Competition and the Phillips Curve

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Introduction

HSA

New Keynesian model under HSA

Competition and the Phillips curve

Steady state analysis

Dynamic analysis

Calvo pricing

Omitted variable bias

Conclusion
Flattening of the Phillips curve

- Federal Reserve Vice Chair: Richard Clarida, on Sept. 26, 2019
  “Another key development in recent decades is that price inflation appears less responsive to resource slack. That is, the short-run price Phillips curve—if not the wage Phillips curve—appears to have flattened, implying a change in the dynamic relationship between inflation and employment”

- San Francisco Fed President: Mary Daly, on Aug. 29, 2019
  “As for the Phillips curve… most arguments today center around whether it’s dead or just gravely ill. Either way, the relationship between unemployment and inflation has become very difficult to spot”

- New York Fed President: John Williams, on Feb. 22, 2019
  “The Phillips curve is the connective tissue between the Federal Reserve’s dual mandate goals of maximum employment and price stability. Despite regular declarations of its demise, the Phillips curve has endured. It is useful, both as an empirical basis for forecasting and for monetary policy analysis”
Market concentration

- **Covarrubias, Gutierrez and Philippon (2019)**
  
  “After 2000, however, the evidence suggests inefficient concentration, decreasing competition and increasing barriers to entry, as leaders become more entrenched and concentration is associated with lower investment, higher prices and lower productivity growth”

- **De Loecker, Eeckhout and Unger (2020)**
  
  “In 1980, aggregate markups start to rise from 21% above marginal cost to 61% now. ... We also find an increase in the average profit rate from 1% to 8%. Although there is also an increase in overhead costs, the markup increase is in excess of overhead”

- **Autor, Dorn, Katz, Patterson and Reenen (2020)**
  
  “sales concentration is rising across a large set of industries. ... aggregate markups have been rising”
What we do

- Argue that market concentration affects the slope of the Phillips curve and the transmission of monetary policy

- Extend the New Keynesian model under CES monopolistic competition
  - Introduce entry and exit as in Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014)
  - Replace CES by HSA (*Homothetic Single Aggregator*) demand system proposed by Matsuyama and Ushchev (2017, 2020a,b, 2022)
    - Robustness check with two alternative homothetic demand systems proposed by Matsuyama and Ushchev (2017, 2020a,b)
HSA

- HSA demand system is *flexible* and *tractable*
  - CES and Translog are special cases
  - HSA can accommodate
    - *Marshall’s Second law*: the price elasticity of demand goes up with its price
      ⇒ Concentration causes higher markup rate
    - *The Third law* (Matsuyama and Ushchev, 2022): the speed of the price elasticity change slows down with its price
      ⇒ Concentration causes lower pass-through rate

- Its single aggregator summarizes all the impacts of market concentration on the price elasticity, the pass-through rate, and hence the flattening of the Phillips curve

- The impact of concentration to the Phillips curve is summarized by two sufficient statistics, both functions of $z = p / A(p)$, the price divided by the single aggregator
  1. the price elasticity: $\zeta(z)$
  2. the pass-through rate: $\rho(z)$
Key takeaways

- **Steady-state effect of concentration**
  - Gross substitutes: $s(z) \uparrow \Leftrightarrow z \downarrow$
  - Under Rotemberg pricing, *the Second law* $\Rightarrow$ lower price elasticity $\Rightarrow$
    higher markup rate $\Rightarrow$ *structurally* flattening the Phillips curve
  - Under Calvo pricing, *the Third law* $\Rightarrow$ lower pass-through rate $\Rightarrow$
    *structurally* flattening the Phillips curve

- **Dynamic effect of endogenous entry**
  - Concentration affects price setting dynamically through strategic complementarity - endogenous cost-push shock
    - The supply side effects of monetary policy through the entry of firms
  - Pass-through rate plays an important role in cyclicality of markup

- **Observational implications of concentration and endogenous cost-push shock under the Second law**
  - A *naive* regression of the inflation rate on the real marginal cost $\Rightarrow$ the *negative* omitted variable bias (OVB): underestimating the slope
  - Under Rotemberg, *the Third law matters for the magnitude*
  - Under Calvo, both *Second and Third laws matter for the magnitude*
NKPC under HSA

- Concentration: $s(z) \uparrow \Leftrightarrow z \downarrow$
  - The Second law: $\zeta'(z) > 0$
  - The Third law: $\rho'(z) > 0$

- Rotemberg pricing

\[
\hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1}{\chi} \frac{1 - \rho(z)}{\rho(z)} \hat{N}_t
\]

- Calvo pricing: $\tilde{z}$ is the average of $z$

\[
\hat{\pi}_{t+1} = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \rho(\tilde{z}) (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \frac{1 - \rho(\tilde{z})}{\zeta(\tilde{z}) - 1} \hat{N}_t
\]
Related literature

- **Flattening of the Phillips curve**
  Mis-measurement (*Goolsbee and Klenow, 2018; Crump et al., 2019*); Labor market (*Daly and Hobijn, 2014*); Policy regime (*McLeay and Tenreyro, 2019; L’Huillier et al., 2022*); Inflation expectation (*Coibion and Gorodnichenko, 2015; Hazell et al., 2022*); Structural (*Sbordone, 2010; Wang and Werning, 2022; Del Negro et al., 2020; Baqae and al., 2021; L’Huillier et al., 2022; Harding et al., 2022; Rubbo, 2022*)

- **Competition and monetary policy (more on the next page)**
  *Wang and Werning (2022), Baqae, Farhi and Sangani (2021)*

- **Business cycle model with entry and exit under monopolistic competition**
  *Bilbiie, Ghironi and Melitz (2012, 2019), Bilbiie, Ghironi and Melitz (2008), Bilbiie, Fujiwara and Ghironi (2014), Bilbiie (2021)*

- **Equivalence/nonequivalence between Rotemberg and Calvo**

- **HSA**
Competition and monetary policy

- CES New Keynesian model, irrespective of entry and exit
  - Competition is irrelevant to the Phillips curve

- Wang and Werning (2022)
  - In an oligopoly model with the strategic interaction
    - $\uparrow$ concentration $\Rightarrow$ the Phillips curve with inflation persistence + endogenous cost-push shock

- Baqee, Farhi and Sangani (2021)
  - $\uparrow$ concentration $\Rightarrow$ $\downarrow$ slope of the Phillips curve + endogenous cost-push shock
  - The supply side effects of monetary policy through the misallocation channel

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- Simple to endogenize entry under HSA; difficult under Kimball
Structure of presentation

1. Introduction
2. HSA
3. New Keynesian model under HSA
4. Competition and the Phillips curve
5. Steady state analysis
6. Dynamic analysis
7. Calvo pricing
8. Omitted variable bias
9. Conclusion
Introduction

2. HSA

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HSA

- A continuum of varieties ($\omega \in \Omega$), gross substitutes and symmetry

- Market share of $\omega = \frac{p_t(\omega)c_t(\omega)}{P(p_t)C_t} = s\left(\frac{p_t(\omega)}{A(p_t)}\right)$ where $\int_{\Omega_t} s\left(\frac{p_t(\omega)}{A(p_t)}\right) d\omega \equiv 1$
  - Gross substitutes $\Rightarrow s(z)$ is decreasing in $z$
  - $A(p_t)$ implicitly defined by the adding-up constraint:
    $\int_{\Omega_t} s\left(\frac{p_t(\omega)}{A(p_t)}\right) d\omega \equiv 1$

- $A(p_t) \neq \text{constant} \times P(p_t)$
  - $A(p_t)$: single aggregator, the inverse measure of competitive pressures, fully captures cross price effects in the demand system
  - $P(p_t)$: theoretical price index, the inverse measure of TFP, captures the productivity consequences of price changes
CES as a special case of HSA:  \( s \left( \frac{p_t(\omega)}{A_t} \right) = \gamma_{CES} \left( \frac{p_t(\omega)}{A_t} \right)^{1-\theta}, \theta > 1 \)

- **Production function**

  \[ C_t = Z_C \left[ \int_{\Omega_t} c_t(\omega)^{1-\frac{1}{\theta}} \, d\omega \right]^\frac{\theta}{\theta-1} \]

- **Hicksian demand function**

  \[ c_t(\omega) = Z_C^{\theta-1} \left( \frac{p_t(\omega)}{P_t} \right)^{-\theta} C_t \]

- **The market share function**

  \[ s \left( \frac{p_t(\omega)}{A_t} \right) = \gamma_{CES} \left( \frac{p_t(\omega)}{A_t} \right)^{1-\theta} = \frac{p_t(\omega) c_t(\omega)}{P_t C_t} = Z_C^{\theta-1} \left( \frac{p_t(\omega)}{P_t} \right)^{1-\theta} \]

-  \( P_t = \text{constant} \times A_t, \text{ iff CES, proved by Matsuyama and Ushchev (2017)} \)
Three price indices

- $P_t$: the final goods price (CPI), which captures the productivity effects of entry – the reference price for consumers

$$\int_{\Omega_t} \frac{p_t(\omega) c_t(\omega)}{P_tC_t} \, d\omega \equiv 1$$

- $A_t$: the single price aggregator, which captures the competitive effects of entry – the reference price for firms

$$\int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) \, d\omega \equiv 1$$

- $p_t$: the average price index (PPI) – the measured price index (without entry effects)

$$p_t = \int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) p_t(\omega) \, d\omega$$
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Model

- Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014) under HSA

  - 4 agents
    1. Consumer
    2. Intermediate goods producer
    3. Final goods producer
    4. Central bank

- Symmetric equilibrium under *monopolistic competition*

- Rotemberg price adjustment cost – Calvo pricing in Section 7

- Endogenous entry but exogenous exit
Timing

- In every period, there is an unbounded mass of prospective entrants

- One-period time-to-build lag: entrants at time $t$ only start producing at time $t + 1$

- Exogenous destruction: all firms are subject to identical probability $\delta$ of exogenous firm destruction at the end of each period, after production and entry
  - A proportion $\delta$ of new entrants will never produce
Consumer

- A representative household maximizes

\[ E_0 \sum_{t=0}^{\infty} \beta^t (u(C_t) - v(L_t)) \]

subject to the budget constraint

\[
\frac{B_{t+1}}{P_t} + x_{t+1} \int_{\Omega_t+\Omega_{E,t}} \frac{V_t(\omega)}{P_t} d\omega + C_t = \frac{(1 + i_{t-1}) B_t}{P_t} + x_t \int_{\Omega_t} \frac{D_t(\omega) + V_t(\omega)}{P_t} d\omega + \frac{W_t}{P_t} L_t
\]

- The final good is produced with intermediate inputs under HSA technology, which leads to the HSA demand for intermediate inputs
**MC intermediate inputs producers**

- Intermediate inputs producer $\omega$ maximizes the value of the firm

$$\frac{V_t(\omega)}{P_t} = \mathbb{E}_0 \sum_{t=0}^{\infty} m_{t,t+1} \left( \frac{D_{t+1}(\omega)}{P_{t+1}} \right)$$

subject to the profit

$$\frac{D_t(\omega)}{P_t} = \frac{p_t(\omega)}{P_t} y_t(\omega) - \frac{W_t}{P_t} l_t(\omega) - \frac{\chi}{2} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \frac{p_t(\omega)}{P_t} y_t(\omega)$$

the linear production technology

$$y_t(\omega) = Z_{P,t} l_t(\omega)$$

the HSA demand curve

$$y_t(\omega) = c_t(\omega) = s \left( \frac{p_t(\omega)}{A_t} \right) \frac{P_tC_t}{p_t(\omega)}$$
Central bank

- A simple feedback rule

\[(1 + i_t) = (1 + i_{t-1})^{\alpha_i} \left( \frac{p_t}{p_{t-1}} - 1 \right)^{(1-\alpha_i)\alpha_\pi} u_t \]
Aggregate conditions

- Free entry condition
  \[
  \frac{W_t}{P_t} \frac{f_{E,t}}{Z_{E,t}} = \frac{V_t}{P_t}
  \]

- Labor market clearing
  \[
  L_t = N_t l_t + N_{E,t} \frac{f_{E,t}}{Z_{E,t}}
  \]

- Firm dynamics
  \[
  N_t = (1 - \delta) (N_{t-1} + N_{E,t-1}) = (1 - \delta) N_{H,t-1}
  \]

- Aggregate accounting
  \[
  \frac{V_t}{P_t} N_{E,t} + C_t = \frac{D_t}{P_t} N_t + \frac{W_t}{P_t} L_t
  \]

- Adding up constraint
  \[
  s\left(\frac{p_t}{A_t}\right) = s(z_t) = \frac{1}{N_t}
  \]

- Final goods price
  \[
  \ln\left(\frac{p_t}{A_t}\right) = \ln\left(\frac{z_t}{\bar{p}_t}\right) = \bar{K} - \frac{1}{s(z_t)} \left[ \int_{z_t}^{\bar{z}} \frac{s(\xi)}{\xi} d\xi \right]
  \]

where
\[
z_t := \frac{p_t}{A_t}, \quad \bar{p}_t := \frac{p_t}{P_t}
\]
Equilibrium

- An equilibrium in this economy is
  - a collection of sequence of aggregate prices \( \{P_t, A_t, W_t, i_t\} \) and the price of intermediate goods \( \{p_t\} \)
  - a collection of sequences of aggregate quantities \( \{Y_t, C_t, L_t\} \) and quantities of intermediate goods \( \{y_t, l_t\} \)
  - a collection of sequences of firm-value functions and profit \( \{V_t, D_t\} \) together with measures of operating firms and entering firms \( \{N_t, N_{E,t}\} \)

- These equilibrium objects satisfy the following conditions
  - households maximize their utility subject to their budget constraints
  - intermediate-good firms maximize the net present value of their per-period profits
  - final-good firms maximize profits
  - all of the feasibility constraints are satisfied
Preference and detrending

- Preference
  
  \[ u(C_t) := \frac{C_t^{1-\sigma} - 1}{1 - \sigma}, \quad \nu(L_t) := \frac{L_t^{1+\psi}}{1 + \psi} \]

- Nominal variables are detrended
  
  \[ w_t := \frac{W_t}{P_t}, \quad d_t := \frac{D_t}{P_t}, \quad v_t := \frac{V_t}{P_t}, \quad z_t := \frac{p_t}{A_t}, \quad \bar{p}_t := \frac{p_t}{P_t}, \quad \pi_t := \frac{p_t}{p_{t-1}} \]
System of equations

1. Taylor rule
   \[(1 + i_t) = (1 + i_{t-1})^{\alpha_i} (\pi_t - 1)^{1-\alpha_i} u_t\]

2. Euler equation for bonds
   \[C_t^{-\sigma} = \beta E_t C_{t+1}^{-\sigma} \frac{1 + i_t}{\pi_{t+1}} \frac{p_{t+1}}{p_t} \]

3. New Keynesian Phillips curve (hereafter, NKPC)
   \[
   \left[1 - \frac{\chi}{2} (\pi_t - 1)^2 \right] \frac{s'(z_t)z_t}{s(z_t)} + \left[1 - \frac{s'(z_t)z_t}{s(z_t)}\right] \frac{L_t^\psi C_t^\sigma}{Z_{P,t} P_t} - \chi (\pi_t - 1) \pi_t + \beta (1 - \delta) E_t \frac{C_{t+1}^- C_t^-}{C_t^-} \chi (\pi_{t+1} - 1) \pi_{t+1}^2 \frac{s(z_{t+1})}{s(z_t)} \frac{\tilde{p}_t}{p_{t+1}} \frac{A_{t+1}}{A_{t+1}} \frac{Y_{t+1}}{Y_t} = 0
   \]

4. Euler equation for equity
   \[L_t^\psi C_t^\sigma \frac{f_{E,t}}{Z_{E,t}} = \beta (1 - \delta) E_t \frac{C_{t+1}^- C_t^-}{C_t^-} \left\{ \left[1 - \frac{L_t^\psi C_{t+1}^-}{Z_{P,t} P_{t+1}} - \frac{\chi}{2} (\pi_{t+1} - 1)^2 \right] s(z_{t+1}) Y_{t+1} + L_t^\psi C_{t+1}^- \frac{f_{E,t+1}}{Z_{E,t+1}} \right\} \]

5. Firm dynamics
   \[\frac{1}{s(z_t)} = (1 - \delta) \left[ \frac{1}{s(z_{t-1})} + \frac{Z_{E,t-1}}{f_{E,t-1}} \left( L_{t-1} - \frac{Y_{t-1}}{\tilde{p}_{t-1} Z_{t-1}} \right) \right] \]

6. \(P_t/A_t\)
   \[\ln \left( \frac{P_t}{A_t} \right) = \ln \left( \frac{z_t}{\tilde{p}_t} \right) = K - \frac{1}{s(z_t)} \left[ \int_{z_t}^z \frac{s(\xi)}{\xi} d\xi \right] \]

7. Resource constraint
   \[C_t = \left[1 - \frac{\chi}{2} (\pi_t - 1)^2 \right] Y_t\]
Steady state

1. Taylor rule

\[ \pi = 1 \]

2. Euler equation for bonds

\[ i = \frac{1 - \beta}{\beta} \]

3. NKPC

\[ L^\Psi Y^\sigma = \frac{\bar{p}}{1 - \frac{s(z)}{s'(z)z}} Z_P \]

4. Euler equation for equity

\[ L = \frac{1}{1 - \delta} \frac{f_E}{Z_E} \left[ \delta - \frac{1 - \beta (1 - \delta)}{\beta} \frac{s'(z)z}{s(z)} \right] \frac{1}{s(z)} \]

5. Firm dynamics

\[ Y = -\frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} Z_P \frac{f_E}{Z_E} \frac{s'(z)z}{s(z)} \frac{z}{s(z)} \exp \left( \frac{f_z}{s(z)} \frac{s(\xi)}{\xi} d\xi \right) \exp \left( K - \frac{f_z}{s(z)} \frac{s(\xi)}{\xi} d\xi \right) \]

6. \( P/A \)

\[ \frac{P}{A} = \frac{z}{\bar{p}} = \exp \left( K - \frac{f_z}{s(z)} \frac{s(\xi)}{\xi} d\xi \right) \]
Linearized system of equations

1. Taylor rule

\[ i_t = \alpha_i i_{t-1} + (1 - \alpha_i) \alpha \hat{\pi}_t + u_t \]

2. Euler equation for bonds

\[ \hat{Y}_t = E_t \hat{Y}_{t+1} - \frac{1}{\sigma} \left[ i_t - \left( \hat{p}_t - E_t \hat{p}_{t+1} \right) - E_t \hat{\pi}_{t+1} \right] \]

3. NKPC

\[ \hat{\pi}_t = \frac{1}{\lambda} \left[ -\frac{s'(z)z}{s(z)} \right] \left( \psi \hat{L}_t + \sigma \hat{Y}_t - \hat{Z}_{P,t} - \hat{\pi}_t \right) + \frac{1}{\lambda} \left[ \frac{s''(z)z}{s(z)} - \frac{s'(z)z}{s(z)} \right] \hat{z}_t + \beta (1 - \delta) E_t \hat{\pi}_{t+1} \]

4. Euler equation for equity

\[
\psi \hat{L}_t = \left\{ \left[ 1 - \beta (1 - \delta) \right] \frac{s'(z)z}{s(z)} + \beta (1 - \delta) \right\} \psi E_t \hat{L}_{t+1} + \left[ 1 - \beta (1 - \delta) \right] \left[ \sigma \frac{s'(z)z}{s(z)} + (1 - \sigma) \right] E_t \hat{Y}_{t+1} \\
- \left( \hat{f}_{E,t} - \hat{Z}_{E,t} \right) + \beta (1 - \delta) \left( E_t \hat{f}_{E,t+1} - \hat{Z}_{E,t+1} \right) - \left[ 1 - \beta (1 - \delta) \right] \frac{s'(z)z}{s(z)} E_t \hat{Z}_{P,t+1} \]

5. Firm dynamics

\[ \hat{z}_t = (1 - \delta) \hat{z}_{t-1} - \left\{ \delta \frac{s(z)}{s'(z)z} - \frac{1 - \beta (1 - \delta)}{\beta} \right\} \hat{L}_t - \hat{Y}_t + \hat{p}_t + \hat{Z}_{P,t} \] + \delta \frac{s(z)}{s'(z)z} \left( \hat{f}_{E,t} - \hat{Z}_{E,t} \right) \]

6. \( \hat{p}_t - \hat{\lambda}_t \)

\[ \hat{p}_t - \hat{\lambda}_t = \hat{z}_t - \hat{p}_t = \left[ 1 + \int_0^\frac{\delta}{\xi} \frac{s(\xi)}{s(z)} d\xi \frac{s'(z)z}{s(z)} \right] \hat{z}_t \]
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NKPC under HSA

- How does market concentration affect the slope of the Phillips curve?
- How does the entry cost \( f_E \) affect the number of firms \( N \) in the steady state, and then the slope of NKPC?

\[
\hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{1}{\chi} \left[ - \frac{s'(z)}{s(z)} \right] (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) + \frac{1}{\chi} \left[ \frac{s'(z)}{s(z)} - \frac{s''(z)}{s'(z)} \right] \left[ - \frac{s'(z)}{s(z)} \right] \hat{z}_t
\]

\[
= \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} \left( \hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t \right) - \frac{1}{\chi} \frac{1 - \rho(z)}{\rho(z)} \hat{N}_t
\]

- Price elasticity: \( \zeta(z) := -\frac{\partial \ln(c_t(\omega))}{\partial \ln(p_t)} = 1 - \frac{s'(z)}{s(z)} > 1 \)
  - Markup rate under flexible price: \( \mu^f(z) = \zeta(z) / (\zeta(z) - 1) \)
  - Pass-through rate under flexible price: \( \rho(z) := \frac{\partial \ln(p_t)}{\partial \ln(W_t/Z_{P,t})} = \left[ 1 - \frac{\ln \left( \frac{\zeta(z)-1}{\zeta(z)} \right)}{\frac{\partial \ln(W_t/Z_{P,t})}{\partial \ln(z)}} \right]^{-1} \)

- The role of pass-through rate in NKPC: Baqee et al. (2021), Auclert et al. (2022)
NKPC under CES

- Under CES
  \[ s(z) = \gamma_{CES} z^{1-\theta} \]

- No effect from competition – the entry cost \( f_{E,t} \) – on parameters in NKPC
  \[ \hat{\pi}_t = \beta (1 - \delta) E_t \hat{\pi}_{t+1} + \frac{\theta - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) \]

- Due to constant price elasticity under CES
  - no effect to the slope of the Phillips curve
  - no dynamic effect of endogenous entry
Implications of the Second law $\zeta'(z) > 0$

$$\hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1}{\chi} \frac{1}{\rho(z)} \hat{N}_t$$

1. Steady-state effect of concentration - flattening of the Phillips curve
   - More concentration $\iff$ higher market share: $s(z) \uparrow \iff z \downarrow$
   - $\zeta'(z) > 0 \iff \zeta(z) \downarrow$, flattening of the Phillips curve

2. Dynamic effect of endogenous entry - endogenous cost-push shock
   - $\zeta'(z) > 0 \iff$ incomplete pass-through: $\rho(z) < 1 \iff$ strategic complementarity
     - The firm reduces its price and markup rate in response to more competitive pressure, a lower $A_t$, when other firms reduce their prices
     - If $\hat{\mu}_t = - (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t)$ and $\hat{N}_t$ move to the opposite directions to a structural shock, its impact on inflation is muted
Supply side effects of monetary policy

- Dynamic effect of endogenous entry $\Rightarrow$ the supply side effects of monetary policy

- Misallocation across heterogenous firms in Baqae, Farhi and Sangani (2021)
  - Monetary easing $\Rightarrow$ production shifts to more efficient firms $\uparrow$ $\Rightarrow$ aggregate TFP $\uparrow$ $\Rightarrow$ inflation-stimulating effect $\downarrow$

- Entry of firms in our model
  - Monetary easing $\Rightarrow$ number of firms $\uparrow$ $\Rightarrow$ markup rate via the Second law $\downarrow$ $\Rightarrow$ inflation-stimulating effect $\downarrow$
HDIA and HIIA

- Matsuyama and Ushchev (2017) characterize three classes of homothetic demand systems
  1. HSA
  2. HDIA (Homothetic with Direct Implicit Additivity): Without endogenous entry, this would be equivalent to Kimball (1995)

\[
\int_{\Omega_t} \varphi \left( \frac{y_t(\omega)}{Y_t(y_t)} \right) d\omega = 1
\]

\[
y_t(\omega) = \varphi'^{-1} \left( \frac{p_t(\omega)}{A_{\text{HDIA}}(p_t)} \right) Y_t(y_t)
\]

3. HIIA (Homothetic with Indirect Implicit Additivity)

\[
\int_{\Omega_t} \theta \left( \frac{p_t(\omega)}{P(p_t)} \right) d\omega = 1
\]

\[
y_t(\omega) = -\theta'^{-1} \left( \frac{p_t(\omega)}{P(p_t)} \right) B_{\text{HIIA}}(y_t)
\]
The same implications on NKPC can be derived by appropriately re-defining $z_t$

$$\hat{\pi}_t = \beta (1 - \delta) \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} \left[ (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho(z)}{\rho(z)} \hat{\pi}_t \right]$$

- Under HSA
  $$z_t = \frac{p_t}{A_t}$$

- Under HDIA
  $$z_t = \frac{p_t}{A_{HDIA,t}}$$

- Under HIIA
  $$z_t = \frac{p_t}{P_t}$$

GE implications, such as entry, productivity, and welfare, can be very different across the three classes
Cyclicality of markup to the technology shock

<table>
<thead>
<tr>
<th>Flexible price</th>
<th>Sticky price</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
<td>constant procyclical</td>
</tr>
<tr>
<td>The Second law</td>
<td>countercyclical procyclical / countercyclical</td>
</tr>
</tbody>
</table>

- In a flexible price equilibrium under CES, constant
- In a sticky price equilibrium under CES, procyclical
  - The marginal cost decreases but the price does not change
- In a flexible price equilibrium under the Second law, countercyclical
  - A positive technology shock increases the number of firms, which causes the markup rate to decline
- In a sticky price equilibrium under the Second law, generally ambiguous and depends on the tension between nominal rigidities and the pass-through rate

\[
\hat{\mu}_t = \frac{1}{\zeta(z) - 1} \left\{ \chi \left[ \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} - \pi_t \right] - \frac{1 - \rho(z)}{\rho(z)} \hat{N}_t \right\}
\]

- Disagreement about the cyclicality of the markup in the literature as surveyed by Nekarda and Ramey (2020)
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## Parametric families of HSA

<table>
<thead>
<tr>
<th></th>
<th>Share</th>
<th>Price elasticity</th>
<th>Pass-through</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CES</strong></td>
<td>( s(z) = \gamma_{CES} z^{1-\theta} )</td>
<td>( \zeta(z) = \theta )</td>
<td>( \rho(z) = 1 )</td>
</tr>
<tr>
<td><strong>Translog</strong></td>
<td>( s(z) = \gamma_{TL} \ln \left( \frac{\bar{z}}{z} \right) )</td>
<td>( \zeta(z) = 1 + \frac{1}{\ln \left( \frac{\bar{z}}{z} \right)} )</td>
<td>( \rho(z) = \frac{1+\ln \left( \frac{\bar{z}}{z} \right)}{2+\ln \left( \frac{\bar{z}}{z} \right)} )</td>
</tr>
<tr>
<td><strong>Co-PaTh</strong></td>
<td>( s(z) = \gamma_{CP} \theta^{1-\rho} \left[ 1 - \left( \frac{\bar{z}}{z} \right)^{\frac{1-\rho}{\rho}} \right]^{\frac{\rho}{1-\rho}} )</td>
<td>( \zeta(z) = \frac{1}{1 - \left( \frac{\bar{z}}{z} \right)^{\frac{1-\rho}{\rho}}} )</td>
<td>( \rho(z) = \rho &lt; 1 )</td>
</tr>
</tbody>
</table>

- \( \bar{z} := \inf \{ z > 0 | s(z) = 0 \} \): if \( \bar{z} < \infty \), \( \bar{z}A_t \) is the choke price

\[
\bar{z} = \left( \frac{\theta}{\theta - 1} \right)^{\frac{\rho}{1-\rho}}
\]
Calibration

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>exit rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\psi$</td>
<td>inverse of labor supply elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$f_E, Z_E, Z_P$</td>
<td>technologies</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>price elasticity under CES</td>
<td>3.8</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Rotemberg adj. cost</td>
<td>77</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>policy inertia</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>policy reaction to $\pi$</td>
<td>1.1 or $\infty$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>pass-through rate</td>
<td>1, 0.9 or 0.5</td>
</tr>
</tbody>
</table>

- Most are taken from Bilbiie, Fujiwara and Ghironi (2014)
- $\bar{K} = 1/ (\theta - 1) \iff Z_C = \gamma_{CES}^{1/(\theta-1)} \iff P_t = A_t$ under CES
- $\gamma_{TL}$ and $\gamma_{CP}$ are calibrated so that $\zeta(z) = \theta$ when $f_E = 1$
Entry cost and the slope of NKPC (Fig. 2)
Concentration and the slope of NKPC (Fig. 2)

- The slope of NKPC: $\frac{\zeta(z) - 1}{\chi}$, where $\frac{dz}{dN} > 0$,
- The Second law

$$\zeta'(z) > 0, \quad \frac{d\ln \left( \frac{\zeta(z)}{\zeta(z) - 1} \right)}{d\ln(z)} = 1 - \frac{1}{\rho(z)} < 0$$
SS with varying entry cost (Fig. 1)

- Increasing barriers to entry: higher entry cost
  - Market concentration: fewer number of firms
    - More concentration ⇔ high market share (concentration), \( s(z) \uparrow ⇔ z \downarrow \)
    - The Second law: \( \zeta'(z) > 0 \Rightarrow \zeta(z) \downarrow \)
  - Higher markup, \( \mu(z) = \frac{\zeta(z)}{\zeta(z) - 1} \uparrow: \) higher profit

\[ \text{Introduction} \quad \text{HSA} \quad \text{NK-HSA} \quad \text{NKPC} \quad \text{SS} \quad \text{Dynamic} \quad \text{Calvo} \quad \text{OVB} \quad \text{Conclusion} \quad \text{References} \]
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Impulse responses

- Two shocks
  1. Technology shock: $Z_{P,t}, Z_{E,t}$
  2. Monetary policy shock: $u_t$

- Two scenarios
  1. Around the same steady state markup
  2. Around the different market concentration reflecting difference in the entry cost
     - Two countries
     - Two regulatory regimes
Technology shock by pass-through rate (Fig. 3)

- Under both sticky and flexible prices
  - Higher return $\Rightarrow C_t = Y_t \uparrow$ and $N_{E,t} \uparrow$ for intertemporal smoothing, but gradual increase in $N_t \Rightarrow z_t \uparrow \Rightarrow y_t \downarrow \Rightarrow d_t \downarrow$
  - Smaller pass-through $\Rightarrow$ lower markup $\Rightarrow$ lower profits $\Rightarrow$ weaker incentive for creation $\Rightarrow$ muted increase in $N_t$

- Sticky vs Flex
  - Countercyclical (procyclical) markup under flexible (sticky) price
    - Cyclicality of markup depends on the pass-through and nominal rigidities
    - The dynamic effect of endogenous entry $\Rightarrow$ deflation with small $\rho(z)$
Technology and monetary policy shocks by entry cost (Fig. 4)

- Higher entry cost ⇒ higher market concentration
  - Response of $\pi_t$ becomes smaller
  - The Second law ⇒ flatter NKPC
Introduction

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NKPC under Calvo pricing

\[
\hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \left[ \rho (\tilde{z}) (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho (\tilde{z})}{\zeta (\tilde{z}) - 1} \hat{N}_t \right]
\]

where \( \hat{\pi}_t := \tilde{p}_t / \tilde{p}_{t-1}, \tilde{z}_t := \tilde{p}_t / A_t \) and \( \tilde{p}_t \) is implicitly given by
\[
\int_{\Omega_t} s \left( p_t (\omega) / A_t \right) d\omega = 1 = N_t s \left( \tilde{p}_t / A_t \right)
\]

- Concentration leads to flattening of NKPC under the Third law (Matsuyama and Ushchev, 2023; Baqae et al., 2023)
  - A higher price leads to a smaller rate of change in the price elasticity
  - A higher entry cost leads to less competitive pressures and lowers the pass-through rate
  - Translog cannot accommodate the Third law

- Strategic complementarity \( \Rightarrow \) real rigidity
Power elasticity of markup rate

- Power Elasticity of Markup rate (hereafter, PEM) proposed by Matsuyama and Ushchev (2023) accommodates the third law
- The market share function: either $\bar{z} = \infty$ and $c \leq 1$ or $\bar{z} < \infty$ and $c = 1$, $\kappa > 0$ and $\lambda > 0 \Rightarrow$ the Second law and the strong Third law

$$s(z) = \exp \left[ \int_{z_0}^{z} \frac{c}{c - \exp \left( -\frac{\kappa \bar{z} - \lambda}{\lambda} \right) \exp \left( \frac{\kappa \xi - \lambda}{\lambda} \right)} \frac{d\xi}{\xi} \right]$$

- Price elasticity
  $$\zeta(z) = \frac{1}{1 - c \exp \left( \frac{\kappa \bar{z} - \lambda}{\lambda} \right) \exp \left( -\frac{\kappa z - \lambda}{\lambda} \right)}$$

- Pass-through
  $$\rho(z) = \frac{1}{1 + \kappa z - \lambda}$$

- PEM collapses to
  - CES with $\kappa = 0$, $c = 1 - 1/\theta$ and $\bar{z} = \infty$
  - Co-PaTh with $\lambda = 0$, $\kappa = (1 - \rho) / \rho$, $c = 1$ and $\bar{z} < \infty$
The markup rate and the slope of NKPC under Calvo

Each locus traces the markup rate $\zeta(\tilde{z}) / [\zeta(\tilde{z}) - 1]$ and the slope of NKPC $(1 - \phi) [1 - \phi \beta (1 - \delta)] \rho(\tilde{z}) / \phi$ by changing $\tilde{z}$ under several $\lambda$ with the reset probability $1 - \phi = 0.25$
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Omitted variable bias

- We now show that under the Second law
  - Concentration affects price setting dynamically through strategic complementarity - endogenous cost-push shock
  - A *naive* regression of the inflation rate on the real marginal cost $\Rightarrow$ the omitted variable bias (OVB): underestimating the slope

- Under Rotemberg pricing, the Third law matters for the magnitude of negative bias

- Under Calvo pricing, both Second and Third laws matter for the magnitude of negative bias
OVB: Rotemberg pricing

- NKPC under Rotemberg pricing

\[
\hat{\pi}_t - \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} = \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho(z)}{\chi \rho(z)} \hat{N}_t
\]

\[
= \kappa^R (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho(z)}{\chi \rho(z)} \hat{N}_t
\]

- Estimation of NKPC

\[
\hat{\pi}_t - \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} = \kappa (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) + \varepsilon_t
\]

- The estimated slope coefficient is biased

\[
\kappa = \kappa^R - \frac{1 - \rho(z)}{\chi \rho(z)} \frac{\text{cov} ( (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t), \hat{N}_t )}{\sigma^2_x}
\]

\[
= \kappa^R + \frac{1 - \rho(z)}{\chi \rho(z)} \frac{\text{cov} (\hat{\mu}_t, \hat{N}_t)}{\sigma^2_x}
\]

- The Second law
  - countercyclical markup \( \Leftrightarrow \text{cov} (\hat{\mu}_t, \hat{N}_t) < 0 \Rightarrow \text{negative bias} \)

- The Third law
  - \( \uparrow \) market concentration \( \Rightarrow \uparrow \left[ 1 - \rho(z) \right] / \rho(z) / \chi \Rightarrow \uparrow \text{negative bias} \)
OVB: Calvo pricing

- **NKPC under Calvo pricing**

\[
\hat{\pi}_t - \beta (1 - \delta) E_t \hat{\pi}_{t+1} = \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)] \rho (\bar{\pi}) (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho (\bar{\pi})}{\zeta (z) - 1} \hat{N}_t}{\phi} \\
= \kappa_C (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \frac{1 - \rho (\bar{\pi})}{\zeta (z) - 1} \hat{N}_t
\]

- **The estimated slope coefficient is biased**

\[
\bar{\kappa} = \kappa_C - \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \frac{\rho (\bar{\pi}) [1 - \rho (\bar{z})]}{\zeta (\bar{z}) - 1} \frac{\text{cov} \left( (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t), \hat{N}_t \right)}{\sigma_x^2} \\
= \kappa_C + \frac{(1 - \phi) [1 - \phi \beta (1 - \delta)]}{\phi} \frac{\rho (\bar{\pi}) [1 - \rho (\bar{z})]}{\zeta (\bar{z}) - 1} \frac{\text{cov} \left( \hat{\mu}_t, \hat{N}_t \right)}{\sigma_x^2}
\]

- **The Second law**
  - countercyclical markup \(\Leftrightarrow\) \(\text{cov} \left( \hat{\mu}_t, \hat{N}_t \right) < 0 \Rightarrow\) negative bias

- **The Second and Third laws**
  - \(\uparrow\) market concentration \(\Rightarrow\) \(\downarrow\) the price elasticity (the Second law) + \(\downarrow\) the pass-through rate (the Third law) \(\Rightarrow\) \(\uparrow\) \(\rho (\bar{\pi}) [1 - \rho (\bar{\pi})] / [\zeta (\bar{\pi}) - 1] \Rightarrow\) \(\uparrow\) negative bias
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Key takeaways

- **Steady-state effect of concentration**
  - Gross substitutes: $s(z) \uparrow \Leftrightarrow z \downarrow$
  - Under Rotemberg pricing, *the Second law* $\Rightarrow$ lower price elasticity $\Rightarrow$ higher markup rate $\Rightarrow$ *structurally* flattening the Phillips curve
  - Under Calvo pricing, *the Third law* $\Rightarrow$ lower pass-through rate $\Rightarrow$ *structurally* flattening the Phillips curve

- **Dynamic effect of endogenous entry**
  - Concentration affects price setting dynamically through strategic complementarity - endogenous cost-push shock
    - The *supply side effects of monetary policy* through the entry of firms
  - Pass-through rate plays an important role in cyclicality of markup

- **Observational implications of concentration and endogenous cost-push shock under the Second law**
  - A *naive* regression of the real marginal cost on the inflation rate $\Rightarrow$ the *negative* OVB: underestimating the slope
  - Under Rotemberg, *the Third law matters for the magnitude*
  - Under Calvo, both *Second and Third laws* matter for the magnitude
Future studies include

- Optimal policy
- Wage Phillips curve
- More general price setting mechanisms such as menu cost
  - We conjecture that results would be a hybrid of Rotemberg and Calvo
- Heterogeneous firm


Daly, Mary C. and Bart Hobijn, “Downward Nominal Wage Rigidities Bend the Phillips Curve,” Journal of Money, Credit and Banking, October 2014, 46 (S2), 51–93.


Harding, Martin, Jesper Linde, and Mathias Trabandt, “Resolving the missing deflation puzzle,” Journal of Monetary Economics, 2022, 126 (C), 15–34.


10 $P(p_t) \text{ vs } A(p_t)$

11 Super-elasticity

12 Flexible price by entry

13 NKP with output gap

14 Computation of $z$
\( P(p_t) \) vs \( A(p_t) \)

- **Under HSA**

\[
\frac{\partial P(p_t)}{P(p_t)} / \frac{\partial p_t(\omega)}{p_t(\omega)} = s\left(\frac{p_t(\omega)}{A(p_t)}\right) \Rightarrow \frac{\partial (P(p_t) / A(p_t))}{P(p_t) / A(p_t)} = \frac{s(z_t(\omega))}{z_t(\omega)} \frac{\partial z_t(\omega)}{z_t(\omega)}
\]

- **By integrating**

\[
\ln\left(\frac{P(p_t)}{A(p_t)}\right) = \bar{K} - \int_{\Omega} \left[ \int_{p_t(\omega)}^{\bar{z}} \frac{s(\zeta)}{\bar{z}} d\zeta \right] d\omega
\]

- \( \bar{z} := \inf\{z > 0 | s(z) = 0\} \): if \( \bar{z} < \infty \), \( \bar{z}A_t \) is the choke price
- \( \bar{K} \): a constant

- \( P(p_t) = \text{constant} \times A(p_t) \) iff CES, because, by differentiating the adding-up constraint

\[
\frac{\partial \ln(A(p_t))}{\partial \ln(p_t(\omega))} = \frac{p_t(\omega)}{A(p_t)} s'\left(\frac{p_t(\omega)}{A(p_t)}\right) \neq s\left(\frac{p_t(\omega)}{A(p_t)}\right) = \frac{\partial \ln(P(p_t))}{\partial \ln(p_t(\omega))}
\]

unless \( s(z) \) is a power function
10. \( P(p_t) \) vs \( A(p_t) \)

11. **Super-elasticity**

12. Flexible price by entry

13. NKP with output gap

14. Computation of \( z \)
Super-elasticity

- Super-elasticity (Klenow and Willis, 2016)

\[ \xi(z) := \frac{\zeta'(z)z}{\zeta(z)} \]

- The relationship between the super-elasticity and the pass-through rate is given by

\[ \xi(z) = \left[ \frac{1}{\rho(z)} - 1 \right] [\zeta(z) - 1] \]

- NKPC under Rotemberg

\[ \hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1}{\chi} \frac{\xi(z)}{\zeta(z) - 1} \hat{N}_t \]

- NKPC under Calvo

\[ \hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{1 - \phi}{\phi [\zeta(z) - 1 + \xi(z)]} \left\{ [\zeta(z) - 1] (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{\xi(z)}{\zeta(z) - 1} \hat{N}_t \right\} \]
10. \( P(p_t) \) vs \( A(p_t) \)

11. Super-elasticity

12. Flexible price by entry

13. NKP with output gap

14. Computation of \( z \)
Flexible price by entry

When entry firms face no nominal frictions, NKPC is given by

\[ \hat{\pi}_t^* = \beta (1 - \delta) \hat{\pi}_{t+1}^* + \frac{\zeta(z) - 1}{\chi} \frac{1}{1 - \delta} [1 - \delta \rho(z)] (\hat{\omega}_t + \hat{p}_t - \hat{Z}_{P,t}) \]

\[ - \frac{1 - \rho(z)}{\chi} \frac{1}{\rho(z)} \frac{1}{1 - \delta} \hat{N}_t \]

where the average inflation rate is implicitly given by

\[ \sum_{\tau=2}^{\infty} (1 - \delta)^{\tau-1} \hat{\pi}_{t, t-\tau} = \sum_{\tau=2}^{\infty} (1 - \delta)^{\tau-1} \hat{\pi}_t^* = \frac{1 - \delta}{\delta} \hat{\pi}_t^* \]
10 $P(p_t)$ vs $A(p_t)$

11 Super-elasticity

12 Flexible price by entry

13 NKP with output gap

14 Computation of $z$
NKP with output gap

To transform the real marginal cost (reciprocal of the markup rate) into the output gap, we extend Bilbiie (2021)’s static entry model to include sticky prices and HSA

\[ \hat{\pi}_t = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{\zeta(z) - 1}{\chi} \frac{1 - \rho(z)}{\rho(z)} \hat{z}_t \]

\[ = \beta \mathbb{E}_t \hat{\pi}_{t+1} + \kappa(z) \hat{Y}_t - \frac{\zeta(z) - 1}{\chi} \frac{1 - \rho(z)}{\rho(z)} \hat{z}_t \]

where

\[ \kappa(z) := \frac{1}{\chi} \frac{\sigma + \psi}{1 + \psi} [\zeta(z) - 1] - \frac{1 - \sigma}{1 + \psi} \frac{\zeta(z) - 1}{s(z)} \int_{\tilde{z}}^{\bar{z}} \frac{s(\xi)}{\xi} d\xi - \frac{\zeta(z) - 1}{s(z)} \int_{z}^{\tilde{z}} \frac{s(\xi)}{\xi} d\xi \]

Flattening of NKP with concentration under benchmark calibration

\[ \frac{d\kappa(z)}{dz} > 0 \]
| 10 | $P(p_t)$ vs $A(p_t)$ |
| 11 | Super-elasticity |
| 12 | Flexible price by entry |
| 13 | NKP with output gap |
| 14 | Computation of $z$ |
Computation of $z$

- Steady state of $z$ is pinned down by
  - Pricing formula: $f(L(z), Y(z), z) = 0$

$$\frac{MRS}{L(z)^\psi Y(z)^\sigma} = \frac{w}{z}$$

- Firm dynamics: $Y = Y(z)$

$$Y(z) = -\frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} Z_P \frac{f_E}{Z_E} \frac{s'(z)z}{s(z)} \frac{1}{s(z)}$$

- Value of the firm: $L = L(z)$

$$L(z) = \frac{1}{1 - \delta} \frac{f_E}{Z_E} \left[ \delta - \frac{1 - \beta (1 - \delta)}{\beta} \frac{s'(z)z}{s(z)} \right] \frac{1}{s(z)}$$
**Computation of \( z \) under CES**

- **Steady state of \( z \) is pinned down by**
  1. **Pricing formula:** \( f(L(z), Y(z)) = \text{constant} \)

\[
\frac{\text{MRS}}{L(z)^{\psi} Y(z)^{\sigma}} = \frac{w}{\frac{\text{markup}}{\theta - 1} \frac{\text{MPL}}{Z_{P}}}.
\]

- **Firm dynamics:** \( Y = Y(z) \):
  \[
  \frac{\partial Y}{\partial z} > 0, \quad \frac{\partial^2 Y}{\partial z \partial f_{E}} > 0
  \]

\[
Y(z) = \frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} Z_{P} \frac{f_{E}}{Z_{E}} (\theta - 1) z^{\theta - 1}
\]

- **Value of the firm:** \( L = L(z) \):
  \[
  \frac{\partial L}{\partial z} > 0, \quad \frac{\partial^2 L}{\partial z \partial f_{E}} > 0
  \]

\[
L(z) = \frac{1}{1 - \delta} \frac{f_{E}}{Z_{E}} \left[ \delta + \frac{1 - \beta (1 - \delta)}{\beta} (\theta - 1) \right] z^{\theta - 1}
\]
Labor market equilibrium with $f_E$: Co-PaTh, $\rho = 0.9$
Analytical integration

- For some special cases, we can obtain an analytical expression for the integral, \( \int_{\bar{z}}^{\tilde{z}} \frac{s(\xi)}{\xi} d\xi \).

- Under Co-PaTh, the integral is given by the hypergeometric function

\[
\int \frac{1 - \left( \frac{\xi}{\bar{z}} \right)^{\frac{1}{\nu}}}{\frac{\xi}{\zeta}} d\xi = \left[ \frac{1 - \left( \frac{\xi}{\bar{z}} \right)^{\frac{1}{\nu}}}{1 + \nu} \right]^{1+\nu} \sum_{n=0}^{\infty} \frac{(1)_n (1 + \nu)_n}{(2 + \nu)_n n!} \left[ 1 - \left( \frac{\xi}{\bar{z}} \right)^{\frac{1}{\nu}} \right]^n.
\]

- Under Translog

\[
\int \frac{\gamma_{TL} \ln \left( \frac{\xi}{\tilde{z}} \right)}{\xi} d\xi = -\frac{\gamma_{TL}}{2} \left[ \ln \left( \frac{\xi}{\tilde{z}} \right) \right]^2.
\]