

A Technology-Gap Model of ‘Premature’ Deindustrialization

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Teaching Slides

Introduction

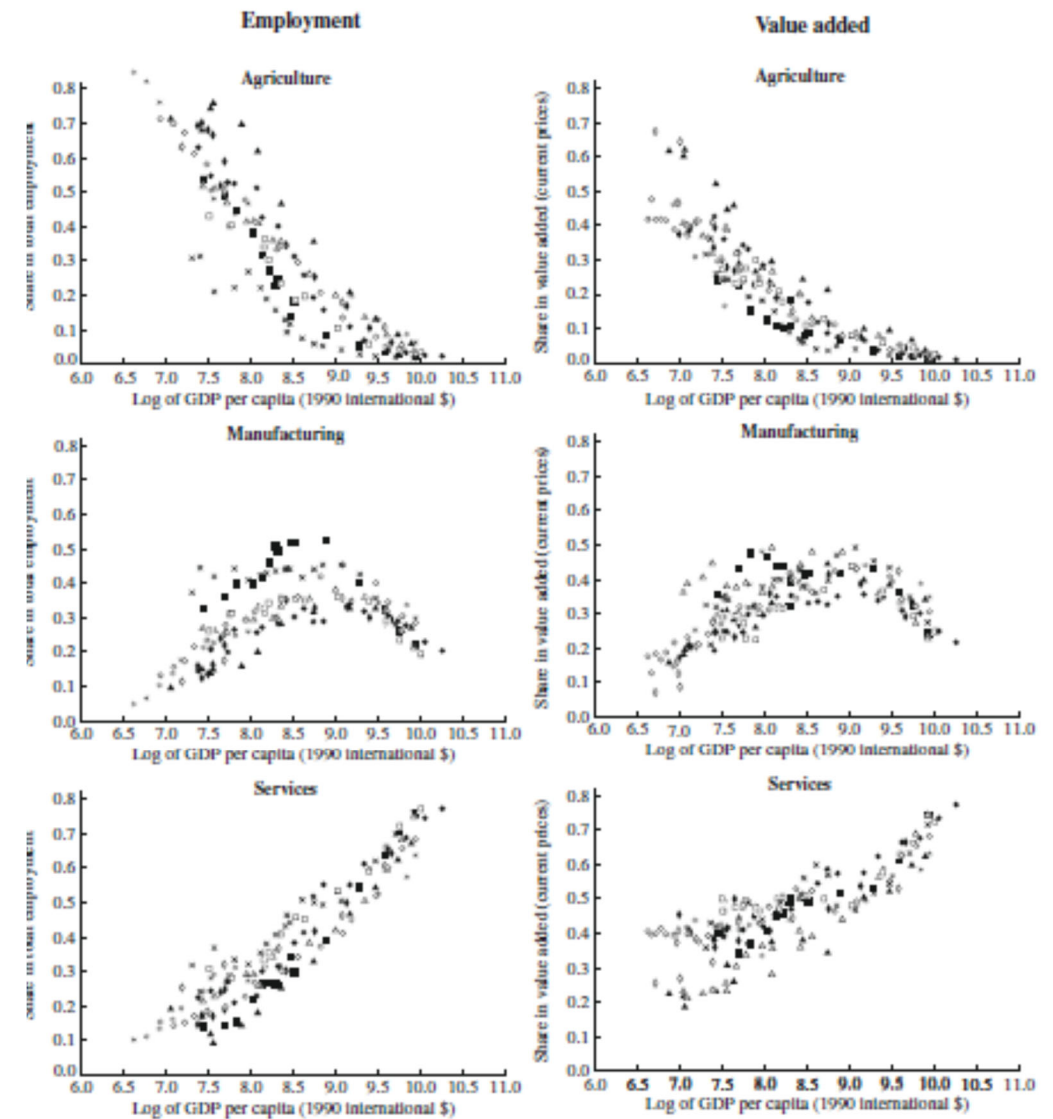
Structural Change

As per capita income rises, the employment or value-added shares

- *Fall* in Agriculture
- *Rise* in Services
- *Rise and Fall* in Manufacturing

From Herrendorf-Rogerson-Valentinyi (2014)

Evidence from Long Time Series for the Currently Rich Countries (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000



Premature Deindustrialization (PD): Rodrik (JEG 2016)

Late industrializers reach their M-peak and start deindustrializing

- *Later* in time
- *Earlier* in per capita income
- with the *lower* peak M-sector shares, compared to early industrializers.

Rodrik (2016) focuses on documenting the patterns, without offering a causal explanation or making normative statements. But

- He speculates that globalization may be a cause.
- He cautions against drawing policy implications, but the word, “premature,” may seem to suggest some types of inefficiency.

In our proposed mechanism,

- PD occurs in the *efficient* equilibrium of a *closed* economy.
- PD is robust to opening up for trade but weakened.

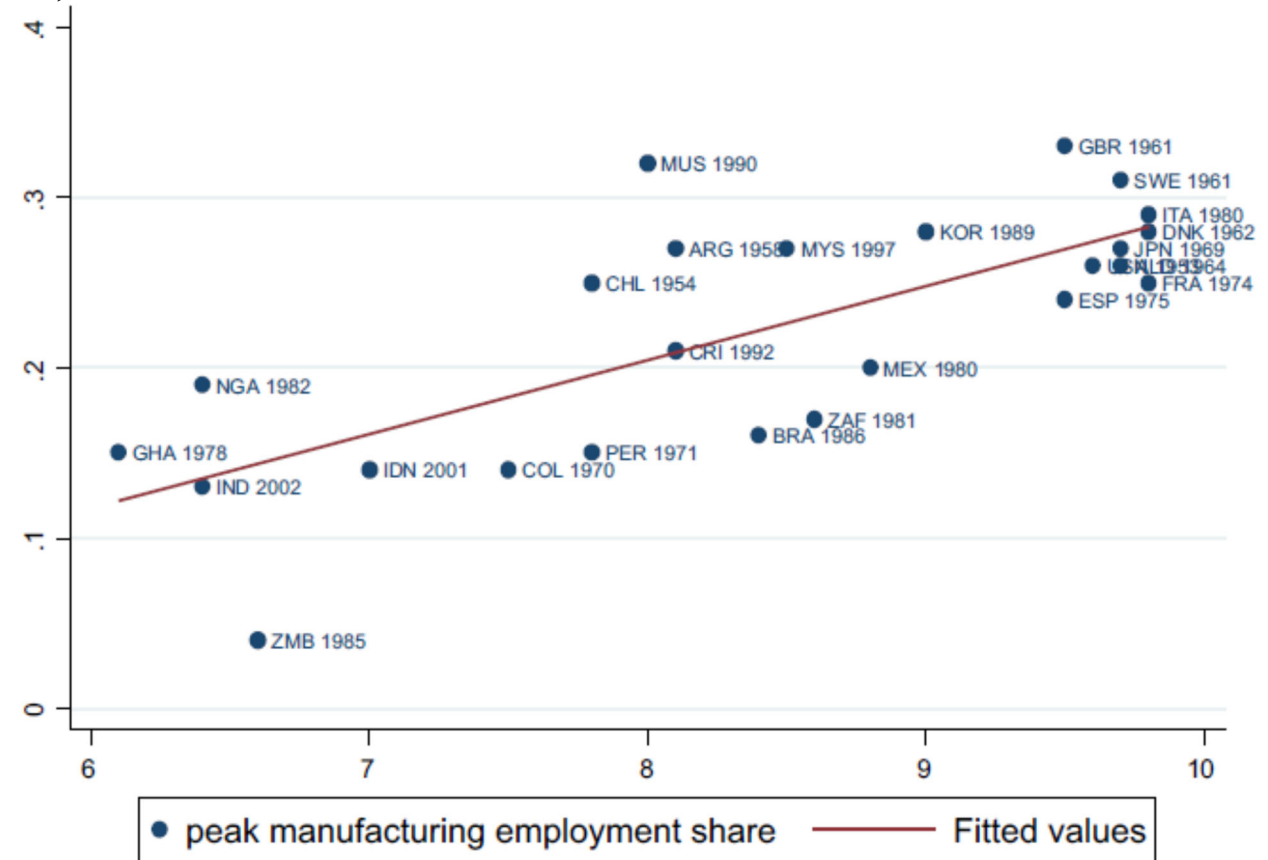


Fig. 5 Income at which manufacturing employment peaks (logs)

This Paper: A Parsimonious Model of Premature Deindustrialization (PD)

In the baseline model,

3 Goods/Sectors: 1=(A)griculture, 2=(M)anufacturing, 3=(S)ervices, *homothetic CES with gross complements* ($\sigma < 1$)

Frontier Technology: $\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$, with $g_1 > g_2 > g_3 > 0 \Rightarrow$ a decline of A, a rise of S, and a hump-shaped of M in each country through the **Baumol (relative price) effect**, as in Ngai-Pissarides (2007)

Countries differ in Actual Technology Used: $A_j(t) = \bar{A}_j(t - \lambda_j)$ due to **Adoption Lags**, $(\lambda_1, \lambda_2, \lambda_3)$.

$$A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{-g_j \lambda_j} e^{g_j t} \Rightarrow \frac{\partial}{\partial \lambda_j} \ln(A_j(t)) = -g_j < 0$$

λ_j has **no “growth” effect, but negative “level” effects**, $e^{-\lambda_j g_j}$, amplified by g_j .

Log-submodularity: g_j magnifies the (negative) impact of the adoption lag on productivity: $\frac{\partial}{\partial g_j} \left(\frac{\partial}{\partial \lambda_j} \ln e^{-g_j \lambda_j} \right) < 0$

We focus on 1-dimension of cross-country heterogeneity: $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$,

- $\lambda \geq 0$, **Technology Gap, country-specific**, as in Krugman (1985); their ability to adopt the frontier technologies.
- $\theta_j > 0$: **sector-specific**, unlike Krugman (1985); how much λ affects the adoption lag and productivity in each sector.

$$A_j(t) = \bar{A}_j(0)e^{-g_j \theta_j \lambda} e^{g_j t} \Rightarrow \frac{\partial}{\partial \lambda} \ln \left(\frac{A_j(t)}{A_k(t)} \right) = -(\theta_j g_j - \theta_k g_k).$$

Main Results: Conditions for PD, defined as “A high- λ country reaches its peak later in time, with lower peak M-share at lower per capita income at its M-peak time.”

i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S.

High relative price of A/low relative price of S in a high- λ country causes a delay.

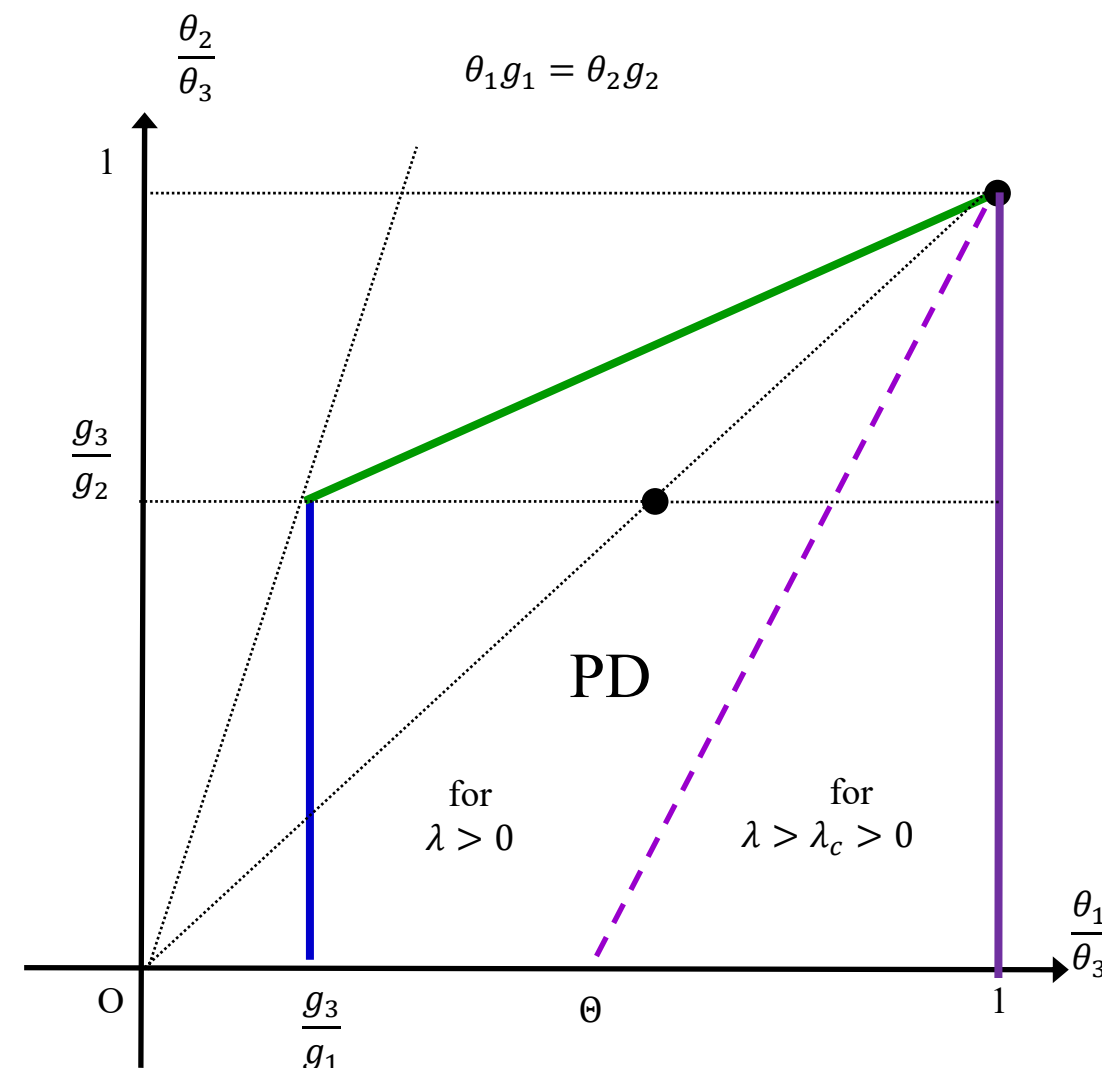
ii) $\left(1 - \frac{g_3}{g_1}\right) \left(\frac{\theta_2}{\theta_3} - 1\right) + \left(1 - \frac{g_3}{g_2}\right) \left(1 - \frac{\theta_1}{\theta_3}\right) < 0$:

Technology adoption takes not too long in M.

Not too high relative price of M in a high- λ country keeps the M-share low.

Under the above conditions,

iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A. Longer adoption lag in S in a high- λ country causes “premature” deindustrialization.

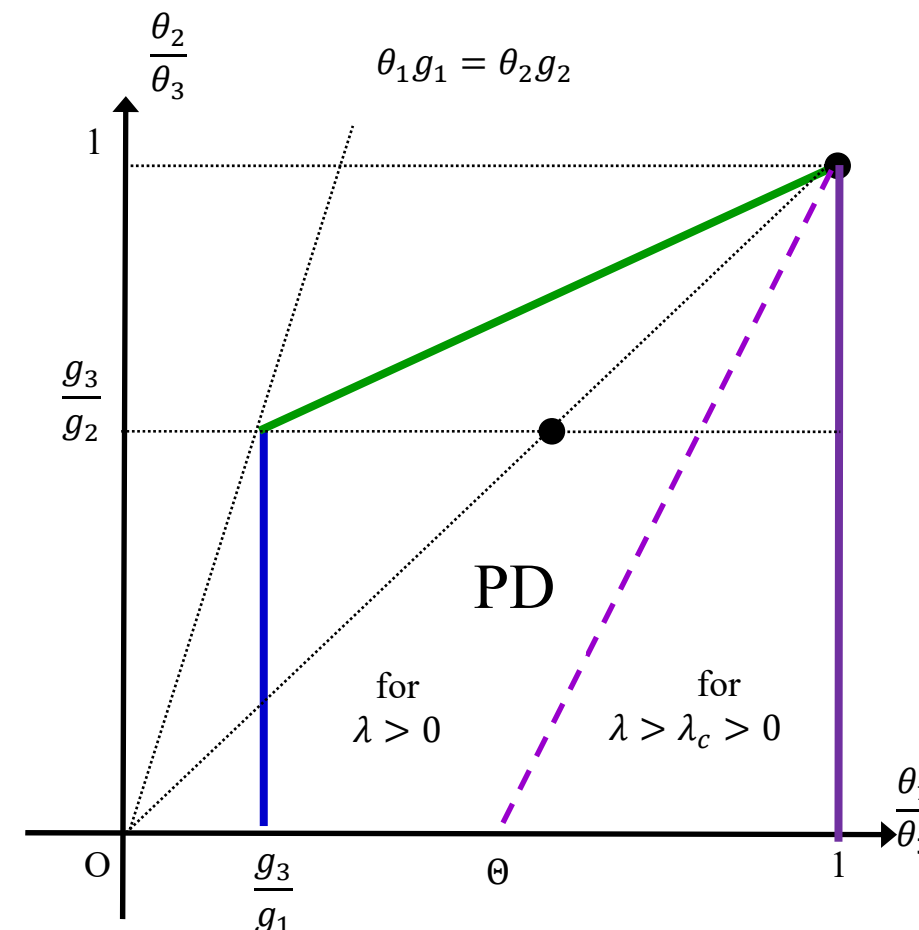


Notes:

- ✓ No PD if $\theta_1 = \theta_2 = \theta_3$. Latecomers would follow the same path with a delay.
- ✓ Conditions i) & ii) $\Rightarrow \theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$.
Cross-country productivity difference is the largest in A.
 $\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative.
negative when calibrated to match Rodrik’s (2016, Table10) findings.

In sum, PD occurs because a high- λ country is:

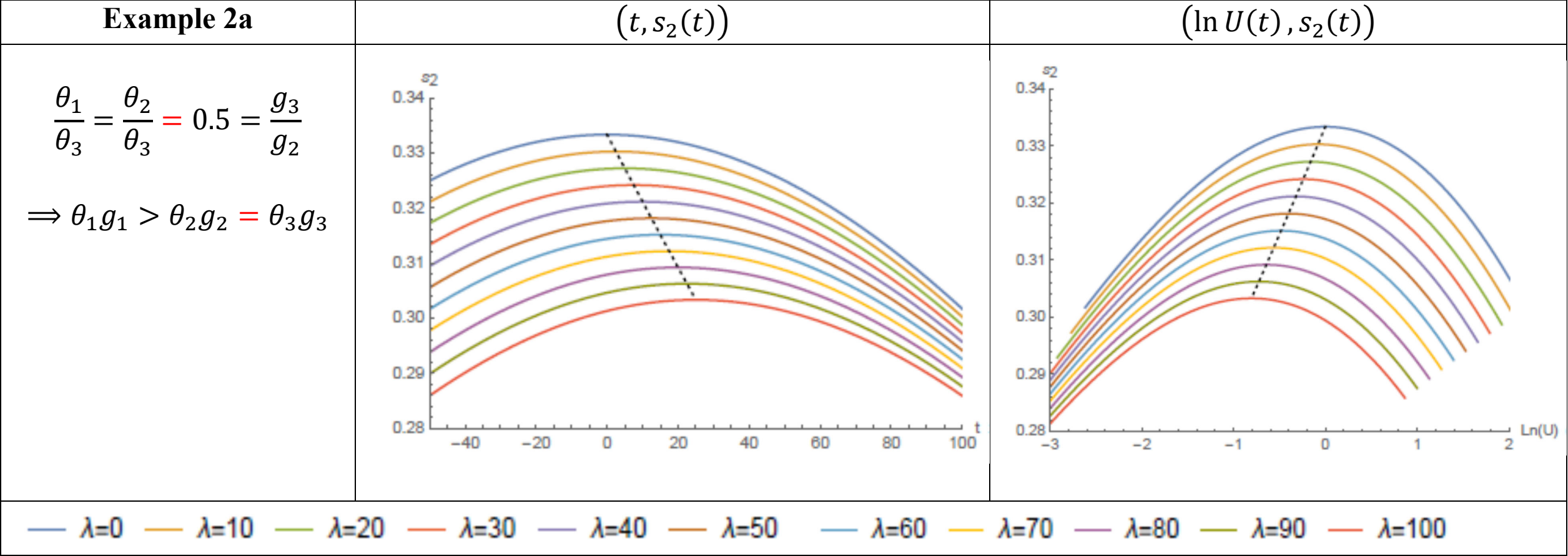
- i) The worse in A than in S, which causes it to peak later in time.
- ii) Its adoption takes the longest in S, which causes it to deindustrialize prematurely.
- iii) Doing ok in M, which explains why its peak M-share is lower.



PD should not be viewed as the prima-facie evidence that the M-sectors in developing countries are doing badly. On the contrary, they are doing fine, relatively to their A and S-sectors, according to this model.

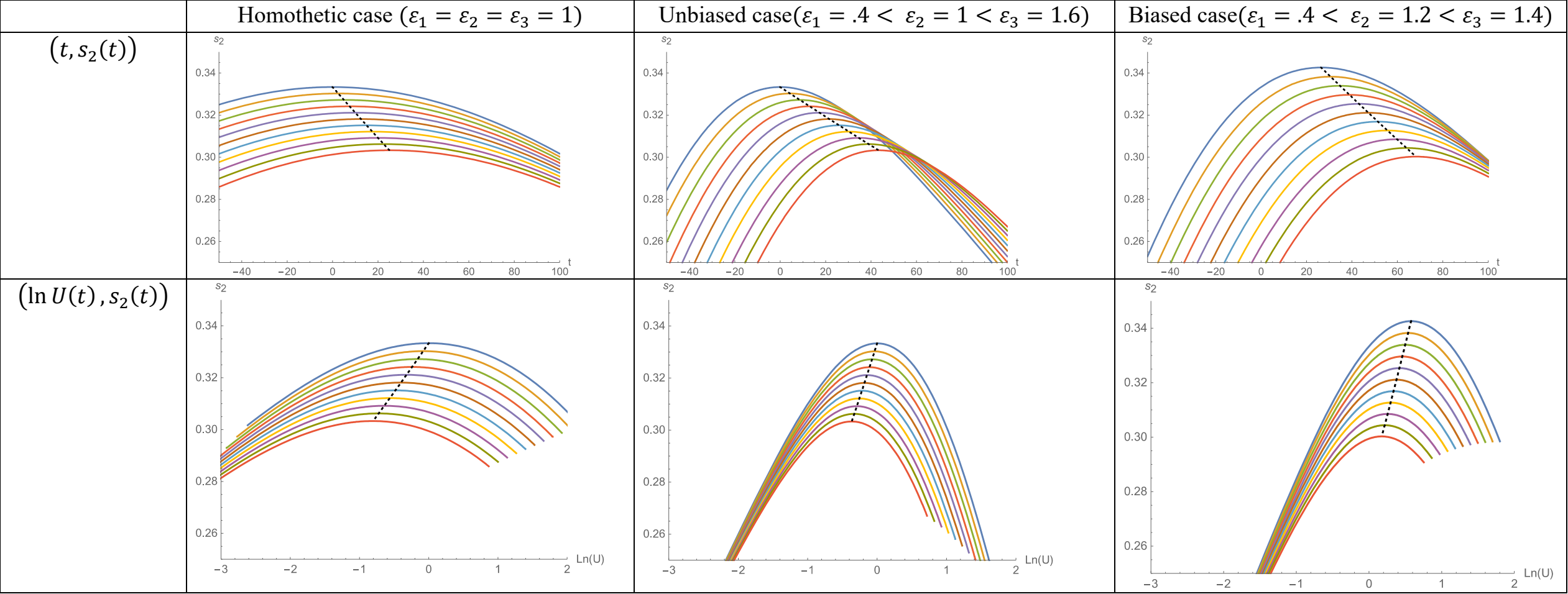
A Numerical Illustration.

$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\sigma = 0.6$; Labor share = $2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.



1st Extension: Adding the Engel Effect with Nonhomothetic CES (a la Comin-Lashkari-Mestieri)

Nonhomotheticity changes the shape of trajectories greatly, but not on how technology gaps, λ , affects the peak values.



We also show that the Engel effect *alone* could not generate PD *without counterfactual implications*.

2nd Extension: International Trade

One implication of our mechanism for PD (consistent with the empirical evidence):

$$\frac{\partial}{\partial \lambda} \ln \left(\frac{A_1(t)}{A_2(t)} \right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.$$

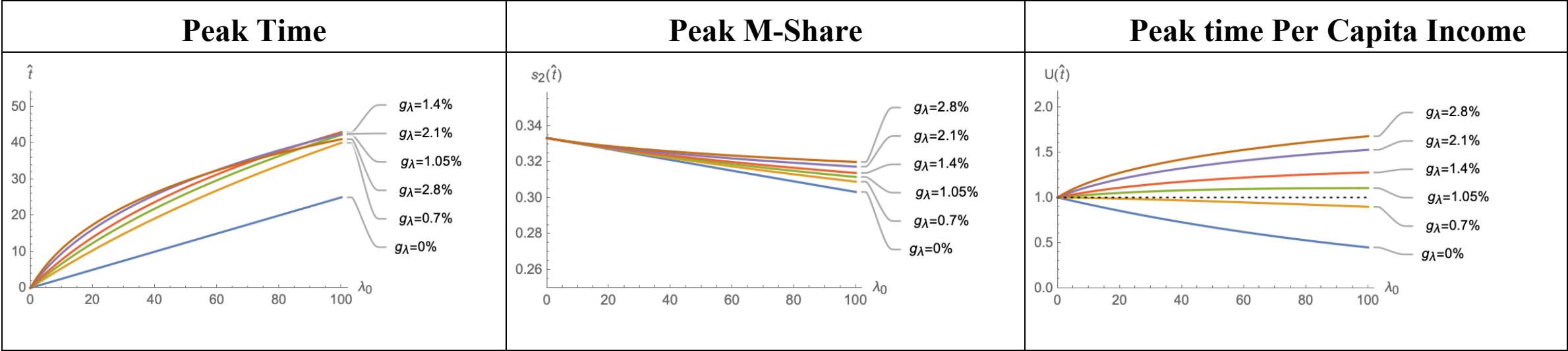
- A low- λ country has comparative advantage in A and a high- λ country has comparative advantage in M.
- Opening up for trade allows a high- λ country to export M to a low- λ country.
- Our mechanism for PD is weakened by opening up for trade, but PD continues to hold, as long as the trade cost is not too small.
- Consistent with the findings that East Asia “suffers” less from PD (Rodrik 2016).

Under our mechanism, PD occurs *not because of*, but *in spite of* international trade.

3rd Extension: Introducing Catching-up

$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)},$ where $\lambda_t = \lambda_0e^{-g_\lambda t},$

Countries differ only in the *initial* value, λ_0 , converging exponentially over time at the same rate, $g_\lambda > 0$



Higher- λ countries

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large.

(Very Selective) Literature Review. Herrendorf-Rogerson-Valentinyi (14) for a survey on structural change.

Related to The Baseline Model

Premature Deindustrialization, Dasgupta-Singh (06), Palma (14), **Rodrik (16)**

The Baumol Effect: Baumol (67), **Ngai-Pissarides (07)**, Nordhaus (08)

Cross-country heterogeneity in technology development

- *Distance to the frontier*: **Krugman** (85), Acemolgu-Aghion-Zilibotti (06)
- *Log-supermodularity*: **Krugman** (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- *Productivity difference across countries the largest in A*: Caselli (05), Gollin et.al. (14, AERP&P)
- *Small adoption lags in M*; Rodrik (2013)

Related to Three Extensions

The Engel Effect (Nonhomotheticity); Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zweimueller (08), Buera-Kaboski (09,12), Boppart (14), **Comin-Lashkari-Mestieri (21)**, Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)

Open Economy Implications: Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (WinP)

Catching-Up/Technology Diffusion: Acemoglu (08), Comin-Mestieri (18)

The Issues We Abstract From

Sector-level productivity growth rate differences across countries: Huneus-Rogerson (20)

Endogenous growth, externalities, Matsuyama (92).

Sectoral wedges/misallocation: Caselli (05), Gollin et.al. (14 QJE) and many others

Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.

Structural Change, the Baumol Effect, and Adoption Lags

Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = (A)griculture, Sector-2 = (M)anufacturing, Sector-3 = (S)ervices.

Demand System: L Identical HH, each endowed with 1 unit of mobile labor, earning the wage w & κ_j units of managerial skills, specific to j , each earning the rent, ρ_j .

Budget Constraint:

$$\sum_{j=1}^3 p_j c_j \leq E \equiv w + \sum_{j=1}^3 \rho_j \kappa_j = \frac{1}{L} \sum_{j=1}^3 p_j Y_j$$

CES Preferences:

$$U(c_1, c_2, c_3) = \left[\sum_{j=1}^3 (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

Expenditure Shares:

$$m_j \equiv \frac{p_j c_j}{E} = \beta_j \left(\frac{p_j}{P} \right)^{1-\sigma}; \quad P = \left[\sum_{k=1}^3 \beta_k (p_k)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}$$

Real Per Capita Income

$$U = \frac{E}{P} = \left[\sum_{k=1}^3 \beta_k \left(\frac{E}{p_k} \right)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}.$$

Three Competitive Sectors: Production

Cobb-Douglas

$$Y_j = \tilde{A}_j (\kappa_j L)^\alpha (L_j)^{1-\alpha} = A_j (L)^\alpha (L_j)^{1-\alpha} = L A_j (s_j)^{1-\alpha}$$

where $A_j \equiv \tilde{A}_j (\kappa_j)^\alpha$. $\alpha \in [0,1)$: the span of control parameter, which introduces diminishing returns in labor.

Labor Share

$$\frac{wL_j}{p_j Y_j} = 1 - \alpha$$

Profit (Managerial Rent) Share

$$\frac{\rho_j \kappa_j L}{p_j Y_j} = \alpha$$

Sectoral Share in Employment

$$s_j \equiv \frac{L_j}{L}; \quad \sum_{j=1}^3 s_j = 1$$

Sectoral Sector in Value-Added

$$\frac{p_j Y_j}{EL} = \frac{p_j Y_j}{\sum_{k=1}^3 p_k Y_k}$$

$$\frac{p_j Y_j}{EL} = s_j = \left(\frac{p_j A_j}{E} \right)^{1/\alpha}; \quad E = \left[\sum_{k=1}^3 (p_k A_k)^{\frac{1}{\alpha}} \right]^\alpha.$$

Equilibrium: The expenditure shares are equal to the employment and value-added shares.

$$\beta_j \left(\frac{p_j}{P} \right)^{1-\sigma} = m_j = \frac{p_j Y_j}{EL} = s_j = \left(\frac{p_j A_j}{E} \right)^{1/\alpha}$$

which lead to

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j \right]^{-a}}{\sum_{k=1}^3 \left[\beta_k^{\frac{1}{\sigma-1}} A_k \right]^{-a}}$$

Per Capita Income

$$U = \left\{ \sum_{k=1}^3 \left[\beta_k^{\frac{1}{\sigma-1}} A_k \right]^{-a} \right\}^{-\frac{1}{a}}$$

where

$$a \equiv \frac{1-\sigma}{1-\alpha(1-\sigma)} = -\frac{\partial \log(s_j/s_k)}{\partial \log(A_j/A_k)} > 0,$$

which captures how much relatively *high* productivity in a sector contributes to its relatively *low* equilibrium share. α magnifies this effect by increasing a .

Productivity Growth:

$$A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{g_j(t-\lambda_j)} = \bar{A}_j(0)e^{-\lambda_j g_j} e^{g_j t}$$

$\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$: **Frontier Technology** in j , with a constant **growth rate** $g_j > 0$.

$A_j(t) = \bar{A}_j(t - \lambda_j)$; λ_j = **Adoption Lag** in j .

- λ_j has **no “growth” effect**, but has a **negative “level” effect**, $e^{-\lambda_j g_j}$, which is proportional to g_j .

Key: Log-submodularity, $\frac{\partial}{\partial g_j} \left(\frac{\partial}{\partial \lambda_j} \ln e^{-\lambda_j g_j} \right) < 0$: g_j magnifies the negative effect of the adoption lag on productivity

- ✓ A large adoption lag doesn’t matter much in a sector with slow productivity growth.
- ✓ Even a small adoption lag matters a lot in a sector with fast productivity growth.

$$U(t) = \left\{ \sum_{k=1}^3 \left[\beta_k^{\frac{1}{\sigma-1}} A_k(t) \right]^{-a} \right\}^{-\frac{1}{a}} = \left\{ \sum_{k=1}^3 \tilde{\beta}_k e^{-a g_k(t-\lambda_k)} \right\}^{-\frac{1}{a}}, \text{ where } \tilde{\beta}_k \equiv \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)} \right)^a > 0.$$

Longer adoption lags would shift down the time path of $U(t)$.

$$g_U(t) \equiv \frac{U'(t)}{U(t)} = \sum_{k=1}^3 g_k s_k(t)$$

The growth rate in per capita income is the weighted average of the sectoral growth rates

Relative Prices:
$$\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \left[\left(\frac{\beta_j}{\beta_k}\right)^{-\alpha} \frac{\bar{A}_j(0)}{\bar{A}_k(0)}\right]^{-a} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Rightarrow \frac{d \ln \left(\frac{p_j(t)}{p_k(t)}\right)}{dt} = \frac{a(g_k - g_j)}{1 - \sigma}$$

Relative Growth Effect: $p_j(t)/p_k(t)$ is de(in)creasing over time if $g_j > (<)g_k$. **Speed independent of λ_j and λ_k .**

Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $p_j(t)/p_k(t)$ at any point in time.

Note: For a fixed $\lambda_j > 0$, a higher g_j makes the relative price of j higher (though declining faster).

Relative Shares:
$$\frac{s_j(t)}{s_k(t)} = \frac{\beta_j}{\beta_k} \left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \frac{\tilde{\beta}_j}{\tilde{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Rightarrow \frac{d \ln \left(\frac{s_j(t)}{s_k(t)}\right)}{dt} = a(g_k - g_j)$$

Relative Growth Effect: $s_j(t)/s_k(t)$ is de(in)creasing over time if $g_j > (<)g_k$. **Speed independent of λ_j and λ_k .**

Shift from faster growing sectors to slower growing sectors over time.

Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $s_j(t)/s_k(t)$ at any point in time.

Note: For a fixed $\lambda_j > 0$, a higher g_j makes the relative share of j higher (though declining faster).

Structural Change with the Baumol (Relative Price) Effect: Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing in t , because

$$\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)} \right] e^{a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)} \right] e^{a(g_1 - g_3)t}$$

Rise of Services: $s_3(t)$ is increasing in t , because

$$\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)} \right] e^{-a(g_1 - g_3)t} + \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)} \right] e^{-a(g_2 - g_3)t}$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped in t , because

$$\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)} \right] e^{-a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)} \right] e^{a(g_2 - g_3)t}.$$

Hump-shaped due to the two opposing forces: $g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

$$s'_2(t) \gtrless 0 \Leftrightarrow (g_1 - g_2)s_1(t) \gtrless (g_2 - g_3)s_3(t) \Leftrightarrow g_U(t) = \sum_{k=1}^3 g_k s_k(t) \gtrless g_2$$

Initially, $\frac{s_1(t)}{s_3(t)}$ is large; the 1st force is stronger. As $\frac{s_1(t)}{s_3(t)}$ declines over time, the 2nd force becomes stronger eventually.

Characterizing Manufacturing Peak: “^” indicates the peak.

$$s_2'(\hat{t}) = 0 \Leftrightarrow (g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t}) \Leftrightarrow g_U(\hat{t}) = g_2$$

Peak Time: From $(g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t})$

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \quad \text{where } \hat{t}_0 \equiv \frac{1}{a(g_1 - g_3)} \ln \left[\left(\frac{g_1 - g_2}{g_2 - g_3} \right) \frac{\tilde{\beta}_1}{\tilde{\beta}_3} \right]$$

Two Normalizations: Without any loss of generality,

$$\hat{t}_0 = 0 \Leftrightarrow \frac{g_2 - g_3}{g_1 - g_2} = \frac{\tilde{\beta}_1}{\tilde{\beta}_3} \equiv \left[\left(\frac{\beta_1}{\beta_3} \right)^{\frac{1}{1-\sigma}} \frac{\bar{A}_3(0)}{\bar{A}_1(0)} \right]^a$$

The calendar time is reset so that its M-peak would be reached at $\hat{t} = 0$ in the absence of the adoption lags.

$$U(0) = 1 \text{ for } \lambda_1 = \lambda_2 = \lambda_3 = 0 \Leftrightarrow \sum_{k=1}^3 \tilde{\beta}_k = \sum_{k=1}^3 \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)} \right)^a = 1.$$

We use the peak time per capita income in the absence of the adoption lags as the *numeraire*.

Note: Under these normalizations, the peak time share of sector- k in the absence of the adoption lags would be $\tilde{\beta}_k$.

Then,

Peak Time

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}.$$

Peak M-Share

$$\frac{1}{\hat{s}_2} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2} \right) e^{a(g_1 - g_2) \left(\frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \hat{t} \right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{a(g_2 - g_3) \left(\hat{t} - \frac{\lambda_2 g_2 - \lambda_3 g_3}{g_2 - g_3} \right)}$$

Peak Time Per Capita Income

$$\hat{U} = \left\{ \sum_{k=1}^3 \tilde{\beta}_k e^{-a g_k (\hat{t} - \lambda_k)} \right\}^{-\frac{1}{a}}$$

So far, we have looked at the impacts of adoption lags in a single country in isolation, without specifying the sources of the adoption lags.

If we allow countries to differ in $(\lambda_1, \lambda_2, \lambda_3)$, we can perfectly account for $(\hat{t}, \hat{s}_2, \hat{U})$,

We now restrict ourselves to one-dimension of country heterogeneity, the technology gap, which generate cross-country variations in adoption lags, and study the cross-country implications.

Technology Gaps and Premature Deindustrialization

Consider the world with many countries with

$$(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$$

$\lambda \geq 0$: **Technology Gap, Country-specific**

$\theta_j > 0$: **Sector-specific**, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in 1-dimension**, λ , in their ability to adopt the frontier technologies.
- $\theta_j > 0$ determines how much the technology gap affects the adoption lag in that sector.

$$\frac{A_j(t)}{A_k(t)} = \frac{\bar{A}_j(0)}{\bar{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)t} \Rightarrow \frac{\partial}{\partial \lambda} \ln \left(\frac{A_j(t)}{A_k(t)} \right) = -(\theta_j g_j - \theta_k g_k)$$

Cross-country productivity difference is larger in sector- j than in sector- k if $\theta_j g_j > \theta_k g_k$.

Proposition 1: Peak Values under the Baumol Effect only

Peak Time:

$$\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda.$$

Peak M-Share:

$$\frac{1}{\hat{s}_2(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2} \right) e^{a(g_1 - g_2) \left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} \lambda - \hat{t}(\lambda) \right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{a(g_2 - g_3) \left(\hat{t}(\lambda) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \lambda \right)}$$

Peak Time Per Capita Income:

$$\hat{U}(\lambda) = \left\{ \sum_{k=1}^3 \tilde{\beta}_k e^{-a g_k [\hat{t}(\lambda) - \theta_k \lambda]} \right\}^{-\frac{1}{a}}$$

Proposition 2: Conditions for PD with the Baumol (Relative Price) Effect

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \theta_1 g_1 > \theta_3 g_3.$$

With $\theta_1 g_1 > \theta_3 g_3$, the price of A is relatively higher than the price of S in a high- λ country, which delays the peak.

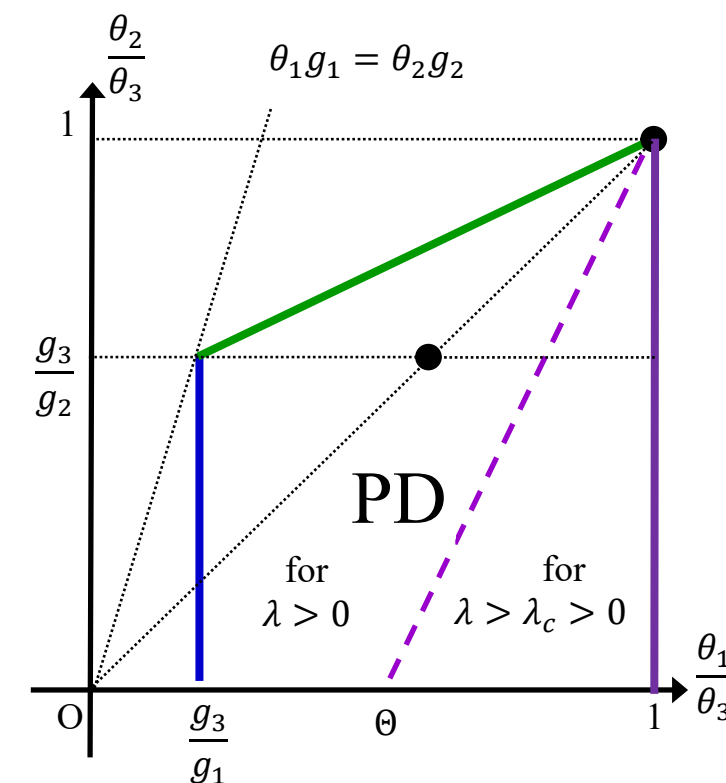
$$\hat{s}_2'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(1 - \frac{g_3}{g_1}\right) \left(\frac{\theta_2}{\theta_3} - 1\right) + \left(1 - \frac{g_3}{g_2}\right) \left(1 - \frac{\theta_1}{\theta_3}\right) < 0$$

With a low θ_2 , which has no effect on \hat{t} , the price of M is low relative to both A & S in a high- λ country, which keeps the M-share low.

Under the above condition,

$$\hat{U}'(\lambda) < 0; \hat{U}(\lambda) < \hat{U}(0) \text{ for } \lambda > \lambda_c \geq 0 \Leftrightarrow \theta_1 < \theta_3 \Leftrightarrow \hat{t}(\lambda) < \theta_1 \lambda < \theta_3 \lambda$$

With $\theta_1 < \theta_3$, the time delay in the peak in a high- λ country is not long enough to make up for the lagging productivity, that is deindustrialization is “premature.”



- $\theta_1 g_1 > \max\{\theta_2 g_2, \theta_3 g_3\}$. (productivity differences the largest in A) .
- $\theta_2 g_2 - \theta_3 g_3$ can be either positive or negative.
- $\max\{\theta_1, \theta_2\} < \theta_3$. (adoption lag the longest in S).

Some Examples

Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

$$\theta_1 = \theta_2 = \theta_3 = 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0$$

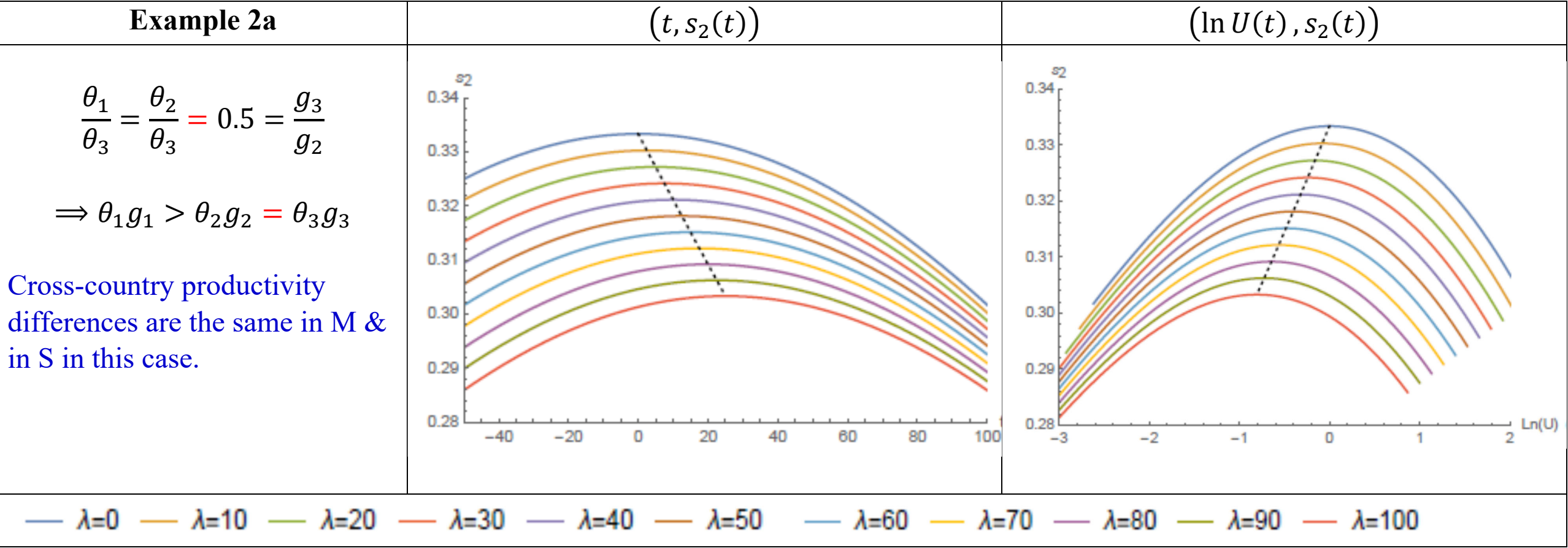
$$\implies \hat{t}(\lambda) = \lambda; \quad \hat{s}_2(\lambda) = \tilde{\beta}_2; \quad \hat{U}(\lambda) = 1$$

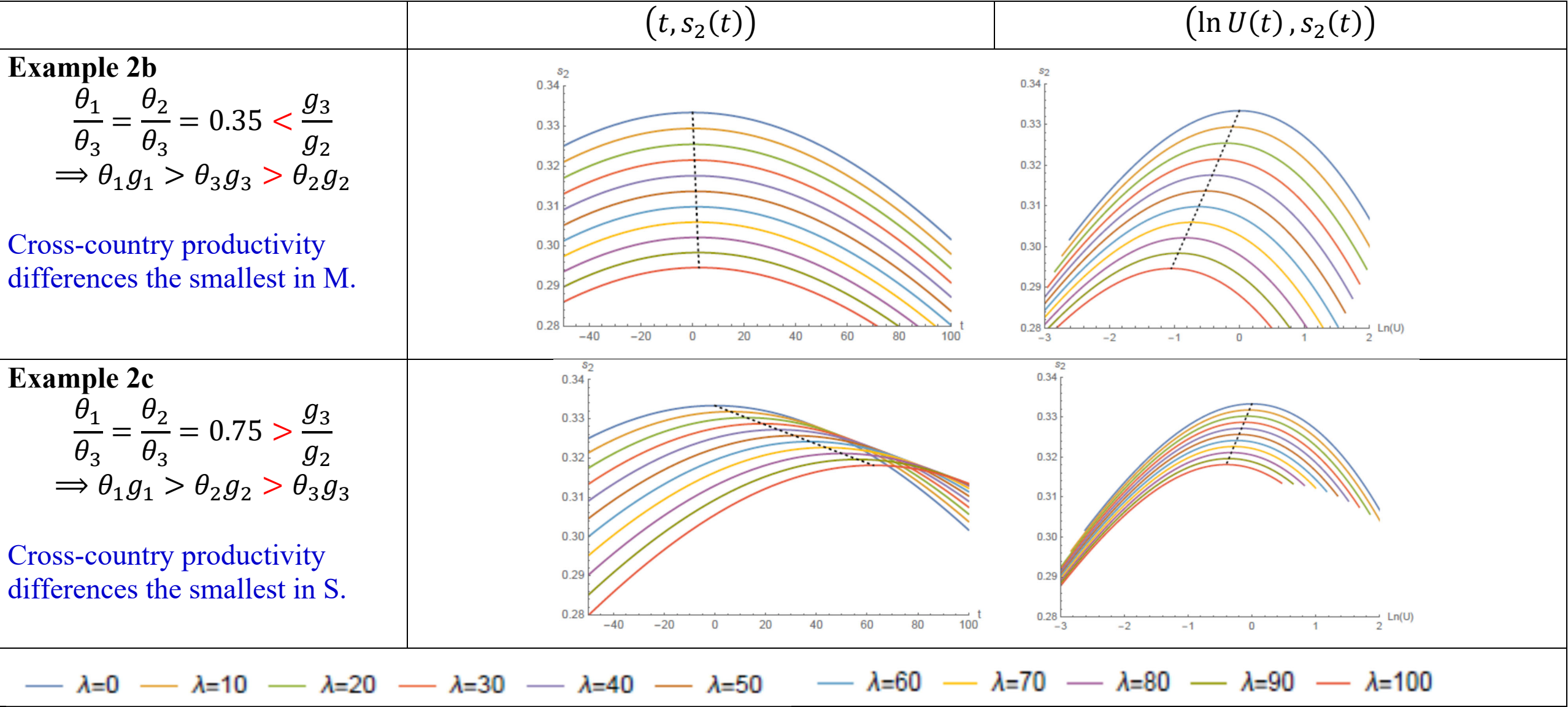
- The country's technology gap causes a delay in the peak time, \hat{t} , by $\lambda > 0$.
- The peak M-share & the peak time per capita income unaffected.

Each country follows the same development path of early industrializers *with a delay*. No PD!!

Thus, the technology gap must have differential impacts on the adoption lags across sectors.

Example 2a-2c: Numerical Illustrations. In all three examples, $\theta_1 = \theta_2 < \theta_3 = 1$ and we use $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\alpha = 1/3$, and $\sigma = 0.6$ (hence $a = 6/13$).
 $\tilde{\beta}_j = 1/3$ for $j = 1, 2, 3 \Rightarrow \hat{s}_2(0) = \tilde{\beta}_2 = 1/3$; $\hat{U}(0) = 1$; $\hat{t}(0) = 0$.





Some Calibrations:

Rodrik (2016) divided countries into pre-1990 peaked vs. post-1990 peaked.

From his Fig.5, $\hat{t}(\lambda) = 25$ years. From his Table 10,

For the employment shares, $\hat{s}_2(0) = 21.5\% > \hat{s}_2(\lambda) = 18.9\%$; $\hat{U}(\lambda)/\hat{U}(0) = \hat{U}(\lambda) = 4273/11048$.

For the value-added shares, $\hat{s}_2(0) = 27.9\% > \hat{s}_2(\lambda) = 24.1\%$. $\hat{U}(\lambda)/\hat{U}(0) = \hat{U}(\lambda) = 20537/47099$.

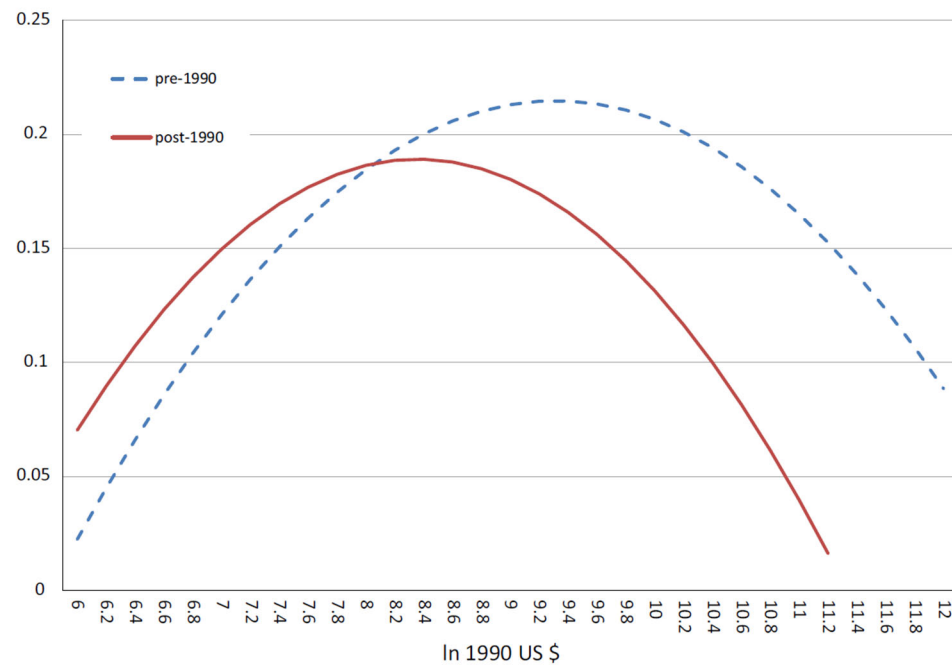


Fig. 6 Simulated manufacturing employment shares

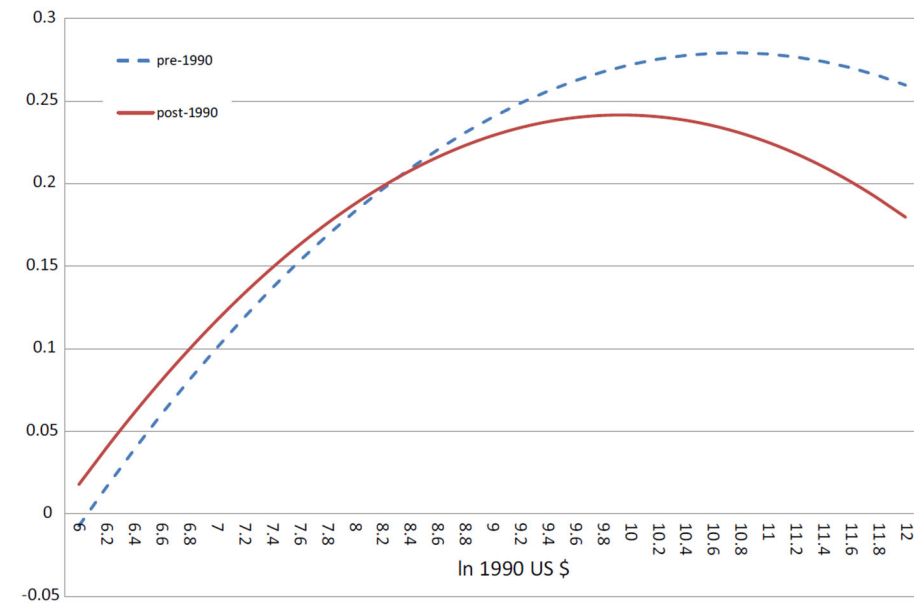


Fig. 7 Simulated manufacturing output shares (MVA/GDP at constant prices)

Peak Time

$$\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda.$$

Peak M-Share

$$\frac{1}{\hat{s}_2(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2} \right) e^{a(g_1 - g_2) \left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} \lambda - \hat{t}(\lambda) \right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{a(g_2 - g_3) \left(\hat{t}(\lambda) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \lambda \right)}$$

Peak Time Per Capita Income

$$\hat{U}(\lambda) = \left\{ \sum_{k=1}^3 \tilde{\beta}_k e^{-a g_k [\hat{t}(\lambda) - \theta_k \lambda]} \right\}^{-\frac{1}{a}}$$

can be inverted into

$$\theta_1 \lambda = \hat{t}(\lambda) - \frac{1}{g_1} \ln \left(\hat{U}(\lambda) \right) + \frac{1}{a g_1} \ln \left(\frac{1 - \hat{s}_2(\lambda)}{1 - \hat{s}_2(0)} \right),$$

$$\theta_2 \lambda = \hat{t}(\lambda) - \frac{1}{g_2} \ln \left(\hat{U}(\lambda) \right) + \frac{1}{a g_2} \ln \left(\frac{\hat{s}_2(\lambda)}{\hat{s}_2(0)} \right).$$

$$\theta_3 \lambda = \hat{t}(\lambda) - \frac{1}{g_3} \ln \left(\hat{U}(\lambda) \right) + \frac{1}{a g_3} \ln \left(\frac{1 - \hat{s}_2(\lambda)}{1 - \hat{s}_2(0)} \right).$$

	Duarte-Restuccia (2010): $g_1 = 3.8\% > g_2 = 2.4\% > g_3 = 1.3\%$	Comin et. al. (2021) $g_1 = 2.9\% > g_2 = 1.3\% > g_3 = 1.1\%$
Empl. Shares	$(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}) \approx (13.9\%, 28.1\%, 26.0\%);$ $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.501, 0.512); \Theta \approx 0.779.$	$(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}) \approx (17.5\%, 36.9\%, 27.4\%);$ $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.511, 0.650)$ and $\Theta \approx 0.848.$
VA Shares	$(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}) \approx (15.1\%, 32.9\%, 28.2\%);$ $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.511, 0.476)$ and $\Theta \approx 0.726.$	$(e^{-g_1\theta_1\lambda}, e^{-g_2\theta_2\lambda}, e^{-g_3\theta_3\lambda}) \approx (18.9\%, 43.3\%, 29.6\%);$ $(\theta_1/\theta_3, \theta_2/\theta_3) \approx (0.520, 0.583)$ and $\Theta \approx 0.805$

$\theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2 \Leftrightarrow e^{-\theta_1 g_1 \lambda} < e^{-\theta_3 g_3 \lambda} < e^{-\theta_2 g_2 \lambda}.$

Cross-country productivity differences not only the largest in A but also the smallest in M.

1st Extension: Introducing the Engel Effect

The Engel Law through Isoelastic Nonhomothetic CES; Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

$$\left[\sum_{j=1}^3 (\beta_j)^{\frac{1}{\sigma}} \left(\frac{c_j}{U^{\varepsilon_j}} \right)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \equiv 1$$

Normalize $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3$; with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, we go back to the standard homothetic CES.

With $\sigma < 1$, $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 \Rightarrow$ **the income elasticity the lowest in A and the highest in S.**

By maximizing U subject to $\sum_{j=1}^3 p_j c_j \leq E$,

Expenditure Shares
$$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j (U^{\varepsilon_j} p_j)^{1-\sigma}}{\sum_{k=1}^3 \beta_k (U^{\varepsilon_k} p_k)^{1-\sigma}} = \beta_j \left(\frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma} \Rightarrow \frac{m_j}{m_k} = \frac{\beta_j}{\beta_k} \left(\frac{p_j}{p_k} U^{\varepsilon_j - \varepsilon_k} \right)^{1-\sigma}$$

Indirect Utility Function:
$$\left[\sum_{j=1}^3 \beta_j \left(\frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \equiv 1$$

Cost-of-Living Index:
$$\left[\sum_{j=1}^3 \beta_j \left(\frac{U^{\varepsilon_j-1} p_j}{P} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \equiv 1 \Leftrightarrow U \equiv \frac{E}{P}$$

Income Elasticity:
$$\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln(U)} = 1 + \frac{\partial \ln m_j}{\partial \ln(E/P)} = 1 + (1 - \sigma) \left\{ \varepsilon_j - \sum_{k=1}^3 m_k \varepsilon_k \right\}$$

Structural Change with the Engel (Income) Effect: Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.

Then, *even with constant relative prices*,

Decline of Agriculture: $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1} \left(\frac{p_2}{p_1} U(t)^{\varepsilon_2 - \varepsilon_1} \right)^{1-\sigma} + \frac{\beta_3}{\beta_1} \left(\frac{p_3}{p_1} U(t)^{\varepsilon_3 - \varepsilon_1} \right)^{1-\sigma}$$

Rise of Services: $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3} \left(\frac{p_1}{p_3} U(t)^{\varepsilon_1 - \varepsilon_3} \right)^{1-\sigma} + \frac{\beta_2}{\beta_3} \left(\frac{p_2}{p_3} U(t)^{\varepsilon_2 - \varepsilon_3} \right)^{1-\sigma}$$

Rise and Fall of Manufacturing: $s_2(t) = m_2(t)$ is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2} \left(\frac{p_1}{p_2} U(t)^{\varepsilon_1 - \varepsilon_2} \right)^{1-\sigma} + \frac{\beta_3}{\beta_2} \left(\frac{p_3}{p_2} U(t)^{\varepsilon_3 - \varepsilon_2} \right)^{1-\sigma}.$$

Hump-shaped due to the two opposing forces: $\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

$$s'_2(t) = m'_2(t) \gtrless 0 \Leftrightarrow (\varepsilon_2 - \varepsilon_1) \frac{m_1(t)}{m_2(t)} \gtrless (\varepsilon_3 - \varepsilon_2) \frac{m_3(t)}{m_2(t)} \Leftrightarrow \eta_2 \gtrless 1$$

Initially, when A is large & S is small, the former effect is stronger. Over time, A shrinks & S expands, and eventually, the latter effect becomes stronger.

The production side is the same as before. By following the same step, we obtain

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j \right]^{-a}}{[U^{\varepsilon_j}]^{-a}}, \quad \text{where } \sum_{k=1}^3 \frac{\left[\beta_k^{\frac{1}{\sigma-1}} A_k \right]^{-a}}{[U^{\varepsilon_k}]^{-a}} \equiv 1$$

With $A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda)}$,

$$s_2(t): \quad \frac{1}{s_2(t)} = U(t)^{a(\varepsilon_1-\varepsilon_2)} \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\theta_1 g_1 - \theta_2 g_2)\lambda} \right] e^{-a(g_1-g_2)t} + 1 + U(t)^{a(\varepsilon_3-\varepsilon_2)} \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\theta_3 g_3 - \theta_2 g_2)\lambda} \right] e^{a(g_2-g_3)t}$$

$$U(t): \quad U(t)^{a\varepsilon_1} \tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + U(t)^{a\varepsilon_2} \tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + U(t)^{a\varepsilon_3} \tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)} \equiv 1$$

$$s_2'(t) = 0: \quad (g_1 - g_2) = (g_2 - g_3) U^{a(\varepsilon_3-\varepsilon_2)} \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} \right] e^{a(\theta_3 g_3 - \theta_1 g_1)\lambda} e^{a(g_1-g_3)t} \\ + \frac{\left\{ (\varepsilon_1 - \varepsilon_2) + (\varepsilon_3 - \varepsilon_2) U^{a(\varepsilon_3-\varepsilon_1)} \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} \right] e^{a(\theta_3 g_3 - \theta_1 g_1)\lambda} e^{a(g_1-g_3)t} \right\} \{ g_1 U^{a(\varepsilon_1-\varepsilon_2)} \tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + g_2 \tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + g_3 U^{a(\varepsilon_3-\varepsilon_2)} \tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)} \}}{\varepsilon_1 U^{a(\varepsilon_1-\varepsilon_2)} \tilde{\beta}_1 e^{-ag_1(t-\theta_1\lambda)} + \varepsilon_2 \tilde{\beta}_2 e^{-ag_2(t-\theta_2\lambda)} + \varepsilon_3 U^{a(\varepsilon_3-\varepsilon_2)} \tilde{\beta}_3 e^{-ag_3(t-\theta_3\lambda)}}.$$

\hat{t} and \hat{U} solve the equation for $U(t)$ and the equation for $s_2'(t) = 0$, simultaneously.

Then, \hat{s}_2 can be obtained by plugging \hat{t} and \hat{U} into the equation for $s_2(t)$

(Analytically Solvable) Case: $0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}},$ where $\bar{g} \equiv \frac{g_1 + g_2 + g_3}{3}$

Proposition 3 (Impact of Adding the Engel Effect on top of the Baumol Effect)

Peak Time
$$\hat{t}(\lambda; \mu) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda - \mu \ln \hat{U}(\lambda; \mu) = \hat{t}(\lambda; 0) - \frac{\mu}{1 + \mu \bar{g}} \ln \hat{U}(\lambda; 0)$$

Peak M-Share
$$\frac{1}{\hat{s}_2(\lambda; \mu)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2} \right) e^{a(g_1 - g_2) \left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} \lambda - \hat{t}(\lambda; 0) \right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2} \right) e^{a(g_2 - g_3) \left(\hat{t}(\lambda; 0) - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \lambda \right)} = \frac{1}{\hat{s}_2(\lambda; 0)}$$

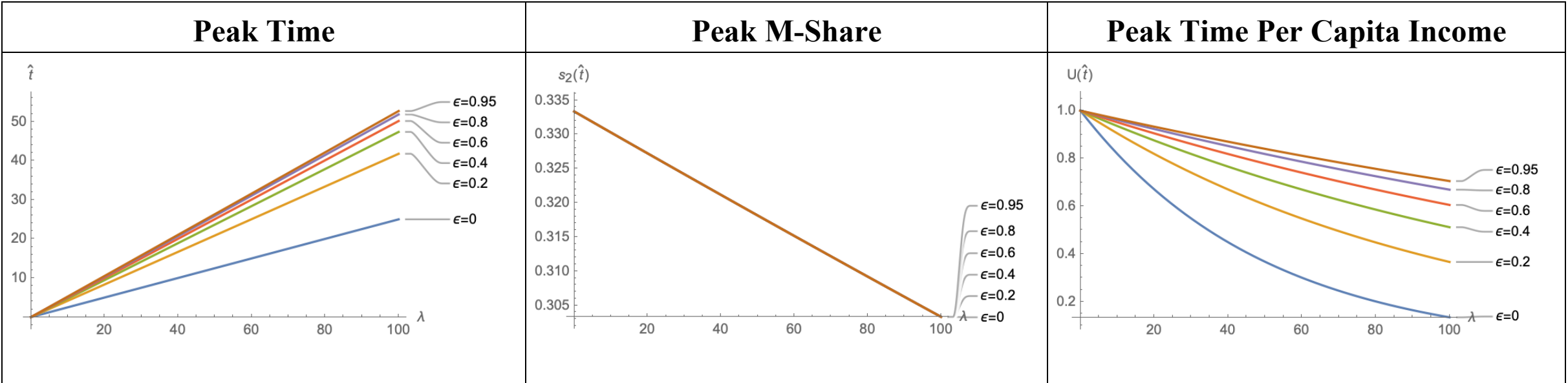
Peak Time Per Capita Income
$$\hat{U}(\lambda; \mu) = \left\{ \sum_{k=1}^3 \tilde{\beta}_k e^{-a g_k [\hat{t}(\lambda; 0) - \theta_k \lambda]} \right\}^{-\frac{1}{a} \left(\frac{1}{1 + \mu \bar{g}} \right)} = \hat{U}(\lambda; 0)^{\left(\frac{1}{1 + \mu \bar{g}} \right)}$$

- $\hat{s}_2'(\lambda; \mu) < 0$; $\hat{U}'(\lambda; \mu) < 0$ under the same condition; $\hat{t}'(\lambda; \mu) > 0$ under a weaker condition.
- Fixing g_1, g_2, g_3 , a higher μ has
 - **No effect** on the peak values of the frontier country, $\hat{t}(0; \mu), \hat{s}_2(0; \mu), \hat{U}(0; \mu)$.
 - **A further delay** in $\hat{t}(\lambda; \mu)$ for every country with $\lambda > 0$.
 - **No effect** on $\hat{s}_2(\lambda; \mu)$ for every country with $\lambda > 0$.
 - **A smaller decline** in $\hat{U}(\lambda; \mu)$ for each country with $\lambda > 0$.

Analytically Solvable Case: A Numerical Illustration

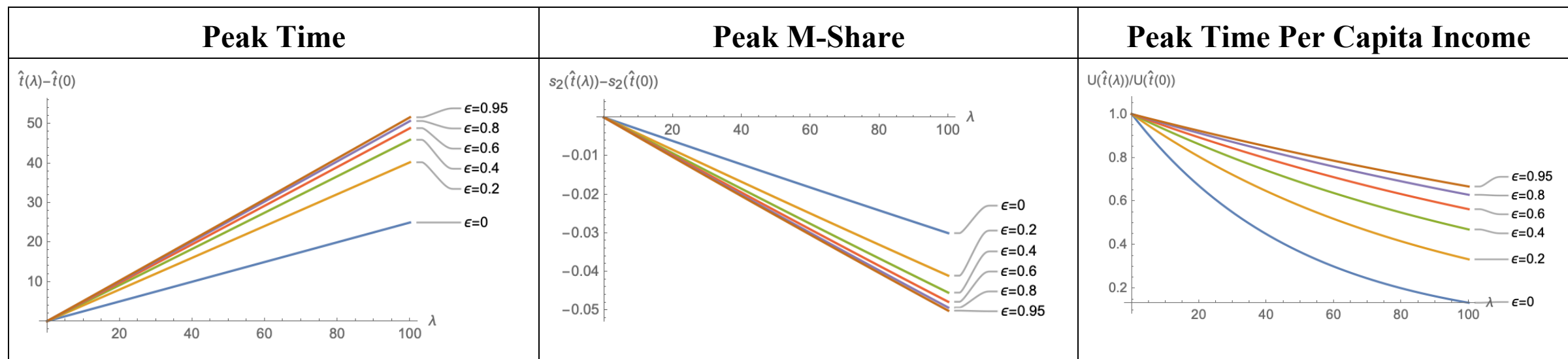
$g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%, \theta = 0.5, a = 6/13; \tilde{\beta}_j = 1/3$ for $j = 1,2,3$.

In this case, $g_1 - g_2 = g_2 - g_3 = \bar{g} = 1.2\% > 0 \Rightarrow \varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$ for $0 < \epsilon = (1.2\%)\mu < 1$



(Empirically More Plausible) Case:

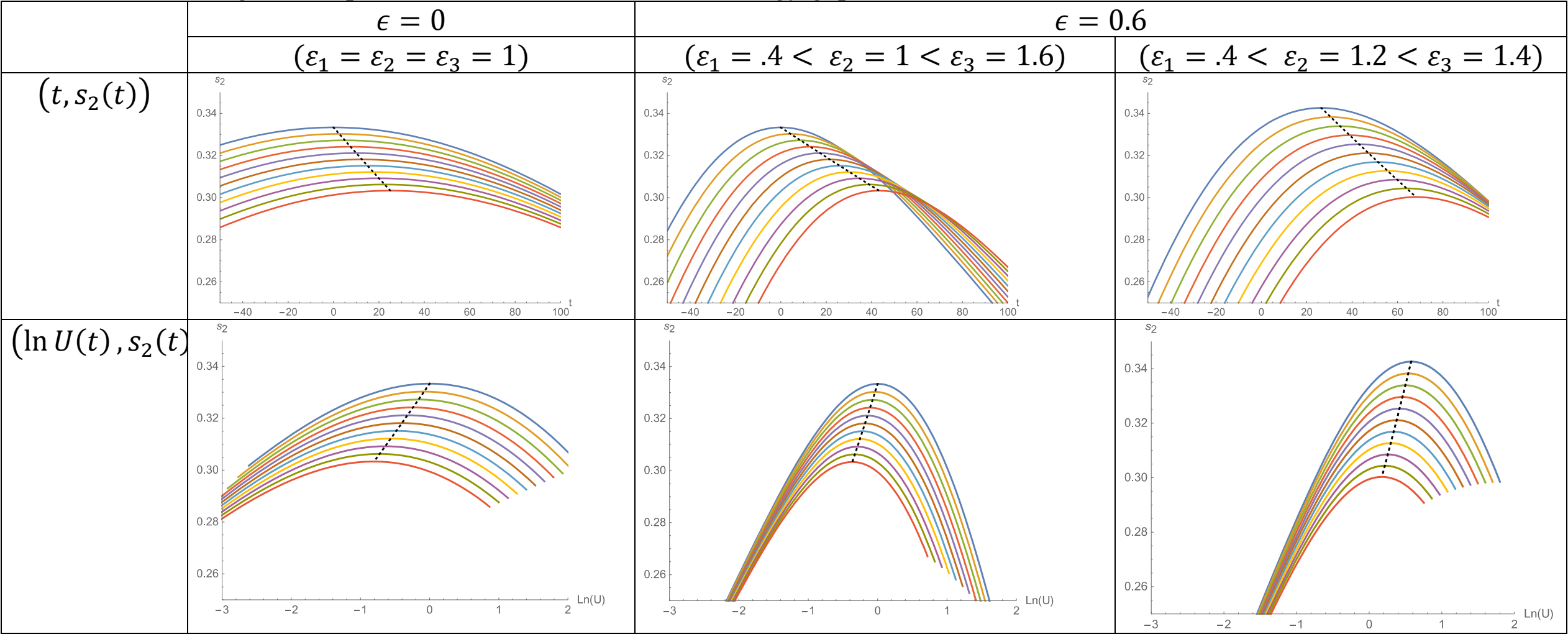
$$\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 + \frac{\epsilon}{3} < \varepsilon_3 = 1 + \frac{2\epsilon}{3} \text{ for } 0 < \epsilon < 1 \Rightarrow \frac{g_1 - g_2}{g_2 - g_3} = 1 < \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} = 4, \text{ as in CLM (2021).}$$



In this case, the frontier country's peak values are affected by ϵ . Relative to the frontier country, a higher ϵ causes a high- λ country to have

- A further delay in $\hat{t}(\lambda; \mu)$
- A *larger* decline in $\hat{s}_2(\lambda; \mu)$
- A smaller decline in $\hat{U}(\lambda; \mu)$.

Stronger nonhomotheticity changes the shape of the time paths significantly.
It does not change the implications on PD, i.e., how technology gaps affect \hat{t} , $s_2(\hat{t})$, and $U(\hat{t})$.



What happens if we had *solely* the Engel effect with $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$, without the Baumol effect, $g_1 = g_2 = g_3 = \bar{g} > 0$?

Under the two normalizations

$$\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2}\right) \frac{\tilde{\beta}_1}{\tilde{\beta}_3} = 1; \quad \tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1$$

which ensures $\hat{U}(0) = 1$ and $\hat{t}(0) = 0$,

Proposition 4: Peak Values under the Engel (Income) Effect only

Peak Time

$$\hat{t}(\lambda) = \frac{1}{a\bar{g}} \ln \left\{ \sum_{k=1}^3 \tilde{\beta}_k e^{a(\theta_k \bar{g} \lambda + \varepsilon_k \ln \hat{U}(\lambda))} \right\}$$

Peak M-Share

$$\frac{1}{\hat{s}_2(\lambda)} = 1 + \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a(\varepsilon_2 - \varepsilon_1) \left(-\frac{\theta_2 - \theta_1}{\varepsilon_2 - \varepsilon_1} \bar{g} \lambda - \ln \hat{U}(\lambda)\right)} + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a(\varepsilon_3 - \varepsilon_2) \left(\ln \hat{U}(\lambda) - \frac{\theta_2 - \theta_3}{\varepsilon_3 - \varepsilon_2} \bar{g} \lambda\right)}$$

Peak Time Per Capita Income

$$\ln \hat{U}(\lambda) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} \bar{g} \lambda$$

Proposition 5: Conditions for PD with the Engel (Income) Effect Only

$$\hat{U}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow 0 < \frac{\theta_1}{\theta_3} < 1$$

With a low θ_1 and a high θ_3 , the price of the income elastic S is high relative to the income inelastic A in a high- λ country, which make it necessary to reallocate labor to S at earlier stage of development.

$$\hat{s}_2'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(1 - \frac{\varepsilon_2}{\varepsilon_3}\right) \left(\frac{\theta_1}{\theta_3} - 1\right) + \left(1 - \frac{\varepsilon_1}{\varepsilon_3}\right) \left(1 - \frac{\theta_2}{\theta_3}\right) > 0.$$

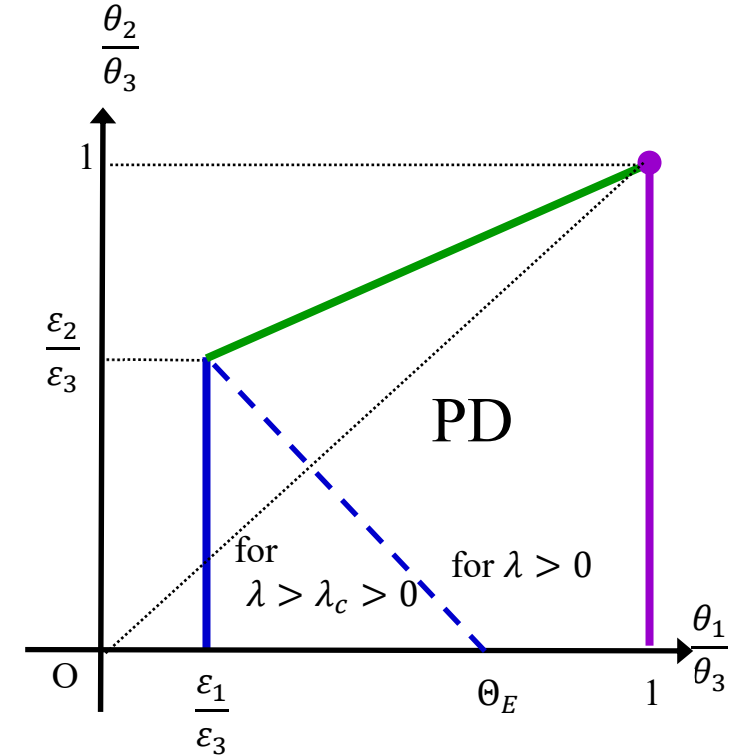
With a low θ_2 , which has no effect on $\hat{U}(\lambda)$, the price of M is low relative to both A & S in a high- λ country, which keeps the M-share low.

Under the above condition,

$$\hat{t}'(\lambda) > 0 \text{ for a sufficiently large } \lambda \Leftrightarrow \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3}$$

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(\Theta_E - \frac{\varepsilon_1}{\varepsilon_3}\right) \left[1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right) \frac{\theta_2}{\theta_3}\right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3}$$

where $\varepsilon_1/\varepsilon_3 < \Theta_E < 1$.



With $g_1 = g_2 = g_3 = \bar{g}$, PD occurs only if $\theta_1 \bar{g}, \theta_2 \bar{g} < \theta_3 \bar{g}$, that is, when cross-country productivity difference is *the largest in S*.

2nd Extension: Introducing International Trade

One Implication of PD (consistent with the empirical evidence):

$$\frac{\partial}{\partial \lambda} \ln \left(\frac{A_1(t)}{A_2(t)} \right) = -(\theta_1 g_1 - \theta_2 g_2) < 0.$$

- A low- λ country has comparative advantage in A and a high- λ country has comparative advantage in M.
- Opening up trade in A and in M would weaken PD by allowing high- λ country to export M.
- Consistent with the findings that East Asia “suffers” less from PD.

A Two-Country Technology Gap Model of PD: $\lambda^1 < \lambda^2$ (Superscript indicates the country)

Trade Cost: Only $e^{-\tau_1} < 1$ fraction of A and only $e^{-\tau_2} < 1$ fraction of M shipped arrive to the destination.

$$m_j^c = \beta_j \left(\frac{p_j^c}{P^c} \right)^{1-\sigma}; \quad P^c = \left[\sum_{k=1}^3 \beta_k (p_k^c)^{1-\sigma} \right]^{1/(1-\sigma)} \quad \& \quad s_j^c = (A_j^c)^{\frac{1}{\alpha}} \left(\frac{p_j^c}{E^c} \right)^{\frac{1}{\alpha}}; \quad E^c = \left[\sum_{k=1}^3 (A_k^c)^{\frac{1}{\alpha}} (p_k^c)^{\frac{1}{\alpha}} \right]^\alpha$$

With $g_1 \theta_1 > g_2 \theta_2$, Leader (Country-1) has CA in A and Laggard (Country-2) has CA in M.

1 may export A to 2: $e^{\tau_1} p_1^1 \geq p_1^2$; $e^{-\tau_1} [A_1^1 (s_1^1)^{1-\alpha} - c_1^1] L^1 = [c_1^2 - A_1^2 (s_1^2)^{1-\alpha}] L^2 \geq 0$. $\rightarrow \#$ $[s_1^1 - m_1^1] E^1 L^1 = [m_1^2 - s_1^2] E^2 L^2 \geq 0$.#

2 may export M to 1: $p_2^1 \leq e^{\tau_2} p_2^2$; $[c_2^1 - A_2^1 (s_2^1)^{1-\alpha}] L^1 = e^{-\tau_2} [A_2^2 (s_2^2)^{1-\alpha} - c_2^2] L^2 \geq 0$. \rightarrow $[m_2^1 - s_2^1] E^1 L^1 = [s_2^2 - m_2^2] E^2 L^2 \geq 0$.

S is nontradeable: $p_3^1 \neq p_3^2$; $c_3^1 = A_3^1 (s_3^1)^{1-\alpha}$; $c_3^2 = A_3^2 (s_3^2)^{1-\alpha}$ \rightarrow $m_3^1 = s_3^1$; $m_3^2 = s_3^2$.

Condition for No Trade Equilibrium:

$$e^{\tau_1 + \tau_2} > \frac{p_2^1(t) p_1^2(t)}{p_1^1(t) p_2^2(t)} = \left[\frac{A_2^1(t) A_1^2(t)}{A_1^1(t) A_2^2(t)} \right]^{-\frac{a}{(1-\sigma)}} = e^{\frac{a(g_1\theta_1 - g_2\theta_2)}{(1-\sigma)}(\lambda^2 - \lambda^1)}$$

$$\Leftrightarrow \tau_1 + \tau_2 > T_+ \equiv \frac{a(g_1\theta_1 - g_2\theta_2)}{(1-\sigma)}(\lambda^2 - \lambda^1) > 0.$$

Trade Equilibrium under

$$0 < \tau_1 + \tau_2 \leq T_+ \equiv \frac{a(g_1\theta_1 - g_2\theta_2)}{(1-\sigma)}(\lambda^2 - \lambda^1).$$

Then, 1 exports A to 2 and imports M from 2.

Equilibrium Conditions:

$$s_1^1 + s_2^1 = m_1^1 + m_2^1; \quad s_1^2 + s_2^2 = m_1^2 + m_2^2$$

$$[s_1^1 - m_1^1]E^1L^1 = [s_2^2 - m_2^2]E^2L^2 > 0$$

$$e^{\tau_1}p_1^1 = p_1^2; \quad p_2^1 = e^{\tau_2}p_2^2$$

Impact of International Trade (Numerical Simulation): $L_1/L_2 = 1$.

$$0 < \tau \equiv \frac{\tau_1 + \tau_2}{T_+} \equiv \frac{(1 - \sigma)(\tau_1 + \tau_2)}{a(g_1\theta_1 - g_2\theta_2)(\lambda^2 - \lambda^1)} < 1$$

$$\Rightarrow 1 < \frac{p_2^1 p_1^2}{p_1^1 p_2^2} = e^{\tau_1 + \tau_2} = e^{\tau T_+} \leq e^{T_+}.$$

We plot how the peak values change in response to τ .

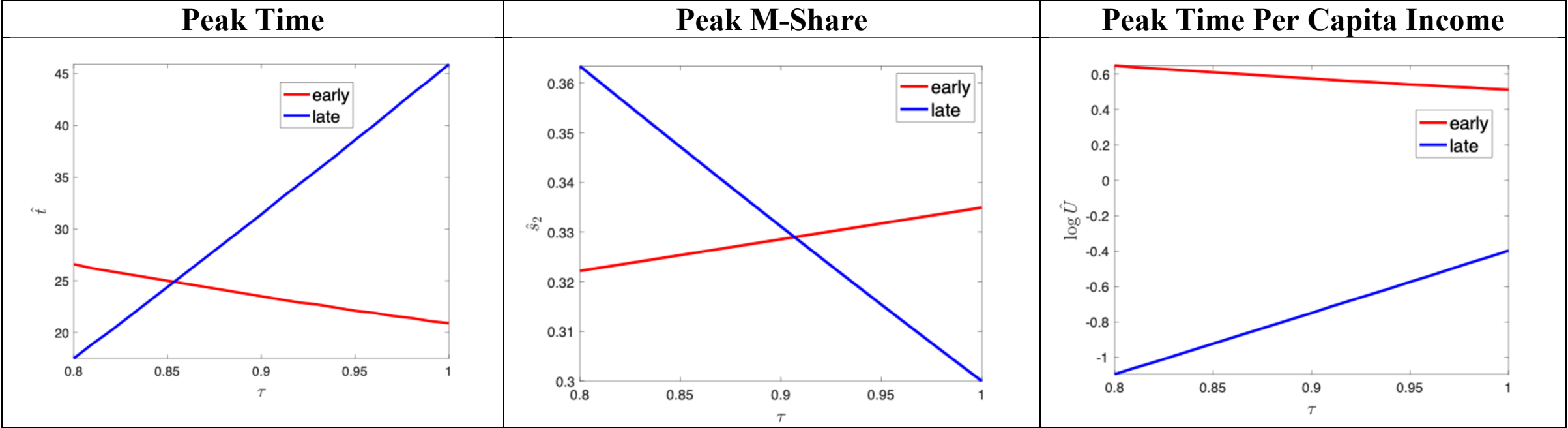
In all four cases, our mechanism for PD is:

- Robust to introducing international trade.
- Weaker in that the differences btw the leader and the laggard in \hat{t} and \hat{s}_2 become smaller (larger in \hat{U} in \hat{m}_2), as τ declines. For a sufficiently small τ , the reversal occurs in \hat{t} and \hat{s}_2 .

PD holds, when the trade cost accounts for more than about 1/3 of the imported goods prices, empirically plausible.

Under our mechanism, PD occurs not because of international trade but in spite of international trade.

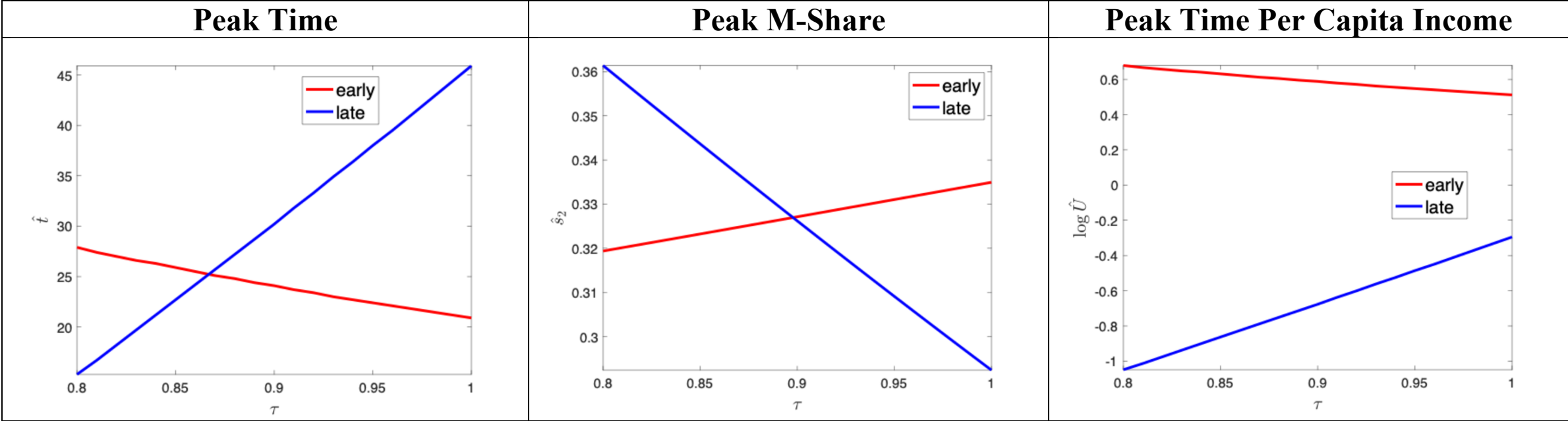
Duarte-Restuccia productivity growth rates;
Employment Shares



Reversal of \hat{t} at $\tau \approx 0.85$, or $e^{\tau_1+\tau_2} = e^{\tau T_+} \approx 1.986 \rightarrow \sqrt{1.986} \approx 1.41$ times higher in the importing country.

Reversal of \hat{s}_2 at $\tau \approx 0.91$ or $e^{\tau_1+\tau_2} = e^{\tau T_+} \approx 2.242 \rightarrow \sqrt{2.242} \approx 1.497$ times higher in the importing country.

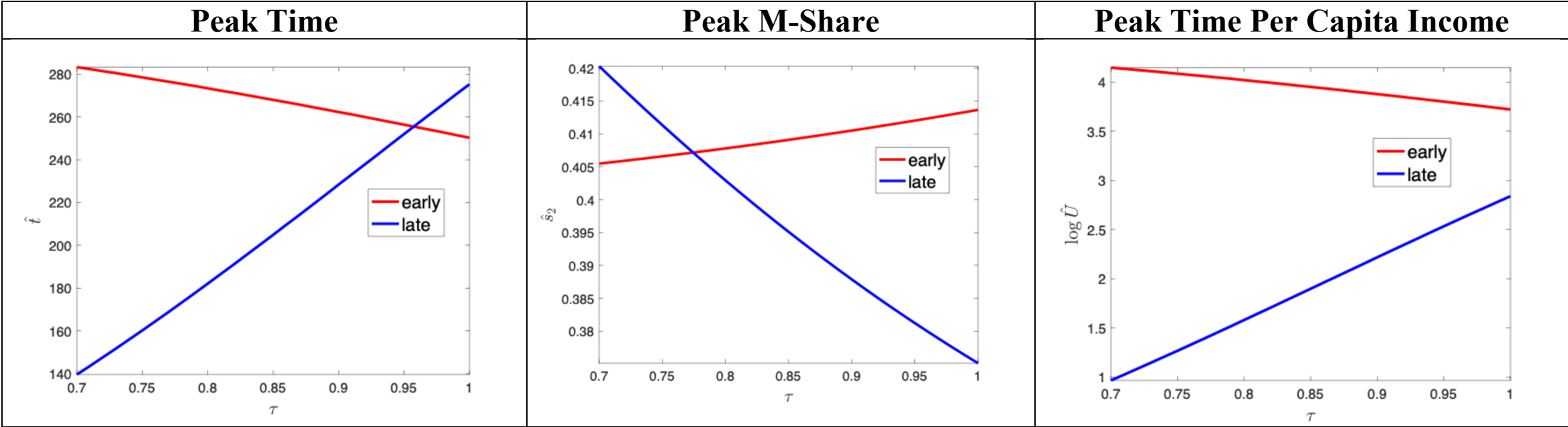
Duarte-Restuccia productivity growth rates;
Value-Added Shares



Reversal of \hat{t} at $\tau \approx 0.87$ or $e^{\tau_1+\tau_2} \approx 2.185 \rightarrow \sqrt{2.185} \approx 1.478$ times higher in the importing country.

Reversal of \hat{s}_2 at $\tau \approx 0.90$ or $e^{\tau_1+\tau_2} \approx 2.244 \rightarrow \sqrt{2.244} \approx 1.498$ times higher in the importing country.

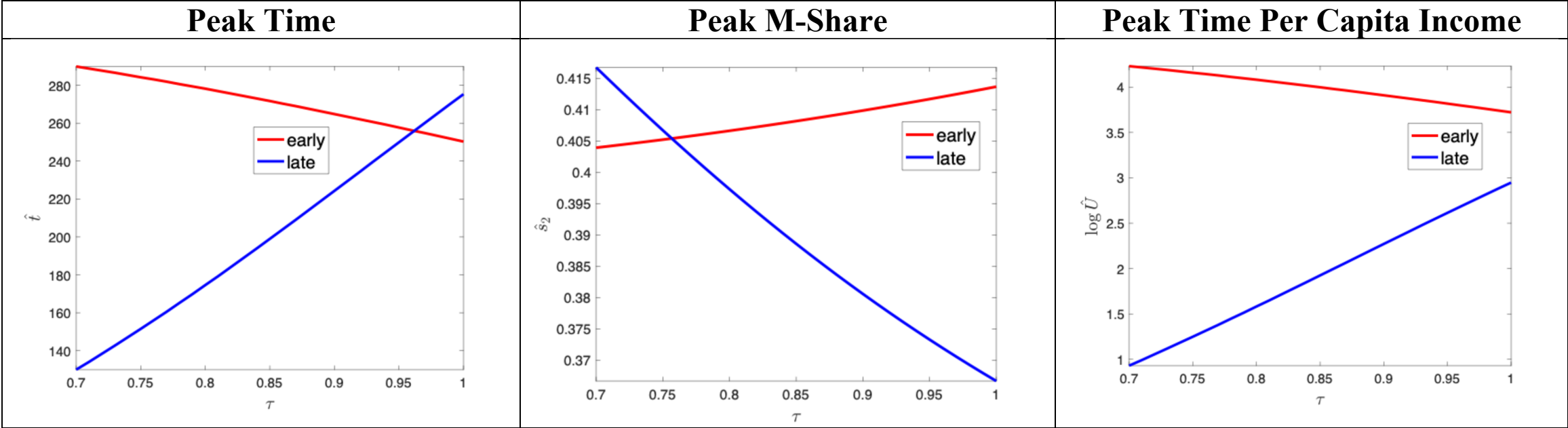
Comin-Lashkari-Mestieri productivity growth rates;
Employment Shares



Reversal of \hat{t} at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.295 \rightarrow \sqrt{2.295} \approx 1.515$ times higher in the importing country.

Reversal of \hat{s}_2 at $\tau \approx 0.77$ or $e^{\tau_1+\tau_2} \approx 1.947 \rightarrow \sqrt{1.947} \approx 1.395$ times higher in the importing country.

Comin-Lashkari-Mestieri productivity growth rates;
Value-Added Shares



Reversal of \hat{t} at $\tau \approx 0.96$, or $e^{\tau_1+\tau_2} \approx 2.504 \rightarrow \sqrt{2.504} \approx 1.582$ times higher in the importing country.

Reversal of \hat{s}_2 at $\tau \approx 0.76$ or $e^{\tau_1+\tau_2} \approx 2.068 \rightarrow \sqrt{2.068} \approx 1.438$ times higher in the importing country.

3rd Extension: Introducing Catching Up

Narrowing a Technology Gap

We assumed that λ is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.

[In contrast, the aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^3 g_k s_k(t)$, declines over time, $g'_U(t) = g_1 s'_1(t) + g_2 s'_2(t) + g_3 s'_3(t) = (g_1 - g_2)s'_1(t) + (g_3 - g_2)s'_3(t) < 0$, the so-called Baumol’s cost disease.]

*What if technological laggards can **narrow a technology gap**, and hence achieve a higher productivity growth in each sector?*

Countries differ only in the *initial* value of lambda, λ_0 , converging exponentially over time at the same rate,

$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where } \lambda_t = \lambda_0 e^{-g_\lambda t}, \quad g_\lambda > 0.$$

$$\Rightarrow \frac{1}{s_2(t)} = \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{a[(\theta_1 g_1 - \theta_2 g_2)\lambda_t - (g_1 - g_2)t]} + 1 + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{a[(\theta_3 g_3 - \theta_2 g_2)\lambda_t + (g_2 - g_3)t]}$$

Again, by setting the calendar time such that $\hat{t}_0 = 0$ for the frontier country with $\lambda_0 = 0$,

Peak Time

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda_{\hat{t}} + D(g_\lambda \lambda_{\hat{t}})$$

Peak Share

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2} \right) \left[\frac{(g_2 - g_3)e^{a(g_2 - g_1)D(g_\lambda \lambda_{\hat{t}})} + (g_1 - g_2)e^{a(g_2 - g_3)D(g_\lambda \lambda_{\hat{t}})}}{g_1 - g_3} \right] \left[e^{\frac{a(g_1 - g_2)(g_2 - g_3)}{(g_1 - g_3)}} \right]^{\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} + \frac{\theta_3 g_3 - \theta_2 g_2}{g_2 - g_3} \right) \lambda_{\hat{t}}}$$

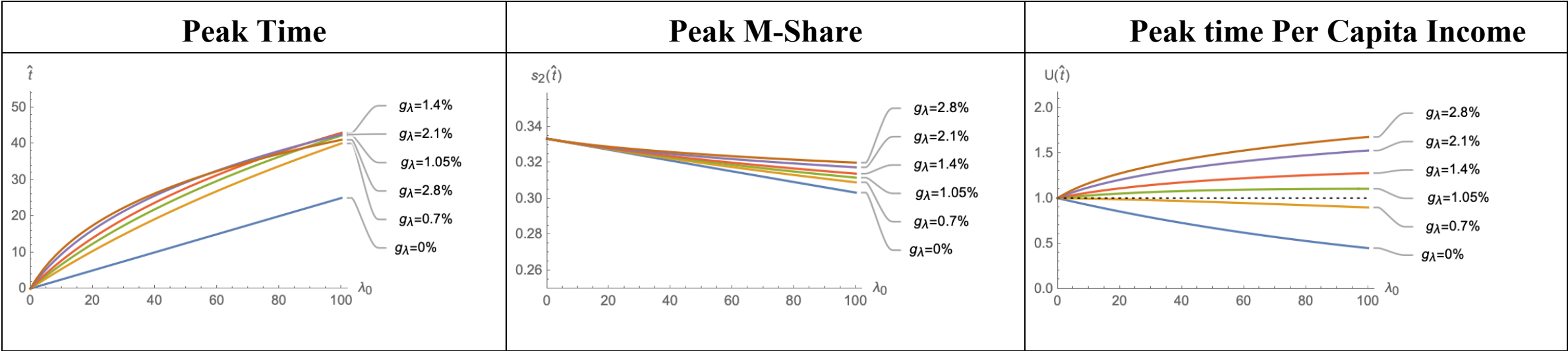
Peak Time Per Capita Income

$$U(\hat{t}) = \left\{ (\tilde{\beta}_1 e^{-a g_1 D(g_\lambda \lambda_{\hat{t}})} + \tilde{\beta}_3 e^{-a g_3 D(g_\lambda \lambda_{\hat{t}})}) e^{-a \frac{(\theta_1 - \theta_3) g_1 g_3}{g_1 - g_3} \lambda_{\hat{t}}} + (\tilde{\beta}_2 e^{-a g_2 D(g_\lambda \lambda_{\hat{t}})}) e^{-a \frac{(\theta_1 - \theta_2) g_1 g_2 + (\theta_2 - \theta_3) g_2 g_3}{g_1 - g_3} \lambda_{\hat{t}}} \right\}^{-\frac{1}{a}}$$

where

$$D(g_\lambda \lambda_{\hat{t}}) = \frac{1}{a(g_1 - g_3)} \ln \left[\left(\frac{g_1 - g_2 + (\theta_1 g_1 - \theta_2 g_2) g_\lambda \lambda_{\hat{t}}}{g_2 - g_3 - (\theta_3 g_3 - \theta_2 g_2) g_\lambda \lambda_{\hat{t}}} \right) \left(\frac{g_2 - g_3}{g_1 - g_2} \right) \right].$$

For $g_\lambda = 0$, $D(g_\lambda \lambda_{\hat{t}}) = D(0) = 0$, and all the parts in red disappear, and we go back to the baseline model.



Technological laggards

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, **unless g_λ is too large**: Comin-Mestieri (2018)

Concluding Remarks

A parsimonious model of Rodrik's (2016) PD based on

- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

In the baseline model. PD occurs for

- cross-country productivity difference larger in A than in S, which explains why a high- λ country peaks later in time.
- technology adoption takes not too long in M, which explains why a high- λ country has a low peak M-share.
- Technology adoption takes longer in S than in A, which explains why a high- λ country peaks prematurely.

The baseline model assumes **homothetic CES, no international trade, no catching up.**

In three extensions, we showed that the results are *robust* against introducing

- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)

The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD.

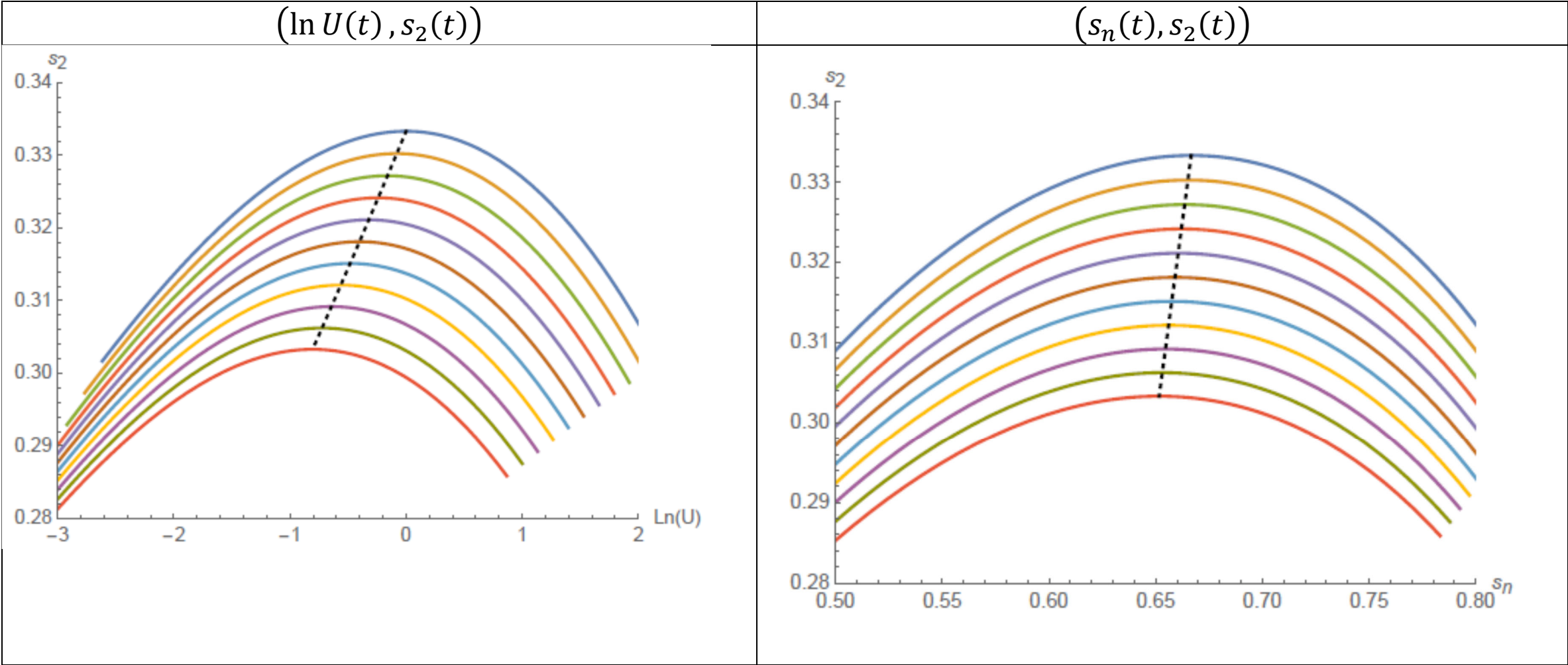
The Engel effect *alone* could not generate PD w/o counterfactual implications on cross-country productivity differences

- **International trade in A and in M, but PD becomes weaker.**
- **Narrowing a technology gap** to allow technological laggards to catch up, unless the catching-up speed is too large.

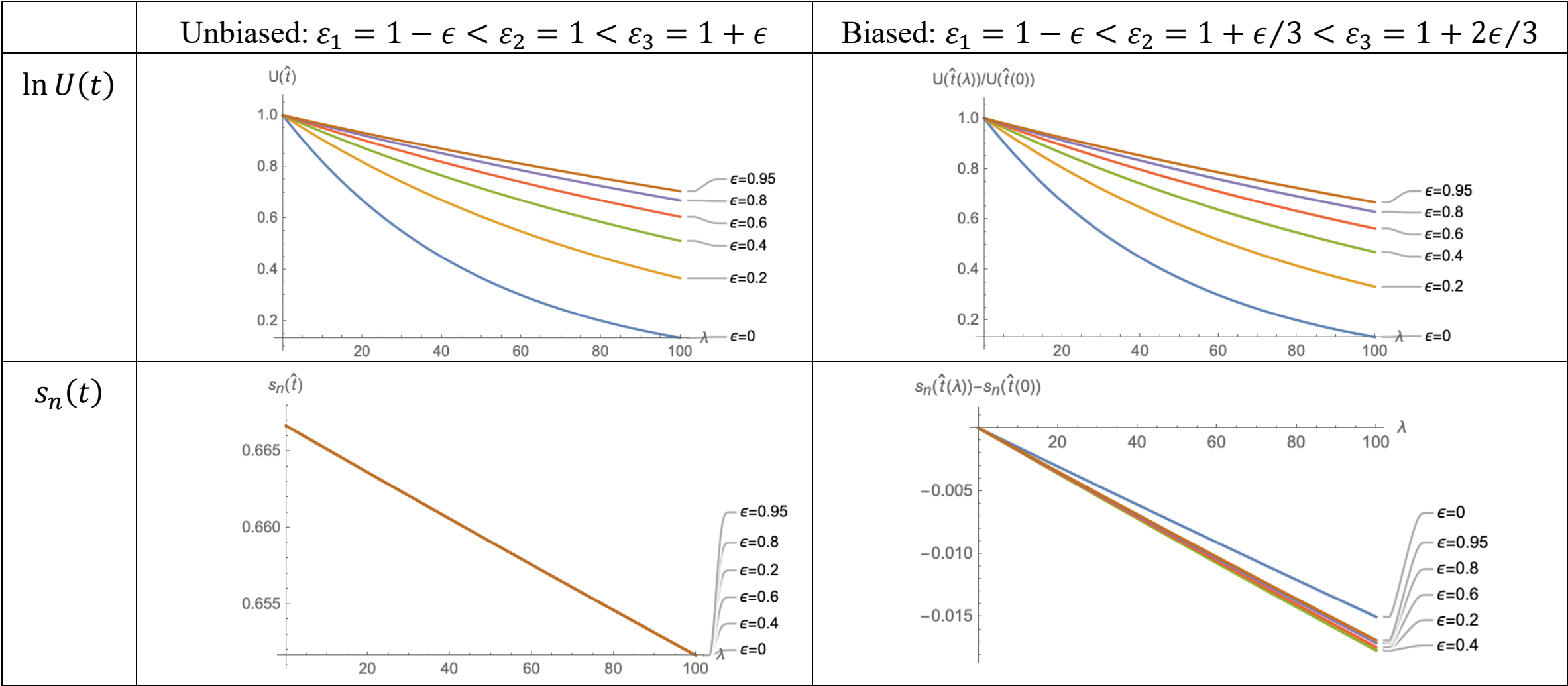
Appendix

Appendix: Non-agricultural share as another measure of development, $1 - s_1(\hat{t}) = s_2(\hat{t}) + s_3(\hat{t}) \equiv s_n(\hat{t})$

Baseline Homothetic Case:



Nonhomothetic Cases:



In the biased case, the frontier country’s peak values are affected by ϵ .