Economics 412
Dynamic Methods for Economics
Winter 2014

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Office hours: TBA

Lectures: Mondays and Wednesdays, from 1pm to 2:50pm
Room: Jacobs 3245

Course Description:

The purpose of the class is to provide students with as many tools and insights as possible to apply optimal control theory to economic problems. The class does *not* replace a rigorous introduction to stochastic processes or optimal control. The rhythm of the class will be determined by registered students.

Evaluation will be based on a few (~3) problem sets, a term paper (project or review) and/or the presentation of one paper of interest to the student either on the theory of optimal control or on some economic application.

There will be lecture notes. Reading material and problem sets will be posted on Blackboard

Textbooks

Øksendal *Stochastic Differential Equations*, 5th Ed.
Duffie, *Dynamic Asset Pricing Theory*, 3rd Ed.

Deterministic optimal control

- Calculus of variation
- Euler equation
- Maximum principle
- Bellman equation
- Equivalent formulations (end value vs. flow)
- Necessary and sufficient conditions for optimality of solution to Bellman equation
    - Blackwell criterion for existence and uniqueness of a solution and contraction mapping theorem
    - Boundedness of value function
- Properties of value function and optimal control
- Operator inheritance
- Benveniste-Scheinkman theorem

Stochastic processes in continuous time

- Stochastic processes
    - Filtration, adapted stochastic process
    - Brownian motion
        - Kolmogorov's Continuity theorem
        - Properties: scaling, volatility identification
    - Martingales, submartingales, supermartingales, local martingales
    - Doob Meyer decomposition theorem
    - Quadratic variation
    - Lévy's characterization theorem
- Stochastic integral
    - Construction
    - Properties: martingale, isometry
    - Dudley's theorem
    - Martingale Representation theorem
    - Malliavin interpretation

- Itô processes
    - Definition
    - Markov property
    - Itô's chain-rule formula
    - Multidimensional version
    - Girsanov's theorem

Reading

Eldredge "Martingale Representation Theorem, and the Lévy characterization"
Karatzas and Shreve, *Brownian motion and stochastic calculus*
Revuz and Yor *Continuous Martingales and Brownian Motion*

Stochastic Differential Equations and Hamilton Jacobi Bellman equation

- Definition, strong and weak solution
    - Existence and uniqueness of strong solution
    - Examples: Geometric Brownian motion, Ornstein-Uhlenbeck processes, Bessel processes
- Feynman Kac formula
- HJB equation: necessity and sufficiency
    - Smoothness of value function
    - Verification argument
    - Transversality condition
    - Existence uniqueness of solution to HJB equation.
- Application: Merton's portfolio optimization.

Optimal Stopping

- Definition, examples of stopping time
- Optional sampling theorem
- Dynkin's formula
- Strong Markov property
- Laplace transform
- HJB equation for combined optimal control and stopping.
- Smooth pasting property: necessity and sufficiency, intuition
- Super-contact condition
- A simple application: Leland 1994
- A more general setting: Manso, Strulovici, and Tchistyi 2008
- Entry/exit problems: repeated stopping problems with coupled value functions. Dixit 87
- Switching models, switching costs
- Portfolio optimization with transaction costs (Davis Norman)
- Comparative statics: discounting

Experimentation

- Multiarmed bandit problem (discounted)
    - Problem statement (discrete time)
    - Whittle's approach
    - Gittins index
    - Sprouting bandits
- Brownian experimentation (Bolton and Harris, 1999)
    - Change of measure
    - Threshold policy
- Exponential bandit (Keller, Rady, and Cripps, 2005)
    - Single decision maker: learing and updating
    - Multi-player experimentation: graphical analysis
- Experimentation with heterogeneous types (Strulovici, 2010)
- Correlated bandits
- Open questions

Principal-Agent problems in continuous time

- i.i.d case
    - Dynamic incentive compatibility
        - Direct approach
        - HJB approach
    - Characterization of optimal contract
    - Verification argument
- Persistent private information
    - Stochastic maximum principle
    - Verification argument

Sannikov "A Continuous-Time Version of the Principal–Agent Problem"
Williams "Persistent Private Information"
Strulovici "Contracts, Information Persistence, and Renegotiation"

HJB equation with jumps and impulse controls

- Lévy processes
    - Poisson random measure
    - Itô-Lévy decomposition
- Girsanov formula for Poisson processes
- Itô-Lévy processes
    - Itô's chain-rule formula for Itô-Lévy processes
- Impulse control
- Viscosity solutions
- HJB equation with impulse control
- Application: capital markets mobility with catastrophe risk (Duffie and Strulovici, 2010)

Possible topics (time allowing / on demand)

Connection to dynamic games

Reading
Abreu Pearce Stacchetti "Toward a theory of discounted repeated games with imperfect monitoring," *Econometrica* 1990
Wiseman "A partial folk theorem for games with unknown payoff distributions," *Econometrica* 2005
Dutta "A Folk Theorem for Stochastic Games," *JET* 1995
Gul Lundholm "Endogenous timing and the clustering of agent decisions" *JPE* 1995
Sannikov
Faingold and Sannikov

Dynamic models of political economy

Reading
Acemoglu Robinson "A theory of political transition" *AER* 2001
Lagunoff "Markov Equilibrium in Models of Dynamic Endogenous Political Institutions" 2008
Roberts, K. "Dynamic Voting in Clubs" 1998
Acemoglu Robinson "Economic backwardness in political perspective" *APSR* 2006
Besley and Coate "Sources of inefficiency in a representative democracy, a dynamic analysis" *AER* 1998

Filtering, Robust control

- Linear Quadratic regulator
- Filtering
    - Kalman filter and the separation principle (Gaussian case)
    - Zakai equation for nonlinear filters
    - solvable case
- Robust control
    - Decision theoretic approach (Epstein and co-authors)
    - Stability
- Open questions