Wedge Dynamics

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This paper

- General **formula** for dynamics of distortions under second-best
  - unifies all special cases in micro/macro literatures
- Novel forces governing dynamics of distortions:
  - "transaction costs" of recouping future rents
  - scope versus urgency of interventions to curtail future rents
- Novel implications for dynamics of distortions:
  - distortions can increase over lifecycle without increase in risk
  - endogenous processes: non-monotonicity of distortions

Plan

1. **Model**
2. First Best
3. Second Best and Wedges
4. Theorem
5. Conclusions

Model

Environment

- Time: $t = 1, ..., T$
- Period-t "productivity": θ_t
  - privately observed by agent at beginning of period t
- F_1: cdf of initial distribution (density f_1)
- $F_{t+1}(\cdot|\theta_t, y_t)$: cdf of θ_{t+1} (density f_{t+1})
  - endogenous process
  - dependence on past decisions: LBD, habit formation, etc.

Environment

- Agent's flow-payoff:
$$v(c_t) - \psi(y_t, \theta_t)$$
- Planner's flow-payoff:
$$v^P(y_t) - c_t$$
- Taxation literature: $\psi(y_t, \theta_t) = \frac{1}{1+\phi}\left(\frac{y_t}{\theta_t}\right)^{1+\phi}$ and $v^P(y_t) = y_t$
- $\theta^t = (\theta_1, ..., \theta_t) = (\theta^{t-1}, \theta_t)$; $\theta = \theta^T$

Environment

- $\lambda[\chi]$: process over Θ under allocation rule $\chi =< c_t(\cdot), y_t(\cdot) >$
- Agent period-t continuation payoff:

$$V_t(\theta^t) \equiv \mathbb{E}^{\lambda[\chi]|\theta^t}\left[\sum_{\tau=t}^{T}\delta^{\tau-t}\left(v(c_\tau(\tilde{\theta}^\tau)) - \psi(y_\tau(\tilde{\theta}^\tau), \tilde{\theta}_\tau)\right)\right]$$

- "Redistribution" constraint:

$$(1-r)V_1(\theta_1) + r\int q(V_1(\theta_1'))dF_1(\theta_1') \geq \kappa \qquad \forall\theta_1$$

  w. $r \in \{0,1\}$.

- Planner maximizes

$$\mathbb{E}^{\lambda[\chi]}\left[\sum_{\tau=1}^{T}\delta^{\tau-1}\left(v^P\left(y_\tau(\tilde{\theta}^\tau)\right) - \tilde{c}_\tau(\tilde{\theta}^\tau)\right)\right]$$

  s.t. IC + redistribution constraint

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First Best

First Best: period-t output

- Principal's continuation payoff

$$V_t^P(\theta^t) \equiv \mathbb{E}^{\lambda[\chi]|\theta^t}\left[\sum_{\tau=t}^{T}\delta^{\tau-t}\left(v^P\left(y_\tau(\tilde{\theta}^\tau)\right)-\tilde{c}_\tau(\tilde{\theta}^\tau)\right)\right]$$

- Optimal χ satisfies

$$v^{P\prime}(y_t(\theta^t)) + LD_t^{FB;\chi}(\theta^t) = \frac{\psi_y(y_t(\theta^t),\theta_t)}{v'(c_t(\theta^t))}$$

$$v'(c_t(\theta^t)) = v'(c_{t+1}(\theta^t,\theta_{t+1}))$$

$$rq\prime(V_1(\theta_1))v\prime(c_1(\theta_1)) = rq\prime(V_1(\theta_1'))v\prime(c_1(\theta_1'))$$

where

$$LD_t^{FB;\chi}(\theta^t) \equiv \delta\frac{\partial}{\partial y_t}\int\left[V_{t+1}^P(\theta^t,\theta_{t+1}) + \frac{V_{t+1}(\theta^t,\theta_{t+1})}{v'(c_t(\theta_t))}\right]dF_{t+1}(\theta_{t+1}|\theta_t, y_t(\theta^t))$$

captures impact of y_t on future surplus via effect on future types

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Second Best and Wedges

Second Best: Incentive Compatibility

IC requires that (at any θ^t)

$$V_t(\theta^{t-1}, \theta_t) = V_t(\theta^{t-1}, \underline{\theta}_t) + \int_{\underline{\theta}_t}^{\theta_t} D_t^{\chi}(\theta^{t-1}, s) ds$$

where

$$\mathcal{D}_t^{\chi}(\theta^t) \equiv -\mathbb{E}^{\lambda[\chi]|\theta^t}\left[\sum_{\tau=t}^{T} \delta^{\tau-t} I_t^{\tau}\left(\tilde{\theta}^{\tau}, y^{\tau-1}(\tilde{\theta}^{\tau-1})\right) \psi_{\theta}(y_{\tau}(\tilde{\theta}^{\tau}), \tilde{\theta}_{\tau})\right]$$

with

$$\boxed{I_t^{\tau}\left(\theta^{\tau}, y^{\tau-1}(\theta^{\tau-1})\right)) : \textbf{impulse response} \text{ of } \theta_{\tau} \text{ to } \theta_t}$$

and

$$\int_{\hat{\theta}_t}^{\theta_t} \left\{\mathcal{D}_t^{\chi}(\theta^{t-1}, s) - \mathcal{D}_t^{\chi \circ \hat{\theta}_t}(\theta^{t-1}, s)\right\} ds \geq 0$$

w. $D_t^{\chi \circ \hat{\theta}_t}(\theta^{t-1}, s)$ analogous to $D_t^{\chi}(\theta^{t-1}, s)$ w. period-t message replaced by $\hat{\theta}_t$

Second Best: Wedges

- Recursive approach
- Period-t wedge

$$W_t(\theta^t) \equiv v^{P'}(y_t(\theta^t)) + LD_t^{FB;\chi}(\theta^t) - \frac{\psi_y(y_t(\theta^t), \theta_t)}{v'(c_t(\theta^t))}$$

- **Relative wedges** (unit-free):

$$\hat{W}_t(\theta^t) \equiv \frac{W_t(\theta^t)}{\frac{\psi_y(y_t(\theta^t), \theta_t)}{v'(c_t(\theta^t))}}$$

- Taxation: wedges related to current marginal tax rates and future (expected) tax bills

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Theorem

Theorem

Under optimal allocations,

$$\boxed{\hat{W}_t(\theta^t) = [RA_t(\theta^t) - D_t(\theta^t)] \left[\hat{W}_t^{RRN}(\theta^t) + \Omega_t(\theta^t)\right]}$$

- $\widehat{W}_t^{RRN}$: Rawlsian-risk-neutral benchmark
- Ω_t: effects of endogenous private information
- RA_t: correction due to non-trasferability of utility
- D_t: attenuation due to planner's inequality aversion

Theorem, cont'd: Rawlsian-risk-neutral benchmark

$$\widehat{W}_t^{RRN} \equiv \frac{\frac{\partial h_t}{\partial y_t}}{\psi_y}$$

where $\psi_y(\theta_t, y_t)$ is type θ_t's marginal disutility of y_t, whereas

$$h_t(\theta^t, y^t) \equiv -\frac{1 - F_1(\theta_1)}{f_1(\theta_1)} I_1^t(\theta^t, y^{t-1}) \psi_\theta(y_t, \theta_t)$$

is period-t handicap (effect of y_t on information rent)

- **DMD literature** (transferable utility): dynamics driven by I_1^t
  - typically declining over time

Theorem, cont'd: endogenous types

$$\Omega_t \equiv \frac{\frac{\partial}{\partial y_t}\mathbb{E}^{\theta^t, y_t}\left[\sum_{\tau=t+1}\delta^{\tau-t}\tilde{h}_\tau\right]}{\psi_y}$$

- monotonicity of handicaps in future types + FOSD:
  - extra benefit of lowering y_t
  - higher wedges
  - declining wedges from T-1 to T
- **scope vs urgency**:
  - Early t: more periods ahead to gain from reduction in rents
  - Early t: less urgency to act
  - non monotonicity of distortions over time

Theorem, cont'd: redistribution

$$D_t \equiv rv'(c_t)\mathbb{E}\left[\frac{1}{v'(\tilde{c}_1)}\right]\left(\frac{\mathbb{E}\left[q'(\tilde{V}_1)|\tilde{\theta}_1 \geq \theta_1\right]}{\mathbb{E}\left[q'(\tilde{V}_1)\right]}\right)$$

- Increasing consumption at θ^t relaxes redistribution constraint (when $r = 1$)
- Lower inequality aversion $\Rightarrow$ lower wedges

Theorem, cont'd: risk aversion and transaction costs

$$RA_t \equiv v'(c_t)\left\{\eta_1 \sum_{\tau=2}^{t} \frac{TC_\tau}{I_1^\tau} + \mathbb{E}\left[\frac{1}{v'(\tilde{c}_1)}|\tilde{\theta}_1 \geq \theta_1\right]\right\}$$

where

$$TC_t = \frac{1-F_t}{f_t}\left\{\mathbb{E}^{\theta^{t-1},y_{t-1}}\left[\frac{1}{v'(\tilde{c}_t)}|\tilde{\theta}_t \geq \theta_t\right] - \mathbb{E}^{\theta^{t-1},y_{t-1}}\left[\frac{1}{v'(\tilde{c}_t)}\right]\right\}$$

Risk aversion and transaction costs

- Non-transferable utility (concavity in c): higher cost of future rents
- Risk increasing over time (e.g., rw) $\rightarrow$ wedges increasing over time
- Piling up of TCs over time can suffice for wedges to increase, even with decreasing risk
  - qualitative implications for wedge dynamics and consumption insurance
  - quantitative implications: **ongoing work**

Recouping rents

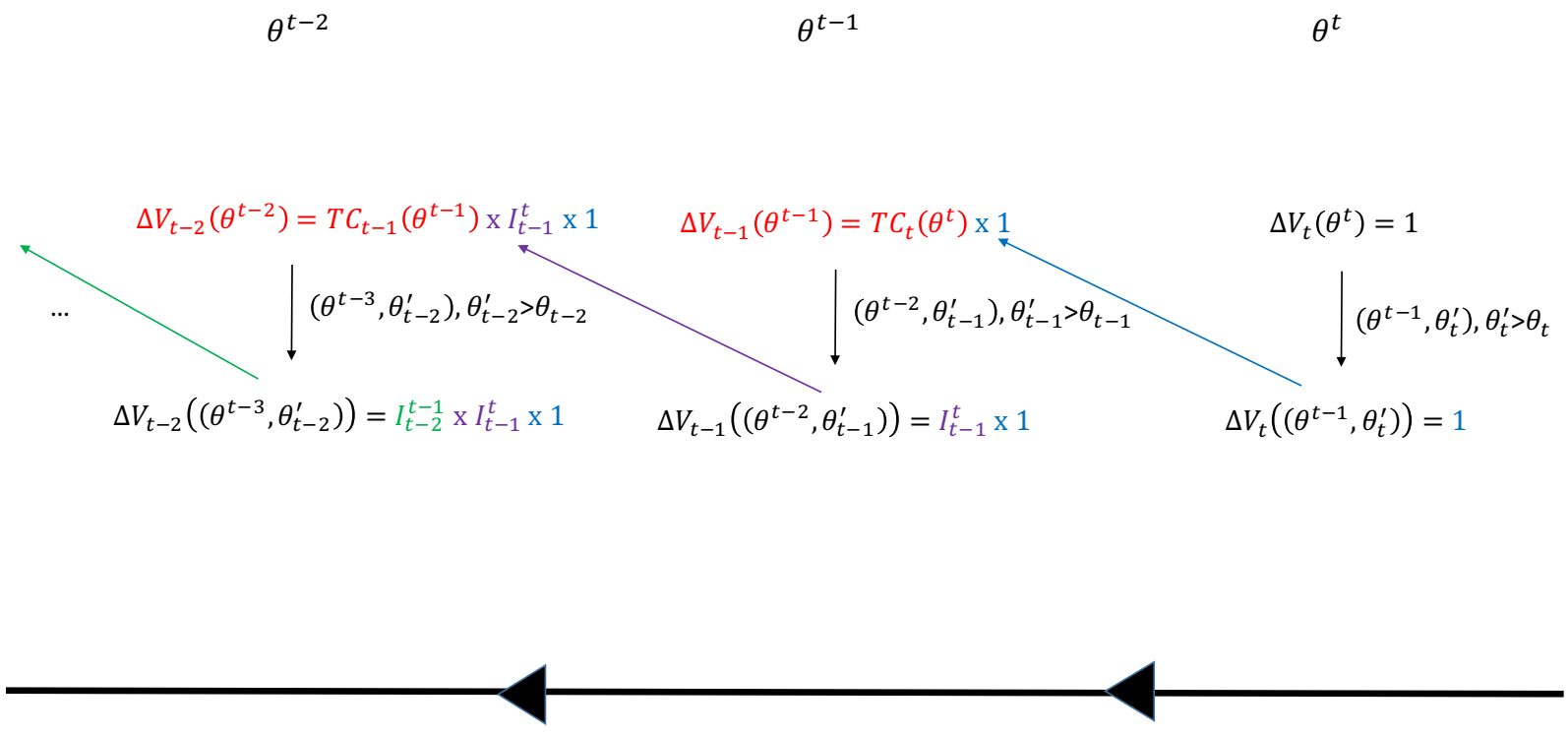

Figure: Accumulation of transaction costs

Recouping rents

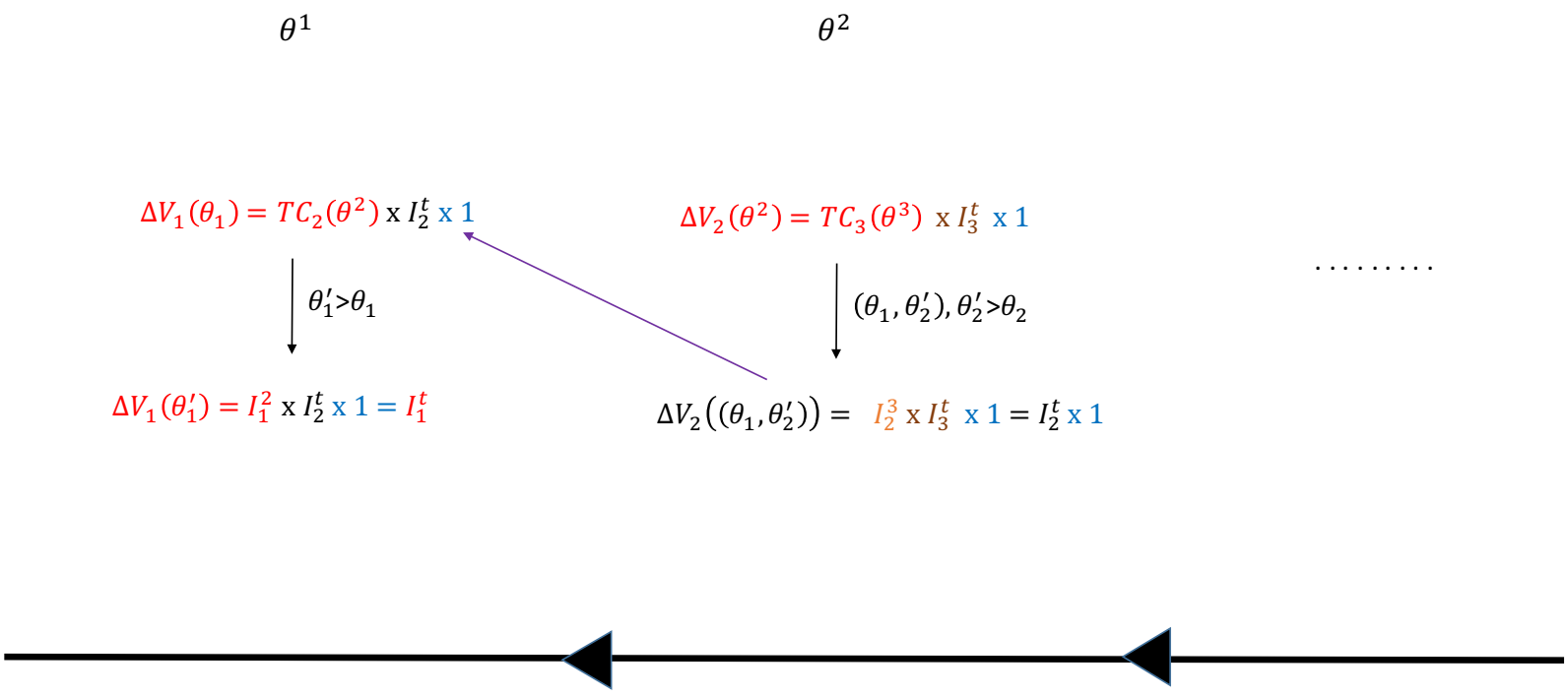

Figure: Accumulation of transaction costs

Conclusions

Conclusions

- general formula for wedge decomposition
  - interaction of forces driving dynamics of distortions
    - endogeneity and persistence of type process
    - risk aversion and accumulation of risk
    - transaction costs of moving rents over time
    - planner's preferences for redistribution
- Ongoing work
  - increasing distortions without increasing risk
  - quantitative implications
- Future work:
  - hidden savings
  - partial commitment
  - ...

Conclusions

THANKS!

Literature

- **Dynamic Mechanism Design**
  - Transferable utility
  - Declining distortions (higher quantity/quality) over time
  - Distortion dynamics driven by impulse responses of future types to initial ones
- **New Dynamic Public Finance**
  - Increasing distortions (and tax rates) over lifecycle
  - Distortions dynamics driven by risk aversion and evolution of risk
- **Taxation w. Human Capital Accumulation**
  - Special processes for productivity shocks

Go back

Recursive Problem

"Promised utility":

$$\Pi_{t+1}(\theta^t) \equiv \int V_{t+1}(\theta^{t+1}) dF_{t+1}(\theta_{t+1} \mid \theta_t, y_t(\theta^t))$$

"Promised Marginal Utility":

$$Z_{t+1}(\theta^t) \equiv -\mathbb{E}^{\lambda[\chi]|\theta^t} \left[\sum_{\tau=t+1}^{T} \delta^{\tau-t-1} I_t^{\tau}(\tilde{\theta}^{\tau}, y^{\tau-1}(\tilde{\theta}^{\tau-1})) \psi_{\theta}(y_{\tau}(\tilde{\theta}^{\tau}), \tilde{\theta}_{\tau}) \right]$$

Recursive Problem

- With above definitions:

$$V_t(\theta^t) = v(c_t(\theta^t)) - \psi(y_t(\theta^t), \theta_t) + \delta \Pi_{t+1}(\theta^t)$$

and

$$\frac{\partial}{\partial \theta_t} V_t(\theta^t) = -\psi_\theta(y_t(\theta^t), \theta_t) + \delta Z_{t+1}(\theta^t)$$

Recursive Problem: $t > 1$

$$Q_t\left(\theta^{t-1}, y_{t-1}, \Pi_t, Z_t\right) \equiv \max_{y_t(\cdot),\, V_t(\cdot),\, \Pi_{t+1}(\cdot),\, Z_{t+1}(\cdot)}$$

$$\int \left\{ v^P(y_t(\theta^t)) - v^{-1}\left(V_t(\theta^t) - \delta\Pi_{t+1}(\theta^t) + \psi(y_t(\theta^t), \theta_t)\right)\right.$$

$$\left. +\delta\ Q_{t+1}\left(\theta^t, y_t(\theta^t), \Pi_{t+1}(\theta^t), Z_{t+1}(\theta^t)\right)\right\} dF(\theta_t \mid \theta_{t-1}, y_{t-1})$$

s.t.

$$\frac{\partial}{\partial \theta_t} V_t(\theta^t) = -\psi_\theta(y_t(\theta^t), \theta_t) + \delta Z_{t+1}(\theta^t)$$

$$\int V_t(\theta^t) dF(\theta_t \mid \theta_{t-1}, y_{t-1}) = \Pi_t$$

with $\Pi_{T+1}(\theta) = 0$ and...

Recursive Problem: $t > 1$

...

$$Z_t = \int I_{t-1}^{t}(\theta^t, y_{t-1}) \left[-\psi_\theta(y_t(\theta^t), \theta_t) + \delta Z_{t+1}(\theta^t) \right] dF_t(\theta_t \mid \theta_{t-1}, y_{t-1})$$

with $\quad Z_{T+1}(\theta) = 0$

Recursive Problem: $t = 1$

Same as above but with no PK and MPK constraints and instead

$$(1 - r)V_1(\underline{\theta}_1) + r \int q(V_1(\theta_1))dF(\theta_1) = \kappa$$

Go back