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A THEORY OF SECTORAL ADJUSTMENT

by

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Abstract

This paper constructs two sector overlapping generations economies where sectoral adjustment of labor takes place slowly in response to sectoral shocks. The dynamics of labor supplies are characterized for a variety of relative price shocks (both exogenous and endogenous) under constant-, decreasing-, and increasing-returns-to-scale technologies. Welfare implications are also discussed.

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1. Introduction

Recent experience in industrial economies suggests that the problems of adjusting to changing conditions of the world economy are made difficult by the limited labor mobility across sectors. Although the significance of sector specificity of labor has often been mentioned, there seems to be few attempts of producing tractable models of sectoral adjustment with perfect foresight. Most previous studies that looked at the effects of sectoral shocks either assume that labor is perfectly mobile or implicitly focus on the long-run impacts of disturbances.

This paper constructs two-sector economies where sectoral adjustment of labor takes place slowly in response to relative price shocks. Each sector produces a good by using labor service specific to the sector. The economy is populated by a continuum of overlapping agents. Each agent at the beginning of her life decides in which sector to work, and once the career decision is made, she will be stuck to the sector of her choice for the rest of her life. Sufficient heterogeneity of individuals (i.e., skill distribution) ensures that aggregate labor supply responds smoothly to price shocks, in spite of the binary nature of individual decision. It is also assumed that every agent faces a constant instantaneous probability of death, p, throughout her lifetime. Sectoral adjustment occurs in this economy due to the demographic change. And the speed of adjustment is endogenously determined by and varies with the fraction of the new cohort attracted to each sector.

The irreversibility of career decision plays a crucial role in this paper. Although the putty-clay assumption on labor is uncommon, it seems no less realistic than the standard assumption of mobile labor. Since so much work has been done under perfect mobility assumption, it would be worthy of going to the other extreme. Furthermore, this assumption helps to focus on the often neglected aspect of sectoral adjustment. The levels of production and employment in depressed industries are determined not only by the rate of exit from, but also by the rate of entry to, the industries.[1] Sectoral adjustment can be therefore achieved by redirecting the flow of new entrants, leaving old workers in place. Most policy debates on sectoral problems are concerned with the exit side only. But the entry side seems to be at least equally important in practice, since the entry decision would be more elastic than the exit decision, and therefore, more responsive to shocks and policy measures.[2] The putty-clay assumption has an advantage of making this point crystallized. Its main drawback is that it makes the models poorly adopted to the issues of adjustment assistance.

Another important assumption is a constant instantaneous probability of death p. This allows one to solve aggregation problems, thereby greatly enhancing the tractability of the models. This assumption makes differential equations of labor supplies so simple that one can easily determine the adjustment path of the economy in response to an intertemporally complex relative price change, either exogenous or endogenous. One can also analyze the case of nonconstant-returns-to-scale technologies. Furthermore, these exercises of comparative dynamics can be performed graphically.

The importance of graphic analysis deserves emphasis here. The standard practice of solving perfect foresight models is to linearize the system around the steady state; see Judd (1985b), for example. While this approach is useful in describing the dynamic paths of outputs and factor movements, it is incapable of capturing the output loss associated with the adjustment process.

For the cost of adjustment has a magnitude of the second order in any model of sectoral adjustment that generates differentiable paths.[3] The graphic analysis, on the other hand, is powerful in that it is capable of dealing with a large change, and therefore, capturing the movement of the economy wandering around inside the long-run production possibility frontier.

Another advantage of the approach adopted in this paper is its flexibility. If one thinks of (1/p), the life expectancy of agents, as the index of the length of the period for which each agent has to make a commitment to her career, then one can choose it anywhere between zero and infinity and examine the effects of specificity on the behavior of the economy. In particular, the models presented below have the pure exchange model (when p goes to zero) and the standard two-sector model (when p goes to infinity) as the two limiting cases.[4]

In the literature of international trade, with its traditional emphasis on sectoral allocations, there have been some attempts of modelling sluggish intersectoral factor movement, such as Lapan (1976), Neary (1978a, 1978b), and Mussa (1978, 1982). Lapan studies the effects of subsidies on labor movement in an economy with relative wage rigidities. He postulates that adjustment of labor depends on the current unemployment in the depressed sector.[5] Neary and Mussa are concerned with short-run capital specificity in a model with two factors of production: capital and labor. Labor is assumed to be perfectly mobile. Neary postulates the adjustment process in which the reallocation of capital stock takes place at a rate determined by the difference between the current rentals on capital in the two sectors. Although both Lapan's and Neary's models have an advantage of being tractable, their adjustment processes implicitly assume that workers and capital owners have static expectations. Mussa develops a perfect foresight model of sectoral adjustment. He introduces the third sector that moves capital from one industry to another by using labor, and then looks at the capital stock adjustment when there is a permanent shock in the relative price. The dynamics in his model are so complicated that it appears quite difficult examining the effect of, say, large temporary relative price changes, let alone endogenizing them.[6] More significantly, however, these studies consider sectoral adjustment through reallocation of existing factors, while this paper considers adjustment through redirecting of new supply of factors. Sectoral adjustment in the real world is, of course, the combination of these two mechanisms.

Before proceeding it should be noted that, in the models presented below, agents are assumed to have access to perfect annuity markets, where they can insure against their uncertain lifetimes, as in Blanchard (1985) and Matsuyama (1987). This assumption is motivated for two reasons. First, the risk of death is purely individualistic and there is no aggregate uncertainty. With a continuum of agents, annuity contracts contingent on death can be offered risklessly.[7] And there is no moral hazard or adverse selection problem to justify the breakdown of annuity markets. Second, the alternative assumption of no annuity markets will not affect the positive results qualitatively, except for the case when increasing-returns-to-scale creates multiple steady states.[8] In this case, the model could generate a variety of interesting dynamics, such as closed orbits, homoclinic orbits (i.e., saddle loops), bifurcations, and a continuum of perfect foresight paths, etc. Although these phenomena would, in themselves, be of some theoretical interest, they are not the main concern of this paper. They will be addressed in a companion paper, Matsuyama (in process). Assuming perfect annuity markets ensures that perfect foresight paths are relatively "well-behaved" even in the presence of increasing returns.

The paper is organized as follows. Sector 2 expounds the basic model, in which technologies are constant returns to scale and a sequence of the relative price and a (constant) interest rate is exogenously given. The model can be thought of either as looking at an open economy or as giving partial equilibrium dynamics of labor adjustment, given the relative price and the interest rate in a closed economy. Section 3 discusses the cases of decreasing and increasing returns to scale. It will be shown that the adjustment process decelerates over time under decreasing returns, while it accelerates under increasing returns. Section 4 constructs a dependent economy model to show how the relative price can be endogenized, and then looks at the effect of sectoral demand shocks. It also shows how the model can be closed. Section 5 discusses the welfare implications. In particular, it is argued that the equilibrium adjustment path is efficient, provided that the technology does not involve externalities, which strongly questions the desirability of government intervention on efficiency grounds. Section 6 suggests desirable directions for future research.

2. The Basic Model: Constant-Returns-to-Scale Technology and Exogenous Relative Prices

Consider a two-sector small open economy. Each sector produces a tradeable good by using the labor service specific to the sector. Let L_t^i

(i=1,2) denote the total supply of labor specific to the i-th sector, measured in efficiency units, at time t. (Time is continuous.) The production technology exhibits constant returns to scale: $X^i = a^i L^i$, where X^i is the output and a^i the average (and marginal) labor productivity in the i-th sector. Throughout the paper it is assumed that there is no technical change. Then, one can set $a^1 = a^2 = 1$ without loss of generality. Take good 1 as a numeraire and let q_t denote the relative price of good 2, which is time-dependent and exogenous. Then, perfect competition ensures that

$$w_t^1 = 1 \quad , \qquad w_t^2 = q_t \quad , \tag{1}$$

where w_t^i is the wage rate in the i-th sector.

The economy is populated by a continuum of overlapping agents. The size of the population is constant over time and normalized to be unity. Every agent throughout her lifetime faces a constant instantaneous probability of death p. The risk of death is individualistic and there exists no aggregate uncertainty. The constant population implies that a new cohort whose size is equal to p is born at each moment of time.[9] Agents within a cohort are not identical, however, differing in their attributes. More specifically, an agent of type α can provide $g^i(\alpha)$ efficiency units of labor service inelastically if she chooses to become a worker specific to the i-th sector. The index of type α is numbered so that $g^1(\alpha)/g^2(\alpha)$ is a strictly increasing function of α. This is to say that an agent with high α has comparative advantage in the first sector relative to an agent with low α. It is convenient to assume that $g^1(\alpha)/g^2(\alpha)$ is differentiable and, therefore, so is its invertible function.[10] Let $\Phi(\alpha)$ be the distribution function of α. It is assumed that $\Phi'(\alpha) > 0$ on the support of Φ, $[\alpha^-,\alpha^+]$. For the reason that will be clear later, it is also assumed that $\{q_t\}$ is bounded and the support of Φ is large enough to satisfy,

$$\frac{g^1(\alpha^+)}{g^2(\alpha^+)} > q_t > \frac{g^1(\alpha^-)}{g^2(\alpha^-)} \quad , \tag{2}$$

for all t.

The crucial assumption in this model is that, at the beginning of her life, every agent needs to decide in which sector to work and, once the career decision is made, she will be stuck to the sector for the rest of her life. Endowed with perfect foresight, an agent chooses her career in order to maximize her human wealth.[11] That is, an agent born at time t decides to go to the first sector if and only if,

$$\int_t^{+\infty} g^1(\alpha) w_s^1 e^{-r(s-t)} ds \geq \int_t^{+\infty} g^2(\alpha) w_s^2 e^{-r(s-t)} ds \quad ,$$

where r is a (constant) exogenous discount rate, which is equal to r^*+p, the sum of the world interest rate r^* (>0) and the annuity premium p. This means that she may invest in foreign assets, which are denominated in good 1 and whose constant interest rate, r^*, is exogenously given in the world capital market. There also exists perfect annuity markets. With a positive asset position, ω, each agent may contract to receive $p\omega$, and to pay ω contingent on her death. With a negative asset position, she is required to pay an insurance premium, $-p\omega$ because she has a default risk with probability p. Thus, the effective rate of return is equal to $r = r^* + p$.

From equation (1), the above condition can be rewritten to:

$$(3) \qquad \frac{g^1(\alpha)}{g^2(\alpha)} \geq Q_t \ ,$$

where

$$(4) \qquad Q_t \equiv r\int_t^{+\infty} q_s e^{-r(s-t)} ds \quad .$$

That is , Q_t is the annuity value of $\{q_s\}_{s=t}^{+\infty}$, a sequence of the relative wage rate in the second sector. Let A denote the inverse function of $g^1(\alpha)/g^2(\alpha)$. Clearly $A' > 0$. Then equation (3) shows that any agent whose type is greater (smaller) than $A(Q_t)$ goes to the first (second) sector. And equation (2) guarantees that $\alpha^- < A(Q_t) < \alpha^+$ so that both sectors always attract some agents. That $A' > 0$ means that, when Q_t is high, the second sector attracts a large fraction of the new cohort.

The total labor supply (and the output) in the first sector thus changes as:

$$\dot{L}_t^1 = p \int_{A(Q_t)}^{\alpha^+} g^1(\alpha) d\Phi(\alpha) - pL_t^1 \quad .$$

or,

$$(5a) \qquad \dot{L}_t^1 = p[\bar{L}^1(Q_t) - L_t^1] \quad ,$$

where $\bar{L}^1(Q) \equiv \int_{A(Q)}^{\alpha^+} g^1(\alpha)d\Phi(\alpha)$ with $\bar{L}^{1\prime}(Q) = -g^1(A(Q))\Phi'(A(Q))A'(Q) < 0$. Similarly, one can show that:

(5b) $$\dot{L}_t^2 = p[\bar{L}^2(Q_t) - L_t^2] \quad ,$$

where $\bar{L}^2(Q) \equiv \int_{\alpha^-}^{A(Q)} g^2(\alpha)d\Phi(\alpha)$ with $\bar{L}^{2\prime}(Q) = g^2(A(Q))\Phi'(A(Q))A'(Q) > 0$. From the definitions, it should be clear that $\bar{L}^i(Q)$ can be interpreted as the labor supply in the i-th sector (and the output of good i) in the steady state associated with $q_t = Q$ for all t. It can be also interpreted as the labor supply in the i-th sector (and the output of good i) at $q_t = Q$ that would result if all workers were mobile across the sectors. The dynamics of sectoral allocations in this economy are fully described by (4), (5a), and (5b), given $\{q_t\}_{t=0}^{+\infty}$ and the initial conditions L_0^1 and L_0^2.

Let us call the locus given by $(\bar{L}^1(Q),\bar{L}^2(Q))$ with $\alpha^- < A(Q) < \alpha^+$, the long-run production possibility frontier. As seen in Figure 1, this locus is strictly concave, downward-sloping and its tangent line at $(\bar{L}^1(Q),\bar{L}^2(Q))$ is orthogonal to (1,Q), as verified as follows.

(6a) $$\frac{d\bar{L}^2}{d\bar{L}^1} = \frac{\bar{L}^{2\prime}(Q)}{\bar{L}^{1\prime}(Q)} = -\frac{g^2(A(Q))}{g^1(A(Q))} = -\frac{1}{Q} < 0 \quad ,$$

and

(6b) $$\frac{d^2(\bar{L}^2)}{(d\bar{L}^1)^2} = \{Q^2\bar{L}^{1\prime}(Q)\}^{-1} < 0 \quad .$$

The last equality in equation (6a) comes from the fact that $\alpha = A(Q)$ satisfies equation (3) with equality. The frontier is concave, since, as one moves along the frontier to increase L^1, one needs to allocate agents with less and less comparative advantage to the first sector, thereby reducing L^2 progressively.[12]

The point representing the actual outputs and labor supplies of this economy $(X_t^1, X_t^2) = (L_t^1, L_t^2)$, which is given at each moment of time, is always located inside or on the production frontier. From equations (5a) and (5b),

$$\frac{dL^2}{dL^1} = \frac{dL^2/dt}{dL^1/dt} = \frac{\bar{L}^2(Q_t) - L_t^2}{\bar{L}^1(Q_t) - L_t^1} \quad .$$

This implies that $(X_t^1, X_t^2) = (L_t^1, L_t^2)$ moves toward $(\bar{L}^1(Q), \bar{L}^2(Q))$ as indicated by the short arrow in Figure 1. But, Q_t generally changes with t. Therefore, the dynamic behavior of $(X_t^1, X_t^2) = (L_t^1, L_t^2)$ can be visualized as a point inside the frontier chasing a moving target on the frontier. In fact, there exists a simple relation between a change in Q_t and a change in the direction of movement of $(X_t^1, X_t^2) = (L_t^1, L_t^2)$. More formally,

<u>Proposition</u>: Let (ν_t, θ_t) be polar coordinates of $(\dot{L}_t^1, \dot{L}_t^2)$: $\dot{L}_t^1 = \nu_t \cos\theta_t$, $\dot{L}_t^2 = \nu_t \sin\theta_t$. Suppose that $(L_t^1, L_t^2) \neq (\bar{L}^1(Q_t), \bar{L}^2(Q_t))$ so that $\nu_t \neq 0$. Then, $\operatorname{sgn}\dot{\theta}_t = \operatorname{sgn}\dot{Q}_t$.

<u>Proof</u>:

From the definition of ν_t and θ_t,

$$(*) \qquad \dot{L}^1\ddot{L}^2 - \dot{L}^2\ddot{L}^1 = \nu\cos\theta(\dot{\nu}\sin\theta + \nu\cos\theta\dot{\theta}) - \nu\sin\theta(\dot{\nu}\cos\theta - \nu\sin\theta\dot{\theta})$$

$$= \nu^2 \ (\cos^2\theta + \sin^2\theta)\dot{\theta} = \nu^2\dot{\theta} \quad .$$

On the other hand, from equations (5a) and (5b),

$$(**) \qquad \dot{L}^1\ddot{L}^2 - \dot{L}^2\ddot{L}^1 = p[\bar{L}^{2\prime}(Q)\dot{Q} - \dot{L}^2]\dot{L}^1 - p[\bar{L}^{1\prime}(Q)\dot{Q} - \dot{L}^1]\dot{L}^2$$

$$= p[\bar{L}^{2\prime}(Q)\dot{L}^1 - \bar{L}^{1\prime}(Q)\dot{L}^2]\dot{Q}$$

$$= p[\dot{L}^1 + Q\dot{L}^2]\bar{L}^{2\prime}(Q)\dot{Q} \quad . \qquad \text{from (6a)}$$

When $(L^1, L^2) \neq (\bar{L}^1(Q), \bar{L}^2(Q))$, $\nu > 0$ and $\dot{L}^1 + Q\dot{L}^2 = p[\bar{L}^1(Q) + Q\bar{L}^2(Q) - (L^1 + QL^2)] > 0$, so that

$$\text{sgn } \dot{\theta} = \text{sgn } \dot{Q} \quad . \qquad \text{from (*) and (**)}$$

Q.E.D.

The proposition states that, when Q_t declines (rises) over time, $(X_t^1, X_t^2) = (L_t^1, L_t^2)$ gradually turns its direction to the right (left). This result will be used repeatedly below.

Comparative Dynamics

Throughout this section it is assumed that the economy is in the steady state at time zero, when the shock unexpectedly occurs. First, consider a permanent increase in q : $q_t = q_o$ for all $t \leq 0$ and $q_t = q^* > q_o$ for all $t \geq 0$. (When good 2 is an export good, an increase in q can be considered either as a terms-of-trade improvement or as an increase in export subsidies, or as a reduction of import tariffs.) Then, $Q_t = q_o$ for all $t \leq 0$ and $Q_t = q^* > q_o$ for all $t \geq 0$. In Figure 2A, the target $(\bar{L}^1(Q), \bar{L}^2(Q))$ jumps from S to S^* and stays there forever. The output vector $(X_t^1, X_t^2) = (L_t^1, L_t^2)$ follows the straight line connecting S and S^*. Note that point S also represents the labor service supplied by the cohorts born before the shock, while S^* represents that by the cohorts born after the shock. The total labor service available in the economy is a weighted average of these two points. As the new generations replace the old, it gradually moves toward S^*. Since the production frontier is strictly concave, the actual production will remain strictly inside the production possibility set throughout the transition period. Algebraically, it can be shown from equations (5a) and (5b) that:

$$L_t^i = \bar{L}^i(q^*) + \{\bar{L}^i(q_o) - \bar{L}^i(q^*)\}e^{-pt} \qquad (i=1,2) \quad .$$

The adjustment cost associated with the change is calculated and shown to decline at the rate equal to p, as follows.

$$\bar{L}^1(q^*) + q^*\bar{L}^2(q^*) - (L_t^1 + q^*L_t^2)$$
$$= \{\bar{L}^1(q^*) + q^*\bar{L}^2(q^*) - \bar{L}^1(q_o) - q^*\bar{L}^2(q_o)\}e^{-pt} > 0 \quad .$$

Note that this has a magnitude of the second order of the price change $q^* - q_o$.

Next, consider a temporary change: $q_t = q^*$ for $0 \le t < T$ and $q_t = q_o$ for all $t \ge T$. Then,

$$Q_t = \begin{cases} q^* + (q_o - q^*)e^{r(t-T)} & 0 \le t < T \\ q_o & t \ge T \end{cases} \quad .$$

As an "average" of q, Q jumps up at time zero, but less than q, and then starts declining immediately and goes back to q_o at $t = T$. In Figure 2B, the target vector $(\bar{L}^1(Q),\bar{L}^2(Q))$ jumps from S to S' at time zero and then returns to S gradually. The output vector follows the adjustment path, P(t), shown in Figure 2B. At time zero, it starts moving toward S', and keeps turning its direction to the right from t = 0 to t = T, while Q declines. After t = T, it follows the straight line (connecting P(T) and S) back to the steady state S.

In order to solve for the path analytically, consider a first-order approximation by assuming that $q^* - q_o$ is sufficiently small. (This is not without a loss, since one cannot calculate the adjustment cost, which has the magnitude of the second order.) Then, some algebra yields,

$$L_t^i - \bar{L}^i(q_o) = \begin{cases} b^i \left\{1-e^{-pt} + \dfrac{p}{r+p}\, e^{-rT}(e^{-pt}-e^{rt})\right\}(q^*-q_o) & (0 \le t \le T) \\[2ex] b^i \left\{\dfrac{re^{pT} + pe^{-rT}}{r+p} - 1\right\} e^{-pt}(q^*-q_o) & (t \ge T) \end{cases}$$

where $b^1 = \bar{L}^{1\prime}(q_o) < 0$ and $b^2 = \bar{L}^{2\prime}(q_o) > 0$. Figure 2C shows the dynamics of L_t^2 along with those of q_t and Q_t. (Under the first order approximation, L_t^1 is simply a mirror image of L_t^2.) Facing a period of high q ahead, a larger fraction of the generation born at time zero goes to the second sector, compared to the older generations. As the young cohort replaces the old, the output of good 2 rises and that of good 1 declines. As time passes, a new generation faces a shorter period of high q and the fraction of a new cohort that goes to the second sector declines. This slows down the sectoral movement and, before t=T, the process will be reversed. After t=T, the economy will converge to the original steady state at the rate equal to p.

The result of the preceding analysis does not depend on finite lifetimes of agents. What does count is that a new cohort enters one after another. To see this, suppose that the size of a new cohort grows at a constant rate, n. Then, the population grows at n and the <u>per capita</u> labor supply in the i-th sector can be shown to satisfy $\dot{L}^i_t = (n+p)[\bar{L}^i(Q_t) - L^i_t]$. The case of n=0 and p>0 has been discussed. It is apparent that the same analysis would go through when n>0 and p=0, in constant-returns-to-scale economies. The next section returns to the case of n=0 and p>0, to discuss nonconstant-returns-to-scale technologies.

3. <u>Non-Constant-Returns-to-Scale Technologies</u>

The assumption of constant returns to scale implies that an individual's career decision depends solely on the future paths of the terms-of-trade. When this assumption is relaxed, an individual needs to take into account the future path of the supply of specific labor, which generates a feedback effect. As shown below, the adjustment process decelerates over time under decreasing returns, while it accelerates under increasing returns.

<u>Decreasing Returns to Scale</u>

Suppose that, while the first sector operates under the constant returns to scale ($X^1 = L^1$), the second sector is subject to the decreasing returns: $X^2 = f(L^2)$ with $f' > 0$, $f'' < 0$. Then, $w^1_t = 1$ and $w^2_t = f'(L^2_t)q_t$.[13] This implies that an individual of type α, born at time t, goes to the first sector if and only if $\alpha \geq A(Q_t)$, where

(4′) $$Q_t \equiv r\int_t^{+\infty} f'(L_s^2)q_s e^{-r(s-t)}ds < +\infty \quad .$$

Therefore, the dynamics of the economy is characterized by the following system of differential equations.

(7a) $$\dot{Q}_t = r[Q_t - q_t f'(L_t^2)] \quad ,$$

(7b) $$\dot{L}_t^1 = p[\bar{L}^1(Q_t) - L_t^1] \quad ,$$

(7c) $$\dot{L}_t^2 = p[\bar{L}^2(Q_t) - L_t^2] \quad .$$

Differentiating (4′) with respect to time yields equation (7a), while equations (7b) and (7c) are the reproduction of equations (5a) and (5b). Note that the dynamics of $\{L^2, Q\}$ is independent of L^1 so that it can be solved diagrammatically. See Figure 3A.[14]

From equation (7c), the $\dot{L}^2 = 0$ locus is given by $\bar{L}^2(Q) = L^2$, which is upward-sloping, if $\alpha^- < A(Q) < \alpha^+$, and it is vertical otherwise. Above this locus, $\dot{L}^2 > 0$ and $\dot{L}^2 < 0$ below it. From equation (7a) the $\dot{Q} = 0$ locus with a constant q is given by $Q = qf'(L^2)$, which slopes downward, and $\dot{Q} > 0$ above the locus and $\dot{Q} < 0$ below it. There exists a unique steady state given by the intersection of the two loci. There exists a unique convergent path to the steady state, which is downward-sloping. Since Q is a jumping variable and L^2 is a predetermined variable, the equilibrium path is determinate for any initial value L_0^2. That is, the system is globally saddlepoint stable.[15] Let us assume that the range of $\{q_t\}$ is limited so that the locus $Q = q_t f'(L^2)$ intersects with the locus $\bar{L}^2(Q) = L^2$ at its upward-sloping part.[16]

Now, consider the effect of a permanent increase in q_t from q_o to q^*, when the economy is initially in the steady state. An increase in q shifts the $\dot{Q} = 0$ locus upward and the economy immediately jumps from A_0 to A_1, and then converges to the new steady state A_2. Notice that Q_t overshoots its new long-run level. This implies that the target vector $(\bar{L}^1(Q), \bar{L}^2(Q))$ jumps from S_o to S′ and then gradually moves back to S^*, in Figure 3B. Therefore, the economy starts moving from S_o toward S′ at time zero, but it continues to bend toward the right until it reaches the new steady state S^*. The intuition should be clear. With a permanent increase in q, a large share of new cohorts goes to the second sector and this increases the supply of labor service specific to the sector. But, this in turn reduces the relative wage rate due to the decreasing returns to scale, which slows down the process of sectoral adjustment.

Next, consider a temporary increase in q. It stays at q^* until t=T, then returns to q_o. In Figure 3A, the economy jumps from A_0 to A_3 at time zero and gradually moves to A_4. By t=T, it reaches to A_5 and then returns to the original steady state. Note that $\dot{Q}$ changes its sign during the transition period. Therefore, the adjustment path of the economy should look like one given in Figure 3C. Along the adjustment path in Figure 3C, point P_4 corresponds to A_4 in Figure 3A and P_5 to A_5. The path bends toward its right until P_5 and then toward its left until it goes back to the steady state.

One can examine the effect of an anticipated increase in q in a similar manner. For example, it can be shown that expectations of future increase in q <u>reduce</u> the relative wage in the second sector in the short run. This is because expected future increases in the relative wage induce a larger fraction of new cohorts to choose the second sector today.[17]

Increasing Returns to Scale

Suppose that, while the first sector operates under the constant returns to scale, the second sector is subject to economies of scale that are external to the firm but internal to the sector: $\chi^2 = h(L^2)\ell^2$, where χ^2 and ℓ^2 are the output and the employment of a firm and L^2 is the total employment in the second sector. The average (and marginal) productivity of labor is positively related to the size of the industry: $h'(L^2) > 0$.[18] This type of external economies of scale, which is usually attributed to Marshall (1920, Book IV, chs. X-XI), is widely used in international economics as a useful way of making increasing returns consistent with perfect competition; see Helpman and Krugman (1985, ch. 3) and Krugman (1987). Then, $w^1_t = 1$ and $w^2_t = h(L^2_t)q_t$. An agent of type α, born at time t, goes to the first sector if and only if $\alpha \geq A(Q_t)$, where

$$(4'') \qquad Q_t \equiv r\int_t^{+\infty} h(L^2_s)q_s e^{-r(s-t)}ds < +\infty \quad .$$

Therefore, the dynamics of the economy are given by:

$$(7a') \qquad \dot{Q}_t = r[Q_t - q_t h(L^2_t)] \quad ,$$

$$(7b) \qquad \dot{L}^1_t = p[\bar{L}^1(Q_t) - L^1_t] \quad ,$$

$$(7c) \qquad \dot{L}^2_t = p[\bar{L}^2(Q_t) - L^2_t] \quad .$$

In the presence of economies of scale, one needs to be concerned with the stability and the determinacy of the equilibrium path.[19] For any constant q, the $\dot{Q} = 0$ locus is given by $Q = qh(L^2)$, which is upward-sloping in the (L^2, Q) space, as is the case with the $\dot{L}^2 = 0$ locus. The local stability of a steady state depends on the relative magnitude of the slopes of these two loci. If the $\dot{Q} = 0$ locus is flatter than the $\dot{L}^2 = 0$ locus, the steady state is saddlepoint stable: if steeper, then it is unstable.[20] The system has always steady states, at least one of which is locally saddlepoint stable (although it may not be in the interior). It is also proved in Appendix that there is no closed orbit. Beyond this, little can be said about the global property of the system, as it depends on the specific functional forms. For example, Figure 4A shows what the dynamics of $\{L^2, Q\}$ look like when there are three steady states, two of which are in the interior and one of which is at the origin; see Matsuyama (in process) for numerical examples that exhibit the phase portrait like this. In this case, the equilibrium path is indeterminate. There exists, however, a sense in which this multiplicity does not really matter. If we ignore the equilibrium path leading to the degenerate zero level steady state--after all, we are interested in "two sector" economies--, all other equilibria, except the one that starts and stays in the lower of the two nondegenerate steady states, eventually converges to the high-level steady state. The low-level steady state would not be observable under occasional perturbations. This obviously reduces the empirical importance of the multiplicity.[21] In what follows, it is assumed that the economy is initially at the locally saddlepoint stable nondegenerate steady state, and in responding to shocks, it jumps to the upward-sloping convergent path leading to the nearby steady state. Alternatively, one can assume that the two loci intersects only once in the interior (and only in the interior). Then, the system is globally saddlepoint stable and the convergent path can be shown to be upward-sloping.

Now, consider the effect of a permanent increase in q. At time zero, Q goes up, but it comes short of its new long run level, and then, it gradually increases over time (under-shooting). In Figure 4B, the target vector $(\bar{L}^1(Q), \bar{L}^2(Q))$ jumps from S_o to S', then moves away to S*. Therefore, the economy starts moving from S_o toward S' at time zero, but it keeps bending toward its left until it reaches the new steady state S*. With a permanent increase in q, a large fraction of new cohorts goes to the second sector, which increases the labor supply in the sector. With economies of scale, this increases the relative wage rate further, thereby accelerating the adjustment process.

One can also analyze the effect of temporary and anticipated increases. In responding to a temporary increase, Q jumps up immediately. The dynamic pattern of $\dot{Q}$ depends on T: the length of the period of high q. If T is small, $\dot{Q}$ is negative throughout the adjustment period. If T is large, then $\dot{Q}$ is positive initially and then becomes negative later.[22] With this information in mind, the corresponding dynamics of (L^1,L^2) can be examined in a similar manner as done in the case of decreasing returns. One can also show that an expected future increase in q raises the relative wage of the second sector even before q eventually rises.

4. The Dependent Economy Model: Endogenizing the Relative Prices

This section shows how the relative price sequence $\{q_t\}$ can be endogenized. To do so, it converts the small open economy model developed in section 2 to a so-called dependent economy model. The economy produces two goods, employing specific labor with constant-returns-to-scale technologies: $X^1 = L^1$, $X^2 = L^2$. Good 1 is tradeable and good 2 is nontradeable. The economy has access to the world capital market, where the interest rate r^* is exogenously given and constant over time. The agents of the economy make their career decision in the same way as in the previous sections.

Aggregate consumption behavior can be formalized as follows. All agents have the identical time-separable preferences. The instantaneous utility is given by $u(C_t+v(N_t))$: C_t and N_t are consumption of the tradeable and the nontradeable at time t, respectively, and $u' > 0$, $v' > 0$, $u'' \leq 0$ and $v'' < 0$. This special functional form implies that the demand for the nontradeable has zero income elasticity. This guarantees that any relative price dynamics comes from sluggish sectoral adjustment. The rate of time preference is constant and equal to δ. Finally, in order to build in aggregate demand shocks, the government is assumed to consume the nontradeable by G_t, which is financed by lump-sum taxes.[23] The government purchases are exogenous and generate utility to the private agents that is separable from the utility of other goods. The market clearing condition for the nontradeable is given by $L_t^2 = X_t^2 = N_t + G_t$.

Under this specification, the inverse demand for the nontradeable is given by $q_t = v'(N_t)$, so that, from the market clearing condition, the equilibrium path of q_t is shown to be $v'(L_t^2 - G_t)$ and the dynamic behavior of the economy is summarized by:

$$\text{(7a")} \qquad \dot{Q}_t = r[Q_t - v'(L_t^2 - G_t)] \quad ,$$

(7b) $$\dot{L}_t^1 = p[\bar{L}^1(Q_t) - L_t^1] \quad ,$$

(7c) $$\dot{L}_t^2 = p[\bar{L}^2(Q_t) - L_t^2] \quad .$$

Notice that the qualitative nature of this system is the same with the case of decreasing returns to scale analyzed in section 3: a rise in G corresponds to an increase in q. Therefore, the analysis of changes in the government spending can be done in terms of Figures 3A, 3B, and 3C. A permanent increase in G shifts the $\dot{Q} = 0$ locus upward and the economy jumps from A_0 to A_1. Since the supply of the nontradeable is inelastic in the short run, its relative price goes up after the government purchase increase, more than its new long run level. A large fraction of the new cohort goes to the nontradeable sector. As time passes, this increases the supply and reduces the relative price gradually, thereby slowing down the sectoral adjustment. A temporary increase in G can be also examined using Figure 3A. The adjustment path of the outputs looks like what is given in Figure 3C.

Up until this point, it has been assumed that the economy has access to the world capital market, where the constant interest rate, r^*, is exogenously given. It should be noted, however, that one can easily close the model by assuming $u'' = 0$: that is, the instantaneous utility is given by $C + v(N)$. Then, the equilibrium interest rate is always equal to the constant discount rate, δ, so that the equilibrium paths of the economy can be obtained by (7a"), (7b) and (7c) with $r = \delta + p$. (Alternatively, one can introduce an constant returns of scale inventory technology, which would help to tie down the interest rate.)

5. Welfare Implications

As seen in the previous sections, the economy stays strictly inside the long run production possibility frontier during the period of adjustment. The existence of such adjustment costs, however, does not necessarily justify the government intervention. In fact, one can demonstrate, in the absence of externalities in production, that the market outcome is efficient in that the present discounted value of the economy's outputs is maximized, and that any government intervention would merely introduce distortions.

For example, consider the case of decreasing returns to scale discussed in the first half of section 3; that is, $X^1 = L^1$ and $X^2 = f(L^2)$. The optimal allocation is the solution to the following social planning problem: Choose α_t to maximize,

$$\int_o^{+\infty} (X_t^1 + q_t X_t^2) e^{-r^* t} dt = \int_o^{+\infty} (L_t^1 + q_t f(L_t^2)) e^{-r^* t} dt$$

subject to

$$\text{(i)} \qquad \dot{L}_t^1 = p\left[\int_{\alpha_t}^{\alpha+} g^1(\alpha) d\Phi(\alpha) - L_t^1\right] ,$$

$$\text{(ii)} \qquad \dot{L}_t^2 = p\left[\int_{\alpha-}^{\alpha_t} g^2(\alpha) d\Phi(\alpha) - L_t^2\right] ,$$

and

$$\text{(iii)} \qquad L_o^1, L_o^2 : \text{given} .$$

Let $\lambda_t^1 e^{-r^* t}$, $\lambda_t^2 e^{-r^* t}$ be the multipliers associated with constraints (i), and (ii), respectively. Then, the necessary conditions are given by, in addition to constraints (i-iii),

$$\text{(iv)} \qquad \lambda_t^1 g^1(\alpha_t) = \lambda_t^2 g^2(\alpha_t)$$

$$\text{(v)} \qquad \dot{\lambda}_t^1 = (r^* + p)\lambda_t^1 - 1 \quad ; \qquad \lim_{t\to+\infty} \lambda_t^1 e^{-r^* t} = 0$$

$$\text{(vi)} \qquad \dot{\lambda}_t^2 = (r^* + p)\lambda_t^2 - q_t f'(L_t^2) \quad ; \qquad \lim_{t\to+\infty} \lambda_t^2 e^{-r^* t} = 0 \quad .$$

Define $Q_t \equiv \lambda_t^2/\lambda_t^1$. Then, (iv) becomes $\alpha_t = A(Q_t)$ so that (i) and (ii) are identical with equations (7b) and (7c). Next, note that (v) implies $\lambda_t^1 = 1/(r^*+p)$, so, from (vi),

$$\dot{Q}_t = (r^* + p)[Q_t - q_t f'(L_t^2)] \quad ; \qquad \lim_{t\to+\infty} Q_t e^{-r^* t} = 0 \quad ,$$

which is identical to (4′), or (7a) plus the transversality condition, hence the efficiency of the equilibrium adjustment path. This result carries over to alternative specifications of production technology, provided that the technology does not involve externalities. It also carries over to the dependent and closed economy models in the last section. Mussa (1978, 1982) demonstrated the same result in his model of sectoral adjustment. His insight has been confirmed in this model, too.

In the presence of external economies in production, the equilibrium path is no longer efficient, so some intervention could enhance efficiency. But this does not justify government intervention in the adjustment process, since this inefficiency does not come from the adjustment process itself. In other words, the equilibrium path is inefficient both in and out of the steady state. For example, consider the case of external economies in the second sector: $X^1 = L^1$ and $\chi^2 = h(L^2)\ell^2$. In this case, the market outcome without intervention leads to a underproduction of good 2. The appropriate policy is to provide a production subsidy on good 2, equal to $h'(L^2)L^2/h(L^2)$. The subsidy rate depends solely on the level of production and it is irrelevant whether the economy is in the adjustment phase.

There is a case where the economy is inefficient only in the adjustment period: the private discount rate used in calculating lifetime income differs from the social discount rate, $r^* + p$. Suppose, for example, that there exist no annuity markets, so that the private discount rate is equal to r^*, instead of $r^* + p$.[24] In the steady state, where wage rates are anticipated to be constant, the discount rate would not affect the career decision, and therefore, the equilibrium without intervention is efficient in the absence of production externalities. The nature of inefficiency out of the steady state depends on the structure of the model and the type of a disturbance. Consider, for example, a permanent increase in q in the model of decreasing returns to scale: $X^1 = L^1$ and $X^2 = f(L^2)$. In this case, one can show that the equilibrium adjustment without intervention is slower than the efficient adjustment. This is because an increase in the relative wage of the second sector is larger in the short run than in the long run. Using a discount rate lower than the social discount rate, agents underestimate Q (from the social point of view) so that a too small fraction of the new cohort goes to the second sector. The government could increase efficiency by speeding up the adjustment. Likewise, a permanent change in G in the dependent economy model of section 4 calls for a measure of increasing the adjustment speed. It should be noted, however, that intervention in the adjustment process is the second best policy. The first best policy is, of course, the setting up of annuity markets.

6. Concluding Remarks

This paper offered overlapping generations models of sectoral adjustment with perfect foresight. It is assumed that, once she made a commitment to a sector, each agent cannot move to the other sector. Aggregate sectoral movement of labor takes place gradually through the demographic change. The main advantage of this approach is its tractability and it is demonstrated that one can easily perform a variety of comparative dynamics.

Because of its simple structure, it seems that this framework can be a useful building block in more complex models. For example, adding stochastic shocks would be an interesting extension. With repeated random disturbances, the economy would be permanently away from the production frontier and the higher the variance of sectoral shocks, the associated output loss, which one may call "the GNP gap," will be higher.

In the last section, only the efficiency criterion is used for the normative analysis. Income distribution is also an important policy issue. The models presented here would be a natural framework in which to consider both inter- and intra-generational equity problems, which have been left for future work. It is only pointed out here that a government policy that attempts to compensate workers in depressed industries would involve time consistency problems. This is because such a policy, if anticipated, would encourage a larger fraction of the new cohorts to enter, thereby perpetuating the problems of the depressed industries.

It is assumed in this paper that labor service is fully employed in each sector. A negative sectoral shock leads to a low wage, not an unemployment. It is highly desirable to extend the model to allow for unemployment. After all, it is the problem of unemployment that spurred interest in sectoral shocks among labor-macro-economists; see Lilien (1982) and Abraham and Katz (1986). One approach would be introducing some sort of wage rigidities that result from asymmetric information, such as Shapiro and Stiglitz (1984). Another approach would be introducing job search activity within a sector coupled with firm-specific shocks.[25] It is hoped that this paper serves as a first step toward a tractable model of sectoral unemployment dynamics with perfect foresight.

Appendix

Proof that there exists no closed orbit

On any solution path of equations (7a') and (7c) with a constant q, we have $dQ/d\bar{L}^2 = r[Q - qh(L^2)]/p[\bar{L}^2(Q) - L^2]$. Therefore, if there exists a closed orbit γ, we have

$$\int_\gamma \left\{ p[\bar{L}^2(Q) - L^2]dQ - r[Q - qh(L^2)]dL^2 \right\} = 0 \quad .$$

This implies, via Green's Theorem (see Rudin [1976, p. 282] for example), that

$$\iint_S (r - p)\ dL^2 dQ = 0 \quad .$$

where S is the interior of γ. But, since $r - p = r^* > 0$, one cannot find a region $S \subset R^2_+$ such that the above equality holds: a contradiction. Hence, there can be no closed orbit.

Q.E.D.

References

Abel, A. and O. Blanchard. "An Intertemporal Model of Saving and Investment." *Econometrica* 51 (May 1983): 675-692.

Abraham, K. and L. Katz. "Cyclical Unemployment: Sectoral Shifts or Aggregate Disturbances?" *Journal of Political Economy* 94 (June 1986): 507-522.

Becker, G. *Human Capital* New York: Columbia University Press, 1964.

Blanchard, O. "Output, the Stock Market, and Interest Rates." *American Economic Review* 71 (March 1981): 132-143.

Blanchard, O. "Debt, Deficits and Finite Horizons." *Journal of Political Economy* 93 (April 1985): 223-247.

Ethier, W. J. "Decreasing Costs in International Trade and Frank Graham's Argument for Protection" *Econometrica* 50 (September 1982): 1243-1268.

Friedman, M. and S. Kuznets *Income from Independent Professional Practice* New York: National Bureau of Economic REsearch, 1945

Grossman, G. M. and C. Shapiro. "A Theory of Factor Mobility" *Journal of Political Economy* 90 (october 1982): 1054-1069.

Grossman, G. M. "Partially Mobile Capital: A General Approach to Two Sector Trade Theory." *Journal of International Economics* 15 (August 1983: 1-17.

Helpman, E. and P. R. Krugman. *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition and the International Economy*, MIT press, 1985.

Judd, K. "The Law of Large Numbers with a Continuum of IID Random Variables." *Journal of Economic Theory* 35 (February 1985): 19-25. (a)

Judd, K. "Short-Run Analysis of Fiscal Policy in a Simple Perfect Foresight Model." *Journal of Political Economy* 93 (April 1985): 298-319. (b)

Kemp, M. and H. Wan. "Hysteresis of Long-Run Equilibrium from Realistic Adjustment Costs." In G. Horwich and P. Samuelson (eds.), *Trade, Stability and Macroeconomics*. New York: Academic Press, 1974.

Krugman, P. R. "Increasing Returns and the Theory of International Trade" in T. F. Bewley (ed.) *Advances in Economic Theory: Fifth World Congress* Cambridge University Press, 1987.

Lapan, H. E. "International Trade, Factor Market Distortions, and the Optimal Dynamic Subsidy." *American Economic Review* 66 (June 1976): 335-346.

Lilien, D. "Sectoral Shifts and Cyclical Unemployment." *Journal of Political Economy* 90 (August 1982): 777-793.

Marshall, A. *Principles of Economics* 8th Edition, MacMillan, London, 1920.

Matsuyama, K. "Current Account Dynamics in a Finite Horizon Model." *Journal of International Economics* 23 (November 1987): 299-313.

Matsuyama, K. "Increasing Returns, Indeterminary of Equilibria and Global Bifurcations in a Perfect Foresight Model." (in process).

Murphy, K. M. and R. H. Topel "The Evolution of Unemployment in the United States: 1968-1985" In S. Fischer (ed.) *NBER Macroeconomics Annual 1987* Cambridge: The MIT Press, 1987.

Mussa, M. "Dynamic Adjustment in the Heckscher-Ohlin-Samuelson Model." *Journal of Political Economy* 86 (October 1978): 775-791.

Mussa, M. "Government Policy and the Adjustment Process." In J. N. Bhagwati (ed.), *Import Competition and Response*. Chicago: University of Chicago Press, 1982.

Mussa, M. "Imperfect Factor Mobility and the Distribution of Income." *Journal of International Economics* 12 (February 1982): 125-141.

Neary, J. P. "Short-Run Capital Specificity and the Pure Theory of International Trade." *Economic Journal* (September 1978): 488-510. (a)

Neary, J. P. "Dynamic Stability and the Theory of Factor-Market Distortions." *American Economic Review* 68 (September 1978): 671-682. (b)

Panagariya, A. "Increasing Returns, Dynamic Stability and International Trade" *Journal of International Economics* 20 (February 1986): 43-63.

Rudin, W. *Principles of Mathematical Analysis* Third Edition, McGraw-Hill Inc. 1976.

Shapiro, C. and J. E. Stiglitz "Equilibrium Unemployment as a Worker Discipline Device." *American Economic Review* 74 (June 1984): 433-444.

Topel, R. H. "Local Labor Markets" *Journal of Political Economy* 94 (Part 2; June 1986): S111-S143.

Weil, P. "Overlapping Families of Infinitely-Lived Agents." H.I.E.R. Discussion Paper No. 1332 (July 1987).

Weiss, L. "Asymmetric Adjustment Costs and Sectoral Shifts." in W. P. Heller, R. M. Starr, and D. Starrett (eds.) *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow, vol II*. Cambridge University Press, 1986.

Notes

1. The organization of a sector in these models is, in a way, reminiscent of Marshall's view of the industry, known for his famous analogy between firms in industry and trees in forest (Marshall [1920, Book IV, Chapter XIII]); Trees (firms) have life-cycles. The forest (industry) remains stationary when their birth (entry) rate and death (exit) rate are in balance. The forest (industry) shrinks, either when the death (exit) rate rises or when the birth (entry) rate declines. It is the latter mechanism that this paper focuses on.

2. For example, Murphy and Topel (1987, sec. 5) reports that the decline in the manufacturing's share of employment in recent years is much more pronounced among the entry cohort than in the aggregate data.

3. This is a direct corollary of the envelope theorem. Of course, the output loss would be of the first-order magnitude in a model where the costs are proportional to the rate of factor movements; see Kemp and Wan (1974). But, with a strictly positive marginal cost of factor movements, the adjustment path would not be differentiable around the steady state, so that linearization cannot be justified. In fact, there would be hysteresis effects which make the steady state path-dependent.

4. This paper has nothing to say about the determinants of specificity, as most studies in the literature. See Grossman and Shapiro (1982), which is the only exception that I am aware of.

5. I wish to thank G. Grossman for bringing Lapan's work to my attention.

6. Weiss (1986) considers aggregate employment dynamics in a model where adjustment costs of changing employment are asymmetric. I wish to thank W. A. Brock for bringing Weiss's work to my attention.

7. The case of a continuum of i.i.d. random variables can cause some conceptual problems, however; see Judd (1985a).

8. How the normative results are affected will be discussed in section 5.

9. Alternatively, one can specify the demography similar to the Weil (1987) economy in the constant-returns-to-scale case, as discussed below. It is necessary, however, to keep the size of the economy constant in order to analyze the nonconstant-returns-to-scale cases.

10. In fact, all functions in the paper, except q_t, are assumed to be sufficiently smooth.

11. This decision rule is, of course, well known in the human capital literature: see Friedman and Kuznets (1945) and Becker (1964).

12. Mussa (1982) shows in a static framework that the imperfect substitutability of labor gives rise to what he called "a convex input transformation curve." Grossman (1983) makes a similar point for capital.

13. One interpretation of $f'' < 0$ is internal diseconomies. Another interpretation is that $f(L^2) = F(K,L^2)$ where F is a linear homogeneous production function and K is a fixed hidden factor of production. The assumption on the distribution of the returns earned by K needs not to be specified, since, in a small open economy, the supply side can be analyzed separately from the demand side.

14. In what follows, some familiarity with the phase diagram analysis of perfect foresight models is assumed: see, for example, Abel and Blanchard (1983) and Blanchard (1981).

15. It is "stable" in the economic sense. Any solution path outside of the stable manifold can be shown to violate either equation (7a) at Q = 0, or equation (4').

16. This condition is given by

$$\left[g^1(\alpha^-)/g^2(\alpha^-)\right]\left[f'(+0)\right]^{-1} < q_t < \left[g^1(\alpha^+)/g^2(\alpha^+)\right]\left[f'\left(\int_{\alpha^-}^{\alpha^+} g^2(\alpha)d\Phi(\alpha)\right)\right]^{-1} ,$$

which can be satisfied by any positive, finite path $\{q_t\}$ if $f'(+0) = +\infty$ and $g^1(\alpha)/g^2(\alpha) \to +\infty$ as $\alpha \to \alpha^+$. The first is a familiar Inada condition. The second implies that at any relative wages, there always exists a positive measure of agents who prefer to work in the first sector.

17. One corollary of this result is that the short run effect on the relative wage is greater for temporary shocks than for permanet ones. Topel (1986) makes a similar point in his model of regionally specific labor. I wish to thank J. Altonji for bringing Topel's work to my attention.

18. Assuming $h'(L^2) < 0$, i.e., external diseconomies, gives another model of decreasing returns to scale. The positive, but not normative, aspects of the model would be qualitatively the same with the internal diseconomy case seen above.

19. Ethier (1982) and Panagariya (1986) examined the dynamic stability of multiple equilibria under increasing returns to scale. Their adjustment processes are, however, inconsistent with perfect foresight.

20. Algebraically, it is saddlepoint stable if $\bar{L}^{2\prime}(q_o)q_o h'(\bar{L}^2(q_o)) < 1$ and unstable if $\bar{L}^{2\prime}(q_o)q_o h'(\bar{L}^2(q_o)) > 1$. Note that the latter is unstable because the trace of the Jacobian matrix that results from the linearization of (7a') and (7c') is equal to $r - p = r^* > 0$.

21. This would not be necessarily the case if the assumption of perfect annuity markets were dropped; see Matsuyama (in process).

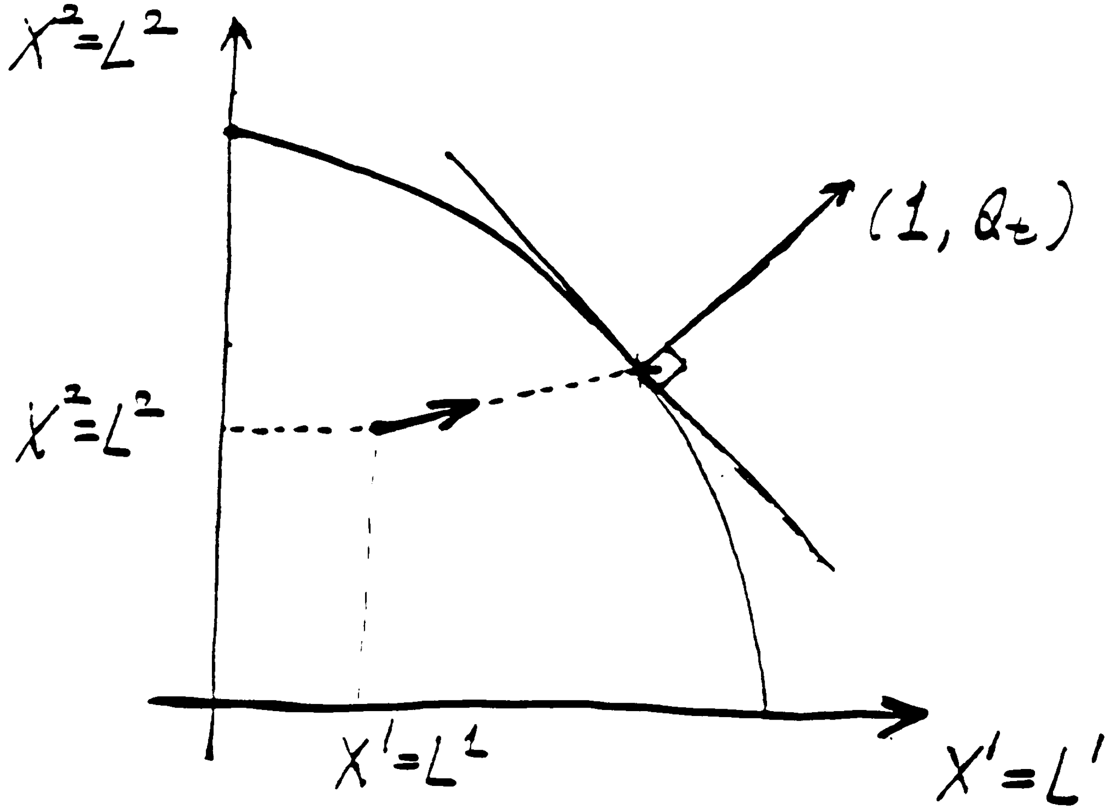
22. The condition that T needs to satisfy in order for this to happen can be obtained algebraically under the linear approximation, by the technique used by Judd (1985b).

23. With the assumed objective function, the government purchase of the tradeable would not affect the price dynamics.

24. This can be justified under the following institutional arrangement. First, accidental bequests are confisticated by the government and then distributed back to those alive in a lump-sum way. Second, when an agent borrows, her loan is guaranteed by the government as long as her consumption plan is consistent with the intertemporal solvency condition, $\lim_{t\to +\infty} \omega_t e^{-r^* t} \geq 0$, where ω_t is her net asset position at time t.

25. Or, one can divide one of the two sectors into multiple subsectors, each of which experiences idiosyncratic shocks.

Fig. 1



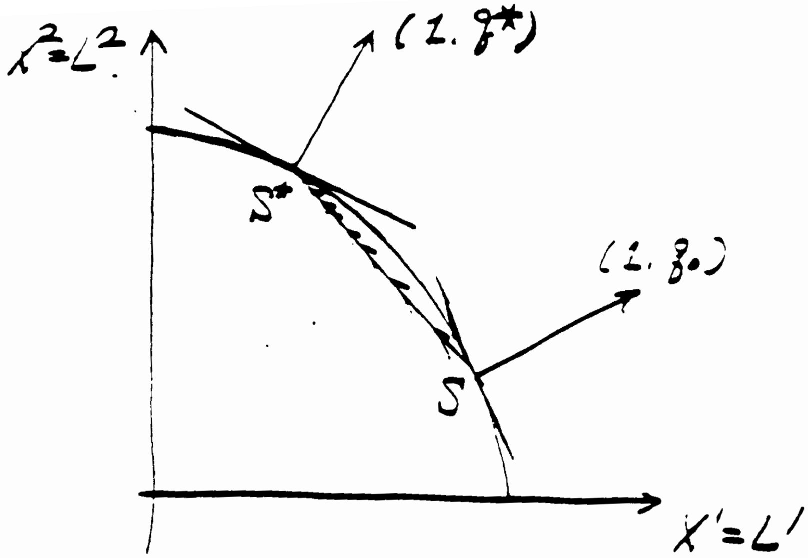


Fig. 2A

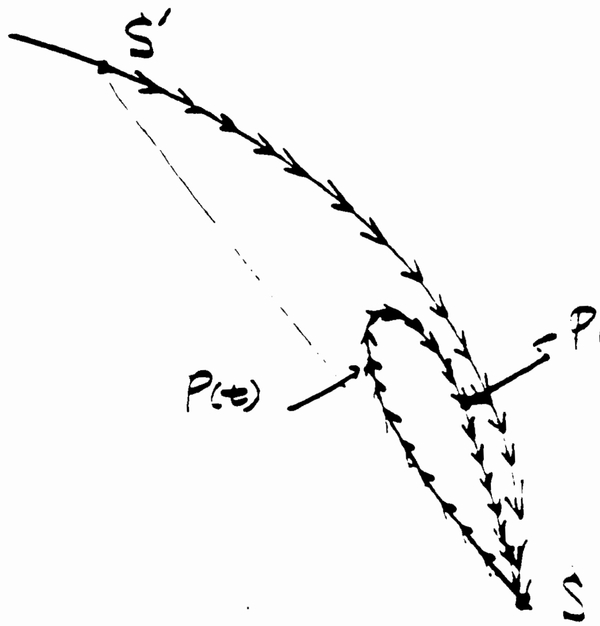


Fig. 2B

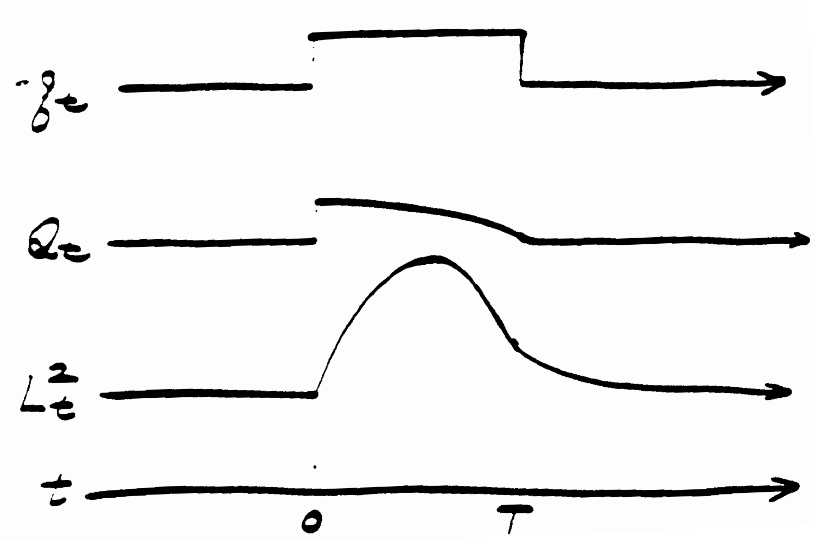


Fig. 2C

Fig. 3A

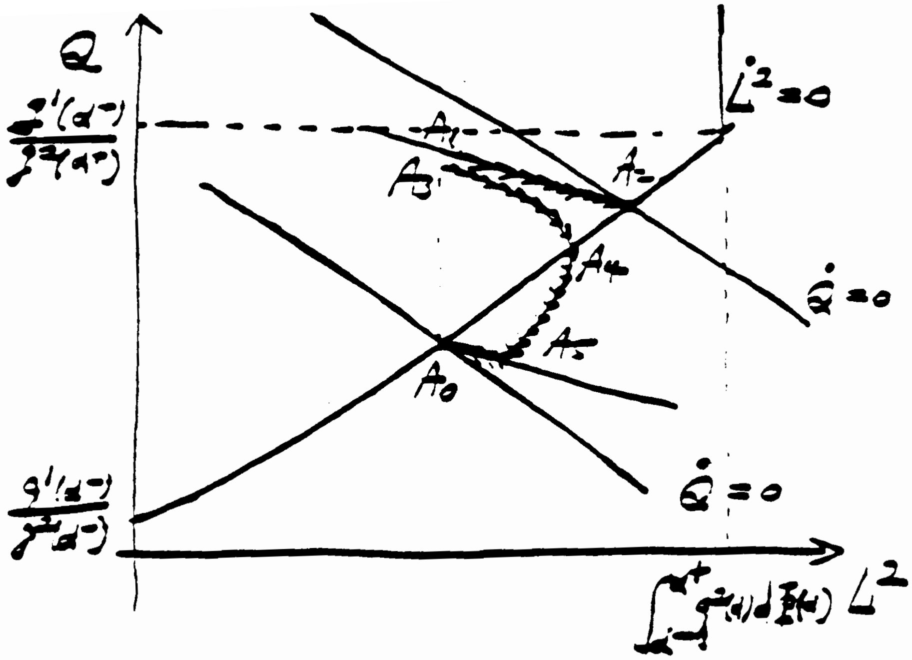


Fig. 3B

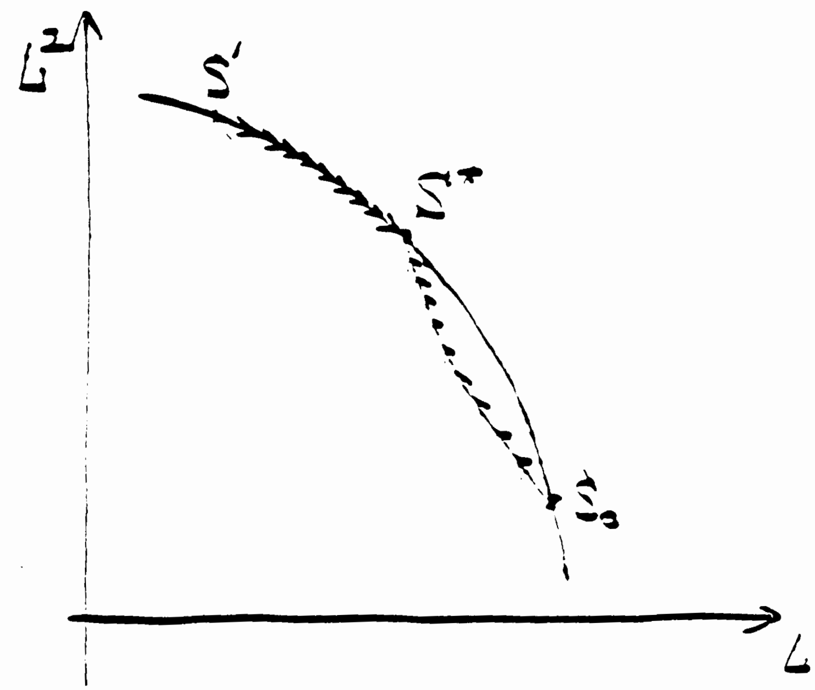


Fig. 3d