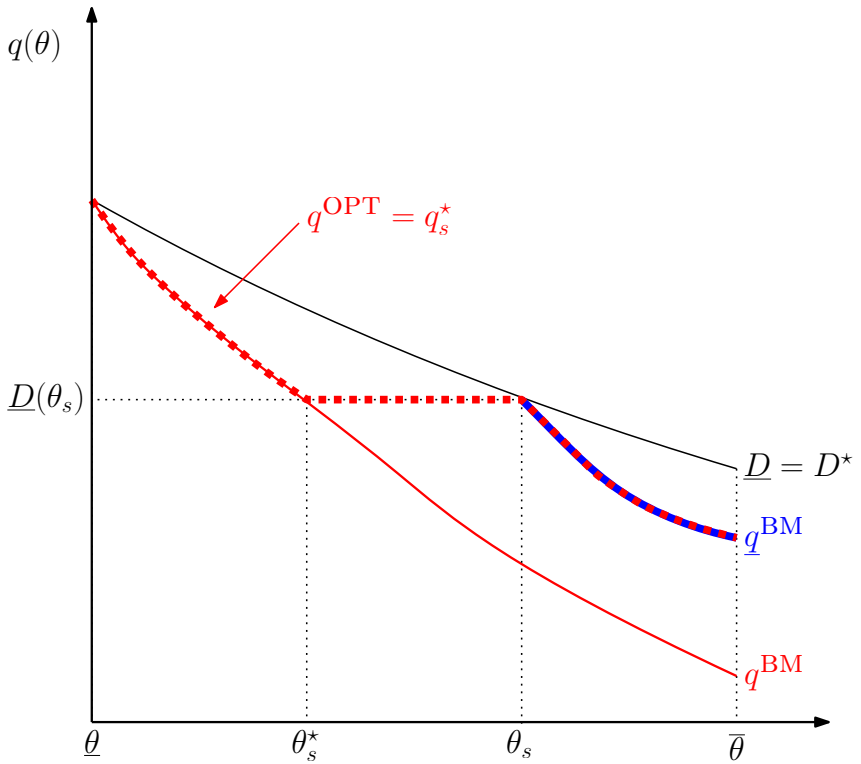
  with $F^\star \in \mathcal{F}$

- So far, $\underline{F}(\theta) = \mathbb{I}(\theta \geq \overline{\theta})$

- Generalization also accommodates for unbounded Θ

General Cost Uncertainty

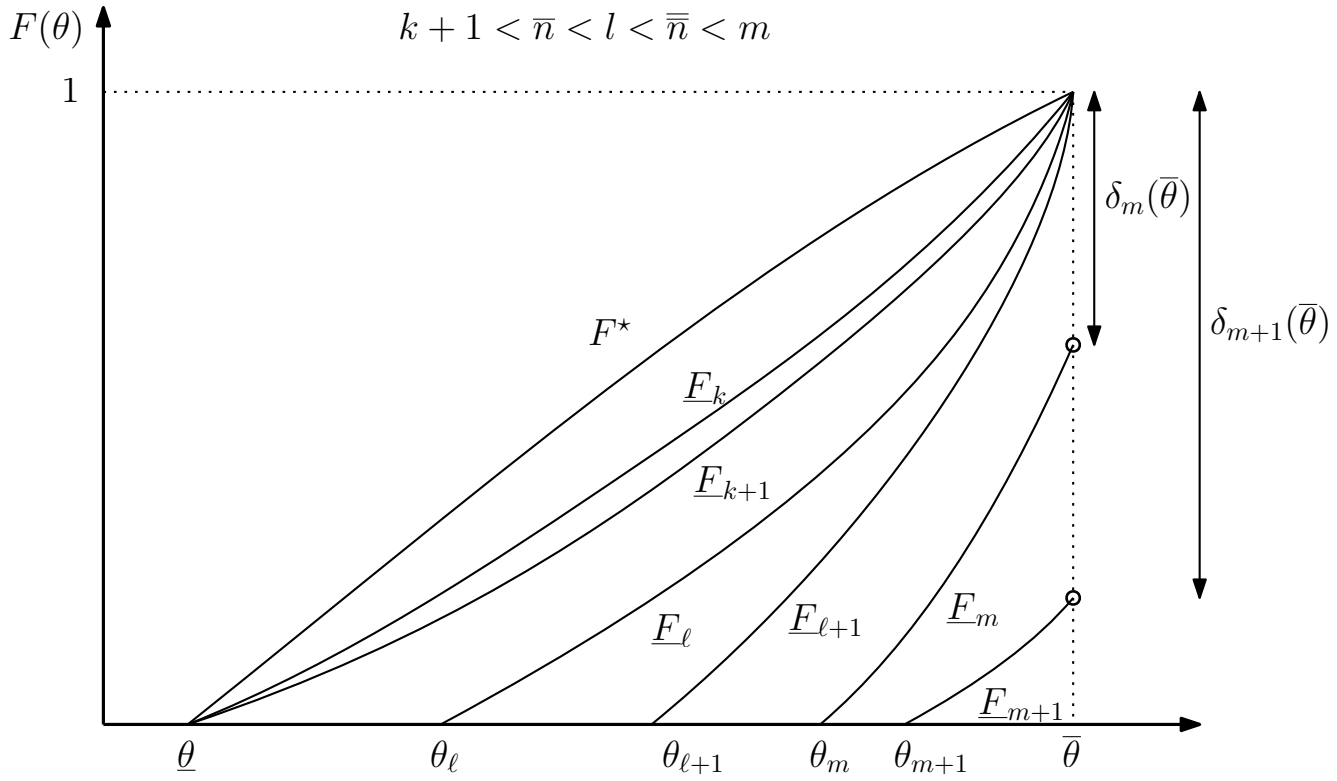
- Suppose $\underline{F}(\theta) = 0$ for all $\theta \leq \theta_s$, with $\theta_s \in (\underline{\theta}, \overline{\theta})$, and regular over $[\theta_s, \overline{\theta})$
- For simplicity, $D^\star = \underline{D}$



- Optimal mechanism bridges two BM schedules
  - Low costs: BM under $(F^*, \underline{V})$
  - High costs: BM under $(\underline{F}, \underline{V})$
  - No distortion at top **and middle** (θ_s)

Variations in Cost Uncertainty

- Suppose $\overline{\theta}$ finite
- (Monotone) sequence $(\underline{F}_n)$ converging to $\mathbb{I}(\theta \geq \overline{\theta})$



Variations in Cost Uncertainty

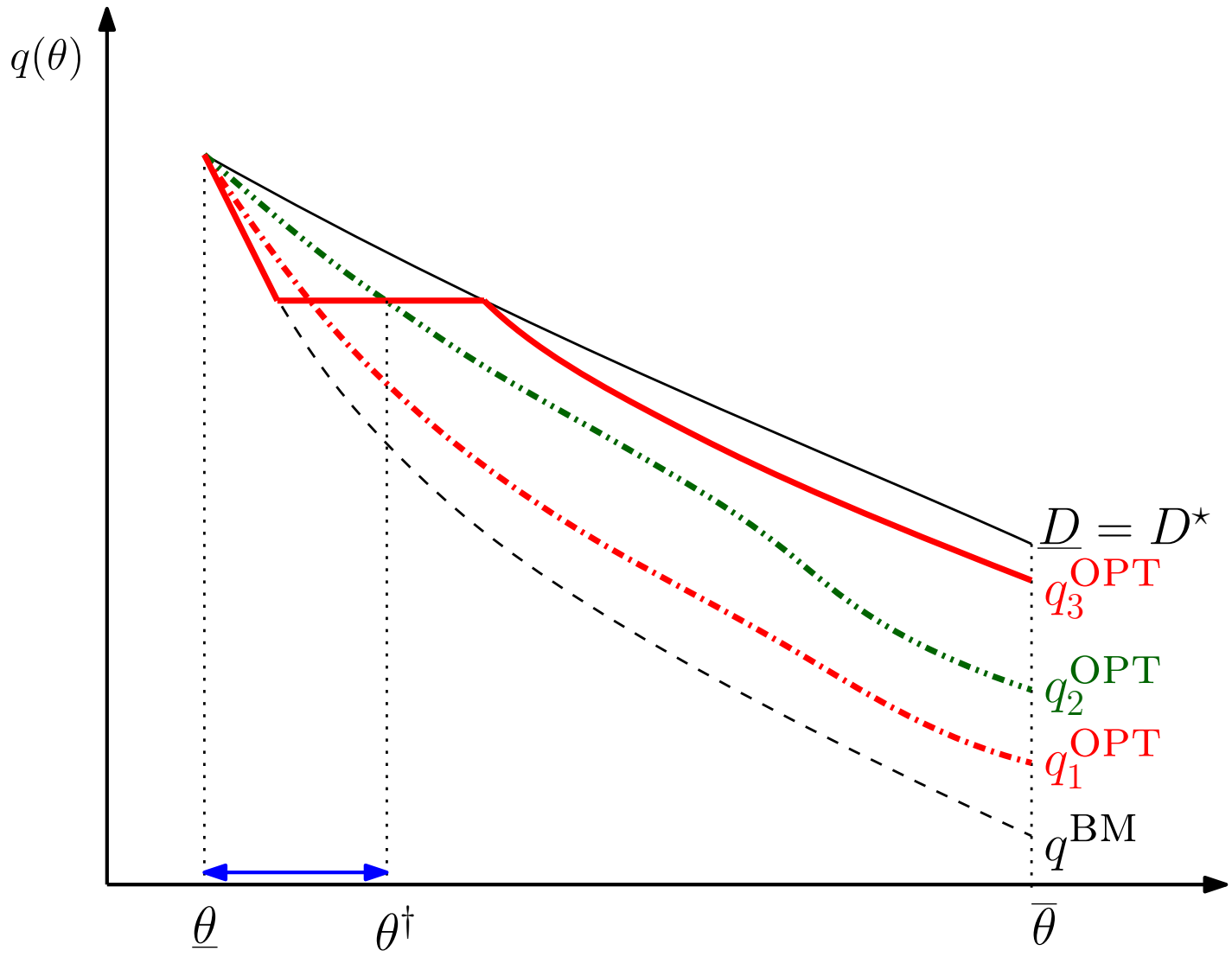
Proposition

Suppose $D^\star = \underline{D}$. *There exist monotone sequences* $(\underline{F}_n)$ *s.t., for every* $\theta \in (\underline{\theta}, \overline{\theta})$, *there exists* $n(\theta) \in \mathbb{N}$ *s.t.*

$$q_{n+1}^{OPT}(\theta) \left\{ \begin{array}{c} \geq \\ \leq \end{array} \right\} q_n^{OPT}(\theta) \quad \textit{for} \quad \left\{ \begin{array}{c} n \leq n(\theta) - 1 \\ n > n(\theta) \end{array} \right\}$$

Moreover, there exists $j, k \in \mathbb{N}$ *with* $j < k$, *s.t.* $q_k^{OPT}(\theta) < q_j^{OPT}(\theta)$

Variations in Cost Uncertainty



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