A Technology-Gap Model of ‘Premature’ Deindustrialization

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Bank of Japan
Introduction
Structural Change

As per capita income rises, the employment or value-added shares

- *Fall* in Agriculture
- *Rise* in Services
- *Rise and Fall* in Manufacturing

*From Herrendorf-Rogerson-Valentinyi (2014)*

*Evidence from Long Time Series for the Currently Rich Countries* (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000
Premature Deindustrialization (PD): Rodrik (JEG 2016)

Late industrializers reach their M-peak and start deindustrializing
- Later in time
- Earlier in per capita income
- with the lower peak M-sector shares, compared to early industrializers.

Rodrik (2016) focuses on documenting the patterns, without offering a causal explanation or making normative statements. But
- He speculates that globalization may be a cause.
- He cautions against drawing policy implications, but the word, “premature,” may seem to suggest some types of inefficiency.

In our proposed mechanism,
- PD occurs in the efficient equilibrium of a closed economy.
- PD is robust to opening up for trade but weakened.
This Paper: A Parsimonious Model of Premature Deindustrialization (PD)

3 Goods/Sectors: 1=(A)griculture, 2=(M)anufacturing, 3=(S)ervices, homothetic CES with gross complements (σ < 1)

Frontier Technology: \( \tilde{A}_j(t) = \tilde{A}_j(0)e^{g_j t} \), with \( g_1 > g_2 > g_3 > 0 \) \( \Rightarrow \) a decline of A, a rise of S, and a hump-shaped of M in each country through the Baumol (relative price) effect, as in Ngai-Pissarides (2007)

Actual Technology Used: \( A_j(t) = \tilde{A}_j(t - \lambda_j) \) due to Adoption Lags, \((\lambda_1, \lambda_2, \lambda_3)\).

\[
A_j(t) = \tilde{A}_j(t - \lambda_j) = \tilde{A}_j(0)e^{-g_j \lambda_j} e^{g_j t} \Rightarrow \frac{\partial}{\partial \lambda_j} \ln \left( A_j(t) \right) = -g_j < 0
\]

\( \lambda_j \) has no “growth” effect, but negative “level” effects, \( e^{-\lambda_j g_j} \), amplified by \( g_j \).

Log-submodularity: \( g_j \) magnifies the (negative) impact of the adoption lag on productivity: \( \frac{\partial}{\partial g_j} \left( \frac{\partial}{\partial \lambda_j} \ln e^{-g_j \lambda_j} \right) < 0 \)

One-dimension of cross-country heterogeneity: For \((\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda\),

- \( \lambda \geq 0 \): Technology Gap, country-specific, as in Krugman (1985); their ability to adopt the frontier technologies.
- \( \theta_j > 0 \): sector-specific, unlike Krugman (1985); how much \( \lambda \) affects the adoption lag and productivity in each sector.

\[
A_j(t) = \tilde{A}_j(0)e^{-g_j \theta_j \lambda} e^{g_j t} \Rightarrow \frac{\partial}{\partial \lambda} \ln \left( \frac{A_j(t)}{A_k(t)} \right) = -(\theta_j g_j - \theta_k g_k).
\]
Main Results: Conditions for PD, defined as “A high-λ country reaches its peak later in time, with lower peak M-share at lower peak time per capita income.”

i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S. High relative price of A/low relative price of S in a high-λ country causes a delay.

ii) $\left(1 - \frac{g_3}{g_1}\right)\left(\frac{\theta_2}{\theta_3} - 1\right) + \left(1 - \frac{g_3}{g_2}\right)\left(1 - \frac{\theta_1}{\theta_3}\right) < 0$: Technology adoption takes not too long in M. Not too high relative price of M in a high-λ country keeps the M-share low.

Under the above conditions,

iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A. Longer adoption lag in S in a high-λ country causes “premature” deindustrialization.

Some Implications
No PD if $\theta_1 = \theta_2 = \theta_3$. Latecomers would follow the same path with a delay.

i) & ii) $\Rightarrow \theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$: Cross-country productivity difference is the largest in A. $\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative; slightly negative when calibrated to match Rodrik’s (2016; Table 10) findings.
A Numerical Illustration.

\(\theta_1 = \theta_2 < \theta_3 = 1\) with \(g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%\); \(\sigma = 0.6\); Labor share = 2/3. We set the other parameters, w.l.o.g., so that the peak time, \(\hat{t} = 0\) and the peak time income per capita, \(U(\hat{t}) = 1\) if \(\lambda = 0\).

<table>
<thead>
<tr>
<th>Example 2a</th>
<th>((t, s_2(t)))</th>
<th>(\ln U(t), s_2(t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2})</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

\(\Rightarrow \theta_1g_1 > \theta_2g_2 = \theta_3g_3\)
1st Extension: Adding the Engel Effect with Nonhomothetic CES (a la Comin-Lashkari-Mestieri)
Nonhomotheticity changes the shape of trajectories greatly, but not on how technology gaps, $\lambda$, affects the peak values.

<table>
<thead>
<tr>
<th>$(t, s_2(t))$</th>
<th>Homothetic ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</th>
<th>Nonhomothetic ($\varepsilon_1 = 4 &lt; \varepsilon_2 = 1 &lt; \varepsilon_3 = 1.6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\ln U(t), s_2(t))$</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
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</tbody>
</table>

We also show that the Engel effect alone could not generate PD without counterfactual implications.
2nd Extension: International Trade

One implication of our mechanism for PD (consistent with the empirical evidence):

\[
\frac{\partial}{\partial \lambda} \ln \left( \frac{A_1(t)}{A_2(t)} \right) = - (\theta_1 g_1 - \theta_2 g_2) < 0.
\]

- A low-\(\lambda\) country has comparative advantage in A and a high-\(\lambda\) country has comparative advantage in M.
- Opening up for trade allows a high-\(\lambda\) country to export M to a low-\(\lambda\) country.
- Our mechanism for PD is weakened by opening up for trade, but PD continues to hold, as long as the trade cost is not too small.
- Consistent with the findings that East Asia “suffers” less from PD (Rodrik 2016).

Under our mechanism, PD occurs not because of, but in spite of international trade.
3rd Extension: Introducing Catching-up

\[ A_j(t) = \tilde{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_{\lambda}t}, \]

Countries differ only in the initial value, \( \lambda_0 \), converging exponentially over time at the same rate, \( g_{\lambda} > 0 \)

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>Peak M-Share</th>
<th>Peak time Per Capita Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_0 )</td>
<td>( g_{\lambda} )</td>
<td>( g_{\lambda} )</td>
</tr>
<tr>
<td>( g_{\lambda}=1.4% )</td>
<td>( g_{\lambda}=2.1% )</td>
<td>( g_{\lambda}=2.8% )</td>
</tr>
<tr>
<td>( g_{\lambda}=1.05% )</td>
<td>( g_{\lambda}=1.4% )</td>
<td>( g_{\lambda}=0.7% )</td>
</tr>
<tr>
<td>( g_{\lambda}=0.7% )</td>
<td>( g_{\lambda}=0% )</td>
<td>( g_{\lambda}=0% )</td>
</tr>
</tbody>
</table>

Higher-\( \lambda \) countries
- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless \( g_{\lambda} \) is too large.
(Very Selective) Literature Review. Herrendorf-Rogerson-Valentinyi (14) for a survey on structural change.

Related to The Baseline Model

Premature Deindustrialization, Dasgupta-Singh (06), Palma (14), Rodrik (16)

The Baumol Effect: Baumol (67), Ngai-Pissarides (07), Nordhaus (08)

Cross-country heterogeneity in technology development

- Distance to the frontier: Krugman (85), Acemolgu-Aghion-Zilibotti (06)
- Log-supermodularity: Krugman (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- Productivity difference across countries the largest in $A$: Caselli (05), Gollin et.al. (14, AERP&P)
- Small adoption lags in $M$: Rodrik (2013)

Related to Three Extensions

The Engel Effect (Nonhomotheticity): Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zweimueler (08), Buera-Kaboski (09,12), Boppart (14), Comin-Lashkari-Mestieri (21), Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)

Open Economy Implications: Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (WinP)

Catching-Up/Technology Diffusion: Acemoglu (08), Comin-Mestieri (18)

The Issues We Abstract From

Sector-level productivity growth rate differences across countries: Huneeus-Rogerson (20)
Endogenous growth, externalities, Matsuyama (92).
Sectoral wedges/misallocation: Caselli (05), Gollin et.al. (14 QJE) and many others
Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.
Structural Change, the Baumol Effect, and Adoption Lags
Three Complementary Goods/Competitive Sectors, \( j = 1, 2, 3 \)

Sector-1 = (A)griculture, Sector-2 = (M)anufacturing, Sector-3 = (S)ervices.

**Demand System:** \( L \) Identical HH, each endowed with 1 unit of mobile labor, earning the wage \( w \) & \( \kappa_j \) units of managerial skills, specific to \( j \), each earning the rent, \( \rho_j \).

**Budget Constraint:**

\[
\sum_{j=1}^{3} p_j c_j \leq E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j = \frac{1}{L} \sum_{j=1}^{3} p_j Y_j
\]

**CES Preferences:**

\[
U(c_1, c_2, c_3) = \left[ \sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}} \right]^{\frac{1}{\sigma-1}}
\]

with \( \beta_j > 0 \) and \( 0 < \sigma < 1 \) (gross complementarity)

**Expenditure Shares:**

\[
m_j \equiv \frac{p_j c_j}{E} = \beta_j \left( \frac{p_j}{P} \right)^{1-\sigma}; \quad P = \left[ \sum_{k=1}^{3} \beta_k \left( p_k \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}
\]

**Real Per Capita Income**

\[
U = \frac{E}{P} = \left[ \sum_{k=1}^{3} \beta_k \left( \frac{E}{p_k} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}.
\]
Three Competitive Sectors: Production

Cobb-Douglas

\[ Y_j = \tilde{A}_j \left( \kappa_j L \right)^\alpha \left( L_j \right)^{1-\alpha} = A_j \left( L \right)^\alpha \left( L_j \right)^{1-\alpha} = LA_j \left( s_j \right)^{1-\alpha} \]

where \( A_j \equiv \tilde{A}_j \left( \kappa_j \right)^\alpha \). \( \alpha \in [0,1) \): the span of control parameter, which introduces diminishing returns in labor.

Labor Share

\[ \frac{wL_j}{p_j y_j} = 1 - \alpha \]

Profit (Managerial Rent) Share

\[ \frac{\rho_j \kappa_j L}{p_j y_j} = \alpha \]

Sectoral Share in Employment

\[ s_j \equiv \frac{L_j}{L}; \quad \sum_{j=1}^{3} s_j = 1 \]

Sectoral Sector in Value-Added

\[ \frac{p_j y_j}{EL} = s_j = \left( \frac{p_j A_j}{E} \right)^{1/\alpha}; \quad E = \left[ \sum_{k=1}^{3} \left( p_k A_k \right)^{1-\alpha} \right]^\alpha. \]
**Equilibrium:** The expenditure shares are equal to the employment and value-added shares.

\[ \beta_j \left( \frac{p_j}{p} \right)^{1-\sigma} = m_j = \frac{p_j Y_j}{EL} = s_j = \left( \frac{p_j A_j}{E} \right)^{1/\alpha} \]

which lead to

**Equilibrium Shares**

\[ s_j = \frac{\left[ \beta_j \frac{1}{\sigma-1} A_j \right]^{-\frac{1}{\alpha}}}{\sum_{k=1}^{3} \left[ \beta_k \frac{1}{\sigma-1} A_j \right]^{-\frac{1}{\alpha}}} \]

**Per Capita Income**

\[ U = \left\{ \sum_{k=1}^{3} \left[ \beta_k \frac{1}{\sigma-1} A_j \right]^{-\frac{1}{\alpha}} \right\}^{-\frac{1}{\alpha}} \]

where

\[ a \equiv \frac{1-\sigma}{(1-\alpha)(1-\sigma)} = -\frac{\partial \log(s_j/s_k)}{\partial \log(A_j/A_k)} > 0, \]

which captures how much relatively high productivity in a sector contributes to its relatively low equilibrium share. \( \alpha \) magnifies this effect by increasing \( a \).
Productivity Growth:

\[ A_j(t) = \tilde{A}_j(t - \lambda_j) = \tilde{A}_j(0)e^{g_j(t - \lambda_j)} = \tilde{A}_j(0)e^{-\lambda_j g_j}e^{g_j t} \]

\[ \tilde{A}_j(t) = \tilde{A}_j(0)e^{g_j t} \]: Frontier Technology in \( j \), with a constant growth rate \( g_j > 0 \).

\[ A_j(t) = \tilde{A}_j(t - \lambda_j); \lambda_j = \text{Adoption Lag} \text{ in } j. \]

- \( \lambda_j \) has no “growth” effect, but has a negative “level” effect, \( e^{-\lambda_j g_j} \), which is proportional to \( g_j \).

Key: Log-submodularity, \( \frac{\partial}{\partial g_j} \left( \frac{\partial}{\partial \lambda_j} \ln e^{-\lambda_j g_j} \right) < 0 \): \( g_j \) magnifies the negative effect of the adoption lag on productivity.

✓ A large adoption lag doesn’t matter much in a sector with slow productivity growth.

✓ Even a small adoption lag matters a lot in a sector with fast productivity growth.

\[ U(t) = \left\{ \sum_{k=1}^{3} \left[ \beta_k \frac{1}{a-1} A_k(t) \right]^{-\frac{1}{a}} \right\}^{-\frac{1}{a}} = \left\{ \sum_{k=1}^{3} \bar{\beta}_k e^{-a g_k(t - \lambda_k)} \right\}^{-\frac{1}{a}}, \text{ where } \bar{\beta}_k = \left( \frac{\beta_k^{1-\sigma}}{A_k(0)} \right)^{\alpha} > 0. \]

Longer adoption lags would shift down the time path of \( U(t) \).

\[ g_U(t) = \frac{U'(t)}{U(t)} = \sum_{k=1}^{3} g_k s_k(t) \]

The growth rate in per capita income is the weighted average of the sectoral growth rates.
Relative Prices: \[
\left( \frac{p_j(t)}{p_k(t)} \right)^{1-\sigma} = \left[ \left( \frac{\beta_j}{\beta_k} \right)^{-\alpha} \frac{\bar{A}_j(0)}{\bar{A}_k(0)} \right]^{-\alpha} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Rightarrow \frac{d \ln \left( \frac{p_j(t)}{p_k(t)} \right)}{dt} = \frac{a(g_k - g_j)}{1 - \sigma}
\]

Relative Growth Effect: \( p_j(t)/p_k(t) \) is decreasing over time if \( g_j > (<) g_k \). Speed independent of \( \lambda_j \) and \( \lambda_k \).

Relative Level Effect: A higher \( \lambda_j g_j - \lambda_k g_k \) raises \( p_j(t)/p_k(t) \) at any point in time.

Note: For a fixed \( \lambda_j > 0 \), a higher \( g_j \) makes the relative price of \( j \) higher (though declining faster).

Relative Shares: \[
\frac{s_j(t)}{s_k(t)} = \frac{\beta_j}{\beta_k} \left( \frac{p_j(t)}{p_k(t)} \right)^{1-\sigma} = \frac{\bar{\beta}_j}{\bar{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Rightarrow \frac{d \ln \left( \frac{s_j(t)}{s_k(t)} \right)}{dt} = a(g_k - g_j)
\]

Relative Growth Effect: \( s_j(t)/s_k(t) \) is decreasing over time if \( g_j > (<) g_k \). Speed independent of \( \lambda_j \) and \( \lambda_k \).

Shift from faster growing sectors to slower growing sectors over time.

Relative Level Effect: A higher \( \lambda_j g_j - \lambda_k g_k \) raises \( s_j(t)/s_k(t) \) at any point in time.

Note: For a fixed \( \lambda_j > 0 \), a higher \( g_j \) makes the relative share of \( j \) higher (though declining faster).
Structural Change with the Baumol (Relative Price) Effect: Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing in $t$, because

$$
\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[ \frac{\bar{\beta}_2}{\bar{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)} \right] e^{a(g_1 - g_2)t} + \left[ \frac{\bar{\beta}_3}{\bar{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)} \right] e^{a(g_1 - g_3)t}
$$

Rise of Services: $s_3(t)$ is increasing in $t$, because

$$
\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[ \frac{\bar{\beta}_1}{\bar{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)} \right] e^{-a(g_1 - g_3)t} + \left[ \frac{\bar{\beta}_2}{\bar{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)} \right] e^{-a(g_2 - g_3)t}
$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped in $t$, because

$$
\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[ \frac{\bar{\beta}_1}{\bar{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)} \right] e^{-a(g_1 - g_2)t} + \left[ \frac{\bar{\beta}_3}{\bar{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)} \right] e^{a(g_2 - g_3)t}
$$

Hump-shaped due to the two opposing forces: $g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

$$s'_2(t) \geq 0 \iff (g_1 - g_2)s_1(t) \geq (g_2 - g_3)s_3(t) \iff g_U(t) = \sum_{k=1}^{3} g_k s_k(t) \geq g_2$$

Initially, $\frac{s_1(t)}{s_3(t)}$ is large; the 1st force is stronger. As $\frac{s_1(t)}{s_3(t)}$ declines over time, the 2nd force becomes stronger eventually.
Characterizing Manufacturing Peak: “^^” indicates the peak.

\[ s'_2(\hat{t}) = 0 \iff (g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t}) \iff g_U(\hat{t}) = g_2 \]

**Peak Time:** From \((g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t})\)

\[ \hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \quad \text{where } \hat{t}_0 \equiv \frac{1}{a(g_1 - g_3)} \ln \left[ \frac{(g_1 - g_2)}{g_2 - g_3} \frac{\bar{\beta}_1}{\bar{\beta}_3} \right] \]

**Two Normalizations:** Without any loss of generality,

\[ \hat{t}_0 = 0 \iff \frac{g_2 - g_3}{g_1 - g_2} = \frac{\bar{\beta}_1}{\bar{\beta}_3} \equiv \left[ \frac{(\beta_1)^{1-\sigma}}{\beta_3} \frac{A_3(0)}{A_1(0)} \right]^a \]

The calendar time is reset so that its M-peak would be reached at \(\hat{t} = 0\) in the absence of the adoption lags.

\[ U(0) = 1 \text{ for } \lambda_1 = \lambda_2 = \lambda_3 = 0 \iff \sum_{k=1}^{3} \tilde{\beta}_k = \sum_{k=1}^{3} \left( \frac{\beta_k^{1-\sigma}}{A_k(0)} \right)^a = 1. \]

We use the peak time per capita income in the absence of the adoption lags as the *numeraire*.

*Note:* Under these normalizations, the peak time share of sector-\(k\) in the absence of the adoption lags would be \(\tilde{\beta}_k\).
Then,

**Peak Time**

\[ t = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}. \]

**Peak M-Share**

\[ \frac{1}{\hat{s}_2} = 1 + \left( \frac{\hat{\beta}_1}{\hat{\beta}_2} \right) e^{a(g_1 - g_2)(\frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \hat{t})} + \left( \frac{\hat{\beta}_3}{\hat{\beta}_2} \right) e^{a(g_2 - g_3)(\frac{\lambda_2 g_2 - \lambda_3 g_3}{g_2 - g_3})} \]

**Peak Time Per Capita Income**

\[ \hat{U} = \left\{ \sum_{k=1}^{3} \hat{\beta}_k e^{-a g_k (\hat{t} - \lambda_k)} \right\}^{-\frac{1}{\alpha}} \]

So far, we have looked at the impacts of adoption lags in a single country in isolation, without specifying the sources of the adoption lags.

If we allow countries to differ in \((\lambda_1, \lambda_2, \lambda_3)\), we can perfectly account for \((\hat{t}, \hat{s}_2, \hat{U})\).

We now restrict ourselves to one-dimension of country heterogeneity, the technology gap, which generate cross-country variations in adoption lags, and study the cross-country implications.
Technology Gaps and Premature Deindustrialization
Consider the world with many countries with 
\[(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda\]

\(\lambda \geq 0\): **Technology Gap**, Country-specific

\(\theta_j > 0\): **Sector-specific**, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in one dimension**, \(\lambda\), in their ability to adopt the frontier technologies.
- **\(\theta_j\) > 0** determines how much the technology gap affects the adoption lag in that sector.

\[\frac{A_j(t)}{A_k(t)} = \frac{\bar{A}_j(0)}{\bar{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)\varepsilon} \Rightarrow \frac{\partial}{\partial \lambda} \ln \left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)\]

Cross-country productivity difference is larger in sector-\(j\) than in sector-\(k\) if \(\theta_j g_j > \theta_k g_k\).

**Proposition 1: Peak Values under the Baumol Effect only**

**Peak Time:**
\[\hat{\tau}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda.\]

**Peak M-Share:**
\[\frac{1}{\bar{S}_2(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1 - g_2) \left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2}\lambda - \hat{\tau}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2 - g_3) \left(\hat{\tau}(\lambda) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\lambda\right)}\]

**Peak Time Per Capita Income:**
\[\bar{U}(\lambda) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k [\hat{\tau}(\lambda) - \theta_k \lambda]}\right\}^{-\frac{1}{a}}\]
Proposition 2: Conditions for PD with the Baumol (Relative Price) Effect

\[ \ell'(\lambda) > 0 \text{ for all } \lambda > 0 \iff \theta_1 g_1 > \theta_3 g_3. \]

With \( \theta_1 g_1 > \theta_3 g_3 \), the price of A is relatively higher than the price of S in a high-\( \lambda \) country, which delays the peak.

\[ s_2'(\lambda) < 0 \text{ for all } \lambda > 0 \iff \left(1 - \frac{g_3}{g_1}\right)\left(\frac{\theta_2}{\theta_3} - 1\right) + \left(1 - \frac{g_3}{g_2}\right)\left(1 - \frac{\theta_1}{\theta_3}\right) < 0 \]

With a low \( \theta_2 \), which has no effect on \( \ell \), the price of M is low relative to both A & S in a high-\( \lambda \) country, which keeps the M-share low.

Under the above condition,

\[ \bar{U}'(\lambda) < 0; \quad \bar{U}(\lambda) < \bar{U}(0) \text{ for } \lambda > \lambda_c \geq 0 \iff \theta_1 < \theta_3 \iff \ell(\lambda) < \theta_1 \lambda < \theta_3 \lambda \]

With \( \theta_1 < \theta_3 \), the time delay in the peak in a high-\( \lambda \) country is not long enough to make up for the lagging productivity, that is deindustrialization is “premature.”

- \( \theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\} \) (productivity differences the largest in A).
- \( \theta_2 g_2 - \theta_3 g_3 \) can be either positive or negative.
- \( \max\{\theta_1, \theta_2\} < \theta_3 \) (adoption lag the longest in S).
Some Examples

Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

\[ \theta_1 = \theta_2 = \theta_3 = 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0 \]

\[ \Rightarrow \hat{t}(\lambda) = \lambda; \quad \hat{s}_2(\lambda) = \bar{\beta}_2; \quad \hat{U}(\lambda) = 1 \]

- The country’s technology gap causes a delay in the peak time, \( \hat{t} \), by \( \lambda > 0 \).
- The peak M-share & the peak time per capita income unaffected.

Each country follows the same development path of early industrializers with a delay. No PD!!

Thus, the technology gap must have differential impacts on the adoption lags across sectors.
A Numerical Illustration.

$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%; \sigma = 0.6$; Labor share = $2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.

<table>
<thead>
<tr>
<th>Example 2</th>
<th>$(t, s_2(t))$</th>
<th>$(\ln U(t), s_2(t))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$</td>
<td>$\Rightarrow \theta_1 g_1 &gt; \theta_2 g_2 = \theta_3 g_3$</td>
<td><img src="image" alt="Graph of Example 2" /></td>
</tr>
</tbody>
</table>
Some Calibrations:

Rodrik (2016) divided countries into pre-1990 peaked vs. post-1990 peaked.
From his Fig.5, $\hat{f}(\lambda) = 25$ years. From his Table 10,
For the employment shares, $s_2(0) = 21.5\% > s_2(\lambda) = 18.9\%$; $\bar{U}(\lambda)/\bar{U}(0) = \bar{U}(\lambda) = 4273/11048$.
For the value-added shares, $s_2(0) = 27.9\% > s_2(\lambda) = 24.1\%$. $\bar{U}(\lambda)/\bar{U}(0) = \bar{U}(\lambda) = 20537/47099$. 

Fig. 6  Simulated manufacturing employment shares
Fig. 7  Simulated manufacturing output shares (MVA/GDP at constant prices)
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_1 = 3.8% &gt; g_2 = 2.4% &gt; g_3 = 1.3%$</td>
<td>$g_1 = 2.9% &gt; g_2 = 1.3% &gt; g_3 = 1.1%$</td>
</tr>
<tr>
<td><strong>Empl. Shares</strong></td>
<td><strong>Empl. Shares</strong></td>
</tr>
<tr>
<td>$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (13.9%, 28.1%, 26.0%);$</td>
<td>$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (17.5%, 36.9%, 27.4%);$</td>
</tr>
<tr>
<td>$(\theta_1 / \theta_3, \theta_2 / \theta_3) \approx (0.501, 0.512); \quad \Theta \approx 0.779.$</td>
<td>$(\theta_1 / \theta_3, \theta_2 / \theta_3) \approx (0.511, 0.650)$ and $\Theta \approx 0.848.$</td>
</tr>
<tr>
<td><strong>VA Shares</strong></td>
<td><strong>VA Shares</strong></td>
</tr>
<tr>
<td>$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (15.1%, 32.9%, 28.2%);$</td>
<td>$\left(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}\right) \approx (18.9%, 43.3%, 29.6%);$</td>
</tr>
<tr>
<td>$(\theta_1 / \theta_3, \theta_2 / \theta_3) \approx (0.511, 0.476)$ and $\Theta \approx 0.726.$</td>
<td>$(\theta_1 / \theta_3, \theta_2 / \theta_3) \approx (0.520, 0.583)$ and $\Theta \approx 0.805.$</td>
</tr>
</tbody>
</table>

\[ \theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2 \iff e^{-\theta_1 g_1 \lambda} < e^{-\theta_3 g_3 \lambda} < e^{-\theta_2 g_2 \lambda}. \]

Cross-country productivity differences not only the largest in A but also the smallest in M.
1st Extension: Introducing the Engel Effect
The Engel Law through Isoelastic Nonhomothetic CES; Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

\[ \left[ \sum_{j=1}^{3} \left( \beta_{j} \right)^{\frac{1}{\sigma}} \left( \frac{c_{j}}{U^{\varepsilon_{j}}} \right)^{1-\frac{1}{\sigma}} \right]^{\sigma-1} \equiv 1 \]

Normalize \( \varepsilon_{1} + \varepsilon_{2} + \varepsilon_{3} = 3 \); with \( \varepsilon_{1} = \varepsilon_{2} = \varepsilon_{3} = 1 \), we go back to the standard homothetic CES.

With \( \sigma < 1 \), \( 0 < \varepsilon_{1} < \varepsilon_{2} < \varepsilon_{3} \) ⇒ the income elasticity the lowest in A and the highest in S.

By maximizing \( U \) subject to \( \sum_{j=1}^{3} p_{j}c_{j} \leq E \),

Expenditure Shares

\[ m_{j} \equiv \frac{p_{j}c_{j}}{E} = \frac{\beta_{j} \left( U_{\varepsilon_{j}} p_{j} \right)^{1-\sigma}}{\sum_{k=1}^{3} \beta_{k} \left( U_{\varepsilon_{k}} p_{k} \right)^{1-\sigma}} = \frac{\beta_{j} \left( U_{\varepsilon_{j}} p_{j} \right)^{1-\sigma}}{E} \Rightarrow \frac{m_{j}}{m_{k}} = \frac{\beta_{j} \left( \frac{p_{j}}{p_{k}} U_{\varepsilon_{j}}^{\varepsilon_{j}-\varepsilon_{k}} \right)^{1-\sigma}}{\beta_{k}} \]

Indirect Utility Function:

\[ \left[ \sum_{j=1}^{3} \beta_{j} \left( \frac{U_{\varepsilon_{j}} p_{j}}{E} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \equiv 1 \]

Cost-of-Living Index:

\[ \left[ \sum_{j=1}^{3} \beta_{j} \left( \frac{U_{\varepsilon_{j}-1} p_{j}}{P} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \equiv 1 \iff U \equiv \frac{E}{P} \]

Income Elasticity:

\[ \eta_{j} \equiv \frac{\partial \ln c_{j}}{\partial \ln (U)} = 1 + \frac{\partial \ln m_{j}}{\partial \ln (E/P)} = 1 + (1 - \sigma) \left\{ \varepsilon_{j} - \sum_{k=1}^{3} m_{k} \varepsilon_{k} \right\} \]
Structural Change with the Engel (Income) Effect: Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.

Then, even with constant relative prices,

**Decline of Agriculture:** $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1} \left( \frac{p_2}{p_1} U(t)^{\varepsilon_2-\varepsilon_1} \right)^{1-\sigma} + \frac{\beta_3}{\beta_1} \left( \frac{p_3}{p_1} U(t)^{\varepsilon_3-\varepsilon_1} \right)^{1-\sigma}$$

**Rise of Services:** $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3} \left( \frac{p_1}{p_3} U(t)^{\varepsilon_1-\varepsilon_3} \right)^{1-\sigma} + \frac{\beta_2}{\beta_3} \left( \frac{p_2}{p_3} U(t)^{\varepsilon_2-\varepsilon_3} \right)^{1-\sigma}$$

**Rise and Fall of Manufacturing:** $s_2(t) = m_2(t)$ is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2} \left( \frac{p_1}{p_2} U(t)^{\varepsilon_1-\varepsilon_2} \right)^{1-\sigma} + \frac{\beta_3}{\beta_2} \left( \frac{p_3}{p_2} U(t)^{\varepsilon_3-\varepsilon_2} \right)^{1-\sigma}$$

Hump-shaped due to the two opposing forces: $\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

$$s_2'(t) = m_2'(t) \geq 0 \iff (\varepsilon_2 - \varepsilon_1) \frac{m_1(t)}{m_2(t)} \geq (\varepsilon_3 - \varepsilon_2) \frac{m_3(t)}{m_2(t)} \iff \eta_2 \geq 1$$

Initially, when A is large & S is small, the former effect is stronger. Over time, A shrinks & S expands, and eventually, the latter effect becomes stronger.
(Analytically Solvable) Case: \(0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}}\), where \(\bar{g} \equiv \frac{g_1 + g_2 + g_3}{3}\)

**Proposition 3 (Impact of Adding the Engel Effect on top of the Baumol Effect)**

**Peak Time**
\[
\hat{t}(\lambda; \mu) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda - \mu \ln \tilde{U}(\lambda; \mu) = \hat{t}(\lambda; 0) - \frac{\mu}{1 + \mu \bar{g}} \ln \tilde{U}(\lambda; 0)
\]

**Peak M-Share**
\[
\frac{1}{\tilde{s}_2(\lambda; \mu)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(g_1 - g_2)\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} \lambda - \hat{t}(\lambda; 0)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(g_2 - g_3)\left(\tilde{t}(\lambda; 0) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\right)} = \frac{1}{\tilde{s}_2(\lambda; 0)}
\]

**Peak Time Per Capita Income**
\[
\tilde{U}(\lambda; \mu) = \left\{\sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k [\tilde{t}(\lambda; 0) - \theta_k \lambda]}\right\} \frac{1}{a(1 + \mu \bar{g})} = \tilde{U}(\lambda; 0)\left(\frac{1}{1 + \mu \bar{g}}\right)
\]

- \(\tilde{s}_2'(\lambda; \mu) < 0; \tilde{U}'(\lambda; \mu) < 0\) under the same condition; \(\hat{t}'(\lambda; \mu) > 0\) under a weaker condition.

- Fixing \(g_1, g_2, g_3\), a higher \(\mu\) has
  - **No effect** on the peak values of the frontier country, \(\hat{t}(0; \mu), \tilde{s}_2(0; \mu), \tilde{U}(0; \mu)\).
  - **A further delay** in \(\hat{t}(\lambda; \mu)\) for every country with \(\lambda > 0\).
  - **No effect** on \(\tilde{s}_2(\lambda; \mu)\) for every country with \(\lambda > 0\).
  - **A smaller decline** in \(\tilde{U}(\lambda; \mu)\) for each country with \(\lambda > 0\).
Analytically Solvable Case: A Numerical Illustration

\[ g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%, \theta = 0.5, \alpha = 6/13; \bar{\beta}_j = 1/3 \text{ for } j = 1,2,3. \]

In this case, \( g_1 - g_2 = g_2 - g_3 = \bar{g} = 1.2\% > 0 \implies \epsilon_1 = 1 - \epsilon < \epsilon_2 = 1 < \epsilon_3 = 1 + \epsilon \text{ for } 0 < \epsilon = (1.2\%)\mu < 1 \)
Stronger nonhomotheticity changes the shape of the time paths significantly. It does not change the implications on PD, i.e., how technology gaps affect $\hat{t}$, $s_2(\hat{t})$, and $U(\hat{t})$.

<table>
<thead>
<tr>
<th></th>
<th>Homothetic ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</th>
<th>Nonhomothetic ($\varepsilon_1 = .4 &lt; \varepsilon_2 = 1 &lt; \varepsilon_3 = 1.6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(t, s_2(t))$</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>$(\ln U(t), s_2(t))$</td>
<td><img src="image3.png" alt="Graph" /></td>
<td><img src="image4.png" alt="Graph" /></td>
</tr>
</tbody>
</table>
What happens if we had *solely* the Engel effect with $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$, without the Baumol effect, $g_1 = g_2 = g_3 = \bar{g} > 0$?

Under the two normalizations

$$
\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2}\right) \frac{\tilde{\beta}_1}{\tilde{\beta}_3} = 1; \quad \tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1
$$

which ensures $\bar{U}(0) = 1$ and $\hat{\lambda}(0) = 0$.

**Proposition 4: Peak Values under the Engel (Income) Effect only**

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>$\tilde{\tan}(\lambda) = \frac{1}{a \bar{g}} \ln \left{ \sum_{k=1}^{3} \tilde{\beta}_k e^{a(\theta_k \bar{g} \lambda + \varepsilon_k \ln \bar{U}(\lambda))} \right}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak M-Share</td>
<td>$\frac{1}{\bar{s}_2(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\bar{\beta}_2}\right) e^{a(\varepsilon_2 - \varepsilon_1)(-\frac{\theta_2 - \theta_1}{\varepsilon_2 - \varepsilon_1}\bar{g} \lambda - \ln \bar{U}(\lambda))} + \left(\frac{\tilde{\beta}_3}{\bar{\beta}_2}\right) e^{a(\varepsilon_3 - \varepsilon_2)(\ln \bar{U}(\lambda) - \frac{\theta_2 - \theta_3}{\varepsilon_3 - \varepsilon_2}\bar{g} \lambda)}$</td>
</tr>
<tr>
<td>Peak Time Per Capita Income</td>
<td>$\ln \bar{U}(\lambda) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} \bar{g} \lambda$</td>
</tr>
</tbody>
</table>
Proposition 5: Conditions for PD with the Engel (Income) Effect Only

\[ \hat{U}'(\lambda) < 0 \text{ for all } \lambda > 0 \iff 0 < \frac{\theta_1}{\theta_3} < 1 \]

With a low \( \theta_1 \) and a high \( \theta_3 \), the price of the income elastic S is high relative to the income inelastic A in a high-\( \lambda \) country, which make it necessary to reallocate labor to S at earlier stage of development.

\[ S_2'(\lambda) < 0 \text{ for all } \lambda > 0 \iff \left( 1 - \frac{\varepsilon_2}{\varepsilon_3} \right) \left( \frac{\theta_1}{\theta_3} - 1 \right) + \left( 1 - \frac{\varepsilon_1}{\varepsilon_3} \right) \left( 1 - \frac{\theta_2}{\theta_3} \right) > 0. \]

With a low \( \theta_2 \), which has no effect on \( \hat{U}(\lambda) \), the price of M is low relative to both A & S in a high-\( \lambda \) country, which keeps the M-share low.

Under the above condition,

\[ \hat{t}'(\lambda) > 0 \text{ for a sufficiently large } \lambda \iff \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3} \]

\[ \hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \iff \left( \theta_E - \frac{\varepsilon_1}{\varepsilon_3} \right) \left[ 1 - \left( \frac{\varepsilon_3}{\varepsilon_2} \frac{\theta_2}{\theta_3} \right) \right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3} \]

where \( \varepsilon_1/\varepsilon_3 < \Theta_E < 1 \).

With \( g_1 = g_2 = g_3 = \bar{g} \), PD occurs only if \( \theta_1 \bar{g}, \theta_2 \bar{g} < \theta_3 \bar{g} \), that is, when cross-country productivity difference is the largest in S.
2<sup>nd</sup> Extension: Introducing International Trade
One Implication of PD (consistent with the empirical evidence):
\[
\frac{\partial}{\partial \lambda} \ln \left( \frac{A_1(t)}{A_2(t)} \right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.
\]

- A low-\(\lambda\) country has comparative advantage in A and a high-\(\lambda\) country has comparative advantage in M.
- Opening up trade in A and in M would weaken PD by allowing high-\(\lambda\) country to export M.
- Consistent with the findings that East Asia “suffers” less from PD.

A Two-Country Technology Gap Model of PD: \(\lambda^1 < \lambda^2\) (Superscript indicates the country)

**Trade Cost:** Only \(e^{-\tau_1} < 1\) fraction of A and only \(e^{-\tau_2} < 1\) fraction of M shipped arrive to the destination.

\[
m_j^c = \beta_j \left( \frac{p_j^c}{P^c} \right)^{1-\sigma} ; \quad P^c = \left[ \sum_{k=1}^{3} \beta_k (p_k^c)^{1-\sigma} \right]^{1/(1-\sigma)} \quad \& \quad s_j^c = (A_j^c)^{\frac{1}{\alpha}} \left( \frac{p_j^c}{E^c} \right)^{\frac{1}{\alpha}} \quad E^c = \left[ \sum_{k=1}^{3} (A_k^c)^{\frac{1}{\alpha}} (p_k^c)^{\frac{1}{\alpha}} \right]^\alpha
\]

With \(g_1 \theta_1 > g_2 \theta_2\), Leader (Country-1) has CA in A and Laggard (Country-2) has CA in M.

1 may export A to 2: \(e^{\tau_1} p_1^1 \geq p_2^1; \quad e^{-\tau_1} [A_1^1(s_1^1)^{1-\alpha} - c_1^1]L^1 = [c_1^2 - A_1^2(s_2^2)^{1-\alpha}]L^2 \geq 0. \rightarrow [s_1^1 - m_1^1]E^1 L^1 = [m_2^1 - s_2^1]E^2 L^2 \geq 0.\)

2 may export M to 1: \(p_2^1 \leq e^{\tau_2} p_2^2; \quad [c_2^1 - A_2^1(s_2^1)^{1-\alpha}]L^1 = e^{-\tau_2} [A_2^2(s_2^2)^{1-\alpha} - c_2^2]L^2 \geq 0. \rightarrow [m_2^1 - s_2^1]E^1 L^1 = [s_2^2 - m_2^2]E^2 L^2 \geq 0.\)

S is nontradeable:

\[
p_3^1 \neq p_3^2; \quad c_3^1 = A_3^1(s_3^1)^{1-\alpha}; \quad c_3^2 = A_3^2(s_3^2)^{1-\alpha} \rightarrow m_3^1 = s_3^1; \quad m_3^2 = s_3^2.
\]
Condition for No Trade Equilibrium:

\[ e^{\tau_1 + \tau_2} > \frac{p_1^2(t)}{p_1^1(t)} \frac{p_2^2(t)}{p_2^1(t)} = \left[ \frac{A_2^1(t)}{A_2^2(t)} \right]^{-\frac{\alpha}{1-\sigma}} = e^{\frac{a(g_1 \theta_1 - g_2 \theta_2)}{(1-\sigma)}(\lambda^2 - \lambda^1)} \]

\[ \iff \tau_1 + \tau_2 > T_+ \equiv \frac{a(g_1 \theta_1 - g_2 \theta_2)}{(1-\sigma)}(\lambda^2 - \lambda^1) > 0. \]

Trade Equilibrium under

\[ 0 < \tau_1 + \tau_2 \leq T_+ \equiv \frac{a(g_1 \theta_1 - g_2 \theta_2)}{(1-\sigma)}(\lambda^2 - \lambda^1). \]

Then, 1 exports A to 2 and imports M from 2.

Equilibrium Conditions:

\[ s_1^1 + s_2^1 = m_1^1 + m_2^1, \quad s_1^2 + s_2^2 = m_1^2 + m_2^2 \]
\[ [s_1^1 - m_1^1] E^1 L^1 = [s_2^2 - m_2^2] E^2 L^2 > 0 \]
\[ e^{\tau_1} p_1^1 = p_1^2; \quad p_2^1 = e^{\tau_2} p_2^2 \]
Impact of International Trade (Numerical Simulation): \( L_1/L_2 = 1 \).

\[
0 < \tau \equiv \frac{\tau_1 + \tau_2}{T^*} \equiv \frac{(1 - \sigma)(\tau_1 + \tau_2)}{a(g_1 \theta_1 - g_2 \theta_2)(\lambda^2 - \lambda^1)} < 1
\]

\[
\Rightarrow 1 < \frac{p^1}{p^1 p^2} = e^{\tau_1 + \tau_2} = e^{\tau T^*} \leq e^{T^*}.
\]

We plot how the peak values change in response to \( \tau \).

In all four cases, our mechanism for PD is:
- Robust to introducing international trade.
- Weaker in that the differences btw the leader and the laggard in \( \hat{t} \) and \( \hat{s}_2 \) become smaller (larger in \( \hat{U} \) in \( \hat{m}_2 \)), as \( \tau \) declines. For a sufficiently small \( \tau \), the reversal occurs in \( \hat{t} \) and \( \hat{s}_2 \).

PD holds, when the trade cost accounts for more than about 1/3 of the imported goods prices, empirically plausible.

Under our mechanism, PD occurs not because of international trade but in spite of international trade.
Duarte-Restuccia productivity growth rates;
Employment Shares

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>Peak M-Share</th>
<th>Peak Time Per Capita Income</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
</tbody>
</table>

Reversal of $\hat{t}$ at $\tau \approx 0.85$, or $e^{\tau_{1}^{+}\tau_{2}} = e^{T_{1}^{+}} \approx 1.986 \rightarrow \sqrt{1.986} \approx 1.41$ times higher in the importing country.

Reversal of $\hat{s}_{2}$ at $\tau \approx 0.91$ or $e^{\tau_{1}^{+}\tau_{2}} = e^{T_{1}^{+}} \approx 2.242 \rightarrow \sqrt{2.242} \approx 1.497$ times higher in the importing country.
Duarte-Restuccia productivity growth rates;
Value-Added Shares

<table>
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<tr>
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<th>Peak Time Per Capita Income</th>
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<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
<td><img src="image3.png" alt="Graph 3" /></td>
</tr>
</tbody>
</table>

Reversal of \( \hat{\tau} \) at \( \tau \approx 0.87 \) or \( e^{\tau_1+\tau_2} \approx 2.185 \to \sqrt{2.185} \approx 1.478 \) times higher in the importing country.
Reversal of \( \hat{s}_2 \) at \( \tau \approx 0.90 \) or \( e^{\tau_1+\tau_2} \approx 2.244 \to \sqrt{2.244} \approx 1.498 \) times higher in the importing country.
Comin-Lashkari-Mestieri productivity growth rates; Employment Shares

<table>
<thead>
<tr>
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<th>Peak M-Share</th>
<th>Peak Time Per Capita Income</th>
</tr>
</thead>
</table>

Reversal of \( \hat{\tau} \) at \( \tau \approx 0.96 \), or \( e^{\tau_1 + \tau_2} \approx 2.295 \rightarrow \sqrt{2.295} \approx 1.515 \) times higher in the importing country.

Reversal of \( \hat{\delta}_2 \) at \( \tau \approx 0.77 \) or \( e^{\tau_1 + \tau_2} \approx 1.947 \rightarrow \sqrt{1.947} \approx 1.395 \) times higher in the importing country.
Comin-Lashkari-Mestieri productivity growth rates; Value-Added Shares

Reversal of $\hat{\tau}$ at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.504 \rightarrow \sqrt{2.504} \approx 1.582$ times higher in the importing country.

Reversal of $\xi_2$ at $\tau \approx 0.76$ or $e^{\tau_1+\tau_2} \approx 2.068 \rightarrow \sqrt{2.068} \approx 1.438$ times higher in the importing country.
3rd Extension: Introducing Catching Up
Narrowing a Technology Gap

We assumed that $\lambda$ is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.
[In contrast, the aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t)$, declines over time, $g_U'(t) = g_1 s_1'(t) + g_2 s_2'(t) + g_3 s_3'(t) = (g_1 - g_2)s_1'(t) + (g_3 - g_2)s_3'(t) < 0$, the so-called Baumol’s cost disease.]

What if technological laggards can narrow a technology gap, and hence achieve a higher productivity growth in each sector?

Countries differ only in the initial value of lambda, $\lambda_0$, converging exponentially over time at the same rate,

$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_\lambda t}, \quad g_\lambda > 0.$$

$$\Rightarrow \frac{1}{s_2(t)} = \left(\frac{\bar{\beta}_1}{\bar{\beta}_2}\right)e^{a[(\theta_1 g_1 - \theta_2 g_2)\lambda_t - (g_1 - g_2)t]} + 1 + \left(\frac{\bar{\beta}_3}{\bar{\beta}_2}\right)e^{a[(\theta_3 g_3 - \theta_2 g_2)\lambda_t + (g_2 - g_3)t]}$$
With double exponentials, we are able to solve the peak values only numerically.

Technological laggards
- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless $g_A$ is too large: Comin-Mestieri (2018)
Concluding Remarks
A Parsimonious model of Rodrik’s (2016) PD based on
- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

We find that PD occurs for
- cross-country productivity difference larger in A than in S.
- technology adoption takes not too long in M.
- Technology adoption takes longer in S than in A.
which implies that cross-country productivity difference the largest in A.

The baseline model assumes **homothetic CES, no international trade, no catching up**.

In three extensions, we showed that the results are **robust** against introducing
- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)
  The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD.
  The Engel effect *alone* could not generate PD w/o counterfactual implications on cross-country productivity differences
- **International trade in A and in M**, but PD becomes weaker.
- **Narrowing a technology gap** to allow technological laggards to catch up,
  unless the catching-up speed is too large.