Discussion Paper No. 1241

A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade

by

Kiminori Matsuyama

March 1999

Math Center web site:
http://www.kellogg.nwu.edu/research/math

A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade[1]

By Kiminori Matsuyama[2]

March 1999

Abstract

This paper develops a Ricardian model with a continuum of goods when consumers have nonhomothetic preferences. Goods are indexed in terms of priority, and the households add higher-indexed goods to their consumption baskets, as they become richer. South (North) has comparative advantage in a lower (higher) spectrum of goods, hence specializing in goods with lower (higher) income elasticities of demand. Due to the income elasticity difference, a variety of exogenous changes have asymmetric effects on the terms of trade, patterns of specialization, and welfare. Product cycles, accompanied by a southern terms of trade deterioration, occurs as a consequence of a faster population growth in South, a uniform productivity growth in South, as well as a global productivity improvement. South's domestic policy to redistribute income from the rich to the poor can improve its terms of trade so much that all the households in South may be better off, at the expense of North.

Keywords: The Ricardian model, The Dornbusch-Fischer-Samuelson Model, The Flam-Helpman-Stokey Models, Technology and Trade, Population Growth and Trade, North-South Trade, Product Cycle, Nonhomothetic Preferences, Demand Complementarities, Immiserizing Growth, Transfer Paradox.

JEL Classification Numbers: F11 (Neoclassical Models of Trade) and O11 (Macroeconomic Analyses of Economic Development)

---

[1] This is a substantially revised version of the paper, previously entitled as "The Dornbusch-Fischer-Samuelson Model without Homotheticity."

[2] Department of Economics, Northwestern University, 2003 Sheridan Road, Evanston, IL 60208, USA.

1. Introduction

A Ricardian model can be defined as a perfectly competitive model of trade, in which each country is endowed with one factor of production, labor, and all the traded goods are produced by constant-returns to scale technologies. International trade takes place due to the technological differences across countries. Because of these features, the Ricardian model is the most natural framework in which to investigate the roles of country size and technology in international trade. Not surprisingly, it is widely used as a building block in the recent literature on technology and trade (see Grossman and Helpman (1995) for a survey.)

The importance of the Ricardian model in the recent literature is due much to Dornbusch, Fischer, and Samuelson (1977) (DFS). Prior to DFS, the use of the Ricardian model was mostly limited to teach a few basic principles of international trade, such as comparative advantage, to undergraduates. It was seldom used to examine the issues that require a careful comparative statics analysis. The problem with the Ricardian model was that, in its classical formulation, a small change in exogenous variables often leads to a discrete change in the patterns of specialization. This discontinuity makes the Ricardian model inconvenient as a tool of comparative statics analysis. Most researchers instead used factor endowment models, such as the Heckscher-Ohlin model and the specific factor model, whose equilibrium depends smoothly on exogenous variables. The presence of multiple factors in these models, however, fundamentally changes the nature of the model, which make them less appropriate for the purpose of investigating the role of technology in trade. The contribution of DFS is that they came up with a way of making the Ricardian model tractable, without modifying its basic features. Instead of working with a finite number of goods, they introduced a continuum of goods and demonstrated, under the assumption of Cobb-Douglas preferences, that a Ricardian model with a continuum of goods is highly tractable and can be used to analyze a variety of issues.

Although DFS did much to enhance the current popularity of the Ricardian model, the assumption of homothetic preferences, implied by the Cobb-Douglas specification, is too restrictive for thinking about many important issues in trade and development, where technological factors play central roles. To give an example, many policymakers and economists in the Third World are concerned with the tendency that the terms of trade deteriorate continuously against poor countries. As the world economy grows, they would argue, the relative demand shifts away from South, which specializes in goods with low income elasticities of demand, and toward industrialized North, which specializes in goods with higher income elasticities. Industrial policy advocates often suggest that a developing country should transform its industrial structure, by targeting sectors with high income elasticities of demand, so as to be able to enjoy the benefits of the growing global economy. Furthermore, South would benefit little from productivity improvement in its export sectors, as the increased purchasing power generated by the lower prices of southern goods would be spent mostly on northern goods. It has also been suggested that new industries are born in North, because only rich consumers can afford to purchase new, often luxury, products. The assumption of homothetic preferences, which implies that all the goods have the same, unitary income elasticities and that the rich and the poor consume all the goods in the same proportion, is simply not appropriate for addressing these issues.

This paper develops an alternative to DFS, an equally tractable Ricardian model with a continuum of goods, in which consumers have nonhomothetic preferences, so as to make it applicable to these issues in trade and development. Goods are ordered in terms of priority. Goods at the lower end of the spectrum are consumed by all the households. As their income levels go up, the households expand their range of consumption, by adding higher-indexed goods to their baskets. In the central case, there are two countries: North and South. The developed, high income North has comparative advantage in a higher spectrum of goods, while the underdeveloped, low income South has comparative advantage in a lower spectrum. This makes South (North) specialize in goods whose demand has lower (higher) income elasticities.

Because of the income elasticity difference, a variety of exogenous changes have asymmetric effects on the terms of trade, patterns of specialization, and welfare. For example, the terms of trade moves against South, and product cycles (i.e., new industries are born in North and old industries move from North to South) occur, as a consequence of a faster population growth in South, a uniform productivity growth in South , as well as a global productivity improvement. The welfare gain of productivity growth is also unevenly distributed. North can capture all the benefits of its own uniform productivity growth, while South may lose from its own uniform productivity growth. The reason for these effects is the asymmetry of *demand complementarities* between goods. When the prices of lower-indexed goods decline, demand for higher-indexed goods will increase. This is because the households respond to the higher real income, resulting from the reduction in lower-indexed goods prices, by adding higher-indexed goods to their consumption baskets. In other words, the income effect makes higher-indexed goods complements to lower-indexed goods in demand. On the other hand, when the prices of higher-indexed goods decline, demand for lower-indexed goods will not increase.

Nonhomotheticity of preferences also implies that transfer payments, which affect *income distribution*, both within and across the two countries, have nontrivial effects. For example, South's domestic policy to redistribute income from the rich, who buys foreign imports, to the poor, who cannot afford to buy them, can lead to a large terms of trade improvement that all the households in South may be better off at the expense of North.[3]

This paper is closely related to Flam and Helpman (1987) and Stokey (1991) (FHS). Both studies presented Ricardian models with a continuum of goods, where consumers have nonhomothetic

---

[3] Another implication of the present framework is that the effects of a population change are no longer isomorphic to those of a uniform change in technology. Even though both changes increase the effective supply of labor, they need

preferences, and applied them to North-South trade. There is the fundamental difference, however. In FHS, goods are indexed according to the product quality, and the assumed preferences imply that different goods are gross substitutes. That is, a reduction in the prices of lower-indexed goods induces the households to *switch* from a higher-indexed good to a lower-indexed good, due to the substitution effect. This may be reasonable, when the goods are vertically differentiated products within an industry, and the models are used to address the issues of intra-industry trade, which is indeed the interpretation offered by FHS. In the present model, on the other hand, goods are not gross substitutes; there are demand complementarities from southern goods to northern goods. That is, a reduction in the prices of lower-indexed goods induces the households to *expand* their consumption set toward higher-indexed goods, due to the income effect. The present model is more appropriate for addressing sectoral issues in trade and development in the presence of significant difference in the income elasticities of demand across sectors.[4],[5]

The rest of the paper is as follows. Section 2 develops a basic model, and highlights its key features in comparison with DFS. Sections 3 and 4 consider the case where each country has a homogenous population. By abstracting from the effects of income distribution, this simplification helps

---

to be treated separately. This point, however, is true for any nonhomothetic preferences, not a unique to the present setting.

[4]Some examples may be useful to illustrate the differences. In Flam and Helpman, each household is restricted to choose only one good from the spectrum (the rich owns a BMW, but not a Yugo). Hence, North-South trade takes place, only when there is nondenegerate income distribution within each country in their model. In Stokey, each household may consume a range of goods from the spectrum (the rich may own both a bike and a car, while the poor owns only a bike). Hence North-South trade takes place even if income distribution is degenerate within each country. (Indeed, her analysis is restricted to the case of homogenous populations). In the present model, the rich buys food and cloths from South, and owns a car made in North, and lower food and cloth prices make a northern-made computer affordable to them (while car production may move to South).

[5] Wilson (1980) considered a variety of extensions to the DFS model, which include some nonhomothetic preferences, and examined the robustness of their results. He basically showed that many comparative statics results obtained by DFS carry over, as long as the difference in income elasticities is not significant and goods are gross substitutes. His analysis, however, offered little insights when there are significant income elasticity differences, or when goods are not gross substitutes. The main problem is that his model, by extending the DFS model, has lost the tractability of DFS. This paper adopts a different strategy from his. Instead of presenting an *extension* of the DFS model, this paper presents an *alternative* to the DFS, which is capable of incorporating significant income effects in a

to focus on the effects of demand complementarities. Section 3 conducts comparative statics under the assumption that the poor country has comparative advantage in lower-indexed goods. This section also offers a more detailed comparision between the present model and FHS. Section 4 briefly discusses how the results change when the poor country has comparative advantage in higher-indexed goods. Section 5 looks at the case of heterogeneous populations and discusses the effect of income redistribution policies. Section 6 extends the model to a multicountry case. Section 7 suggests the direction for future research.

2. The Model

This section develops the basic model, in which there are two countries, Home and Foreign. An extension to a multicountry case is done in Section 6.

Technology:

There is a continuum of competitive industries, indexed by $z \in [0,\infty)$, each producing a homogenous good, also indexed by z. Labor is the only factor of production. Let $a(z)$ and $a^*(z)$ be the Home and Foreign unit labor requirements of sector z, i.e., labor input required to produce one unit of output z in Home and Foreign. Following DFS, it is assumed that

(A1). $a^*(z)/a(z)$ is continuous and strictly decreasing in z.

Thus, Home has comparative advantage in a lower spectrum of goods, and Foreign in a higher spectrum. By taking Foreign labor as a *numeraire*, and denoting the price of a Home labor by w, the price of good z is given by $p(z) = \min\{wa(z), a^*(z)\}$. Given (A1), there is a marginal good, m, the switching point in the chain of comparative advantage, so that Home produces only goods in $[0, m]$, and Foreign produces only goods in $[m, \infty)$, and the prices are determined by

(1) $p(z) = wa(z),\ z \in [0,m]; \qquad p(z) = a^*(z),\ z \in [m,\infty).$

---

tractable manner. The goal of this paper is not to examine the robustness of the DFS model, but to develop a Ricardian model applicable to many issues that are central in trade and development.

The marginal good is inversely related to w, according to

(2) $w = a^*(m)/a(m)$.

This relation is depicted by the downward sloping curve in Figure 1.

Households:

There are N households at Home and N^* households at Foreign. There may be a nondegenerate income distribution due to skill differences, reflected in differences in the effective labor supply. Let $F(h)$ and $F^*(h^*)$ be the distributions of effective labor supply across the households at Home and Foreign respectively. A home household with h units of effective labor earns wh, while a foreign household with h^* earns h^*.

All the households share the same preferences. The present model differs critically from DFS in the structure of preferences. A household with the income I seeks to maximize $V = \int_0^{\infty} b(z)x(z)dz$, subject to the budget constraint, $\int_0^{\infty} p(z)x(z)dz \leq I$, where $b(z) > 0$ is the utility weight attached to good z, and $x(z)$ is an indicator function, with $x(z) = 1$ if good z is consumed and $x(z) = 0$ if it is not. The assumption that goods come in discrete units and that each household's desire of a particular good satiates after one unit, adopted from Murphy, Shleifer and Vishny (1989), has a strong implication. An increase in the utility takes the form of increased diversity, and not of increased consumption of the same good, and wealthier households consume all the goods consumed by poorer households, plus some.[6] Furthermore, it is assumed that the order in which each household purchases goods is the same with the order of goods given by (A1).

(A2). $b(z)/p(z) = b(z)/\min\{wa(z), a^*(z)\}$ is strictly decreasing in z.

[6] One may call lower-indexed goods "necessities," and higher-indexed goods "luxuries." However, none of the goods satisfies the standard definition of "a necessity" nor "a luxury," based on the property of demand function. Any particular good is a luxury for a sufficiently poor household, and a necessity for a sufficiently rich household.

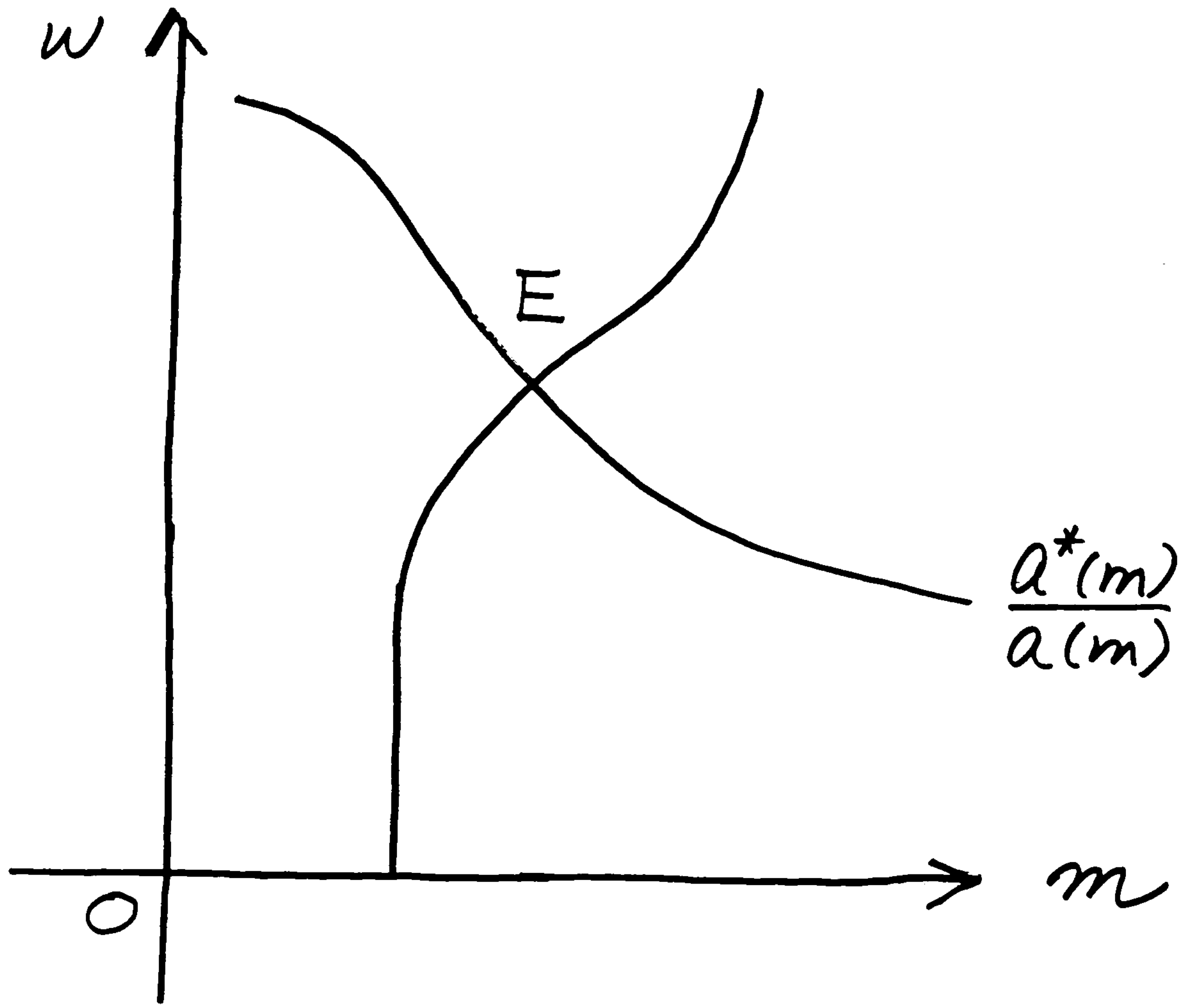


Figure 1

These two assumptions, (A1) and (A2), jointly imply that Home has comparative advantage in lower-indexed goods, which even poor households purchase and that Foreign has comparative advantage in higher-indexed goods, those purchased by wealthier households.[7] Let us define

(3) $$E(z) \equiv \int_0^z p(s)ds = \int_0^z \min\{wa(s), a^*(s)\}ds .$$

Since the household purchases all the lower indexed goods and expands its range of consumption upward as far as it can afford, a Home household with income, wh, chooses $x(z) = 1$ for $z \in [0, u(h)]$ and $x(z) = 0$ for $z \in (u(h), \infty)$, where $u(h)$ is given by

(4) $$E(u(h)) = wh$$

and attains the utility level, $V(h) = B(u(h))$, where $B(z) \equiv \int_0^z b(s)ds$. Similarly, a Foreign household with income, h^*, chooses $x(z) = 1$ for $z \in [0, u^*(h^*)]$ and $x(z) = 0$ for $z \in (u^*(h^*), \infty)$, where

(5) $$E(u^*(h^*)) = h^* .$$

and attains the utility level, $V^*(h^*) = B(u^*(h^*))$. In this model, there is a one-to-one mapping between the level of utility attained by a household, $V(h)$ *and* $V^*(h^*)$, and the highest indexed good they consume, $u(h)$ and $u^*(h^*)$. For this reason, $u(h)$ and $u^*(h^*)$ may also be viewed as utility measures.

The Labor Market Equilibriums and the Balanced Trade:

[7] What is critical here is that there is *some* (either positive or negative) correlation between the ordering of goods based on the pattern of comparative advantage and the order in which the households purchase goods. In DFS, the correlation is assumed to be *zero* (i.e., homotheticity), which helps to make their model tractable. The present model assumes that the correlation is perfect, in order to consider the implications that there is *some* correlation, and at the same time, to keep the tractablity of the model. Note that no assumption has yet been made in terms of the direction of correlation (whether positive or negative). An additional assumption is needed to make this model one of North-South Trade, where the poorer country produces goods with lower income elasticities of demand. This will be done later in section 3, in the form of (A3).

Since Home produces only the goods in [0,m], and the goods in [0,m) are produced only by Home, the Home labor market equilibrium condition is given by $L = N\int_0^\infty hdF(h) = \int_0^m a(z)Q(z)dz$, where $Q(z)$ is the total demand for good z. Since all the household whose income is greater than or equal to $E(z)$ consumers one unit of good z, $Q(z) = N[1-F(E(z)/w)] + N^*[1-F^*(E(z))]$. Hence, the Home labor market equilibrium can be rewritten to

$$wN\int_0^\infty hdF(h) = \int_0^m wa(z)Q(z)dz = \int_0^m Q(z)dE(z) = \int_0^m \left\{ N\left[\int_{E(z)/w}^\infty dF(h)\right] + N^*\left[\int_{E(z)}^\infty dF^*(h^*)\right]\right\} dE(z)$$

$$= N\int_0^\infty \left[\int_0^{\min\{E^{-1}(wh),m\}} dE(z)\right] dF(h) + N^*\int_0^\infty \left[\int_0^{\min\{E^{-1}(h^*),m\}} dE(z)\right] dF^*(h^*),\text{ or}$$

$$\text{(6)}\qquad wL = wN\int_0^\infty hdF(h) = N\int_0^\infty \min\{wh, E(m)\}dF(h) + N^*\int_0^\infty \min\{h^*, E(m)\}dF^*(h^*).$$

The left-hand side is the total labor income at Home, equal to the Home national income. The right hand is the Home GNP, equal to the total spending on the Home goods; the first term is Home's expenditure on the Home goods, the second term is Foreign expenditure on the Home goods. Note that a Home household spends $\min\{wh, E(m)\}$ and a Foreign household spends $\min\{h^*, E(m)\}$ on the Home products. Similarly, the Foreign labor market equilibrium condition can be written to

$$\text{(7)}\qquad L^* = N^*\int_0^\infty h^* dF^*(h^*) = \int_m^\infty a^*(z)Q(z)dz$$

$$= N\int_0^\infty \max\{wh - E(m), 0\}dF(h) + N^*\int_0^\infty \max\{h^* - E(m), 0\}dF^*(h^*)$$

The two labor market equilibrium conditions, (6) and (7), are indeed identical, due to Walras's Law, and they can be rearranged to obtain

(8) $$N\int_0^{\infty}\max\{h-\int_0^m a(s)ds,0\}dF(h) = N^*\int_0^{\infty}\min\{\frac{h^*}{w},\int_0^m a(s)ds\}dF^*(h^*).$$

where use has be made of $E(m)=\int_0^m p(s)ds = w\int_0^m a(s)ds$. Equation (8) states that, given the static nature of the model, the trade is balanced. That is, the value of the Home imports, the left-hand side, must be equal to the value of the Home exports, the right-hand side. The balanced trade condition, (8), is depicted in Figure 1. It is upward-sloping, as long as some Foreign households are poor enough to consume only the Home goods. An increase in the relative wage of the Home labor, a rise in w, would force such poor Foreign households to cut their spending on the Home goods, thereby reducing indirect demand for Home labor. To restore the equilibrium, Home must expand the range of production, a rise in m. When w is sufficiently small that all the Foreign households are rich enough to consume some Foreign goods, a small change in w does not affect the demand for Home labor. In this case, the balanced trade condition, (8), becomes $(N+N^*)\int_0^m a(s)ds = L$ and it is vertical, as depicted in Figure 1.

Equations (2) and (8) jointly determine the equilibrium value of m and w. Then, from (4) and (5), one can determine the equilibrium range of goods consumed by different households, as well as their utility levels.

A Comparison with the Dornbusch-Fischer-Samuelson (DFS) model:

Before proceeding, it is worth comparing the present model with the DFS model. DFS assumed that all the households have the identical Cobb-Douglas preferences over a fixed range of goods, say [0,1], with $V=\int_0^1\beta(z)\ln(x(z))dz$, where $\beta(\bullet) > 0$ satisfies $\int_0^1\beta(s)ds = 1$, and $x(z)$ can be any positive real number. This assumption implies that each household spends the fraction, $\vartheta(z) \equiv \int_0^z \beta(s)ds$, of income on the goods in [0,z], regardless of the income level. As a result, the labor market condition, or equivalently the balanced trade condition, becomes

$$(9) \qquad w = \frac{\vartheta(m)}{1-\vartheta(m)} \frac{N^* \int_0^\infty h^* dF^*(h^*)}{N \int_0^\infty h dF(h)} = \frac{\vartheta(m)}{1-\vartheta(m)} \frac{L^*}{L},$$

which yields a positive relation between w and m. In DFS, the equilibrium levels of w and m are determined jointly by (2) and (9). Thus, understanding the differences between (8) and (9) is the key for understanding the results below.

First, as seen in eq. (9), DFS, due to their homothetic preferences, is independent of income distribution within each country, and hence they cannot affect the aggregate variables, such as m and w. On the other hand, as shown in eq. (8), the equilibrium values of m and w depend on distribution of h, and h^*, due to the nonhomothetic preferences. Second, eq. (9) passes through the origin. That is, $w \to 0$ implies $m \to 0$. On the other hand, m approaches a positive number, satisfying $(N+N^*)\int_0^m a(s)ds = L$, according to eq. (8). In DFS, as the Home wage rate and the price of Home goods become cheaper, all the households increase the amount of consumption of Home goods, through substitution effects, which increase demand for Home labor. To keep the labor market equilibrium and the balanced trade, Home's production must keep shifting toward the bottom end of the goods spectrum. In the present model, the households do not increase the amount of consumption of lower-indexed goods, when their prices go down. The aggregate demand for each good is bounded by $N+ N^*$. For this reason, the Home must continue to produce a certain range of lower indexed goods to keep all the Home labor employed. Third, the unit labor requirements, $a(z)$, and $a^*(z)$, do not appear in eq. (9). That is, at constant relative wages, neither a change in $a(z)$ nor a change in $a^*(z)$ affect the labor market equilibrium in DFS, due to the Cobb-Douglas preferences.[8] In the present model, there is asymmetry between $a(z)$ and $a^*(z)$. Reducing $a(z)$ and hence the prices of the Home goods shift the household spending away from Home goods toward Foreign goods, thereby increasing the relative demand for Foreign labor, due to demand complementarities. To restore the balance, Home must expand its range of production. On the other hand, $a^*(z)$ does not appear in eq. (8), because a reduction in $a^*(z)$, and the prices of Foreign goods only induce the household to buy other Foreign goods with higher indices, and hence does not cause a spending shift between Home and Foreign goods. What matters for the following analysis is not so much that $a^*(z)$ does not affect eq. (8), but rather that there is asymmetry in which $a(z)$, and $a^*(z)$ affect eq. (8).

3. North-South Trade: The Case of Homogeneous Populations

In the next two sections, it is assumed that households are homogenous in each country. By abstracting the effect of income distribution, this assumption helps to focus on the implications of asymmetric demand complementarities. Let all the households endowed with one unit of effective labor. Then, eq. (8) becomes $\int_0^m a(s)ds = N/(N+N^*)$, if $w \leq 1+N^*/N$, and $\int_0^m a(s)ds = 1 - N^*/(wN)$, if $w > 1+N^*/N$, as depicted in Figure 2.

In this section (but not in the next), we further assume

(A3) $a(z) > a^*(z)$ for all z.

which means that Foreign has absolute advantage in all the industries. This assumption ensures that the foreign households are richer than the home households, $w < 1$, in equilibrium. Combined with (A1) and (A2), (A3) implies that the poor (rich) country specializes in production of goods, with lower (higher)

[8] As Wilson (1980) demonstrated, this feature of the DFS model does not hold even for general homothetic preferences.

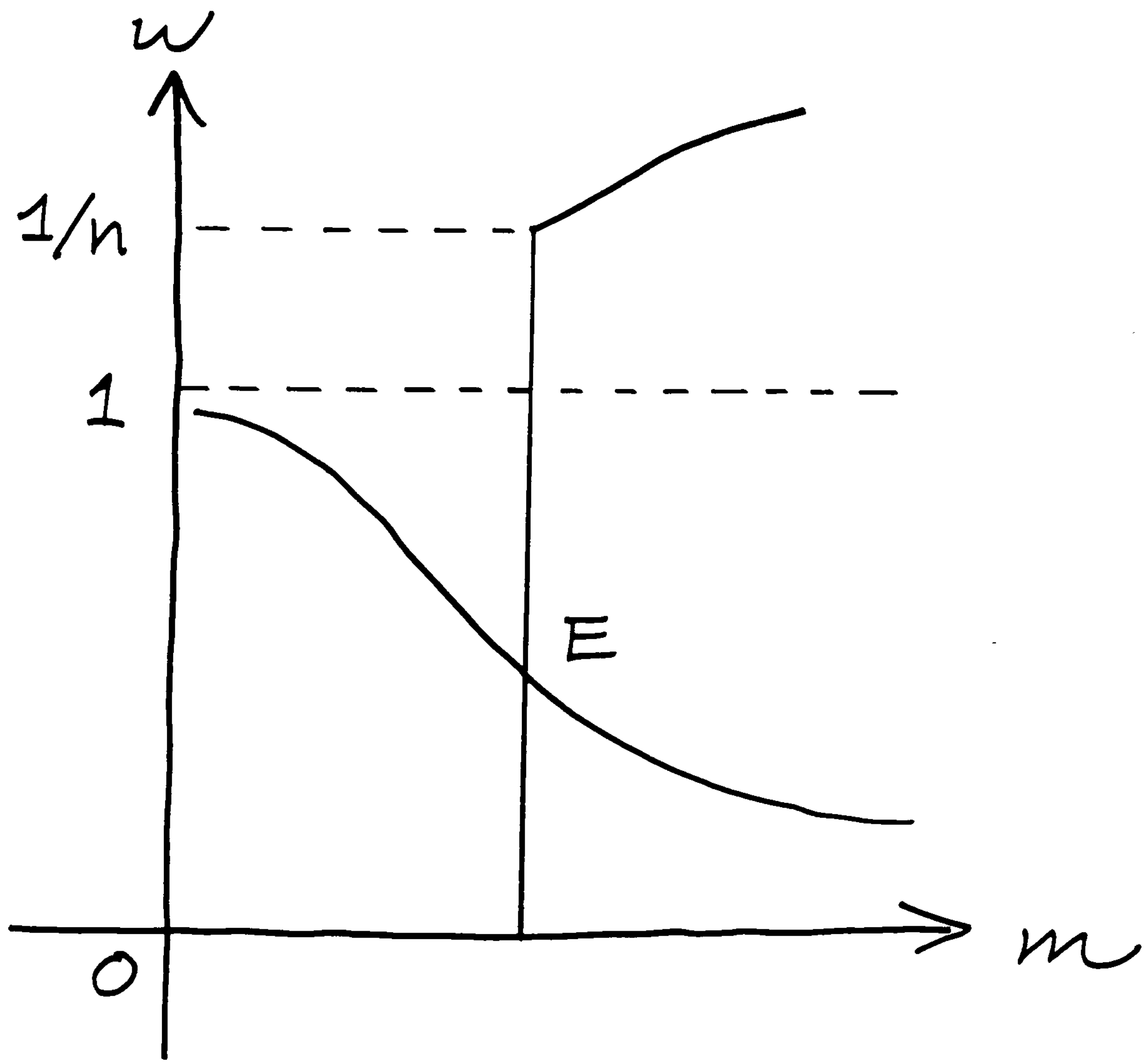


Figure 2

income elasticities of demand. It is the combination of (A2) and (A3) that makes this model one of North-South trade.[9] We identify hereafter Foreign as developed, high income North, and Home as less developed, low-income South.

The equilibrium conditions, eq. (2) and eq. (8), now become

$$(10) \qquad \int_0^m a(s)ds = n,$$

$$(11) \qquad wa(m) = a^*(m)$$

where n is the share of South in the world population. Equation (11) is a reproduction of eq. (2). These conditions are depicted in Figure 2. Note that the downward-sloping curve intersects with the vertical section of the balanced trade/labor market equilibrium condition, which is why eq. (10) is independent of w. This feature of equilibrium greatly simplifies the comparative statics exercises conducted below, but not essential for the central results.[10]

Equations (4) and (5) now become

$$(12) \qquad w\int_0^m a(s)ds + \int_m^u a^*(s)ds = w$$

$$(13) \qquad w\int_0^m a(s)ds + \int_m^{u^*} a^*(s)ds = 1$$

[9]The joint satisfaction of (A1), (A2), and (A3) can be justified as follows. Imagine that technology is subject to sector-specific, country-specific, learning-by-doing, similar to Krugman (1987), and that, initially, the two countries are in autarky, with North having overall better technologies than South. Then, with the preferences structure assumed here, technology gaps between North and South would become larger in higher-indexed sectors under autarky. (See Matsuyama (1999) for such a closed economy model.) The model of this section can be interpreted as describing the situation where these two countries start trading after these patterns of technologies have developed.

[10] What is essential is that, for a given w, a decline in $a(z)$ increases m more than a decline in $a^*(z)$ of the equal magnitude decreases m along the balanced trade/labor equilibrium condition. The exact form of eq. (10), its independence of $a^*(z)$ and of w, is not essential. For example, one could extend the present model by putting leisure into the utility function, thereby endogenizing labor supply. (Or equivalently, one could introduce a nontradable goods sector.) Such an extension make the balanced trade/labor equilibrium condition dependent of $a^*(z)$ and of w, but do not change the essential features of the model. Those who remain skeptical should also consult section 5, which show the robustness of the results, even though nondegenerate income distribution makes the balanced trade/labor equilibrium condition dependent of w.

where u and u^* are the highest-indexed good consumed, hence the utility level attained, by southern and northern households. They satisfy $m < u < u^*$. Since North imports all the goods produced by South, South imports some northern goods in exchange. Hence, in this equilibrium, all the southern households consume all the basic goods produced at South, plus some goods produced at North ($m < u$). Northern households, who are richer than southern households, consume a wider range of their own goods and a higher level of utility ($u < u^*$).

The volume of trade per household is

$$(14) \qquad 2N^* \int_0^m a(s)ds/(N+N^*) = 2n(1-n),$$

measured in the unit of Home labor. The volume of trade is thus independent of the terms of trade, w, and of the patterns of production, m.

A Comparison with the Flam-Helpman and Stokey (FHS) Models:

Before proceeding, it might be instructive to compare the present model with the North-South Trade models of Flam and Helpman (1987) and of Stokey (1991), which have apparent similarities. First, their models have a continuum of goods supplied competitively. Second, the preferences are nonhomothetic in such a way that only high income households demand for higher-indexed goods, and the set of goods produced in equilibrium is endogenous. Third, they make the assumptions analogous to (A1) through (A3). The country with comparative advantage in higher indexed goods, is the developed North, having absolute advantage in all the indexed goods, and the less developed South has comparative advantage in lower-indexed goods.

There is the fundamental difference, however, between FHS and the present model. In FHS, goods are indexed according to the product quality, and the assumed preferences imply that different goods are gross substitutes. That is, a reduction in the prices of lower-indexed goods induces the households to *switch* from a higher-indexed good to a lower-indexed good, due to the substitution effect. This may be reasonable, when the goods are vertically differentiated products within an industry, and the models are used to address the issues of intra-industry trade, which is indeed the interpretation offered by FHS. In the present model, on the other hand, goods are not gross substitutes. A reduction in the prices of lower-indexed goods induces the households to *expand* their consumption set toward higher-indexed goods, due to the income effect. (Goods are not, however, Pareto-Edgeworth complements; there is no greater benefit of consuming the goods together than separately.) The present model should thus be interpreted as addressing intersectoral trade, where different sectors produce goods, whose demand have different income elasticities.

Due to the above-mentioned difference in the demand structures, the equilibrium in FHS has the following features. First, the goods at the bottom end of the spectrum are not produced. Second, there is a gap between the range of goods produced in the two countries. That is, there is a range of goods not produced in equilibrium, which are of higher quality than the highest-quality good produced by South, and of lower quality than the lowest-quality good produced by North. In the present model, no gap exists in the range of goods produced. Third, a deterioration of South's terms of trade, which makes southern goods cheaper and causes a shift of production of some goods from North to South, also tends to discourage North from producing the upper end of the spectrum. In the present model, North introduces new goods at the upper end when South's terms of trade deteriorate, as will be seen below.

3A. Population Size

This section discusses the effects of a change in the population sizes in the two countries. An increase in N means that the southern population and labor supply increase at the same rate. The vertical line, the balanced trade condition (10), shifts to the right in Figure 2. The factor terms of trade move against South (a decline in w), and some industries move from North to South (an increase in m). A differentiation of (10) and (11) yields

(15) $a(m)dm = dn > 0$

(16) $dw = -\xi(m)dm < 0,$

where $-\xi(m)$ is the slope of eq. (11) at m. Note that $\xi(m)$ can be any positive number. A total differentiation of (12) and (13), and use of (10) and (11), yields

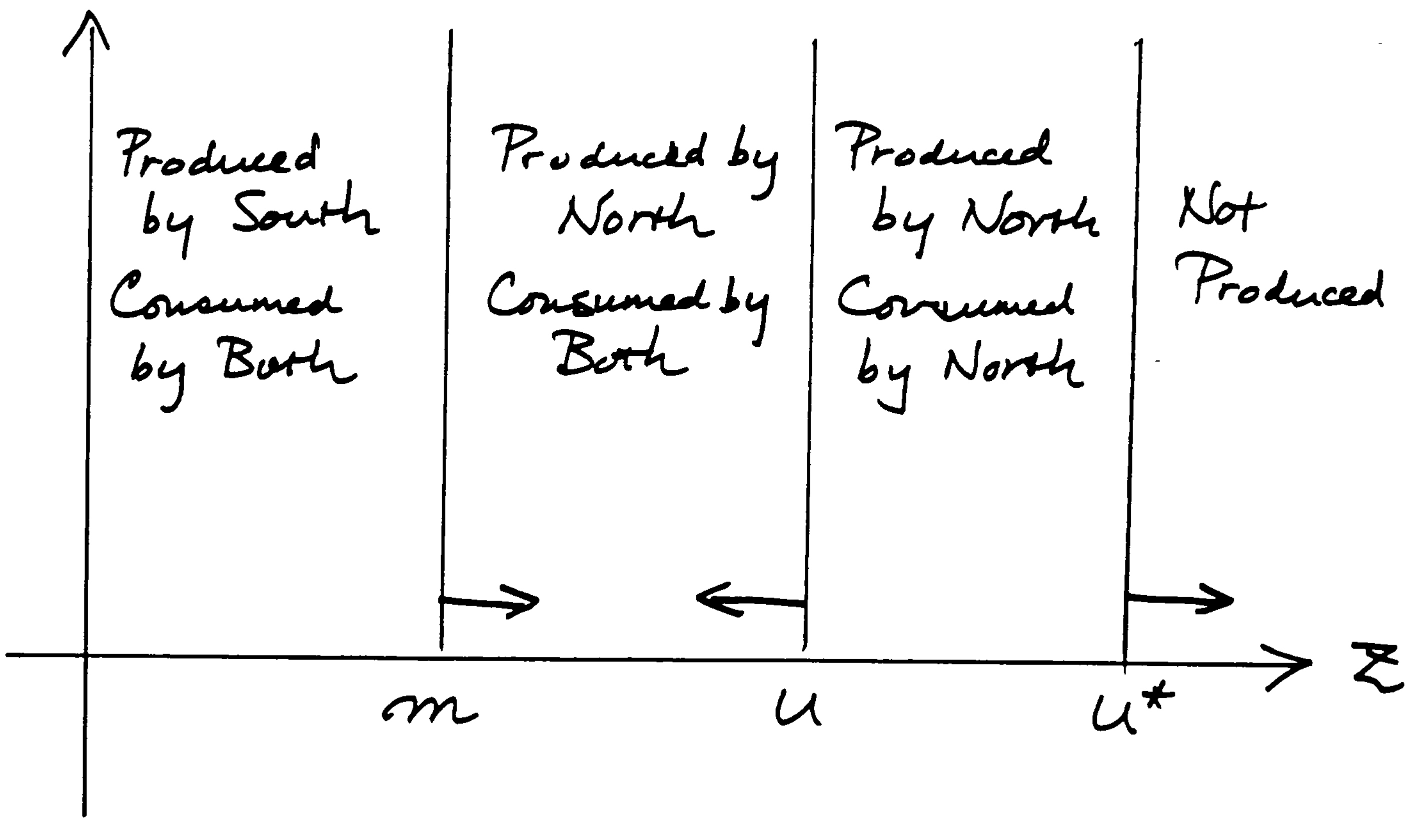
(17) $a^*(u)du = (1-n)dw < 0,$

(18) $a^*(u^*)du^* = -ndw > 0.$

The effects of a change in m cancel out due to (11). Note also that $a^*(u)ndu + a^*(u^*)(1-n)du^* = 0,$ hence the effect is purely distributional. An increase in N^* has exactly the opposite effects, because the system, (10)-(13), depends solely on the relative size of the two countries. Figure 3 summarizes the above results, showing how the range of goods consumed and produced in each country as the southern share of the population, n, changes. The volume of trade per household is, from (14), reaches the highest level when n = 1/2, the result previously obtained in models of trade in horizontal differentiated products (see Helpman and Krugman 1985, Ch. 8).

In DFS, an increase in the Home country size, which shifts *down* the balanced trade curve at the same rate, reduces the Home relative wage proportionally less than the size increase. Hence, the share of Home income in the world rises. In the present model, an increase in the Home country size, which shifts the balanced trade line *to the right*, may reduce the Home relative wage more than the size increase, because there is no restriction on the magnitude of $\xi(m)$, the slope of the downward-sloping curve, (11). Therefore, the share of Home income in the world may go down.

Unlike in the DFS model, and as in the FHS models, the range of goods produced in the world economy changes in this model. Imagine that the population grows faster in South than in North over



Produced by South
Consumed by Both
Produced by North
Consumed by Both
Produced by North
Consumed by North
Not Produced
m
u
u*
z

Figure 3

time. South experiences a secular decline in its terms of trade, and the lower end of industries in North move continuously to South. As the prices of imports from South decline, the northern households expand its range of consumption continuously toward higher-indexed goods, thereby giving birth to new industries in North. The faster population growth in South can hence generate product cycle phenomena, similar to those discussed by Linder (1961) and Vernon (1966).

The above result is in sharp contrast with FHS. First, in their models, new goods do not appear in North in response to the faster population growth in South. The upper end of the spectrum of goods produced does not change in the Flam & Helpman model. In Stokey, the highest indexed goods are abandoned, as the northern households switch to cheaper goods produced in South. Second, although FHS also predict that North abandons the lower end of the goods in response, these goods are not immediately produced in South. Only after a time lag, South starts producing the goods previously produced in North.

3B. Productivity Changes

This section examines the effects of productivity improvement, which means a reduction in $a(z)$, and $a^*(z)$. Let $g(z) \equiv -dlog(a(z))$ and $g^*(z) \equiv -dlog(a^*(z))$ be the rate of productivity growth in sector z in North and in South, respectively. Then, from (10)-(13),

(19) $$a(m)dm = \int_0^m g(s)a(s)ds,$$

(20) $$dw = -\xi(m)dm + w\{g(m) - g^*(m)\}$$

(21) $$a^*(u)du = (1-n)dw + w\int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds$$

(22) $$a^*(u^*)du^* = -ndw + w\int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds$$

The first terms in the right-hand sides of (21) and (22) represents the (factor) terms of trade effect. Although an overall impact of productivity growth is positive $(a^*(u)ndu + a^*(u^*)(1-n)du^* > 0)$, the welfare gain can be unevenly distributed between North and South due to the terms of trade effect.

Northern productivity improvement: $(g(z) = 0$ *and* $g^*(z) > 0)$

First, let us consider the effect of a productivity improvement in North. In Figure 2, this implies that the downward sloping curve shifts down, while the vertical line is unaffected. Hence, m remains unchanged and w declines at the rate equal to $g^*(m)$: see eqs. (19) and (20). Hence, from (22), $du^* > 0$; the northern household expands its range of consumption and their welfare improves. New industries are born in North. The effect on southern household is subtler. From (19) and (20), (21) becomes $a^*(u)du = -(1-n)wg^*(m) + \int_m^u g^*(s)a^*(s)ds$. From (10) and (12), this can be further rewritten to

$$a^*(u)du = \int_m^u \{g^*(s) - g^*(m)\}a^*(s)ds.$$

If the productivity change is uniform across the export sectors, $g^*(z) = g^*$ for all $z \in [m, u]$, the South's factor terms of trade deterioration (the decline in w) offsets exactly the productivity improvement in all the northern export sectors, and hence, the South's terms of trade measured in goods remains the same, and therefore, the southern household's budget constraint remains intact. As a result, $du = 0$. This case thus serves as a useful benchmark. The result that North captures the entire gain of productivity improvement, without any spillover effect to South, offers a strong contrast with the effect of a northern population growth, examined in the previous section, even though both changes imply an increase in the aggregate supply of effective labor in North. Population growth in North would lead to a proportional increase in northern demand for all the goods that they consume, so that some demand increase go to southern goods and South is better off. On the other hand, productivity improvement leads to an increase in income earned by each northern household, and hence leads to an increase in demand only for northern products. This result also differs sharply from DFS, where a uniform improvement in northern labor productivity has the same effect with an increase in the population, because of the homotheticity (see eq. (9)). The equivalence is lost when preferences are nonhomothetic, as in the present model.

The result that $du = 0$ critically depends on the uniformity of the change. If $g^*(z)$ is increasing over $[m, u]$, then $du > 0$, because the southern terms of trade improves. On the other hand, if $g^*(z)$ is decreasing in z, then $du < 0$, because the southern terms of trade deteriorates. In other words, South benefits when the change in North amplifies the existing patterns of comparative advantage, and loses otherwise. It is, however, wrong to interpret this result in terms of "export-biased" or "import-biased" growth, a common distinction in trade theory, because what matters here is the bias in northern productivity growth within the goods, $[m, u]$, all of which are exported to South.

Southern productivity improvement: ($g(z) > 0$ *and* $g^*(z) = 0$)

Let us now consider the effect of a productivity improvement in South. In Figure 2, this shifts the vertical line to the right and the downward-sloping curve upward, and hence m increases unambiguously. That is, with an improved technology in South, some industries migrate from North to South. The effect on w, u and u^* are, on the other hand, ambiguous. To see what is involved, note that, from (19)-(21),

$$a(m)dm = \int_0^m g(s)a(s)ds > 0 \tag{23}$$

$$dw = -\xi(m)dm + wg(m) \tag{24}$$

$$a^*(u)du = (1-n)dw + w\int_0^m g(s)a(s)ds \tag{25}$$

Eq. (22) becomes, by using (23), (24) and (10),

(26) $$a^*(u^*)du^* = \frac{n\xi(m)}{a(m)}\int_0^m g(s)a(s)ds + w\int_0^m \{g(s) - g(m)\}a(s)ds$$

If the change is uniform across the export sectors, $g(z) = g$ for all $z \in [0,m]$, the above expressions are simplified to

$$a(m)dm = ng > 0,$$

$$\frac{dw}{w} = -\frac{\xi(m)}{w}dm + g = \left[1 - \frac{n\xi(m)}{wa(m)}\right]g < g$$

$$a^*(u)du = (1-n)dw + nwg = \left[w - \frac{n(1-n)\xi(m)}{a(m)}\right]g$$

$$a^*(u^*)du^* = \frac{n^2\xi(m)}{a(m)}g > 0$$

With a uniform technological improvement in South, the terms of trade move in favor of North (since $dw/w < g$), and the cheaper southern goods allow the households in North to expand their consumption. The patterns of product cycles, the birth of new industries in North, $du^* > 0$, and the migration of some industries from North to South, $dm > 0$, thus emerge. Such product cycles do not appear in FHS. In Flam and Helpman, new goods are not introduced in North. In Stokey, the products at the upper end are dropped, as the northern households switch to cheaper southern products.

Even when the change is uniform across sectors, the effects on w and u are ambiguous. This is because the model imposes no restriction on $\xi(m)$, the slope of the downward sloping curve in Figure 2, and hence dw/w could take a value anywhere between $-\infty$ to g. If $\xi(m) > a(m)w/n = a^*(m)/n$, the South's factor terms of trade deteriorates. If $\xi(m) > a^*(m)/n(1-n)$, the deterioration is so large that the southern welfare declines and they are forced to cut back their consumption at the higher end, $du < 0$, generating the situation of immiserizing growth (see Bhagwati 1958).

Note the asymmetric effects of productivity improvements in North and in South. North cannot lose from its own productivity improvement, while South may lose from its own productivity improvement. Immiserizing growth is a possibility for South, because it specializes in goods, whose demand does not go up in response to a rising income. South's productivity improvement, without generating an increase in demand for their goods, reduces demand for the southern labor. In order to keep its workers fully employed, South must move into industries, in which it has less comparative advantage, which could lead to a deterioration of the factor terms of trade. When South experiences an immiserizing growth, North captures more than 100% of the world's productivity gain. This cannot happen in FHS, where goods are gross substitutes and the lower prices, due to productivity growth, lead to a demand increase. In their models, the southern household income and welfare rise after a uniform productivity growth in South.

The result that North benefits from the southern productivity growth depends on the uniformity of the change. As seen in eq. (26), if $g(z)$ is sufficiently small over $[0,m]$, relative to $g(m)$, North could lose, $du^* < 0$, and South captures more than 100% of all the world's productivity gain. In this case, North loses its industries at both ends of its spectrum. Such a situation may arise, when the southern productivity growth is due to the technology transfer from North, because South has more to learn from North in sectors where North has greater absolute advantage, i.e., a higher z. North could be worse off from such a technology transfer, because there is less room left for taking advantage of the differences, if South "narrows the technology gap" and becomes similar to North. (In the extreme case, if South succeeded in catching up North completely and its technology became identical, the Northern welfare would go down to the autarky level.) This point is general, and holds true in any Ricardian model, including DFS and FHS. It should be pointed out, however, that what matters here is the bias across sectors within [0,m], all of which are exported to North. Hence, it is wrong to interpret in terms of "export-biased" or "import-biased".

Global productivity improvement: ($g(z) = g^*(z) > 0$)

Finally, let us consider the global change, in which both North and South experience the same rate of productivity improvements in each sector, but the impact is not necessarily uniform across sectors. This can be analyzed by shifting the vertical line to the right, with the downward-sloping curve unperturbed in Figure 2. Hence, $dw < 0$, and $dm > 0$. From (19)-(22),

$$a(m)dm = \int_0^m g(s)a(s)ds > 0$$

$$dw = -\xi(m)dm < 0$$

$$a^*(u)du = \left[w - (1-n)\frac{\xi(m)}{a(m)}\right]\int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds$$

$$a^*(u^*)du^* = \left[w + n\frac{\xi(m)}{a(m)}\right]\int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds > 0.$$

The effect on u is ambiguous, because $\xi(m)$ and hence the effect on w can be arbitrarily large. On the other hand, $du^* > 0$, unambiguously. In spite that productivity improvement takes place worldwide, the asymmetry of demand response lead to a terms of trade movement against South, and the patterns of product cycles, where some industries move from North to South and new industries are born in North, emerge (dm, $du^* > 0$).

3C. Transfer Payments

Suppose that the transfer payments are made from North to South, financed by lump-sum taxes in North, and distributed by lump-sum transfers in South. It has no effect on m and w. The new equilibrium would involve a trade deficit for South, equal to the transfer, and $du > 0$ and $du^* < 0$. The transfer has no effect on the prices because all the households, both in North and in South, spend their last income on northern goods. This would be different, if there are poor households who cannot consume northern made goods, as will be seen in the following sections.

4. A Digression: a Plantation Economy

While keeping the homogeneity of populations, let us ask what happens if (A3) does not hold. As long as $w < 1$ holds in equilibrium, all the results in the previous section carry over. Even if $w > 1$ so that South is richer than North, all the comparative statics results on m, w, u, and u^*, carry over, as long as $w < 1/n$, so that the northern household is rich enough to consume some of the goods that they produce. In this case, $m < u^* < u$. The only modification is that, because $u > u^*$, it is the southern household that consume the highest-indexed good. This means that the patterns of product cycles emerge when an exogenous change leads to dm, $du > 0$, not to dm, $du^* > 0$.

The properties of equilibrium differ significantly if $w > 1/n$ in equilibrium, i.e., when the downward sloping curve intersects the upward-sloping part of the balanced trade condition. In this case, (10) and (13) need to be replaced by $\int_0^m a(s)ds = 1 - (1-n)/(wn)$ and $w\int_0^{u^*} a(s)ds = 1$, respectively, and the volume of trade per household is equal to $2(1-n)$, if measured in Foreign labor. In this case, $u^* < m < u$. That is, not only the foreign household is poorer, but it is so poor that their household income would be exhausted before consuming all the Home goods, and they cannot afford to consume any of the goods that they are specialized in producing. All the high-indexed goods made in the poor Foreign are exported to the rich, Home country. Such a situation may arise in some plantation economies. The rest of this section briefly discusses the comparative statics results in such a plantation economy, to the extent that they differ from the case under (A3). (Throughout this discussion, Home is no longer identified as South, nor is Foreign as North.)

One case that has a different effect is a Foreign productivity improvement ($g(z)= 0$, $g^*(z) > 0$). As before, the factor terms of trade move against Home and in favor of Foreign ($dw < 0$). Foreign gains unambiguously ($du^* > 0$). Unlike before, a rise in Foreign income, which allow the foreign household to consume more Home goods, leads to an increase in demand for Home labor. To accommodate such an increase in demand, Home loses some industries, which migrate to Foreign ($dm < 0$). If the productivity gains are uniform, the change in the factor terms of trade is less than proportional than the productivity gains, which leads to an improvement in Home's terms of trade, measured in goods, hence $du > 0$.

The transfer payment also has different effects, because the Home household spends an additional income on Foreign goods, while the Foreign household spends an additional income on Home goods. The transfer from Home to Foreign, financed and distributed by a lump-sum manner, increases (reduces) the demand for Home (Foreign) labor, which lead to $dw > 0$ and $dm < 0$. The terms of trade moves against Foreign (the recipient). This negative secondary effect of transfer is not large enough to generate a "transfer paradox." Formally, suppose that the Home government taxes a faction, T, of its resident's income and transfers the revenue to the Foreign government, which divides the transfer equally. Then, the home household income is $w(1-T)$, while the foreign household income becomes $1 + wNT/N^*$. Suppose that T is small enough that $u^* < m < u$ holds in the presence with the transfer. Then, the equilibrium condition for Home labor becomes $wN = NE(m) + N^*(1 + wNT/N^*)$, or $\int_0^m a(s)ds = 1-T-(1-n)/(wn)$. Hence, the transfer leads to a shifts the upward-sloping curve up, and as a result, $dm < 0$

and $dw > 0$. The Home and Foreign budget constraints become $w\int_0^m a(s)ds + \int_m^u a^*(s)ds = w(1-T)$, and $w\int_0^{u^*} a(s)ds = 1 + wnT/(1-n)$. Hence, the effect of an increase in T, evaluated at $T = 0$, is

$$\left[\frac{a(m)}{\xi(m)} + \frac{(1-n)}{nw^2}\right]dw = dT > 0$$

$$-na^*(u)du = (1-n)a(u^*)du^* = wndT - (1-n)\frac{dw}{w} = wn\frac{a(m)}{\xi(m)}dw > 0.$$

Hence, the donor (Home) loses and the recipient (Foreign) gains from the transfer, in spite that it causes an adverse terms of trade change. The above expression also shows that the terms of trade effect can be powerful enough to eliminate the primary effect of the transfer, i.e., as $\xi \to \infty$, $du \to 0$ and $du^* \to 0$.

5. North-South Trade: The Case of Heterogeneous Populations

Let us now go back to North-South trade, by reintroducing (A3), and examine the case, where there are nondegenerate distributions, $F(h)$ and $F^*(h^*)$, of incomes within each economy. Thus, w and m are determined jointly by (2) and (8), and the consumption set and the utility level of each household is determined by (4) and (5). The case of heterogeneous populations is interesting, when some households are so poor that that, in equilibrium, they cannot afford to consume goods produced in North, that is, $wh < E(m)$ or $h^* < E(m)$, or equivalently $u(h) < m$ or $u^*(h^*) < m$. The existence of the poor households in North, those with $h^* < E(m)$, implies that in Figure 1, the downward-sloping curve (2) intersects at the upward-sloping part of the balanced trade curve (8). Let us also assume that the richest household in the world, whose consumption set determines the upward end of the goods produced in North, resides in North.

Population Size: An increase in m, or a faster population growth in South, shifts the balanced trade curve to the right in Figure 1, which leads to $dm > 0$ and $dw < 0$. The North's terms-of-trade improves and all the northern households, are better off, and new industries are born in North. Product cycle appears. The rich southern households, who consume foreign imports, are worse off because of the terms of trade deterioration. On the other hand, the poor southern households, who cannot afford to buy foreign imports, are unaffected, because they essentially live in autarky, and their welfare is insulated from the terms of trade change.

Northern productivity improvement ($g(z) = 0$, $g^*(z) > 0$): This shifts the downward-sloping curve down in Figure 1, which leads to $dw < 0$. Unlike in the case of homogenous populations, this also leads to $dm < 0$, and the rate of decline in w is less than $g^*(m)$. This is because the poor northern households, whose marginal consumption good is a southern good, consume more southern goods, when their income goes up. This increases demand for labor in South. To keep South's labor market in balance, South specializes in a narrower spectrum of goods, by abandoning the upper end of industries, which move from South to North. All the northern households are better off. Unlike the case of homogenous populations, the rich southern households gain if North's productivity growth is uniform across the sectors, which produce goods they import. This is because the decline in w is proportionately less than the productivity growth. If the productivity is biased, they can be worse off. The poor southern households are unaffected.

Southern productivity improvement ($g(z) > 0$, $g^*(z) = 0$): This shifts the downward-sloping curve up and the upward-sloping curve to the right, which leads to $dm > 0$, while dw/w can be anywhere between $-\infty$ to $g(m)$. The poor southern households, insulated from the terms of trade change, are better off unambiguously. If productivity growth is uniform over $[0,m]$, then all the northern households are better off. In this case, new industries are born in North, and the patterns of product cycle emerge. If productivity growth is faster at m than $[0,m)$, then North can be worse off. The effect on the rich southern household is ambiguous even if the change is uniform. If the terms of trade deterioration is large, they can be worse off.

Global productivity improvement ($g(z) = g^*(z) > 0$): This shifts the balanced trade curve to the right, while the downward-sloping curve is unaffected. Therefore, $dm > 0$ and $dw < 0$. All the northern households are better off, and new industries are born. Again, product cycle appears. The poor southern households are better off. The effect on the rich southern households is, however, ambiguous.

Income Transfers: Because of the coexistence of poor and rich households in each country,--the poor who spends their additional income on southern goods and the rich on northern goods--, aggregate variables, m and w, are generally affected, when the transfer is made, whether it is across countries or within countries. Furthermore, such a transfer can have perverse welfare effects, in which donors may gain and recipients may lose. (For the literature of transfers and welfare, see Bhagwati, Brecher and Hatta (1983) and the work cited therein.)

Let us consider South's domestic transfer policy, which redistributes income from the rich, whose marginal consumption good is an import from North, to the poor, whose marginal consumption good is a domestic good. This policy shifts the upward-sloping curve up, which leads to $dm < 0$ and $dw > 0$. The southern poor households are better off unambiguously, because they are insulated from the terms of trade change. All the northern households are worse off, because the terms of trade move against North. How about the rich household in South, whose income are taken away. Perhaps paradoxically, they may end up better off due to the improved terms of trade. To see this formally, suppose that northern households are homogenous, with $h^* = 1$. The southern households consists of two types; there are $N/2$ households with h_L and $N/2$ households with h_H, where $h_L < h_H$. Then, if a transfer per household, equal to T measured in Home labor, is made from the rich to the poor in South, the South's labor market equilibrium is, $w(h_L+h_H)N/2 = \{w(h_L+T)+E(m)\}N/2 + N^*E(m)$, or $\int_0^m a(s)ds = (h_H - T)(2-n)/n$. Since the rich southern household's budget constraint is $w\int_0^m a(s)ds + \int_m^{u_H} a^*(s)ds = w(h_H - T)$, the effect of an increase in T, evaluated at $T = 0$, is

$$dw = -\xi(m)dm = \frac{\xi(m)}{a(m)}\left(\frac{2-n}{n}\right)dT > 0$$

$$a^*(u_H)du_H = -wdT + \left[\int_m^{u_H} a^*(s)ds\right]dw = \left\{\frac{\xi(m)}{a(m)}\left(\frac{2-n}{n}\right)\left[\int_m^{u_H} a^*(s)ds\right] - w\right\}dT$$

Therefore, with a sufficiently large $\xi(m)$, the positive terms of trade effect offsets more than the primary effect of transfer. South's government can thus improve the welfare of all the households in South, by adopting a "domestic" redistribution policy, which transfer income from the rich, who consume imports on the margin, to the poor who consume domestic goods on the margin. Just by reversing, the argument above also shows that a redistribution from the poor to the rich in South can make all the southern households, including the rich, worse off.

Other types of transfer policies can be analyzed in a similar manner. For example, if North's government adopts a domestic policy of redistributing income from the rich to the poor, the resulting terms of trade deterioration can make all the households in North worse off, including the poor, who receives the transfer. South benefits from the terms of trade change. The effect of an international aid, made from North to South, also depends critically on how the transfer is distributed within South. If it is distributed only to the rich households in South, an adverse terms-of-trade effect can eliminate much of the transfer's benefit to South.

6. A Multicountry World

One advantage of the present model over DFS is that it is relatively straightforward to extend the model to incorporate more than two countries. This section offers a sketch of how the analysis can proceed when there are many countries and points out some new issues that arise in a multilateral world.

Let J be the number of countries, with j being an index of country, $j = 1, 2, \ldots, J$, and $a_j(z)$ be country j's unit labor requirement in sector z, with the following property.

(A4). For all $j = 1, 2, \ldots, J-1$, $a_{j+1}(z)/a_j(z)$ is continuous, strictly decreasing in $z \in [0,\infty)$.

By denoting the wage rate in country j by w_j. Then, country j produces only goods in $[m_{j-1}, m_j)$, where m_j is an increasing sequence, satisfying

$m_0 = 0$,

(27) $\quad w_j/w_{j+1} = a_{j+1}(m_j)/a_j(m_j) \qquad (j = 1, 2, \ldots, J-1).$

and $m_J = \infty$. The prices are given by

$$p(z) = w_j\, a_j(z) \text{ for } z \in [m_{j-1}, m_j).$$

If there are N_j households in country j, with the distribution of skills given by $F^j(h_j)$, then the labor market equilibrium conditions in all the countries are given by

(28) $$w_j N_j \int_0^\infty h_j dF^j(h_j) = \sum_k N_k \int_0^\infty \min\{w_k h_k - E(m_{j-1}), E(m_j) - E(m_{j-1})\} dF^k(h_k),$$

$$(j = 1, 2, \ldots, J-1.)$$

where $E(z) = \int_0^z p(s)ds$. Eqs.(27) and (28) jointly determine the equilibrium values of m_j and w_j $(j=1, 2, \ldots, J-1.)$ The budget constraint of a household in country j with h_j units of labor endowment is given by $E(u_j(h_j)) = w_j h_j$, which also determines the set of goods consumed by such a household , as well as the utility level achieved.

In what follows, the analysis is focused on the three country case, $J = 3$, and each country is populated by homogenous households with $h_j = 1$ ($j = 1, 2, 3.$). Furthermore,

(A5) $a_2(z)/a_1(z)$, $a_3(z)/a_2(z) < 1$ for all $z \in [0,\infty)$.

This assumption is ensured that $w_1 < w_2 < w_3$. That is, a low-income country, 1, specializes in a lower spectrum of goods; a high-income country, 3, specializes in a higher spectrum; and a middle-income country, 2, specializes in an intermediate range. By choosing country 3's labor as a numeraire, $w_3 = 1$, the initial task is to determine the equilibrium values of $m_1 < m_2$, $w_1 < w_2 < 1$, as well as u_1, u_2, and u_3.

From the budget constraint, $E(u_j) = w_j$ and $w_1 < w_2 < 1$, $u_1 < u_2 < u_3$. Since country 3 imports from country 1, country 1 must import some goods produced in country 2, $(m_1 < u_1)$, and country 2 must import from country 3 in equilibrium, ($m_2 < u_2$). Hence, there are two possible equilibrium configurations, depending on whether $u_1 > m_2$ or $u_1 < m_2$. In the first case, i.e.,

$$m_1 < m_2 < u_1 < u_2 < u_3,$$

all the households spend their marginal income on goods produced in country 3, and there is a two-way bilateral trade flow between each pair of countries. This case is similar in many ways with the case of two countries. On the other hand, in the second case,

$$m_1 < u_1 < m_2 < u_2 < u_3,$$

country 1 is not rich enough to be able to consume goods produced in country 3. Hence, country 1 runs a bilateral trade surplus vis-à-vis country 3, which in turn runs a bilateral trade surplus vis-à-vis country 2, which in turn runs a bilateral trade surplus vis-à-vis country 1. Furthermore, households in country 1 spend their marginal income on goods produced in country 2, while households in countries 2 and 3 spend their marginal income on goods produced in country 3.

The labor market equilibrium condition in country 1 is given by

$w_1 N_1 = (N_1 + N_2 + N_3)E(m_1)$. In the second case, country 2's the labor market equilibrium is

$w_2 N_2 = N_1(w_1 - E(m_1)) + (N_2 + N_3)(E(m_2) - E(m_1))$. By using $E(m_1) = w_1 \int_0^{m_1} a_1(s)ds$ and

$E(m_2) - E(m_1) = w_2 \int_{m_1}^{m_2} a_2(s)ds$. The equilibrium is determined jointly by the following four conditions,

which can be solved recursively:

$$\int_0^{m_1} a_1(s)ds = n_1,$$

$$\int_{m_1}^{m_2} a_2(s)ds + \frac{w_1}{w_2} n_1 = \frac{n_2}{n_2 + n_3},$$

$$\frac{w_1}{w_2} = \frac{a_2(m_1)}{a_1(m_1)}.$$

and

$$w_2 = \frac{a_3(m_2)}{a_2(m_2)}.$$

where n_j is the share of country j. By using these conditions, $E(m_2) > E(u_1) = w_1$ can be shown to be equivalent to

$$\frac{n_2}{n_2 + n_3} > \frac{w_1}{w_2} = \frac{a_2(m_1)}{a_1(m_1)}$$

Since m_1 is increasing in n_1, the right-hand side is decreasing in n_1. Hence, the second case, where the low-income country cannot afford to import from the high-income country and there are bilateral trade imbalances, is likely to occur, if country 1 is larger, which lowers its income, and/or if country 2 is larger relative to country 3, which means that country 2 produces a wide range of goods in the middle spectrum.

Fixing the country sizes, making country 1's technology worse or country 2's better has the same effect, thereby making the second case more likely.

In the second case, transfers from country 2 or 3 to country 1 affects the terms of trade, because country 1 spends the marginal income to country 2's goods, and countries 2 and 3 spend the marginal income to country 3's goods. The detailed analysis is left for the exercise to the reader.

7. Suggestions for Further Extensions

There are obviously many ways in which the above analysis can be extended. This section suggests some. First, the model can easily be extended to endogenize labor supply by putting leisure in the utility function (or equivalently, one can introduce the nontradeable sector, which produces a homogenous divisible good.) Then, even in the case of homogeneous populations, the model becomes similar to the case discussed in section 5, by making the balanced trade condition upward-sloping. To the extent that southern households respond to an additional real income by increasing leisure (or by consuming more of the nontraded good), South also benefits from its productivity improvement in the traded sectors. It also makes the blanced trade/labor market equilibrium condition dependent of Northern technologies. However, as long as the traded sectors have the same structure, much of the results obtained above will carry over. In particular, any exogenous change in South that causes a terms-of-trade deterioration for South leads to product cycles, in which the range of goods produced in North move up the spectrum, and North benefits more from productivity improvement than South. Second, the model can also be extended to include monetary factors, as done by DFS themselves. This allows a temporary separation of income and spending by the households. It would be interesting to see how the asymmetry of demand response may interfere the smooth operations of the price-specie-mechanism portrayed by DFS. Third, the model can of course be used to address many standard issues in trade theory, such as the effect of tariffs. It should be pointed out that the effects of tariffs depend crucially on how the revenue is distributed. Fourth, one may incorporate more than one factor. For example, if higher-indexed goods use capital more intensively than lower-indexed goods, then North would accumulate more capital than South, and a higher level of nonlabor income in North further affects the trade patterns though income effects. Fifth, the model assumed that each sector produces a homogenous good. If each sector produces a continuum of goods, which are indexed in a manner similar to Flam and Helpman (1987), and Stokey (1991), then there will be North-South trade both across and within sectors. This seems to a most natural way of integrating the FHS models into the present framework. Finally, perhaps, the most interesting extension would be to endogenize productivity changes. Throughout this paper only the question of how technology affects trade is discussed. The exogeneity of technology makes it impossible to address the question of how trade affects technology. It is highly desirable to introduce learning-by-doing and R&D in the present model, along the lines pursued by the recent literature.[11] Such an extension is indeed essential to examine the validity of the argument made by industrial policy advocates, who believe that the income elasticity of demand is one of the key criteria for industrial targeting.

[11] See Lucas (1993) and Grossman and Helpman (1995), which discuss various alternatives of endogenizing technologies. See also Matsuyama (1999), which developed a closed economy model of sector-specific learning-by-doing, with the preference structure similar to the present model.

References:

Bhagwati, Jagdish N., "Immiserizing Growth: A Geometrical Note," *Review of Economic Studies*, 1958, 25, 201-205.

Bhagwati, Jagdish N., Richard A. Brecher, and Tatsuo Hatta, "The Generalized Theory of Transfers and Welfare: Bilateral Transfers in a Multilateral World," *American Economic Review*, 73, September 1983, 606-618.

Dornbusch, Rudiger, Stanley Fischer and Paul A. Samuelson, "Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods," *American Economic Review*, December 1977, 67, 823-839.

Flam, Harry, and Elhanan Helpman, "Vertical Product Differentiation and North-South Trade," *American Economic Review*, December 1987, 77, 810-822.

Grossman, Gene, and Elhanan Helpman, "Technology and Trade," in Gene Grossman and Kenneth Rogoff, eds., *Handbook of International Economics*, vol. III, North-Holland, 1995.

Helpman, Elhanan, and Paul R. Krugman, *Market Structure and Foreign Trade*, Cambridge, MIT Press, 1985.

Krugman, Paul R., "The Narrow Moving Band, The Dutch Disease, and the Competitive Consequences of Mrs. Thatcher: Notes on Trade in the Presence of Dynamic Scale Economies," Journal of Development Economics, 27, 1987, 41-55.

Linder, Staffan Burenstam, *An Essay on Trade and Transformation*, Uppsala: Almqvist & Wiksell, 1961.

Lucas, Robert E. Jr., "Making a Miracle," *Econometrica*, 61, March 1993, 251-272.

Matsuyama, Kiminori, "A Theory of Mass Consumption Society," Work in Progress, 1999.

Murphy, K. M., A. Shleifer, and R. Vishny, "Income Distribution, Market Size, and Industrialization," *Quarterly Journal of Economics*, August 1989, 537-564.

Stokey, Nancy L., "The Volume and Composition of Trade Between Rich and Poor Countries," *Review of Economic Studies*, January 1991, 58, 63-80.

Vernon, Raymond, "International Investment and International Trade in the Product Cycle," *Quarterly Journal of Economics*, May 1966, 80, 190-207.

Wilson, Charles A., "On the General Structure of Ricardian Models with a Continuum of Goods," *Econometrica*, 48(7), November 1980, pp. 1675-1702.