Introduction
Structural Change

As per capita income rises, employment or value-added shares

- fall in Agriculture
- rise in Services
- rise and fall in Manufacturing

*From Herrendorf-Rogerson-Valentinyi (2014)*

*Evidence from Long Time Series for the Currently Rich Countries* (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000
Premature Deindustrialization (Rodrik, J. Econ Growth, 2016)

Late industrializers reach their M-peak and start deindustrializing

- Later in time
- Earlier in per capita income
- with the lower peak M-sector shares, compared to early industrializers.

By “premature” no welfare connotations intended.

Fig. 5 Income at which manufacturing employment peaks (logs)
This Paper: A Parsimonious Model of Premature Deindustrialization (PD)

Key Ingredients

3 Goods/Sectors: 1=(A)griculture, 2=(M)anufacturing, 3=(S)ervices, homothetic CES with gross complements ($\sigma < 1$)

Frontier Technology: $\tilde{A}_j(t) = \tilde{A}_j(0)e^{g_jt}$, with $g_1 > g_2 > g_3 > 0 \Rightarrow$ a decline of A, a rise of S, and a hump-shaped of M in each country through the Baumol (relative price) effect, as in Ngai-Pissarides (2007)

Actual Technology Used: $\hat{A}_j(t) = \tilde{A}_j(t - \lambda_j)$ due to Adoption Lags, $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$

$\lambda \geq 0$: Technology Gap: country-specific, as in Krugman (1985)
$\theta_j > 0$: sector-specific, unlike Krugman (1985), common across countries

- Countries differ only in one dimension, $\lambda \geq 0$, in their ability to adopt the frontier technologies.
- $\theta_j > 0$ controls how much the technology gap affects the adoption lag and hence productivity in each sector.

$$\hat{A}_j(t) = \tilde{A}_j(t - \lambda_j) = \tilde{A}_j(0)e^{-\lambda_j g_j e^{g_jt}} = \tilde{A}_j(0)e^{-g_j \theta_j \lambda e^{g_jt}} \implies \frac{\partial}{\partial \lambda} \ln \left( \frac{\hat{A}_j(t)}{\hat{A}_k(t)} \right) = -\left( \theta_j g_j - \theta_k g_k \right)$$

$\lambda$ has no “growth” effect, but negative “level” effects proportional to $\theta_j g_j$ in sector-$j$
Key Mechanisms:
• $\theta_j$ magnifies the impact of the technology gap on the adoption lag: $\frac{\partial}{\partial \theta_j} \left( \frac{\partial \lambda_j}{\partial \lambda} \right) > 0$ (supermodularity)

• $g_j$ magnifies the (negative) impact of the adoption lag on productivity: $\frac{\partial}{\partial g_j} \left( \frac{\partial \ln e^{-\lambda_j g_j}}{\partial \lambda_j} \right) < 0$ (log-submodularity)

Main Results: PD occurs (defined as “A high-$\lambda$ country reaches its peak later, with lower peak M-share at lower peak time per capita income”) under the conditions:

i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S.
High relative price of A/low relative price of S in a high-$\lambda$ country causes a delay.

ii) $\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} > \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}$: technology adoption takes not too long in M.
Not too high relative price of M in a high-$\lambda$ country keeps the M-share low.

iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A.
Longer adoption lag in S in a high-$\lambda$ country causes deindustrialization “prematurely.”

Implications of the conditions for PD
i) & ii) $\Rightarrow \theta_1 g_1 > \theta_2 g_2, \theta_3 g_3$: cross-country productivity difference the largest in A.
ii) & iii) $\Rightarrow \theta_1, \theta_2 < \theta_3$: Technology adoption takes longest in S.
A Numerical Illustration.

$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\sigma = 0.6$; Labor share = $2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.

<table>
<thead>
<tr>
<th>Example 2a</th>
<th>$(t, s_2(t))$</th>
<th>$(\ln U(t), s_2(t))$</th>
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</thead>
<tbody>
<tr>
<td>$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$</td>
<td><img src="image1.png" alt="Graph" /></td>
<td><img src="image2.png" alt="Graph" /></td>
</tr>
<tr>
<td>$\Rightarrow \theta_1 g_1 &gt; \theta_2 g_2 = \theta_3 g_3$</td>
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### 1st Extension: Adding the Engel Effect with Nonhomothetic CES (Comin-Lashkari-Mestieri)

Nonhomotheticity changes the shape of trajectories greatly, but not on how technology gaps, $\lambda$, affects the peak values.

<table>
<thead>
<tr>
<th>$(t, s_2(t))$</th>
<th>Homothetic case ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</th>
<th>Unbiased case ($\varepsilon_1 = .4 &lt; \varepsilon_2 = 1 &lt; \varepsilon_3 = 1.6$)</th>
<th>Biased case ($\varepsilon_1 = .4 &lt; \varepsilon_2 = 1.2 &lt; \varepsilon_3 = 1.4$)</th>
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<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
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<td><img src="image5" alt="Graph" /></td>
<td><img src="image6" alt="Graph" /></td>
<td><img src="image7" alt="Graph" /></td>
<td><img src="image8" alt="Graph" /></td>
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</table>

We also show that the Engel effect alone could not generate PD without counterfactual implications.
2nd Extension: Introducing Catching-up

\[ \tilde{A}_j(t) = \tilde{A}_j(0) e^{g\lambda_j(t - \theta_j \lambda t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_\lambda t}, \]

Countries differ only in the initial value, \( \lambda_0 \), converging exponentially over time at the same rate, \( g_\lambda > 0 \)

<table>
<thead>
<tr>
<th>Peak Time</th>
<th>Peak M-Share</th>
<th>Peak time Per Capita Income</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="graph1.png" alt="Graph" /></td>
<td><img src="graph2.png" alt="Graph" /></td>
<td><img src="graph3.png" alt="Graph" /></td>
</tr>
</tbody>
</table>

Higher-\( \lambda \) countries
- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless \( g_\lambda \) is too large.
(Very Selective) Literature Review. Herrendorf-Rogerson-Valentinyi (14) for a survey on structural change

Related to The Baseline Model

Premature Deindustrialization, Rodrik (16)

The Baumol Effect: Baumol (67), Ngai-Pissarides (07), Nordhaus (08)

Sectoral implications of cross-country heterogeneity in technology development

- Log-supermodularity: Krugman (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- Productivity difference across countries the largest in A: Caselli (05), Gollin et.al. (14, AERP&P)
- Small adoption lags in M; Rodrik (2013)

Related to Two Extensions

The Engel Effect (Nonhomotheticity): Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zeitmueler (08), Buera-Kaboski (09,12), Boppart (14), Comin-Lashkari-Mestieri (21), Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)

Catching-Up/Technology Diffusion: Acemoglu (08), Comin-Mestieri (18)

The Issues We Abstract From

Sector-level productivity growth rate differences across countries: Huneeus-Rogerson (20)

Open economy implications: Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (WinP)

Endogenous growth, externalities, Matsuyama(92),

Sectoral wedges/misallocation: Caselli(05), Gollin et.al. (14 QJE) and many others

Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.
Structural Change, the Baumol Effect, and Adoption Lags
Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = (A)griculture, Sector-2 = (M)anufacturing, Sector-3 = (S)ervices.

**Demand System:** $L$ Identical HH, each supplies 1 unit of mobile labor at $w$; $\kappa_j$ units of factor specific to $j$ at $\rho_j$.

**Budget Constraint:**

$$\sum_{j=1}^{3} p_j c_j \leq E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j$$

**CES Preferences:**

$$U(c) = \left[ \sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}}, \frac{\sigma}{\sigma-1} \right]$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

**Expenditure Shares:**

$$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j (p_j)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k (p_k)^{1-\sigma}} = \beta_j \left( \frac{E/p_j}{U} \right)^{\sigma-1}$$
Three Competitive Sectors: Production

Cobb-Douglas

\[ Y_j = A_j (\kappa_j L_j)^\alpha (L_j)^{1-\alpha} \]

\( A_j > 0 \): the TFP of sector-\( j \); \( \alpha \in [0,1) \) the share of specific factor.

Employment Share

\[ s_j \equiv \frac{L_j}{L} ; \quad \sum_{j=1}^{3} s_j = 1 \]

Output per worker

\[ \frac{Y_j}{L_j} = \bar{A}_j (s_j)^{-\alpha} ; \quad \frac{Y_j}{L} = \bar{A}_j (s_j)^{1-\alpha} \]

where \( \bar{A}_j \equiv A_j (\kappa_j)^\alpha \).

With Cobb-Douglas, \( wL_j = (1 - \alpha)p_j Y_j \), implying the employment shares equal to

Value-Added Shares

\[ \frac{p_j Y_j}{EL} = \frac{p_j Y_j}{\sum_{k=1}^{3} p_k Y_k} = s_j = \frac{L_j}{L} \]
Equilibrium: The expenditure shares are equal to the employment and value-added shares.

\[ m_j = \frac{p_j Y_j}{EL} = s_j \]

which lead to

Equilibrium Shares

\[ s_j = \frac{\left[ \beta_j \right]^{1 \sigma^{-1} A_j} - a}{\sum_{k=1}^{3} \left[ \beta_k \right]^{1 \sigma^{-1} A_k} - a} \]

Per Capita Income

\[ U = \left\{ \sum_{k=1}^{3} \left[ \beta_k \right]^{1 \sigma^{-1} A_k} \right\}^{-\frac{1}{a}} \]

where

\[ a \equiv \frac{1 - \sigma}{1 - \alpha(1 - \sigma)} = - \frac{\partial \log(s_j/s_k)}{\partial \log(A_j/A_k)} > 0, \]

which captures how much relatively high productivity in a sector contributes to its relatively low equilibrium share. \( \alpha \) magnifies this effect by increasing \( a \).
Productivity Growth:

\[ \tilde{A}_j(t) = \tilde{A}_j(t - \lambda_j) = \tilde{A}_j(0)e^{g_j(t-\lambda_j)} = \tilde{A}_j(0)e^{-\lambda_j g_j e^{g_j t}} \]

\[ \tilde{A}_j(t) = \tilde{A}_j(0)e^{g_j t} \]: Frontier Technology in \( j \), with a constant growth rate \( g_j > 0 \).

\[ \tilde{A}_j(t) = \tilde{A}_j(t - \lambda_j) ; \lambda_j = \text{Adoption Lag} \text{ in } j. \]

- \( g_j \) and \( \lambda_j \) are sector-specific.
- \( \lambda_j \) has no “growth” effect.
- \( \lambda_j \) has the “level” effect, \( e^{-\lambda_j g_j} \), which is decreasing in \( \lambda_j \) and the effect is proportional to \( g_j \)

Key: Log-submodularity, \( \frac{\partial}{\partial g_j} \left( \frac{\partial}{\partial \lambda_j} \ln e^{-\lambda_j g_j} \right) < 0 \): \( g_j \) magnifies the negative effect of the adoption lag on productivity

A large adoption lag would not matter much in a sector with slow productivity growth.

Even a small adoption lag would matter a lot in a sector with fast productivity growth.

\[ U(t) = \left\{ \sum_{k=1}^{3} \left[ \beta_k \frac{1}{\sigma - 1} \tilde{A}_k \right] \right\}^{-1/a} = \left\{ \sum_{k=1}^{3} \tilde{\beta}_k e^{-a g_k(t-\lambda_k)} \right\}^{-1/a} , \text{ where } \tilde{\beta}_k \equiv \left( \beta_k \frac{1}{\sigma - 1} \tilde{A}_k(0) \right)^{-1} = \left( \frac{\beta_k^{1-\sigma}}{\tilde{A}_k(0)} \right)^{a} > 0. \]

Longer adoption lags would shift down the time path of \( U(t) \).

\[ g_U(t) \equiv \frac{U'(t)}{U(t)} = \sum_{k=1}^{3} g_k s_k(t) \]

The aggregate growth rate is the weighted average of the sectoral growth rates
Relative Prices: 
\[
\left( \frac{p_j(t)}{p_k(t)} \right)^{1-\sigma} = \left[ \left( \frac{\beta_j}{\beta_k} \right)^{-\alpha} \frac{\bar{A}_j(0)}{\bar{A}_k(0)} \right]^{-\alpha} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} 
\Rightarrow \frac{d \ln \left( \frac{p_j(t)}{p_k(t)} \right)}{dt} = \frac{a(g_k - g_j)}{1 - \sigma}
\]

Relative Growth Effect: \( p_j(t)/p_k(t) \) is de(in)creasing over time if \( g_j > (<) g_k \).

Relative Level Effect: A higher \( \lambda_j g_j - \lambda_k g_k \) raises \( p_j(t)/p_k(t) \) at any point in time.

Note: For a fixed \( \lambda_j \), a higher \( g_j \) makes the relative price of \( j \) higher (though declining faster).

Relative Shares: 
\[
\frac{s_j(t)}{s_k(t)} = \frac{\beta_j}{\beta_k} \left( \frac{p_j(t)}{p_k(t)} \right)^{1-\sigma} = \frac{\bar{\beta}_j}{\bar{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} 
\Rightarrow \frac{d \ln \left( \frac{s_j(t)}{s_k(t)} \right)}{dt} = \frac{a(g_k - g_j)}{1 - \sigma}
\]

Relative Growth Effect: \( s_j(t)/s_k(t) \) is de(in)creasing over time if \( g_j > (<) g_k \).

Shift from faster growing sectors to slower growing sectors over time.

Relative Level Effect: A higher \( \lambda_j g_j - \lambda_k g_k \) raises \( s_j(t)/s_k(t) \) at any point in time.

Note: For a fixed \( \lambda_j \), a higher \( g_j \) makes the relative share of \( j \) higher (though declining faster).
Structural Change with the Baumol (Relative Price) Effect: Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing in $t$, because

$$\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[ \frac{\tilde{\beta}_2}{\tilde{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)} \right] e^{a(g_1 - g_2)t} + \left[ \frac{\tilde{\beta}_3}{\tilde{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)} \right] e^{a(g_1 - g_3)t}$$

Rise of Services: $s_3(t)$ is increasing in $t$, because

$$\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[ \frac{\tilde{\beta}_1}{\tilde{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)} \right] e^{-a(g_1 - g_3)t} + \left[ \frac{\tilde{\beta}_2}{\tilde{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)} \right] e^{-a(g_2 - g_3)t}$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped in $t$, because

$$\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[ \frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)} \right] e^{-a(g_1 - g_2)t} + \left[ \frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)} \right] e^{a(g_2 - g_3)t}.$$  

Hump-shaped due to the two opposing forces: $g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

$$s_2'(t) \preceq 0 \iff (g_1 - g_2) \frac{s_1(t)}{s_2(t)} \preceq (g_2 - g_3) \frac{s_3(t)}{s_2(t)} \iff g_U(t) = \sum_{k=1}^{3} g_k s_k(t) \preceq g_2$$
Characterizing Manufacturing Peak: “^^” indicates the peak.

\[ s_2'(\hat{t}) = 0 \iff (g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t}) \iff g_U(\hat{t}) = g_2 \]

Peak Time: From \((g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t})\)

\[ \hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \quad \text{where} \quad \hat{t}_0 = \frac{1}{a(1-g_3)} \ln \left( \frac{g_1 - g_2}{g_2 - g_3} \frac{\bar{\beta}_1}{\bar{\beta}_3} \right) \]

Two Normalizations: Without any loss of generality,

\[ \hat{t}_0 = 0 \iff \frac{g_2 - g_3}{g_1 - g_2} = \frac{\bar{\beta}_1}{\bar{\beta}_3} \equiv \left( \frac{\beta_1}{\beta_3} \right)^{1-\sigma} \left( \frac{\bar{A}_3(0)}{\bar{A}_1(0)} \right) \]

The calendar time is reset so that its M-peak would be reached at \(\hat{t} = 0\) in the absence of the adoption lags.

\[ U(0) = 1 \text{ for } \lambda_1 = \lambda_2 = \lambda_3 = 0 \iff \sum_{k=1}^{3} \tilde{\beta}_k = \sum_{k=1}^{3} \left( \frac{\beta_k}{\bar{A}_k(0)} \right)^{1-\sigma} = 1. \]

We use the peak time per capita income in the absence of the adoption lags as the numeraire.

Note: Under these normalizations, the peak time share of sector-\(k\) in the absence of the adoption lags would be \(\tilde{\beta}_k\).
Then,

**Peak Time**

\[ \hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}. \]

**Peak M-Share**

\[ \frac{1}{s_2(\hat{t})} = 1 + \left( \frac{1}{\beta_2} - 1 \right) e^{a(g_1-g_2)(g_2-g_3)/(g_1-g_3)} \left( \frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \frac{\lambda_3 g_3}{g_2 - g_3} \right) \]

**Peak Time Per Capita Income**

\[ U(\hat{t}) = \left\{ \left( 1 - \bar{\beta}_2 \right) e^{-a g_1 g_3 \left( \frac{\lambda_1 - \lambda_3}{g_1 - g_3} \right)} + \bar{\beta}_2 e^{-a g_2 \left( \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} - \lambda_2 \right)} \right\}^{-\frac{1}{a}} \]

So far, we have looked at the impacts of adoption lags in a single country in isolation, without specifying the sources of the adoption lags.

Next, we introduce cross-country heterogeneity, the technology gap, which generate cross-country variations in adoption lags, and study the cross-country implications.
Technology Gaps and Premature Deindustrialization
Consider the world with many countries with 
\[(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda\]

\(\lambda \geq 0\): **Technology Gap, Country-specific**

\(\theta_j > 0\): **Sector-specific**, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in one dimension**, \(\lambda\), in their ability to adopt the frontier technologies.
- \(\theta_j > 0\) determines how the technology gap affects the adoption lag in that sector.

\[
\frac{\dot{A}_j(t)}{\dot{A}_k(t)} = \frac{\dot{A}_j(0)}{\dot{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)\epsilon} = \frac{\partial}{\partial \lambda} \ln \left( \frac{\dot{A}_j(t)}{\dot{A}_k(t)} \right) = -(\theta_j g_j - \theta_k g_k)
\]

Cross-country productivity difference is larger in sector-\(j\) than in sector-\(k\) if \(\theta_j g_j > \theta_k g_k\).

**Peak Time**

\[\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda.\]

**Peak M-Share**

\[
\frac{1}{S_2(\lambda)} = 1 + \left( \frac{1}{\beta_2} - 1 \right) e^{(g_2 - g_3)} \left( \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \right) a \lambda
\]

**Peak Time Per Capita Income**

\[
\hat{U}(\lambda) = \left\{ (1 - \tilde{\beta}_2) e^{-g_1 g_3 (\frac{\theta_1 - \theta_3}{g_1 - g_3}) a \lambda} + \tilde{\beta}_2 e^{-g_2 (\frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \theta_2) a \lambda} \right\} \frac{1}{a}
\]
Figure 1: Conditions for Premature Deindustrialization (PD) only with the Baumol (Relative Price) Effect

\[ \hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \iff \theta_1 g_1 > \theta_3 g_3. \]

With \( \theta_1 g_1 > \theta_3 g_3 \), the price of A is high and the price of S is low relative to M in a high-\( \lambda \) country, which delays structural change.

\[ \hat{s}_2'(\lambda) < 0 \text{ for all } \lambda > 0 \iff \frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} > \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}. \]

With a low \( \theta_2 \), which has no effect on \( \hat{t} \), the price of M is low relative to both A & S in a high-\( \lambda \) country, which keeps the M-share low.

Under the above condition,

\[ \hat{U}'(\lambda) < 0; \ U(\lambda) < U(0) \text{ for } \lambda > \lambda_c > 0 \iff \theta_1 < \theta_3 \iff \hat{t}(\lambda) < \theta_1 \lambda < \theta_3 \lambda \]

\[ \hat{U}'(\lambda) < 0 \text{ for all } \lambda > 0 \iff (1 - \Theta) \left( 1 - \frac{\theta_2}{\theta_3} \right) < 1 - \frac{\theta_1}{\theta_3}, \]

where \( g_3 / g_1 < \Theta < 1. \)

These conditions jointly imply \( \theta_1 g_1 > \theta_2 g_2, \theta_3 g_3 \) (productivity differences the largest in A) and \( \theta_1, \theta_2 < \theta_3 \) (adoption lag the longest in S).
Some Examples

Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

\[ \theta_1 = \theta_2 = \theta_3 = 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0 \]

\[ \implies \hat{t}(\lambda) = \lambda; \quad \hat{s}_2(\lambda) = \bar{\beta}_2; \quad \bar{U}(\lambda) = 1 \]

- The country’s technology gap causes a delay in the peak time, \( \hat{t} \), by \( \lambda > 0 \).
- The peak M-share & the peak time per capita income unaffected.

Each country follows exactly the same development path of early industrializers with a delay. No PD!!

Thus, the technology gap must have differential impacts on the adoption lags across sectors.
**Example 2a-2c: Numerical Illustrations.** In all three examples, $\theta_1 = \theta_2 < \theta_3 = 1$ and we use $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\alpha = 1/3$, and $\sigma = 0.6$ (hence $\alpha = 6/13$).

$\tilde{\theta}_j = 1/3$ for $j = 1,2,3 \Rightarrow \tilde{s}_2(0) = \tilde{s}_2 = 1/3$; $\tilde{U}(0) = 1$; $\hat{t}(0) = 0$.

\[
\begin{align*}
\frac{\theta_1}{\theta_3} &= \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2} \\
\Rightarrow \theta_1 g_1 &> \theta_2 g_2 = \theta_3 g_3
\end{align*}
\]

Cross-country productivity differences are the same in M & in S in this case.
### Example 2b
\[
\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.35 < \frac{g_3}{g_2} \\
\implies \theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2
\]
Cross-country productivity differences the smallest in M.

### Example 2c
\[
\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.75 > \frac{g_3}{g_2} \\
\implies \theta_1 g_1 > \theta_2 g_2 > \theta_3 g_3
\]
Cross-country productivity differences the smallest in S.
Introducing the Engel Effect
The Engel Law through Isoelastic Nonhomothetic CES; Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

\[
\left[ \sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} \left( \frac{c_j}{U^{\varepsilon_j}} \right)^{\frac{1-\sigma}{\sigma-1}} \right]^{\sigma-1} \equiv 1
\]

Normalize \( \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3 \); with \( \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1 \), we go back to the standard homothetic CES.

With \( \sigma < 1 \), \( 0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 \Rightarrow \) the income elasticity the lowest in A and the highest in S.

By maximizing \( U \) subject to \( \sum_{j=1}^{3} p_j c_j \leq E \),

Expenditure Shares

\[
m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j \left( \frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k \left( \frac{U^{\varepsilon_k} p_k}{E} \right)^{1-\sigma}} \Rightarrow \frac{m_j}{m_k} = \frac{\beta_j \left( \frac{p_j U^{\varepsilon_j-\varepsilon_k}}{P} \right)^{1-\sigma}}{\beta_k \left( \frac{p_k U^{\varepsilon_j-\varepsilon_k}}{P} \right)^{1-\sigma}}
\]

Indirect Utility Function:

\[
\left[ \sum_{j=1}^{3} \beta_j \left( \frac{U^{\varepsilon_j} p_j}{E} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \equiv 1
\]

Cost-of-Living Index:

\[
\left[ \sum_{j=1}^{3} \beta_j \left( \frac{U^{\varepsilon_j-1} p_j}{P} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \equiv 1 \Leftrightarrow U \equiv \frac{E}{P}
\]

Income Elasticity:

\[
\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln(U)} = 1 + \frac{\partial \ln m_j}{\partial \ln(E/P)} = 1 + (1 - \sigma) \left\{ \varepsilon_j - \sum_{k=1}^{3} m_k \varepsilon_k \right\}
\]
Structural Change with the Engel (Income) Effect: Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.

Then, even with constant relative prices,

**Decline of Agriculture:** $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1} \left( \frac{p_2}{p_1} U(t)^{\varepsilon_2 - \varepsilon_1} \right)^{1-\sigma} + \frac{\beta_3}{\beta_1} \left( \frac{p_3}{p_1} U(t)^{\varepsilon_3 - \varepsilon_1} \right)^{1-\sigma}$$

**Rise of Services:** $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3} \left( \frac{p_1}{p_3} U(t)^{\varepsilon_1 - \varepsilon_3} \right)^{1-\sigma} + \frac{\beta_2}{\beta_3} \left( \frac{p_2}{p_3} U(t)^{\varepsilon_2 - \varepsilon_3} \right)^{1-\sigma}$$

**Rise and Fall of Manufacturing:** $s_2(t) = m_2(t)$ is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2} \left( \frac{p_1}{p_2} U(t)^{\varepsilon_1 - \varepsilon_2} \right)^{1-\sigma} + \frac{\beta_3}{\beta_2} \left( \frac{p_3}{p_2} U(t)^{\varepsilon_3 - \varepsilon_2} \right)^{1-\sigma}.$$  

Hump-shaped due to the two opposing forces: $\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

$$s_2'(t) = m_2'(t) \geq 0 \iff \left( \varepsilon_2 - \varepsilon_1 \right) \frac{m_1(t)}{m_2(t)} \geq \left( \varepsilon_3 - \varepsilon_2 \right) \frac{m_3(t)}{m_2(t)} \iff \eta_2 \geq 1$$

with constant relative prices.
The production side is the same as before. By following the same step, we obtain

**Equilibrium Shares**

\[ s_j = \frac{1}{U^{\bar{A}_j}} \left( \beta_j^{\sigma-1} \right)^{-a}, \quad \text{where} \quad \sum_{k=1}^{3} \frac{1}{U^{\bar{A}_k}} = 1 \]

With \( \bar{A}_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{g_j(t - \theta_j\lambda)} \),

\[ s_2(t): \quad \frac{1}{s_2(t)} = U(t)^{a(\varepsilon_1 - \varepsilon_2)} \left[ \frac{\bar{b}_2}{\bar{b}_1} e^{a(\theta_1 g_1 - \theta_2 g_2)\lambda} \right] e^{-a(g_1 - g_2)t} + 1 + U(t)^{a(\varepsilon_3 - \varepsilon_2)} \left[ \frac{\bar{b}_3}{\bar{b}_2} e^{a(\theta_3 g_3 - \theta_2 g_2)\lambda} \right] e^{a(g_2 - g_3)t} \]

\[ U(t): \quad U(t)^{a\varepsilon_1} \bar{b}_1 e^{-ag_1(t - \theta_1\lambda)} + U(t)^{a\varepsilon_2} \bar{b}_2 e^{-ag_2(t - \theta_2\lambda)} + U(t)^{a\varepsilon_3} \bar{b}_3 e^{-ag_3(t - \theta_3\lambda)} \equiv 1 \]

\[ s'_2(t) = 0: \quad (g_1 - g_2) = (g_2 - g_3)U^{a(\varepsilon_3 - \varepsilon_2)} \left[ \frac{\bar{b}_3}{\bar{b}_1} e^{a(\theta_3 g_3 - \theta_1 g_1)\lambda} e^{a(g_1 - g_2)t} \right] + \left\{ \varepsilon_1 U^{a(\varepsilon_1 - \varepsilon_2)} \bar{b}_1 e^{-ag_1(t - \theta_1\lambda)} + \varepsilon_2 U^{a(\varepsilon_3 - \varepsilon_2)} \bar{b}_2 e^{-ag_2(t - \theta_2\lambda)} + \varepsilon_3 U^{a(\varepsilon_3 - \varepsilon_2)} \bar{b}_3 e^{-ag_3(t - \theta_3\lambda)} \right\} \]

\( \hat{t} \) and \( \hat{U} \) solve the equation for \( U(t) \) and the equation for \( s'_2(t) = 0 \), simultaneously.

Then, \( \hat{s}_2 \) can be obtained by plugging \( \hat{t} \) and \( \hat{U} \) into the equation for \( s_2(t) \).
A Technology-Gap Model of Premature Deindustrialization

I. Fujiwara and K. Matsuyama

(Analytically Solvable)

“Unbiased” Case

\[ 0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}} \]

where \( \bar{g} \equiv \frac{g_1 + g_2 + g_3}{3} \)

Peak Time

\[ \hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda - \ln \left\{ \left( 1 - \bar{\beta}_2 \right) e^{-g_1 g_3 \left( \frac{\theta_1 - \theta_3}{g_1 - g_3} \right) a \lambda} + \tilde{\beta}_2 e^{-g_2 \left( \frac{\theta_1 g_1 - \theta_3 g_3 - \theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \right) a \lambda} \right\}^{-\frac{1}{a \left( 1 + \mu g \right)}} \]

Peak M-Share

\[ \frac{1}{s_2(\hat{t})} = 1 + \left( 1 - \frac{1}{\tilde{\beta}_2} \right) e^{(g_2 - g_3) \left( \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \right) a \lambda} \]

Peak Time Per Capita Income

\[ U(\hat{t}) = \left\{ \left( 1 - \tilde{\beta}_2 \right) e^{-g_1 g_3 \left( \frac{\theta_1 - \theta_3}{g_1 - g_3} \right) a \lambda} + \tilde{\beta}_2 e^{-g_2 \left( \frac{\theta_1 g_1 - \theta_3 g_3 - \theta_2 g_2 - \theta_3 g_3}{g_2 - g_3} \right) a \lambda} \right\}^{-\frac{1}{a \left( 1 + \mu g \right)}} \]

\[ \frac{\partial s_2(\hat{t})}{\partial \lambda} < 0; \quad \frac{\partial U(\hat{t})}{\partial \lambda} < 0 \text{ under the same condition; } \frac{\partial \hat{t}}{\partial \lambda} > 0 \text{ under a weaker condition.} \]

With \( g_1, g_2, g_3 \) fixed, a higher \( \mu \) has

- **No effect** on \( \hat{t}, s_2(\hat{t}), U(\hat{t}) \) for the country with \( \lambda = 0 \).
- A further delay in \( \hat{t} \) for every country with \( \lambda > 0 \).
- **No effect** on \( s_2(\hat{t}) \) for every country with \( \lambda > 0 \).
- A smaller decline in \( U(\hat{t}) \) for each country with \( \lambda > 0 \).
(Analytically Solvable) “Unbiased” Case: A Numerical Illustration

\[ g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%, \theta = 0.5, a = 6/13; \tilde{\beta}_j = 1/3 \text{ for } j = 1, 2, 3. \]

In this case, \( g_1 - g_2 = g_2 - g_3 = \tilde{g} = 1.2\% > 0 \Rightarrow \epsilon_1 = 1 - \epsilon < \epsilon_2 = 1 < \epsilon_3 = 1 + \epsilon \text{ for } 0 < \epsilon = (1.2\%)\mu < 1 \)
(Empirically More Plausible) Biased Case:

\[ \varepsilon_1 = 1 - \varepsilon < \varepsilon_2 = 1 + \frac{\varepsilon}{3} < \varepsilon_3 = 1 + \frac{2\varepsilon}{3} \text{ for } 0 < \varepsilon < 1 \Rightarrow \frac{g_1 - g_2}{g_2 - g_3} = 1 < \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} = 4, \text{ as in CLM (2021)}. \]

In this case, the frontier country’s peak values are affected by \( \varepsilon \). Relative to the frontier country, a higher \( \varepsilon \) causes a high-\( \lambda \) country to have

- A further delay in \( \hat{t} \)
- A larger decline in \( s_2(\hat{t}) \).
- A smaller decline in \( U(\hat{t}) \).
Stronger nonhomotheticity changes the shape of the time paths significantly. It does not change the implications on PD, i.e., how technology gaps affect $\hat{t}$, $s_2(\hat{t})$, and $U(\hat{t})$.

<table>
<thead>
<tr>
<th>$\varepsilon = 0$</th>
<th>$\varepsilon = 0.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homothetic case ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</td>
<td>Unbiased case ($\varepsilon_1 = 0.4 &lt; \varepsilon_2 = 1 &lt; \varepsilon_3 = 1.6$)</td>
</tr>
</tbody>
</table>

$$(t, s_2(t))$$

$$(\ln U(t), s_2(t))$$
Premature Deindustrialization (PD) through the Engel (Income) Effect Only

What happens if we had *solely* the Engel effect with $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$, without the Baumol effect, $g_1 = g_2 = g_3 = \bar{g} > 0$?

**Peak Time**

$$\hat{t} = \frac{1}{a\bar{g}} \ln \left\{ (1 - \bar{\beta}_2) e^{(\varepsilon_3 \theta_1 - \varepsilon_1 \theta_3) a\bar{g} \lambda} + \bar{\beta}_2 e^{(\theta_2 + (\theta_1 - \theta_3) \varepsilon_2) a\bar{g} \lambda} \right\}$$

**Peak M-Share**

$$\frac{1}{s_2(\hat{t})} - 1 = \left( \frac{1}{\bar{\beta}_2} - 1 \right) e^{(\varepsilon_3 - \varepsilon_2)(\theta_1 - \theta_3) a\bar{g} \lambda}$$

**Peak Time Per Capita Income**

$$\ln U(\hat{t}) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} \bar{g} \lambda$$

with the two normalizations

$$\left( \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} \right) \bar{\beta}_1 = 1; \; \bar{\beta}_1 + \bar{\beta}_2 + \bar{\beta}_3 = 1$$

which ensures $U(\hat{t}) = 1$ and $\hat{t} = 0$ for $\lambda = 0$. 
Conditions for Premature Deindustrialization (PD) only with the Engel Effect

\[
\frac{\partial U(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \iff 0 < \frac{\theta_1}{\theta_3} < 1
\]

With a low \( \theta_1 \) and a high \( \theta_3 \), the price of the income elastic \( S \) is high relative to the income inelastic \( A \) in a high-\( \lambda \) country, which make it necessary to reallocate labor to \( S \) at earlier stage of development.

\[
\frac{\partial s_2(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \iff \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} > \frac{\theta_2 - \theta_3}{\varepsilon_3 - \varepsilon_2}
\]

With a low \( \theta_2 \), which has no effect on \( U(\hat{t}) \), the price of \( M \) is low relative to both \( A \) & \( S \) in a high-\( \lambda \) country, which keeps the M-share low.

Under the above condition,

\[
\frac{\partial \hat{t}}{\partial \lambda} > 0 \text{ for a sufficiently large } \lambda \iff \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3}
\]

\[
\frac{\partial \hat{t}}{\partial \lambda} > 0 \text{ for all } \lambda > 0 \iff \left( \theta_E - \frac{\varepsilon_1}{\varepsilon_3} \right) \left[ 1 - \left( \frac{\varepsilon_3}{\varepsilon_2} \right) \frac{\theta_2}{\theta_3} \right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3}
\]

where \( \varepsilon_1/\varepsilon_3 < \Theta_E < 1 \).

With \( g_1 = g_2 = g_3 = \bar{g} \), PD occurs only if \( \theta_1 \bar{g}, \theta_2 \bar{g} < \theta_3 \bar{g} \), that is, when cross-country productivity difference is the largest in \( S \).
Introducing Catching Up
Narrowing a Technology Gap

We assumed that \( \lambda \) is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.

[In contrast, the aggregate growth rate, \( g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t) \), declines over time, \( g'_U(t) = g_1 s'_1(t) + g_2 s'_2(t) + g_3 s'_3(t) = (g_1 - g_2)s'_1(t) + (g_3 - g_2)s'_3(t) < 0 \), the so-called Baumol’s cost disease.]

What if technological laggards can narrow a technology gap, and hence achieve a higher productivity growth in each sector?

Countries differ only in the initial value of lambda, \( \lambda_0 \), converging exponentially over time at the same rate,

\[
\tilde{A}_j(t) = \tilde{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where} \quad \lambda_t = \lambda_0 e^{-g_t}, \quad g_\lambda > 0.
\]

\[
\Rightarrow \frac{1}{s_2(t)} = \left( \frac{\bar{\beta}_1}{\bar{\beta}_2} \right) e^{a[(\theta_1 g_1-\theta_2 g_2)\lambda_t-(g_1-g_2)t]} + 1 + \left( \frac{\bar{\beta}_3}{\bar{\beta}_2} \right) e^{a[(\theta_3 g_3-\theta_2 g_2)\lambda_t+(g_2-g_3)t]}
\]
Again, by setting the calendar time such that \( \hat{t}_0 = 0 \) for the frontier country with \( \lambda_0 = 0 \),

**Peak Time**

\[
\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} \lambda_{\hat{t}} + D(g\lambda_{\hat{t}})
\]

**Peak Share**

\[
\frac{1}{s_2(\hat{t})} = 1 + \left( \frac{\hat{\beta}_1 + \hat{\beta}_3}{\hat{\beta}_2} \right) \left[ (g_2 - g_3)e^{a(g_2-g_1)D(g\lambda_{\hat{t}})} + (g_1 - g_2)e^{a(g_2-g_3)D(g\lambda_{\hat{t}})} \right] \left[ e^{\frac{a(g_1-g_2)(g_2-g_3)}{g_1-g_3}} \right] \left( \frac{\theta_1 g_1 - \theta_2 g_2 + \theta_3 g_3 - \theta_2 g_2}{g_1-g_2} \right) \lambda_{\hat{t}}
\]

**Peak Time Per Capita Income**

\[
U(\hat{t}) = \left\{ \left( \hat{\beta}_1 e^{-ag_1D(g\lambda_{\hat{t}})} + \hat{\beta}_3 e^{-ag_3D(g\lambda_{\hat{t}})} \right) e^{\frac{a(\theta_1-\theta_3)g_1 g_3 \lambda_{\hat{t}}}{g_1-g_3}} + \left( \hat{\beta}_2 e^{-ag_2D(g\lambda_{\hat{t}})} \right) e^{\frac{a(\theta_1-\theta_2)g_1 g_2 + (\theta_2-\theta_3)g_3 g_2 \lambda_{\hat{t}}}{g_1-g_3}} \right\}^{\frac{1}{a}}
\]

where

\[
D(g\lambda_{\hat{t}}) = \frac{1}{a(g_1 - g_3)} \ln \left[ \frac{g_1 - g_2 + (\theta_1 g_1 - \theta_2 g_2) g\lambda_{\hat{t}}}{g_2 - g_3 - (\theta_3 g_3 - \theta_2 g_2) g\lambda_{\hat{t}}} \left( \frac{g_2 - g_3}{g_1 - g_2} \right) \right].
\]

For \( g\lambda = 0, D(g\lambda_{\hat{t}}) = D(0) = 0 \), and all the parts in red disappear, and we go back to the baseline model.
Technological laggards
- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless $g_A$ is too large: Comin-Mestieri (2018)
Concluding Remarks
A Parsimonious model of Rodrik’s (2016) PD based on
- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

We find that PD occurs for
- cross-country productivity difference larger in A than in S.
- technology adoption takes not too long in M.
- Technology adoption takes longer in S than in A.
which implies that cross-country productivity difference the largest in A; that technology adoption the longest in S.

The baseline model assumes **homothetic CES** (to focus on the Baumol effect) and **no catching up** (to isolate the level effect from the growth effect).

In two extensions, we showed that the results are **robust** against introducing
- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)
  The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD
  The Engel effect alone could not generate PD w/o counterfactual implications on cross-country productivity differences
- **Narrowing a technology gap** to allow technological laggards to catch up
  unless the catching-up speed is too large.
Appendix
Appendix: Non-agricultural share as another measure of development, \(1 - s_1(\hat{t}) = s_2(\hat{t}) + s_3(\hat{t}) \equiv s_n(\hat{t})\)

Baseline Homothetic Case:

\[
\begin{array}{l}
\ln U(t), s_2(t) \\

\end{array}
\]

\[
\begin{array}{l}
(s_n(t), s_2(t)) \\

\end{array}
\]
Nonhomothetic Cases:

<table>
<thead>
<tr>
<th></th>
<th>Unbiased: $\varepsilon_1 = 1 - \varepsilon &lt; \varepsilon_2 = 1 &lt; \varepsilon_3 = 1 + \varepsilon$</th>
<th>Biased: $\varepsilon_1 = 1 - \varepsilon &lt; \varepsilon_2 = 1 + \varepsilon/3 &lt; \varepsilon_3 = 1 + 2\varepsilon/3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln U(t)$</td>
<td><img src="image" alt="Graph of $\ln U(t)$" /></td>
<td><img src="image" alt="Graph of $\ln U(t)$" /></td>
</tr>
<tr>
<td>$s_n(t)$</td>
<td><img src="image" alt="Graph of $s_n(t)$" /></td>
<td><img src="image" alt="Graph of $s_n(t)$" /></td>
</tr>
</tbody>
</table>

In the biased case, the frontier country’s peak values are affected by $\varepsilon$. 

\[
\ln U(t) = \ln U(0) - \varepsilon t
\]

\[
s_n(t) = s_n(0) - \frac{\varepsilon}{1 - \varepsilon} t
\]