Adversarial Coordination and Public Information Design

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Motivation

- Coordination: central to many socio-economic environments

- Damages to society of mkt coordination on undesirable actions can be severe
  - Monte dei Paschi di Siena (MPS)
  - creditors + speculators with heterogenous beliefs about size of nonperforming loans
  - default by MPS: major crisis in Eurozone (and beyond)

- Government intervention
  - limited by legislation passed in 2015

- Information Design (e.g., stress testing): instrument of last resort
Questions

- Structure of optimal policy?
  - What information should be passed on to mkt?
- “Right” notion of transparency?
- Optimality of
  - pass/fail policies
  - monotone rules
- Properties of persuasion in global games?
Related literature

- **Adversarial Coordination/Unique Implementation:** Segal (2003), Winter (2004), Sakovics and Steiner (2012), Frankel (2017), Halac et al. (2020), Halac et al. (2021)...


Plan

- Baseline Model
- Perfect Coordination Property
- Pass/Fail
- Monotone Policies
- Enrichments
- Micro-foundations
Global Games of Regime Change

- Specific game in spirit of Rochet and Vives (2004)
- Information designer: Policy maker (PM)
- Agents: investors, $i \in [0, 1]$
- Actions
  
  $a_i = \begin{cases} 
  1 & \text{(pledge)} \\
  0 & \text{(not pledge)}
  \end{cases}$

- $A \in [0, 1]$: aggregate pledge
- Regime change: default
- Default outcome: $r \in \{0, 1\}$, with
  
  $r = \begin{cases} 
  0 \text{ (default)} & \text{if } A \leq 1 - \theta \\
  1 & \text{if } A > 1 - \theta
  \end{cases}$

- “fundamentals” $\theta$: liquidity, performing loans, etc.
- Supermodular game w. dominance regions: $(-\infty, 0]$ and $(1, +\infty)$
- $\theta$ drawn from abs. continuous cdf $F$, smooth density $f$
PM’s payoff

\[ U^P(\theta, A) = \begin{cases} 
W(\theta) > 0 & \text{if } r = 1 \\
L(\theta) < 0 & \text{if } r = 0
\end{cases} \]

Agents’ payoff from not pledging (safe action) normalized to zero

Agents’ payoff from pledging

\[ u = \begin{cases} 
g(\theta) > 0 & \text{if } r = 1 \\
b(\theta) < 0 & \text{if } r = 0
\end{cases} \]
Beliefs

- \( x \equiv (x_i)_{i \in [0,1]} \in X \): signal profile with each
  \[
  x_i \sim p(\cdot|\theta)
  \]
i.i.d., given \( \theta \).

- \( p(x|\theta) \) strictly positive over an open interval \( \varrho_{\theta} \equiv (\underline{\varrho}_{\theta}, \bar{\varrho}_{\theta}) \) containing \( \theta \).

- \( X(\theta) \subset \mathbb{R}^{[0,1]} \): collection of signal profiles consistent with \( \theta \)

Example 1: \( x_i = \theta + \sigma \xi_i \) with \( \xi_i \sim N(0, 1) \)
Example 2: \( x_i = \theta + \sigma \zeta_i \) with \( \zeta_i \sim U(-1, 1) \)
Disclosure Policies (Stress Tests)

- **Stress Test** $\Gamma = (S, \pi)$
  - $S$: set of scores/grades/disclosures
  - $\pi: \Theta \rightarrow \Delta(S)$
Timing

1. PM announces $\Gamma = (S, \pi)$ and commits to it

2. $(\theta, x)$ realized

3. $s \in S$ drawn from $\pi (\theta)$ and publicly announced

4. Agents simultaneously choose whether or not to pledge

5. Default outcome and payoffs
Solution Concept: MARP

- Robust/adversarial approach

- PM does not trust her ability to coordinate mkt on her favorite course of action

- **Most Aggressive Rationalizable Profile (MARP):**
  - minimizes PM’s payoff across all profiles surviving *iterated deletion of interim strictly dominated strategies* (IDISDS)

- \( a^\Gamma \equiv (a_i^\Gamma)_{i \in [0,1]} \): MARP consistent with \( \Gamma \) (\( a_i^\Gamma \): complete plan of action)
Perfect Coordination Property [PCP]

Definition 1

\( \Gamma = \{ S, \pi \} \) satisfies PCP if, for any \( \theta \in \Theta \), any exogenous information \( x \in X(\theta) \), any \( s \in \text{supp}(\pi(\theta)) \), and any pair of individuals \( i, j \in [0, 1] \),

\[ a_\Gamma^i(x_i, s) = a_\Gamma^j(x_j, s), \]

where \( a_\Gamma \equiv (a_\Gamma^i)_{i \in [0,1]} \) is MARP consistent with \( \Gamma \).
Theorem 1

Given any (regular) $\Gamma$, there exists (regular) $\Gamma^*$ satisfying PCP and s.t., at any $\theta$, default probability under $\Gamma^*$ same as under $\Gamma$.

- Regularity: MARP well defined

(formal proof)
Perfect Coordination Property [PCP]

- Policy $\Gamma^* = (S^*, \pi^*)$ removes any **strategic uncertainty**
- It preserves **structural uncertainty**
- Under $\Gamma^*$, agents know actions all other agents but not **beliefs rationalizing such actions**
- Inability to predict beliefs that rationalize other agents’ actions essential to minimization of default risk
- “Right” form of transparency
  - conformism in beliefs about mkt response
  - ...not in beliefs about “fundamentals”
Optimal policy combines:

- public **Pass/Fail** announcement

- eliminate strategic uncertainty

- additional disclosures necessary to guarantee that, when $r = 1$ announced (i.e., when bank passed test), all agents pledge under MARP
When is optimal policy binary?

**Theorem 2**

Assume \( p(x|\theta) \) satisfies MLRP. Given any policy \( \Gamma \) satisfying PCP, there exists **binary policy** \( \Gamma^* = (\{0, 1\}, \pi^*) \) also satisfying PCP and s.t., for any \( \theta \), prob of default under \( \Gamma^* \) same as under \( \Gamma \).

- MARP in threshold strategies: signals other than regime outcome can be dropped (averaging over \( s \)) without affecting incentives
- Result hinges on Log-SM of \( p(x|\theta) \), i.e., on MLRP
  - co-movement between state \( \theta \) and belies

(Example-PF)
Optimality of Monotone Tests

Figure: Optimal Monotone Policy.
\[ \bar{x}_G \equiv \sup \left\{ x : \int_{\Theta} u(\theta, 1 - P(x|\theta)) \mathbb{I}(\theta \geq 0)p(x|\theta) \, dF(\theta) \leq 0 \right\} \]

**Condition M**: Following properties hold:

1. \( \inf \{ \theta \in \Theta : \bar{x}_G \in \varrho_\theta \} \leq 0; \)
2. \( p(x|\theta) \) and \( |u(\theta, 1 - P(x|\theta))| \) (weakly) log-supermodular over \( \{(\theta, x) \in [0, 1] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\} ; \)
3. \( \forall \theta_0, \theta_1 \in [0, 1] \), with \( \theta_0 < \theta_1 \), \( \forall x \leq \bar{x}_G \) s.t. (a) \( u(\theta_1, 1 - P(x|\theta_1)) \leq 0 \) and (b) \( x \in \varrho_{\theta_0}, \)

\[
\frac{U^P(\theta_1, 1) - U^P(\theta_1, 0)}{U^P(\theta_0, 1) - U^P(\theta_0, 0)} > \frac{p(x|\theta_1) u(\theta_1, 1 - P(x|\theta_1))}{p(x|\theta_0) u(\theta_0, 1 - P(x|\theta_0))}
\]

**Theorem 3**

Suppose \( p(x|\theta) \) log-supermodular, Condition M holds. Given any \( \Gamma \), there exists deterministic binary monotone \( \Gamma^* = (\{0, 1\}, \pi^*) \) satisfying PCP and yielding payoff weakly higher than \( \Gamma \).
Sub-optimality of Monotone Tests

Example 1

Suppose that, for any $\theta$,

(a) $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$, and $L(\theta) = L$;

(b) $\theta \sim U[-K, 1+K]$, $K \in \mathbb{R}_{++}$;

(c) $x_i = \theta + \sigma \epsilon_i$, with $\sigma \in \mathbb{R}_+$ and $\epsilon_i \sim U[-1, 1]$, with $\sigma < K/2$.

There exists $\sigma^\# \in (0, K/2)$ such that, for all $\sigma \in (0, \sigma^\#)$, there exists (deterministic) non-monotone policy satisfying PCP that yields payoff strictly higher than optimal monotone policy.
Optimality of Monotone Tests
Sub-optimality of Monotone Tests

- Let $\theta^{MS} \in (0, 1)$ be implicitly defined by $\int_0^1 u(\theta^{MS}, l) dl = 0$

- $D^\Gamma \equiv \{d_i = (\theta_i, \bar{\theta}_i) : i = 1, \ldots, N\}$: partition of $[0, \theta^{MS}]$ induced by deterministic $\Gamma$

- $\Delta (\Gamma) \equiv \max_{i=1,\ldots,N} |\bar{\theta}_i - \theta_i|$: mesh of $D^\Gamma$

Example 2

Suppose $\theta \sim U[\mathbb{R}]$ and $x_i = \theta + \sigma \varepsilon_i$, with $\varepsilon_i \sim N(0, 1)$. Assume that, for any $\theta$, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$ and $L(\theta) = L$. There exists $\bar{\sigma} > 0$ and $\mathcal{E} : (0, \bar{\sigma}] \rightarrow \mathbb{R}_+$, with $\lim_{\sigma \rightarrow 0^+} \mathcal{E}(\sigma) = 0$, s.t., for any $\sigma \in (0, \bar{\sigma}]$, the following is true: given any deterministic binary $\Gamma$ satisfying PCP and s.t. $\Delta (\Gamma) > \mathcal{E}(\sigma)$, there exists another deterministic binary $\Gamma^*$ with $\Delta (\Gamma^*) < \mathcal{E}(\sigma)$ that also satisfies PCP and yields payoff strictly higher than $\Gamma$. 
Extensions

- Default iff $R(\theta, A, z) \leq 0$
  - $z$ drawn from $Q_\theta$: residual uncertainty

- PM’s payoff
  $$\hat{U}^P(\theta, A, z) = \begin{cases} 
  \hat{W}(\theta, A, z) & \text{if } r = 1 \\
  \hat{L}(\theta, A, z) & \text{if } r = 0
  \end{cases}$$

- Agents’ payoffs
  $$\hat{u}(\theta, A, z) = \begin{cases} 
  \hat{g}(\theta, A, z) & \text{if } r = 1 \\
  \hat{b}(\theta, A, z) & \text{if } r = 0
  \end{cases}$$

- Expected payoff differential: $u(\theta, A)$
**Generalizations**

**Condition FB.** For any \( x \), \( u(\theta, 1 - P(x|\theta)) \geq 0 \) (alternatively, \( u(\theta, 1 - P(x|\theta)) \leq 0 \)) implies \( u(\theta'', 1 - P(x|\theta'')) > 0 \) for all \( \theta'' > \theta \) (alternatively, \( u(\theta', 1 - P(x|\theta')) < 0 \) for all \( \theta' < \theta \)).

**Condition PCP.** For any \( \Lambda \in \Delta(\Delta(\Theta)) \) consistent with \( F \)

\[
\int \left( \int_{-\infty}^{\theta^G} U^P(\theta, 0) G(d\theta) + \int_{\theta^G}^{+\infty} U^P(\theta, 1) G(d\theta) \right) \Lambda(dG) \geq \\
\int \left( \int U^P(\theta, 1 - P(\xi^G|\theta)) G(d\theta) \right) \Lambda(dG)
\]

\( \xi^G \): MARP given \( G \)
\( \theta^G \equiv \inf \{ \theta : u(\theta, 1 - P(\xi^G|\theta)) \geq 0 \} \)
Generalizations

**Theorem 4**

(a) Given any $\Gamma$, there exists $\Gamma^*$ satisfying PCP and s.t., for any $\theta$, agents' expected payoff under $\sigma^{\Gamma^*}$ is at least as high as under $\sigma^{\Gamma}$. PM’s payoff under $\Gamma^*$ at least as high as under $\Gamma$.

(b) Suppose $p(x|\theta)$ satisfies MLRP; then $\Gamma^*$ binary.

(c) Suppose condition $M$ holds. Then $\Gamma^*$ monotone.

- PCP: announcement of sign of agents’ expected payoff under MARP
Comparative statics: increase in uncertainty

- Former liabilities: $D$

- Bank’s legacy asset delivers
  - $l(\theta) \in \mathbb{R}$ end of period 1
  - $C(\theta)$ end of period 2

- Bank can issue (i) **new shares** OR (ii) **short-term debt**

- Potential investors submit market orders

- Noise traders $z \sim Q_\theta$
Comparative statics: increase in uncertainty

- $Y(p, \theta, z)$: exogenous demand for shares (alternatively, debt)

- Market clearing price $p^*(\theta, A, z)$ solves

  $$q + 1 - A = A + Y(p^*, \theta, z).$$

- Default:

  $$R(\theta, A, z) = l(\theta) + \rho_s q p^*(\theta, A, z) - D \leq 0$$
Comparative statics: increase in uncertainty

Analysis can be used to study

- effect of different recapitalization policies
  - \((q_E, q_D)\)

- role of uncertainty for toughness of optimal stress tests
  - uncertainty about bank’s profitability: \(\sigma\)
  - uncertainty about macro variables: \(z\)

Proposition 1

There exists \(\bar{\sigma} > 0\) such that, for any \(\sigma, \sigma' \in (0, \bar{\sigma}]\), with \(\sigma' > \sigma\):

\[
\theta^*_E(\sigma') < \theta^*_E(\sigma) \text{ and } \theta^*_D(\sigma') > \theta^*_D(\sigma).
\]
Conclusions

- Public information design under adversarial coordination

- Key properties:
  - Perfect coordination property ("right" notion of transparency)
  - Optimality of Pass/Fail policies
  - Monotone rules

- Extension 1: PM uncertain about mkt’s beliefs
  - robust-undominated design (see also Dworczak & Pavan (2021))

- Extension 2: Elicitation and persuasion (see also Inostroza (2021))
THANKS!
Let \( r(\omega; a^\Gamma) \in \{0, 1\} \) be default outcome at \( \omega \equiv (\theta, x, s) \) when agents play according to \( a^\Gamma \).

Let \( \Gamma^* = \{S^*, \pi^*\} \) be s.t. \( S^* = S \times \{0, 1\} \) and
\[
\pi^*((s, r(\omega; a^\Gamma))|\theta) = \pi(s|\theta), \text{ all } (\theta, s) \text{ s.t. } \pi(s|\theta) > 0
\]

After receiving \( s^* \equiv (s, 1) \), agents use Bayes' rule to update beliefs about \( \omega \equiv (\theta, x, s) \):
\[
\partial \Lambda^\Gamma_i(\omega|x_i, (s, 1)) = \frac{1\{r(\omega; a^\Gamma) = 1\}}{\Lambda^\Gamma_i(1|x_i, s)} \partial \Lambda^\Gamma_i(\omega|x_i, s)
\]

where
\[
\Lambda^\Gamma_i(1|x_i, s) \equiv \int_{\{\omega:r(\omega;a^\Gamma)=1\}} \ d\Lambda^\Gamma_i(\omega|x_i, s)
\]
Let $a_{\Gamma}^{(n)}, a_{\Gamma^*}^{(n)}$ be most aggressive profile surviving $n$ round of IDISDS under $\Gamma$ and $\Gamma^*$, respectively.

**Definition 2**

Strategy profile $a_{\Gamma^*}^{(n)}$ less aggressive than $a_{\Gamma}^{(n)}$ iff, for any $i \in [0, 1]$,  

$$a_{\Gamma}^{(n),i}(x_i, s) = 1 \implies a_{\Gamma^*}^{(n),i}(x_i, (s, 1)) = 1$$

**Lemma 1**

For any $n$, $a_{\Gamma^*}^{(n)}$ less aggressive than $a_{\Gamma}^{(n)}$
Induction

Let $a^\Gamma_0 = a^\Gamma_0^*$ be strategy profile where all agents refrain from pledging, regardless of their (endogenous and exogenous) information.

Suppose that $a^\Gamma_{(n-1)}^*$ less aggressive than $a^\Gamma_{(n-1)}$

Note that $r(\omega|a^\Gamma) = 0 \Rightarrow r(\omega|a^\Gamma_{(n-1)}) = 0$

Hence, $r(\omega; a^\Gamma) = 1$ “removes” from support of agents’ beliefs states $\omega = (\theta, x, s)$ for which default occurs under $a^\Gamma_{(n-1)}$
Payoffs from pledging in case of default are negative
Payoff from \textbf{pledging} under $\Gamma^*$ when agents follow $a_{(n-1)}^\Gamma$

\[
U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^\Gamma) = \frac{\int_\omega u(\theta, A(\omega; a_{(n-1)}^\Gamma))1\{r(\omega; a^\Gamma)=1\}d\Lambda_i^\Gamma(\omega|x_i,s)}{\Lambda_i^\Gamma(1|x_i,s)} > \frac{\int_\omega u(\theta, A(\omega; a_{(n-1)}^\Gamma))d\Lambda_i^\Gamma(\omega|x_i,s)}{\Lambda_i^\Gamma(1|x_i,s)} = \frac{U_i^\Gamma(x_i, s; a_{(n-1)}^\Gamma)}{\Lambda_i^\Gamma(1|x_i,s)}
\]

Hence, $U_i^\Gamma(x_i, s; a_{(n-1)}^\Gamma) > 0 \Rightarrow U_i^{\Gamma^*}(x_i, (s, 1); a_{(n-1)}^\Gamma) > 0$
That \( a_{(n-1)}^{\Gamma^*} \) less aggressive than \( a_{(n-1)}^{\Gamma} \) along with supermodularity of game implies that

\[
U^*_{i} (x_i, (s, 1); a_{(n-1)}^{\Gamma}) > 0 \Rightarrow U^*_{i} (x_i, (s, 1); a_{(n-1)}^{\Gamma^*}) > 0
\]

As a consequence,

\[
a_{(n),i}^{\Gamma} (x_i, s) = 1 \Rightarrow a_{(n),i}^{\Gamma^*} (x_i, (s, 1)) = 1
\]

This means that \( a_{(n)}^{\Gamma^*} \) less aggressive than \( a_{(n)}^{\Gamma} \).
Above lemma implies MARP under $\Gamma^*$, $a^\Gamma^* \equiv a^\Gamma^{(\infty)}$, less aggressive than MARP under $\Gamma$, $a^\Gamma \equiv a^{(\infty)}$

In turn, this implies that $r(\omega; a^\Gamma) = 1$ makes it common certainty that $r(\omega; a^\Gamma^*) = 1$

Hence, all agents pledge after hearing that $r(\omega; a^\Gamma) = 1$

Similarly, $r(\omega; a^\Gamma) = 0$ makes it common certainty that $\theta \leq 1$. Under MARP, all agents refrain from pledging when hearing that $r(\omega; a^\Gamma) = 0$
Assume $g(\theta) = k, b(\theta) = -k$

Pledging rationalizable iff $Pr(r = 1) \geq 1/2$
Example PF/Suboptimality

No disclosure: under MARP, $a_i^\Gamma(x_i) = 0$, all $x_i$
Example P/F Suboptimality

- Suppose PM informs agents of whether \( \theta \) is extreme or intermediate
- \( a_i^\Gamma(x_i, s) = 1 \), all \((x_i, s)\)
Example P/F Suboptimality

If, instead, PM only recommends to pledge (equivalently, $\Gamma$ is pass/fail):

$$a_i^\Gamma(x_i, 1) = 0 \text{ for all } x_i$$

Suboptimality of P/F policies (+ failure of RP)