The Generalized Engel’s Law: In Search for A New Framework

By

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The Canon Institute for Global Studies
January 14, 2016

Introduction:

- **Engel's Law:** the expenditure share of food declines as household income rises.
- **Generalized Engel's Law**: the *aggregate* expenditure shares of many consumption goods change systematically with *per capita* income, both in times series and in cross-section, suggesting ***differential income elasticity*** across goods
- With profound implications on structural change and international trade, for example,
  - The decline of agriculture, rise & fall of manufacturing, rise of service sectors
  - US, EU, and Japan are the three biggest markets for SUV; China, India, and Indonesia are the three biggest markets for motorbikes
- Most formal models of **structural change** and **international trade** assume ***homothetic preferences***, which implies
  - Every good has the identical income elasticity that is equal to one.
  - The rich & the poor consume goods in the same proportions.
- In this talk, I will
  - Discuss why existing attempts to incorporate ***nonhomothetic preferences*** in formal models are too restrictive and too inflexible to capture rich implications of the generalized Engel's law.
  - Propose a new form of nonhomothetic preferences and demonstrate its power with some applications to structural change and international trade.

Empirical Regularities

Goods vs. Services (from Boppart 2014)

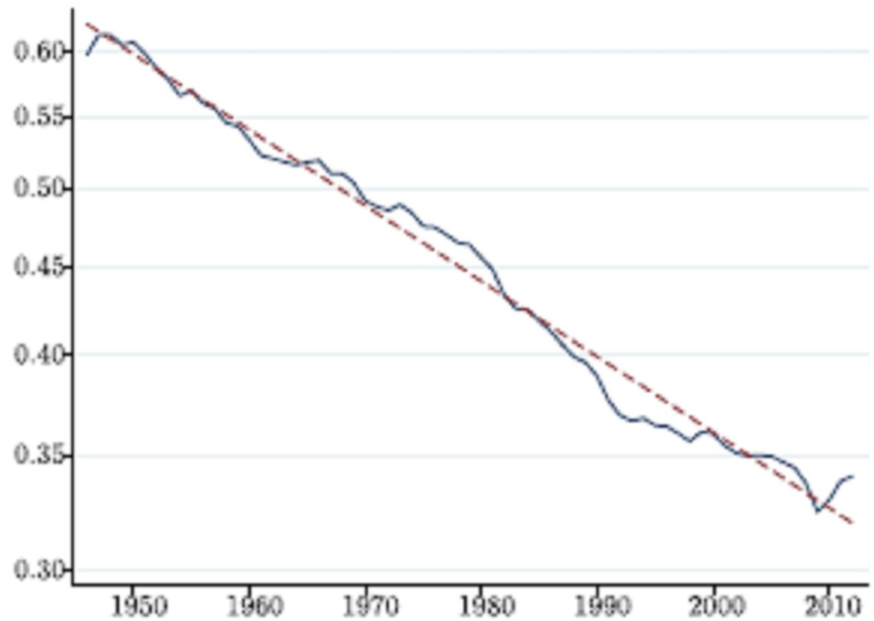
The share of "Goods" in total personal consumption expenditure declines.

Log of expenditure share of goods in post-war US

According to Bureau of Economic Analysis (BEA),

"a good" is "a tangible commodity that can be stored and inventoried,"

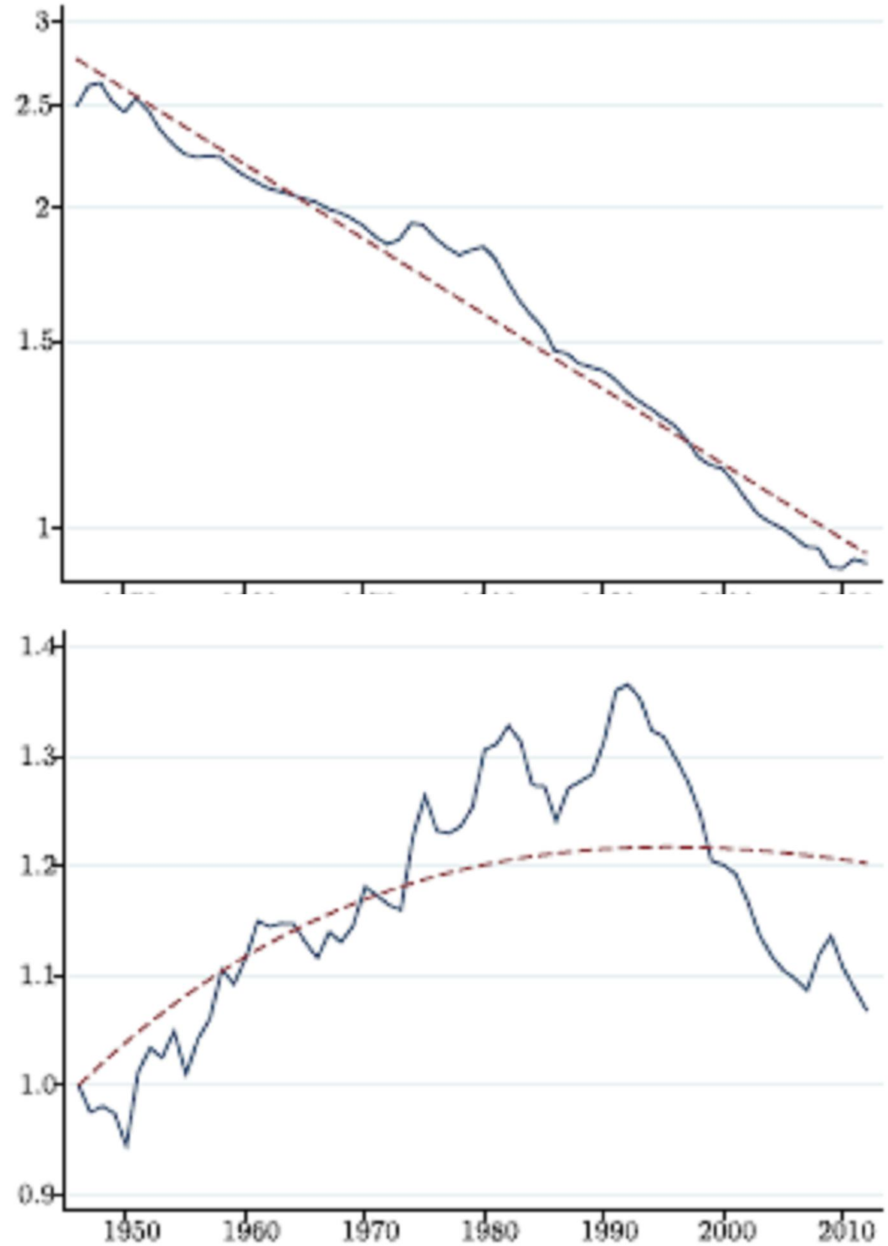
"a service" is "a commodity that cannot be stored or inventoried and that is usually consumed at the place and time of purchase"



The main categories of "goods" are "Motor vehicles and parts," "furnishings and durable household equipment," "recreational goods and vehicles," "food and beverages purchased for off-premises consumption," "clothing and footwear" "gasoline and other energy goods," "other durable/nondurable goods."

The price of Goods relative to Services declines.

Log of relative price of goods to services



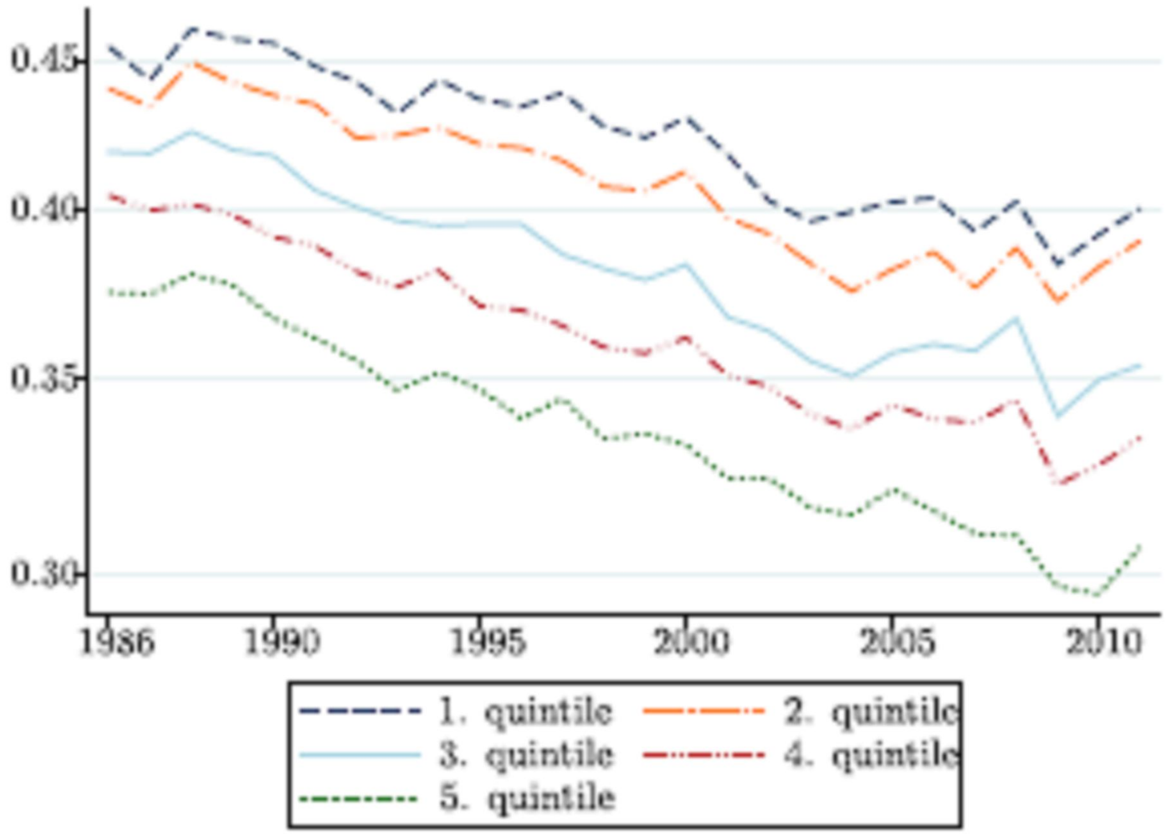
This could cause the share of goods to decline, if goods and services are not very substitutable.

If the change in the shares is driven *solely* by the relative price change, **the real quantity of services relative to goods** *should* go down through the substitution effect.

However, it **is non-monotonic, increasing until early 1990s,** suggesting the differential income effects.

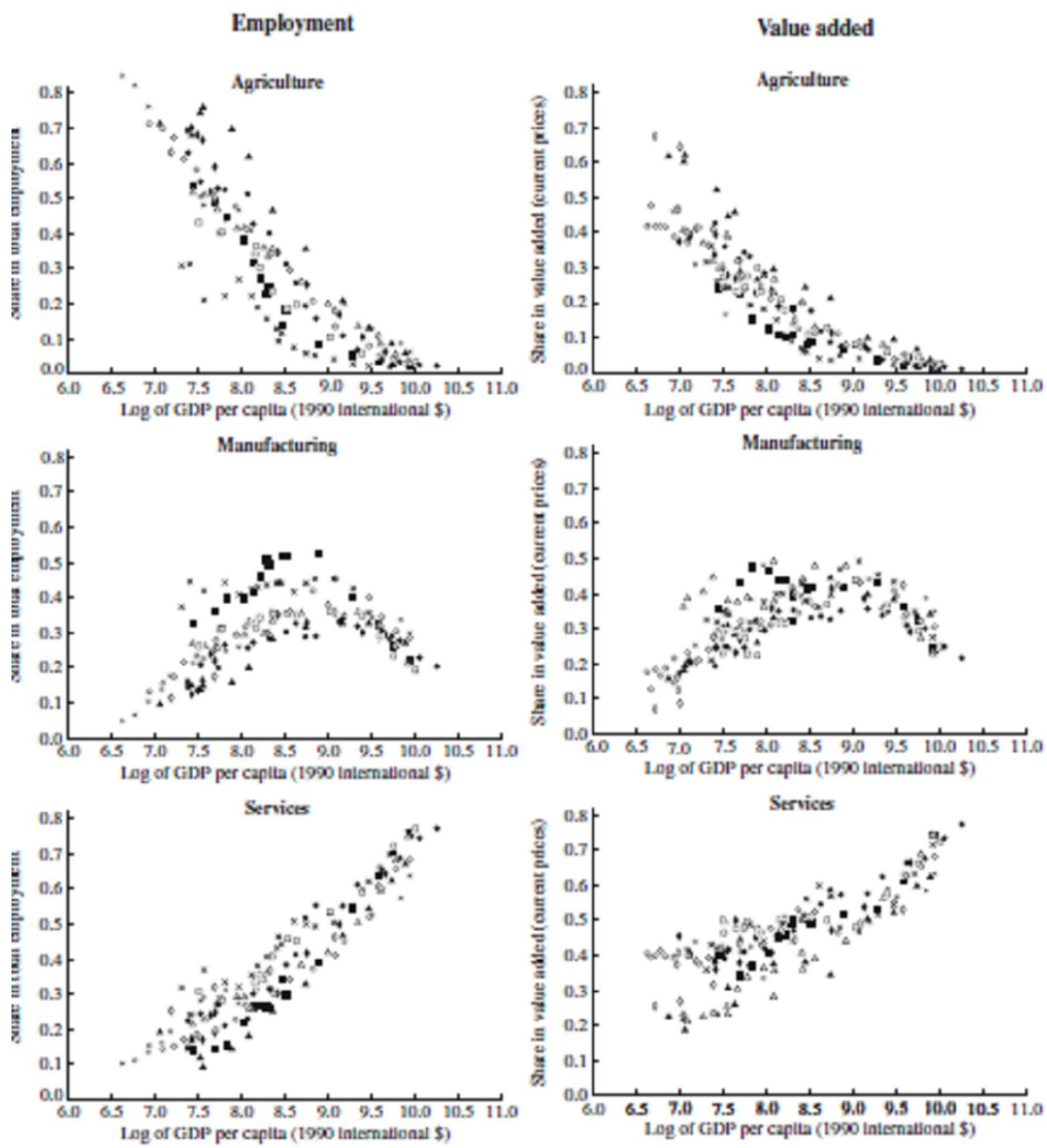
Next, some micro-evidence.

Poorer households spend a larger fraction of their budget on goods than richer households



Agriculture, Manufacturing, Service (from Herrendorf-Rogerson-Valentinyi 2014)

Evidence from Long Time Series for the Currently Rich Countries (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000



Measured by the shares in

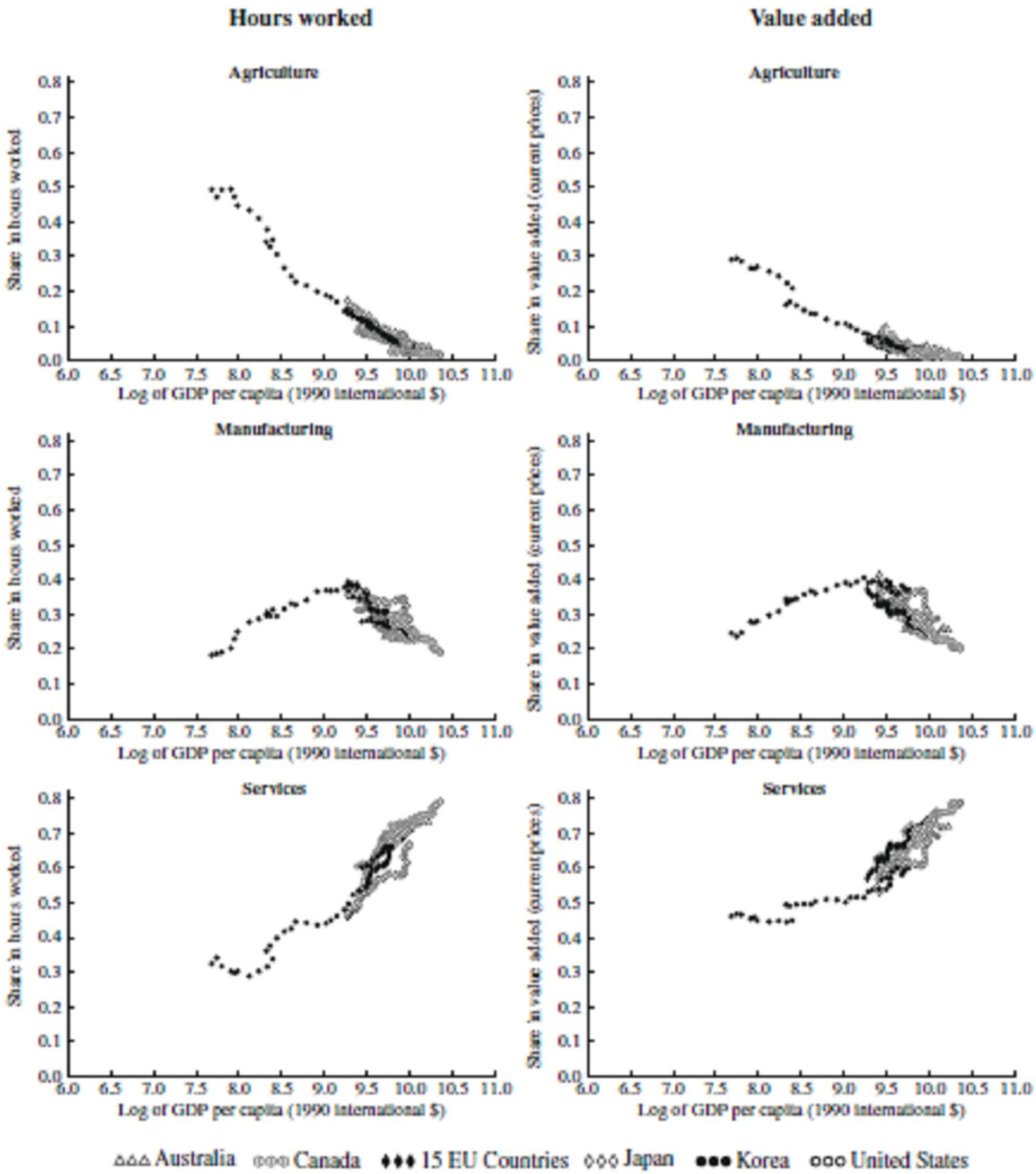
- Employment
- Nominal Value-Added

- Decline of Agriculture
- Rise of Services
- Hump shape of Manufacturing

Evidence from Recent Panels (Australia, Canada, 15 EU-countries, Japan, Korea, and United States) 1970-2007

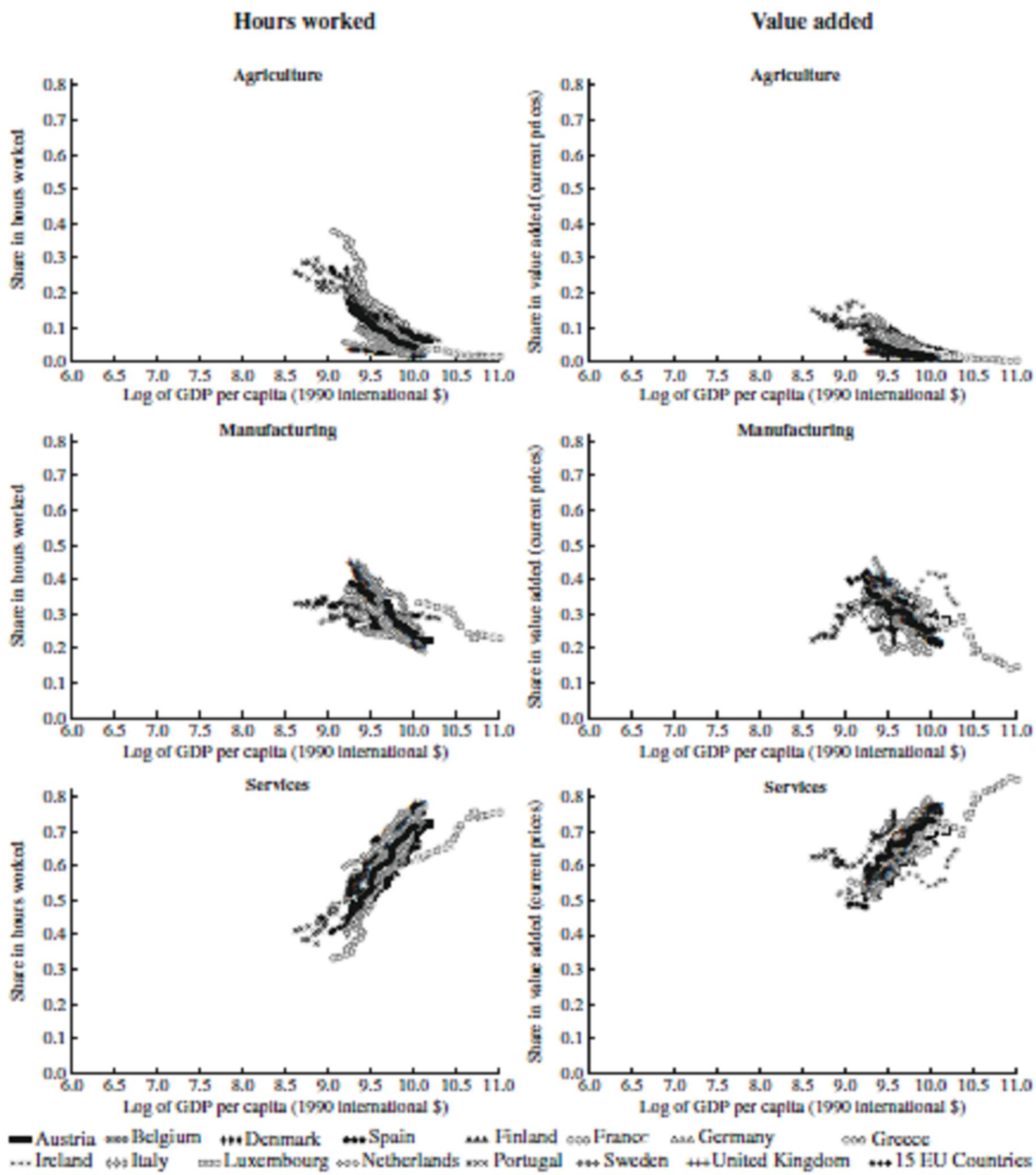
The same patterns, when measured by the shares in
- Hours worked
- Nominal Value-Added

But, manufacturing has been decreasing since 1970 for all countries except Korea.



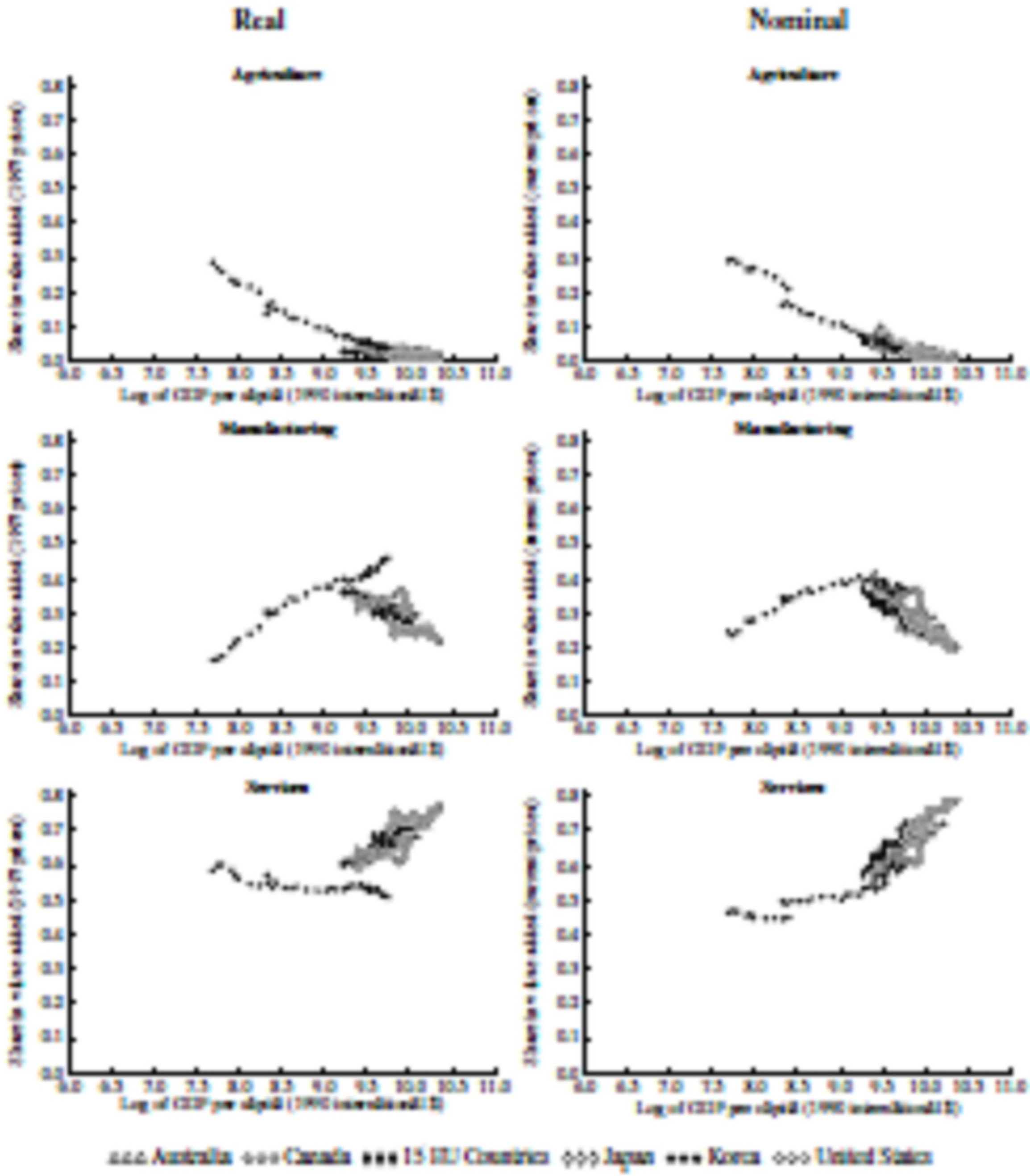
The same evidence for 15 EU-countries separately

Substantial heterogeneity across countries with similar per capita income, suggesting that openness (and specialization) matters



Evidence from Recent Panels (Australia, Canada, 15 EU-countries, Japan, Korea, and United States) 1970-2007

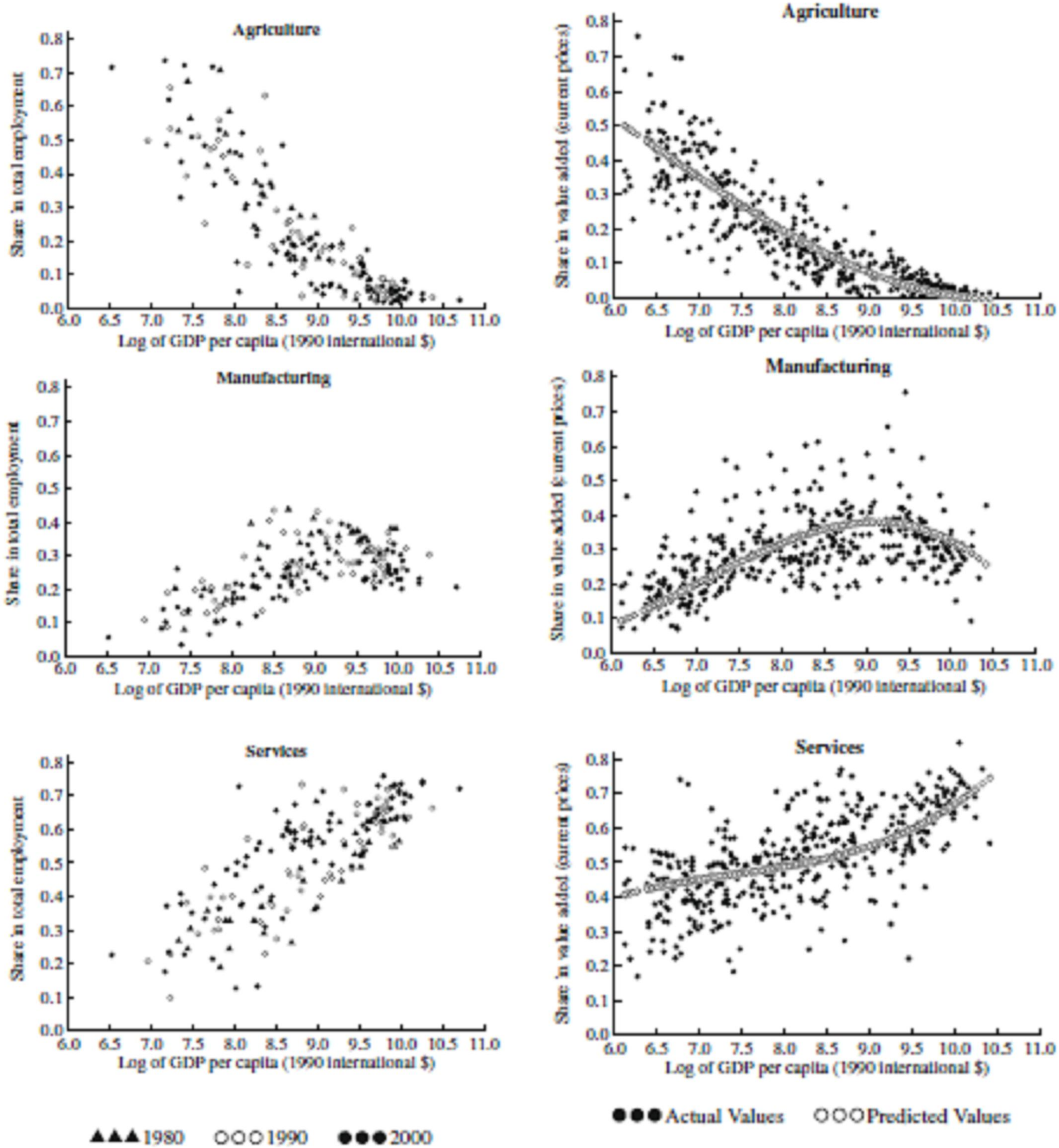
Real and Nominal Value-Added have similar patterns (except Korea), which is inconsistent with homothetic CES, suggesting differential income effects.



Evidence for Both Currently Rich and Poor Countries

Left:
Employment Share (World Development Indicators),
1980, 1990, 2000

Right:
Nominal Value-Added (UN national accounts),
1975-2005
with fitted curves

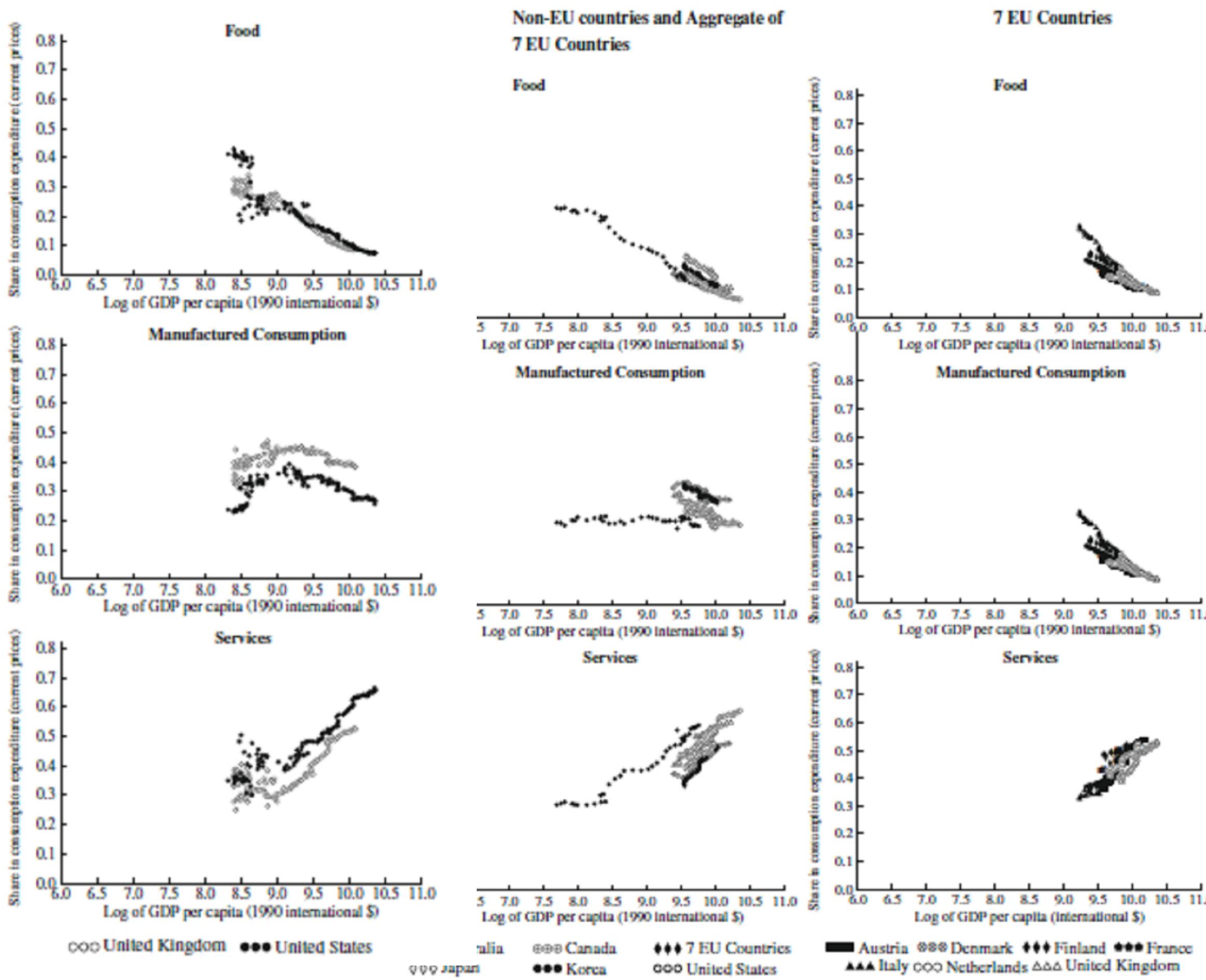


Nominal Consumption

- *Food*
- *Manufactured Consumption (incl retail)*
- *Services*

Left: *UK and US, 1900-2008*

Middle & Right
Australia, Canada, 7 EU (Austria, Denmark, Finland, France, Italy, Netherlands, UK), Japan, Korea, US, 1970-2007

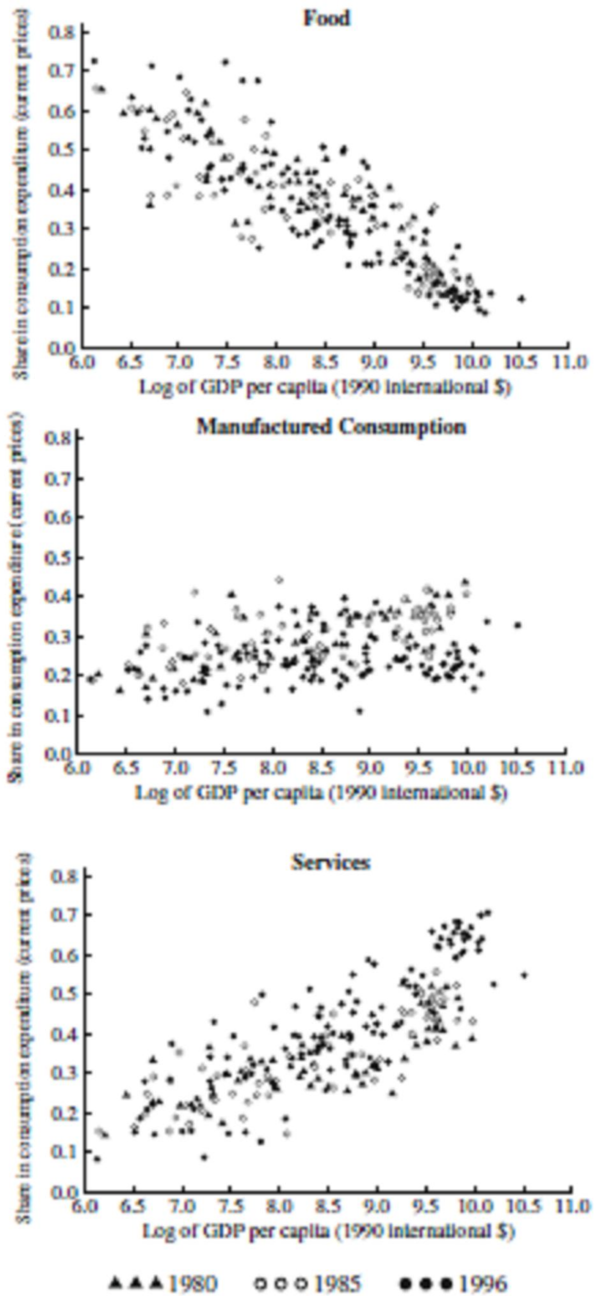


Nominal Consumption for More Countries (from PWT), 1980, 1985, 1996

- *Food,*
- *Manufactured Consumption (incl retail),*
- *Services*

Decline in Food Consumption
Rise in Service Consumption

Manufacturing is less clear.



Differential income elasticities across consumption categories (Aguiar-Bils 2015)

TABLE 2—ENGEL CURVES FROM 1994–1996 EXPENDITURE SURVEY

Good category	CE share 1994–1996	(I) Elasticity	(I) SE	(II) Elasticity	(II) SE
Housing	27.3	0.92	(0.02)	0.93	(0.02)
Food at home	11.7	0.37	(0.02)	0.47	(0.02)
Vehicle purchasing, leasing, insurance	13.2	1.02	(0.08)	0.72	(0.1)
All other transportation	7.4	0.89	(0.03)	0.91	(0.04)
Utilities	5.2	0.47	(0.02)	0.55	(0.02)
Health expenditures including insurance	5.0	0.91	(0.06)	1.11	(0.08)
Appliances, phones, computers with associated services	4.9	0.87	(0.04)	0.94	(0.05)
Food away from home	4.6	1.33	(0.06)	1.32	(0.07)
Entertainment equipment and subscription television	4.1	1.26	(0.07)	1.22	(0.08)
Men's and women's clothing	2.6	1.35	(0.05)	1.38	(0.06)
Entertainment fees, admissions, reading	2.2	1.74	(0.06)	1.65	(0.07)
Cash contributions (not for alimony/support)	2.2	1.81	(0.18)	1.26	(0.12)
Furniture and fixtures	1.5	1.39	(0.1)	1.55	(0.15)
Education	1.3	1.63	(0.18)	1.88	(0.23)
Shoes and other apparel	1.5	1.09	(0.09)	1.19	(0.11)
Domestic services and childcare	1.5	1.60	(0.13)	1.80	(0.13)
Tobacco, other smoking	1.0	−0.26	(0.09)	−0.05	(0.08)
Alcoholic beverages	1.0	1.14	(0.09)	1.14	(0.08)
Children's clothing (up to age 15)	1.0	0.67	(0.07)	0.83	(0.09)
Personal care	1.0	0.96	(0.05)	0.96	(0.05)

Notes: The first column presents each good's average share of total expenditure for 1994–1996. The remaining columns report estimates of each good's expenditure elasticity, with associated standard errors in parentheses. Specification (I) sums each household's expenditure (on each good and in total) over all four interviews and instruments log total expenditure with dummy variables indicating the household's income category as well as the continuous variable of log real after-tax income. Specification (II) splits the four interviews into two subsamples. Each household's expenditure is computed using the sum of the final two interviews. Log total expenditure summed over the first two interviews is used as the instrument. The correlation of the two specifications is 0.93. See text for details of sample construction and regression specification. All specifications include demographic control dummies for age, household size, and number of earners.

TABLE II

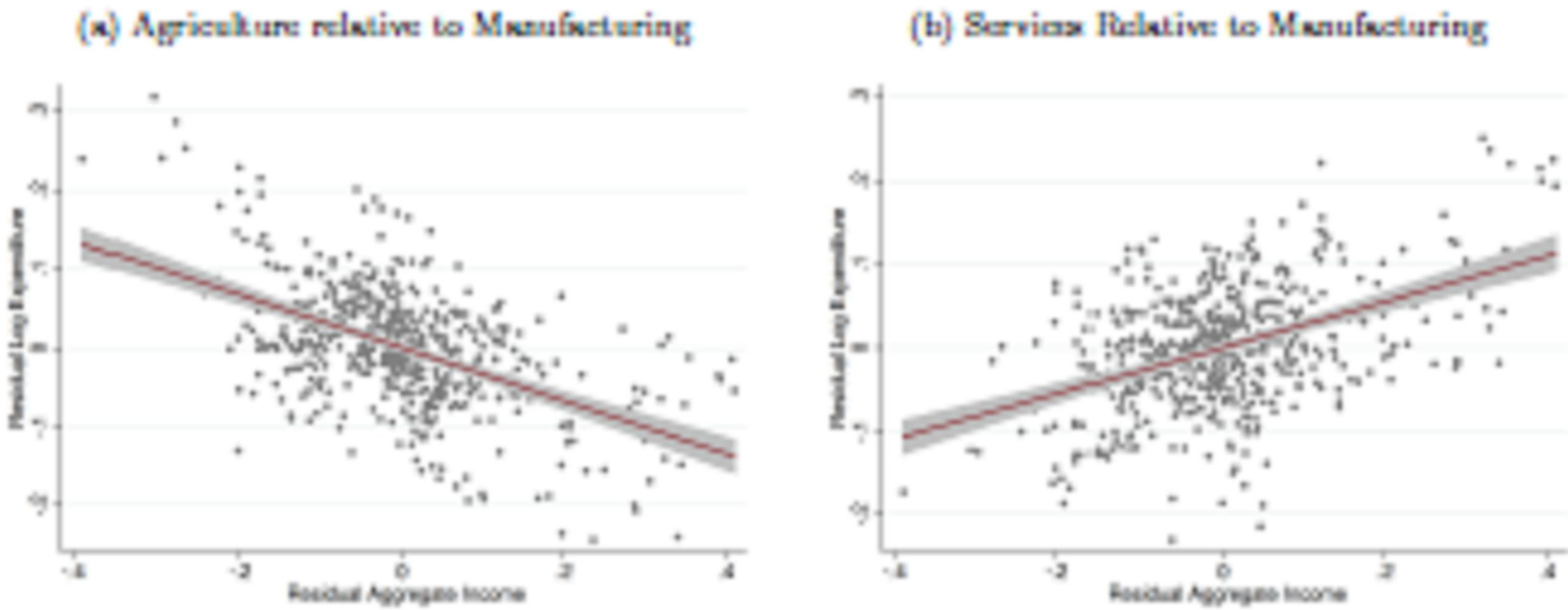
ESTIMATED INCOME ELASTICITY BY SECTOR

GTAP code	Sector name	Income elast.	Std. error	Skill intensity
gro	Cereal grains nec	0.110*	0.133	0.135
pdr	Paddy rice	0.254*	0.199	0.061
pcr	Processed rice	0.352*	0.113	0.130
c_b	Sugar cane, sugar beet	0.433*	0.233	0.091
oap	Animal products nec	0.444*	0.098	0.132
ctl	Bovine cattle, sheep and goats, horses	0.458*	0.137	0.164
vol	Vegetable oils and fats	0.545*	0.063	0.217
sgr	Sugar	0.588*	0.085	0.221
frs	Forestry	0.623*	0.121	0.118
v_f	Vegetables, fruit, nuts	0.640*	0.136	0.095
p_c	Petroleum, coal products	0.664*	0.052	0.313
b_t	Beverages and tobacco products	0.667*	0.079	0.297
tex	Textiles	0.707*	0.064	0.231
ofd	Food products nec	0.777*	0.063	0.268
mil	Dairy products	0.826*	0.077	0.248
ely	Electricity	0.848*	0.073	0.372
nmm	Mineral products nec	0.874	0.097	0.281
crp	Chemical, rubber, plastic products	0.880	0.067	0.356
cns	Construction	0.880	0.061	0.294
wht	Wheat	0.883	0.202	0.117
fsh	Fishing	0.886	0.139	0.124
osd	Oil seeds	0.889	0.194	0.119
ocr	Crops nec	0.893	0.144	0.115
atp	Air transport	0.929	0.070	0.313
wtp	Water transport	0.932	0.100	0.299
ome	Machinery and equipment nec	0.938	0.066	0.372
lum	Wood products	0.970	0.103	0.248
otn	Transport equipment nec	0.981	0.076	0.343
lea	Leather products	0.981	0.066	0.212
otp	Transport nec	0.990	0.074	0.296
fmp	Metal products	0.992	0.077	0.297
cmt	Bovine meat products	1.023	0.078	0.238
osg	Public Administration and services	1.033	0.049	0.503
mvh	Motor vehicles and parts	1.034	0.066	0.341
wtr	Water	1.039	0.087	0.378
ppp	Paper products, publishing	1.044	0.093	0.340
omt	Meat products nec	1.052	0.096	0.233
wap	Wearing apparel	1.057	0.069	0.247
ros	Recreational and other services	1.075	0.067	0.475
ele	Electronic equipment	1.094	0.070	0.358
omf	Manufactures nec	1.095	0.065	0.279
trd	Trade	1.106	0.070	0.308
rmk	Raw milk	1.118	0.145	0.152
cmn	Communication	1.152*	0.078	0.485
obs	Business services nec	1.324*	0.059	0.504
ofi	Financial services nec	1.331*	0.090	0.546
pfb	Plant-based fibers	1.339*	0.193	0.167
isr	Insurance	1.392*	0.104	0.533
wol	Wool, silk-worm cocoons	1.426*	0.177	0.089
gdt	Gas manufacture, distribution	2.221*	0.260	0.362

Differential income elasticities across consumption categories (Caron-Fally-Markusen 2014)

Income elasticity differences are stable across a wide range of per capita income levels (Comin-Lashkari-Mestieri 2015)

Figure 1: Partial Correlations of Sectoral Expenditure and Aggregate Consumption



Notes: Data for OECD countries, 1970-2005. Each point corresponds to a country-year observation after partialling-out sectoral prices and country fixed effects. The red line depicts the OLS fit, the shaded regions, the 95% confidence interval.

The Rise of Service Sector is driven by the growth of skill-intensive services.

The Rise of Skill-Premium is demand driven.
(Buera-Kaboski 2012)

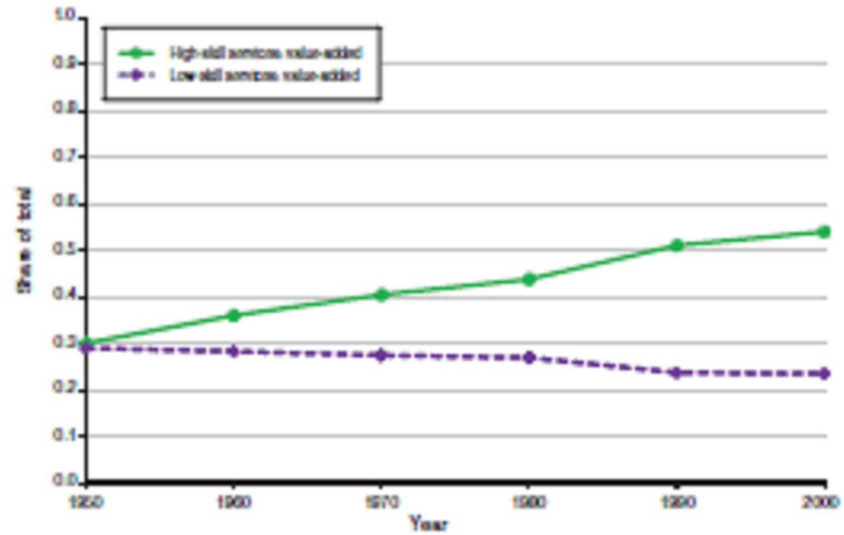


Figure 3. Growth of Low- and High-Skill Service Shares

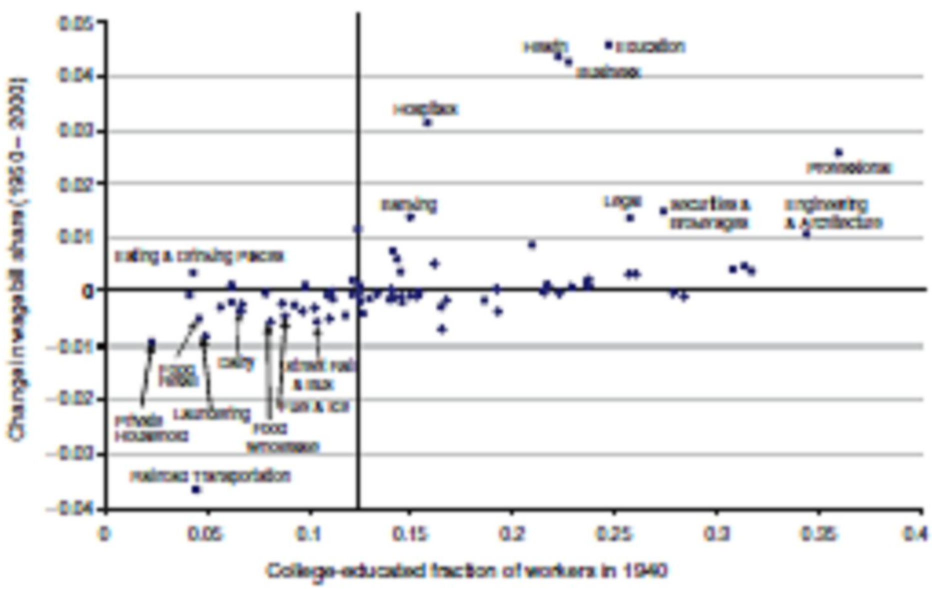


Figure 4. Growth versus Skill Intensity of Disaggregate Service Industries

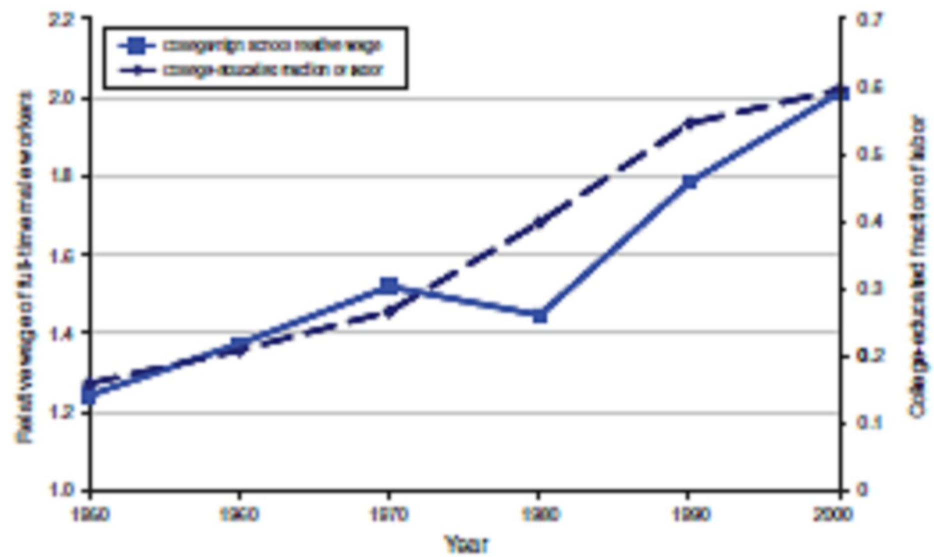


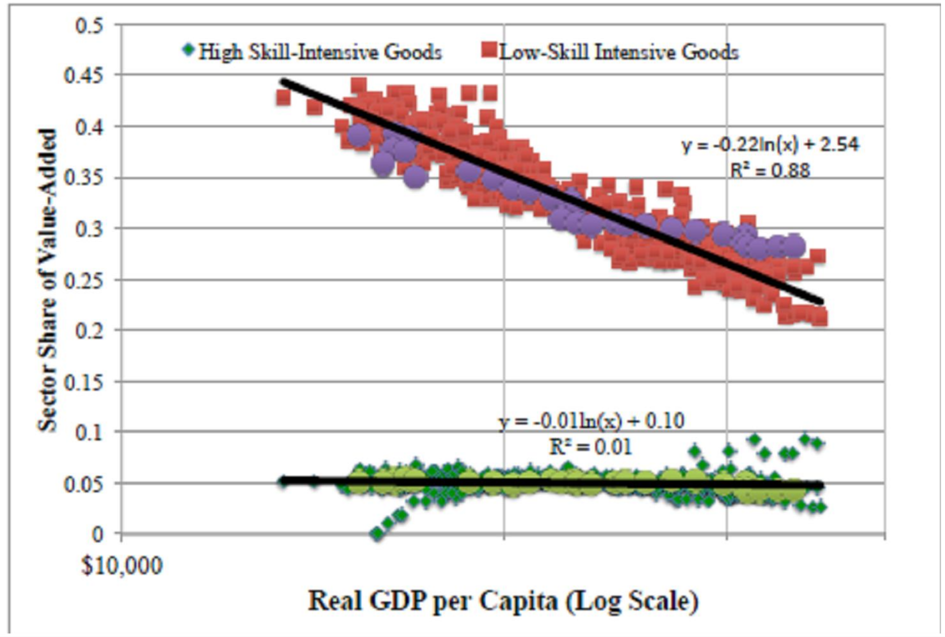
Figure 5. Growth of College Premium and Fraction College-Educated

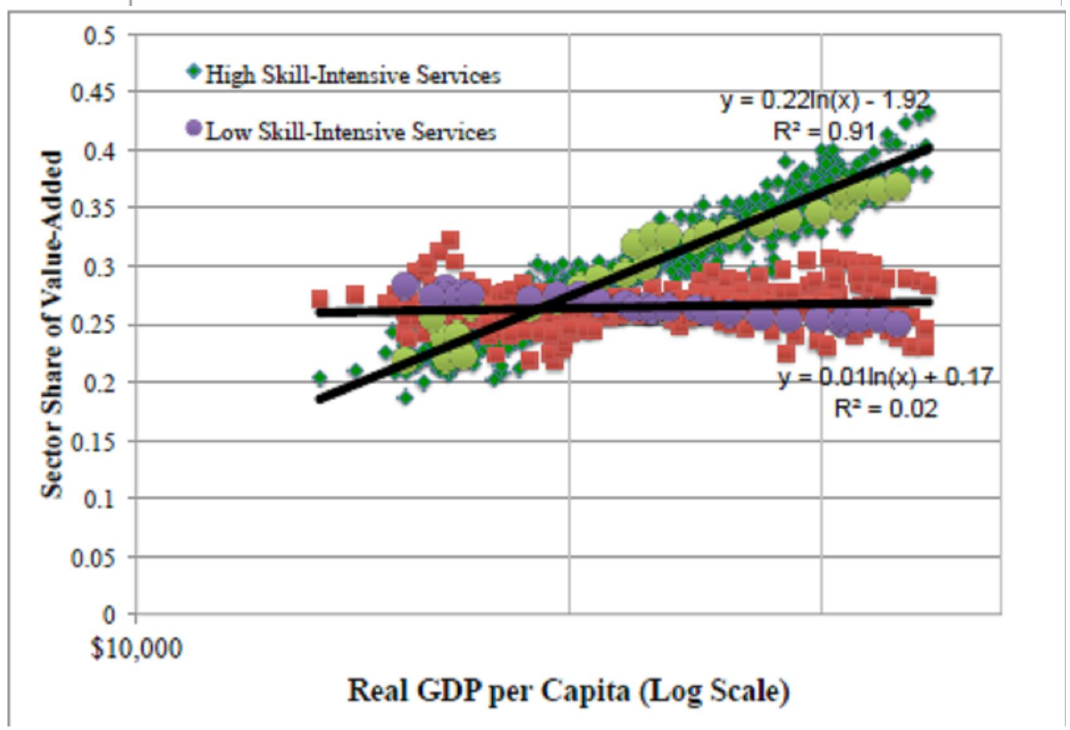
The same is true more broadly (Buera-Kaboski-Rogerson 2015)

12 rich countries (Austria, Denmark, France, Germany, Holland, Italy, Japan, Korea, Spain, UK, US) 1970-2005

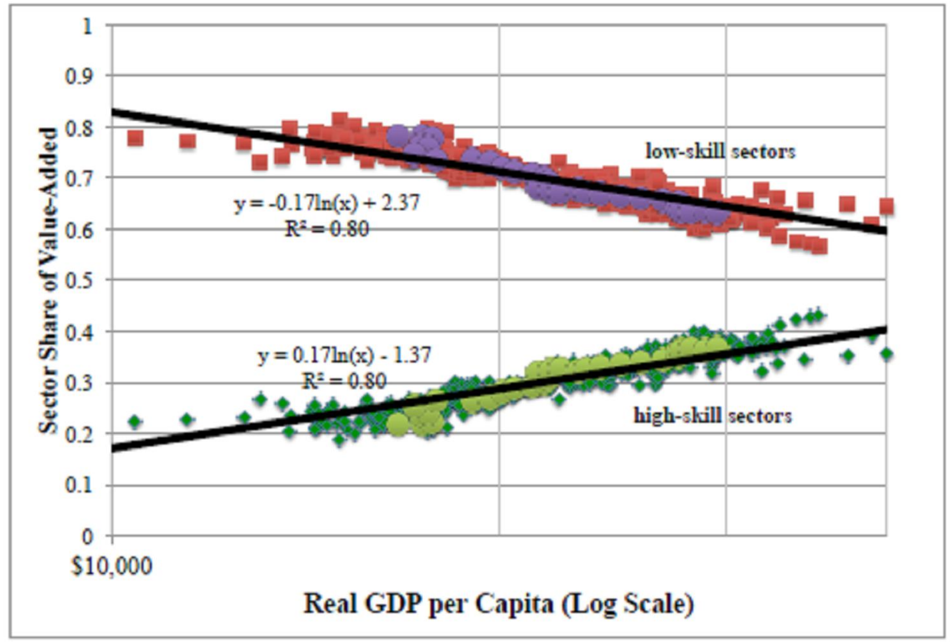
Skill-intensity: share of labor income to high skilled (college graduate and above)

Sector shares of value-added:
Top: Within Manufactures
Bottom: Within Services



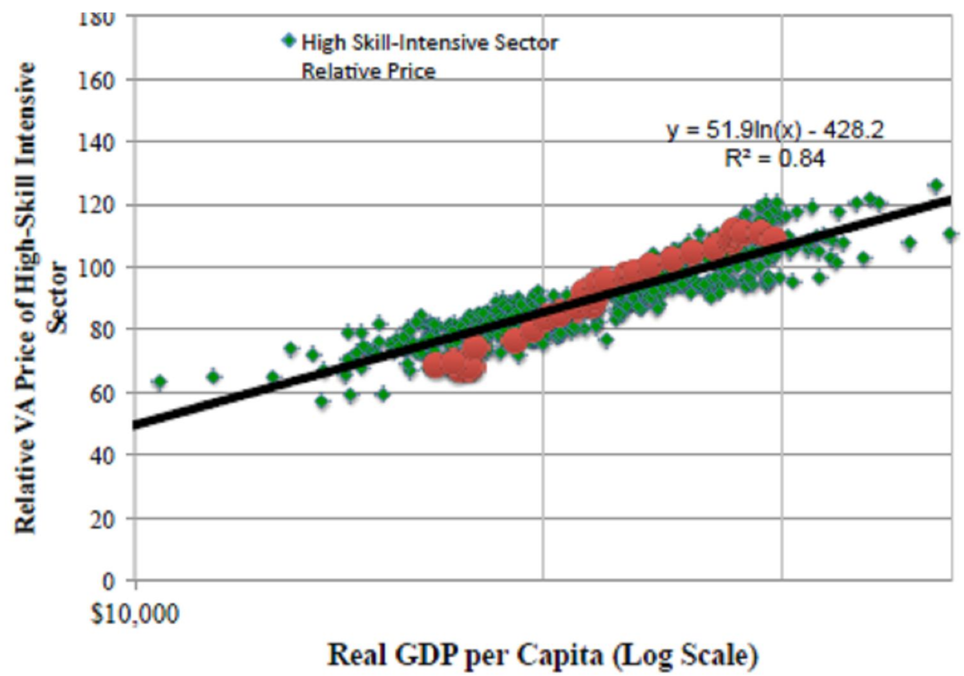


Right: Both Manufactures and Services



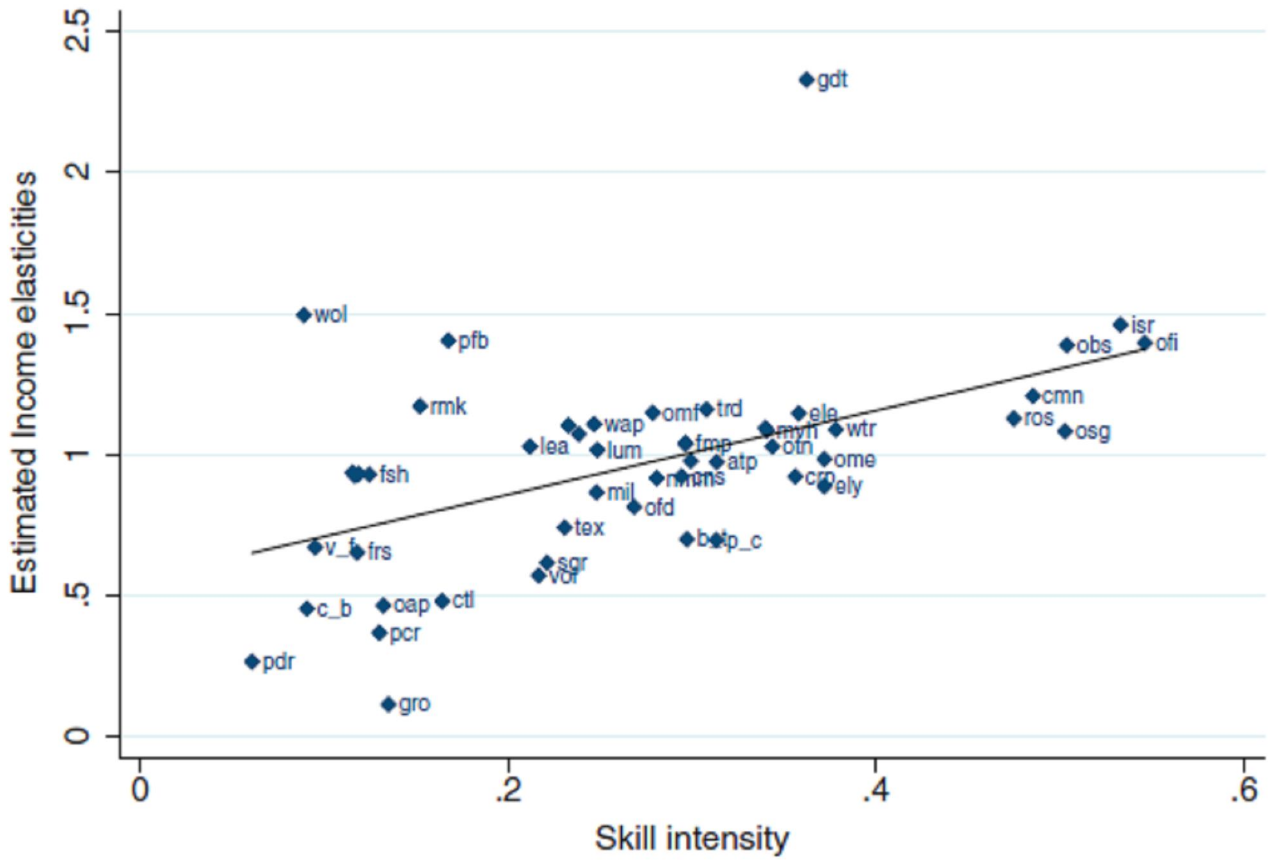
Rise of Skill-Intensive Sectors
Fall of Skill-Unintensive Sectors

in spite of the higher relative price of skill intensive sectors,



suggesting that skill-intensive sectors are more income-elastic.

Income elasticities are positively correlated with skill-intensities (from Caron-Fally-Markusen 2014)



Also, rich countries are net-exporters of *high-income elastic* and *skill-intensive* goods

A Simple Model of Structural Change

***J*-consumption goods sectors.**

Representative Household: $U(x)$, where $x = (x_1, x_2, ..., x_J)$,

Technologies: CRS with $x_j = A_j L_j(t)$ where A_j is the productivity of sector-j.

Resource Constraint: $\sum_{j=1}^{J} L_j(t) = L(t)$, where $L(t)$ is the supply of the unique factor, increasing over time, & $\lim_{t\to\infty} L(t) = \infty$.

Notes:

- Alternatively, $L(t)$ can also be interpreted as Hicks-neutral technical change.
- Relative prices are determined solely by A_j's.
- No means for intertemporal resource allocation, so that the equilibrium is a sequence of the static equilibrium at each t, which changes as $L(t)$ changes.

Example 1A: CES

$$U(x) = \sum_{j=1}^{J} \alpha_j \frac{(x_j)^{1-1/\sigma}}{1-1/\sigma} \text{ for } \sigma > 0, \neq 1; \quad \text{or } U(x) = \sum_{j=1}^{J} \alpha_j \log(x_j),$$

where $\alpha_j > 0$ for all j.

Then,

$$\frac{L_j(t)}{L(t)} = \frac{(\alpha_j)^{\sigma}(A_j)^{\sigma-1}}{\sum_{k=1}^{J}(\alpha_k)^{\sigma}(A_k)^{\sigma-1}} \equiv \beta_j .$$

With **homothetic preferences** and **constant relative prices,** sectoral compositions remain constant as the economy grows.

What if the prices change?

Example 1B: CES with different productivity growth: Baumol (1969), Ngai-Pissaridis (2007). Let $A_j(t) = \exp(g_j t)$. Then,

$$\frac{L_j(t)}{L(t)} = \beta_j(t) \equiv \frac{(\alpha_j)^\sigma (A_j(t))^{\sigma-1}}{\sum_{k=1}^{J} (\alpha_k)^\sigma (A_k(t))^{\sigma-1}} = \left[1 + \sum_{k\neq j} (\alpha_k/\alpha_j)^\sigma e^{(g_k - g_j)(\sigma-1)t}\right]^{-1}.$$

Consider $J = 3$, with $g_1 > g_2 > g_3$ and $\sigma < 1$:

- $\frac{L_1(t)}{L(t)}$ is monotone decreasing and $\lim_{t\to\infty} \frac{L_1(t)}{L(t)} = 0$;
- $\frac{L_3(t)}{L(t)}$ is monotone increasing and $\lim_{t\to\infty} \frac{L_3(t)}{L(t)} = 1$;
- $\lim_{t\to\infty} \frac{L_2(t)}{L(t)} = 0$. $\frac{L_2(t)}{L(t)}$ is hump-shaped (inverted U), if $(\alpha_1)^\sigma (g_1 - g_2) > (\alpha_3)^\sigma (g_2 - g_3)$.

➢ This captures the fall in agriculture, the rise of services, and the rise of fall of industry.

➢ But, with $\sigma < 1$, the real and nominal expenditure shares move in the *opposite direction*, contrary to the empirical evidence.

Explicit Additivity, Homotheticity, and CES

Under explicit additive separability, preferences are homothetic iff CES.

Bergson's Law: Suppose that the utility function, $U : R_+^J \to R$, is quasi-concave, increasing, and explicitly additively separable, $U(x) = \sum_{j=1}^{J} u_j(x_j)$. Then, it is homothetic if & only if

$$u_j(x) = \alpha_j \frac{(x_j)^{1-\theta}}{1-\theta} + \beta_j \text{ for } \theta > 0, \neq 1; \text{ or } u_j(x) = \alpha_j \log(x_j) + \beta_j, \text{ where } \alpha_j > 0 \text{ for all } j.$$

Notes: Explicit Additivity is essential. First, easy to construct homothetic preferences that are not explicitly additively separable. To see this, let $f_k : R_+^J \to R_+$ ($k = 1, 2, \ldots, K$) is linear homogeneous and $g : R_+^K \to R$ is homothetic. Then, $U : R_+^J \to R$ defined by:

$$U(x) = g\left(f_1(x), f_2(x), \ldots, f_K(x)\right),$$

is also homothetic. But, it is not generally explicitly additively separable.

Ex.1: $U(x) = A(x_1)^{\alpha}(x_2)^{1-\alpha} + Bx_1$; **Ex.2:** $U(x) = \alpha \log(x_1 + x_2) + \beta \log(x_2)$, and so on.

Second, a utility function $U(x)$ is said to be **implicitly additively separable** when it can be defined implicitly by $\sum_{j=1}^{J} f_j(x_j;U)=1$. Within this class, it is easy to see that

- $\sum_{j=1}^{J} f_j(x_j/U)=1$ is homothetic but generally not CES.
- $\sum_{j=1}^{J} \alpha_j(U)(x_j)^{1-\theta}=1$ is CES but generally not homothetic.

(Additional conditions are required for this function to be increasing and quasi-concave.)

Third, explicit additivity has the following (undesirable) implications.

Pigou's Law: $\dfrac{\text{Income Elasticity of Good s}}{\text{Price Elasticity of Good s}}$ = constant

(Bergson's Law is a special case.)

In spite of all these, most existing models of nonhomothetic preferences use explicitly additively separable ones, particularly Stone-Geary preferences.

Stone-Geary Preferences:

$$U(x)=\sum_{j=1}^{J}\alpha_j \frac{(x_j-\gamma_j)^{1-1/\sigma}}{1-1/\sigma} \text{ for } \sigma>0, \neq 1; \quad \text{or} \qquad U(x)=\sum_{j=1}^{J}\alpha_j \log(x_j-\gamma_j),$$

where $\alpha_j > 0$ for all *j*.

The household demand, under the budget constraint, $\sum_{j=1}^{J} p_j x_j^h \leq I^h$, takes form of:

$$p_j x_j^h = \Gamma_j(p) + B_j(p) I^h \text{ for each } j.$$

Notes:

- For $\gamma_j > 0$, it is interpreted as the subsistent level of consumption. For $\gamma_j < 0$, $-\gamma_j > 0$ is interpreted as the endowment of good j.
- With $\Gamma_j(p) \neq 0$, the *average* propensity to consume, $p_j x_j^h / I^h$, monotonically decreasing (a necessity) or, monotonically increasing (a luxury) in I^h. **(i.e., non-homothetic).**
- But, the *marginal* propensity to consume, $\partial p_j x_j^h / \partial I^h = B_j(p)$, is independent of I^h, which allow for aggregation across households. Thus, we can talk about the representative household or the Household Sector, even if they may differ in their income and expenditure.

When applied to our simple model of structural change,

$$\frac{L_j(t)}{L(t)} = \beta_j \left(1 - \frac{\sum_{k=1}^{J} \left(\gamma_k / A_k \right)}{L(t)} \right) + \frac{\gamma_j}{A_j L(t)}, \quad \text{where } \beta_j \equiv \frac{\left(\alpha_j \right)^{\sigma} \left(A_j \right)^{\sigma-1}}{\sum_{k=1}^{J} (\alpha_k)^{\sigma} (A_k)^{\sigma-1}}.$$

- $\frac{L_j(t)}{L(t)}$ is monotonically increasing (decreasing) if $(1-\beta_j)\frac{\gamma_j}{A_j} < (>) \beta_j \sum_{k \neq j}^{J} \left(\frac{\gamma_k}{A_k} \right)$;
- $\frac{L_j(t)}{L(t)}$ is constant, iff $(1-\beta_j)\frac{\gamma_j}{A_j} = \beta_j \sum_{k \neq j}^{J} \left(\frac{\gamma_k}{A_k} \right)$;

and

- $\lim_{t \to \infty} \frac{L_j(t)}{L(t)} = \beta_j$.

Example 2A: $J = 2$; $\gamma_1 > 0$ & $\gamma_2 = 0$.

- $\dfrac{L_1(t)}{L(t)} = \beta_1 + \beta_2 \dfrac{\gamma_1}{A_1 L(t)}$, decreasing over time with $\lim_{t\to\infty} \dfrac{L_1(t)}{L(t)} = \beta_1$;
- $\dfrac{L_2(t)}{L(t)} = \beta_2 \left(1 - \dfrac{\gamma_1}{A_1 L(t)}\right)$, increasing over time with $\lim_{t\to\infty} \dfrac{L_2(t)}{L(t)} = \beta_2$.

Interpretation: Sector-1 is the food sector; and Sector-2 is everything else; or Sector-2 is service and Sector-1 is everything else.

Example 2B: $J = 3$; $\gamma_1 > 0$, $\gamma_2 = 0$, & $\gamma_3 < 0$. Then,

- $\dfrac{\dot{L}_1(t)}{L(t)} < 0$, as $(1-\beta_1)\dfrac{\gamma_1}{A_1} > 0 > \beta_1 \dfrac{\gamma_3}{A_3}$;
- $\dfrac{\dot{L}_3(t)}{L(t)} > 0$, as $(1-\beta_3)\dfrac{\gamma_3}{A_3} < 0 < \beta_3 \dfrac{\gamma_1}{A_1}$;
- $\dfrac{\dot{L}_2(t)}{L(t)} > 0$ if $\dfrac{\gamma_1}{A_1} + \dfrac{\gamma_3}{A_3} > 0$; $\dfrac{\dot{L}_2(t)}{L(t)} < 0$ if $\dfrac{\gamma_1}{A_1} + \dfrac{\gamma_3}{A_3} < 0$; $\dfrac{\dot{L}_2(t)}{L(t)} = 0$ if $\dfrac{\gamma_1}{A_1} + \dfrac{\gamma_3}{A_3} = 0$.

Interpretation: Sector-1, agriculture; Sector-2 manufacturing; Sector-3 services.

Notes:

- Kongsamut, Rebelo, Xie (2001) embedded this type of preferences into a standard growth model to reconcile the Fisher-Clark-Kuznets thesis with the Kaldor's balance growth view. However, Example 2B suggests that this was a futile attempt.
  - We can never have the rise and fall of manufacturing (the inverted U-patterns), because its share has to be rising, declining, or constant.
  - The share of every sector will eventually converge to a constant.
- One could obviously try to combine differential productivity growth with Stone-Geary.

However, Stone-Geary have several fundamental flaws.

- *Asymptotically homothetic*, suggesting that non-homotheticity is important only for poor households and poor countries. This feature
  - contradicts with stable slopes of Engel's curves (e.g., Comin-Lashkari-Mestieri 2015)
  - makes it difficult to fit the long-run data (e.g.,Buera-Kaboski 2009)
- Due to its explicit additivity, it is subject to Pigou's Law.
- The key parameters, γ_j, are given by a quantity of good j, hence not unit-free. Indeed, one could choose a unit of each good such that γ_j = 1, 0, or –1, w.l.o.g. In other words, it cannot meaningfully distinguish more than three sectors in terms of their income elasticities.

Some Alternative Nonhomothetic Preferences:

ConstantRatio of IncomeElasticity (CRIE): Fieler(2011),Caron-Fally-Markusen(2014)

$$U(x) = \sum_{j=1}^{J} \alpha_j \frac{(x_j)^{1-1/\sigma_j}}{1-1/\sigma_j} \text{ for } \sigma_j \neq 1$$

- Allow for nonhomothecity that are independent of income levels.
- Can accommodate any number of sectors.
- But, due to its explicit additivity, it is subject to Pigou's law; $\varepsilon_j / \varepsilon_k = \sigma_j / \sigma_k$.

Price Independent Generalized Linearity (PIGL) Preferences: Boppart (2014).
Its indirect utility function is given by:

$$V(p_1, p_2, E) = \frac{1}{\varepsilon}\left\{\left[\frac{E}{p_1}\right]^{\varepsilon} - 1\right\} - \frac{\nu}{\gamma}\left\{\left[\frac{p_2}{p_1}\right]^{\gamma} - 1\right\}$$

- Allow for nonhomotheticty that do not disappear asymptotically.
- But, it can accommodate only two sectors
- Though not subject to Pigou's law, it still imposes functional relations between the income elasticity and price elasticity.

Hierarchical Preferences: Matsuyama (2000, 2002)**,** Foellmi-Zweimueller (2008)**,** Buera and Kaboski (2012)

$$U(x) = \sum_{j=1}^{\infty} \alpha_j \min\{x_j, \bar{x}_j\}, \quad \text{for the case of a finite number of goods}$$

or $$U(x) = \int_{j=0}^{\infty} \alpha_j \min\{x_j, \bar{x}_j\} dj, \quad \text{for the case of a continuum of goods}$$

where $\bar{x}_j$ is the saturation level of good j.

If α_j declines sufficiently rapidly so that α_j / p_j is monotone decreasing,

- The households buy only $j \in [0, J]$ up to their saturation levels, where $\int_{j=0}^{J} p_j \bar{x}_j dj = E$.
- As E rises, they expand the range of goods purchased.
- Each good is a luxury for poor households, and a necessity for rich households.
- If the goods are classified into the three sectors such that the density of agriculture goods, manufacturing goods, and services, are monotone decreasing, single-peak, and monotone increasing, this generates the observed patterns of structural change.

Implicitly Additive Nonhomothetic CES: Hanoch (1975)

$$\sum_{j=1}^{J} \alpha_j(U)(x_j)^{1-1/\sigma} = 1 ,$$

where $\alpha_j(U)$, a function of *U*, is a weight on sector j

Isoelastic Case: Comin, Lashkari, and Mestieri (2015); Matsuyama (2015)

$$\sum_{j=1}^{J} (\beta_j)^{\frac{1}{\sigma}} (U)^{\frac{\varepsilon_j - \sigma}{\sigma}} (x_j)^{\frac{\sigma-1}{\sigma}} = 1, \qquad (\beta_j > 0,\ \sigma \neq 1),$$

where global monotonicity and quasi-concavity requires $(\varepsilon_j - \sigma)/(1-\sigma) > 0$.

By solving Max *U*, subject to $\sum_{j=1}^{J} (\beta_j)^{\frac{1}{\sigma}} (U)^{\frac{\varepsilon_j - \sigma}{\sigma}} (x_j)^{\frac{\sigma-1}{\sigma}} = 1$ and $\sum_{j=1}^{J} p_j x_j = E$,

$$\frac{p_j x_j}{E} = \frac{\beta_j (U)^{\varepsilon_j - \sigma} (p_j)^{1-\sigma}}{\sum_{j=1}^{J} \beta_k (U)^{\varepsilon_k - \sigma} (p_k)^{1-\sigma}},$$

where *U* is given implicitly by $(E)^{1-\sigma} \equiv \sum_{k=1}^{J} \beta_k (U)^{\varepsilon_k - \sigma} (p_k)^{1-\sigma}$.

Applying this to our simple model of structural change with constant productivity,

$$\frac{L_j(t)}{L(t)} = \frac{\beta_j (U(t))^{\varepsilon_j - \sigma} (A_j)^{\sigma-1}}{\sum_{k=1}^{J} \beta_k (U(t))^{\varepsilon_k - \sigma} (A_k)^{\sigma-1}}, \text{ with } \left(L(t)\right)^{1-\sigma} \equiv \sum_{k=1}^{J} \beta_k (U(t))^{\varepsilon_k - \sigma} (A_k)^{\sigma-1}.$$

- $U(t)$ is monotone increasing in $L(t)$ under the global monotonicity condition, $(\varepsilon_j - \sigma)/(1-\sigma) > 0$.

- $\dfrac{L_j(t)/L(t)}{L_k(t)/L(t)} = \left(\dfrac{\beta_j}{\beta_k}\right)\left(\dfrac{A_j}{A_k}\right)^{\sigma-1} \left(U(t)\right)^{\varepsilon_j - \varepsilon_k}$ is decreasing over time if $\varepsilon_j < \varepsilon_k$.

- For the three sectors with $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$, $\dfrac{L_1(t)}{L_2(t)}$ & $\dfrac{L_2(t)}{L_3(t)}$ are decreasing. Furthermore, $\dfrac{L_1(t)}{L(t)}$ decreasing; $\dfrac{L_2(t)}{L(t)}$ hump-shaped; and $\dfrac{L_3(t)}{L(t)}$ increasing.
- One could also accommodate differential productivity growth. See Comin-Lashkari-Mestieri (2015).

An Application to International Trade: Matsuyama (2015); "The Home Market Effect and Patterns of Trade Between Rich and Poor Countries"

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