Competition and the Phillips Curve

Ippei Fujiwara and Kiminori Matsuyama

Discussion Paper DP17521
Published 31 July 2022
Submitted 29 July 2022

Centre for Economic Policy Research
33 Great Sutton Street, London EC1V 0DX, UK
Tel: +44 (0)20 7183 8801
www.cepr.org

This Discussion Paper is issued under the auspices of the Centre’s research programmes:

- International Macroeconomics and Finance
- Monetary Economics and Fluctuations

Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as an educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Ippei Fujiwara and Kiminori Matsuyama
Competition and the Phillips Curve

Abstract

It has been well-documented that the Phillips curve has flattened, making central bankers wary of the reduced effectiveness of monetary policy to achieve price stability. There has also been a growing concern about higher market concentration and the rising profit margins and markup rates. Are these two events observed in recent years merely coincidental? Or, are they causally related? To address this issue, this paper extends the canonical New Keynesian model by introducing markup-rate changes caused by entry and exit, under Homothetic Single Aggregator (hereafter, HSA), a class of homothetic demand systems, which contain CES and Translog as special cases. We use HSA because its single aggregator summarizes all the impacts of market concentration on the pricing behavior of monopolistically competitive firms, and thus it serves as a sufficient statistic. We show that, under Marshall’s second law of demand (i.e., the price elasticity of demand goes up with its price), market concentration leads to a rise of the markup rate, and that, under the third law of demand (i.e., the rate of increase in the price elasticity goes down with its price), market concentration leads to a decline of the pass-through rate. We demonstrate analytically that these changes in the markup rate and the pass-through rate cause the flattening of the Phillips curve. Furthermore, Marshall’s second law of demand generates the dynamic effect of competition. That is, a change in the number of firms through endogenous entry directly affects inflation rates in the New Keynesian Phillips curve, which can be interpreted as an endogenous cost-push shock.

JEL Classification: E31, E52, L16

Keywords: New Keynesian Phillips Curve, Market concentration, monopolistic competition, Endogenous entry, HSA, variable markup and pass-through rates, Marshall’s second law of demand, the third law of demand

Ippei Fujiwara - ippei.fujiwara@keio.jp
Keio University, Australian National University and CEPR

Kiminori Matsuyama - k-matsuyama@northwestern.edu
Northwestern University and CEPR

Acknowledgements
We thank Fabio Ghironi and seminar participants at (chronologically) HKUST, VAMS (Virtual Australian Macroeconomics Seminar) and University of Washington for their feedback. Fujiwara is grateful for financial support from JSPS KAKENHI Grant-in-Aid for Scientific Research (B) No. 21H00698.
Competition and the Phillips Curve

Ippei Fujiwara† Kiminori Matsuyama‡

July 29, 2022

Abstract

It has been well-documented that the Phillips curve has flattened, making central bankers wary of the reduced effectiveness of monetary policy to achieve price stability. There has also been a growing concern about higher market concentration and the rising profit margins and markup rates. Are these two events observed in recent years merely coincidental? Or, are they causally related? To address this issue, this paper extends the canonical New Keynesian model by introducing markup-rate changes caused by entry and exit, under Homothetic Single Aggregator (hereafter, HSA), a class of homothetic demand systems, which contain CES and Translog as special cases. We use HSA because its single aggregator summarizes all the impacts of market concentration on the pricing behavior of monopolistically competitive firms, and thus it serves as a sufficient statistic. We show that, under Marshall’s second law of demand (i.e., the price elasticity of demand goes up with its price), market concentration leads to a rise of the markup rate, and that, under the third law of demand (i.e., the rate of increase in the price elasticity goes down with its price), market concentration leads to a decline of the pass-through rate. We demonstrate analytically that these changes in the markup rate and the pass-through rate cause the flattening of the Phillips curve. Furthermore, Marshall’s second law of demand generates the dynamic effect of competition. That is, a change in the number of firms through endogenous entry directly affects inflation rates in the New Keynesian Phillips curve, which can be interpreted as an endogenous cost-push shock.

Keywords: New Keynesian Phillips curve, market concentration, monopolistic competition, endogenous entry, HSA, variable markup and pass-through rates, Marshall’s second law of demand, the third law of demand

JEL codes: E31, E52, L16

*We thank Fabio Ghironi and seminar participants at (chronologically) HKUST, VAMS (Virtual Australian Macroeconomics Seminar) and University of Washington for their feedback. Fujiwara is grateful for financial support from JSPS KAKENHI Grant-in-Aid for Scientific Research (B) No. 21H00698.
†Keio University, ANU, ABFER and CEPR. E-mail: ippei.fujiwara@keio.jp
‡Northwestern University and CEPR. E-mail: k-matsuyama@northwestern.edu
1 Introduction

One of the major issues central banks have been facing in recent years is the flattening of the Phillips curve. For example, Federal Reserve Vice Chair, Richard Clarida said on Sept. 26, 2019, that “Another key development in recent decades is that price inflation appears less responsive to resource slack. That is, the short-run price Phillips curve – if not the wage Phillips curve – appears to have flattened”. San Francisco Fed President, Mary Daly stated on Aug. 29, 2019, that “As for the Phillips curve… most arguments today center around whether it’s dead or just gravely ill”. New York Fed President, John Williams explained on Feb. 22, 2019, that “The Phillips curve is the connective tissue between the Federal Reserve’s dual mandate goals of maximum employment and price stability. Despite regular declarations of its demise, the Phillips curve has endured. It is useful, both as an empirical basis for forecasting and for monetary policy analysis.” Central banks are wary of it since achieving price stability becomes more challenging due to the reduced impact of monetary policy on inflation rates through real economic activities.

Another significant development that the U.S. economy has experienced in recent years is market concentration. There is a growing concern about the adverse impacts of the resulting increase in profit margins and markups on the macroeconomy. Recent studies provide empirical evidence of increasing market concentration. Covarrubias, Gutierrez and Philippon (2019) show that “After 2000, however, the evidence suggests inefficient concentration, decreasing competition and increasing barriers to entry, as leaders become more entrenched and concentration is associated with lower investment, higher prices and lower productivity growth.” According to Loecker, Eeckhout and Unger (2020), “In 1980, aggregate markups start to rise from 21% above marginal cost to 61% now. ... We also find an increase in the average profit rate from 1% to 8%. Although there is also an increase in overhead costs, the markup increase is in excess of overhead.” In addition, Autor, Dorn, Katz, Patterson and Reenen (2020) state that “sales concentration is rising across a large set of industries. ... aggregate markups have been rising.”

These two events observed in recent years seem unrelated through the lens of the textbook New Keynesian model (see, for example, Gali, 2015, Walsh, 2010 and Woodford, 2003). Under the CES demand system extensively used in the literature, market concentration neither leads to the flattening of the Phillips curve nor directly impacts the inflation rate through the Phillips curve. This irrelevance result, we argue, can be overturned once we depart from CES and adopt more flexible demand systems. The contribution of this paper is to offer a theoretical framework that reveals the causality from market concentration to the flattening of the Phillips curve. To this end, we extend the canonical New Keynesian model under CES monopolistic competition in two respects. First, we incorporate entry and exit as in Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014). Second, we replace CES by Homothetic Single Aggregator (hereafter, HSA), a class of homothetic demand systems, proposed by Matsuyama and Ushchev (2017). HSA is fully characterized by the market share function. We use HSA for
several reasons. First, HSA contains CES and Translog as special cases. Second, it is tractable due to its single aggregator, which summarizes all the impacts of market concentration on the pricing behavior of monopolistically competitive firms, and thus it serves as a sufficient statistic. Third, it is straightforward to ensure the uniqueness of equilibrium in spite of endogenous change in the number of firms. Fourth, HSA is flexible. For example, HSA can accommodate Marshall’s second law of demand (hereafter, the Second law) and as well as what Matsuyama and Ushchev (2022b) call the third law of demand (hereafter, the Third law). To the best of our knowledge, this paper is the first attempt to incorporate the HSA demand system into the New Keynesian framework.

Under the Second law, higher entry cost, by increasing market concentration, leads to lower price elasticity (hence higher markups), and larger profits, as observed in recent years. We first show that this leads to the flattening of the Phillips curve under Rotemberg (1982) pricing. To see why, note that the slope of the New Keynesian Phillips curve (hereafter, NKPC) increases with the price elasticity under Rotemberg due to its state dependence in pricing. The incentive to set the price closer to the target level, which prevails in the flexible price equilibrium, is stronger with higher price elasticity. Consequently, as market concentration causes lower price elasticity, firms adjust their prices more gradually, which leads to the flattening of the Phillips curve. We call this the steady-state effect of competition.

With parameters calibrated following Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014), market concentration generates a sizable reduction in the slope of NKPC. We show that the rise in the markup rate as observed in the data can halve the slope of NKPC. As a result, in impulse responses, the responses of the inflation rate to both technology and monetary policy shocks become smaller as market concentration deepens.

The Second law (or equivalently, incomplete pass-through) also implies strategic complementarity in price setting and procompetitive effect of entry under HSA. Thus, firms reduces their prices and markup rates in response to more competitive pressures caused by a decline in the competing prices and more entry. Therefore, a change in market concentration directly affects price setting over the business cycle, thereby generating endogenous fluctuations in the markup rate, i.e., the endogenous cost-push shock, which we call the dynamic effect of competition.

The pass-through rate affects the impact of market concentration on the Phillips curve through two channels. On one hand, lower pass-through rates imply greater changes in the price elasticity. Hence, due to the steady-state effect of competition, the flattening effect of market concentration is stronger. On the other hand, the dynamic effect of competition, i.e., the endogenous cost-push shock, is larger with the lower pass-through rate. These two effects go

---

1The Second law states that the price elasticity of demand goes up with its price; For the empirical evidence on its implications under monopolistic competition, see Campbell and Hopenhayn (2005), Burstein and Gopinath (2014), Loecker and Goldberg (2014), Feenstra and Weinstein (2017), Amiti, Itskhoki and Konings (2019) and Baqee et al. (2021a). The third law states that the rate of increase in the price elasticity goes down with its price; For the empirical evidence, see Berman et al. (2012), Amiti, Itskhoki and Konings (2014) and Baqee et al. (2021a).
in the opposite directions. However, we find numerically that the steady-state effect of competition dominates the dynamic effect of competition. In other words, the impulse response of the inflation rate to technology and monetary policy shocks is smaller with higher market concentration.

In the main model considered in this paper, price stickiness is based on the Rotemberg (1982) price adjustment costs. We also explore how market concentration leads to the flattening of the Phillips curve under the staggered pricing a la Calvo (1983). It turns out that market concentration leads to the flattening of the Phillips curve under Calvo pricing when HSA satisfies what Matsuyama and Ushchev (2022b) call the third law (higher price leads to a smaller rate of change in the price elasticity, i.e., lower pass-through rate).

In summary, the causal effects from market concentration to the flattening of the Phillips curve are captured by two statistics: the price elasticity under Rotemberg pricing and the pass-through rate under Calvo pricing. Under HSA, the market share function fully characterizes both the price elasticity and the pass-through rate and its single aggregator summarizes all the impacts of market concentration on the flattening of the Phillips curve.

It should be also pointed out that under the Second law, the cyclicality of the markup rate is determined by the tension between nominal rigidities and the pass-through rate. Suppose a positive technology shock hits the economy. As is well-known, in a sticky price equilibrium under the CES demand system, the markup rate is procyclical, because the price does not respond in the short run to the decline in the marginal cost. But, under the Second law, a positive technology shock increases the number of firms, which work in the direction of reducing the markup rate and making it countercyclical.

**Layout** The structure of this article is as follows. After the literature review, Section 2 formally defines the HSA demand system and explains its key properties. Section 3 introduces the New Keynesian model with Rotemberg (1982) pricing under the HSA demand system. Section 4 discusses the relationship between competition and the Phillips curve. Sections 5 and 6 provide the simulation analysis using parametric families of HSA: CES, Translog and Co-Path. Section 5 shows how higher entry cost leads to concentration and the flattening of the Phillips curve, while Section 6 investigates impulse responses to technology and monetary policy shocks by different pass-through rate and by different entry cost. Section 7 studies the relationship between competition and the Phillips curve under Calvo (1983) pricing. There, we show simulation results using another parametric family of HSA called PEM, which accommodates the third law. Section 8 concludes.

---

2Co-Path, proposed by Matsuyama and Ushchev (2020a), stands for Constant Pass-Through. It satisfies the Second law and contains CES as a limit case.

3PEM, proposed by Matsuyama and Ushchev (2022b), stands for Power Elasticity of Markup rate. It satisfies the third law and contains Co-Path as a limit case, which in turn contains CES as a limit case.
Table 1: Monetary policy and competition

<table>
<thead>
<tr>
<th></th>
<th>comp.</th>
<th>entry</th>
<th>pref.</th>
<th>nominal friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wang and Werning (2020)</td>
<td>oligo.</td>
<td>exo.</td>
<td>Kimball</td>
<td>Calvo</td>
</tr>
<tr>
<td>Baqaee, Farhi and Sangani (2021b)</td>
<td>mono.</td>
<td>exo.</td>
<td>Kimball</td>
<td>Calvo</td>
</tr>
<tr>
<td>our paper</td>
<td>mono.</td>
<td>end.</td>
<td>HSA</td>
<td>Rotemberg / Calvo</td>
</tr>
</tbody>
</table>

oligo.: oligopolistic competition, mono.: monopolistic competition, exo.: exogenous entry, end.: endogenous entry

**Literature** This paper is related to several strands of literature. The most closely related is the recent studies inquiring into the relationship between monetary policy and competition. It is well-known that in the prototypical New Keynesian model under the CES demand system, competition is irrelevant to the Phillips curve, irrespective of entry and exit. It neither changes the slope of the Phillips curve nor exert any direct impact on the inflation rate. Several recent studies challenge this irrelevance of competition. Wang and Werning (2020) show that in a dynamic oligopoly model with strategic interaction, higher concentration leads to amplified real effects of monetary policy, the Phillips curve with inflation persistence, and the endogenous cost-push shock. Baqaee, Farhi and Sangani (2021b) demonstrate that higher concentration leads to the flattening of the Phillips curve through the misallocation channel. Table 1 compares the differences in approaches among these two and our studies. We use HSA instead of the Kimball (1995) aggregator, partly because it is easier to ensure the existence and the uniqueness of equilibrium with endogenous entry, and partly because it is more analytically tractable.

Our paper is also related to monopolistic competition models with entry and exit. Bilbiie, Ghironi and Melitz (2012, 2019) construct the flexible price model while Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014) set up the sticky price model. Bilbiie et al. (2012) shows that the endogenous entry models can account for business cycles as well as the standard RBC models. Bilbiie, Fujiwara and Ghironi (2014) and Bilbiie et al. (2019) characterize the optimal policy under entry and exit. Unlike these previous studies, this study reveals how changes in the competitive environment affect the transmission mechanisms of shocks in a model with endogenous price elasticity under the Second law.

The core ingredient of this paper is HSA. Since HSA was proposed by Matsuyama and Ushchev (2017), a growing number of studies have been applying it to explain a variety of economic phenomena. Matsuyama and Ushchev (2020b) investigate how the condition for procompetitive vs. anticompetitive entry – the markup rate either goes down or goes up as more firms enter – is related to the condition for excessive vs. insufficient entry. In order to understand how competitive pressures affect selection and sorting of firms with heterogeneous productivity, Matsuyama and Ushchev (2022b) extend Melitz (2003) model to incorporate the endogenous markup embedded in the HSA demand system. Matsuyama and Ushchev (2022a) extend Judd (1985) model of endogenous innovation cycles by replacing CES with HSA to show how market size has the destabilizing effects on the dynamics of innovation. Grossman,
Helpman and Lhuillier (2021) discuss whether the policy should promote diversification or reshoring in the face of insecure supply chains and seek for the policy instruments to achieve efficient supply chains under the Second law. Baqaee, Farhi and Sangani (2021a) study how an increase in market size, say, due to globalization, affects welfare in a monopolistically competition model with heterogenous markups and endogenous entry. Kasahara and Sugita (2020) propose a nonparametric estimation approach of the HSA demand system, which is applied to Fujiwara, Kimoto, Shiratsuka and Shirota (2021) to estimate the robot price index. Our study is the first to apply the HSA demand system to the New Keynesian model, the workhorse model for policy simulations and forecasting in central banks and international organizations.

2 HSA

This section explains the HSA demand system, originally proposed by Matsuyama and Ushchev (2017) and restricted to a continuum of varieties \((\omega \in \Omega_t)\), gross substitutes, and symmetry to be applied for monopolistic competition by Matsuyama and Ushchev (2020b, 2022a,b).

Consider the single final good, which is produced competitively by assembling differentiated intermediate inputs with the constant-returns-to-scale technology, characterized by the unit cost function \(P(p_t)\), where \(p_t\) is the vector of intermediate inputs prices. We call the demand system for intermediate inputs by the final goods producers HSA if the market share for the input variety \(\omega\) depends solely on its single relative price, i.e., its own price over the single price aggregator \(A(p_t)\):

\[
\frac{\partial \ln P(p_t)}{\partial \ln p_t(\omega)} = \frac{p_t(\omega) c_t(\omega)}{P(p_t) C_t} = s \left( \frac{p_t(\omega)}{A(p_t)} \right),
\]

where

\[
\int_{\Omega_t} s \left( \frac{p_t(\omega)}{A(p_t)} \right) d\omega \equiv 1.
\]

\(s: \mathbb{R}_{++} \to \mathbb{R}_+\) is the market share function, decreasing in the relative price \(z_t(\omega) := p_t(\omega) / A(p_t)\) for \(s(z) > 0\) with \(\lim_{z \to z^*} s(z) = 0\). If \(\bar{z} := \inf \{ z > 0 | s(z) = 0 \} < \infty, \bar{z}A(p_t)\) is the choke price. Because varieties are gross substitutes, the market share is decreasing in \(z: s'(z) < 0\).

The single price aggregator \(A_t = A(p_t)\) is implicitly defined by the adding-up constraint in equation (2). By construction, market shares add up to one. As is evident from equation (2), \(A(p_t)\) is linear homogenous in \(p_t\) for fixed \(\Omega_t\). A larger \(\Omega_t\), namely, more variety, reduces \(A(p_t)\).

By integrating equation (1), one can show that \(A(p_t)\) and \(P(p_t)\) are structurally related as follows:

\[
P(p_t) = A(p_t) \exp \left\{ \bar{K} - \int_{\Omega_t} \int_{z_t(\omega) \Omega_t} \frac{s(\xi)}{\xi} \frac{d\omega}{d\xi} \right\},
\]

\(\bar{K}\)From Proposition 1 in Matsuyama and Ushchev (2017), one can show that the HSA demand system as described above can be derived from a unique consumer preference.
where $\bar{K}$ is a constant.\textsuperscript{5}

$A (p_t)$ is the inverse measure of competitive pressures and fully captures the cross price effects in the demand system. It is different from the unit cost function of the final goods $P_t = P (p_t)$, which is the inverse measure of TFP and captures the productivity consequences of price changes. In general, they do not move together: $A (p_t) \neq \text{constant} \times P (p_t)$. This can be seen by differentiating the adding-up constraint in equation (2):

$$\frac{\partial \ln (A (p_t))}{\partial \ln (p_t)} = \frac{p_t (\omega) p_t (\omega)}{A (p_t)} = \frac{s (p_t (\omega))}{A (p_t)} = \frac{\partial \ln (P (p_t))}{\partial \ln (p_t)},$$

which shows that $A (p_t) \neq \text{constant} \times P (p_t)$ unless $zs' (z) / s (z) < 0$ is constant, in other words, $s (z)$ is a decreasing power function. This corresponds to CES because plugging $s (z) = \gamma_{CES} z^{1-\theta}$, where $\theta > 1$, into equations (1) to (3) can verify

$$C_t = Z_C \left[ \int_{\Omega_t} c_t (\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}, \quad (4)$$

$$c_t (\omega) = Z_C^{\theta-1} \left( \frac{p_t (\omega)}{P (p_t)} \right)^{-\theta} C_t,$$

$$Z_C^{\theta-1} \left( \frac{p_t (\omega)}{P (p_t)} \right)^{1-\theta} = \frac{p_t (\omega) c_t (\omega)}{P (p_t) C_t} = s \left( \frac{p_t (\omega)}{A (p_t)} \right) = \gamma_{CES} \left( \frac{p_t (\omega)}{A (p_t)} \right)^{1-\theta},$$

where $Z_C$ is the TFP of the final goods production, and related to $\bar{K}$ and $\gamma_{CES}$ as follows:

$$\frac{P (p_t)}{A (p_t)} = \exp \left( \bar{K} - \frac{1}{\theta - 1} \right) = \frac{\gamma_{CES}}{Z_C}. \quad (5)$$

Three price indices We consider endogenous entry (variety). Even under symmetric situation, individual price is not equal to the aggregate price due to entry effects. In addition, we have two aggregate prices: the final goods price and the single price aggregate. As a result, there are three aggregate prices.

The first one is the final goods price or CPI $P_t$, which captures the productivity effects of entry, and is the reference price for consumers, implicitly given by

$$\int_{\Omega_t} p_t (\omega) c_t (\omega) P_t C_t d\omega \equiv 1.$$ 

The second is the single price aggregate $A_t$, which captures the competitive effects of entry, and is the reference price for firms, implicitly given by

$$\int_{\Omega_t} s \left( \frac{p_t (\omega)}{A_t} \right) d\omega \equiv 1.$$

\textsuperscript{5}For the detailed derivation of equation (3), see Matsuyama and Ushchev (2017).
The third one is the average price index or PPI $p_t$:
\[ p_t := \frac{1}{N_t} \int_{\Omega_t} p_t(\omega) \, d\omega. \]

Unlike $P_t$ and $A_t$, the average price index $p_t$ is not affected by entry effects and therefore is the measured price index. Thus, in what follows, we consider the inflation rate in terms of $p_t$ when evaluating the responsiveness of inflation rates to macroeconomics variables in NKPC.

3 New Keynesian model under HSA

We consider a closed economy populated by four agents: household, final goods producer, intermediate inputs producer, and the central bank. Intermediate inputs producers are under monopolistic competition and set prices subject to nominal rigidities. They are also subject to the endogenous entry but exogenous exit.

Our model is an extension of those in Bilbiie, Ghironi and Melitz (2008) and Bilbiie, Fujiwara and Ghironi (2014) to incorporate the HSA demand system.

Timing  Time is discrete: $t = 0, 1, 2, 3, \ldots$ There is an unbounded mass of potential entrants in every period. They are subject to one-period time-to-build lag. That is, entrants at time $t$ only start producing at time $t+1$. Entry is determined endogenously by the free entry condition, but exit is exogenous. All firms face the same probability $\delta$ of exogenous firm destruction at the end of each period, after production and entry. As a result, a proportion $\delta$ of new entrants never produces.

3.1 Household

A representative household chooses consumption $C_t$, labor supply $L_t$, the nominal bond $B_t$, and the equity of intermediate inputs producer $x_t$, in order to maximize welfare:
\[ \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left( u(C_t) - v(L_t) \right), \]
subject to the budget constraint:
\[ \frac{B_{t+1}}{P_t} + x_{t+1} \int_{\Omega_t + \Omega_{E,t}} \frac{V_t(\omega)}{P_t} \, d\omega + C_t = (1 + i_{t-1}) \frac{B_t}{P_t} + x_t \int_{\Omega_t} \frac{D_t(\omega) + V_t(\omega)}{P_t} \, d\omega + \frac{W_t}{P_t} L_t. \] (6)

where $\beta$ and $\delta$ denote the subjective discount factor and the probability of exogenous termination. $W_t$ and $i_t$ denote the nominal wage and the nominal interest rate. $V_t(\omega)$ and $D_t(\omega)$ denote the equity price (the value) and the profit of the intermediate inputs producer $\omega \in \Omega_t$. 
3.2 Final goods producer

Final goods producers are under perfect competition and produce the final good by assembling intermediate inputs using the constant-returns-to-scale technology that generates the HSA demand system in equations (1) and (2).

3.3 Intermediate inputs producer

Intermediate inputs producer \(\omega\) uses labor \(l_t(\omega)\) to produce its output \(y_t(\omega)\) with the linear technology:

\[
y_t(\omega) = Z P_t l_t(\omega),
\]

where \(Z_{P,t}\) is the common technology in production. Intermediate inputs producer \(\omega\) chooses the price path \(p_t(\omega)\) to maximize the value of the firm:

\[
V_t(\omega) P_t = \mathbb{E}_t \sum_{i=1}^{\infty} \beta (1 - \delta)^i u'(C_{t+i}) \left( \frac{D_{t+i}(\omega)}{P_{t+i}} \right),
\]

subject to the HSA demand curve in equation (1), :

\[
y_t(\omega) = c_t(\omega) = s \left( \frac{p_t(\omega)}{A_t} \right) \frac{P_t C_t}{p_t(\omega)},
\]

and the Rotemberg (1982) quadratic price adjustment cost, where \(\chi\) scales the size of the cost:

\[
D_t(\omega) = \frac{p_t(\omega)}{P_t} y_t(\omega) - \frac{W_t}{P_t} l_t(\omega) - \frac{\chi}{2} \left( \frac{p_t(\omega)}{p_{t-1}(\omega)} - 1 \right)^2 \frac{p_t(\omega)}{P_t} y_t(\omega),
\]

In the main part of our analysis in Sections 3 to 6, we use Rotemberg (1982) pricing for the simplicity, but we will also consider Calvo (1983) pricing in Section 7.

In this model, we depart from CES only in the direction of the Second law. As shown in Proposition 1 in Matsuyama and Ushchev (2020b), this guarantees that the equilibrium exists and is symmetric.

3.4 Aggregate conditions and others

Monetary policy  The central bank sets the nominal interest rate following the simple feedback rule reacting to the PPI inflation rate:

\[
(1 + i_t) = (1 + i_{t-1}) a_i \left( \frac{p_t}{p_{t-1}} - 1 \right)^{(1 - a_i) \alpha_i} u_t,
\]

where \(\alpha_i > 1\) to satisfy the Taylor principle. \(a_i\) represents the policy inertia. \(u_t\) denotes the monetary policy shock.
Free entry Entrants pay the entry cost $W_t/Z_{E,t}f_{E,t}$, where $Z_{E,t}$ and $f_{E,t}$ are the common technology to facilitate entry and the labor demand for entry purpose, respectively. Since the equilibrium is symmetric, entry occurs until the firm value becomes equal to the entry cost, resulting in the free entry condition:

$$\frac{W_t f_{E,t}}{P_t Z_{E,t}} = \frac{V_t}{P_t}. \quad (11)$$

Firm dynamics

$$N_t = (1 - \delta) (N_{t-1} + N_{E,t-1}) = (1 - \delta) N_{H,t-1}, \quad (12)$$

$N_t$ is the number of firms (varieties) and the mass of $\Omega_t$. $N_{E,t}$ is the number of entrants and the mass of $\Omega_{E,t}$.

Market clearing Financial market clearing conditions for the nominal bond and the equity are given by $B_t = 0$ and $x_t = 1$, respectively.

The labor market clearing condition is given by

$$L_t = N_t l_t + N_{E,t} f_{E,t} Z_{E,t}. \quad (13)$$

The left hand side is the supply of the labor. The first and second terms on the right hand side are the labor demands for production and entry, respectively.

Substitution of equation (12) into the budget constraint in equation (6) leads to

$$\frac{V_t}{P_t} N_{E,t} + C_t = \frac{D_t}{P_t} N_t + \frac{W_t}{P_t} L_t.$$ 

Substitution of equation (10), (11) and (13) into the above aggregate accounting equation yields the resource constraint:

$$C_t = 1 - \frac{X}{2} (\pi_t - 1)^2 N_t \frac{p_t}{P_t} y_t = \left[ 1 - \frac{X}{2} (\pi_t - 1)^2 \right] Y_t.$$ 

Output is either consumed or used for the price adjustment costs.

HSA demand system As the equilibrium is symmetric, the market share is simply expressed by the inverse of the number of firms:

$$s \left( \frac{p_t}{A_t} \right) = \frac{1}{N_t}. \quad (14)$$

The equation for the reference prices for households and intermediate inputs producers in equation (3) simplifies to

$$\ln \left( \frac{P_t}{A_t} \right) = \bar{K} - \frac{1}{s \left( \frac{p_t}{A_t} \right)} \int_{\Omega_t} \left[ \frac{\xi}{\xi_t} s(\xi) d_\xi \right] d\omega. \quad (15)$$
3.5 Equilibrium

**Definition.** An *equilibrium* in this economy is a collection of sequence of aggregate prices \( \{P_t, A_t, W_t, \ldots\} \) and the price of intermediate goods \( \{p_t\} \); a collection of sequences of aggregate quantities \( \{Y_t, C_t, L_t\} \) and quantities of intermediate goods \( \{y_t, l_t\} \); and a collection of sequences of firm-value functions and profit \( \{V_t, D_t\} \) together with measures of operating firms and entering firms \( \{N_t, N_{E,t}\} \). These equilibrium objects satisfy the following conditions:

- Households maximize their utility subject to their budget constraints, intermediate-good firms maximize the net present value of their per-period profits, final-good firms maximize profits, all of the feasibility constraints are satisfied.

**Preference**  Utility and disutility functions are set as

\[
u(C_t) := \frac{C_1^{1-\sigma} - 1}{1-\sigma}, \quad v(L_t) := \frac{L_1^{1+\psi}}{1+\psi},
\]

where \(\sigma\) and \(\psi\) denote the relative risk aversion (inverse of the intertemporal elasticity of substitution) and the inverse of Frisch elasticity, respectively.

In Section 4, we will show analytical results without specific functional forms for the market share function in the HSA demand system in equation (1). In numerical analysis provided in Sections 5, 6 and 7, parametric families of the HSA demand system are examined.

**Detrending**  Nominal variables are detrended as follows:

\[
w_t := \frac{W_t}{P_t}, \quad d_t := \frac{D_t}{P_t}, \quad v_t := \frac{V_t}{P_t}, \quad z_t := \frac{p_t}{A_t}, \quad \bar{p}_t := \frac{p_t}{P_t}, \quad \pi_t := \frac{p_t}{p_{t-1}}.
\]

Notice that both \(P_t\) and \(A_t\), i.e., the reference prices for households and firms, include entry effects. Since entry effects are not included in the official price statistics, the inflation rate is measured by PPI \(p_t\) throughout this paper. Since the equilibrium is symmetric, this is equal to the individual prices of intermediate inputs producers.

**System of equations**  Appendix A displays the system of nonlinear as well as log-linearly approximated equations in addition to the steady state conditions.

4 Competition and the Phillips curve

Now, we are ready to derive NKPC under HSA. By solving the intermediate inputs producers’ problem, which is to maximize the value in equation (8) subject to equations (10) to (9),

\[
\hat{\pi}_t = \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} + \frac{1}{\lambda} \left[ -s'(z) \hat{z}_{t} \right] (\hat{W}_t - \hat{Z}_{p,t} - \hat{p}_t) + \frac{1}{\lambda} \left[ \frac{s'(z) \hat{z}_{t}}{s(z)} - \frac{s'(z) \hat{z}_{t}}{s'(z)} \right] \hat{z}_{t}, \quad (16)
\]
where the circumflex (^) denotes the percentage deviation from the steady state value. From equation (14), \( \hat{z}_t \) can be shown to be positively correlated with \( \hat{N}_t \) as follows:

\[
\hat{N}_t = \frac{s'(z)\hat{z}_t}{s(z)}
\]  

(17)

Then, using this equation, we can further simplify equation (16) to

\[
\hat{\pi}_t = \beta (1 - \delta) E_t \hat{\pi}_{t+1} + \frac{\tilde{\zeta}(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{1 - \rho(z)}{\chi \rho(z)} \hat{N}_t,
\]

(18)

where

\[
\tilde{\zeta}(z) := \frac{\partial \ln (c_t (\omega))}{\partial \ln (p_t)} = 1 - \frac{s'(z)z}{s(z)} > 1,
\]

(19)

is the price elasticity function (hence, the markup rate under the flexible price would be \( \mu_f(z) = \tilde{\zeta}(z) / (\tilde{\zeta}(z) - 1) \)), and

\[
\rho(z) := \frac{\partial \ln (p_t)}{\partial \ln (W_t / Z_{P,t})} = \left[1 - \frac{d \ln \left( \frac{\tilde{\zeta}(z)}{s(z) - 1} \right)}{d \ln (z)} \right]^{-1},
\]

(20)

is the pass-through rate function. We call equation (18) NKPC under HSA.\(^6\)

Equation (18) contains the standard textbook NKPC under CES, such as Gali (2015), Walsh (2010) and Woodford (2003). It is obtained by setting \( s(z_t) = \gamma_{CES} (z_t)^{1-\theta} \), where \( \theta \) denotes the constant price elasticity, and \( \tilde{\zeta}(z) = \theta \) and \( \rho(z) = 1 \):

\[
\hat{\pi}_t = \beta (1 - \delta) E_t \hat{\pi}_{t+1} + \frac{\theta - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t),
\]

where \( \hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t \) is the inverse of the markup rate, and, therefore comparable to the real marginal cost measure in the canonical New Keynesian model. As is well-known, competition does not affect the Phillips curve even with endogenous entry under CES. Market concentration neither changes the slope of the Phillips curve nor exerts any direct impact on the inflation rate. The constant elasticity makes the competitive environment irrelevant to price dynamics.

By extending NKPC from CES to HSA, equation (18) introduces two channels through which competition affects price dynamics. First, the slope of the Phillips curve is no longer constant and determined by the steady state conditions. Thus, changes in the competitive environment will affect the slope of the Phillips curve. We call this the steady-state effect of competition. Second, endogenous changes in \( N_t \) by causing fluctuations in \( z_t \), affect the inflation rate directly in NKPC, which acts like the endogenous cost-push shock. We call this term in equation (18) the dynamic effects of competition.

\(^6\)Appendix B shows that the slope of NKPC remains the same even when inflation rates are measured in terms of \( P_t \) and \( A_t \).
Even though HSA endogenizes the price elasticity, the reason why the slope of NKPC in equation (18) depends on the price elasticity is the same with the textbook NKPC under CES. Recall that under CES, the optimal price setting condition is given by

\[(1 - \theta) + \theta mc_t - \chi (\pi_t - 1)^2 \pi_t + \mathbb{E}_t \beta (1 - \delta) \frac{u'(C_{t+1})}{u'(C_t)} \chi (\pi_{t+1} - 1) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0,\]

where \(mc_t\) denotes the real marginal cost. In response to price increase, the second term shows how much costs decline, which is proportional to the price elasticity, while the third and fourth terms represent losses and gains from price adjustments, which are expressed by inflation rates. Consequently, the higher the the price elasticity, the more the demand declines and the higher the impact of the marginal cost on inflation rates becomes. In other words, with an exogenous change in the real marginal cost, the incentive to set the price closer to the target level (i.e., the price prevailing under the flexible price) becomes stronger with higher price elasticity. This motive stems from the state dependency in Rotemberg (1982) pricing and therefore is absent in Calvo (1983) pricing.

### 4.1 Implications of the Second law

Under the Second law, \(\zeta' (z) > 0\) and hence the pass-through is incomplete: \(\rho (z) < 1\). Therefore, higher cost implies lower markup rates.

More concentration unambiguously result in higher market share \(s (z)\) and lower \(z\). The Second law, \(\zeta' (z) > 0\), causes a smaller slope coefficient \((\zeta (z) - 1) / \chi\) in NKPC as equation (18) shows.

Also, the Second law yields the endogenous cost-push shock, i.e., the dynamics effect of competition. From equation (20), the markup rate under the flexible price is not constant:

\[\hat{\mu}_t = -\frac{1 - \rho (z)}{\rho (z)} \hat{z}_t = -\frac{1 - \rho (z)}{\rho (z)} (\hat{p}_t - \hat{A}_t), \tag{21}\]

The firm reduces its price and the markup rate in response to higher competitive pressures, i.e., a lower \(A_t\), when other firms reduce their prices. This is indeed strategic complementarity in price setting under HSA due to the Second law or, equivalently, incomplete pass-through.

This endogenous cost-push shock has an additional implication to aggregate fluctuations. If \(\hat{\mu}_t = - (\hat{W}_t - \hat{Z}_{p,t} - \hat{p}_t)\) and \(\hat{z}_t\) (and therefore \(\hat{N}_t\), see equation (17)) move to the opposite directions (same direction) to a structural shock, its impact on inflation rates is muted (amplified). As the analyses in Section 6 reveal, whether they are positively or negatively correlated depends on the nature of structural shocks. For example, to a positive technology shock, the number of firms \(\hat{N}_t\) and therefore \(\hat{z}_t\) increase through the wealth effect, but the strategic complementarity in price setting reduces the markup rate \(\hat{\mu}_t\) as shown in equation (21). As a result, the dynamic effect of competition weakens the responses of inflation rates to real economic variables.
Table 2: Cyclicality of the markup rate to the technology shock

<table>
<thead>
<tr>
<th>Flexible price</th>
<th>Sticky price</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
<td>countercyclical / procyclical</td>
</tr>
<tr>
<td>the Second law</td>
<td>procyclical</td>
</tr>
</tbody>
</table>

Table 3: Parametric families of HSA

<table>
<thead>
<tr>
<th>Market share</th>
<th>Price elasticity</th>
<th>Pass-through</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES</td>
<td>$s(z) = \gamma_{CES}z^{1-\theta}$</td>
<td>$\zeta(z) = \theta$</td>
</tr>
<tr>
<td>Translog</td>
<td>$s(z) = \gamma_{TL} \ln \left( \frac{z}{\bar{z}} \right)$</td>
<td>$\zeta(z) = 1 + \frac{1}{\ln(\bar{z})} \quad \rho(z) = \frac{1 + \ln(z)}{2 + \ln(z)}$</td>
</tr>
<tr>
<td>Co-Path</td>
<td>$s(z) = \gamma_{CP} \theta^{2 \rho} \left[ 1 - \left( \frac{z}{\bar{z}} \right)^{1-\rho} \right] \frac{\rho}{\bar{z}}$</td>
<td>$\zeta(z) = \frac{1}{1 - \left( \frac{z}{\bar{z}} \right)} \frac{\rho}{\bar{z}} \quad \rho(z) = \rho &lt; 1$</td>
</tr>
</tbody>
</table>

4.2 Cyclicality of the markup rate

Table 2 summarizes the cyclicality of the markup rate to the positive technology shock. In a flexible price equilibrium under CES, the markup rate is constant. In a sticky price equilibrium under CES, the markup rate becomes procyclical. This is because the marginal cost decreases but the price does not change, at least, for a short-run. In a flexible price equilibrium under the Second law, the markup rate becomes countercyclical because a positive technology shock increases the number of firms (varieties), which causes the markup rate to decline.

This explains why the cyclicality of the markup rate in a sticky price equilibrium under the Second law is generally ambiguous and depends on the tension between nominal rigidities and the pass-through rate. This can be seen by rewriting equation (18) in terms of the markup rate under the sticky price:

$$\hat{\mu}_t = \frac{X}{\zeta(z) - 1} \left[ \beta (1 - \delta) \pi_t \hat{A}_{t+1} - \pi_t \right] - 1 - \frac{\rho(z)}{\rho(z)} \frac{1 - \rho(z)}{\rho(z)} \hat{z}_t. \quad (22)$$

As the pass-through rate becomes larger (smaller) and prices become stickier (more flexible), the markup rate becomes more procyclical (countercyclical).

5 Steady state analysis

In next two sections, we simulate the New Keynesian model under HSA. For this purpose, we use three parametric families of HSA: CES, Translog and Co-Path. Co-Path is a parametric family of the HSA demand system proposed by Matsuyama and Ushchev (2020a). This parametric family is characterized by the property that under the flexible price equilibrium, the pass-through rate is given by a constant parameter between zero and unity. Table 3 shows the market share function, the price elasticity, and the pass-through rate for each family. We set
Table 4: Calibrated parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>definition</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>subjective discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>relative risk aversion</td>
<td>1</td>
</tr>
<tr>
<td>$\delta$</td>
<td>exit rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\psi$</td>
<td>inverse of labor supply elasticity</td>
<td>1</td>
</tr>
<tr>
<td>$f_E, Z_E, Z_P$</td>
<td>technologies</td>
<td>1</td>
</tr>
<tr>
<td>$\theta$</td>
<td>price elasticity under CES</td>
<td>3.8</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Rotemberg adj. cost</td>
<td>77</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>policy inertia</td>
<td>0.9</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>policy reaction to $\pi$</td>
<td>1.5 or $\infty$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>pass-through rate</td>
<td>1, 0.9 or 0.5</td>
</tr>
</tbody>
</table>

$z = (\theta / (\theta - 1))^{\rho/(1-\rho)}$ so that at the limit: $\rho \to 1$, Co-Path converges to CES. The inequality constraint that the price must be lower than the choke price need not be taken into account, since firms are symmetric. The demand for all firms, i.e., the aggregate demand, cannot be zero.

Under CES, the price elasticity is constant and, hence, the markup rate is constant and the pass-through rate is unity. Under both Translog and Co-Path, the price elasticity is increasing in $z$. Thus, both satisfy the Second law.

Parameters are calibrated as in Table 4. Most of them are taken from Bilbiie, Fujiwara and Ghironi (2014). When the Taylor rule coefficient is infinity, namely, $\alpha_\pi = \infty$, the model replicates the flexible price equilibrium. We examine several pass-through rates. We set $\tilde{K}$ so that $p_t/A_t = z_t = \tilde{p}_t = p_t/P_t$ under CES. From equation (5), this can be achieved by setting $\tilde{K} = 1/ (\theta - 1)$.\(^7\) $\gamma_{TL}$ and $\gamma_{CP}$ in Table 3 are calibrated so that $\zeta(z) = \theta$ when $f_E = 1$.\(^8\) $\gamma_{TL}$ and $\gamma_{CP}$ affect $z$ in the steady state.

In Figure 1, we replicate higher market concentration, and the rising profit margins and markups reported by Covarrubias, Gutierrez and Philippon (2019), Loecker, Eeckhout and Unger (2020), and Autor, Dorn, Katz, Patterson and Reenen (2020). The horizontal axis displays the entry cost; the further to the right, the greater the barriers to entry. The green line corresponds to CES, the red line to Translog, and the blue lines to Co-Path, when $\rho = 0.9$ (dotted) and $\rho = 0.5$ (dashed), respectively. In all cases, an increase in the entry cost leads to market concentration (decline in the number of firms). Under Translog and Co-Path, this also leads to higher markup rates and profits.\(^9\) The Second law embedded in Translog and Co-Path enables the model to replicate the stylized facts as observed in the data. Notice also that Translog is similar to Co-Path with $\rho = 0.5$.\(^9\)

---

\(^7\)This is equivalent to setting $Z_C = \gamma_{CES}^{1/(\theta-1)}$.\(^8\)Price elasticity is given by $\theta$ under CES and thus determined independently from $\gamma_{CES}$. We set $\gamma_{CES}$ at the same value of $\gamma_{CP}$ when $\rho = 0.5$.

\(^9\)The bottom right panel in Figure 1 shows a nonlinear relationship between the entry cost and the profit in the steady state. This is because the entry cost entails wage payments, as shown in equation (11).
Figure 2 illustrates how market concentration affects the slope of the Phillips curve. The horizontal axis shows the entry cost in the upper panel. There, the greater the barriers to entry, the flatter the Phillips curve. The lower panel displays how the slope of the Phillips curve changes with the number of firms. Since the greater the cost of entry, the fewer the number of firms, the lower panel is just like a mirror image of the upper panel. Fewer firms in the market result in the flattening of the Phillips curve.

Fewer firms (higher market share) corresponds to lower $z_t$, as shown in equation (17). Under the Second law, $\zeta' (z) > 0$, this leads to lower price elasticity and consequently, the slope of the Phillips curve $(\zeta (z) - 1) / \chi$ declines with higher market concentration.

The slope of the Phillips declines more to market concentration as the pass-through rate becomes smaller. This is because, as implied in equation (21), the smaller pass-through rate causes the markup rate and therefore the price elasticity to react more strongly to competitive pressures.

Loecker, Eeckhout and Unger (2020) argue that “aggregate markups start to rise from 21% above marginal cost to 61% now.” This increase in the markup rate of about 40 percentage points suggests that the entry cost would have increased by about 3.5 times in Translog, and by about 2.5 times in Co-Path with $\rho = 0.5$ in Figure 1. According to Figure 2, the accompanying market concentration can halve the slope of the Phillips curve. The Second law is a
Figure 2: Concentration and the slope of the Phillips curve

$slope of NKPC$

$entry cost$

$number of firms$

CES: $\rho=1.0$

Co-Path: $\rho=0.9$

Translog

Co-Path: $\rho=0.5$
well-established fact provided by many empirical studies. Thus, it could be said that changes in the competitive environment in recent years have exerted a significant impact on the slope of the Phillips curve.

6 Dynamic analysis

This section describes how the economy under the HSA demand system reacts to technology and monetary policy shocks. First, to understand dynamic properties under the Second law, we look at how different pass-through rates affect the responses to the technology shock under both flexible and sticky price. For this purpose, we calibrate parameters so that markup rates are the same at the steady state across preferences, and then compare the impulse responses to technology shocks. Second, to understand the impact of market concentration, we also look at how the different entry cost affect the responses to both technology and monetary policy shocks under the sticky price. Under Co-Path with $\rho = 0.5$, we examine how the entry cost, i.e., the reduction in the number of firms, modifies the response of macroeconomic variables to technology and monetary policy shocks.

Impulse responses to the technology shock by pass-through rate (Figure 3) The green line corresponds to CES, the red line to Translog, and the blue lines to Co-Path, when $\rho = 0.9$ (dotted) and $\rho = 0.5$ (dashed), respectively. The upper panel represents the impulse response under the sticky price model and the lower panel represents the impulse response under the flexible price model.

Responses to the technology shocks here are similar to those obtained in Bilbiie, Ghironi and Melitz (2008, 2012, 2019) and Bilbiie, Fujiwara and Ghironi (2014). A positive technology shock increases consumption and then output by inducing positive wealth effects. In the mean time, since it is a temporary shock, it increases savings, i.e., investment in new firms, reflecting the consumption smoothing motive. Then, the number of firms gradually increases. Subsequently, the increase in the number of firms leads to a contraction of market share. As a result, profits become smaller, and the economy returns to its original level.

Next, let’s look at how impulse responses differ by pass-through rate, in particular focusing on the differences among when $\rho = 0.5, 0.9$, and 1. At a lower $\rho$, the markup rate declines faster. As a result, profits are smaller, and a fewer firms enter after a positive technology shock hits the economy.

As suggested in Section 4.2, a comparison of the sticky and flexible price cases shows that the pass-through rate makes a significant difference in the responses of the markup rate. As is well-known, in the sticky price and the constant price elasticity, the markup rate is pro-cyclical

---

10This is achieved by calibrating $\gamma_{TL}$ and $\gamma_{CP}$ in Table 3 so that $\xi(z) = \theta$ in the steady state.
11To fix the idea, imagine that we are comparing two countries or two time periods with different entry costs. The aim is to understand how responses to shocks differ under different competitive regimes.
12Notice that Translog is similar to Co-Path with $\rho = 0.5$ even in impulse responses.
Figure 3: Impulse responses to the technology shock by pass-through rate

**sticky price**

- Output
- Number of firms
- Markup
- Inflation rate
- Nominal interest rate

**flexible price**

- Output
- Number of firms
- Markup
- Inflation rate

Legend:
- CES $\rho = 1.0$
- Co-Path $\rho = 0.9$
- Co-Path $\rho = 0.5$
- Translog
in response to positive technology shocks. Marginal costs fall from higher efficiency, but prices remain unchanged in the short run, resulting in higher markup rates. On the other hand, in the case of flexible price and the endogenous price elasticity under the Second law, an increase in the number of firms leads to higher price elasticity, resulting in smaller markup rates. This tendency becomes stronger as the pass-through rate becomes smaller. As a result, consistent with equation (22), in the sticky price case, as the pass-through rate becomes smaller, the markup rate moves from procyclical to countercyclical. Cyclicality of the markup rate depends on the pass-through rate and nominal rigidities.

Also, as equation (18) implies, the dynamic effect of competition leads to more deflation with smaller pass-through rates.

Impulse responses to technology and monetary policy shocks by entry cost (Figure 4) Cases shown here are for under Co-Path. The red line indicates the entry cost of 0.5 (half the benchmark), the green line indicates this is 1 (benchmark case), and the blue line indicates this is 2 (twice the benchmark). As the color of the line changes from red to green to blue, the barrier to entry becomes larger.

The responses of the inflation rate to both technology and monetary policy shocks become smaller as the barrier to entry becomes larger. This reflects that the slope of the Phillips curve becomes smaller with higher market concentration, as analyzed in Sections 4 and 5.

The markup rate \( \mu_t \) and \( z_t \) move to the opposite directions to a technology shock. Thus, its impact on inflation is muted by the dynamic effect of competition. The opposite has occurred for monetary policy shocks. Equation (18) illustrates that the size of the endogenous cost-push shock becomes larger as the pass-through rate become smaller. Taken together, these imply that market concentration yields stronger dynamic effects of competition and amplify the impact of monetary policy shocks. However, the effect of the flattening of the Phillips curve to weakening monetary policy dominates the dynamic effect of competition. As a result, the impulse responses of the inflation rate to the monetary policy shock becomes smaller as market concentration develops.

7 Calvo pricing

So far, we have looked at the case of the Rotemberg (1982) price adjustment cost as the source of sticky prices. Do the implications of the competitive environment on the Phillips curve depend on the assumption of nominal rigidities? To address this question, this section derives NKPC under HSA and Calvo (1983) pricing. Details of the derivation are given in Appendix C.

NKPC under HSA and Calvo pricing is given by

\[
\hat{\pi}_t = \beta (1 - \delta) E_t \hat{\pi}_{t+1} + \frac{(1 - \alpha) [1 - \alpha^\beta (1 - \delta)]}{\alpha} \left[ \rho (\hat{z}) (\hat{\tilde{W}_t} - \hat{\tilde{Z}_{P,t}} - \hat{\tilde{p}_t}) - (1 - \rho (\hat{z})) \hat{N_t} \right], \quad (23)
\]
Figure 4: Impulse responses to technology and monetary policy shocks by entry cost
where $\alpha$ is the probability that a firm can adjust its price. Under Calvo pricing, unlike Rotemberg pricing, not all firms set the same prices. Therefore, we define the average price under Calvo friction denoted by $\tilde{p}_t$, to distinguish it from the case with Rotemberg pricing, where the price of individual firms is equal to the average price. It is implicitly given by

$$\int_{\Omega_t} s \left( \frac{p_t(\omega)}{A_t} \right) d\omega = 1 = N_t s \left( \frac{\tilde{p}_t}{A_t} \right).$$

Accordingly, $\tilde{z}_t := \tilde{p}_t / A_t$ and $\tilde{r}_t := \tilde{p}_t / \tilde{p}_{t-1}$.

Under Calvo pricing in equation (23), the slope of the Phillips curve is affected by the pass-through rate. Recall that under Rotemberg pricing in equation (18), the slope is affected by the price elasticity. Here, market concentration leads to the flattening of the Phillips curve under the Third law (Matsuyama and Ushchev, 2022b). According to the Third law, a higher price leads to a smaller rate of change in the price elasticity. As a result, a higher entry cost leads to less competitive pressures and lowers the pass-through rate. This is because the firm increases its price and the markup rate in response to less competitive pressures, a higher $A_t$, when other firms increase their prices. Under this strategic complementarity embedded in HSA, higher concentration results in the flattening of the Phillips curve. This strategic complementarity also induces the dynamic effect of competition in the same manner as under Rotemberg pricing in equation (18).

The intuition behind why the pass-through rate matters in the slope under Calvo pricing is as follows. The optimal reset price $p_t^*$ satisfies

$$E_0 \sum_{i=0}^{\infty} [\alpha \beta (1 - \delta)]^i u' (C_{t+i}) Y_{t+i} s \left( \frac{p_t^*}{A_{t+i}} \right) \left[ 1 - \frac{\zeta \left( \frac{p_t^*}{A_{t+i}} \right)}{\zeta \left( \frac{p_t^*}{A_{t+i}} \right) - 1} \frac{W_{t+i}}{Z_{p_t^*,t+i}} \right] = 0.$$

Each firm aims at achieving the Lerner pricing formula in the present discounted value with the reset probability taken into account. Since at time $t$, a proportion $1 - \psi$ of firms can reset the price and a proportion $\psi$ of firms keep the previous price, the reset price can be expressed as a function of inflation rates. The impact of the marginal cost on the reset price and therefore inflation rates depends on how the markup rate changes with the reset price, which is pinned down by the pass-through rate as shown in equation (20).$$\begin{align*}
\sum_{i=0}^{\infty} [\alpha \beta (1 - \delta)]^i u' (C_{t+i}) Y_{t+i} s \left( \frac{p_t^*}{A_{t+i}} \right) \left[ 1 - \frac{\zeta \left( \frac{p_t^*}{A_{t+i}} \right)}{\zeta \left( \frac{p_t^*}{A_{t+i}} \right) - 1} \frac{W_{t+i}}{Z_{p_t^*,t+i}} \right] = 0.
\end{align*}$$

13In this regard, this paper also has implications for the literature on the equivalence and the nonequivalence between the two pricing arrangements. As is well-known and explicitly shown in Roberts (1995), both Calvo pricing and Rotemberg pricing result in a common formulation of NKPC to the first order approximation. Nistico (2007) shows that these two price settings yield the same welfare losses to a second order of approximation around the efficient steady state, while Lombardo and Vestin (2008) show that they may entail different welfare costs at higher order approximation around the inefficient steady state. Ascari and Rossi (2012) provides that the long-run Phillips curve, i.e., the long-run relationship between inflation and output, and the dynamics in the presence of the trend inflation are different between these two price settings. This paper represents that how market concentration affects the slope of the Phillips curve is qualitatively different between Calvo pricing and Rotemberg pricing.

14Notice that as evident in Table 3, Translog cannot accommodate the Third law.
Market share function and the slope of the Phillips curve  Equations (18) and (23) show that the causal impact from market concentration to the flattening of the Phillips curve is summarized by two statistics: the price elasticity \( \zeta(z) \) under Rotemberg pricing and the pass-through rate \( \rho(z) \) under Calvo pricing. In the HSA demand system, the market share function \( s(z) \) fully characterizes both the price elasticity and the pass-through rate as in equations (19) and (20) and its single aggregator summarizes all the impacts of market concentration on the flattening of the Phillips curve.

Power elasticity of markup rate (PEM)  Co-Path satisfies the Third law only in the weak form: the pass-through rate is constant. PEM, another parametric family of HSA, satisfies the Third law in the strong form: a higher price leads to a smaller rate of change in the price elasticity.\(^{15}\) PEM contains CES and Co-Path as limit cases.

\[
\rho(z) := \frac{\partial \ln (p_t)}{\partial \ln (W_t/Z_{P,t})} = \left[ 1 - \frac{d \ln \left( \frac{\zeta(z)}{\zeta(z) - 1} \right)}{d \ln (z)} \right]^{-1}, \tag{24}
\]

The market share function in PEM is given by

\[
s(z) = \exp \left[ \int_{z_0}^{z} \frac{c}{c - \exp \left( -\frac{\kappa z - \lambda}{A} \right) \exp \left( \frac{\kappa z - \lambda}{A} \right) \frac{d \zeta}{\zeta}} \right], \tag{25}
\]

with either \( z = 0 \) and \( c \leq 1 \) or \( z < 0 \) and \( c = 1 \). Then, \( \kappa > 0 \) and \( \lambda > 0 \) ensure that both the Second law and the strong Third law are satisfied. By using equations (19) and (20), the price elasticity and the pass-through rate under PEM are, respectively, given by

\[
\zeta(z) = \frac{1}{1 - c \exp \left( \frac{\kappa z - \lambda}{A} \right) \exp \left( -\frac{\kappa z - \lambda}{A} \right)}, \tag{26}
\]

and

\[
\rho(z) = \frac{1}{1 + \kappa z^{-\lambda}}. \tag{27}
\]

Notice that by comparing equations (20) and (27), one can immediately see

\[
\kappa z^{-\lambda} = -\frac{d \ln \left( \frac{\zeta(z)}{\zeta(z) - 1} \right)}{d \ln (z)},
\]

which means that the elasticity of the markup rate under the flexible price is a power function of \( z \). This is why Matsuyama and Ushchev (2022b) call this family Power Elasticity of Markup rate (PEM). The market share function in equation (25) collapses to that under Co-Path with \( \lambda = 0, \kappa = (1 - \rho) / \rho, c = 1 \) and \( z < \infty \), and to that under CES with \( \kappa = 0, c = 1 - 1/\theta \) and \( z = \infty \). When PEM collapses to CES, the market share function is given by \( s(z) = (z/z_0)^{1-\theta} \).

\(^{15}\)For more details, see Appendix D.3. in Matsuyama and Ushchev (2022b).
Figure 5: Markup and the slope of the Phillips curve under Calvo pricing

To be comparable between CES and PEM, we set \( z_0 = \gamma^{\frac{1}{1-\gamma}} \).\(^{16}\)

Figure 5 illustrates the relationship between the markup rate and the slope of NKPC in equation (23) under several \( \lambda \).\(^{17}\) The black dot indicates the slope of the Phillips curve and the steady state markup under CES. There, the price elasticity, and therefore the markup rate, is not affected by competitive pressures, so the markup rate is \( \theta \) / \( \theta - 1 \) and the slope of the Phillips curve is \( (1 - a) \left[ 1 - a \beta (1 - \delta) \right] / a \). Equation (27) shows that as \( \lambda \) approaches to zero, the pass-through rate hardly moves, i.e., PEM collapses to Co-Path. Therefore, variations in the pass-through rate become smaller. \( \lambda \) controls how variable the pass-through rate is to changes in \( z \) as implied in equation (27), while \( \kappa \) determines the level of the pass-through rate.

According to Loecker, Eeckhout and Unger (2020), the markup rate increased to 21% to 61% in recent years. Although it is difficult to draw quantitative conclusions since there are no estimated values for \( \kappa \) and \( \lambda \), Figure 5 shows that even under Calvo pricing, changes in the competitive environment among firms can result in a much smaller slope of the Phillips curve through the Third law.

\(^{16}\)\( \gamma \) is set at a value so that \( \zeta(z) = \theta \) when \( f_E = 1 \) and \( \rho = 0.5 \). Notice that \( \gamma \) does not affect the price elasticity and the pass-through rate as evident in equations (26) and (27).

\(^{17}\)Here, the relationship between the markup rate and the slope of NKPC is drawn by changing the steady state value of \( z \). As in Figure 2 in Section 4, it is also possible to consider this as a case in which the entry cost changes.
8 Conclusion

Under Rotemberg pricing, the Second law implies that higher market concentration leads to higher markup rates and eventually the flattening of the Phillips curve. This results in muted impacts of structural shocks on inflation rates with higher market concentration. In addition, market concentration directly affects price setting over the business cycle through strategic complementarity. Thus, the endogenous cost-push shock emerges under the Second law. We also show that the pass-through rate plays an important role in the cyclicality of the markup rate. As the pass-through rate becomes larger (smaller) and prices become stickier (more flexible), the markup rate becomes more procyclical (countercyclical). Under Calvo pricing, the Third law plays a similar role in the flattening of the Phillips curve.

Thus, the causal impacts from market concentration to the flattening of the Phillips curve are summarized by two statistics: the price elasticity under Rotemberg pricing and the pass-through rate under Calvo pricing. Under the HSA demand system, the market share function fully characterizes both the price elasticity and the pass-through rate and its single aggregator summarizes all the impacts of market concentration on the flattening of the Phillips curve.

Because HSA is tractable and intuitive, there are still a number of interesting research topics that could be addressed by applying HSA to the New Keynesian framework, to name a few, analysis on optimal policy, incorporating the wage Phillips curve, exploring the menu cost model under heterogeneous firms. We are also interested in rigorous empirical investigation on the HSA demand system, say, using the administrative data. They all are left for our future research.
References


A System of equations

A.1 Nonlinear

1. Taylor rule
\[(1 + i_t) = (1 + i_{t-1})^{\alpha_i} (\pi_t - 1)^{(1-\alpha_i)\alpha_{\pi}} u_t\]

2. Euler equation for bonds
\[C_{t-\sigma} = \beta E_t C_{t+1}^{\sigma} \frac{1 + i_t}{\bar{p}_{t+1}} \frac{\bar{p}_{t+1}}{\bar{p}_t}\]

3. NKPC
\[\left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] \frac{s'(z_t) z_t}{s(z_t)} + \left[1 - \frac{s'(z_t) z_t}{s(z_t)}\right] \frac{L_t^\psi C_t^{\sigma}}{Z_{P,t}^\psi \bar{p}_t} - \chi (\pi_t - 1) \pi_t + \beta (1 - \delta) E_t \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}} \chi (\pi_{t+1} - 1) \pi_{t+1} \frac{s(z_{t+1})}{s(z_t)} Y_{t+1} = 0\]

4. Euler equation for equity
\[L_t^\psi C_t^{\sigma} \frac{f_{E,t}}{Z_{E,t}} = \beta (1 - \delta) E_t \frac{C_{t+1}^{\sigma}}{C_t^{\sigma}} \left\{\left[1 - \frac{L_t^\psi C_t^{\sigma}}{Z_{P,t}^\psi \bar{p}_t} - \frac{\chi}{2} (\pi_t - 1)^2\right] s(z_{t+1}) Y_{t+1} + L_t^\psi C_t^{\sigma} \frac{f_{E,t+1}}{Z_{E,t+1}}\right\}\]

5. Firm dynamics
\[\frac{1}{s(z_t)} = (1 - \delta) \left[\frac{1}{s(z_{t-1})} + Z_{E,t-1} f_{E,t-1} \left(L_t - \frac{Y_{t-1}}{\bar{p}_{t-1} Z_{t-1}}\right)\right]\]

6. Relative price\(^{18}\)
\[\ln \left(\frac{z_t}{\bar{p}_t}\right) = \bar{K} - \frac{1}{s(z_t)} \left[\int_{z_t}^\xi s(\xi) d\xi\right]\]

7. Resource constraint
\[C_t = \left[1 - \frac{\chi}{2} (\pi_t - 1)^2\right] Y_t\]

\(^{18}\)We need to solve the integral in
\[\bar{p} = \exp \left(\bar{K} - \frac{L_t^\psi}{Z_{E,t}^\psi} \frac{\xi}{s(z_t)}\right)\]

Under Co-Path, the integral is given analytically by the hypergeometric function when \(v := 1/\bar{p}\) is integer:
\[\int \left[1 - \left(\frac{\xi}{z}\right)^v\right] d\xi = v \left[1 - \left(\frac{\xi}{z}\right)^v\right]^{1+v} \sum_{n=0}^{\infty} (1)_{n} (1+v)_{n} \left[1 - \left(\frac{\xi}{z}\right)^v\right]^n\]

Under Translog, the integral is also analytically given by
\[\int \frac{1 - \gamma_{TL} \ln \left(\frac{\xi}{z}\right)}{\xi} d\xi = \frac{\gamma_{TL}}{2} \left[\ln \left(\frac{\xi}{z}\right)\right]^2\]

28
A.2 Steady state

1. Taylor rule
   \[ \pi = 1 \]

2. Euler equation for bonds
   \[ i = \frac{1 - \beta}{\beta} \]

3. NKPC
   \[ L^\psi\gamma^\sigma = \frac{\bar{p}}{1 - \frac{s'(z)}{s(z)^2}} Z_p \]

4. Euler equation for equity
   \[ L = \frac{1}{1 - \delta} \frac{f_E}{Z_E} \left[ \delta - \frac{1 - \beta (1 - \delta)}{\beta} \frac{s'(z) z}{s(z)} \right] \frac{1}{s(z)} \]

5. Firm dynamics
   \[ Y = -\frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} Z_p \frac{f_E}{Z_E} \frac{s'(z) z}{s(z)^2} \exp \left( K - \frac{\int s'(z) dt}{s(z)} \right) \]

6. Relative price
   \[ \bar{p} = \frac{z}{\exp \left( K - \frac{\int s'(z) dt}{s(z)} \right)} \]

A.3 Log-linearly approximated

1. Taylor rule
   \[ i_t = \alpha_i i_{t-1} + (1 - \alpha_i) \alpha \pi \hat{\alpha}_t + u_t \]

2. Euler equation for bonds
   \[ \hat{Y}_t = \mathbb{E}_t \hat{Y}_{t+1} - \frac{1}{\sigma} [i_t - (\hat{p}_t - \mathbb{E}_t \hat{p}_{t+1}) - \mathbb{E}_t \hat{\pi}_{t+1}] \]

3. NKPC
   \[ \hat{\pi}_t = \frac{1}{\lambda} \left[ -\frac{s'(z) z}{s(z)} (\psi \hat{L}_t + \sigma \hat{Y}_t - \hat{Z}_{P,t} - \hat{p}_t) + \frac{1}{\lambda} \left[ \frac{s'(z) z}{s(z)} - \frac{s'(z) z}{s'(z)} \right] \right] \hat{z}_t + \beta (1 - \delta) \mathbb{E}_t \hat{\pi}_{t+1} \]

4. Euler equation for equity
   \[ \psi \hat{L}_t = \left\{ [1 - \beta (1 - \delta)] \frac{s'(z) z}{s(z)} + \beta (1 - \delta) \right\} \psi \mathbb{E}_t \hat{L}_{t+1} + [1 - \beta (1 - \delta)] \left[ \frac{s'(z) z}{s(z)} + (1 - \sigma) \right] \mathbb{E}_t \hat{Y}_{t+1} \]
   \[ \quad - \left( f_{E,t} - \hat{Z}_{E,t} \right) + \beta (1 - \delta) \mathbb{E}_t \left( f_{E,t+1} - \hat{Z}_{E,t+1} \right) - [1 - \beta (1 - \delta)] \frac{s'(z) z}{s(z)} \mathbb{E}_t \hat{Z}_{P,t+1} \]
5. Firm dynamics

\[ \dot{z}_t = (1 - \delta) \dot{z}_{t-1} - \left\{ \left[ \delta \frac{s(z)}{s'(z)} z - \frac{1 - \beta (1 - \delta)}{\beta} \right] \dot{L}_t - \hat{Y}_t + \hat{Z}_{P,t} \right\} + \delta \frac{s(z)}{s'(z)} (\hat{f} - \bar{Z}_{E,t}) + \text{Calvo pricing frictions.} \]

They are randomly assigned the prices that the existing firms set in the previous period.

Some of new entrants cannot change their prices upon entry due to Calvo pricing frictions.

\[ \text{Note that } \pi_{NKPC} \text{ in terms of CPI } P_t \text{ inflation rates and inflation rates in the single price aggregate } A_t \text{ are given by} \]

\[ \pi_{C,t} = \beta (1 - \delta) E_t \pi_{C,t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{[\zeta(z) - 1] [1 - \rho(z)]}{\chi \rho(z)} \dot{z}_t \]

\[ + \int_z^s \frac{s(\xi)}{\xi} d\xi \frac{\zeta(z) - 1}{s(z)} \left\{ \beta (1 - \delta) E_t \dot{z}_{t+1} - \left[ 1 + \beta (1 - \delta) \right] \dot{z}_t + \dot{z}_{t-1} \right\} , \]

\[ \pi_{A,t} = \beta (1 - \delta) E_t \pi_{A,t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{[\zeta(z) - 1] [1 - \rho(z)]}{\chi \rho(z)} \dot{z}_t \]

\[ + \beta (1 - \delta) E_t \dot{z}_{t+1} - \left[ 1 + \beta (1 - \delta) \right] \dot{z}_t + \dot{z}_{t-1} . \]

Note that \( \pi_{C,t} := \frac{\hat{p}_t}{\hat{p}_{t-1}} \pi_t \) and \( \pi_{A,t} := \frac{A_t}{A_{t-1}} \pi_t \).

NKPC in terms of the real marginal cost \( \pi_c := \frac{W_t}{z_{P,t}} \) is given by

\[ \pi_t = \beta (1 - \delta) E_t \pi_{t+1} + \frac{\zeta(z) - 1}{\chi} \left[ \hat{m}_c - \hat{p}_t - \frac{1 - \rho(z)}{\rho(z)} \hat{z}_t \right] \]

\[ = \beta (1 - \delta) E_t \pi_{t+1} + \frac{\zeta(z) - 1}{\chi} \left\{ \hat{m}_c - \left[ \frac{1 - \rho(z)}{\rho(z)} + \int_z^s \frac{s(\xi)}{\xi} d\xi \frac{\zeta(z) - 1}{s(z)} \right] \hat{z}_t \right\} . \]

The slope of NKPC remains the same.

6. Relative price

\[ \hat{p}_t = -\frac{1}{s(z)} \int_z^s \frac{s(\xi)}{\xi} d\xi \frac{s'(z)}{s(z)} \dot{z}_t \]

B Phillips curves with \( P_t \) and \( A_t \) inflation rates

NKPC in terms of CPI \( P_t \) inflation rates and inflation rates in the single price aggregate \( A_t \) are given by

\[ \hat{\pi}_{C,t} = \beta (1 - \delta) E_t \hat{\pi}_{C,t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{[\zeta(z) - 1] [1 - \rho(z)]}{\chi \rho(z)} \hat{z}_t \]

\[ + \int_z^s \frac{s(\xi)}{\xi} d\xi \frac{\zeta(z) - 1}{s(z)} \left\{ \beta (1 - \delta) E_t \hat{z}_{t+1} - \left[ 1 + \beta (1 - \delta) \right] \hat{z}_t + \hat{z}_{t-1} \right\} , \]

\[ \hat{\pi}_{A,t} = \beta (1 - \delta) E_t \hat{\pi}_{A,t+1} + \frac{\zeta(z) - 1}{\chi} (\hat{W}_t - \hat{Z}_{P,t} - \hat{p}_t) - \frac{[\zeta(z) - 1] [1 - \rho(z)]}{\chi \rho(z)} \hat{z}_t \]

\[ + \beta (1 - \delta) E_t \hat{z}_{t+1} - \left[ 1 + \beta (1 - \delta) \right] \hat{z}_t + \hat{z}_{t-1} . \]

Note that \( \pi_{C,t} := \frac{P_t}{P_{t-1}} \pi_t \) and \( \pi_{A,t} := \frac{A_t}{A_{t-1}} \pi_t \).

NKPC in terms of the real marginal cost \( \pi_c := \frac{W_t}{P_t} \) is given by

\[ \hat{\pi}_t = \beta (1 - \delta) E_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} \left[ \hat{m}_c - \hat{p}_t - \frac{1 - \rho(z)}{\rho(z)} \hat{z}_t \right] \]

\[ = \beta (1 - \delta) E_t \hat{\pi}_{t+1} + \frac{\zeta(z) - 1}{\chi} \left\{ \hat{m}_c - \left[ \frac{1 - \rho(z)}{\rho(z)} + \int_z^s \frac{s(\xi)}{\xi} d\xi \frac{\zeta(z) - 1}{s(z)} \right] \hat{z}_t \right\} . \]

The slope of NKPC remains the same.

C NKPC under Calvo pricing and HSA

Some of new entrants cannot change their prices upon entry due to Calvo pricing frictions. They are randomly assigned the prices that the existing firms set in the previous period.
C.1 Price setting

Intermediate goods producer $\omega$ maximizes the value of the firm:

$$
\frac{V_t(\omega)}{P_t} = E_0 \sum_{i=0}^{\infty} \left[ \psi \beta (1 - \delta) \right]^i \frac{u'(C_{t+i})}{u'(C_t)} \left( \frac{D_{t+i}(\omega)}{P_{t+i}} \right),
$$

where

$$
\frac{D_t(\omega)}{P_t} = \frac{p_t(\omega)}{P_t} y_t(\omega) - \frac{W_t}{P_t} l_t(\omega),
$$

subject to equations (7) and (9).

The optimal price setting condition is given by

$$
E_0 \sum_{i=0}^{\infty} \left[ \psi \beta (1 - \delta) \right]^i \frac{u'(C_{t+i})}{u'(C_t)} Y_{t+i} \left( \frac{p^*_t}{A_{t+i}} \right) - \frac{W_{t+i}}{Z_{P,t+i}} \left[ s' \left( \frac{p^*_t}{A_{t+i}} \right) - s \left( \frac{p^*_t}{A_{t+i}} \right) \right] \left( \frac{A_{t+i}}{p^*_t} \right) = 0,
$$

which can be be rewritten as

$$
X_{1,t} = X_{2,t},
$$

where

$$
X_{1,t} := E_0 \sum_{i=0}^{\infty} \left[ \psi \beta (1 - \delta) \right]^i \frac{u'(C_{t+i})}{u'(C_t)} Y_{t+i} \left( \frac{p^*_t}{A_{t+i}} \right) \left( \frac{p^*_t}{A_{t+i}} \right) p^*_t,
$$

$$
X_{2,t} := E_0 \sum_{i=0}^{\infty} \left[ \psi \beta (1 - \delta) \right]^i \frac{u'(C_{t+i})}{u'(C_t)} Y_{t+i} \frac{W_{t+i}}{Z_{P,t+i}} \left[ s' \left( \frac{p^*_t}{A_{t+i}} \right) - s \left( \frac{p^*_t}{A_{t+i}} \right) \right] \left( \frac{A_{t+i}}{p^*_t} \right) \left( \frac{A_{t+i}}{p^*_t} \right).
$$

C.1.1 Log-linearization

Above system can be log-linearized as follows:

$$
\hat{X}_{1,t} = \hat{X}_{2,t},
$$

$$
\hat{X}_{1,t} = u'(C) \frac{Y}{A} s' \left( \frac{p^*_t}{A} \right) p^* \left[ \frac{1}{1 - \psi \beta (1 - \delta)} \hat{A}_t + \hat{Y}_{1,t} \right],
$$

where

$$
\hat{Y}_{1,t} = \frac{u''(C_t) C_t}{u'(C)} \hat{C}_t + \hat{Y}_t - \left[ 1 + \frac{s'' \left( \frac{p^*_t}{A} \right) p^*_t}{s' \left( \frac{p^*_t}{A} \right)} \right] \hat{A}_t + \psi \beta (1 - \delta) E_t \hat{Y}_{1,t+1},
$$
and
\[
\hat{X}_{2,t} = u'(C) \frac{W}{A} \frac{Z}{p^*} \left[ s' \left( \frac{p^*}{A} \right) - s \left( \frac{p^*}{A} \right) \right] + \begin{bmatrix} \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})} \frac{1}{1 - \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})}} - 1 \end{bmatrix} - 1 \psi \beta \frac{1}{1 - \delta} \beta_t + \hat{Y}_{2,t},
\]

where
\[
\hat{Y}_{2,t} = u''(C_t) C_t \hat{C}_t + \hat{Y}_t + W_t - Z_{t,P,t} - \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})} \hat{A}_t + \psi \beta (1 - \delta) E_t \hat{Y}_{t-1}.
\]

Combining above five equations yields
\[
\frac{1}{1 - \psi \beta (1 - \delta)} \begin{bmatrix} \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})} \frac{1}{1 - \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})}} - 1 \end{bmatrix} \hat{\beta}_t = \begin{bmatrix} \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})} \frac{1}{1 - \frac{s''(\frac{p^*}{A}) \frac{p^*}{A}}{s'(\frac{p^*}{A})}} \end{bmatrix} \hat{A}_t + \psi \beta (1 - \delta) E_t \hat{\beta}_{t-1},
\]

C.2 Aggregate price

Under the Calvo pricing, the adding-up constraint in equation (2) is given by
\[
(1 - \psi) N_t s \left( \frac{p_t}{A_t} \right) + \psi \int_0^{N_t} s \left( \frac{p_{t-1}(\omega)}{A_t} \right) d\omega = 1.
\]

In the steady state,
\[
(1 - \psi) N s \left( \frac{p^*}{A} \right) + \psi N s \left( \frac{p^*}{A} \right) = 1.
\]

Since \( s (\hat{p}/A) = 1/N, s (p^*/A) = 1/N \). As a result,
\[
p^* = \hat{p}.
\]

C.2.1 Log linearization

Log-linear approximation of equation (29) yields
\[
(1 - \psi) s' \left( \frac{p^*}{\bar{A}} \right) \frac{p^*}{\bar{A}} \hat{\hat{p}}^*_t = s' \left( \frac{p^*}{\bar{A}} \right) \frac{p^*}{\bar{A}} \hat{A}_t - \hat{N}_t - \psi s' \left( \frac{p^*}{\bar{A}} \right) \frac{p^*}{\bar{A}} \int_0^N \hat{p}_{t-1} (\omega) \, d\omega,
\]
where we define \( \hat{p}_t (\omega) := \ln (p_t (\omega) / \hat{p}_t) \).

Log-linear approximation of \( \int_0^N s (p_t (\omega) / \hat{p}_t) \, d\omega = 1 \) leads to

\[
\int_0^N \hat{p}_t (\omega) \, d\omega = \frac{1}{s' \left( \frac{p^*}{\bar{A}} \right)} \hat{A}_t - \frac{1}{s' \left( \frac{p^*}{\bar{A}} \right)} \hat{N}_t.
\]

By combining above two equations together, we have

\[
\hat{p}_t^* = \frac{1}{1 - \psi} \hat{A}_t - \frac{\psi}{1 - \psi} \hat{A}_{t-1} - \frac{1}{1 - \psi} \frac{s \left( \frac{p^*}{\bar{A}} \right)}{s' \left( \frac{p^*}{\bar{A}} \right)} (\hat{N}_t - \psi \hat{N}_{t-1}). \tag{31}
\]

### C.3 NKPC

Substituting equation (31) into equation (28) leads to NKPC in terms of \( \hat{A}_t \):

\[
\hat{\hat{A}}_{A,t} = \beta (1 - \delta) \mathbb{E}_t \hat{\hat{A}}_{A,t+1} + \frac{s' \left( \frac{p^*}{\bar{A}} \right) s' \left( \frac{p^*}{\bar{A}} \right)}{s \left( \frac{p^*}{\bar{A}} \right) s' \left( \frac{p^*}{\bar{A}} \right)} \hat{A}_t - \frac{\psi}{1 - \psi} \hat{A}_{t-1} - \frac{1}{1 - \psi} \frac{s \left( \frac{p^*}{\bar{A}} \right)}{s' \left( \frac{p^*}{\bar{A}} \right)} (\hat{N}_t - \psi \hat{N}_{t-1}) \left[ \frac{1}{\psi} (\hat{N}_t - \psi \hat{N}_{t-1}) - \beta (1 - \delta) (\mathbb{E}_t \hat{N}_{t+1} - \psi \hat{N}_t) \right]. \tag{32}
\]

Log-linearization of \( N_t s (\hat{p}_t / \hat{A}_t) = 1 \) yields

\[
\hat{A}_t = \hat{p}_t + \frac{s \left( \frac{p^*}{\bar{A}} \right)}{s' \left( \frac{p^*}{\bar{A}} \right)} \hat{N}_t,
\]

and

\[
\hat{\hat{A}}_{A,t} = \hat{A}_t + \frac{s \left( \frac{p^*}{\bar{A}} \right)}{s' \left( \frac{p^*}{\bar{A}} \right)} (\hat{N}_t - \hat{N}_{t-1}).
\]

Substituting these into equation (32) leads to

\[
\hat{A}_t = \beta (1 - \delta) \mathbb{E}_t \hat{A}_{t+1} + \frac{1 - \psi}{\psi} \left[ \frac{1 - \psi \beta (1 - \delta)}{\psi} \right] \rho (\hat{z}) \left[ (\hat{W}_t - \hat{Z}_t - \hat{p}_t) - \frac{1}{\hat{z}} (1 - \rho (\hat{z}) \hat{N}_t) \right],
\]

where we use equations (19) and (20) and define \( \hat{z}_t := \hat{p}_t / \hat{A}_t = p_t^* / \hat{p}_t \). The final equality comes from equation (30).