Online Appendix of
"Early-Career Discrimination: Spiraling or Self-Correcting?"

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C Proofs for section 3

C.2 Proofs for section 3.3 (Large market with flexible wages)

C.2.4 Relaxing limited liability

We have shown that a-workers and b-workers fare quite differently under breakdown learning even if wages are flexible. One might conjecture that this result relies on the assumption that wages have to be nonnegative (that is, the minimum wage must equal the payoff from remaining unemployed): if b-workers could offer negative wages, they would do so and "steal" employment opportunities away from a-workers. In this section, we show that relaxing the limited liability assumption does not guarantee that b-workers have similar employment opportunities as a-workers do, because a-workers will also lower their wages and outbid b-workers. As a result, relaxing the limited liability assumption intensifies competition among workers and thus only benefits the employers.

In this section, we assume that there exists a fixed bound $LB > 0$ such that wages have to be at least $-LB$. We will focus on breakdown learning and show that the disparity between the two groups persists when LB is small enough. (For larger LB, we conjecture that b-workers compete all of their surplus away and have a zero expected lifetime payoff.)

We assume that $\alpha > 1$ and $\alpha p_a < 1$, so according to the dynamic matching μ^* in Proposition C.2 there exists a time $0 < T_b < \infty$ such that b-workers are hired starting from T_b. The marginal productivity $p^M(G_{h_t})$ is p_a for $t \leqslant T_b$. We also assume that $\alpha p_a + \beta p_b > 1$, so there are more high-type workers than tasks. Due to this assumption, the marginal productivity $p^M(G_{h_t})$ is p_b for $t \in (T_b, \infty)$.

We revise the dynamic matching μ^* in Proposition C.2 by lowering the wage for a matched worker k at time t from $(p_k - p^M(G_{h_t}))v$ to $(p_k - p^M(G_{h_t}))v - LB$. Hence, at any time t, the marginal worker's wage is $-LB < 0$. This revised wage function captures the idea that workers

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benefit from the opportunities to be learnt, so they compete against each other by lowering the wage until the marginal worker's wage drops to the bound $-LB$. This is the only change we made to μ^*. In particular, at any time t, all employers originally had the same flow profit by Lemma C.2. Their flow profit now increases by LB, so all employers continue to have the same flow profit.

We let $\mu^*(LB)$ denote this revised dynamic matching. We next show that if LB is small enough, $\mu^*(LB)$ is dynamically stable.

Proposition C.7. *Assume that $\alpha > 1$, $\alpha p_a < 1$, and $\alpha p_a + \beta p_b > 1$. Under breakdown learning, $\mu^*(LB)$ is dynamically stable for any*

$$LB < \frac{v(\lambda_\ell((2-p_b)p_b - p_a) + r(p_b - p_a))}{\lambda_\ell p_b + r}.$$

In the limit of $p_a \downarrow p_b$, this condition reduces to:

$$LB < \frac{\lambda_\ell(1-p_b)p_b v}{\lambda_\ell p_b + r},$$

which is equivalent to the condition that a b-worker's continuation payoff at time 0 is strictly positive.

Proof. Pick any $h_t \in \mathcal{H}_t$. We want to show that conditions (i)-(iii) in Definition 2 are satisfied.

(i) If employer j is matched to a worker under $\mu^*(LB)_{h_t}$, her flow payoff on path is at least s. The distribution $G(h_{t+\mathrm{d}t})$, and hence j's continuation payoff from $t+\mathrm{d}t$ on, does not depend on j's deviation. Hence, she does not strictly prefer to take a safe arm over $[t, t+\mathrm{d}t)$ and then revert to $\mu^*(LB)_{h_{t+\mathrm{d}t}}$.

(ii) Suppose that worker k is matched at history h_t according to $\mu^*(LB)$. Let $p(t)$ be this worker's expected productivity at history h_t. We next show that he does not strictly prefer to stay unmatched for $[t, t+\mathrm{d}t)$ and then revert to $\mu^*(LB)_{h_{t+\mathrm{d}t}}$.

Pick any $\tau > t$. Let $Q(\tau)$ denote the probability that this worker has generated a breakdown in $[t, \tau)$, and $p(\tau)$ denote the worker's expected productivity at time τ conditional on no breakdown in $[t, \tau)$. By Bayes rule,

$$(1 - Q(\tau))p(\tau) = p(t).$$

The worker's expected flow-earnings at time τ are

$$(1 - Q(\tau))\left(\left(p(\tau) - p^M(G_{h_\tau})\right)v - LB\right) = p(t)\frac{\left(p(\tau) - p^M(G_{h_\tau})\right)v - LB}{p(\tau)} \tag{1}$$

which is strictly increasing in $p(\tau)$. Staying unmatched over $[t, t+\mathrm{d}t)$ and then reverting to $\mu^*(LB)_{h_{t+\mathrm{d}t}}$ only makes $p(\tau)$ lower than its value on path. Hence, the worker's expected flow-earnings at time $\tau \geqslant t + \mathrm{d}t$ is higher on path than if he is unmatched over $[t, t+\mathrm{d}t)$.

However, the worker's flow-earnings over $[t, t+\mathrm{d}t)$ can be negative if he is matched, so they can be lower than his flow-earnings if he is unmatched.

For any $\tau \geqslant t + \mathrm{d}t$, we now compare the worker's expected flow-earnings at time τ on and off path. Let $p^{\text{on}}(\tau)$ and $p^{\text{off}}(\tau)$ be, respectively, the probabilities of a high type conditional on no breakdown on path and and off path. Then we have:

$$
\begin{aligned}
p^{\text{on}}(\tau) &= \frac{p(t)}{p(t) + (1-p(t))e^{-\lambda_\ell(\tau - t)}} \\
p^{\text{off}}(\tau) &= \frac{p(t)}{p(t) + (1-p(t))e^{-\lambda_\ell(\tau - t - \mathrm{d}t)}}.
\end{aligned}
$$

Substituting $p^{\text{on}}(\tau)$ and $p^{\text{off}}(\tau)$ into (1), we obtain the difference between on-path flow-earnings and off-path flow-earnings at time τ:

$$
\begin{aligned}
&p(t)\frac{\left(p^{\text{on}}(\tau) - p^M(G_{h_\tau})\right)v - LB}{p^{\text{on}}(\tau)} - p(t)\frac{\left(p^{\text{off}}(\tau) - p^M(G_{h_\tau})\right)v - LB}{p^{\text{off}}(\tau)} = \\
&\left(e^{\lambda_\ell \mathrm{d}t} - 1\right)(1-p(t))e^{-\lambda_\ell(\tau-t)}(LB + p^M(G_{h_\tau})v) \geqslant \left(e^{\lambda_\ell \mathrm{d}t} - 1\right)(1-p(t))e^{-\lambda_\ell(\tau-t)}(LB + p_b v),
\end{aligned} \tag{2}
$$

where the inequality follows from the fact that this payoff difference increases in $p^M(G_{h_\tau})$ and that $p^M(G_{h_\tau}) \geqslant p_b$. We now integrate the right-hand side of (2) and obtain that the difference between on-path and off-path continuation payoffs at time $t + \mathrm{d}t$ is at least:

$$
\begin{aligned}
&\int_{t+\mathrm{d}t}^{\infty} e^{-r(\tau-t)}\left(e^{\lambda_\ell \mathrm{d}t} - 1\right)(1-p(t))e^{-\lambda_\ell(\tau-t)}(LB + p_b v)\,\mathrm{d}\tau \\
&= \frac{\left(e^{\lambda_\ell \mathrm{d}t} - 1\right)e^{-(\lambda_\ell + r)\mathrm{d}t}(1-p(t))(LB + p_b v)}{\lambda_\ell + r} = \frac{\lambda_\ell(1-p(t))(LB + p_b v)}{\lambda_\ell + r}\mathrm{d}t + o(\mathrm{d}t).
\end{aligned} \tag{3}
$$

The worker's total discounted earnings in $[t, t+\mathrm{d}t)$ if he stays on path and being matched are:

$$
\begin{aligned}
&\int_t^{t+\mathrm{d}t} e^{-r(\tau-t)}p(t)\frac{\left(p^{\text{on}}(\tau) - p^M(G_{h_\tau})\right)v - LB}{p^{\text{on}}(\tau)}\mathrm{d}\tau \geqslant \int_t^{t+\mathrm{d}t} e^{-r(\tau-t)}p(t)\frac{\left(p^{\text{on}}(\tau) - p_a\right)v - LB}{p^{\text{on}}(\tau)}\mathrm{d}\tau \\
&= \frac{(p(t)-1)\left(1 - e^{-(\lambda_\ell + r)\mathrm{d}t}\right)(LB + p_a v)}{\lambda_\ell + r} - \frac{\left(1 - e^{-r\mathrm{d}t}\right)p(t)(LB - (1-p_a)v)}{r} \\
&= \left((p(t) - p_a)v - LB\right)\mathrm{d}t + o(\mathrm{d}t),
\end{aligned} \tag{4}
$$

where the inequality follows from the fact that (1) decreases in $p^M(G_{h_\tau})$ and that $p^M(G_{h_\tau}) \leqslant p_a$. If the worker deviates and stays unmatched, his total discounted earnings in $[t, t+\mathrm{d}t)$ are zero.

The worker prefers to be matched than being unmatched over $[t, t+\mathrm{d}t)$ if the sum of (3) and (4) is positive. For small dt, this is satisfied if

$$LB < \frac{v(p(t)(\lambda_\ell(1-p_b)+r)+\lambda_\ell(p_b-p_a)-p_a r)}{\lambda_\ell p(t)+r}. \tag{5}$$

The right-hand side increases in $p(t)$, which is the worker's expected productivity at t conditional on no breakdown being realized. Since $p(t)$ is at least p_b, the right-hand side is the smallest when $p(t)$ equals p_b. Hence, the condition (5) is satisfied if

$$LB < \frac{v(\lambda_\ell((2-p_b)p_b-p_a)+r(p_b-p_a))}{\lambda_\ell p_b+r}.$$

In the limit of $p_a \downarrow p_b$, this condition reduces to:

$$LB < \frac{\lambda_\ell(1-p_b)p_b v}{\lambda_\ell p_b+r}. \tag{6}$$

(iii) Suppose that worker k and employer j are not matched to each other under $\mu^*(LB)_{h_t}$. We next show that there is no wage $w \geqslant -LB$ such that both k and j strictly prefer to be matched to each other at flow wage w over $[t, t+\mathrm{d}t)$ and then revert to $\mu^*(LB)_{h_{t+\mathrm{d}t}}$.

If k is matched to another employer under $\mu^*(LB)_{h_t}$, w needs to be strictly higher than worker k's current wage. This implies that employer j's flow payoff will be strictly lower than his current flow payoff. Hence, j does not strictly prefer to pair with k over $[t, t+\mathrm{d}t)$.

If k is not matched, this means that $p_k \leqslant p^M(G_{h_t})$. On the other hand, worker k's wage is at least $-LB$, so employer j's flow payoff from being matched to worker i is at most $p_k v + LB$. But employer j's flow payoff on path is at least $p^M(G_{h_t})v + LB$. So employer j will not find it strictly profitable to be matched to i.

Lastly, we calculate a b-worker's lifetime earnings. Suppose that this worker starts being hired at $t \geqslant T_b$. His expected productivity at time $\tau \geqslant t$ conditional on no breakdown is $p(\tau)$:

$$p(\tau) = \frac{p(t)}{p(t)+(1-p(t))e^{-\lambda_\ell(\tau-t)}} = \frac{p_b}{p_b+(1-p_b)e^{-\lambda_\ell(\tau-t)}}.$$

Substituting this $p(\tau)$, $p(t)=p_b$, and $p^M(G_{h_\tau})=p_b$ into the worker's expected flow-earnings (1) at time τ and integrating the flow-earnings over all $\tau \geqslant t$, we obtain the worker's expected lifetime earnings:

$$e^{-rt}\int_t^\infty e^{-r(\tau-t)}p_b\frac{(p(\tau)-p_b)\,v-LB}{p(\tau)}\mathrm{d}t = e^{-rt}\frac{\lambda_\ell p_b((1-p_b)v-LB)-LBr}{r(\lambda_\ell+r)},$$

which is strictly positive if and only if (6) holds. ■

D Proofs and additional results for section 4

D.1 Auxiliary discussion for section 4.2

Proof of Lemma 4.1. We first show the inequality for the breakdown environment. Suppose $q_a > q_b$, and let $\mu_\ell := \lambda_\ell / r$. The expected payoff of each type of each worker is given by

$$U_a(\theta_a; q_a, q_b) = \begin{cases} 1 & \text{if } \theta_a = h \\ \dfrac{1}{\mu_\ell + 1} & \text{if } \theta_a = \ell, \end{cases} \qquad U_b(\theta_b; q_a, q_b) = \begin{cases} \dfrac{\mu_\ell(1-q_a)}{\mu_\ell + 1} & \text{if } \theta_b = h \\ \dfrac{\mu_\ell(1-q_a)}{(\mu_\ell + 1)^2} & \text{if } \theta_b = \ell. \end{cases}$$

The benefit of investment is given by $B_i(q_a, q_b) = \pi\left(U_i(h; q_a, q_b) - U_i(\ell; q_a, q_b)\right)$. Therefore, given $q_a > q_b$, the benefit of investment is:

$$B_a(q_a, q_b) = \pi \frac{\mu_\ell}{\mu_\ell + 1} > B_b(q_a, q_b) = \pi \left(\frac{\mu_\ell}{1 + \mu_\ell}\right)^2 (1 - q_a).$$

Hence, the benefit to the worker who is favored post-investment is strictly higher. Again, the benefit of investment for worker i is:

$$B_i(q_a, q_b) = \begin{cases} \pi \dfrac{\mu_\ell}{\mu_\ell + 1} & \text{if } q_i > q_{-i} \\ \pi \left(\dfrac{\mu_\ell}{1 + \mu_\ell}\right)^2 (1 - q_{-i}) & \text{if } q_i < q_{-i}. \end{cases}$$

Hence, the benefit of investment for worker i is discontinuous at $q_i = q_{-i}$. We now show the inequality for the breakthrough environment. Let $q_a > q_b$. The employer uses worker a exclusively for a period of length $t^* = \frac{1}{\lambda_h} \log\left(\frac{q_a(1-q_b)}{(1-q_a)q_b}\right)$ and then splits the task equally among the two workers for a subsequent period of length $t_s := \frac{2}{\lambda_h} \log\left(\frac{q_b(1-\underline{p})}{(1-q_b)\underline{p}}\right)$. Let $S(h, q_b)$ and $S(\ell, q_b)$ denote the payoffs to a high-type worker and a low-type worker, respectively, if (i) his competitor has a high type with probability q_b; (ii) the employer holds the same belief about both workers and hence splits the task equally between the two workers until the belief for both workers drops to $\underline{p}$. The post-investment payoff for each type of each worker is:

$$\begin{aligned} U_a(h; q_a, q_b) &= 1 - e^{-rt^*} + e^{-rt^*}\left(1 - e^{-\lambda_h t^*} + e^{-\lambda_h t^*} S(h, q_b)\right), \\ U_a(\ell; q_a, q_b) &= 1 - e^{-rt^*} + e^{-rt^*} S(\ell, q_b), \\ U_b(h; q_a, q_b) &= e^{-rt^*}\left(1 - q_a + q_a e^{-\lambda_h t^*}\right) S(h, q_b), \\ U_b(\ell; q_a, q_b) &= e^{-rt^*}\left(1 - q_a + q_a e^{-\lambda_h t^*}\right) S(\ell, q_b). \end{aligned}$$

Note that $U_a(h; q_a, q_b) - U_a(\ell; q_a, q_b) > e^{-rt^*}(S(h, q_b) - S(\ell, q_b))$ whereas $U_b(h; q_a, q_b) - U_b(\ell; q_a, q_b) < e^{-rt^*}(S(h, q_b) - S(\ell, q_b))$. Hence, $B_a(q_a, q_b) > B_b(q_a, q_b)$.

To characterize $S(h, q_b)$ and $S(\ell, q_b)$, let t_1 be the arrival time of a breakthrough for a high-type worker and let t_2 be the arrival time of his competitor's breakthrough when the task is split equally between workers. For a low type, a breakthrough never arrives. In the absence of any breakthroughs, the employer experiments with the workers until the belief hits $\underline{p}$. The length of this experimentation period is given by t_s as defined above. The CDFs of t_1 and t_2 for $t_1, t_2 \leqslant t_s$ are:

$$F_1(t_1) = 1 - e^{-\frac{\lambda_h t_1}{2}}, \quad F_2(t_2) = q_b(1 - e^{-\frac{\lambda_h t_2}{2}}),$$

with corresponding density functions f_1 and f_2 respectively. Therefore,

$$S(\ell, q_b) = \int_0^{t_s} f_2(t_2)\frac{1-e^{-rt_2}}{2}\,\mathrm{d}t_2 + (1 - F_2(t_s))\frac{1-e^{-rt_s}}{2},$$

$$\begin{aligned} S(h, q_b) = &\int_0^{t_s} f_1(t_1)\left(\int_0^{t_1} f_2(t_2)\frac{1-e^{-rt_2}}{2}\,\mathrm{d}t_2 + (1-F_2(t_1))\left(\frac{1-e^{-rt_1}}{2} + e^{-rt_1}\right)\right)\mathrm{d}t_1 \\ &+ (1 - F_1(t_s))\left(\int_0^{t_s} f_2(t_2)\frac{1-e^{-rt_2}}{2}\,\mathrm{d}t_2 + (1-F_2(t_s))\frac{1-e^{-rt_s}}{2}\right). \end{aligned}$$

This allows us to obtain explicit expressions for B_a and B_b. Letting $\mu_h := \lambda_h/r$, we have

$$\begin{aligned} B_a(q_a, q_b) = \pi &\left(\frac{q_b(\underline{p}-1)}{(q_b-1)\underline{p}}\right)^{-2/\mu_h}\left(\frac{(q_b-1)q_a}{q_b(q_a-1)}\right)^{-1/\mu_h} \\ &\frac{(1-\underline{p})^2\left(\frac{q_b(1-\underline{p})}{(1-q_b)\underline{p}}\right)^{\frac{2}{\mu_h}}(q_b(\mu_h q_b + 2) - (\mu_h+2)q_a) - (1-q_b)^2(\underline{p}(\mu_h(\underline{p}-2)-2) + (\mu_h+2)q_a)}{2(\mu_h+2)(q_b-1)(1-\underline{p})^2 q_a} \end{aligned}$$

if $q_a > q_b$, and

$$\begin{aligned} B_a(q_a, q_b) = \pi &\left(\frac{q_a(\underline{p}-1)}{(q_a-1)\underline{p}}\right)^{-2/\mu_h}\left(\frac{(q_a-1)q_b}{q_a(q_b-1)}\right)^{-1/\mu_h} \\ &\frac{(1-\underline{p})^2\left(\frac{q_a(1-\underline{p})}{(1-q_a)\underline{p}}\right)^{2/\mu_h}\mu_h q_a(q_b-1) - (q_a-1)(q_b-1)\left(\underline{p}(\mu_h(\underline{p}-2)-2) + (\mu_h+2)q_a\right)}{2(\mu_h+2)(q_a-1)(1-\underline{p})^2 q_a} \end{aligned}$$

if $q_a \leqslant q_b$. It is immediate that B_a is continuously differentiable at any (q_a, q_b) such that $q_a \neq q_b$. Moreover,

$$\lim_{q_a \to q_b^+} B_a(q_a, q_b) = \lim_{q_a \to q_b^-} B_a(q_a, q_b)$$

$$\lim_{q_a \to q_b^+}\frac{\partial B_a(q_a, q_b)}{\partial q_a} = \lim_{q_a \to q_b^-}\frac{\partial B_a(q_a, q_b)}{\partial q_a}, \quad \lim_{q_a \to q_b^+}\frac{\partial B_a(q_a, q_b)}{\partial q_b} = \lim_{q_a \to q_b^-}\frac{\partial B_a(q_a, q_b)}{\partial q_b}.$$

Hence, B_a is continuously differentiable at $q_a = q_b$ as well.[1]

[1] For detailed calculations, see the online supplement at http://yingniguo.com/wp-content/uploads/2020/06/differentiability.pdf.

$\blacksquare$

Proof of Proposition 4.1. A post-investment belief pair (q_a, q_b) and a cost-threshold pair (c_a, c_b) constitute an equilibrium if and only if $\forall i \in \{a, b\}$:

$$B_i(q_a, q_b) = c_i, \text{ and } q_i = p_i + (1 - p_i)F(c_i)\pi.$$

From the second condition, we have $c_i = F^{-1}\left(\frac{q_i - p_i}{(1-p_i)\pi}\right)$. Hence, a belief pair (q_a, q_b) constitutes an equilibrium if and only if:

$$\begin{cases} \frac{1}{\pi} B_a(q_a, q_b) - \frac{1}{\pi} F^{-1}\left(\frac{q_a - p_a}{(1-p_a)\pi}\right) & = 0 \\ \frac{1}{\pi} B_b(q_a, q_b) - \frac{1}{\pi} F^{-1}\left(\frac{q_b - p_b}{(1-p_b)\pi}\right) & = 0. \end{cases} \tag{7}$$

Let $g_a(p_a, p_b, q_a, q_b)$ and $g_b(p_a, p_b, q_a, q_b)$ denote respectively the LHS of each equation in (7). Both g_a and g_b are continuously differentiable, because B_a, B_b and F are continuously differentiable and F' is strictly positive.

Existence of symmetric equilibrium. We first show that if workers have the same prior belief, there is a symmetric equilibrium in which they have the same post-investment belief. Let $\hat{p}$ denote the two workers' prior belief and define

$$g(q, \pi) := \frac{1}{\pi} B_i(q, q) - \frac{1}{\pi} F^{-1}\left(\frac{q - \hat{p}}{(1-\hat{p})\pi}\right).$$

A symmetric equilibrium exists if there exists $\hat{q} \in [\hat{p}, \hat{p} + (1-\hat{p})\pi]$ such that $g(\hat{q}, \pi) = 0$, or equivalently,

$$\frac{\pi\left(\mu_h + \frac{\left(\frac{\hat{q}(1-\underline{p})}{(1-\hat{q})\underline{p}}\right)^{-\frac{\mu_h+2}{\mu_h}}((\mu_h+2)\hat{q}+\underline{p}(\mu_h(\underline{p}-2)-2))}{(1-\underline{p})\underline{p}}\right)}{2(\mu_h+2)} = F^{-1}\left(\frac{\hat{q} - \hat{p}}{\pi(1-\hat{p})}\right). \tag{8}$$

Such a $\hat{q}$ exists because for $\hat{q} \in [\hat{p}, \hat{p} + (1-\hat{p})\pi]$: (i) $B_i(\hat{q}, \hat{q})$ is continuous, strictly positive, and strictly less than one; and (ii) $F^{-1}\left(\frac{\hat{q}-\hat{p}}{(1-\hat{p})\pi}\right)$ is strictly increasing, equals 0 if $\hat{q} = \hat{p}$, and equals 1 if $\hat{q} = \hat{p} + (1-\hat{p})\pi$. Therefore, there exists $\hat{q} \in (\hat{p}, \hat{p} + (1-\hat{p})\pi)$ such that $F^{-1}\left(\frac{\hat{q}-\hat{p}}{(1-\hat{p})\pi}\right)$ crosses $B_i(\hat{q}, \hat{q})$ from below. Hence, $g_a(\hat{p}, \hat{p}, \hat{q}, \hat{q}) = g_b(\hat{p}, \hat{p}, \hat{q}, \hat{q}) = 0$.

Non-singularity of the Jacobian at $(\hat{p}, \hat{p}, \hat{q}, \hat{q})$. We next show that the Jacobian matrix evaluated at $(\hat{p}, \hat{p}, \hat{q}, \hat{q})$ is invertible for a generic set of parameters, where the Jacobian is given by:

$$J = \left.\begin{pmatrix} \frac{\partial g_a}{\partial q_a} & \frac{\partial g_a}{\partial q_b} \\ \frac{\partial g_b}{\partial q_a} & \frac{\partial g_b}{\partial q_b} \end{pmatrix}\right|_{(\hat{p}, \hat{p}, \hat{q}, \hat{q})}.$$

Note that J is symmetric: $\frac{\partial g_a}{\partial q_a} = \frac{\partial g_b}{\partial q_b}\Big|_{(\hat{p},\hat{p},\hat{q},\hat{q})}$ and $\frac{\partial g_a}{\partial q_b} = \frac{\partial g_b}{\partial q_a}\Big|_{(\hat{p},\hat{p},\hat{q},\hat{q})}$. Hence, we only need to show that:

$$\frac{\partial g_a}{\partial q_a} + \frac{\partial g_a}{\partial q_b}\Big|_{(\hat{p},\hat{p},\hat{q},\hat{q})} \neq 0 \tag{9}$$

$$\frac{\partial g_a}{\partial q_a} - \frac{\partial g_a}{\partial q_b}\Big|_{(\hat{p},\hat{p},\hat{q},\hat{q})} \neq 0. \tag{10}$$

Claim (9) holds because

$$\frac{\partial g(q,\pi)}{\partial q}\Big|_{q=\hat{q}} < 0.$$

This inequality follows from the fact that $\frac{1}{\pi}F^{-1}\left(\frac{q-\hat{p}}{(1-\hat{p})\pi}\right)$ generically crosses $\frac{1}{\pi}B_i(q,q)$ transversally from below at $q = \hat{q}$, as shown in the following lemma.

Lemma D.1. *There exists a set* $\Pi \subset (0,1)$ *of measure one such that* $g(q,\pi)$ *intersects zero transversally at each intersection point for any* $\pi \in \Pi$.

Proof. First, $g(q,\pi)$ is strictly increasing in π because the term $\frac{1}{\pi}B_i(q,q)$ is independent of π and F^{-1} is strictly increasing in $[0,1]$. Therefore 0 is a regular value of $g(q,\pi)$. By the Transversality Theorem (**?**), there exists a set $\Pi \in (0,1)$ of values for π such that $(0,1)\setminus\Pi$ has measure zero and for any $\pi \in \Pi$, 0 is a regular value of $g(q,\pi)$. Hence, generically the derivative of $g(q,\pi)$ with respect to q at any intersection point $q = \hat{q}$ such that $g(\hat{q},\pi) = 0$ is non-zero. ■

Claim (10) holds unless:

$$\frac{\frac{\left(\frac{\hat{q}(1-\underline{p})}{(1-\hat{q})\underline{p}}\right)^{-2/\mu_h}\left((\mu_h+2)\hat{q}^2+\mu_h(2\hat{q}-1)\underline{p}^2-2(\mu_h+1)(2\hat{q}-1)\underline{p}\right)}{(\underline{p}-1)^2} + \frac{2\hat{q}(\mu_h\hat{q}+1)}{1-\hat{q}}}{2(\mu_h+2)\hat{q}^2} = \frac{1}{\pi^2(1-\hat{p})F'\left(F^{-1}\left(\frac{\hat{q}-\hat{p}}{\pi(1-\hat{p})}\right)\right)}. \tag{11}$$

Fix $(F,\underline{p},\mu_h)$. The following lemma shows that for almost any $(\pi,\hat{p})$ claim (10) holds.

Lemma D.2. *Suppose that* F *is weakly convex. Then, claim* (10) *is satisfied in equilibrium for almost all* $(\pi,\hat{p})$.

Proof. The system of equations (8) and (11) is equivalent to:

$$g_1(\hat{p},\hat{q},\pi) := \frac{1}{\pi}F^{-1}\left(\frac{\hat{q}-\hat{p}}{(1-\hat{p})\pi}\right) - h_1(\hat{q}) = 0$$
$$g_2(\hat{p},\hat{q},\pi) := \frac{1}{\pi(1-\hat{p})F'(\pi h_3(\hat{q}))} - h_2(\hat{q}) = 0,$$

where h_1, h_2, h_3 are functions of $\hat{q}$ only and h_3 is defined from the equilibrium condition (7) as:

$$h_3(\hat{q}) := \frac{1}{\pi}F^{-1}\left(\frac{\hat{q}-\hat{p}}{(1-\hat{p})\pi}\right) = \frac{1}{\pi}B_a(\hat{q},\hat{q}).$$

Note that g_1 is strictly decreasing in $\hat{p}$ and π, whereas g_2 is strictly increasing in $\hat{p}$ but decreasing in π, by the convexity of F. Therefore, the determinant of the Jacobian matrix of this system with respect to $(\pi, \hat{p})$ is strictly negative. So the Jacobian matrix is invertible. This implies that for almost all $(\pi, \hat{p})$, the function $g = (g_1, g_2)(\hat{p}, \hat{q}, \pi)$ crosses $(0,0)$ transversally: there exists a set $\Pi \times P \subset (0,1) \times (\underline{p}, 1)$ of measure one such that for any $(\pi, \hat{p}) \in \Pi \times P$, the values of q that sustain a symmetric equilibrium satisfy claim (10). ■

Implicit function theorem. We apply the implicit function theorem for any parameter values assumed in the model except for the set of measure zero of parameters identified above. Therefore, by the implicit function theorem, there exists a neighborhood $B \subset [0,1]^2$ of $(\hat{p}, \hat{p})$ and a unique continuously differentiable map $\mathbf{q} : B \to [0,1]^2$ such that $g_a(\hat{p}, \hat{p}, \mathbf{q}(\hat{p}, \hat{p})) = 0$, $g_b(\hat{p}, \hat{p}, \mathbf{q}(\hat{p}, \hat{p})) = 0$ and for any $(p_a, p_b) \in B$

$$g_a(p_a, p_b, \mathbf{q}(p_a, p_b)) = g_b(p_a, p_b, \mathbf{q}(p_a, p_b)) = 0.$$

By the continuity of the map $\mathbf{q}$, $\mathbf{q}(p_a, p_b)$ converges to $\mathbf{q}(\hat{p}, \hat{p}) = (\hat{q}, \hat{q})$ as $p_a \to \hat{p}$ and $p_b \to \hat{p}$. Hence, the workers' post-investment probabilities of having a high type converge as well.

■

Proof of Proposition 4.2. Throughout the proof, a "worker's type" refers to the worker's pre-investment type. We focus on the equilibrium with post-investment beliefs $q_a > q_b$ and cost thresholds $c_a > c_b$ as $p_b \uparrow p_a$. The argument for the equilibrium with $q_b > q_a$ is similar.

We first characterize this equilibrium. Using B_a and B_b derived in the proof of Lemma 4.1, the cost thresholds are:

$$c_a = \pi \frac{\mu_\ell}{\mu_\ell + 1} > c_b = \pi \frac{\mu_\ell^2 (1 - q_a)}{(\mu_\ell + 1)^2}.$$

where the post investment belief pair (q_a, q_b) is given by $q_a = p_a + (1 - p_a)\pi F(c_a)$ and $q_b = p_b + (1 - p_b)\pi F(c_b)$. Note that $c_i \in (0,1)$ for each $i \in \{a, b\}$. Given that $c_a > c_b$ and $p_a > p_b$, the employer is indeed willing to favor worker a.

Let $\kappa := \frac{\mu_\ell (1 - q_a)}{\mu_\ell + 1} < 1$. Since worker a is favored post-investment, a high-type worker a obtains payoff 1, while a high-type worker b obtains payoff κ. Hence, the ratio of worker b's to worker a's payoff, conditional on each being a high type, is exactly κ.

We next argue that for any realized cost c, a low-type worker b's payoff is *at most* a fraction κ of the low-type worker a's payoff. Hence, the same holds when taking the expectation with respect to c.

1. If $c \geqslant c_a$, neither low-type worker a nor low-type worker b invests. The ratio of low-type worker b's payoff to low-type worker a's payoff is exactly κ.

2. If $c_b < c < c_a$, a low-type worker a is willing to invest but a low-type worker b is not. If the low-type worker a deviates to no investment, the ratio of low-type worker b's payoff

to low-type worker a's payoff is κ. By investing worker a obtains a strictly higher payoff. Therefore, the payoff ratio must be strictly lower when the low-type worker a invests.

3. If $c \leqslant c_b$, both the low type of worker a and of worker b invest. Ignoring investment cost $c > 0$, the payoff ratio of the low-type worker b to that of the low-type worker a is κ. Once the investment cost is subtracted from both the numerator and the denominator, the payoff ratio becomes strictly smaller.

■

Proposition D.1 (Investment polarization under breakdown learning). *Fixing all else but λ_h and λ_ℓ, there exists $\bar{\lambda} > 0$ such that for any $\lambda_h, \lambda_\ell \geqslant \bar{\lambda}$ and in any pair of equilibria, one from each environment, the worker favored post-investment invests strictly more in the breakdown environment than in the breakthrough one, whereas the worker discriminated against post-investment invests strictly less in breakdown environment than in the breakthrough one.*

Proof. Throughout the proof, we set $\pi = 1$ without loss, as π merely scales the benefit from investment $B_i(q_a, q_b)$ and the threshold for investment for each i. Let i denote the worker favored post-investment, and $-i$ be the worker discriminated against post-investment.

As we take λ_ℓ, λ_h to infinity, worker i's benefit from investment converges to 1 under breakdown learning, while it converges to

$$\bar{B}_i(q_i, q_{-i}) := \frac{(1 - q_{-i})^2 q_i + q_i - q_{-i}^2}{2q_i(1 - q_{-i})},$$

under breakthrough learning, where we use the fact that $\underline{p} \to 0$ as $\lambda_h \to \infty$. The function $\bar{B}_i(q_i, q_{-i})$ increases in q_i, and decreases in q_{-i}. Since q_i is bounded above by $p_a + (1 - p_a)\pi$ and q_{-i} is bounded below by p_b, $\bar{B}_i(q_i, q_{-i})$ is bounded from above by

$$\bar{B}_i(p_a + (1 - p_a)\pi, p_b) = \frac{(p_a + (1 - p_a)\pi)((p_b - 2)p_b + 2) - {p_b}^2}{2(p_a + (1 - p_a)\pi)(1 - p_b)} < 1.$$

By continuity of worker i's benefit from investment in λ_ℓ, λ_h, when λ_ℓ, λ_h are sufficiently large, the worker favored post-investment invests more under breakdown learning than under breakthrough learning.

As we take λ_ℓ, λ_h to infinity, worker $-i$'s benefit from investment converges to $(1 - q_i)$ under breakdown learning, while it converges to

$$\bar{B}_{-i}(q_i, q_{-i}) := \frac{(1 - q_i)(2 - q_{-i})}{(2 - 2q_{-i})} > 1 - q_i,$$

under breakthrough learning. Here, the inequality follows from $0 < q_{-i} < 1$. Given that the favored worker i invests more under breakdown than under breakthrough learning, q_i is higher under breakdown learning as well. Hence, the benefit from investment for the worker who is discriminated against is higher under breakthrough learning than under breakdown learning when λ_h, λ_ℓ are large enough. ■

D.2 Proofs for section 4.3

Proof of Proposition 4.3. Let $U_i(p_a, p_b)$ be worker i's payoff given the belief pair (p_a, p_b). For any $p_a > p_b$, the employer first uses worker a for a period of length t^*. If no signal occurs in $[0, t^*)$, the employer's belief toward worker a drops to p_b. Let $f(s)$ for $s \in [0, t^*)$ be the density of the random arrival time of the first signal from worker a. We let $p_a(s)$ be the belief that $\theta_a = h$ if there is no signal up to time s, and let $j(p_a(s))$ be the belief that $\theta_a = h$ right after the first signal at time s. Worker a's payoff is given by

$$\int_0^{t^*} f(s)\left(1 - e^{-rs} + e^{-rs}U_a(j(p_a(s)), p_b)\right) \mathrm{d}s \\ + \left(1 - \int_0^{t^*} f(s)\,\mathrm{d}s\right)\left(1 - e^{-rt^*} + e^{-rt^*}U_a(p_b, p_b)\right).$$

Worker b's payoff is given by

$$\int_0^{t^*} f(s)e^{-rs}U_b(j(p_a(s)), p_b)\,\mathrm{d}s + \left(1 - \int_0^{t^*} f(s)\,\mathrm{d}s\right) e^{-rt^*}U_b(p_b, p_b).$$

As $p_a \downarrow p_b$, t^* converges to zero. Both workers' payoffs converge to $U_a(p_b, p_b) = U_b(p_b, p_b)$. ■

Proof of Proposition 4.4. Let $U_i(q_a, q_b)$ be worker i's payoff given the belief pair (q_a, q_b). We let $p_a(s)$ be the belief toward worker a if there is no signal up to time s, and let $j(p_a(s))$ be the belief toward him right after the first signal at time s.

Given that $p_a > p_b$, the employer begins with worker a, and uses worker a exclusively if no signal occurs. We let $f(s) = p_a\lambda_h e^{-\lambda_h s} + (1 - p_a)\lambda_\ell e^{-\lambda_\ell s}$ be the density of the arrival time $s \in [0, \infty)$ of the first signal from worker a. We can write worker a's payoff as follows:

$$\int_0^{\infty} f(s)\left(1 - e^{-rs} + e^{-rs}U_a\left(j(p_a(s)), p_b\right)\right) \mathrm{d}s.$$

We can write worker b's payoff as follows:

$$\int_0^{\infty} f(s)e^{-rs}U_b\left(j(p_a(s)), p_b\right) \mathrm{d}s.$$

The payoff difference between a and b is:

$$\int_0^{\infty} f(s)\left(1 - e^{-rs} + e^{-rs}\left(U_a\left(j(p_a(s)), p_b\right) - U_b\left(j(p_a(s)), p_b\right)\right)\right) \mathrm{d}s.$$

We claim that $U_a(q_a, q_b) - U_b(q_a, q_b) \geqslant -1$ for any q_a, q_b, since $U_i(q_a, q_b)$ is in the range $[0, 1]$ for any i, q_a, q_b. Therefore, the payoff difference is at least:

$$\int_0^\infty f(s)\left(1 - 2e^{-rs}\right)\,\mathrm{d}s.$$

This term is greater than 0 if and only if $r^2 - (1-2p_a)r(\lambda_\ell - \lambda_h) - \lambda_h\lambda_\ell > 0$. ■