Constant Pass-Through

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Last Updated: 2021-09-24, 1:47

Introduction

Monopolistic competition (MC) under CES

- Strong restrictions on the pricing behavior:
  - exogenously constant markup rate common across MC firms
  - complete pass-through
- **In multi-sector settings** (with nested CES):
  - markup rates can differ across sectors but not among MC firms within each sector
  - pass-through rate = 1 for every MC firm in every sector
- Various types of heterogeneity across MC firms isomorphic to each other

We propose and characterize *parametric families,* **CoPaTh (**and its special case, **CoCoPaTh,** and its special case, **CPE)**

- feature a *constant pass-through rate* as a parameter for each MC firm
- accommodate
  - a single measure of "toughness of competition"
  - endogenous markup rates/incomplete pass-through/strategic complementarity
  - various types of heterogeneity across MC firms, *not isomorphic to each other*
- **CoCoPaTh,** with a constant pass-through rate, *sector-specific* parameter, *common* across MC firms within a sector
  - Tough competition
    - *has no effects on their relative prices* (as the markup rates decline uniformly across all MC firms)
    - reduces the relative revenue/profit of those *with lower markup rates*, *not* necessarily smaller or less productive.
  - Retain much of tractability of CES, a useful building block for a wide range of MC models

    the average pass-through rate in the economy changes endogenously through sectoral composition!

Notes:

- Our goal is *not* to propose a model of an economy. Instead, it is to propose a building block, which we hope some find useful when they construct their models of an economy.
- In some ways, we are motivated by similar considerations that led Arrow-Chenery-Minhas-Solow (ACMS) to generalize Cobb-Douglas by proposing CES.

	Expenditure share	Elasticity of Substitution	Price elasticity under MC	Pass-through rate under MC
Cobb-Douglas	constant	1	Not applicable	Not applicable
CES	variable	constant & common within sector sector-specific (with nested CES)	constant & common within sector sector-specific (with nested CES)	1
CPE	variable	variable	constant product-specific	1
CoCoPaTh	variable	variable	variable	constant & common within sector sector-specific (with nested CoCoPaTh)
CoPaTh	variable	variable	variable	constant product-specific

Three Families of CoPaTh

We characterize *parametric* families of

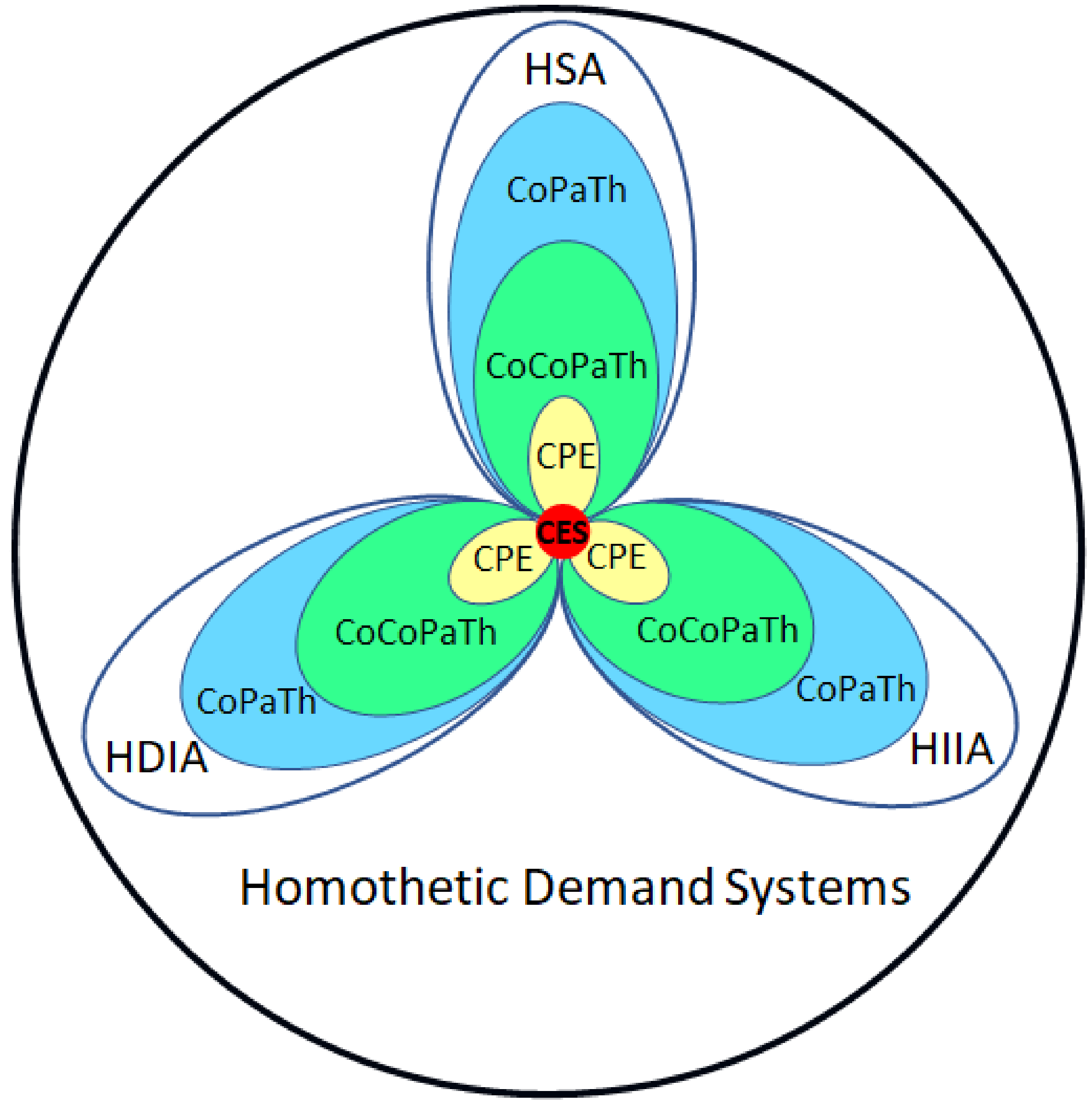
- Constant Price Elasticity **(CPE)**
- Common Constant Path-Through (**CoCoPaTh**)
- Constant Path Through(**CoPaTh**)

$$\text{CES} \subset \text{CPE} \subset \text{CoCoPaTh} \subset \text{CoPaTh}$$

within each of the 3 *nonparametric* classes of the demand systems:

- Homothetic with a Single Aggregator **(H.S.A.)**
- Homothetic Direct Implicit Additivity (**HDIA**)
- Homothetic Indirect Implicit Additivity (**HIIA**)

studied by Matsuyama-Ushchev (2017)



A Frequently Asked Question

In light of some empirical evidence (e.g., Amiti-Itskhoki-Konings, Berman-Martin-Meyer) that larger firms tend to have lower pass-through rates, how good is the assumption of CoCoPaTh (a constant & common pass-through rate)?

- Large firms may have low PaTh rates due to **oligopolistic behaviors** (Atkeson-Burstein, Edmond-Midrigan-Xu)

- Even when you want to assume that all firms are MC,
  - CoCoPaTh allow **sector-specific** PaTh rates; the average size of firms may differ across sectors.
  - CoCoPaTh are **better than homothetic translog**, which implies *higher* PaTh rates among larger MC firms (if MC firms differ only in productivity).

- We do not believe PaTh rates are *literally* constant and common among MC firms even within a sector. But
  - This assumption is no worse than the assumption that firms are heterogeneous only in productivity.
  - CoCoPaTh provides a useful benchmark for those who believe that it is not endogenous markup rate heterogeneity but endogenous PaTh rate heterogeneity that is important for understanding the data.

In summary, we think
- *Oligopoly models are better suited for explaining lower PaTh rates among larger firms within a sector.*
- *For any situation where you want to assume that some or all firms are MC, assuming a constant & common PaTh rate among them within a sector is a small price to pay for the tractability.*

General Setup

A Monopolistically Competitive (MC) Sector (as a Building Block)

A Production Sector consists of

- **Competitive firms:** produce a single good by assembling intermediate inputs $\omega \in \Omega$,, using **CRS technology**

CRS Production Function: $$X = X(\mathbf{x}) \equiv \min_{\mathbf{p}} \left\{ \mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega \,\middle|\, P(\mathbf{p}) \geq 1 \right\}$$

Unit Cost Function: $$P = P(\mathbf{p}) \equiv \min_{\mathbf{x}} \left\{ \mathbf{px} = \int_{\Omega} p_\omega x_\omega d\omega \,\middle|\, X(\mathbf{x}) \geq 1 \right\}$$

Duality Principle: Either $X = X(\mathbf{x})$ or $P = P(\mathbf{p})$ can be used as a primitive of the CRS technology, as long as linear homogeneity, monotonicity, and quasi-concavity are satisfied.

- A subset of *intermediate inputs varieties*, $\Omega^M \subset \Omega$, produced by profit-maximizing MC firms

$\Omega \,/\, \Omega^M$, may be supplied competitively, by oligopolists or by non-maximizing MC firms, etc.

We can also allow multi-product MC firms, as long as they do not produce a positive measure of products.

Demand Curve for ω

$$x_\omega = X(\mathbf{x})\frac{\partial P(\mathbf{p})}{\partial p_\omega}$$

Inverse Demand Curve for ω

$$p_\omega = P(\mathbf{p})\frac{\partial X(\mathbf{x})}{\partial x_\omega}$$

Market Size of the Sector

$$\mathbf{px} = \int_\Omega p_\omega x_\omega d\omega = P(\mathbf{p})X(\mathbf{x})$$

Revenue Share of Firm ω,

$$s_\omega = \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{p_\omega x_\omega}{P(\mathbf{p})X(\mathbf{x})}$$

$$s_\omega(p_\omega, \mathbf{p}) = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}; \;\; s_\omega(x_\omega, \mathbf{x}) = \frac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega}$$

Price Elasticity of ω:

$$\zeta_\omega = -\frac{\partial \ln x_\omega}{\partial \ln p_\omega}$$

$$\zeta_\omega(p_\omega, \mathbf{p}) = 1 - \frac{\partial \ln\left(\frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega}\right)}{\partial \ln p_\omega}; \; \zeta_\omega(x_\omega, \mathbf{x}) = \left[1 - \frac{\partial \ln\left(\frac{\partial \ln X(\mathbf{x})}{\partial \ln x_\omega}\right)}{\partial \ln x_\omega}\right]^{-1}$$

For general CRS, little restrictions on ζ_ω, beyond the homogeneity of degree zero in $(p_\omega, \mathbf{p})$ or in $(x_\omega, \mathbf{x})$.

Three Classes of CRS Production Functions: from Matsuyama-Ushchev (2017)

Homothetic with a Single Aggregator (H.S.A.)

$$\frac{P(\mathbf{p})}{cA(\mathbf{p})} = \exp\left[-\int_{\Omega}\left[\int_{p_\omega/A(\mathbf{p})}^{\bar{z}} \frac{s_\omega(\xi)}{\xi}d\xi\right]d\omega\right] \quad \Longleftarrow s_\omega = s_\omega\left(\frac{p_i}{A(\mathbf{p})}\right), \quad where \int_{\Omega} s_\omega\left(\frac{p_\omega}{A(\mathbf{p})}\right)d\omega \equiv 1$$

or

$$\frac{X(\mathbf{x})}{cA^*(\mathbf{x})} = \exp\left[\int_{\Omega}\left[\int_{0}^{x_\omega/A^*(\mathbf{x})} \frac{s_\omega^*(\xi)}{\xi}d\xi\right]d\omega\right] \quad \Longleftarrow s_\omega = s_\omega^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right), \quad where \int_{\Omega} s_\omega^*\left(\frac{x_\omega}{A^*(\mathbf{x})}\right)d\omega = 1$$

Homothetic Direct Implicit Additivity (HDIA): $X(\mathbf{x})$ ***implicitly additive & linear homogeneous***

$$\int_{\Omega} \phi_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right)d\omega \equiv 1 \quad \Longrightarrow s_\omega = \frac{x_\omega}{C^*(\mathbf{x})}\phi_\omega'\left(\frac{x_\omega}{X(\mathbf{x})}\right), \quad where \quad C^*(\mathbf{x}) \equiv \int_{\Omega} x_\omega\phi_\omega'\left(\frac{x_\omega}{X(\mathbf{x})}\right)d\omega$$

Homothetic Indirect Implicit Additivity (HIIA): $P(\mathbf{p})$ ***implicitly additive & linear homogeneous***

$$\int_{\Omega} \theta_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right)d\omega \equiv 1 \quad \Longrightarrow s_\omega = \frac{p_\omega}{C(\mathbf{p})}\theta_\omega'\left(\frac{p_\omega}{P(\mathbf{p})}\right), \quad where \quad C(\mathbf{p}) \equiv \int_{\Omega} p_\omega\theta_\omega'\left(\frac{p_\omega}{P(\mathbf{p})}\right)d\omega$$

with some restrictions on $s_\omega(\cdot)$ or $s_\omega^*(\cdot), \phi_\omega(\cdot), \theta_\omega(\cdot)$ to ensure monotonicity and quasi-concavity $P(\mathbf{p})$ and $X(\mathbf{x})$

H.S.A., HDIA, HIIA are disjoint with the sole exception of CES.

Appealing Features of the 3 Classes when Applied to Monopolistic Competition

	$P(\mathbf{p})$ or $X(\mathbf{x})$	**Revenue Share:** s_ω	**Price Elasticity:** ζ_ω	For CES
H.S.A.	$\frac{P(\mathbf{p})}{cA(\mathbf{p})} = \exp\left[-\int_\Omega \left[\int_{p_\omega/A(\mathbf{p})}^{\bar{z}} \frac{s_\omega(\xi)}{\xi} d\xi\right] d\omega\right]$	$s_\omega\left(\frac{p_\omega}{A(\mathbf{p})}\right)$ with $\int_\Omega s_\omega\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1$	$\zeta_\omega\left(\frac{p_\omega}{A(\mathbf{p})}\right) \equiv 1 - \frac{z s'_\omega(z)}{s_\omega(z)}\Big\vert_{z=\frac{p_\omega}{A(\mathbf{p})}} > 1$	$\frac{P(\mathbf{p})}{A(\mathbf{p})} = \frac{A^*(\mathbf{x})}{X(\mathbf{x})} = const.$ $\Leftrightarrow s_\omega(\cdot)$ or $s^*_\omega(\cdot)$ is a power function
	$\frac{X(\mathbf{x})}{cA^*(\mathbf{x})} = \exp\left[\int_\Omega \left[\int_0^{x_\omega/A^*(\mathbf{x})} \frac{s^*_\omega(\xi)}{\xi} d\xi\right] d\omega\right]$	$s^*_\omega\left(\frac{x_\omega}{A^*(\mathbf{x})}\right)$ with $\int_\Omega s^*_\omega\left(\frac{x_\omega}{A^*(\mathbf{x})}\right) d\omega = 1$	$\zeta^*_\omega\left(\frac{x_\omega}{A^*(\mathbf{x})}\right) \equiv \left[1 - \frac{y s^{*\prime}_\omega(y)}{s^*_\omega(y)}\Big\vert_{y=\frac{x_\omega}{A^*(\mathbf{x})}}\right]^{-1} > 1$	
HDIA Kimball	$\int_\Omega \phi_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right) d\omega \equiv 1$	$\frac{x_\omega}{C^*(\mathbf{x})}\phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right)$ with $C^*(\mathbf{x}) \equiv \int_\Omega x_\omega \phi'_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right) d\omega$	$\zeta^D_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right) \equiv -\frac{\phi'_\omega(y)}{y\phi''_\omega(y)}\Big\vert_{y=\frac{x_\omega}{X(\mathbf{x})}} > 1$	$\frac{C^*(\mathbf{x})}{X(\mathbf{x})} = const.$ $\Leftrightarrow \phi_\omega(\cdot)$ is a power function.
HIIA	$\int_\Omega \theta_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right) d\omega \equiv 1$	$\frac{p_\omega}{C(\mathbf{p})}\theta'_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right)$ with $C(\mathbf{p}) \equiv \int_\Omega p_\omega \theta'_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right) d\omega$	$\zeta^I_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right) \equiv -\frac{z\theta''_\omega(z)}{\theta'_\omega(z)}\Big\vert_{z=\frac{p_\omega}{P(\mathbf{p})}} > 1$	$\frac{C(\mathbf{p})}{P(\mathbf{p})} = const.$ $\Leftrightarrow \theta_\omega(\cdot)$ is a power function.

with further restrictions on $s_\omega(\cdot)$ or $s^*_\omega(\cdot), \phi_\omega(\cdot), \theta_\omega(\cdot)$ to ensure i) the gross substitutability and ii) the existence & the uniqueness of the free-entry equilibrium.

- **Revenue share** depends on a single aggregator under H.S.A; on two aggregators under HDIA. & HIIA.
- **The price elasticity** is a function of a single aggregator.
  - A single aggregator captures the effect of competition on the markup rate.
  - Comparative statics results dictated by the derivative of the price elasticity function.
  - Marshall's 2nd Law ⇔ Procompetitive effect ⇔ Strategic complementarity, not true in general.

Another Frequently Asked Question

What is the relative advantage of the three classes?

We believe that H.S.A. has advantage over HDIA and HIIA, because

- the revenue share functions, $s_\omega(\cdot)$, are the primitive of H.S.A. and hence it can be readily identified by typical firm level data, which has revenues but not output.

- With free-entry, easier to ensure the existence and uniqueness of equilibrium and characterize the equilibrium and conduct comparative statics under H.S.A., because

    - Under H.S.A., one need to pin down the equilibrium value of only one aggregator in each sector.

    - Under HDIA and HIIA, one need to pin down the equilibrium values of two aggregators in each sector.

MC Firm's pricing behavior; p_ω profit-maximizing price; ψ_ω: marginal cost

In all three classes,

FOC (Lerner Formula)

$$p_\omega\left[1-\frac{1}{\zeta_\omega(p_\omega/\mathcal{A}(\mathbf{p}))}\right]=\psi_\omega; \qquad \zeta_\omega(\cdot)>1$$

If LHS is monotone increasing in p_ω

$$\Longrightarrow \frac{p_\omega}{\mathcal{A}(\mathbf{p})}=\mathcal{G}_\omega\left(\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})}\right); \qquad \mathcal{G}'_\omega(\cdot)>0$$

Markup rate μ_ω for $\omega\in\Omega^M$

$$\mu_\omega\equiv\frac{p_\omega}{\psi_\omega}=\frac{\mathcal{G}_\omega(\psi_\omega/\mathcal{A}(\mathbf{p}))}{\psi_\omega/\mathcal{A}(\mathbf{p})}$$

Pass-through rate ρ_ω for $\omega\in\Omega^M$

$$\rho_\omega\equiv\frac{\partial\ln p_\omega}{\partial\ln\psi_\omega}=\frac{\partial\ln\mathcal{G}_\omega(\psi_\omega/\mathcal{A}(\mathbf{p}))}{\partial\ln(\psi_\omega/\mathcal{A}(\mathbf{p}))}=1+\frac{\partial\ln\mu_\omega}{\partial\ln\psi_\omega}$$

CoPaTh Pricing formula $\omega \in \Omega^M$

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega} = \left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega}\right)^{\rho_\omega} \Leftrightarrow \zeta_\omega\left(\frac{p_\omega}{\mathcal{A}(\mathbf{p})}\right) = \frac{1}{1 - \left(1 - \frac{1}{\sigma_\omega}\right)\left(\frac{p_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega}\right)^{\frac{1}{\rho_\omega}-1}}$$

- **Product-specific**
  - $\sigma_\omega > 1$: markup (substitutability) shifter
  - $\beta_\omega > 0$: price (quality) shifter
  - $\psi_\omega > 0$: marginal cost (inverse of productivity)
  - $\rho_\omega \leq 1$: pass-through rate
  - $\gamma_\omega > 0$: market-size shifter (does not appear in the pricing formula)

- **Common across products within a sector**
  - $\mathcal{A} = \mathcal{A}(\mathbf{p})$: linear homogeneous in **p,** common price aggregator capturing ``toughness of competition''

Complete pass-through case ($\rho_\omega = 1$)

CoPaTh Pricing formula $\omega \in \Omega^M$

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega} = \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega} \Longrightarrow p_\omega = \frac{\sigma_\omega}{\sigma_\omega - 1}\psi_\omega$$

- **CPE** – Constant product-specific Price Elasticity

- **Product-specific markup rate:** depends solely on σ_ω, not on
  - marginal cost ψ_ω
  - price shifter β_ω
  - common aggregator $\mathcal{A} = \mathcal{A}(\mathbf{p})$

- **CES:** special case of CPE with $\sigma_\omega = \sigma$

Incomplete pass-through case $(0 < \rho_\omega < 1)$

CoPaTh Pricing formula for $\omega \in \Omega^M$

$$\frac{p_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega} = \left(\left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)\frac{\psi_\omega}{\mathcal{A}(\mathbf{p})\beta_\omega}\right)^{\rho_\omega} \implies \ln p_\omega = (1-\rho_\omega)\ln \bar{p}_\omega + \rho_\omega \ln \psi_\omega$$

Choke price for $\omega \in \Omega^M$

$$\bar{p}_\omega \equiv \bar{\beta}_\omega = \mathcal{A}(\mathbf{p})\beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{\frac{\rho_\omega}{1-\rho_\omega}} < \infty$$

where $\bar{\beta}_\omega = \beta_\omega \left(\frac{\sigma_\omega}{\sigma_\omega - 1}\right)^{\frac{\rho_\omega}{1-\rho_\omega}}$ is the "relative" choke price. (Note $\bar{\beta}_\omega \to \infty$, as $\rho_\omega \nearrow 1$.)

Price of each product $\omega \in \Omega^M$

- strategic complementarity in pricing. $\mathcal{A}(\mathbf{p}) \uparrow \Rightarrow \bar{p}_\omega \uparrow \Rightarrow p_\omega \uparrow$
- sector-wide pass-through rate is one, if all firms/products are MC and hit by proportional cost shocks.
- log-linear in marginal cost and choke price
  - Under CoCoPaTh $(0 < \rho_\omega = \rho < 1)$, common coefficients across all products in Ω^M, as in **the standard pass-through regression** (Gopinath-Rigobon 2008; Nakamura-Zerom 2010)

Under CoCoPaTh $(0 < \rho_\omega = \rho < 1)$

Price and markup ratios for $\omega_1, \omega_2 \in \Omega^M$

- **Price ratio** for $\omega_1, \omega_2 \in \Omega^M$

$$\frac{p_{\omega_1}}{p_{\omega_2}} = \left(\frac{\beta_{\omega_1}}{\beta_{\omega_2}}\right)^{1-\rho} \left(\frac{\sigma_{\omega_1}/(\sigma_{\omega_1}-1)}{\sigma_{\omega_2}/(\sigma_{\omega_2}-1)}\frac{\psi_{\omega_1}}{\psi_{\omega_2}}\right)^{\rho}$$

- **Markup ratio** for $\omega_1, \omega_2 \in \Omega^M$

$$\frac{\mu_{\omega_1}}{\mu_{\omega_2}} = \left(\frac{\sigma_{\omega_1}/(\sigma_{\omega_1}-1)}{\sigma_{\omega_2}/(\sigma_{\omega_2}-1)}\right)^{\rho} \left(\frac{\beta_{\omega_1}/\psi_{\omega_1}}{\beta_{\omega_2}/\psi_{\omega_2}}\right)^{1-\rho}$$

Note: both independent of $\mathcal{A}(\mathbf{p})$

- A great advantage when studying the GE effects of shocks that change the relative cost across MC firms (e.g., the exchange rate, the tariffs, the energy prices).
- Under CoCoPaTh, the impact of such shocks on the markup rates and relative prices can be calculated without worrying about the general equilibrium feedback effect.

Under CoCoPaTh ($0 < \rho_\omega = \rho < 1$)

Sales ratio for $\omega_1, \omega_2 \in \Omega^M$

γ_ω = quantity shifter or *market size for* $\omega \in \Omega$

- **Incomplete pass-through case ($\mathbf{0 < \boldsymbol{\rho} < 1}$)** for $\omega_1, \omega_2 \in \Omega^M$

$$\frac{p_{\omega_1} x_{\omega_1}}{p_{\omega_2} x_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}} \frac{\bar{\beta}_{\omega_1}}{\bar{\beta}_{\omega_2}} \left(\frac{\sigma_{\omega_1} - 1}{\sigma_{\omega_2} - 1}\right)^{\frac{\rho}{1-\rho}} \left[\frac{1 - \left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}{1 - \left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}\right]^{\frac{\rho}{1-\rho}}$$

- **Complete pass-through case ($\boldsymbol{\rho} \to \mathbf{1}$)** for $\omega_1, \omega_2 \in \Omega^M$:

$$\frac{p_{\omega_1} x_{\omega_1}}{p_{\omega_2} x_{\omega_2}} = \frac{\gamma_{\omega_1} \beta_{\omega_1} \left(\frac{\sigma_{\omega_1}}{\sigma_{\omega_1} - 1} \frac{\psi_{\omega_1}}{\mathcal{A}(\mathbf{p}) \beta_{\omega_1}}\right)^{1-\sigma_{\omega_1}}}{\gamma_{\omega_2} \beta_{\omega_2} \left(\frac{\sigma_{\omega_2}}{\sigma_{\omega_2} - 1} \frac{\psi_{\omega_2}}{\mathcal{A}(\mathbf{p}) \beta_{\omega_2}}\right)^{1-\sigma_{\omega_2}}} \propto [\mathcal{A}(\mathbf{p})]^{\sigma_{\omega_1} - \sigma_{\omega_2}}$$

Note: both are increasing with $\mathcal{A}(\mathbf{p}) \Leftrightarrow \mu_{\omega_1} < \mu_{\omega_2}$

MC firms with lower markups (not necessarily smaller firms) suffer more from tougher competition.

Under CoCoPaTh ($0 < \rho_\omega = \rho < 1$)

Profit ratio for $\omega_1, \omega_2 \in \Omega^M$

γ_ω = quantity shifter or *market size for* $\omega \in \Omega$

- **Incomplete pass-through case ($\mathbf{0} < \boldsymbol{\rho} < \mathbf{1}$)**:

$$\frac{\pi_{\omega_1}}{\pi_{\omega_2}} = \frac{\gamma_{\omega_1}}{\gamma_{\omega_2}} \frac{\bar{\beta}_{\omega_1}}{\bar{\beta}_{\omega_2}} \left(\frac{\sigma_{\omega_1} - 1}{\sigma_{\omega_2} - 1}\right)^{\frac{\rho}{1-\rho}} \left[\frac{1 - \left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}{1 - \left(\psi_{\omega_1}/\mathcal{A}(\mathbf{p})\bar{\beta}_{\omega_1}\right)^{1-\rho}}\right]^{\frac{1}{1-\rho}}$$

- **Complete pass-through case ($\boldsymbol{\rho} \to \mathbf{1}$)**:

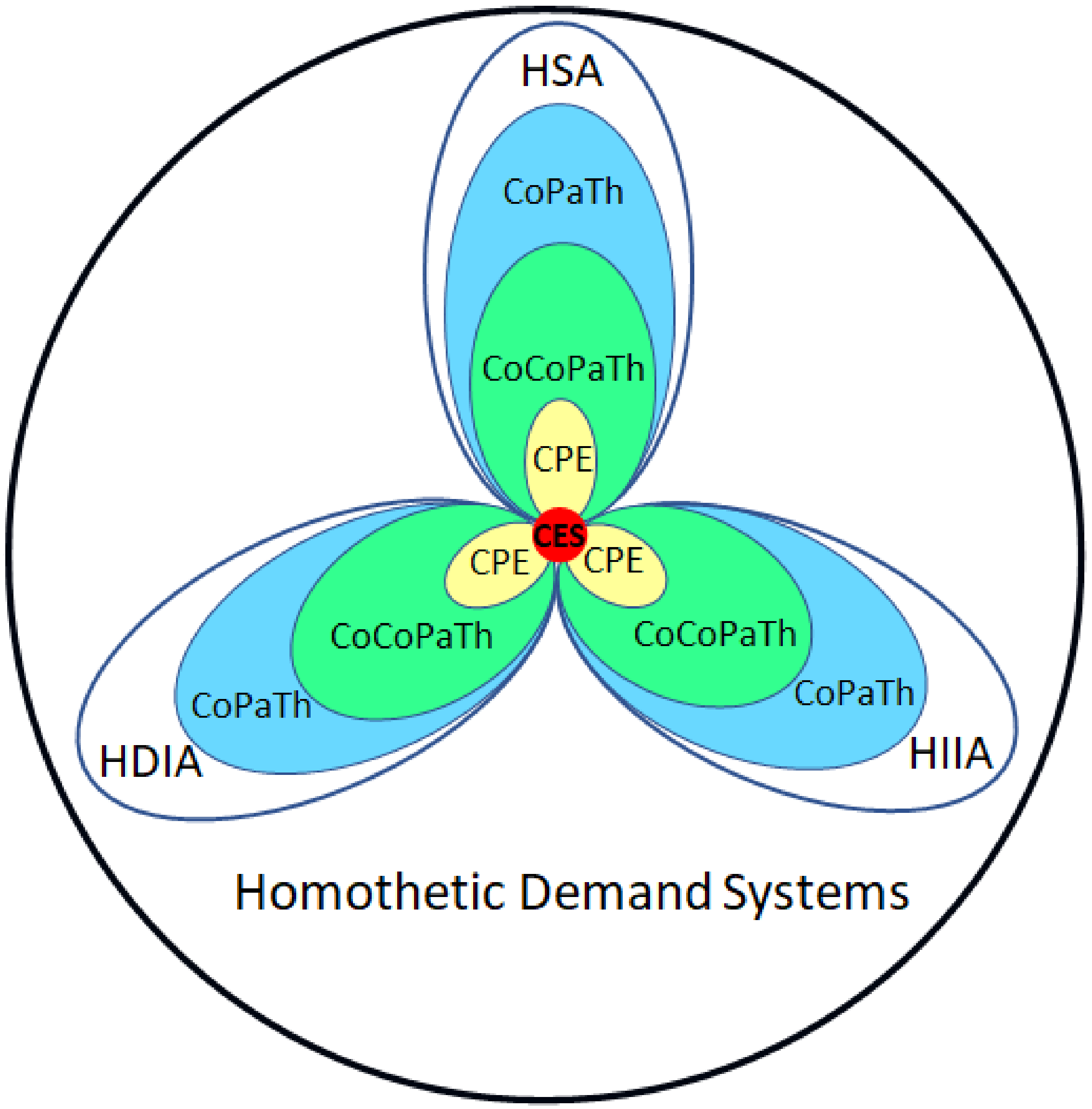
$$\frac{\pi_{\omega_1}}{\pi_{\omega_2}} = \frac{\frac{\gamma_{\omega_1}\beta_{\omega_1}}{\sigma_{\omega_1}} \left(\frac{\sigma_{\omega_1}}{\sigma_{\omega_1} - 1} \frac{\psi_{\omega_1}}{\mathcal{A}(\mathbf{p})\beta_{\omega_1}}\right)^{1-\sigma_{\omega_1}}}{\frac{\gamma_{\omega_2}\beta_{\omega_2}}{\sigma_{\omega_2}} \left(\frac{\sigma_{\omega_2}}{\sigma_{\omega_2} - 1} \frac{\psi_{\omega_2}}{\mathcal{A}(\mathbf{p})\beta_{\omega_2}}\right)^{1-\sigma_{\omega_2}}} \propto [\mathcal{A}(\mathbf{p})]^{\sigma_{\omega_1} - \sigma_{\omega_2}}$$

Note: both are increasing with $\mathcal{A}(\mathbf{p}) \Leftrightarrow \mu_{\omega_1} < \mu_{\omega_2}$

MC firms with lower markups (not necessarily smaller firms) suffer more from tougher competition.

CoPaTh: Three Classes

The Three Families of CoPaTh Demand Systems



Homothetic Demand with a Single Aggregator (H.S.A.); $\mathcal{A}(\mathbf{p}) = A(\mathbf{p}) \neq cP(\mathbf{p})$

$$\int_{\Omega} s_{\omega}\left(\frac{p_{\omega}}{A(\mathbf{p})}\right) d\omega \equiv 1 \quad \Longrightarrow \quad \zeta_{\omega}\left(\frac{p_{\omega}}{A(\mathbf{p})}\right) \equiv 1 - \left.\frac{z s'_{\omega}(z)}{s_{\omega}(z)}\right|_{z=\frac{p_{\omega}}{A(\mathbf{p})}}$$

CoPaTh under H.S.A.

$$s_{\omega}(z) = \gamma_{\omega}\beta_{\omega}\left[\sigma_{\omega} - (\sigma_{\omega} - 1)\left(\frac{z}{\beta_{\omega}}\right)^{\frac{1-\rho_{\omega}}{\rho_{\omega}}}\right]^{\frac{\rho_{\omega}}{1-\rho_{\omega}}} = \gamma_{\omega}\bar{\beta}_{\omega}(\sigma_{\omega} - 1)^{\frac{\rho_{\omega}}{1-\rho_{\omega}}}\left[1 - \left(\frac{z}{\bar{\beta}_{\omega}}\right)^{\frac{1-\rho_{\omega}}{\rho_{\omega}}}\right]^{\frac{\rho_{\omega}}{1-\rho_{\omega}}}$$

$$\zeta_{\omega}(z) \equiv 1 - \frac{z s'_{\omega}(z)}{s_{\omega}(z)} = \frac{1}{1 - \left(1 - \frac{1}{\sigma_{\omega}}\right)\left(\frac{z}{\beta_{\omega}}\right)^{\frac{1-\rho_{\omega}}{\rho_{\omega}}}} = \frac{1}{1 - \left(\frac{z}{\bar{\beta}_{\omega}}\right)^{\frac{1-\rho_{\omega}}{\rho_{\omega}}}}$$

Notes:

- CPE is obtained as $\rho_{\omega} \to 1$, holding β_{ω} fixed, which causes $\bar{\beta}_{\omega} \to \infty$.
- These expressions hold for $\varepsilon < z < \bar{\beta}_{\omega}$ where $\varepsilon > 0$ is arbitrarily small!

Homothetic Direct Implicit Additivity (HDIA); $\mathcal{A}(\mathbf{p}) = B(\mathbf{p}) \neq \boldsymbol{c}P(\mathbf{p})$

$$\int_{\Omega} \phi_\omega\left(\frac{x_\omega}{X(\mathbf{x})}\right) d\omega \equiv 1 \quad \Longrightarrow \quad \zeta_\omega^D\left(\frac{x_\omega}{X(\mathbf{x})}\right) = \zeta_\omega^D\left((\phi_\omega')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)\right) \equiv -\left.\frac{\phi_\omega'(y)}{y\phi_\omega''(y)}\right|_{y=\frac{x_\omega}{X(\mathbf{x})}=(\phi_\omega')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)}$$

where

$$\frac{x_\omega}{X(\mathbf{x})} = (\phi_\omega')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right); \int_{\Omega} \phi_\omega\left((\phi_\omega')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)\right) d\omega \equiv 1.$$

CoPaTh under HDIA

$$\phi_\omega(y) = \bar{\beta}_\omega \int_0^{y} \left(1 + \frac{1}{\sigma_\omega - 1}\left(\frac{\xi}{\gamma_\omega}\right)^{\frac{1-\rho_\omega}{\rho_\omega}}\right)^{-\frac{\rho_\omega}{1-\rho_\omega}} d\xi$$

$$\zeta_\omega^D(y) \equiv -\frac{\phi_\omega'(y)}{y\phi_\omega''(y)} = 1 + (\sigma_\omega - 1)\left(\frac{y}{\gamma_\omega}\right)^{-\frac{\rho_\omega}{1-\rho_\omega}} > 1$$

Notes:

- CPE is obtained as $\rho_\omega \to 1$, holding β_ω fixed, which causes $\bar{\beta}_\omega \to \infty$.
- These expressions hold for all $y > 0$!

Homothetic Indirect Implicit Additivity (HIIA); $\mathcal{A}(\mathbf{p}) = P(\mathbf{p})$

$$\int_{\Omega} \theta_\omega\left(\frac{p_\omega}{P(\mathbf{p})}\right) d\omega \equiv 1 \quad \Longrightarrow \quad \zeta_\omega^I\left(\frac{p_\omega}{P(\mathbf{p})}\right) \equiv -\left.\frac{z\theta_\omega''(z)}{\theta_\omega'(z)}\right|_{z=\frac{p_\omega}{P(\mathbf{p})}}$$

CoPaTh under HIIA

$$\theta_\omega(z) = \gamma_\omega(\sigma_\omega - 1)^{\frac{\rho_\omega}{1-\rho_\omega}} \int_z^{\bar{\beta}_\omega} \left(\left(\frac{\xi}{\bar{\beta}_\omega}\right)^{-\frac{1-\rho_\omega}{\rho_\omega}} - 1\right)^{\frac{\rho_\omega}{1-\rho_\omega}} d\xi$$

$$\zeta_\omega^I(z) \equiv -\frac{z\theta_\omega''(z)}{\theta_\omega'(z)} = \frac{1}{1 - \left(1 - \frac{1}{\sigma_\omega}\right)\left(\frac{z}{\bar{\beta}_\omega}\right)^{\frac{1-\rho_\omega}{\rho_\omega}}} = \frac{1}{1 - \left(\frac{z}{\bar{\beta}_\omega}\right)^{\frac{1-\rho_\omega}{\rho_\omega}}}$$

Notes:

- CPE is obtained as $\rho_\omega \to 1$, holding β_ω fixed, which causes $\bar{\beta}_\omega \to \infty$.
- These expressions hold for $0 < z < \bar{\beta}_\omega$!