Social Custom vs. Fashion Cycle: Path-Dependence and Limit Cycle in the Best Response Dynamics

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Introduction

"Fashion is custom in the guise of departure from custom."
Edward Sapir (1930, p.140)
"Fashion is evolution without destination."
Agnes Brooks Young (1937, p.5)

- **Fashion as a *Collective Process of Evolving Tastes***
  - A process of *continuous change* in which some forms of social behavior enjoy temporary popularity only to be replaced by others, unlike **Social Custom.**
  - A *recurring* process, in which many "new" styles are not so much born as rediscovered: the cyclical and regular nature, unlike **Fads**.

- Many areas of human activity are under the sway of fashion.
  - Not only in the area of dress, but also architecture, music, painting, literature, business practice, political doctrines, scientific ideas (not least in economic theory).
  - Also every stage of our lives: from names given at birth to the forms of gravestones.

Economics: few, if any, attempts to identify mechanisms behind such fashion cycles, where it is hard to believe that they occur due to some marketing ploys by “fashion industries”. (Nobody owns the patent on the style of architecture, literature, music and painting. Even in fashion industries, nobody owns the patent on color, the length of skirts, the width of neckties, etc.)

Psychology: Two forces behind continuous recurrent patterns of fashion

- **Conformity:** the desire to adopt and imitate others, or to join the crowd.
  - People may imitate others out of admiration or by the desire to assert equality with them.
  - They may follow with enthusiasm or be coerced through ridicule and ostracism
- **Nonconformity:** the desire to acquire individuality and personal distinction, or to disassociate one's self from the masses.

My Hypothesis: These two fundamentally irreconcilable human desires-- the desire to act or look the same, and the desire to act or look different—both must operate for the recurring process of fashion to emerge and persist.

- Conformity *alone* would lead to an emergence of the social custom, or convention.
- Nonconformity *alone* would prevent any discernible patterns from emerging.

Related but different from
Simmel ([1904]1957, p.546) "two social tendencies are essential to the establishment of fashion, namely, the need of union on the one hand and the need of isolation on the other. Should one of these be absent, fashion will not be formed--its sway will abruptly end."

- We study the conditions for an emergence of social customs and fashion cycles in a pairwise random matching game with 2 types of players: C(onformists) & N(onconformists)

  - Each player is matched with both types with some probability
  - Each player chooses one of two actions before matching.
  - When matched, two players observe the choice made by their partners.
    - C gains a higher payoff if he and his partner have made the *same* choice.
    - N gains a higher payoff if she and her partner have made the *different* choices.

Two actions are **strategic complements** for a C, and **strategic substitutes** for a N

- As a static (one-shot) game, the set of Nash equilibria depends on the relative size of the two groups and the relative frequency of Across-types vs. Within-types matching.
  - If played only by a pair of C, the game of strategic complements, with two stable Nash in pure strategy and one unstable Nash in mixed strategies like in *Pure Coordination Game*.
  - If played only between a C and a N, the game of strategic substitutes, with a unique Nash in mixed strategies, like *Matching Pennies*.
  - A unique Nash in mixed strategies also if played only by a pair of N's.

- **Dynamic game in continuous time:** the *best response dynamics* (a constant fraction of players can switch to the best response to the current distribution of strategies)
  - **Global convergence** to the stable distribution of strategies, common across the types No discernible patterns, or the state of **disorder**, emerge.
  - **Path-dependence:** C set **the social custom**, and N revolt against it; the initial condition or "history" determines which action becomes the custom.
  - **Global convergence to Limit cycle:** N become fashion leaders and switch their actions periodically, while C follow with delay.

- **Two kinds of bifurcation** leads to the emergence of the limit cycle
  - **1st:** an increase in the share of C leads to a loss of stability in the unique mixed-strategy Nash in the game with strategic substitutes, *like Hopf Bifurcation*
    The regular patterns of fashion cycles emerge from disorder.
  - **2nd:** a decrease in the share of C eliminates the two locally stable Nash in pure strategy in the game with strategic complements, *like Heteroclinic Bifurcation*.
    Fashion cycles emerge as departure from custom.

  In summary,
  - Too strong nonconformity lead to **disorder**.
  - Too strong conformity lead to the emergence of **the social custom**.
  - **Transition from disorder to fashion cycles to social custom** occurs as the share of C goes up, when Across-types matchings are more frequent to Within-types.

The Matching Game

The Matching Game

Continuum of Anonymous Players: Conformists, $\theta \in (0,1)$, & Nonconformists, $1-\theta$

Pairwise Matching in Continuous Time: Poisson Arrival Rates

	C	N
C	$(1-\beta)\theta dt$	$\beta(1-\theta)dt$
N	$\beta\theta dt$	$(1-\beta)(1-\theta)dt$

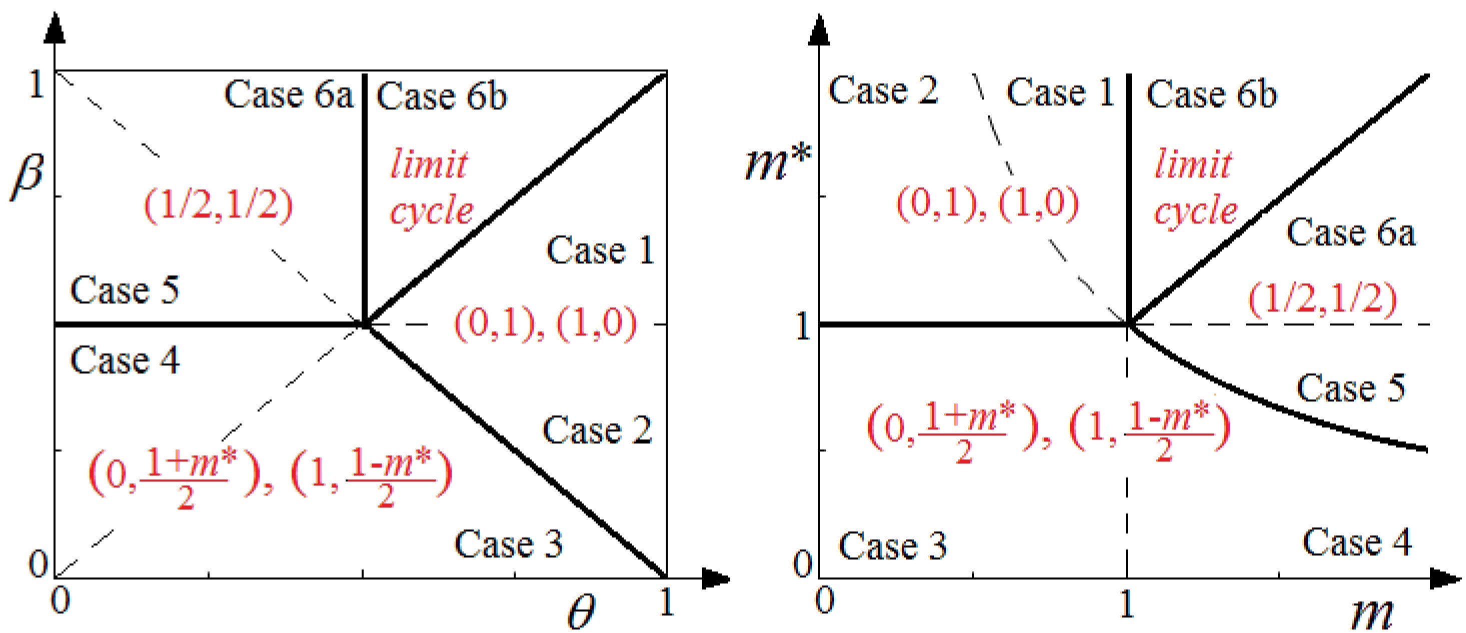
- the prob of being matched with a particular type is proportional to the size of that type.
- $\beta \in (0,1)$: relative match frequency of Across-vs-Within types;
  - All matches are Within types if $\beta \to 0$;
  - All matches are Across types if $\beta \to 1$.

Relative Frequency of Being Matched to A Different Type:

$m = \frac{\beta(1-\theta)}{(1-\beta)\theta}$ for C; $m^* = \frac{\beta\theta}{(1-\beta)(1-\theta)}$ for N

Diverse Matching Patterns:

One-to-one correspondence: $(\theta, \beta) \in [0,1]^2 \leftrightarrow (m, m^*) \in R_+^2$



Strategies: Blue (B) or Red (R): Each player must choose without knowing the match.

Strategy Distributions (Behavioral Patterns): $\lambda_t(\lambda_t^*)$: the fraction of C(N) choosing B

Payoff: 0 when not matched. When matched,

Conformist's Payoff ($S > D$)

	B	R
B	S	D
R	D	S

Nonconformist's Payoff ($S^* < D^*$)

	B	R
B	S^*	D^*
R	D^*	S^*

C's Expected Payoffs: $\Pi_{Bt} = p_{Bt}S + p_{Rt}D$; $\Pi_{Rt} = p_{Bt}D + p_{Rt}S$ with

$$p_{Bt} = \lambda_t(1-\beta)\theta + \lambda_t^*\beta(1-\theta);$$

$$p_{Rt} = (1-\lambda_t)(1-\beta)\theta + (1-\lambda_t^*)\beta(1-\theta);$$

N's Expected Payoffs: $\Pi_{Bt}^* = p_{Bt}^*S^* + p_{Rt}^*D^*$; $\Pi_{Rt}^* = p_{Bt}^*D^* + p_{Rt}^*S^*$ with

$$p_{Bt}^* = \lambda_t\beta\theta + \lambda_t^*(1-\beta)(1-\theta);$$

$$p_{Rt}^* = (1-\lambda_t)\beta\theta + (1-\lambda_t^*)(1-\beta)(1-\theta);$$

Static (One-Shot) Game

Static (One-Shot) Game: If each player were free to choose between B & R at any time, being anonymous and atomistic, they would play this game as if a static, one-shot game.

- C would choose B (R) if he expects $p_B > p_R$ ($p_B < p_R$);
- N would choose R (B) if she expects $p_B^* > p_R^*$ ($p_B^* < p_R^*$)

Best Response Correspondences:

(2a) $$\lambda \in \begin{cases} \{1\} & if \quad (\lambda - 1/2) + m(\lambda^* - 1/2) > 0 \\ [0,1] & if \quad (\lambda - 1/2) + m(\lambda^* - 1/2) = 0 \\ \{0\} & if \quad (\lambda - 1/2) + m(\lambda^* - 1/2) < 0 \end{cases}$$

(2b) $$\lambda^* \in \begin{cases} \{0\} & if \quad m^*(\lambda - 1/2) + (\lambda^* - 1/2) > 0 \\ [0,1] & if \quad m^*(\lambda - 1/2) + (\lambda^* - 1/2) = 0 \\ \{1\} & if \quad m^*(\lambda - 1/2) + (\lambda^* - 1/2) < 0 \end{cases}$$

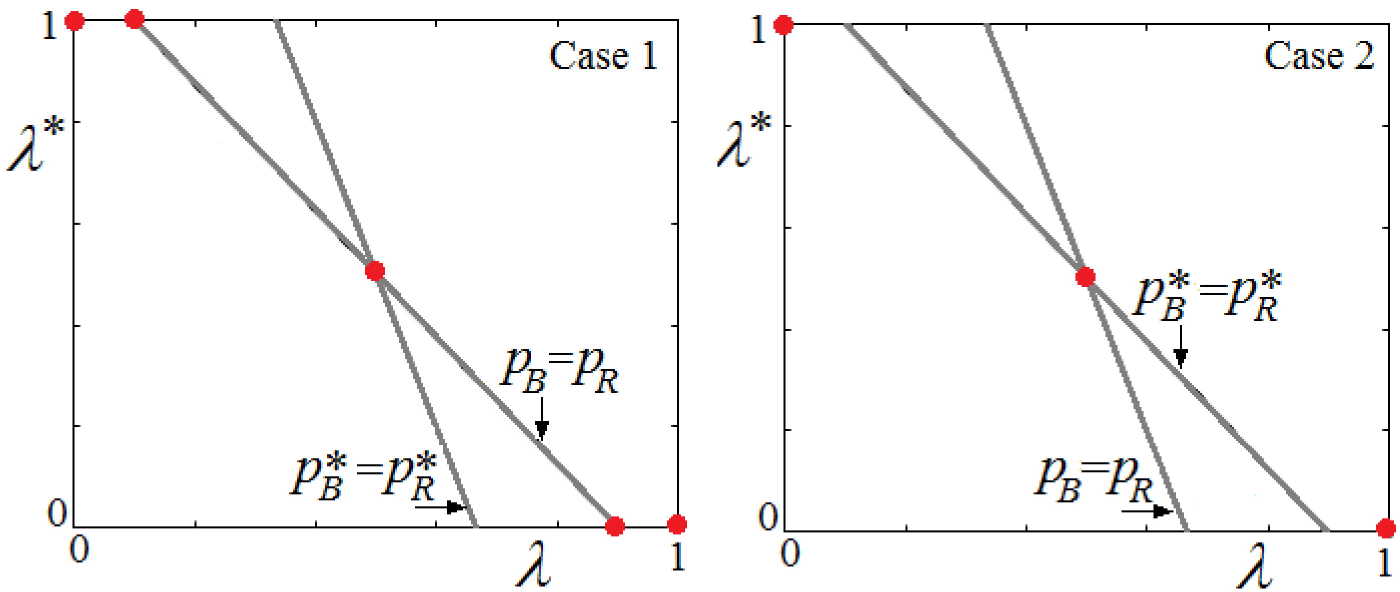
<u>Proposition 1:</u> The Nash equilibria of the static game are:

Case 1$(m < 1 < mm^*;\ 1/2 < \beta < \theta)$;

$$(\lambda, \lambda^*) = (1/2,1/2), (1,0), (0,1), ((1-m)/2,1) \text{ and } ((1+m)/2,0)$$

Case 2 $(m < mm^* < 1;\ 1-\theta < \beta < 1/2)$;

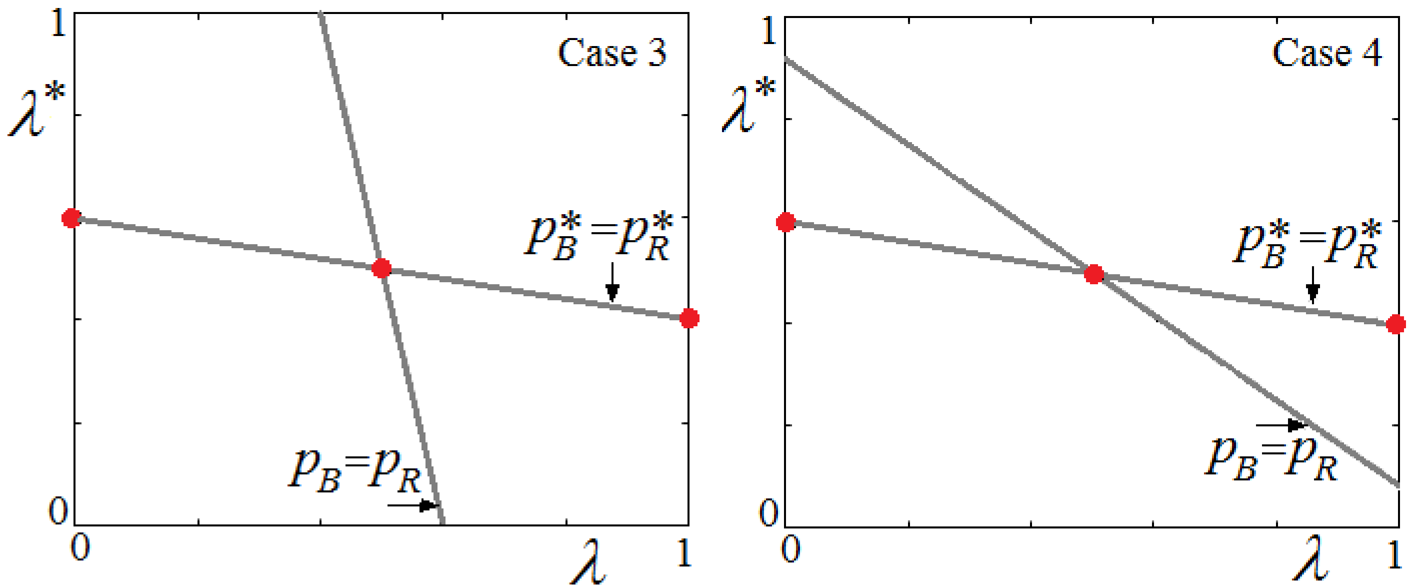
$$(\lambda, \lambda^*) = (1/2,1/2), (1,0), \text{ and } (0,1).$$



The share of C is sufficiently large $(\theta > \beta, 1-\beta)$, so that all players are matched more often with a C, rather than with a N $(m^* > 1 > m)$.

Case 3 $(mm^* < m < 1;\ \beta < \theta < 1-\beta)$ and Case 4 $(m > 1 > mm^*;\ 1/2 > \beta > \theta)$:

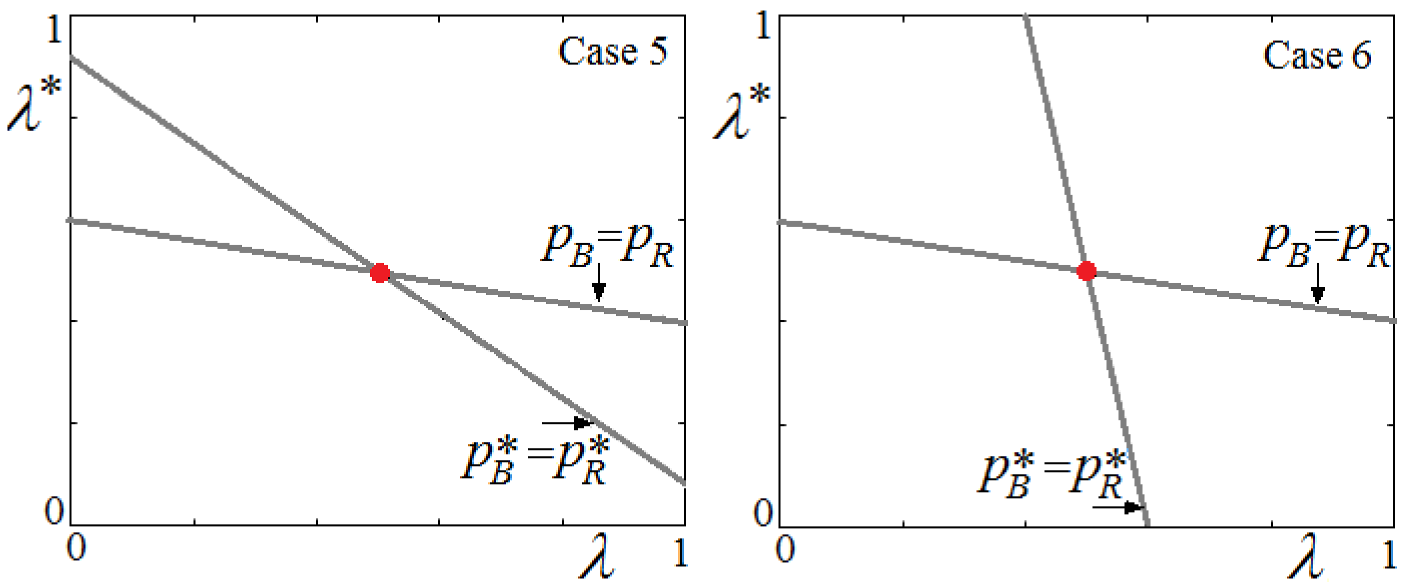
$$(\lambda, \lambda^*) = (1/2, 1/2), (1, (1-m^*)/2) \text{ and } (0, (1+m^*)/2)$$



With a large share of N and matching biased toward within-types $(\beta < 1-\theta)$, a N is matched with a N more often than with a C $(m^* < 1)$; a C meets a C more often than a N does $(\beta < 1/2$ or $m < 1/m^*)$.

Case 5($m > mm^* > 1$; $1 - \theta > \beta > 1/2$) and Case 6($mm^* > m > 1$; $\beta > \theta > 1 - \beta$):

$$(\lambda, \lambda^*) = (1/2, 1/2).$$



Case 5: N play primarily among themselves ($m^* < 1$). With matching biased toward Across-types ($\beta > 1/2$), a C meets a N more often than a N does ($m > 1/m^*$).

Case 6: With matching biased toward Across-types, C mees N more often and N meets C more often. ($\beta > \theta, 1 - \theta$ or $m, m^* > 1$).

Best Response Dynamics

Best Response Dynamics: Gilboa and Matsui (1991)

At each time, the suboptimal strategy switches to the optimal strategy at the rate, $\alpha > 0$.

$$\text{(3a)}\quad \dot{\lambda}_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & if \quad (\lambda_t - 1/2) + m(\lambda_t^* - 1/2) > 0 \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & if \quad (\lambda_t - 1/2) + m(\lambda_t^* - 1/2) = 0 \\ \{-\alpha\lambda_t\} & if \quad (\lambda_t - 1/2) + m(\lambda_t^* - 1/2) < 0 \end{cases}$$

$$\text{(3b)}\quad \dot{\lambda}_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & if \quad m^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) > 0 \\ [-\alpha\lambda_t^*, \alpha(1-\lambda_t^*)] & if \quad m^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) = 0 \\ \{\alpha(1-\lambda_t^*)\} & if \quad m^*(\lambda_t - 1/2) + (\lambda_t^* - 1/2) < 0 \end{cases}$$

One Justification:

- The opportunity to switch actions follows Poisson process with the arrival rate, $\alpha > 0$
- When the opportunity arrives, each player chooses the action that results in a higher expected discounted payoff over the next expected duration of commitment, with the discount rate, $\delta > 0$.

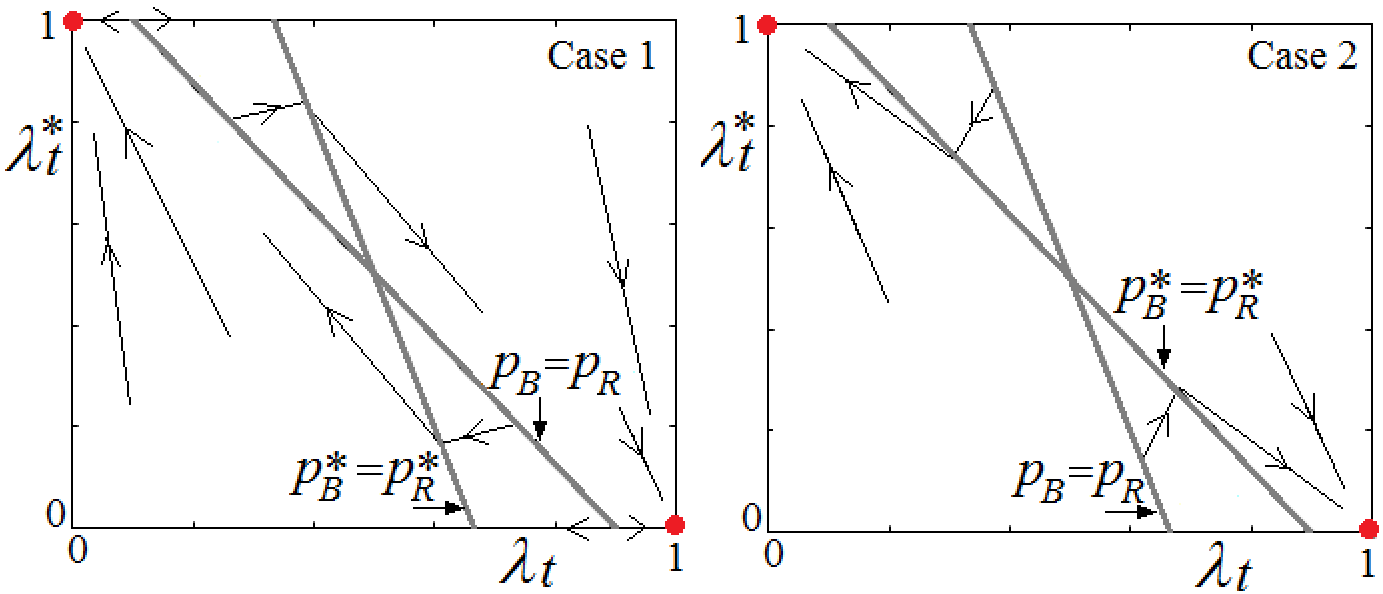
$$\text{(4a)}\quad \dot{\lambda}_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & if \quad \int_t^\infty (\Pi_{Bs} - \Pi_{Rs})\, e^{(\alpha+\delta)(t-s)} ds > 0 \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & if \quad \int_t^\infty (\Pi_{Bs} - \Pi_{Rs})\, e^{(\alpha+\delta)(t-s)} ds = 0 \\ \{-\alpha\lambda_t\} & if \quad \int_t^\infty (\Pi_{Bs} - \Pi_{Rs})\, e^{(\alpha+\delta)(t-s)} ds < 0 \end{cases}$$

$$\text{(4b)}\quad \dot{\lambda}_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & if \quad \int_t^\infty (\Pi_{Bs}^* - \Pi_{Rs}^*)\, e^{(\alpha+\delta)(t-s)} ds > 0 \\ [-\alpha\lambda_t^*, \alpha(1-\lambda_t^*)] & if \quad \int_t^\infty (\Pi_{Bs}^* - \Pi_{Rs}^*)\, e^{(\alpha+\delta)(t-s)} ds = 0 \\ \{\alpha(1-\lambda_t^*)\} & if \quad \int_t^\infty (\Pi_{Bs}^* - \Pi_{Rs}^*)\, e^{(\alpha+\delta)(t-s)} ds < 0 \end{cases}$$

Eqs. (3a)-(3b) are the limit of eqs. (4a)-(4b), where $\delta/\alpha \to \infty$ or $\alpha/\delta \to 0$.

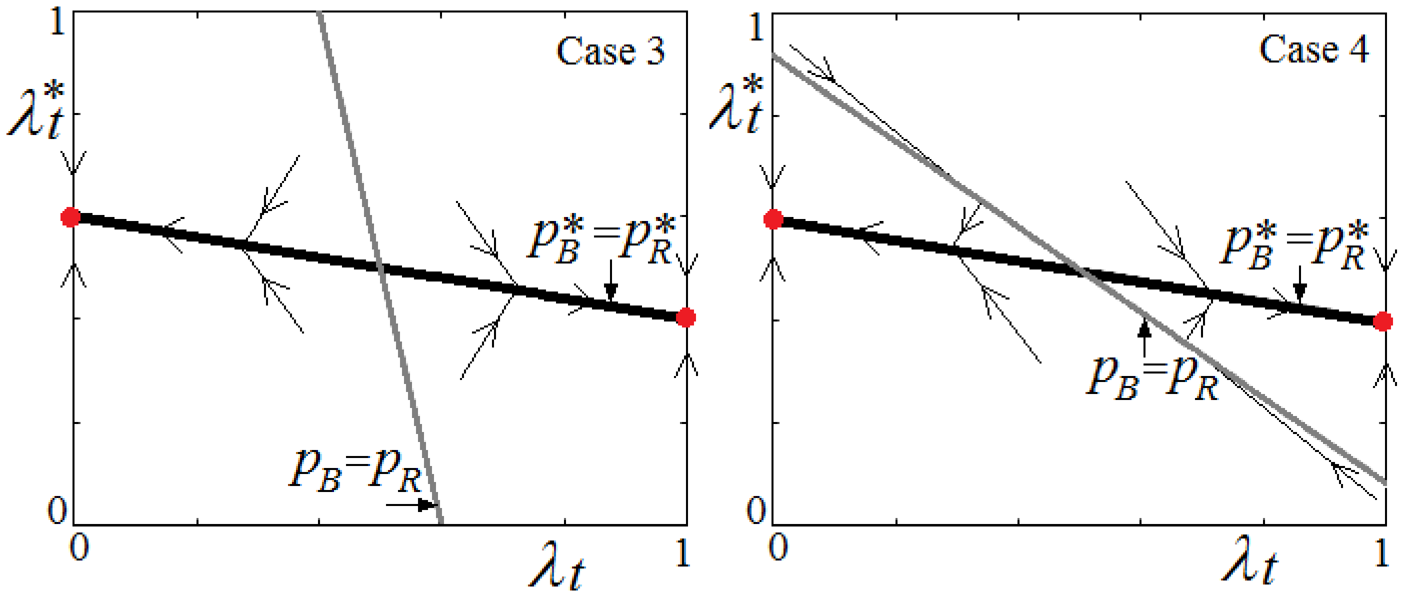
Proposition 2.The stable behavior patterns of the best response dynamics (3) are:

Cases 1 and 2 $(m^* > 1 > m)$: $(\lambda, \lambda^*) = (1,0)$ and $(0,1)$.



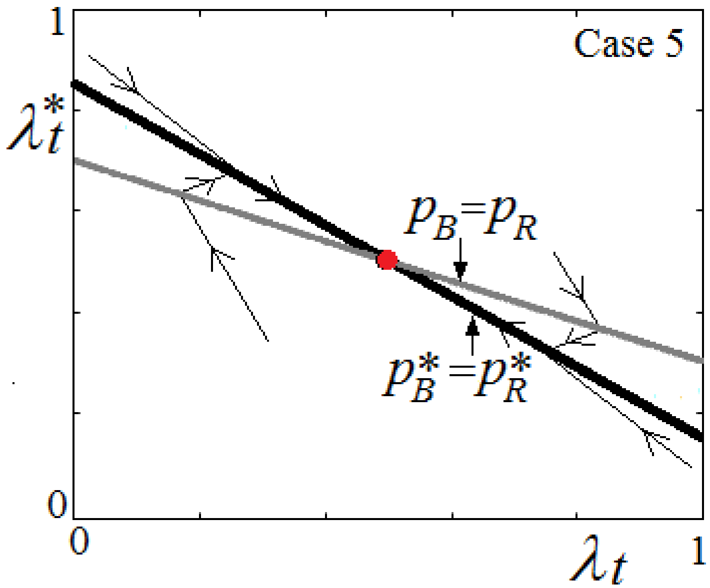
Both types are matched primarily with C; C sets the social custom, and N revolts against it. History (the initial condition) determines which action becomes the custom.
Basins of attraction separated by $p_B = p_R$.

Cases 3 and 4 $(m^* < 1;\ m < 1/m^*)$: $(\lambda, \lambda^*) = (1, (1 - m^*)/2)$ and $(0, (1 + m^*)/2)$



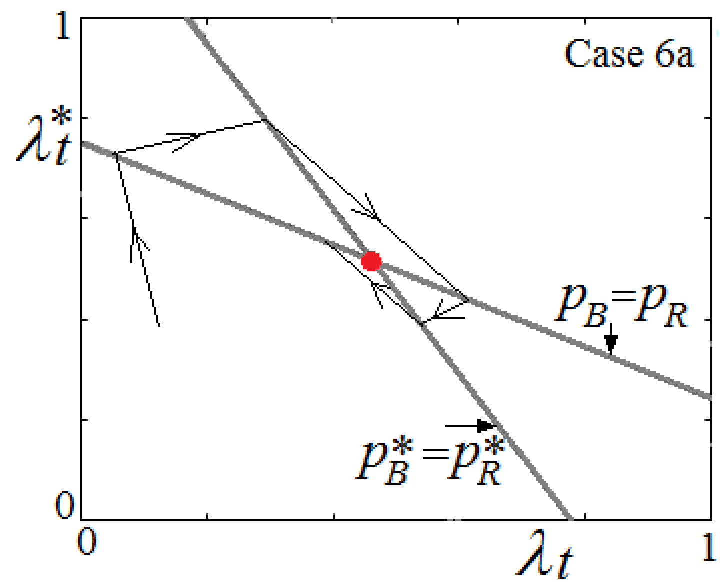
C sets the social custom, but with enough within-type matching, N plays mix-strategies among them.
Basins of attraction separated by $p_B = p_R$.

Case 5 ($m > mm^* > 1$): $(\lambda, \lambda^*) = (1/2, 1/2)$.



N play primarily among themselves ($m^* < 1$), which implies mixing. With matching biased toward Across-types ($\beta > 1/2$), a C meets a N more often than a N does ($m > 1/m^*$), so that they also end up mixing.

Case 6:$(mm^* > m > 1)$
With matching biased toward Across-types, both types play primarily against a different type ($\beta > \theta, 1-\theta$ or $m, m^* > 1$), generating **spiral pattern.**



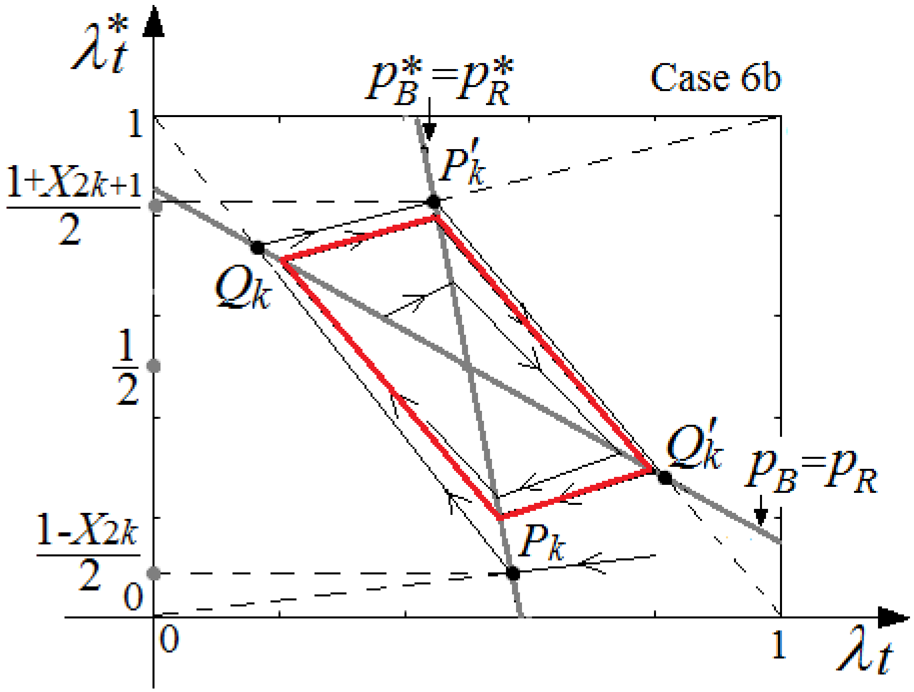
Case 6a $(m \geq m^* > 1/m)$: $(\lambda, \lambda^*) = (1/2, 1/2)$.
N is the majority $(1/2 \geq \theta > 1-\beta)$ and $m \geq m^* > 1$, implying C catch up with N.

Case 6b $(m^* > m > 1)$: **The limit cycle,** which is the parallelogram defined by the four vertices,

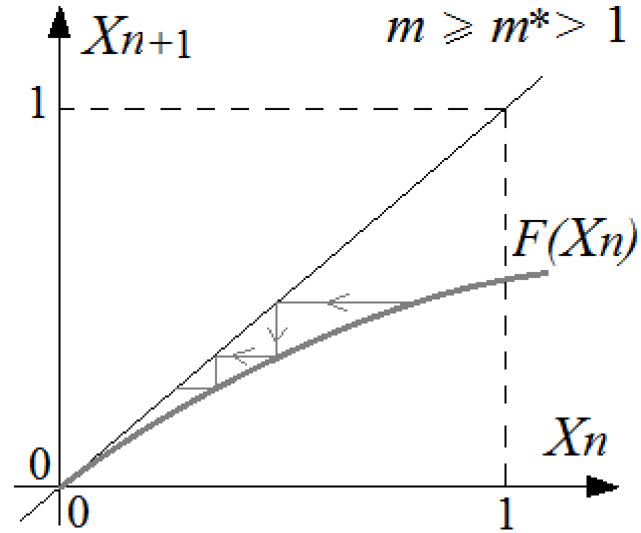
$$P = ((1 + X_\infty/m^*)/2, (1 - X_\infty)/2);$$
$$Q = ((1 - X_\infty)/2, (1 + X_\infty/m)/2);$$
$$P' = ((1 - X_\infty/m^*)/2, (1 + X_\infty)/2);$$
$$Q' = ((1 + X_\infty)/2, (1 - X_\infty/m)/2);$$

where $X_\infty \equiv (m^* - m)/(mm^* - 1) = (2\theta - 1)\beta(1-\beta)/(2\beta - 1)\theta(1-\theta)$.
C is the majority $(\beta > \theta > 1/2)$ and $m^* > m > 1$, which implying C will not catch up with N.

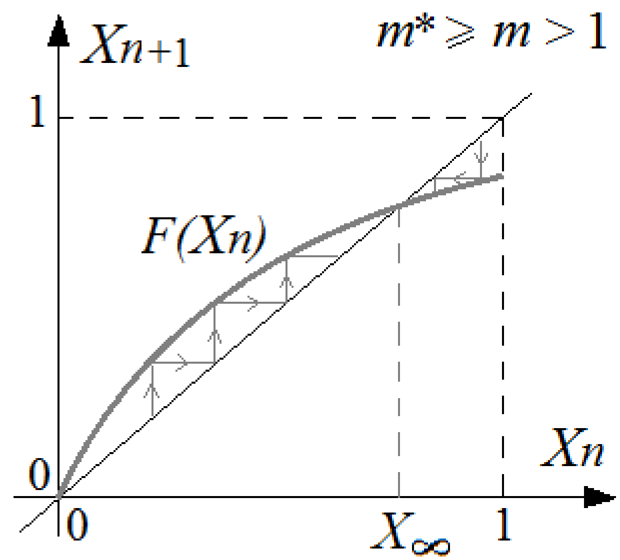
Detailed Look at Cases 6a and 6b:



Case 6a



Case 6b

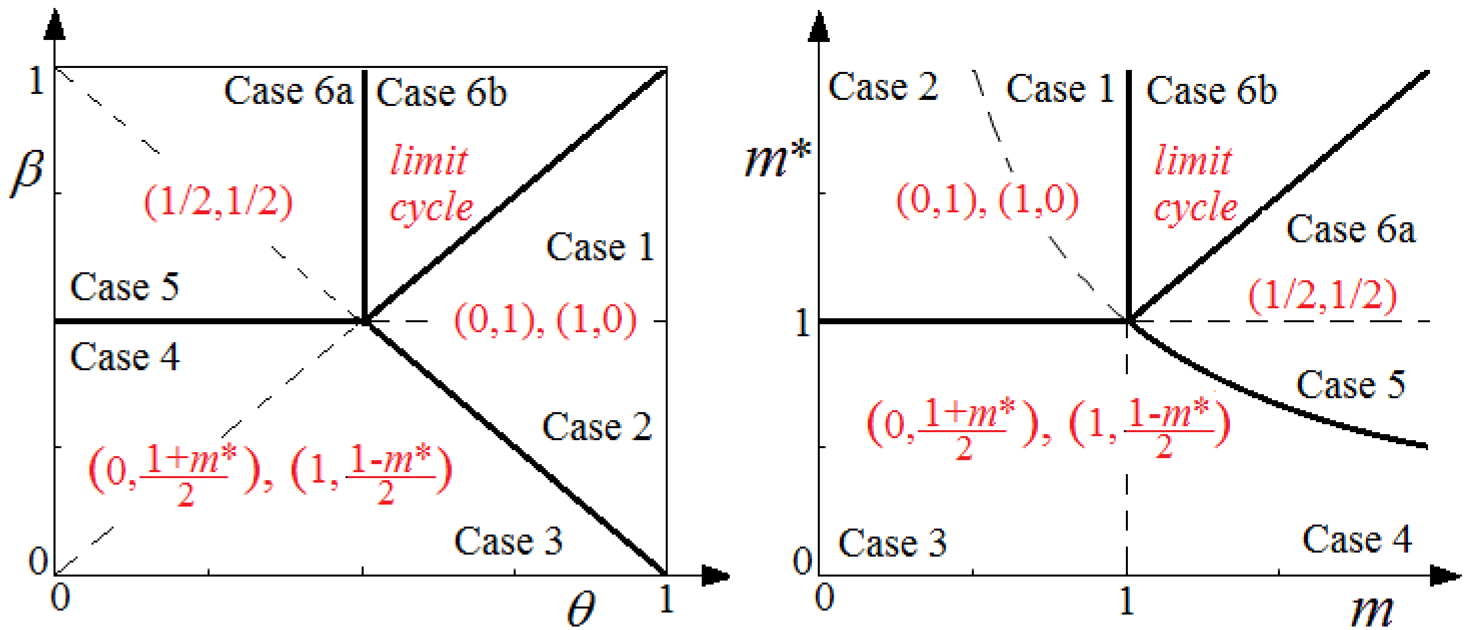


X_n : the path's distance from $(\lambda, \lambda^*) = (1/2, 1/2)$ at its n-th crossing of $p_B^* = p_R^*$.

Poincare Map:

$$X_{n+1} = F(X_n) \equiv \frac{(1+m)(m^*-1)X_n}{(m-1)(1+m^*) + 2(mm^*-1)X_n}$$

Stable Behavioral Patterns in the Parameter Spaces:



In all cases, the socially stable patterns (the attractors) are independent of α.

Two Bifurcations Causing the Limit Cycle

"Hopf" Bifurcation:

Case 6a ($1 - \beta < \theta < 1/2;\ m > m^* > 1$) → Case 6b ($\beta > \theta > 1/2;\ m^* > m > 1$)

- An increase in θ causes the unique globally stable Nash, $(\lambda, \lambda^*) = (1/2, 1/2)$, to lose its stability and bifurcate into the limit cycle, at the line ($\theta = 1/2;\ m = m^*$)
- C is still concerned more Across-Types than Within-Types, but less so than the N is. This enables N to take the lead in switching actions.

Fashion cycle emerges out of disorder, with stronger conformity.

"Heteroclinic" Bifurcation:

Case 1 ($\theta > \beta > 1/2;\ mm^* > 1 > m$) → Case 6b ($\beta > \theta > 1/2;\ m^* > m > 1$)

- A decline in θ or an increase in β causes two locally stable Nash, $(\lambda, \lambda^*) = (1, 0)$ and $(\lambda, \lambda^*) = (0, 1)$, to get connected with "heteroclinic" orbits, at the line ($\theta = \beta$ or $m = 1$), which then become the limit cycle
- C start paying more attention to a N than to another C, and start imitating her.

Fashion cycle emerges as departure from the custom, with stronger nonconformity.

An Alternative Model of Fashion Cycle

Trickle-Down Theory; Simmel([1904]1957); indirectly Veblen (1899)

Key Idea: Fashion is driven by an imitation of the elite class by the masses.
- The elite class seeks to set itself apart from the masses by adopting a new style
- This in turn leads to a new wave of imitation.

Formalizing the Trickle-Down Theory: (Non-Random Matching) Game

- Continuum of (not Anonymous) Players in Two classes: L(ower) and U(pper)
- Strategies: Two Actions, B and R, with λ_t (λ_t^*) the fraction of L (U) choosing B
- **Payoffs:** *Assume* to be a linear function of λ_t and λ_t^*.

$$\Pi_{Bt} - \Pi_{Rt} = \left(\lambda_t - \frac{1}{2}\right) + \mu\left(\lambda_t^* - \frac{1}{2}\right),$$
$$\Pi_{Bt}^* - \Pi_{Rt}^* = -\mu^*\left(\lambda_t - \frac{1}{2}\right) + \left(\lambda_t^* - \frac{1}{2}\right),$$

where $\mu > 0$ and $\mu^* > 0$ capture the extent to which each player pays attention to the distinct class relative to its own.

Best Response Dynamics

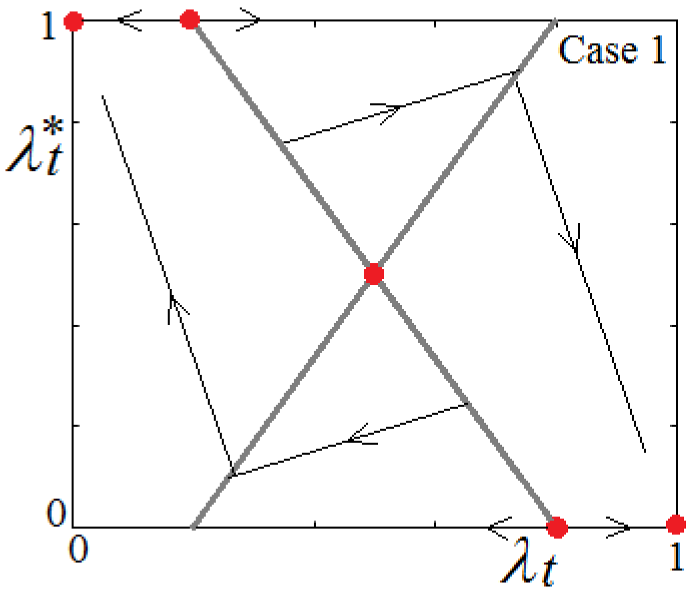
(5a) $$\dot{\lambda}_t \in \begin{cases} \{\alpha(1-\lambda_t)\} & if \quad (\lambda_t - 1/2) + \mu(\lambda_t^* - 1/2) > 0 \\ [-\alpha\lambda_t, \alpha(1-\lambda_t)] & if \quad (\lambda_t - 1/2) + \mu(\lambda_t^* - 1/2) = 0 \\ \{-\alpha\lambda_t\} & if \quad (\lambda_t - 1/2) + \mu(\lambda_t^* - 1/2) < 0 \end{cases}$$

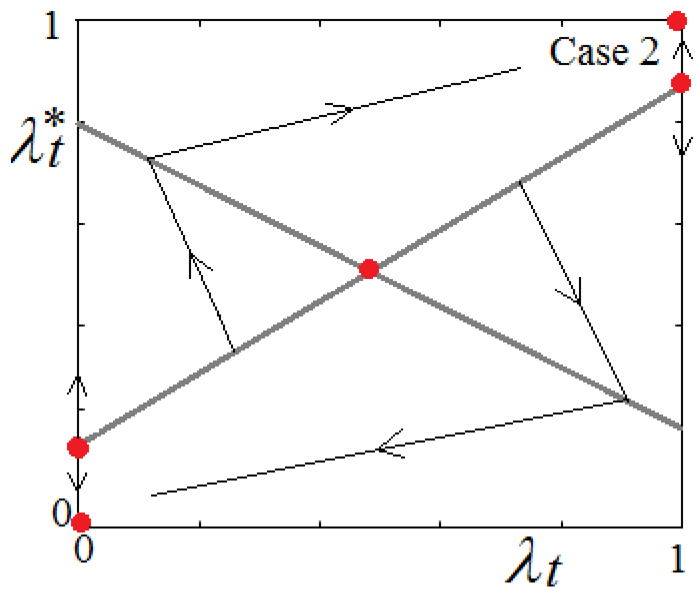
(5b) $$\dot{\lambda}_t^* \in \begin{cases} \{-\alpha\lambda_t^*\} & if \quad \mu^*(\lambda_t - 1/2) - (\lambda_t^* - 1/2) > 0 \\ [-\alpha\lambda_t^*, \alpha(1-\lambda_t^*)] & if \quad \mu^*(\lambda_t - 1/2) - (\lambda_t^* - 1/2) = 0 \\ \{\alpha(1-\lambda_t^*)\} & if \quad \mu^*(\lambda_t - 1/2) - (\lambda_t^* - 1/2) < 0 \end{cases}$$

Analytically, (3a)-(3b) and (5a)-(5b) differ only in that starred players are affected negatively by its own group in (3), and positively in (5).

Proposition 3: The stable behavior patterns of (5) are:

Case 1 ($\mu^* > 1 > \mu$): $(\lambda, \lambda^*) = (1,0)$ and $(0,1)$,
Case 2 ($\mu^* < 1 < \mu$): $(\lambda, \lambda^*) = (1,1)$ and $(0,0)$.





Case 1 (Segregation)

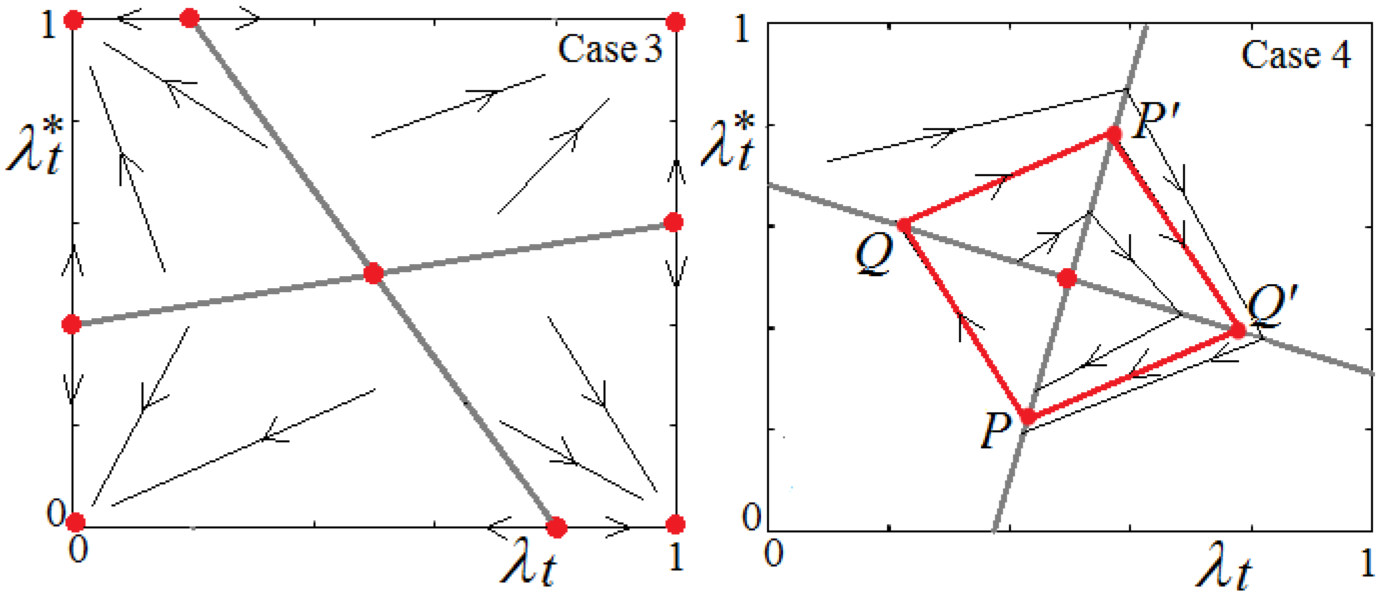
Case 2 (Integration)

Case 3 $(\mu, \mu^* < 1)$: $(\lambda, \lambda^*) = (1,0), (0,1), (1,1)$ and $(0,0)$,

Case 4 $(\mu, \mu^* > 1)$: The limit cycle, which is the parallelogram defined by the four vertices,

$$P = ((1 - X_\infty/\mu^*)/2, (1 - X_\infty)/2);$$
$$Q = ((1 - X_\infty)/2, (1 + X_\infty/\mu)/2);$$
$$P' = ((1 + X_\infty/\mu^*)/2, (1 + X_\infty)/2);$$
$$Q' = ((1 + X_\infty)/2, (1 - X_\infty/\mu)/2);$$

where $X_\infty \equiv (\mu + \mu^*)/(1 + \mu\mu^*)$.



Case 3(Segrega-/Integration)   Case 4 (Limit Cycle)

- Case 1 (Segregation): U's strong desire to separate & L's weak desire to imitate.
- Case 2 (Integration): U's weak desire to separate & L's strong desire to imitate
- Case 3(Segrega-/Integration); U's weak desire to separate & L's weak desire to imitate
- Case 4 (Limit Cycle): U's strong desire to separate & L's strong desire to imitate

- The limit cycle occurs through 2 types of "heteroclinic" bifurcations
  - Case 1 → Case 4: An increase in L's desire to imitate causes two segregation locally stable Nash to get connected with heteroclinic orbits, which then become the cycle.
  - Case 2 → Case 4: An increase in U's desire to separate causes two integration locally stable Nash to get connected with heteroclinic orbits, which then become the cycle.

Trickle-Down theory

- Might be well suited to explain fashion in dress from 17th to 19th century Europe.
- Some predicted the end of fashion cycles as the social structure changed.
- Media exposure and mass production make it harder for U to separate by early adoption of a new style. Their effort for a distinction manifest in the product quality.

Conformist-Nonconformist Theory

- A theory of changing tastes, changing *Zeitgeist*, the spirit of the times.
- More universal, transcending across periods and societies with different social structures.