Economics 412-1
Dynamic Methods for Economics
Fall 2024

Instructor: Bruno Strulovici
KGH 3225
b-strulovici@northwestern.edu
Office hours: by appointment

Lectures: Tuesdays and Thursdays from 1:30pm to 3:20pm
Room: KGH 3301

Course Description:

The purpose of the class is to provide students with as many tools and insights as possible to apply optimal control and stopping theories to economic problems. The class does *not* replace a rigorous introduction to stochastic processes or optimal control. The rhythm of the class will be determined by registered students.

Evaluation is based on 3 problem sets and either a term paper (project or review) or the presentation of one paper of interest to the student either on the theory of optimal control or on some economic application.

Lecture notes will be provided. Reading material and problem sets will be posted on Canvas.

Recommended Textbooks

Øksendal *Stochastic Differential Equations*, Springer
Duffie, *Dynamic Asset Pricing Theory*, Princeton University Press (self-contained appendix)

More advanced:
Karatzas and Shreve, *Brownian Motion and Stochastic Calculus,* Springer
Revuz and Yor. *Continuous Martingales and Brownian Motion,* Springer

Overview of the Course

Deterministic optimal control

- Calculus of variation
- Euler equation
- Maximum principle
- Dynamic programming and the Bellman equation
- Equivalent formulations (end value vs. flow)
- Necessary and sufficient conditions for optimality of solution to Bellman equation
    - Blackwell criterion for existence and uniqueness of a solution and contraction mapping theorem
    - Boundedness of value function
- Properties of value function and optimal control
- Operator inheritance
- Benveniste-Scheinkman theorem

Stochastic processes in continuous time

- Stochastic processes
    - Filtration, adapted stochastic process
    - Brownian motion
        - Kolmogorov's continuity theorem
        - Properties: scaling, volatility identification
    - Martingales, submartingales, supermartingales, local martingales
    - Doob Meyer decomposition theorem
    - Quadratic variation
    - Lévy's characterization theorem
- Stochastic integral
    - Construction
    - Properties: martingale, isometry
    - Dudley's theorem
    - Martingale Representation theorem
    - Malliavin interpretation

- Itô processes
    - Definition
    - Markov property
    - Itô's chain-rule formula
    - Multidimensional version
    - Girsanov's theorem

Stochastic Differential Equations and Hamilton Jacobi Bellman equation

- Definition of an SDE, strong and weak solutions
    - Existence and uniqueness of a strong solution
    - Examples: Geometric Brownian motion, Ornstein-Uhlenbeck processes, Bessel processes
- Feynman Kac formula
- HJB equation: necessity and sufficiency
    - Smoothness of value function
    - Verification argument
    - Transversality condition
    - Existence uniqueness of solution to HJB equation.
- Application: Merton's portfolio optimization.

Optimal Stopping

- Definition, examples of stopping time
- Optional sampling theorem
- Dynkin's formula
- Strong Markov property
- Laplace transform
- HJB equation for combined optimal control and stopping.
- Smooth pasting property: necessity and sufficiency, intuition
- Super-contact condition
- Application to corporate finance and optimal default: Leland (1994)
- Entry/exit problems: repeated stopping problems with coupled value functions. Dixit (1987)
- Switching models, switching costs
- Comparative statics of discounting (Quah and Strulovici, 2013)

Experimentation

- Multi-armed bandit problem (discounted)
    - Problem statement (discrete time)
    - Whittle's approach
    - Gittins index
    - Sprouting bandits
- Brownian experimentation (Bolton and Harris, 1999)
    - Change of measure
    - Threshold policy
- Exponential bandit (Keller, Rady, and Cripps, 2005)
    - Single decision maker: learning and updating
    - Multi-player experimentation: graphical analysis
- Experimentation with heterogeneous types (Strulovici, 2010)
- Correlated bandits
- Open questions

Principal-Agent problems in continuous time

- IID shocks
    - Dynamic incentive compatibility
        - Direct approach
        - HJB approach
    - Characterization of optimal contract
    - Verification argument
- Persistent private information
    - Stochastic maximum principle
    - Verification argument

Sannikov “A Continuous-Time Version of the Principal–Agent Problem”
Williams “Persistent Private Information”
Bloedel, Krishna, and Strulovici “Persistent Private Information Revisited”
Strulovici “Renegotiation-Proof Contracts with Persistent Private Information”

HJB equation with jumps and impulse controls

- Lévy processes
  - Poisson random measure
  - Itô-Lévy decomposition
- Itô-Lévy processes
  - Chain-rule formula for Itô-Lévy processes
- Impulse control
- Viscosity solutions
- HJB equation with impulse control
- Application: capital markets mobility with catastrophe risk (Duffie and Strulovici, 2010)

Recent Topics

Information structure and dynamic signal acquisition
Zhong (2019), Morris and Strack (WP 2019)

Decision theory
Relation between timing and accuracy of decisions: Fudenberg, Strack, Strzalecki (AER, 2018), Camboni and Durandard (WP)
Relation between uncertainty and discounting (Gabaix Laibson “Myopia and discounting”)

Time dimension as policy space
Callander (AER, 2011), Garfagnini and Strulovici (Restud, 2016), Bardhi (2020,2021)

Dynamic Persuasion, Approval Mechanisms, and Liability Rules
Escudé and Sinander (2019)
McClellan (Econometrica, 2021)
Henry Ottaviani (AER 2019)
Poggi Strulovici (WP 2021)

Dynamic games, higher-order beliefs, and filtering
Cisternas and Kolb (WP, 2019)
Bonatti, Cisternas, Toikka (Restud, 2017)

Stopping games
Boli Xu

Mean field games
Ben Moll and co.

Relation between discrete and continuous time
Kreps (2021), Sadzik Stacchetti (2015)

Directed learning
Ke and Villas-Boas (JET, 2019)

Epidemiologic Model
Xiaoyun Qiu (WinP)

Discrimination
Bardhi, Guo, Strulovici (WP 2020)

Two-Dimensional Theories
Durandard Strulovici (2021)
Bardhi (2021)