Economics D10-1: Lecture 4

Classical Demand Theory:
Preference-based approach to consumer behavior (MWG 3)

Logical structure of the preference-based approach

- Assumptions on preferences:
  - Substantive
    - Rational
      - Complete
      - Transitive
  - Tractability
    - Monotonic
    - Continuous
    - Convex
  - Convenient special cases
    - Homothetic
    - Quasi-linear
- Utility function representation and equivalencies
  - Existence of continuous, increasing function u representing preferences
  - Quasi-concavity of u
  - Linear homogeneity of u
  - Quasi-linearity of u
- Testable implications of utility maximization:
  - Law of Compensated Demand
  - Slutsky symmetry

Consumer Preferences: Rationality and Desirability

- Binary *preference relation* = on consumption set X.
- Rationality: a preference relation = is *rational* if it is
  - *Complete:* ∀**x**,**y** ∈ X, either **x = y** or **y = x**, or both.
  - *Transitive:* **x = y** and **y = z** ⇒ **x = z**
- Desirability assumptions:
  - Monotonicity: = is *monotone* on X if, for **x**,**y** ∈ X, **y**>>**x** implies **y** > **x**.
  - Strong monotonicity: = is *strongly monotone* if **y**=**x** and **y**?**x** imply that **y** > **x**.
  - Local nonsatiation: = is *locally nonsatiated* if for every **x** ∈ X and ε > 0 ∃ **y** ∈ X s.t.ô**y**-**x**ô = ε and **y** > **x**.

Consumer Preferences: other properties

- <u>Convexity</u>: Let X be a convex set. The preference relation = is *convex* if, ∀x ∈ X, the upper contour set {**y** ∈ X: **y = x**} is convex
- <u>Strict convexity</u>: Let X be a convex set. The preference relation = is *strictly convex* if, ∀**x**,**y**,**z** ∈ X, **y = x**, **z = x**, and **y** ? **z** implies t**y**+(1-t)**z** > **x** for all 0<t<1.
- <u>Homotheticity</u>: A monotone preference relation = on X=$\Re^{L}_{+}$ is *homothetic* if **x = y** ⇔ t**x** = t**y** for all t=0.
- <u>Quasilinearity</u>: The monotone preference relation = on X = (-∞,∞)×$\Re^{L-1}_{+}$ is *quasilinear* with respect to commodity 1 if **x** ³ **y** ⇔ (**x** + t$\mathbf{e}_1$) ³ (**y** +t$\mathbf{e}_1$), where $\mathbf{e}_1$ = (1, 0, …, 0).

Consumer Preferences: Continuity

- Continuity (Alternative Definition):
  - The preference relation = is *continuous* if, $\forall$**x** $\in$ X=$\Re^L_+$, the "at least as good as sets" =(**x**)={**z** $\in$ X: **z = x**} and the "no better than sets" =(**x**)={**z** $\in$ X: **x = z**} are closed in $\Re^L_+$. (Equivalently, define the "better than" and "worse than" sets to open in $\Re^L_+$.)
- Lexicographic (strict) preference ordering L.
  - For all **x**,**y** $\in$ X=$\Re^L_+$, **x**L**y** if $x_1>y_1$, or if $x_1=y_1$ and $x_2>y_2$, or if $x_i=y_i$ for i = 1, …, k-1 $\leq$ L-1 and $x_k > y_k$.
- L is not continuous.

Consumer Preferences: utility function representations

- Definition: The utility function u:X$\to\Re$ *represents* = if, $\forall$**x**,**y** $\in$ X, u(**x**) $\geq$ u(**y**) $\Leftrightarrow$ **x** = **y**.
- Theorem: The preference ordering = on $\Re^L_+$ can be represented by a *continuous* utility function u:$\Re^L_+\to\Re$ if it is rational, monotone, and continuous.
- Proof:

  (i) Let **e** = (1, 1, …, 1). Define u(**x**) so that u(**x**)**e** » **x**. (The completeness, monotonicity, and continuity of = ensures that this number exists and is unique for every **x**.)

  (ii) Let **x** = **y**. Then, by construction, u(**x**)**e** = u(**y**)**e**. By monotonicity, u(**x**) $\geq$ u(**y**). Let u(**x**) $\geq$ u(**y**). Again by construction, **x** » u(**x**)**e** = u(**y**)**e** » **y**. Then, by transitivity, **x** = **y**.

Consumer Preferences: continuity of the utility function

- Proof (cond.):

  (iii) For continuity of u, we need to show that the inverse image sets under u of every open ball in $\Re_+$ are open in $\Re^L_+$. Now,

  $u^{-1}((a,b)) = \{\mathbf{x} \in \Re^L_+\text{: } a < u(\mathbf{x}) < b\}$

  $= \{\mathbf{x} \in \Re^L_+\text{: } a\mathbf{e} \mathbf{<} u(\mathbf{x})\mathbf{e} \mathbf{<} b\mathbf{e}\}$

  $= \{\mathbf{x} \in \Re^L_+\text{: } a\mathbf{e} \mathbf{<} \mathbf{x} \mathbf{<} b\mathbf{e}\}$

  $= \mathbf{>}(a\mathbf{e}) \cap \mathbf{>}(b\mathbf{e})$

  By the continuity of $\mathbf{=}$, the above sets are open in $\Re^L_+$, as is there intersection.