# Imperfect Credit Markets, Household Wealth Distribution, and Development

(Prepared for *Annual Review of Economics*)

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## 1. Introduction

**Topic:** Understand *theoretically* how credit constraints affect the intertwining nature of the dynamics of wealth distribution and development.

**Objective:** *Highlight* some key results by using a series of illustrative models in an attempt to dispel some *commonly held misconceptions* in the literature.

**Strategy:** Conduct 3 types of models, which progressively build on one another.

**1. Single Dynasty Model**
- Can explain individual poverty traps
- Not useful for understanding the effects of distribution on development.

**2. Model of Interacting Dynasties with a Fixed Threshold**
- Can explain collective poverty traps and path-dependence in distribution dynamics
- Not useful for understanding the effects of inequality on development

**3. Models of Interacting Dynasties with a Variable Threshold**
- Offer a rich and flexible framework for understanding the interactive nature of inequality and development under credit constraints.

**Single Dynasty Models and Individual Poverty Traps:** e.g., Galor & Zeira (1993)

*Key Features*:

- It looks at the wealth dynamics of a *single* household in *isolation*.
- Due to credit constraints, the household needs to be wealthy enough to be able to finance the profitable investment (e.g., set up a firm or obtain higher education).

*Key Results*:

- A poor household, who starts below the threshold, may be caught in vicious circles of poverty, or at least will have difficulty getting out of poverty.
- Wealth inequality might rise at least initially over the course of development, generating something akin to the Kuznets Inverted U-curve.

*Caution:*

- Contrary to the claim often made in the literature, this type of analysis cannot make any clear prediction regarding the effects of wealth distribution in general, and wealth inequality in particular, on the development process.
- This is because household wealth dynamics are independent of household wealth distribution in the economy.

**Models of interacting dynasties with fixed thresholds:** e.g., Banerjee-Newman (1993)

*Key Features*:

- The rich, whose wealth is above the (fixed) threshold, become employers, and the poor, whose wealth is below, work for the rich.
- More poor mean a lower wage, making it harder for them to get out of poverty. More rich mean a higher wage, helping poor workers to get out of poverty.

*Key Results:*

- *Collective poverty traps;* the poor remain poor, because there are *so many* of them.
- *Path-dependence*; Wealth distribution dynamics depends on the initial condition.

*Cautions:* This should not be interpreted as "persistence of inequality,"

- Greater initial inequality does not necessarily mean greater long run inequality.
- More initial inequality could mean more rich, which could mean long run equality.
- No clear prediction about the effects of wealth inequality, due to the ***absolute notion*** of the rich and the poor
- *Fixed* threshold also creates an "underdevelopment trap." If everyone is poor, nobody hires, and hence everyone remains poor. *Trivial and Not Robust.*

**Models of Interacting Dynasties with Variable Thresholds**: e.g., Aghion-Bolton (1997), Freeman (1996), Matsuyama (2000, 2006), Moohkerjee-Ray (2003, 2010).

*Key Feature: **Relative notion** of the rich and the poor*;
For example, let the borrowing limit go up with the profitability of the investment. Then,

More poor → lower wage → investment more profitable, easing the credit constraint → lower threshold, giving the relatively poor access to credit

*Key Results:*

- No underdevelopment trap, everybody remaining poor is not an equilibrium.
- Two new cases
  - *Symmetry-breaking*: Even if initial distribution is perfectly equal, there will be long run inequality. A one-shot redistribution would be ineffective.
  - *Global Convergence*: Even if initial distribution is highly unequal, there will be long run equality. Redistribution is unnecessary to achieve long run equality.

**Thought experiment: Effects of Penniless Immigrants on Poor households**

- Single Dynasty Model → No Effect
- Interacting Dynasties with Fixed Threshold → Negative Effect
- Interactive Dynasties with Variable Threshold → Possibly Positive Effect

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## 2. *A Single Dynasty Model: Individual Poverty Traps*

**Time:** Discrete (t = 0, 1, 2,…)

**Final (Numeraire) Good**: used for both Consumption and Investment

**A Dynasty:** Infinite-sequence of one-period lived agents linked by inheritance

**An Agent** (living in period t):

- Inherit his wealth, $w_t$, in the form of bequest at the beginning of the period
- Make investment "choices" to maximize the end-of-the period wealth.
- Earn some additional income, $y$.
- Consume by $c_t$ and bequests $w_{t+1}$ at the end of the period

**Two Ways of Allocating the Inheritance, $w_t$.**

- Run a **non-divisible investment project**, which converts $F$ units of the input at the beginning of period t into $R$ units in **Final Good** at the end of period t, by **borrowing** $F-w_t$ at the market rate of return equal to $r$.
- **Lend** $x_t \leq w_t$ units of the input at the beginning of period t for $rx_t$ units of the final good at the end of period t. (Or, **Storage** with the rate of return, $r$.)

**Agent's End-of-Period Wealth:**

$U_t = y + R - r(F-w_t) = y + R - rF + rw_t,$ if borrow and run the project,

$U_t = y + rw_t$ if lend (or put in storage).

***Profitability Constraint*** **(PC):** $R \geq rF$

***Borrowing Constraint*** **(BC):** $\lambda R \geq r(F-w_t) \rightarrow w_t \geq w_c \equiv F - \lambda R/r.$

Let $R > rF$. Then, the agent invests if and only if (BC) holds.

**Agent's Consumption and Bequest Decisions:**

$Max\left(\frac{c_t}{1-\beta}\right)^{1-\beta}\left(\frac{w_{t+1}}{\beta}\right)^{\beta}$ s.t. $c_t + w_{t+1} \leq U_t \rightarrow c_t = (1-\beta)U_t, w_{t+1} = \beta U_t$

**Dynasty's Wealth Accumulation:**

$$w_{t+1} = \beta U_t = \begin{cases} \beta(y + rw_t) & \text{if} \quad w_t < w_c \equiv F - \lambda R/r, \\ \beta[y + rw_t + (R-rF)] & \text{if} \quad w_t \geq w_c \equiv F - \lambda R/r. \end{cases}$$

**Figure 1: Wealth Dynamics in a Single Dynasty Model:**

Let $\beta r < 1$, $W_L \equiv \dfrac{\beta y}{1-\beta r}$ and $W_H \equiv \dfrac{\beta(y+R-rF)}{1-\beta r}$.

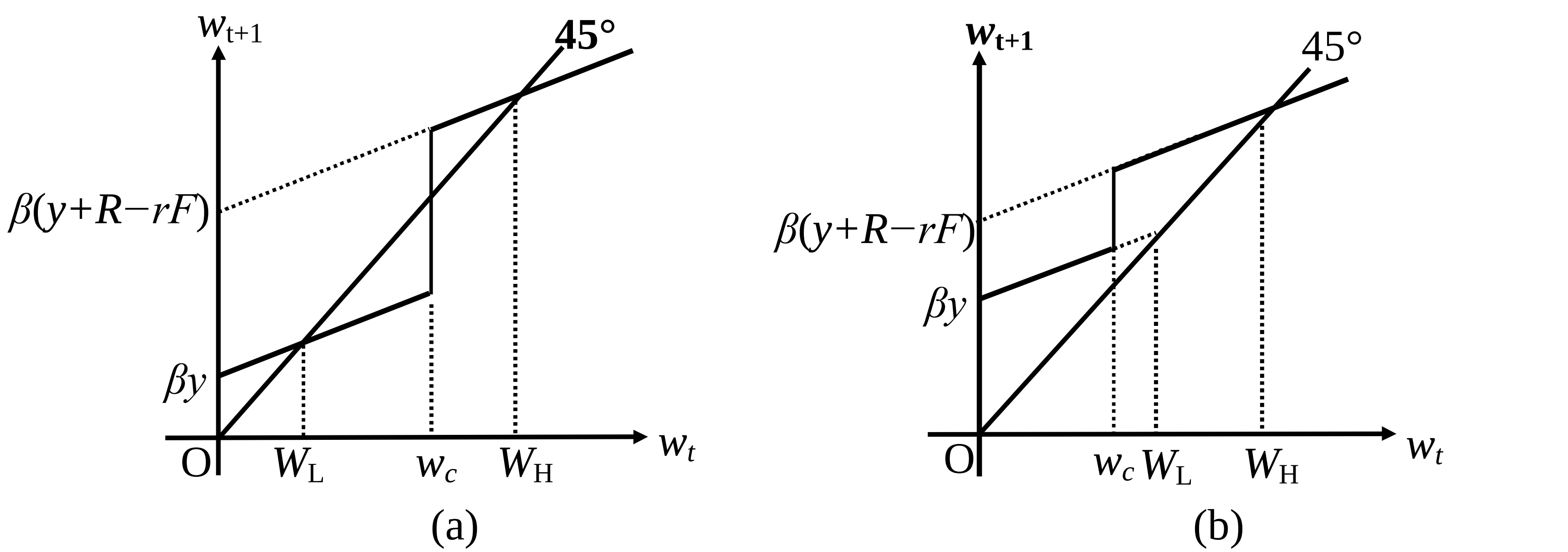



Fig 1a, $W_L < w_c < W_H$.
Long run household wealth depends on its initial wealth.→ **Individual Poverty Traps**

Fig 1b, $w_c < W_L < W_H$.
It could take a long time for the poor to get out of poverty. → Wealth inequality may rise initially and then go down, as **Kuznets Inverted U-hypothesis**.

*Some Notes:*

- This model describes one household in isolation.
  - Each household's wealth dynamics is independent of initial wealth distribution, $G_0(w)$.
  - A change in $G_0(w)$ has no effect, unless it also changes the initial wealth of this household.

- No clear prediction on the relation b/w initial distribution and long run wealth.

- In Fig.1(a), the average wealth converges to $X_0W_L + (1-X_0)W_H$, increasing in $X_0 \equiv G_0(w_c)$. Eliminating initial inequality would maximize (minimize) it, if the initial average is higher (lower) than $w_c$.
- Some might argue that more inequality is growth-enhancing in poor countries and growth-retarding in rich countries. This is not robust, either, because it is a mere artifact of the assumption that there is only one investment project.

## *3. Interacting Dynasties; Labor Market Channel*

**Time:** Discrete (t = 0, 1, 2,…)

**Final Good**: used both for Consumption and Investment

**A Continuum of Inherently-Identical Infinitely-Lived Dynasties:**

- Each is linked by one period lived agent through inheritance
- In each period, they differ only in inheritance. $w_t \sim G_t(w)$.

**An Agent of a Particular Dynasty, living in period t**:

- Receives the initial wealth, $w_t$, in bequest at the beginning of the period
- Make occupational & investment "choices" to maximize the end-of-the period wealth.
- Consume by $c_t$ and bequest $w_{t+1}$ at the end of the period

**Occupational and Investment Choices:**

- *Worker*: Earns the wage rate, $v_t$; lends $w_t$ at the gross return $r$
- *Entrepreneur:* Borrows $F - w_t$ at the gross rate of return, $r$, and sets up a firm, which requires $F$ units of the final good at the beginning of period. The firm hires labor at the wage rate, $v_t$, and produces the final good at the end of period, with the technology, $\varphi(n)$; $\varphi'(n) > 0 > \varphi''(n)$; $\varphi(0) = 0$ and $\varphi(\infty) = \infty$.

Labor Employment: $n(v_t) \equiv \text{Argmax}_n\{\varphi(n) - v_t n\}$

Gross Profit: $\pi(v_t) \equiv \text{Max}_n\{\varphi(n) - v_t n\} \equiv \varphi(n(v_t)) - v_t n(v_t))\}$

$$\pi'(v) = -n(v) < 0,\ \pi''(v) = -n'(v) > 0$$

$$\pi(0) = n(0) = \varphi(\infty) = \infty.$$

**Agent's End-of-Period Wealth:**

$U_t = v_t + rw_t,$ by becoming a worker

$U_t = \pi(v_t) + r(w_t - F)$ by becoming an entrepreneur

***Profitability Constraint*** **(PC):** $\pi(v_t) - v_t \geq rF \leftrightarrow v_t \leq V$, with $\pi(V) - V \equiv rF$.

- $v_t < V$, every agent wants to be an employer.
- $v_t = V$, indifferent.
- $v_t > V$, every agent wants to a worker.

***Borrowing Constraint* (BC):** $\lambda\pi(v_t) \geq r(F - w_t) \rightarrow w_t \geq C(v_t) \equiv \text{Max}\{0, F - \lambda\pi(v_t)/r\}$

- $C'(v) > 0$ and $C''(v) < 0$, if $C(v) > 0$ and $\lambda > 0$.
- $C(v) = 0$ for a small v if $\lambda > 0$.

Combining (PC) and (BC):

- $v_t > V$, then $v_t > \pi(v_t) - rF$:
  - nobody sets up a firm, no demand for labor; cannot be an equilibrium.

- $v_t < V$, then $v_t < \pi(v_t) - rF$:
  - The agents with $w_t < C(v_t)$ have no choice but to become workers.
  - The agents with $w_t \geq C(v_t)$ become employers and hire $n(v_t)$ each.

- $v_t = V$, then $v_t = \pi(v_t) - rF$:
  - The agents with $w_t < C(V)$ have no choice but to become workers.
  - The agents with $w_t \geq C(V)$ are willing to be employers and hire $n(v_t)$ each.

**Labor Market Equilibrium (LME):** $\dfrac{G_t\left(C(v_t)\right)}{1-G_t\left(C(v_t)\right)} \leq n(v_t)$; $\quad v_t \leq V$ ,

$G_t(\bullet) \Rightarrow \mathbf{v_t}, \boldsymbol{\pi}(\mathbf{v_t})$

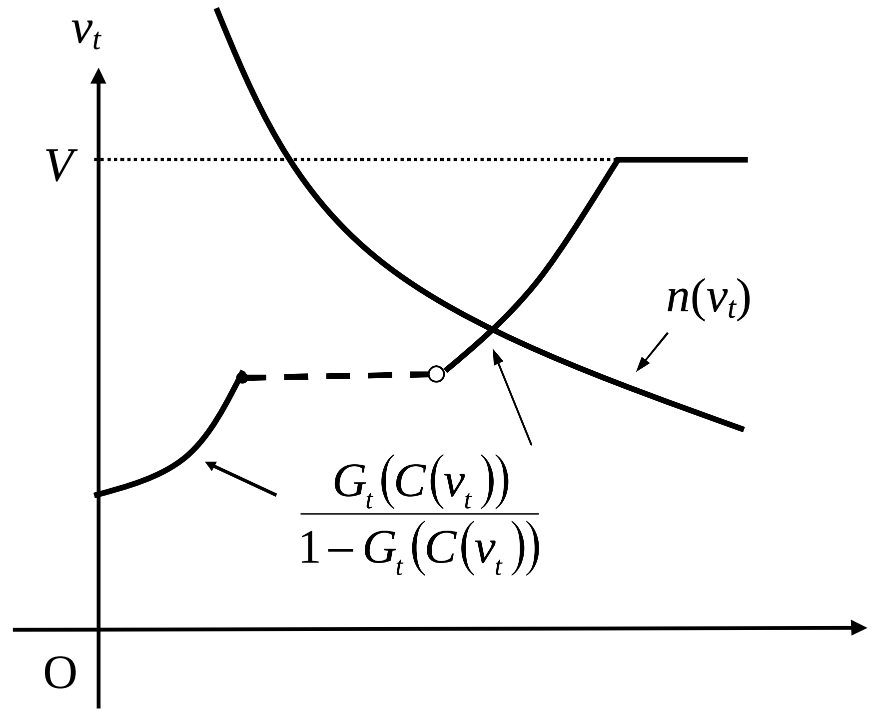



Figure 2a)

## Wealth Accumulation (WA)

$$w_{t+1} = \begin{cases} \beta(v_t + rw_t) & \text{if } w_t < C(v_t), \\ \beta(\pi(v_t) - rF + rw_t) & \text{if } w_t \geq C(v_t). \end{cases}$$

The arrows show
the effects of a higher $v_t$.

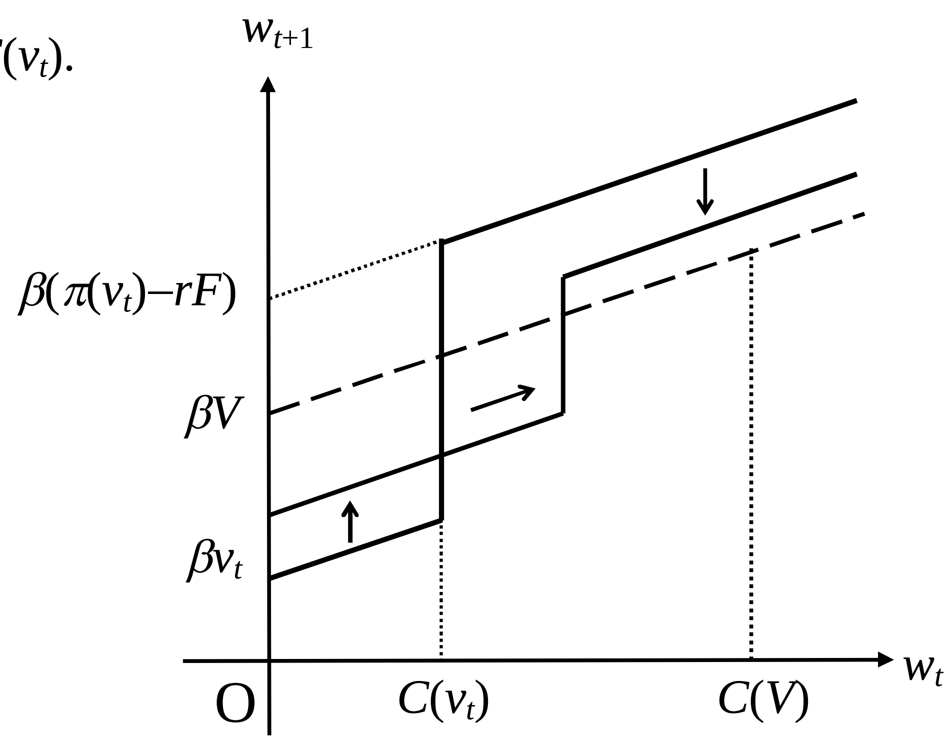



Figure 2b)

**$G_t(\bullet) \Rightarrow v_t, \pi(v_t)$**
**$\Rightarrow G_{t+1}(\bullet) \Rightarrow v_{t+1}, \pi(v_{t+1})$**
**$\Rightarrow \ldots$**

### 3.1 Interacting Dynamics with a Fixed Threshold: $\lambda = 0 \rightarrow C(v_t) = F$.

**(LME):** $\frac{X_t}{1-X_t} \leq n(v_t);\ v_t \leq V$

where $X_t \equiv G_t(F)$

**(WA):** $w_{t+1} = \begin{cases} \beta(v_t + rw_t) & \text{if } w_t < F, \\ \beta(\pi(v_t) - rF + rw_t) & \text{if } w_t \geq F. \end{cases}$

*Note:* Dynamics depends solely on $X_t$

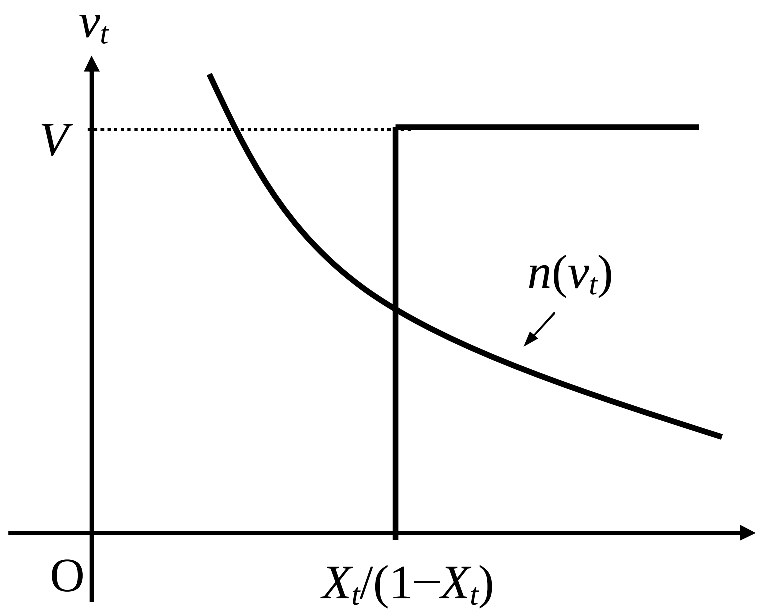



Figure 3a)

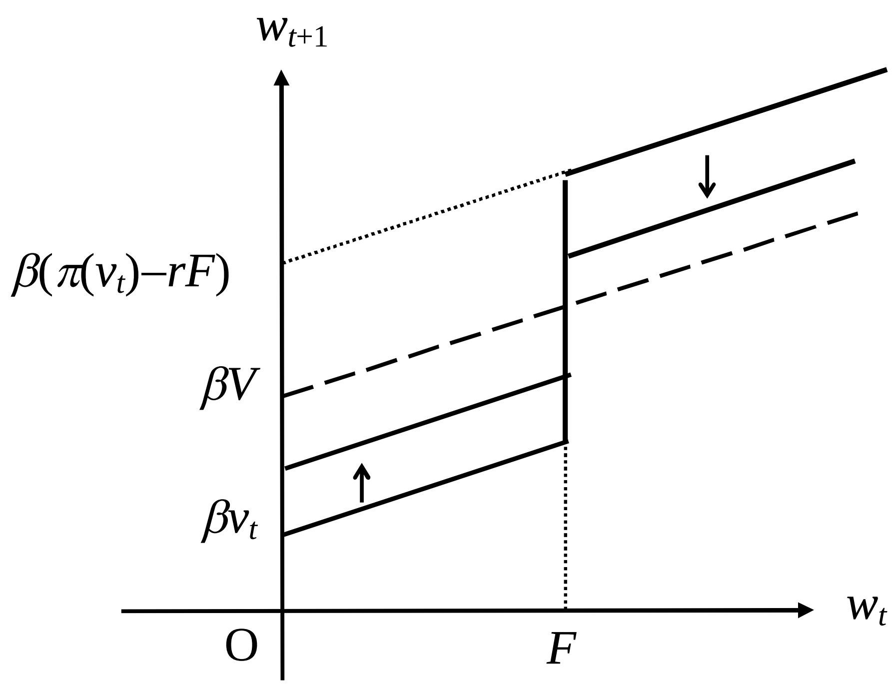
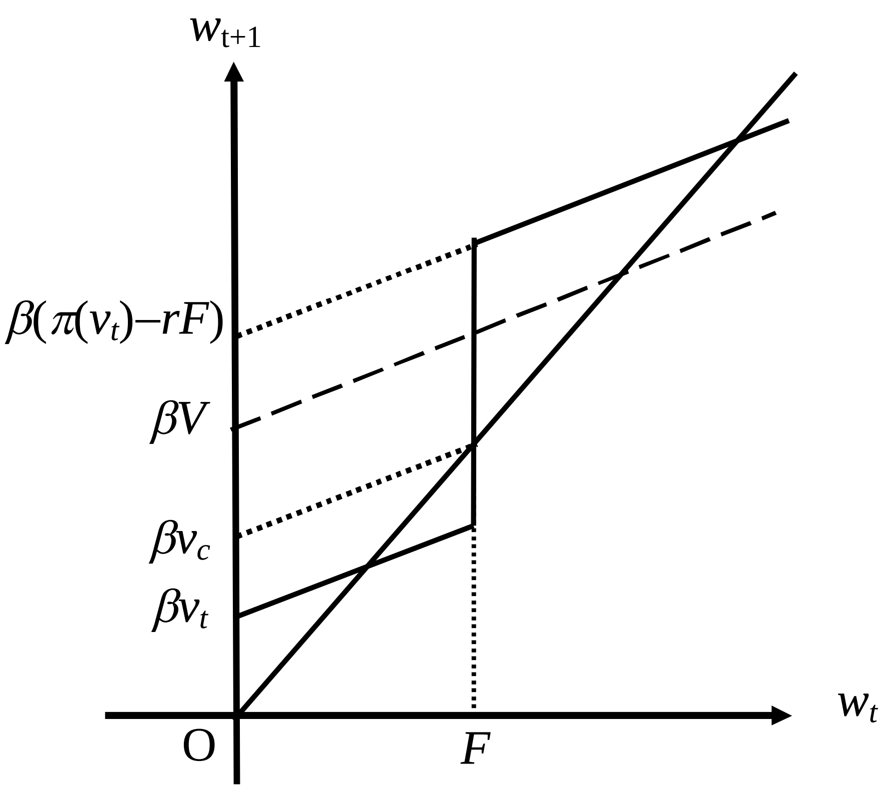



Figure 3b)

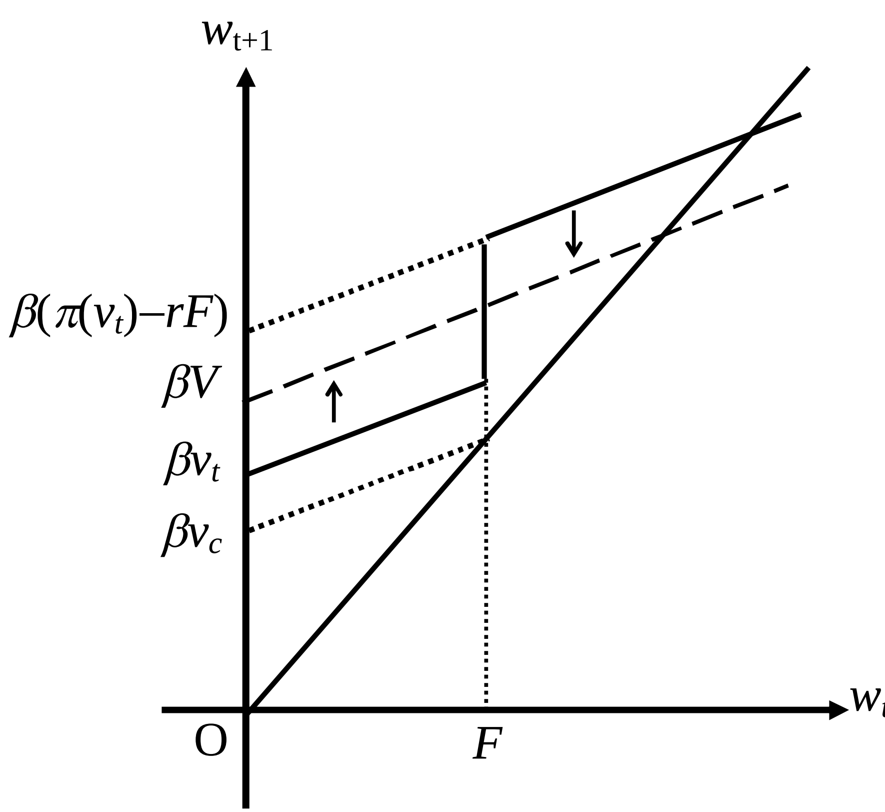
Suppose $v_c \equiv (1-\beta r)F/\beta < V$. Then,

Either $\quad v_t \le v_c < V < \pi(v_t) - rF \quad$ OR $\quad v_c < v_t \le V \le \pi(v_t) - rF$.




$X_t = X_{t+1} = \ldots = X_\infty$; $v_t = v_{t+1} = \ldots = v_\infty$.

Figure 4a)




$X_t$ declines til $X_\infty = 0$; $v_t$ rises til $v_\infty = V$.

Figure 4b)

In period 0, the wage rate, $v_0$, is given by $X_0 = n(v_0)/[1+n(v_0)]$.

- A fraction, $G_0(F) = X_0$, of the agents becomes workers;
- A fraction, $1-X_0$, of the agents becomes entrepreneurs;

If $G_0(F) = X_0 \geq X_c \equiv n(v_c)/[1+n(v_c)]$, this is a steady state.

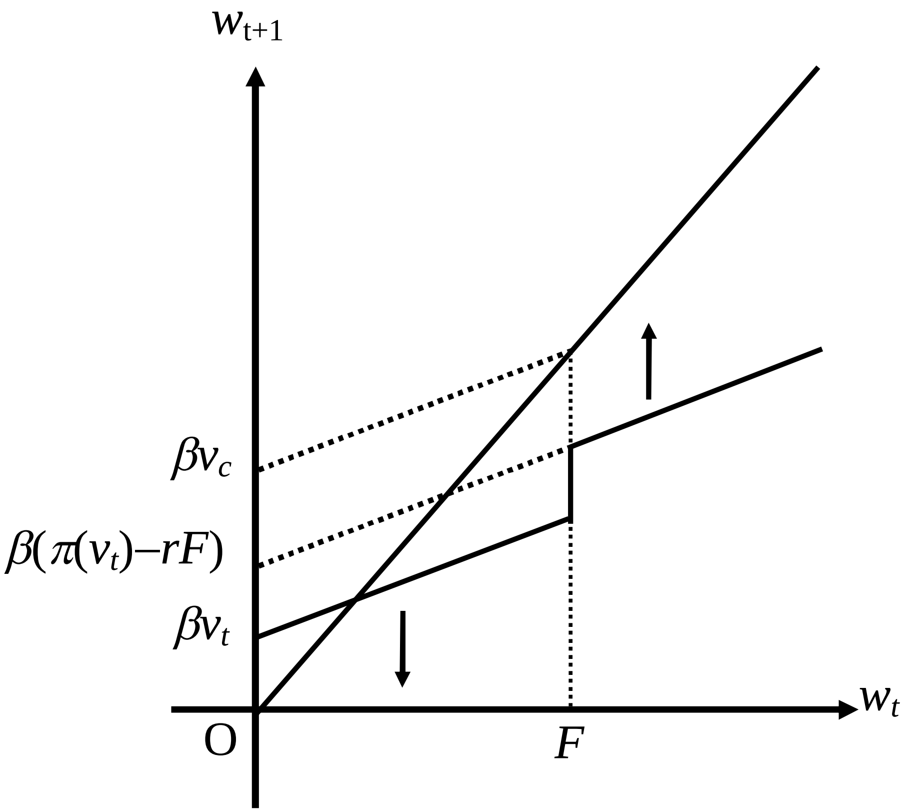
- A fraction, $X_0$, of the dynasties becomes the proletariat; their wealth converges to $\beta v_0/(1-\beta r)$. They are trapped in poverty.
- A fraction $1-X_0$ of the dynasties becomes the bourgeoisie; their wealth converges to $\beta[\pi(v_0)-rF]/(1-\beta r)$.

If $G_0(F) = X_0 < X_c \equiv n(v_c)/[1+n(v_c)]$, the wage rate is sufficiently high that some workers leave enough wealth that allows their descendants to become entrepreneurs, which further raise the wage rate. In steady state, $v = V$ and each dynasty's wealth converges to $\beta V/(1-\beta r) > F$; The workers and entrepreneurs are equally wealthy; the class distinction disappears.

- *Collective poverty trap;* the poor are trapped because there are so many of them.
- *Path-dependence*, in the distribution dynamics.
- *Caution:* This should not be interpreted as "persistence of inequality." A higher initial inequality, by reducing $X_0$, can eliminate collective poverty trap and long run inequality.

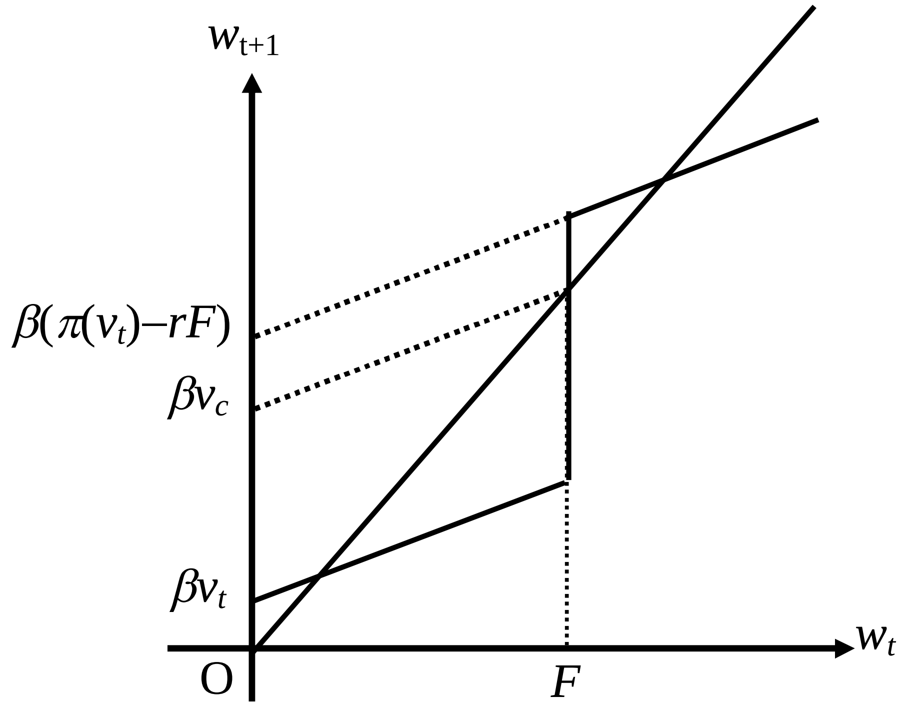
For $v_c \equiv (1-\beta r)F/\beta > V$,

Either $v_t \le \pi(v_t) - rF < v_c$ OR $v_t < v_c \le \pi(v_t) - rF$.




$X_t$ rises and $v_t$ declines until

Figure 4c)




$X_t = X_{t+1} = \ldots = X_\infty$; $v_t = v_{t+1} = \ldots = v_\infty$.

Figure 4d)

## 3.2 Interacting Dynasties with Variable Threshold: $\lambda>0 \rightarrow C(v_t) = \mathrm{Max}\{0, F-\lambda\pi(V)/r\}$.

**Steady State Analysis**

*Classless Society: Steady State with Wealth Equality:* $v_\infty = V$.

$$w_\infty = \beta V/(1-\beta r) \geq C(V) = \mathrm{Max}\{0, F - \lambda\pi(V)/r\}.$$

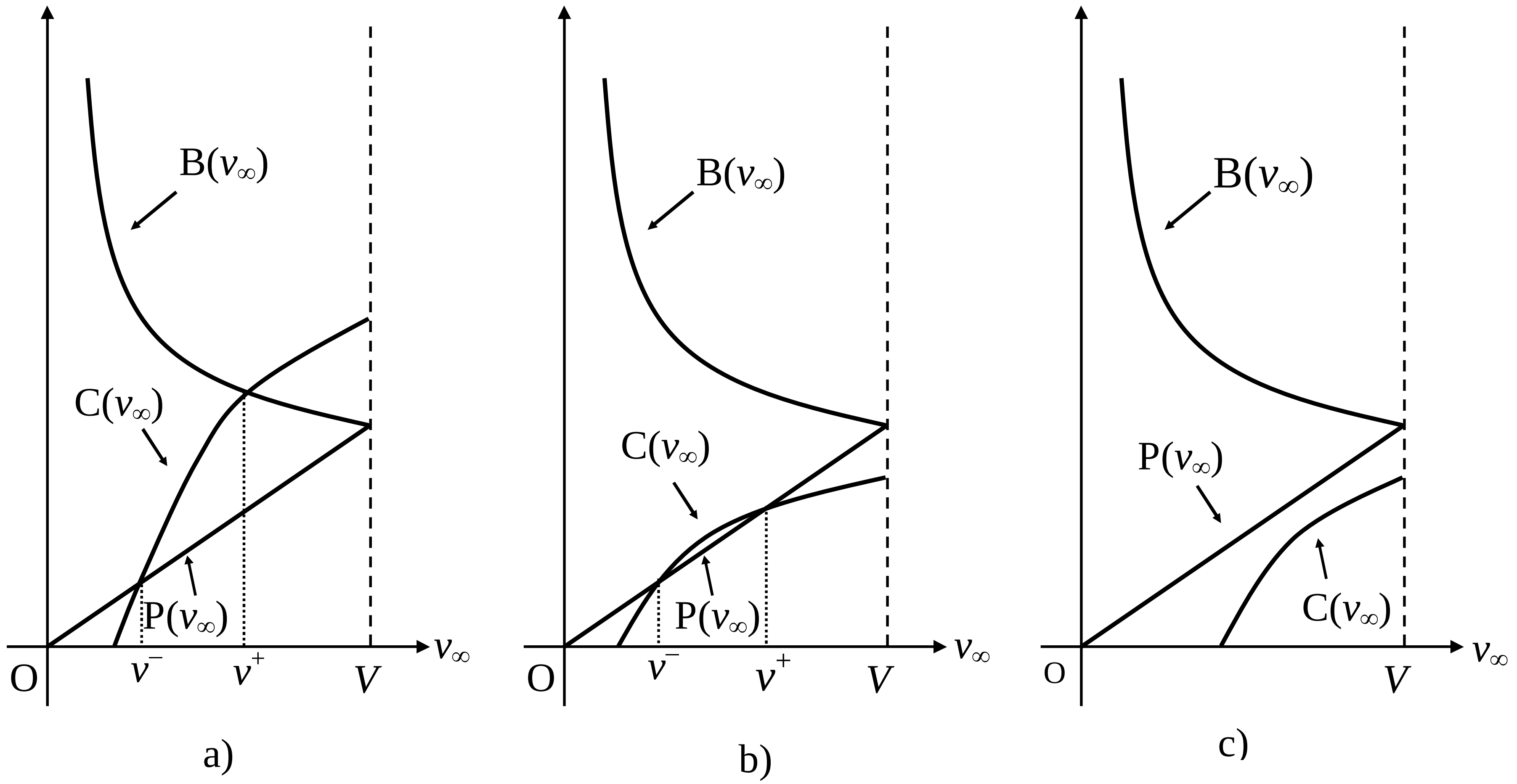
Labor market clears because the agents are indifferent.

*Class Society: Steady States with Wealth Inequality:* $v_\infty < V$

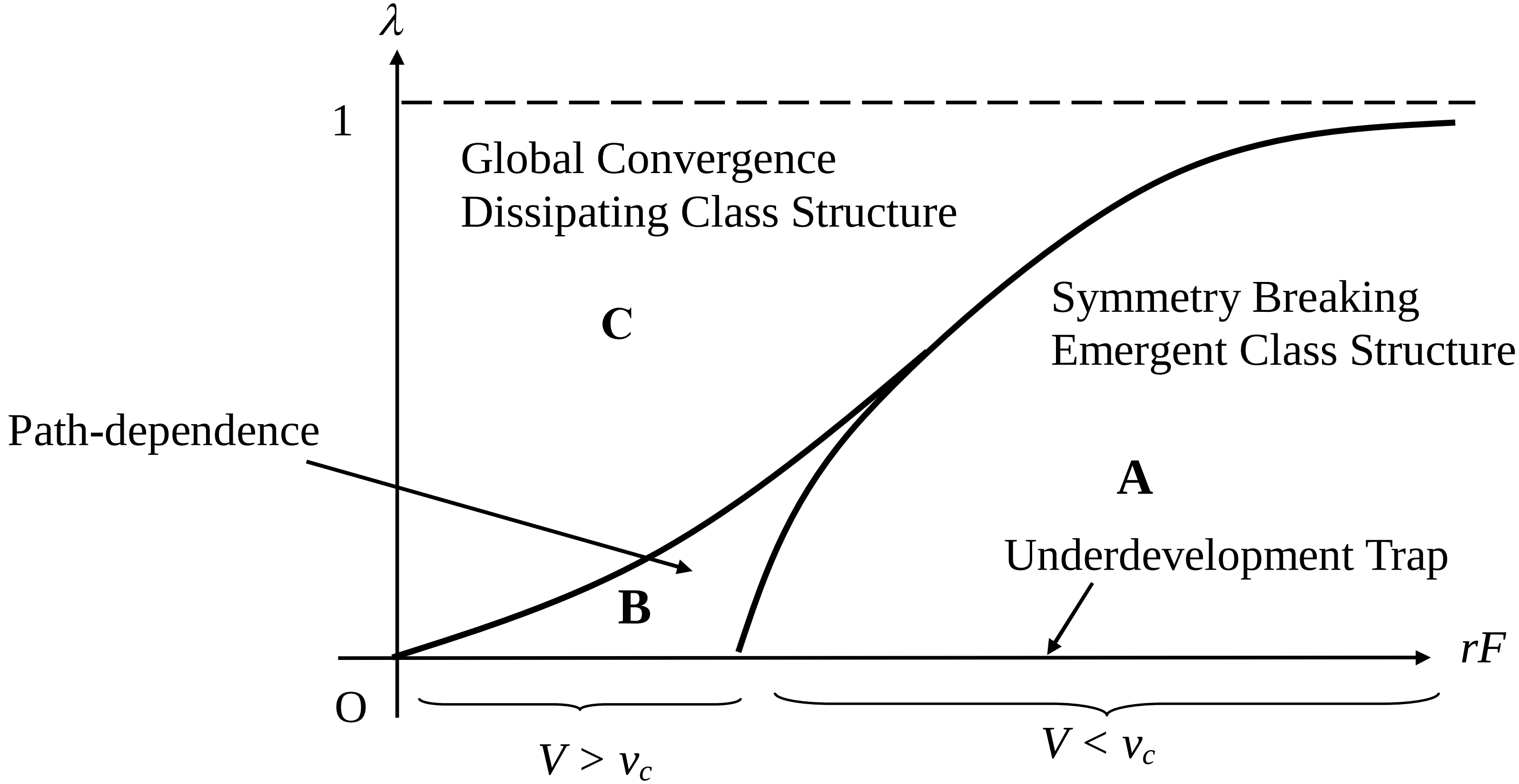
Bourgeoisie's wealth converges to: $B(v_\infty) \equiv \beta(\pi(v_\infty) - rF)/(1-\beta r) \geq C(v_\infty)$,
Proletariat's wealth converges to: $P(v_\infty) \equiv \beta v_\infty/(1-\beta r) < C(v_\infty)$,

Labor Market Equilibrium; $X_\infty/(1-X_\infty) = \mathrm{n}(v_\infty)$

## Three Generic Cases:



**Three Generic Cases in the Parameter Space**




**A:** Unequal Steady States Only (Symmetry-breaking): *Emergent* Class Structure
**Long Run Inequality** for any Initial Distribution, One-Time Redistribution Ineffective
**B:** Equal and Unequal Steady States Co-Exist. **Path-dependence;** Initial Distribution Matters; One-Time Redistribution Effective
**C:** Equal Steady State Only: **Long Run Equality** for any Initial Distribution

## 4. Interacting Dynasties with a Variable Threshold: Credit Market Channel

They now interact through the credit market, where the rate of return, $r_t$, is determined endogenously to keep the balance between the supply and the demand for credit.

**Three Ways of Allocating the Inheritance, $w_t$.**

- Run a **non-divisible investment project**, which converts $F$ units of the input at the beginning of period t into $\boldsymbol{R}$ units in **Final Good** at the end of period t, by **borrowing** $F-w_t$ at the market rate of return equal to $\boldsymbol{r_t}$**.**
- **Divisible storage technology**, with a small rate of return, $\rho > 0$.
- **Lend** $x_t \leq w_t$ units of the input at the beginning of period t for $r_t x_t$ units of the final good at the end of period t.

**Agent's End-of-Period Wealth:**

$U_t = y + R - r_t(F-w_t) = y + R - r_tF + r_tw_t$, if borrow and run the project,
$U_t = y + r_tw_t$ if lend (or put in storage).

***Profitability Constraint* (PC):** $R/F \geq r_t \geq \rho$

***Borrowing Constraint* (BC):** $w_t \geq C(r_t) \equiv \text{Max}\ \{0, F - \lambda R/r_t\}$.

Suppose $\rho < r_t < R/F$. Then,

- Agents with $w_t < C(r_t)$ become (reluctant) lenders because they cannot borrow;
- Agents with $C(r_t) \leq w_t < F$ become borrowers, since they need to and can borrow;
- Agents with $w_t > F$, become lenders, because they have more than enough to self-finance their own investment.

(CME):Credit supply = $\int_0^\infty w dG_t(w) = [1 - G_t(C(r_t))]F$ =Credit demand for $\rho < r_t < R/F$

Otherwise, $r_t = \rho$ with LHS > RHS or $r_t = R/F$ with LHS < RHS.

(WA) $$w_{t+1} = \begin{cases} \beta(y + r_t w_t) & \text{if } w_t < C(r_t), \\ \beta(y + R - r_t F + r_t w_t) & \text{if } w_t \geq C(r_t). \end{cases}$$

Again, we can iterate (CME) and (WA) to obtain $\boldsymbol{G_t(\bullet) \Rightarrow r_t \Rightarrow G_{t+1}(\bullet) \Rightarrow r_{t+1} \Rightarrow \ldots}$

## *Steady States*

***For a sufficiently large R and/or β,*** those rich enough to invest will accumulate enough wealth to become lenders.

- This will lower $r_t$, and hence $C(r_t)$, allowing the poor to start borrowing and investing.
- This process continues until every household will succeed in getting out of poverty. ***Global Convergence & Long Run Equality* with $\mathbf{r_\infty = \rho}$**, *a la* Aghion-Bolton.

***For R and β not high enough***, those rich enough invest will not accumulate enough wealth, so that they continue to borrow. Three different cases:

*A.* ***Symmetry-breaking & Long run Inequality*** *(For small R and/or small λ)*

The rich remain rich, in part because they have access to the cheap supply of credit offered by the poor, who is excluded from the profitable investment and have no choice to lend their tiny wealth to the rich. One-shot redistribution ineffective in eliminating inequality

B. ***Path-dependence*;**

One-shot redistribution can be effective in eliminating inequality

*C.* ***Global convergence & Long Run Equality with $\mathbf{r_\infty = R/F}$*** *(For relatively large R or λ)*

Strong credit demand by the rich drives up the rate of return, which helps the poor lenders to get out of poverty