Attention, Coordination, and Bounded Recall

Online Supplement

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Abstract

This supplement contains the formal statement (together with its proof) of a claim at the end of Section 3 in the main text that "Results qualitatively similar to those discussed in Section 3.2 (Proposition 3) obtain in economies in which, for any z, the planner expects the agents to take their actions according to the equilibrium rule $\mathrm{k}(\cdot;z)$ of Proposition 1 instead of the efficient rule $\mathrm{k}^*(\cdot;z)$ of Proposition 2."

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A Private vs Social Value of Attention when Information is Used According to Equilibrium Rule

Claim. *Suppose that, for any allocation of attention z, the planner expects the agents to use the information according to the equilibrium rule* $\mathrm{k}(\cdot|z)$ *instead of the efficient rule* $\mathrm{k}^*(\cdot|\mathrm{z})$. *Let $\hat{z}$ be the equilibrium allocation of attention.*

(a) Consider economies in which the inefficiencies (in the allocation of attention and in the subsequent usage of information) originate in the discrepancy between the sensitivity of the complete-information equilibrium actions and the first-best actions to the fundamentals ($u_{\sigma\sigma} = 0$, $\alpha = \alpha^*$, but $\kappa_1 \neq \kappa_1^*$). Starting from $\hat{z}$, having the agents increase their attention to any source for which $\hat{z}_n > 0$ increases welfare if $(\kappa_1^* - \kappa_1)/\kappa_1 > 0$, and decreases it if the inequality is reversed.

(b) Consider economies in which the inefficiencies (in the allocation of attention and in the subsequent usage of information) originate in the discrepancy between the equilibrium and the socially-optimal degrees of coordination ($u_{\sigma\sigma} = 0$, $\kappa_1 = \kappa_1^*$, but $\alpha \neq \alpha^*$). There exists a threshold $R^* > 0$ such that, starting from $\hat{z}$, having the agents increase their attention to any source for which $\hat{z}_n > 0$ increases welfare if

$$\alpha - \alpha^* \overset{\text{sgn}}{=} \{R^* - r_n(\hat{z})\},$$

and decreases it otherwise, where, for any $n = 1, ..., N$, any $z \in \mathbb{R}_+^N$, $r_n(z) \equiv t_n / C_n'(z)$.

Proof of the Claim. The proof is in two steps. Step 1 characterizes the gross marginal benefit of increasing the attention to any given source of information, accounting for the effects of such an increase on the subsequent usage of information. Step 2 uses the result in step 1 to establish the results in the proposition.

Step 1. First observe that, for any given z, welfare under the equilibrium strategy $\mathrm{k}(\cdot; z)$ is given by[1]

$$w(z) \equiv \mathbb{E}[u(\tilde{k}, \tilde{K}, \sigma_k, \tilde{\theta}) \mid z] - C(z) = \mathbb{E}[W(\tilde{\kappa}, 0, \tilde{\theta})] - \mathcal{L}(z) - C(z), \tag{A.1}$$

where $\tilde{\kappa} = \kappa_0 + \kappa_1 \tilde{\theta}$. The function $W(K, 0, \theta) \equiv u(K, K, 0, \theta)$ is the payoff that each agent obtains when, given the fundamentals θ, all agents take the same action, i.e., $k^i = k^j = K$ all $i, j \in [0, 1]$ ($W(\kappa, 0, \theta)$ is thus welfare under the complete-information equilibrium allocation), whereas

$$\begin{aligned}\mathcal{L}(z) \equiv \frac{|u_{kk} + u_{\sigma\sigma}|}{2} \cdot Var[\tilde{k}^j - \tilde{K} \mid z)] + \frac{|u_{kk} + 2u_{kK} + u_{KK}|}{2} \cdot Var[\tilde{K} - \tilde{\kappa} \mid z] \\ - Cov\left[\tilde{K} - \tilde{\kappa}, W_K(\tilde{\kappa}, 0, \tilde{\theta}) \mid z\right]\end{aligned}$$

are the welfare losses due to the asymmetry of information. The first two terms in $\mathcal{L}$ measure the welfare losses due to, respectively, the dispersion of individual actions around the aggregate action and the volatility of the aggregate action around its complete-information counterpart. The last term captures losses (or gains) due to the correlation between the 'aggregate error' in the agents'

[1]The representation of equilibrium welfare in (A.1) follows from the same steps as in Angeletos and Pavan (2007); the proof is thus omitted.

actions due to asymmetric information, $K-\kappa$, and W_K, the social return to aggregate activity. Following steps similar to those in Angeletos and Pavan (2007), one can show that

$$Cov\left[\tilde{K}-\tilde{\kappa}, W_K(\tilde{\kappa},0,\tilde{\theta}) \mid z\right] = |u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\frac{\sum_n \gamma_n(z)-1}{\pi_\theta},$$

$$Var[\tilde{K}-\tilde{\kappa} \mid z] = \kappa_1^2\frac{\left(\sum_{s=1}^N \gamma_s(z)-1\right)^2}{\pi_\theta} + \kappa_1^2\sum_{s=1}^N \frac{(\gamma_s(z))^2}{\eta_s},$$

and

$$Var[\tilde{k}^j-\tilde{K} \mid z)] = \kappa_1^2\sum_{s=1}^N \frac{(\gamma_s(z))^2}{z_s t_s}.$$

For any z, welfare when all actions are taken under the equilibrium rule $\mathrm{k}(\cdot;z)$ can thus be expressed as

$$\begin{aligned} w(z) = \mathbb{E}[W(\tilde{\kappa},0,\tilde{\theta})] &- \frac{|u_{kk}+u_{\sigma\sigma}|\kappa_1^2}{2}\sum_{s=1}^N \frac{(\gamma_s(z))^2}{z_s t_s} \\ &- \frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{2}\left[\frac{\left(\sum_{s=1}^N \gamma_s(z)-1\right)^2}{\pi_\theta} + \sum_{s=1}^N \frac{(\gamma_s(z))^2}{\eta_s}\right] \\ &+ |u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2 \cdot \left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\cdot\frac{\sum_{s=1}^N \gamma_s(z)-1}{\pi_\theta} - C(z). \end{aligned}$$

The (gross) marginal effect on welfare of an increase in the attention to the n-th source is thus equal to

$$\begin{aligned} &\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(z))^2}{(z_n)^2 t_n} - |u_{kk}+u_{\sigma\sigma}|\kappa_1^2\sum_{s=1}^N \frac{\gamma_s(z)}{z_s t_s}\frac{\partial\gamma_s(z)}{\partial z_n} \\ &- |u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2\left[\frac{\left(\sum_{s=1}^N \gamma_s(z)-1\right)\sum_{s=1}^N \frac{\partial\gamma_s(z)}{\partial z_n}}{\pi_\theta} + \sum_{s=1}^N \frac{\gamma_s(z)}{\eta_s}\frac{\partial\gamma_s(z)}{\partial z_n}\right] \\ &+ \frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\sum_{s=1}^N \frac{\partial\gamma_s(z)}{\partial z_n}. \end{aligned} \tag{A.2}$$

Substituting $|u_{kk}+2u_{kK}+u_{KK}| = (1-\alpha^*)|u_{kk}+u_{\sigma\sigma}|$, I can rewrite the sum of the second and third addendum in (A.2) as

$$\begin{aligned} &- |u_{kk}+u_{\sigma\sigma}|\kappa_1^2\left[\frac{(1-\alpha^*)\left(\sum_{s=1}^N \gamma_s(z)-1\right)\sum_{s=1}^N \frac{\partial\gamma_s(z)}{\partial z_n}}{\pi_\theta} + \sum_{s=1}^N \left(\frac{1-\alpha^*}{\eta_s}+\frac{1}{z_s t_s}\right)\gamma_s\frac{\partial\gamma_s(z)}{\partial z_n}\right] \\ &= -|u_{kk}+u_{\sigma\sigma}|\kappa_1^2\left[\sum_{s=1}^N \left(\frac{\left(\frac{1-\alpha^*}{\eta_s}+\frac{1}{z_s t_s}\right)\frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)} - (1-\alpha^*)}{\pi_\theta + \sum_{n=1}^N \frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}}\right)\frac{\partial\gamma_s(z)}{\partial z_n}\right]. \end{aligned}$$

Using

$$\pi_s(z) \equiv \frac{\eta_s z_s t_s}{z_s t_s+\eta_s} \text{ and } \rho_s(z) = \frac{z_s t_s}{z_s t_s+\eta_s},$$

I then have that

$$\left(\frac{1-\alpha^*}{\eta_s}+\frac{1}{z_s t_s}\right)\frac{(1-\alpha)\pi_s(z)}{1-\alpha\rho_s(z)}-(1-\alpha^*)=-\frac{\eta_s(\alpha-\alpha^*)}{(1-\alpha)z_s t_s+\eta_s}.$$

The sum of the second and third addendum in (A.2) can thus be rewritten as

$$|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\sum_{s=1}^N\frac{\eta_s}{[(1-\alpha)z_s t_s+\eta_s]\left[\pi_\theta+\sum_{n=1}^N\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}\right]}\frac{\partial\gamma_s(z)}{\partial z_n}.$$

Next, note that

$$\frac{\eta_s}{[(1-\alpha)z_s t_s+\eta_s]\left[\pi_\theta+\sum_{n=1}^N\frac{(1-\alpha)\pi_n(z)}{1-\alpha\rho_n(z)}\right]}=\frac{\gamma_s(z)}{(1-\alpha)z_s t_s}.$$

I conclude that, given any z, the net social marginal benefit of having the agents pay more attention to any source n is equal to

$$\begin{aligned}\frac{\partial w(z)}{\partial z_n}=&\frac{|u_{kk}+u_{\sigma\sigma}|}{2}\frac{(\kappa_1\gamma_n(z))^2}{(z_n)^2 t_n}+|u_{kk}+u_{\sigma\sigma}|\kappa_1^2(\alpha-\alpha^*)\sum_{s=1}^N\frac{\gamma_s(z)}{(1-\alpha)z_s t_s}\frac{\partial\gamma_s(z)}{\partial z_n}\\&+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\sum_{s=1}^N\frac{\partial\gamma_s(z)}{\partial z_n}-C_n'(z).\end{aligned}\tag{A.3}$$

The first term in (A.3) is the direct marginal effect of a reduction in the cross-sectional dispersion of individual actions that obtains as a result of an increase in the attention z_n, holding fixed the equilibrium rule $\mathrm{k}(\cdot;z)$. The second term combines the marginal effects of changing the equilibrium rule $\mathrm{k}(\cdot;z)$ on (a) the volatility of the aggregate action K around its complete-information counterpart κ and (b) the dispersion of individual actions around the mean action. The third term, which is relevant only in economies that are inefficient under complete information, captures the effect of changing the rule $\mathrm{k}(\cdot;z)$ on the way the "error" due to incomplete information $K-\kappa$ covaries with the inefficiency of the complete-information allocation. Clearly, by usual envelope arguments, these last two terms are equal to zero in economies in which the equilibrium use of information is efficient (that is, in which $\mathrm{k}(\cdot;z)=\mathrm{k}^*(\cdot;z)$, which happens if, and only if, $\alpha=\alpha^*$ and $\kappa_1=\kappa_1^*$) or, alternatively, when the planner can induce the agents to use their information efficiently.

Step 2. First consider the economies in part (a) of the proposition. Recall that these are economies in which the inefficiencies in the allocation of attention and in the subsequent usage of information originate in the discrepancy between the sensitivity of the complete-information actions and the first-best actions to the fundamentals (i.e., $u_{\sigma\sigma}=0$, $\alpha=\alpha^*$, but $\kappa_1\neq\kappa_1^*$)). Using (A.3), I have that, in these economies, the social net marginal effect of having the agents increase their attention to any source n, starting from the equilibrium level $\hat{z}$, is equal to[2]

$$\frac{\partial w(\hat{z})}{\partial z_n}=\frac{|u_{kk}|}{2}\frac{(\kappa_1\gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n}+\frac{|u_{kk}+2u_{kK}+u_{KK}|\kappa_1^2}{\pi_\theta}\left(\frac{\kappa_1^*-\kappa_1}{\kappa_1}\right)\sum_{s=1}^N\frac{\partial\gamma_s(\hat{z})}{\partial z_n}-C_n'(\hat{z}).\tag{A.4}$$

[2]If $\hat{z}_n=0$, then interpret the derivative as the right-hand derivative.

The result in part (a) then follows from the fact that, for any z,

$$\sum\nolimits_{s=1}^{N}\gamma_s(z)=\frac{1}{\frac{\pi_\theta}{\sum_{s=1}^{N}\frac{(1-\alpha)\eta_s z_s t_s}{(1-\alpha)z_s t_s+\eta_s}}+1}$$

is increasing in z_n, along with the fact that, in equilibrium, any source n for which $\hat{z}_n>0$ is such that

$$\frac{|u_{kk}|}{2}\frac{\left(\kappa_1\gamma_n(\hat{z})\right)^2}{(\hat{z}_n)^2\,t_n}=C_n'(\hat{z}).$$

Next, consider the economies in part (b). Recall that these are economies in which the inefficiencies in the equilibrium allocation of attention and in the subsequent usage of information originate in the discrepancy between the equilibrium and the socially-optimal degrees of coordination ($u_{\sigma\sigma}=0$, $\kappa_1=\kappa_1^*$, but $\alpha\neq\alpha^*$).

Using (A.3), I have that, in these economies, the net benefit of having the agents increase their attention to any source n evaluated at $z=\hat{z}$ is equal to

$$\frac{\partial w(\hat{z})}{\partial z_n}=\frac{|u_{kk}|}{2}\frac{\left(\kappa_1\gamma_n(\hat{z})\right)^2}{(\hat{z}_n)^2\,t_n}+|u_{kk}|\kappa_1^2(\alpha-\alpha^*)\sum\nolimits_{s=1}^{N}\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\frac{\partial\gamma_s(\hat{z})}{\partial z_n}-C_n'(\hat{z}).$$

Using the fact that, for any source n, irrespective of whether $\hat{z}_n>0$ or $\hat{z}_n=0$, the private net marginal benefit of increasing the attention to the source is equal to

$$\frac{|u_{kk}|}{2}\frac{\left(\kappa_1\gamma_n(\hat{z})\right)^2}{(\hat{z}_n)^2\,t_n}-C_n'(\hat{z}),$$

I then have that the social benefit exceeds the private benefit if and only if

$$\alpha-\alpha^*\overset{\text{sgn}}{=}\sum\nolimits_{s=1}^{N}\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\frac{\partial\gamma_s(\hat{z})}{\partial z_n}.$$

Next, observe that

$$\begin{aligned}
&\sum\nolimits_{s=1}^{N}\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\frac{\partial\gamma_s(\hat{z})}{\partial z_n}\\
&=-\sum\nolimits_{s=1}^{N}\frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s}\frac{\frac{(1-\alpha)\eta_s\hat{z}_s t_s}{(1-\alpha)\hat{z}_s t_s+\eta_s}}{\left(\pi_\theta+\sum_{l=1}^{N}\frac{(1-\alpha)\eta_l\hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l+\eta_l}\right)^2}\frac{\partial}{\partial\hat{z}_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_n t_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\right)\\
&+\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n}\frac{\frac{\partial}{\partial\hat{z}_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_n t_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^{N}\frac{(1-\alpha)\eta_l\hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l+\eta_l}}\\
&=\frac{\frac{\partial}{\partial\hat{z}_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_n t_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^{N}\frac{(1-\alpha)\eta_l\hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l+\eta_l}}\left[\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n}-\sum\nolimits_{s=1}^{N}\left(\frac{(\gamma_s(\hat{z}))^2}{(1-\alpha)\hat{z}_s t_s}\right)\right].
\end{aligned}$$

Clearly,

$$\frac{\frac{\partial}{\partial\hat{z}_n}\left(\frac{(1-\alpha)\eta_n\hat{z}_n t_n}{(1-\alpha)\hat{z}_n t_n+\eta_n}\right)}{\pi_\theta+\sum_{l=1}^{N}\frac{(1-\alpha)\eta_l\hat{z}_l t_l}{(1-\alpha)\hat{z}_l t_l+\eta_l}}>0.$$

Hence,

$$\sum\nolimits_{s=1}^{N} \frac{\gamma_s(\hat{z})}{(1-\alpha)\hat{z}_s t_s} \frac{\partial \gamma_s(\hat{z})}{\partial z_n} \overset{\text{sgn}}{=} \frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} - \sum\nolimits_{s=1}^{N} \frac{(\gamma_s(\hat{z}))^2}{(1-\alpha)\hat{z}_s t_s}.$$

This means that the social benefit exceeds the private benefit if and only if

$$\alpha - \alpha^* \overset{\text{sgn}}{=} \frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} - M_0(\hat{z}),$$

where, for any z,

$$M_0(z) \equiv \sum\nolimits_{s=1}^{N} \frac{(\gamma_s(z))^2}{(1-\alpha) z_s t_s} > 0$$

does not depend on the specific source of information under consideration. Now observe that

$$\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} = \frac{\eta_n}{(1-\alpha)\hat{z}_n t_n + \eta_n} \frac{1}{M_1(\hat{z})}$$

where, for any z,

$$M_1(z) \equiv \pi_\theta + \sum_{s=1}^{N} \frac{(1-\alpha)\eta_s z_s t_s}{(1-\alpha) z_s t_s + \eta_s}.$$

Lastly, use the fact that, for any n such that $\hat{z}_n > 0$,

$$C_n'(\hat{z}) = \frac{|u_{kk}|}{2} \frac{(\kappa_1 \gamma_n(\hat{z}))^2}{(\hat{z}_n)^2 t_n}$$

to note that, for any source that receives strictly positive attention in equilibrium,

$$(1-\alpha)\hat{z}_n t_n + \eta_n = M_2(\hat{z}) \sqrt{\frac{t_n \eta_n^2}{C_n'(\hat{z})}}$$

where, for any z,

$$M_2(z) \equiv \sqrt{\frac{|u_{kk}|}{2}} \kappa_1 M_1(z)(1-\alpha).$$

I conclude that there exists a constant

$$M(\hat{z}) \equiv [M_0(\hat{z}) M_1(\hat{z}) M_2(\hat{z})]^2 > 0$$

such that

$$\frac{\gamma_n(\hat{z})}{(1-\alpha)\hat{z}_n t_n} - M_0(\hat{z}) \overset{\text{sgn}}{=} \frac{C_n'(\hat{z})}{t_n} - M(\hat{z}).$$

Part (b) of the claim then follows from the above results by letting $R^* \equiv 1/M(\hat{z})$. Q.E.D.