Robust Predictions in Dynamic Screening

Daniel Garrett, Alessandro Pavan, Juuso Toikka

March 2018

Long-term contracting

- Benefits of long-term contracting
  - prices/incentives set efficiently over course of relationship
  - agent residual claimant on joint surplus
  - principal extracts rents through "fixed fees"
- In practice: private information (at time of contracting) prevents full surplus extraction
  - limiting agents' rents calls for **distortions**
  - Optimal (profit maximizing) mechanisms trade off **efficiency** and **rent extraction**
- Dynamics of distortions when private info evolves over time?

Long-term contracting

- Benefits of long-term contracting
  - prices/incentives set efficiently over course of relationship
  - agent residual claimant on joint surplus
  - principal extracts rents through "fixed fees"
- In practice: private information (at time of contracting) prevents full surplus extraction
  - limiting agents' rents calls for **distortions**
  - Optimal (profit maximizing) mechanisms trade off **efficiency** and **rent extraction**
- Dynamics of distortions when private info evolves over time?

Long-term contracting

- Benefits of long-term contracting
  - prices/incentives set efficiently over course of relationship
  - agent residual claimant on joint surplus
  - principal extracts rents through "fixed fees"
- In practice: private information (at time of contracting) prevents full surplus extraction
  - limiting agents' rents calls for **distortions**
  - Optimal (profit maximizing) mechanisms trade off **efficiency** and **rent extraction**
- Dynamics of distortions when private info evolves over time?

Dynamic Mechanism Design

- **revenue management** (Courty and Li, 2000, Battaglini 2005, Boleslavsky and Said, 2013, Ely, Garrett and Hinnosaar, 2014, Board and Skrzypacz, 2015, Akan, Ata, and Dana, 2015,..

- **disclosure in auctions** (Eso and Szentes, 2007, Bergemann and Wambach (2015), Li and Shi (2017)...)

- **experimentation** (Bergemann and Välimäki, 2010, Pavan, Segal, and Toikka, 2014, Fershtman and Pavan, 2017...)

- **life-cycle taxation** (Farhi and Werning, 2012, Kapicka, 2013, Stantcheva, 2014, Makris and Pavan,2017,...)

- **managerial compensation** (Garrett and Pavan, 2012, 2014,...)

- **insurance** (Hendel and Lizzeri, 2003, Handel, Hendel, Whinston, 2015,...)

Relaxed Approach

- Standard approach: "**relaxed program**"
  - necessary conditions for IC ("**local**" constraints)
  - ex-post verification of remaining IC constraints
- Relaxed approach (when valid)
  - complete characterization of optimal mechanism
  - optimal mechanism often derived in closed form
  - approach mimics tractability of Myerson's (1981) auctions

Relaxed Approach

- Standard approach: "**relaxed program**"
  - necessary conditions for IC ("**local**" constraints)
  - ex-post verification of remaining IC constraints
- Relaxed approach (when valid)
  - complete characterization of optimal mechanism
  - optimal mechanism often derived in closed form
  - approach mimics tractability of Myerson's (1981) auctions

Existing predictions: stringent conditions

- **Reverse engineering**:
  - conditions (process and payoffs) guaranteeing policies appropriately monotone
    - monotone hazard rate
    - monotone impulse responses
    - decreasing hazard rates
    - dynamic supermodularity
- Central prediction: "**vanishing distortions**
- Prediction hinges on "relaxed approach"?

Existing predictions: stringent conditions

- **Reverse engineering**:
  - conditions (process and payoffs) guaranteeing policies appropriately monotone
    - monotone hazard rate
    - monotone impulse responses
    - decreasing hazard rates
    - dynamic supermodularity
- Central prediction: "**vanishing distortions**
- Prediction hinges on "relaxed approach"?

Existing predictions: stringent conditions

- **Reverse engineering**:
  - conditions (process and payoffs) guaranteeing policies appropriately monotone
    - monotone hazard rate
    - monotone impulse responses
    - decreasing hazard rates
    - dynamic supermodularity
- Central prediction: "**vanishing distortions**
- Prediction hinges on "relaxed approach"?

This paper: Variational Approach

- Variational approach
  - IC-preserving perturbations of putative optimal policies
- **Robust predictions**
  - convergence to efficiency
- **Bounds** on distortions (all horizons)

This paper: Variational Approach

- Variational approach
  - IC-preserving perturbations of putative optimal policies
- **Robust predictions**
  - convergence to efficiency
- **Bounds** on distortions (all horizons)

Plan

- Discrete types
  - dynamics of "wedges" (MB - MC)
  - dynamic of allocations
- Continuum of types
  - payoff equivalence
  - wedges as "handicaps"
  - convergence of expected wedges

Plan

- Discrete types
  - dynamics of “wedges” (MB - MC)
  - dynamic of allocations
- Continuum of types
  - payoff equivalence
  - wedges as “handicaps”
  - convergence of expected wedges

MODEL

Procurement environment

- Canonical procurement a' la Baron-Myerson (1982)
- Players:
  - principal: procurer
  - agent: supplier
- $t = 1, 2, \ldots$ (many results also for finite horizon)
- $q_t \in (0, \bar{q})$: period-t output supplied by agent
- p_t: total period-t payment from principal
- $\delta \in (0, 1)$: common discount factor

Payoffs and information

- Gross value of output to principal: $B : (0, \bar{q}) \to \mathbb{R}$, strictly increasing, strictly concave, twice-continuously differentiable,

$$\lim_{q \searrow 0} B(q) = -\infty.$$

- Agent's period-t cost $C(q_t, h_t)$ with

$$C(q_t, h_t) = h_t q_t + c(q_t)$$

with $c(\cdot) : (0, \bar{q}) \to \mathbb{R}_+$ strictly increasing, strictly convex, twice-continuously differentiable,

$$\lim_{q \nearrow \bar{q}} c(q) = +\infty$$

- Agent period-t "type": $h_t \in \Theta = \{\theta_1, \ldots, \theta_N\}$
  - $0 < \theta_1 < \cdots < \theta_N$
  - h_t privately observed at beginning of period t
  - process $F = (F_t(\cdot|h^{t-1}))$

Payoffs and information

- Gross value of output to principal: $B : (0, \bar{q}) \to \mathbb{R}$, strictly increasing, strictly concave, twice-continuously differentiable,

$$\lim_{q \searrow 0} B(q) = -\infty.$$

- Agent's period-t cost $C(q_t, h_t)$ with

$$C(q_t, h_t) = h_t q_t + c(q_t)$$

with $c(\cdot) : (0, \bar{q}) \to \mathbb{R}_+$ strictly increasing, strictly convex, twice-continuously differentiable,

$$\lim_{q \nearrow \bar{q}} c(q) = +\infty$$

- Agent period-t "type": $h_t \in \Theta = \{\theta_1, \ldots, \theta_N\}$
  - $0 < \theta_1 < \cdots < \theta_N$
  - h_t privately observed at beginning of period t
  - process $F = (F_t(\cdot | h^{t-1}))$

Payoffs and information

- Gross value of output to principal: $B : (0, \bar{q}) \to \mathbb{R}$, strictly increasing, strictly concave, twice-continuously differentiable,

$$\lim_{q \searrow 0} B(q) = -\infty.$$

- Agent's period-t cost $C(q_t, h_t)$ with

$$C(q_t, h_t) = h_t q_t + c(q_t)$$

  with $c(\cdot) : (0, \bar{q}) \to \mathbb{R}_+$ strictly increasing, strictly convex, twice-continuously differentiable,

$$\lim_{q \nearrow \bar{q}} c(q) = +\infty$$

- Agent period-t "type": $h_t \in \Theta = \{\theta_1, \ldots, \theta_N\}$
  - $0 < \theta_1 < \cdots < \theta_N$
  - h_t privately observed at beginning of period t
  - process $F = (F_t(\cdot | h^{t-1}))$

Payoffs

- Principal's intertemporal payoff

$$U^P = \sum_{t \geq 1} \delta^{t-1} \left(B\left(q_t\right) - p_t \right)$$

- Agent's intertemporal payoff

$$U^A = \sum_{t \geq 1} \delta^{t-1} \left(p_t - C\left(q_t, h_t\right) \right)$$

- Agent's outside option: zero
- Principal's payoff in case she fails to procure output: $-\infty$

Mechanisms

- Direct mechanism: $\langle q_t(h^t), p_t(h^t) \rangle_{t=1}^{\infty}$ where

$$h^t = (h_1, h_2, \ldots, h_t) \in \Theta^t$$

Incentive compatibility

- Agent payoff from t onwards under truth-telling

$$V_t\left(h^t\right) = \mathbb{E}\left[\sum_{s=t}^{\infty} \delta^{s-t}\left(p_s\left(\tilde{h}^s\right) - C\left(q_s\left(\tilde{h}^s\right), \tilde{h}_s\right)\right) | h^t\right]$$

- **IC**: for all t, all $h^t \in \Theta^t$, all reporting strategies σ,

$$V_t\left(h^t\right) \geq \mathbb{E}\left[\sum_{s=t}^{\infty} \delta^{s-t}\left(\ p_s^{\sigma}\left(\tilde{h}^s\right) - C\left(q_s^{\sigma}\left(\tilde{h}^s\right), \tilde{h}_s\right) \ \right) | h^t\right]$$

- **IR:** participation requires

$$V_1\left(h_1\right) \geq 0 \text{ all } h_1 \in \Theta.$$

Principal's problem

- Principal chooses $\langle q_t(h^t), p_t(h^t) \rangle_{t=1}^{\infty}$ to maximize

$$\mathbb{E}\left[\sum_{t \geq 1} \delta^{t-1} \left(B\left(q_t\left(\tilde{h}^t\right)\right) - p_t\left(\tilde{h}^t\right)\right)\right]$$

subject to IC and IR constraints.

- $\langle q_t^*(h^t), p_t^*(h^t) \rangle_{t=1}^{\infty}$: solution to principal's problem
- Solution always exist and q^* unique

Principal's problem

- Principal chooses $\langle q_t(h^t), p_t(h^t)\rangle_{t=1}^{\infty}$ to maximize

$$\mathbb{E}\left[\sum_{t\geq 1}\delta^{t-1}\left(B\left(q_t\left(\tilde{h}^t\right)\right)-p_t\left(\tilde{h}^t\right)\right)\right]$$

subject to IC and IR constraints.

- $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$: solution to principal's problem

- Solution always exist and q^* unique

Principal's problem

- Principal chooses $\langle q_t(h^t), p_t(h^t)\rangle_{t=1}^{\infty}$ to maximize

$$\mathbb{E}\left[\sum_{t\geq 1} \delta^{t-1}\left(B\left(q_t\left(\tilde{h}^t\right)\right) - p_t\left(\tilde{h}^t\right)\right)\right]$$

subject to IC and IR constraints.

- $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$: solution to principal's problem

- Solution always exist and q^* unique

Efficiency

- Efficient policy: all t, all $h_t \in \Theta$,

$$B'\left(q^E(h_t)\right) = C_q\left(q^E(h_t), h_t\right)$$

- Payment scheme implementing efficient allocation

$$p^E(h_t) = B\left(q^E(h_t)\right)$$

all t, all $h_t \in \Theta_t$

WEDGES

Convergence of expected "wedges"

Definition

Process satisfies **"Long-run Independence"** if

$$\lim_{t\to\infty} \max_{h_1, h_1', h_t \in \Theta} \left| \Pr\left(\tilde{h}_t \leq h_t | h_1\right) - \Pr\left(\tilde{h}_t \leq h_t | h_1'\right) \right| = 0.$$

Proposition

Suppose F satisfies "Long-run independence."
As $t \to +\infty$,

$$\mathbb{E}\left[B'\left(q_t^*\left(\tilde{h}^t\right)\right) - C_q\left(q_t^*\left(\tilde{h}^t\right), \tilde{h}_t\right)\right] \to 0$$

Suppose distortions always of same sign. Then $q_t^*(\cdot)$ converge to $q^E(\cdot)$ in probability

Convergence of expected "wedges"

Definition

Process satisfies **"Long-run Independence**" if

$$\lim_{t\to\infty} \max_{h_1, h_1', h_t \in \Theta} \left| \Pr\left(\tilde{h}_t \leq h_t | h_1\right) - \Pr\left(\tilde{h}_t \leq h_t | h_1'\right) \right| = 0.$$

Proposition

Suppose F satisfies "Long-run independence."
As $t \to +\infty$,

$$\mathbb{E}\left[B'\left(q_t^*\left(\tilde{h}^t\right)\right) - C_q\left(q_t^*\left(\tilde{h}^t\right), \tilde{h}_t\right)\right] \to 0$$

Suppose distortions always of same sign. Then $q_t^*(\cdot)$ converge to $q^E(\cdot)$ in probability

Proof Sketch

- Difficulty: direction in which IC and IR binds unknown
- Suppose $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$ is optimal (hence IC and IR)
  - then $q_t^*(h^t) \in (0, \bar{q})$ all t, all h^t.
- **Idea**: IC preserving perturbations
  - increase $q_t^*(\cdot)$ uniformly by small amount $\nu > 0$
  - increase period-t payments by $c(q_t^*(h^t)+\nu) - c(q_t^*(h^t))$
  - increase period-1 payments $p_1^*(\cdot)$ uniformly by $\delta^{t-1}\nu \max_{h_1\in\Theta} \mathbb{E}\left[\tilde{h}_t | h_1\right]$
- New mechanism IC and IR
  - IC: additional quantity ν produced *irrespective of reports*!
  - Each type h_1 expects additional rent $\delta^{t-1}\nu\left(\max_{\hat{h}_1\in\Theta} \mathbb{E}\left[\tilde{h}_t | \hat{h}_1\right] - \mathbb{E}\left[\tilde{h}_t | h_1\right]\right)$

Proof Sketch

- Difficulty: direction in which IC and IR binds unknown
- Suppose $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$ is optimal (hence IC and IR)
  - then $q_t^*(h^t) \in (0, \bar{q})$ all t, all h^t.
- **Idea**: IC preserving perturbations
  - increase $q_t^*(\cdot)$ uniformly by small amount $\nu > 0$
  - increase period-t payments by $c(q_t^*(h^t) + \nu) - c(q_t^*(h^t))$
  - increase period-1 payments $p_1^*(\cdot)$ uniformly by $\delta^{t-1}\nu \max_{h_1 \in \Theta} \mathbb{E}\left[\tilde{h}_t | h_1\right]$
- New mechanism IC and IR
  - IC: additional quantity ν produced *irrespective of reports*!
  - Each type h_1 expects additional rent $\delta^{t-1}\nu\left(\max_{\hat{h}_1 \in \Theta} \mathbb{E}\left[\tilde{h}_t | \hat{h}_1\right] - \mathbb{E}\left[\tilde{h}_t | h_1\right]\right)$

Proof Sketch

- Difficulty: direction in which IC and IR binds unknown
- Suppose $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$ is optimal (hence IC and IR)
  - then $q_t^*(h^t) \in (0, \bar{q})$ all t, all h^t.
- **Idea**: IC preserving perturbations
  - increase $q_t^*(\cdot)$ uniformly by small amount $\nu > 0$
  - increase period-t payments by $c(q_t^*(h^t)+\nu) - c(q_t^*(h^t))$
  - increase period-1 payments $p_1^*(\cdot)$ uniformly by $\delta^{t-1}\nu \max_{h_1\in\Theta} \mathbb{E}\left[\tilde{h}_t | h_1\right]$
- New mechanism IC and IR
  - IC: additional quantity ν produced *irrespective of reports*!
  - Each type h_1 expects additional rent $\delta^{t-1}\nu\left(\max_{\hat{h}_1\in\Theta} \mathbb{E}\left[\tilde{h}_t | \hat{h}_1\right] - \mathbb{E}\left[\tilde{h}_t | h_1\right]\right)$

Proof by contradiction

- Suppose result not true. There exists increasing sequence (t_n) s.t. either

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] > \zeta$$

for all t_n in sequence, or

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] < -\zeta$$

for an appropriate $\zeta > 0$.

- Focus on first case. Increase $q^*_{t_n}(\cdot)$ uniformly at arbitrary date t_n in sequence by arbitrarily small amount $\nu_n > 0$

- Adjust payments as described above
- New mechanism IC and IR

Proof by contradiction

- Suppose result not true. There exists increasing sequence (t_n) s.t. either

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] > \zeta$$

for all t_n in sequence, or

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] < -\zeta$$

for an appropriate $\zeta > 0$.

- Focus on first case. Increase $q^*_{t_n}(\cdot)$ uniformly at arbitrary date t_n in sequence by arbitrarily small amount $\nu_n > 0$

- Adjust payments as described above
- New mechanism IC and IR

Proof by contradiction

- Suppose result not true. There exists increasing sequence (t_n) s.t. either

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] > \zeta$$

  for all t_n in sequence, or

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] < -\zeta$$

  for an appropriate $\zeta > 0$.

- Focus on first case. Increase $q^*_{t_n}(\cdot)$ uniformly at arbitrary date t_n in sequence by arbitrarily small amount $\nu_n > 0$

- Adjust payments as described above

- New mechanism IC and IR

Proof by contradiction

- Suppose result not true. There exists increasing sequence (t_n) s.t. either

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] > \zeta$$

for all t_n in sequence, or

$$\mathbb{E}\left[B'\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right)\right) - C_q\left(q^*_{t_n}\left(\tilde{h}^{t_n}\right), \tilde{h}_{t_n}\right)\right] < -\zeta$$

for an appropriate $\zeta > 0$.

- Focus on first case. Increase $q^*_{t_n}(\cdot)$ uniformly at arbitrary date t_n in sequence by arbitrarily small amount $\nu_n > 0$
- Adjust payments as described above
- New mechanism IC and IR

Reaching a contradiction...

- New mechanism increases expected surplus by at least $\delta^{t_n-1}\zeta\nu_n$
  - (ν_n small enough)
- New mechanism leaves additional expected rent

$$\delta^{t_n-1}\nu_n\left(\max_{\hat{h}_1}\left\{\mathbb{E}\left[\tilde{h}_{t_n}|\hat{h}_1\right]\right\}-\mathbb{E}\left[\tilde{h}_{t_n}|h_1\right]\right)$$

  to each initial type h_1
- Since, for all $h_1 \in \Theta$,

$$\max_{\hat{h}_1}\left\{\mathbb{E}\left[\tilde{h}_{t_n}|\hat{h}_1\right]\right\}-\mathbb{E}\left[\tilde{h}_{t_n}|h_1\right]\to 0 \text{ as } t_n\to\infty$$

  increase in surplus dominates for t_n large enough.
- When distortions always of same sign, convergence of wedges implies convergence of surplus and hence of policies (in probability) Q.E.D.

Reaching a contradiction...

- New mechanism increases expected surplus by at least $\delta^{t_n-1}\zeta v_n$
  - (v_n small enough)

- New mechanism leaves additional expected rent

$$\delta^{t_n-1} v_n \left(\max_{\hat{h}_1} \left\{ \mathbb{E}\left[\tilde{h}_{t_n} | \hat{h}_1 \right] \right\} - \mathbb{E}\left[\tilde{h}_{t_n} | h_1 \right] \right)$$

to each initial type h_1

- Since, for all $h_1 \in \Theta$,

$$\max_{\hat{h}_1} \left\{ \mathbb{E}\left[\tilde{h}_{t_n} | \hat{h}_1 \right] \right\} - \mathbb{E}\left[\tilde{h}_{t_n} | h_1 \right] \to 0 \text{ as } t_n \to \infty$$

increase in surplus dominates for t_n large enough.

- When distortions always of same sign, convergence of wedges implies convergence of surplus and hence of policies (in probability) Q.E.D.

Reaching a contradiction...

- New mechanism increases expected surplus by at least $\delta^{t_n-1}\zeta\nu_n$
  - (ν_n small enough)
- New mechanism leaves additional expected rent

$$\delta^{t_n-1}\nu_n\left(\max_{\hat{h}_1}\left\{\mathbb{E}\left[\tilde{h}_{t_n}|\hat{h}_1\right]\right\}-\mathbb{E}\left[\tilde{h}_{t_n}|h_1\right]\right)$$

  to each initial type h_1
- Since, for all $h_1 \in \Theta$,

$$\max_{\hat{h}_1}\left\{\mathbb{E}\left[\tilde{h}_{t_n}|\hat{h}_1\right]\right\}-\mathbb{E}\left[\tilde{h}_{t_n}|h_1\right]\rightarrow 0 \text{ as } t_n\rightarrow\infty$$

  increase in surplus dominates for t_n large enough.
- When distortions always of same sign, convergence of wedges implies convergence of surplus and hence of policies (in probability) Q.E.D.

Reaching a contradiction...

- New mechanism increases expected surplus by at least $\delta^{t_n-1}\zeta\nu_n$
  - (ν_n small enough)

- New mechanism leaves additional expected rent

$$\delta^{t_n-1}\nu_n\left(\max_{\hat{h}_1}\left\{\mathbb{E}\left[\tilde{h}_{t_n}|\hat{h}_1\right]\right\}-\mathbb{E}\left[\tilde{h}_{t_n}|h_1\right]\right)$$

  to each initial type h_1

- Since, for all $h_1\in\Theta$,

$$\max_{\hat{h}_1}\left\{\mathbb{E}\left[\tilde{h}_{t_n}|\hat{h}_1\right]\right\}-\mathbb{E}\left[\tilde{h}_{t_n}|h_1\right]\to 0 \text{ as } t_n\to\infty$$

  increase in surplus dominates for t_n large enough.
- When distortions always of same sign, convergence of wedges implies convergence of surplus and hence of policies (in probability) Q.E.D.

Additional predictions

Definition

Process satisfies "**FOSD**" if $\breve{h}^{t-1} \geq \bar{h}^{t-1}$implies $F_t\left(h_t|\breve{h}^{t-1}\right) \leq F_t\left(h_t|\bar{h}^{t-1}\right)$, all h_t.

Process satisfies "**Markov**" if evolution of h_t governed by time-invariant irreducible transition matrix A with $A_{ij} = Pr(\theta_i|\theta_j) > 0$ all i, j.

Process satisfies "**Stationary Markov**" if F_1 coincides with ergodic distribution

Additional predictions

Proposition

Suppose F satisfies "FOSD". Then, for all t,

$$\mathbb{E}\left[B'\left(q_t^*\left(\tilde{h}^t\right)\right) - C_q\left(q_t^*\left(\tilde{h}^t\right), \tilde{h}_t\right)\right] \geq 0.$$

If, in addition, F satisfies "Stationary Markov", expected wedges decrease with t.

- FOSD:
  - IR binds only for θ_N
  - cut output and adjust p so that IR continues to bind for θ_N
  - perturbation increases surplus and reduces rents, hence profitable
- FOSD + Stationary Markov:
  - shift output uniformly towards later dates
  - smaller rents due to declining persistence of initial types

Additional predictions

Proposition

Suppose F satisfies "FOSD". Then, for all t,

$$\mathbb{E}\left[B'\left(q_t^*\left(\tilde{h}^t\right)\right) - C_q\left(q_t^*\left(\tilde{h}^t\right), \tilde{h}_t\right)\right] \geq 0.$$

If, in addition, F satisfies "Stationary Markov", expected wedges decrease with t.

- FOSD:
  - IR binds only for θ_N
  - cut output and adjust p so that IR continues to bind for θ_N
  - perturbation increases surplus and reduces rents, hence profitable
- FOSD + Stationary Markov:
  - shift output uniformly towards later dates
  - smaller rents due to declining persistence of initial types

Additional predictions

Proposition

Suppose F satisfies "FOSD". Then, for all t,

$$\mathbb{E}\left[B'\left(q_t^*\left(\tilde{h}^t\right)\right)-C_q\left(q_t^*\left(\tilde{h}^t\right),\tilde{h}_t\right)\right]\geq 0.$$

If, in addition, F satisfies "Stationary Markov", expected wedges decrease with t.

- FOSD:
  - IR binds only for θ_N
  - cut output and adjust p so that IR continues to bind for θ_N
  - perturbation increases surplus and reduces rents, hence profitable
- FOSD + Stationary Markov:
  - shift output uniformly towards later dates
  - smaller rents due to declining persistence of initial types

ALLOCATIONS

"Sufficient patience"

- Assume F is "Markov"
- $\alpha \equiv \min_{i,j} A_{ij} > 0$
  - high α: little persistence
- $b \equiv \sum_{i=1}^{N} \theta_i$
- $\kappa \equiv \underset{i,j}{min} \left\{ \; B\left(q^E\left(\theta_i\right)\right) - C\left(q^E\left(\theta_i\right), \theta_i\right) - \left[B\left(q^E\left(\theta_j\right)\right) - c\left(q^E\left(\theta_j\right), \theta_i\right)\right] \; \right\}$
- Patience threshold:

$$\bar{\delta} \equiv \begin{cases} \frac{2\bar{q}b-\kappa}{2\bar{q}b-\kappa+2\kappa\alpha} \text{ if } \kappa < 2\bar{q}b \\ 0 \text{ otherwise} \end{cases} .$$

Vanishing distortions

Proposition

Suppose $T = +\infty$ *and* F *is "Markov". For any* $\delta \in (\bar{\delta}, 1)$,

1. $\lim_{t\to\infty} \mathbb{E}\left[B\left(q_t^*\left(\tilde{h}^t\right)\right) - C\left(q_t^*\left(\tilde{h}^t\right), \tilde{h}_t\right)\right]$
   $= \lim_{t\to\infty} \mathbb{E}\left[B\left(q^E\left(\tilde{h}_t\right)\right) - C\left(q^E\left(\tilde{h}_t\right), \tilde{h}_t\right)\right]$.
2. *For any* $\eta > 0$, $\lim_{t\to\infty} \Pr\left(\left|q_t^*\left(\tilde{h}^t\right) - q^E\left(\tilde{h}_t\right)\right| > \eta\right) = 0$.

Bounds on distortions

Corollary

Suppose F is Markov and let $\lambda = \frac{\bar{q}b}{1-\delta(1-2\alpha)}$. *Irrespective of time horizon and of patience, for any* t,

$$\left(\begin{array}{c} \mathbb{E}\left[B\left(q^E\left(\tilde{h}_t\right)\right) - C\left(q^E\left(\tilde{h}_t\right), \tilde{h}_t\right)\right] \\ -\mathbb{E}\left[B\left(q^*_t\left(\tilde{h}^t\right)\right) - C\left(q^*_t\left(\tilde{h}^t\right), \tilde{h}_t\right)\right] \end{array} \right) \leq \frac{2\lambda}{\left(\delta + \frac{\kappa}{2\lambda}\right)^{t-1}},$$

with $\delta + \frac{\kappa}{2\lambda} > 1$ *for* $\delta \in \left(\bar{\delta}, 1\right)$.

- Result also provides conservative **bound on rate of convergence**

Proof: Idea

- Idea: under efficient mechanism with

$$p^E(h_t) = B\left(q^E(h_t)\right)$$

  IC slack all histories
- Perturbations obtained by combining putative mechanism with efficient one guarantee slack in IC

Proof: approching efficiency “too fast”

- **Idea 1 (FAILED!):** Suppose $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$ is optimal and convergence to efficiency does not hold.
  - Replace payment and allocation rules with those of efficient mechanism from t onwards.
  - Adjust payments, to ensure satisfaction of IR constraints.
  - Such adjustment can be made s.t. (expected) increase in surplus dominates expected increase in rents (when t large enough)
  - *If IC, new mechanism improves upon* $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$
- **Problem:** New mechanism need not be IC!
  - IC from date t onwards, but not necessarily at earlier dates.

Proof: approching efficiency "too fast"

- **Idea 1 (FAILED!):** Suppose $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$ is optimal and convergence to efficiency does not hold.
  - Replace payment and allocation rules with those of efficient mechanism from t onwards.
  - Adjust payments, to ensure satisfaction of IR constraints.
  - Such adjustment can be made s.t. (expected) increase in surplus dominates expected increase in rents (when t large enough)
  - *If IC, new mechanism improves upon* $\langle q_t^*(h^t), p_t^*(h^t)\rangle_{t=1}^{\infty}$
- **Problem:** New mechanism need not be IC!
  - IC from date t onwards, but not necessarily at earlier dates.

Proof: Approaching efficiency "gradually"

- Any linear convex combination of $\langle q_t^*(h^t)\rangle_{t=\tau}^{\infty}$ and $\left\langle q^E(h_t)\right\rangle_{t=\tau}^{\infty}$ can be implemented with payments that make IC slack at all histories Amount of slack determined by κ and linear weights
- Gradual growth in weights on efficiency

$$q_1^{new}(h_1) = \left(1-\alpha^1\right) q_1^*(h_1) + \alpha^1 q^E(h_1)$$

and, for any $t \geq 2$,

$$q_t^{new}\left(h^t\right) = \left(1-\alpha^{\geq 2}\right) q_t^*\left(h^t\right) + \alpha^{\geq 2} q^E(h_t)$$

with $0 < \alpha^1 \leq \alpha^{\geq 2} \leq 1$.

  - new mechanism IC if $\alpha^{\geq 2}$ not too much larger than α^1
  - for fixed $\alpha^{\geq 2}$, mechanism IC from $t=2$ onwards
  - if $\alpha^1 = \alpha^{\geq 2}$, then there is slack in IC at $t=1$. Hence, can decrease α^1 below $\alpha^{\geq 2}$ by small amount and preserve IC

Proof: Approaching efficiency "gradually"

- Any linear convex combination of $\langle q_t^*(h^t)\rangle_{t=\tau}^{\infty}$ and $\left\langle q^E(h_t)\right\rangle_{t=\tau}^{\infty}$ can be implemented with payments that make IC slack at all histories Amount of slack determined by κ and linear weights
- Gradual growth in weights on efficiency

$$q_1^{new}(h_1) \;\; = \;\; (1-\alpha^1)\, q_1^*(h_1) + \alpha^1 q^E(h_1)$$

and, for any $t \geq 2$,

$$q_t^{new}\left(h^t\right) = \left(1-\alpha^{\geq 2}\right) q_t^*\left(h^t\right) + \alpha^{\geq 2} q^E(h_t)$$

with $0 < \alpha^1 \leq \alpha^{\geq 2} \leq 1$.

  - new mechanism IC if $\alpha^{\geq 2}$ not too much larger than α^1
  - for fixed $\alpha^{\geq 2}$, mechanism IC from $t = 2$ onwards
  - if $\alpha^1 = \alpha^{\geq 2}$, then there is slack in IC at $t = 1$. Hence, can decrease α^1 below $\alpha^{\geq 2}$ by small amount and preserve IC

Role of discount factor

- **Why δ large?**
  - Proposed "new" mechanism approaches efficiency gradually.
  - Positive weight on efficient policy at early dates may increase information rents (by relatively large amount)
  - When δ small, gains in surplus at later dates need not compensate for increased rents at earlier periods.
- However, convergence to efficiency for **all** δ if process not very persistent

General cost functions

- Convergence results extend to general $C(q, h)$ satisfying mild regularity conditions
- Deterministic mechanisms need not be optimal
  - arguments related to Strausz (2006)
  - violation of integral mon.
  - "relaxed program" need not be valid
- Perturbations involve *randomizations* between putative optimal and efficient allocations

CONTINUUM OF TYPES

Continuum of types

- Markov chain $F = (F_t)$; $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$
- F_1 (abs. continuous) cdf of initial distribution (density f_1)
- $F_t(\cdot|h_{t-1})$ cdf of h_t given $h_{t-1} \in \Theta$ ($f_t(h_t|h_{t-1}) > 0$ all $h_t, h_{t-1} \in \Theta$)
- *Stochastic monotonicity*: $F_t(\cdot|h'_{t-1})$ first-order stochastically dominates $F_t(\cdot|h_{t-1})$ for $h'_{t-1} > h_{t-1}$
- *Time-invariance*: $F_t(\cdot|\theta) = F_s(\cdot|\theta)$ all $t, s > 1$, all $\theta \in \Theta$
- *Ergodicity*: $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} \left| F^t(A; \theta) - \pi(A) \right| \to 0 \text{ as } t \to \infty.$$

Continuum of types

- Markov chain $F = (F_t)$; $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$
- F_1 (abs. continuous) cdf of initial distribution (density f_1)
- $F_t(\cdot|h_{t-1})$ cdf of h_t given $h_{t-1} \in \Theta$ ($f_t(h_t|h_{t-1}) > 0$ all $h_t, h_{t-1} \in \Theta$)
- *Stochastic monotonicity*: $F_t(\cdot|h'_{t-1})$ first-order stochastically dominates $F_t(\cdot|h_{t-1})$ for $h'_{t-1} > h_{t-1}$
- *Time-invariance*: $F_t(\cdot|\theta) = F_s(\cdot|\theta)$ all $t, s > 1$, all $\theta \in \Theta$
- *Ergodicity*: $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} \left| F^t(A; \theta) - \pi(A) \right| \to 0 \text{ as } t \to \infty.$$

Continuum of types

- Markov chain $F = (F_t)$; $\Theta = \left[\underline{\theta}, \bar{\theta}\right] \subset \mathbb{R}_+$
- F_1 (abs. continuous) cdf of initial distribution (density f_1)
- $F_t(\cdot|h_{t-1})$ cdf of h_t given $h_{t-1} \in \Theta$ ($f_t(h_t|h_{t-1}) > 0$ all $h_t, h_{t-1} \in \Theta$)
- *Stochastic monotonicity*: $F_t\left(\cdot|h'_{t-1}\right)$ first-order stochastically dominates $F_t(\cdot|h_{t-1})$ for $h'_{t-1} > h_{t-1}$
- *Time-invariance*: $F_t(\cdot|\theta) = F_s(\cdot|\theta)$ all $t, s > 1$, all $\theta \in \Theta$
- *Ergodicity*: $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} \left|F^t(A;\theta) - \pi(A)\right| \to 0 \text{ as } t \to \infty.$$

Continuum of types

- Markov chain $F = (F_t)$; $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}_+$
- F_1 (abs. continuous) cdf of initial distribution (density f_1)
- $F_t(\cdot|h_{t-1})$ cdf of h_t given $h_{t-1} \in \Theta$ ($f_t(h_t|h_{t-1}) > 0$ all $h_t, h_{t-1} \in \Theta$)
- *Stochastic monotonicity*: $F_t\left(\cdot|h'_{t-1}\right)$ first-order stochastically dominates $F_t(\cdot|h_{t-1})$ for $h'_{t-1} > h_{t-1}$
- *Time-invariance*: $F_t(\cdot|\theta) = F_s(\cdot|\theta)$ all $t, s > 1$, all $\theta \in \Theta$
- *Ergodicity*: $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} \left|F^t(A;\theta) - \pi(A)\right| \to 0 \text{ as } t \to \infty.$$

Continuum of types

- Markov chain $F = (F_t)$; $\Theta = \left[\underline{\theta}, \bar{\theta}\right] \subset \mathbb{R}_+$
- F_1 (abs. continuous) cdf of initial distribution (density f_1)
- $F_t(\cdot|h_{t-1})$ cdf of h_t given $h_{t-1} \in \Theta$ ($f_t(h_t|h_{t-1}) > 0$ all $h_t, h_{t-1} \in \Theta$)
- *Stochastic monotonicity*: $F_t\left(\cdot|h'_{t-1}\right)$ first-order stochastically dominates $F_t(\cdot|h_{t-1})$ for $h'_{t-1} > h_{t-1}$
- *Time-invariance*: $F_t(\cdot|\theta) = F_s(\cdot|\theta)$ all $t, s > 1$, all $\theta \in \Theta$
- *Ergodicity*: $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} \left|F^t(A;\theta) - \pi(A)\right| \to 0 \text{ as } t \to \infty.$$

Continuum of types

- Markov chain $F = (F_t)$; $\Theta = \left[\underline{\theta}, \bar{\theta}\right] \subset \mathbb{R}_+$
- F_1 (abs. continuous) cdf of initial distribution (density f_1)
- $F_t(\cdot|h_{t-1})$ cdf of h_t given $h_{t-1} \in \Theta$ ($f_t(h_t|h_{t-1}) > 0$ all $h_t, h_{t-1} \in \Theta$)
- *Stochastic monotonicity*: $F_t\left(\cdot|h'_{t-1}\right)$ first-order stochastically dominates $F_t(\cdot|h_{t-1})$ for $h'_{t-1} > h_{t-1}$
- *Time-invariance*: $F_t(\cdot|\theta) = F_s(\cdot|\theta)$ all $t, s > 1$, all $\theta \in \Theta$
- *Ergodicity*: $\exists!$ invariant distribution π s.t., for all $\theta \in \Theta$

$$\sup_{A \in \mathscr{B}(\Theta)} \left|F^t(A;\theta) - \pi(A)\right| \to 0 \text{ as } t \to \infty.$$

Continuous types: auxiliary shocks

- Stochastic process can be represented by "auxiliary shocks" independent of initial private information
  - e.g., Eso, Szentes (2007), Pavan, Segal, Toikka (2014)
- $h_t = z(h_{t-1}, \varepsilon_t)$, where $\varepsilon = (\varepsilon_t)$ are i.i.d. random variables
- E.g., ε_t drawn from $U(0,1)$ and $z(h_{t-1}, \varepsilon_t) = F^{-1}(\varepsilon_t | h_{t-1})$ with

$$F^{-1}(\varepsilon_t | h_{t-1}) \equiv \inf\{\theta_t : \mathrm{F}(\theta_t | h_{t-1}) \geq \varepsilon_t\}$$

  - "probability integral transform"
- *Regularity*: $\frac{\partial z(h_{t-1}, \varepsilon_t)}{\partial h_{t-1}}$ exists, continuous and bounded

Continuous types: auxiliary shocks

- Let $(Z_{\tau,t})_{t\geq\tau}$ be a collection of functions s.t. $h_t = Z_{\tau,t}(h_\tau, \varepsilon)$ for $t \geq \tau$

- **Impulse responses**:

$$I_{\tau\to t}\left(h^t\right) = \frac{\partial Z_{t,\tau}(h_\tau, \varepsilon)}{\partial h_\tau}$$

  (where vector ε derived from h^t using function $z(\cdot)$)

- AR(1) example (violates full support):

$$\begin{aligned} h_t &= \gamma h_{t-1} + \varepsilon_t \\ &= Z_{\tau,t}(h_\tau, \varepsilon) = \gamma^{t-\tau} h_\tau + \gamma^{t-\tau-1}\varepsilon_{\tau+1} + \cdots + \gamma\varepsilon_{t-1} + \varepsilon_t \\ &\to I_{\tau\to t}\left(h^t\right) = \gamma^{t-\tau}. \end{aligned}$$

Characterization of IC

Theorem

Suppose F satisfies "Markov", "FOSD," and "regularity". Mechanism $\langle q_t(h^t), p_t(h^t)\rangle_{t=1}^{\infty}$ IC iff, for all $t \geq 0$, all h^{t-1}, $V_t(h^t)$ Lipschitz continuous in h_t with

$$\frac{\partial V_t(h^t)}{\partial h_t} = -\mathbb{E}\left[\sum_{s \geq t} \delta^{s-t} I_{t \to s}\left(\tilde{h}^s\right) q_s\left(\tilde{h}^s\right) | h^t\right] \text{ a.e. } h_t,$$

and, for all h^{t-1}, h_t, $\hat{h}_t$,

$$\int_{\hat{h}_t}^{h_t} \left[D_t\left(\left(h^{t-1}, x\right); x\right) - D_t\left(\left(h^{t-1}, x\right); \hat{h}_t\right)\right] dx \geq 0$$

where $D_t(h^t; y) \equiv -\mathbb{E}\left[\sum_{s \geq t} \delta^{s-t} I_{t \to s}\left(\tilde{h}^s\right) q_s\left(\tilde{h}^s_{-t}, y\right) | h^t\right]$.

Dynamic virtual surplus

- Previous result implies principal's payoff equals "dynamic virtual surplus"

$$\mathbb{E}\left[\sum_{t\geq 1}\delta^{t-1}\begin{pmatrix} B\left(q_t\left(\tilde{h}^t\right)\right)-C\left(q_t\left(\tilde{h}^t\right),\tilde{h}_t\right) \\ -\left(\frac{F_1\left(\tilde{h}_1\right)}{f_1\left(\tilde{h}_1\right)}I_{1\to t}\left(\tilde{h}^t\right)\right)q_t\left(\tilde{h}^t\right)\end{pmatrix}\right]$$

$$-V_1\left(\bar{\theta}\right)$$

(FOSD ensures IC binds at $\bar{\theta}$, so, at optimum, $V_1\left(\bar{\theta}\right)=0$).

Relaxed approach

- "**Relaxed/First-order approach**": Pointwise maximization

$$B'\left(q_t^*\left(h^t\right)\right) = C_q\left(q_t^*\left(h^t\right), h_t\right) + \frac{F_1\left(h_1\right)}{f_1\left(h_1\right)} I_{1\to t}\left(h^t\right)$$

- FOSD ($I_{1\to t} \geq 0$) $\Rightarrow$ downward distortions
  - Distortions driven by impulse responses
  - Validity of FOA: above policies must satisfy "integral monotonicity" constraints

- Condition for convergence to efficiency (point-wise)
  - vanishing impulse responses

Beyond FOA: Convergence of wedges

Definition

"Policies eventually interior" if $\exists T$ and $(\underline{b}_t)$ and $(\bar{b}_t)$, with $0 < \underline{b}_t < \bar{b}_t < \bar{q}$, s.t., for all $t \geq T$, $q_t^*(h^t) \in [\underline{b}_t, \bar{b}_t]$.

Theorem

Assume F satisfies "Markov", "FOSD" and "Regularity". If optimal policies "eventually interior",

$$\mathbb{E}\left[B'\left(q_t^*\left(\tilde{h}^t\right)\right) - C_q\left(q_t^*\left(\tilde{h}^t\right), \tilde{h}_t\right)\right] = \mathbb{E}\left[\frac{F_1\left(\tilde{h}_1\right)}{f_1\left(\tilde{h}_1\right)} I_t(\tilde{h}^t)\right]$$

If, in addition, F satisfies "ergodicity", then $\mathbb{E}\left[\frac{F_1(\theta_1)}{f_1(\theta_1)} I_t(\theta^t)\right] \to 0$ *as* $t \to \infty$ *and convergence from above and monotone in* t*.*

Proof Sketch

- Observe that $\mathbb{E}\left[I_t\left(\tilde{h}^t\right)|h_1\right] = \frac{d}{dh_1}\mathbb{E}\left[\tilde{h}_t|h_1\right]$.
- Thus,

$$\mathbb{E}\left[\frac{F_1\left(\tilde{h}_1\right)}{f_1\left(\tilde{h}_1\right)}I_t\left(\tilde{h}^t\right)\right] = \mathbb{E}\left[\frac{F_1\left(\tilde{h}_1\right)}{f_1\left(\tilde{h}_1\right)}\mathbb{E}\left[I_t\left(\tilde{h}^t\right)|\tilde{h}_1\right]\right]$$

$$= \int_{\underline{\theta}}^{\bar{\theta}} F_1(h_1)\mathbb{E}\left[I_t\left(\tilde{h}^t\right)|h_1\right]dh_1$$

$$= F_1(\theta_1)\mathbb{E}[\tilde{h}_t \mid h_1]\Big|_{h_1=\underline{\theta}}^{h_1=\bar{\theta}}$$

$$+ \int_{\underline{\theta}}^{\bar{\theta}} f_1(h_1)\mathbb{E}[\tilde{h}_t \mid h_1]dh_1$$

$$= \mathbb{E}[\tilde{h}_t \mid \bar{\theta}] - \mathbb{E}[\tilde{h}_t\ ] \to 0$$

by ergodicity.

Proof Sketch

- If F monotone (FOSD),

$$\mathbb{E}[\tilde{h}_t \mid \bar{\theta}] - \mathbb{E}[\tilde{h}_t\ ] \geq 0$$

implying that convergence is from above.

- If, in addition, $F_1 = \pi$, then

$$\mathbb{E}\left[\frac{F_1(\tilde{h}_1)}{f_1(\tilde{h}_1)} I_{1\to t}\left(\tilde{h}^t\right)\right] - \mathbb{E}\left[\frac{F_1(\tilde{h}_1)}{f_1(\tilde{h}_1)} I_{1\to s}\left(\tilde{h}^s\right)\right]$$

$$= \mathbb{E}[\tilde{h}_t \mid \bar{\theta}] - \mathbb{E}[\tilde{h}_s \mid \bar{\theta}] \leq 0$$

for $t > s$, implying convergence is monotone in time.

SIMPLE MECHANISMS

Simple mechanisms

- Results apply to settings where mechanisms constrained to be "simple" provided above perturbations are admissible (i.e., preserve simplicity)
- "Simple" might mean
  - continuity restrictions on allocations
  - measurability restrictions
    - e.g. allocations depend only on last few reported types

CONCLUSIONS

Summary

- DMD literature$\longrightarrow$ "relaxed" approach (as in Myerson)
- This paper: variational approach
  - IC-preserving perturbations
  - Idea related to Rogerson (1985) for moral hazard settings

Summary

- Convergence of wedges (in expectation)
  - fairly robust property (long-run independence)
- Convergence of allocations (in probability)
  - enough patience
- Additional "economic" properties (e.g., FOSD and stationarity)
  - convergence from above and monotone in time
- Results apply to environments where we don't know which IC and IR constraints bind

THANK YOU!!!