Economic Growth & Development: Part 1
Endogenous Technological Change in Static GE Models with Monopoly

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Understanding Technological Change

We analyze R&D activities (innovation, technological adoption, product development, etc.) as something that are driven by profit-motives.

R&D create know-how, technologies, and more generally knowledge, using labor, equipments, and many other inputs. Why can't we treat this as simple input-output processes? Or why can't we view them as human capital accumulation?

Special Features of knowledge creation:

Non-rivalry of knowledge: Large fixed cost of creation, small (often negligible) replication cost; Increasing returns; Market-size effect: direction of technological change
Non-excludability of knowledge; the problems with intellectual property right protection (patents and copyright, trade secrets, etc.)
Appropriate effect: Rewards earned by innovators generally are much less than social value of R&D. This suggests not enough R&D activities
Creative destruction; business stealing effect; new technologies often make existing technologies obsolete. This suggests too much R&D.

Need to introduce monopoly powers in general equilibrium to model the process technological change driven by profit-motives.

They would help us to understand

- How differences in market structure and public policies affect productivity growth
- How equilibrium growth differ from "optimal" growth; in what sense, the market may fail to provide the right incentive for technological change?

We now look at two examples from Matsuyama (JEL 1995):

- A Model of Technology Adoption (Section 2)
- A Model of Product Development (Section 3)

Both feature simple static general equilibrium models with some monopoly power. Monopoly profits generate an incentive for technological change. At the same time, they show how the market could fail, demonstrating the possibility of **Poverty Traps** and **Uneven Development** due to strategic complementarities in technology adoption and product development.

A Model of Technology Adoption; Matsuyama (JEL 1995; Sec.2); adopted from "Industrialization and the Big Push," Murphy-Shleifer-Vishny (JPE 1989)

Overview: In this model, the *monopoly profit* offers an incentive to adopt new technologies in each industry. There is also *demand linkage* across industries. Through such *aggregate demand spillovers*, an incentive to adopt new technologies in a particular industry is larger when new technologies are adopted in other industries. In other words, technology adoptions across different sectors are *strategic complements*. This could create multiple equilibriums. The Pareto-dominated equilibrium may be viewed as an underdevelopment trap caused by *coordination failure.*

Labor: Only Factor of Production and the Numeraire

A Continuum of Goods (and Industries), indexed by $z \in [0,1]$

Representative Consumer: endowed with L units of labor and own all firms, receiving the aggregate profit, Π, and pay taxes, T.

$$MaxU = \alpha\int_0^1 \ln(c(z))dz + (1-\alpha)\ln(N) \quad \text{s.t.} \quad \int_0^1 p(z)c(z)dz + N \leq Y - T\,;$$

➔ $c(z) = \dfrac{\alpha(Y-T)}{p(z)} = \dfrac{\alpha(L+\Pi-T)}{p(z)}$.

Government: spend G on each good and hire labor by N', financed by $T = G + N'$

➔ $g(z) = \dfrac{G}{p(z)}$.

Industries: Each industry has **Two** technologies.

- **Cottage industry technique**; one unit of labor produces one unit of its output, available to anyone;
- (potential) monopolist, which has sole access to **Modern Technology**; which saves μ unit of labor per output (0 < μ < 1). However, it requires purchasing units of goods from every industry to set up as follows: $\exp\left[\int_0^1 \ln(f(z))dz\right] \geq F$

Cost Minimization: $Min\int_0^1 p(z)f(z)dz \quad \text{s.t.} \int_0^1 \ln(f(z))dz \geq \ln F$

➔ $f(z) = \dfrac{F}{p(z)}$.

Aggregate Demand Curve for good z: $q(z) = c(z) + sf(z) + g(z) = \dfrac{\alpha(Y-T) + sF + G}{p(z)}$

s: fraction of industries, where monopolists adopt the modern technology.

Equilibrium Goods Price; $p(z) = 1$ for all $z \in [0,1]$

Why?

- If a monopolist in Industry z chooses not to adopt the modern technology, the good is supplied competitively by cottage industry technique, so that $p(z) = 1$
- If a monopolist in Industry z chooses to adopt the modern technology, it has an incentive to raise the price, as long as $p(z) < 1$, but at $p(z) = 1$. Limit pricing as the cottage industry act as a competitive fringe.

Aggregate Demand for good z: $q(z) = \alpha(Y-T) + sF + G \equiv q$ for all z

Monopoly Profit, or Incentive to Adopt the Modern Technology

Each investing monopolist set $f(z) = F$ and hence, the fixed cost is $\int_0^1 p(z) f(z) dz = F$

Thus, the monopoly profit is

$$\pi = \mu q - F \;= \mu[\alpha(Y - T) + sF + G] - F \;= \mu[\alpha(L + s\pi - T) + G] - (1 - \mu s)F$$

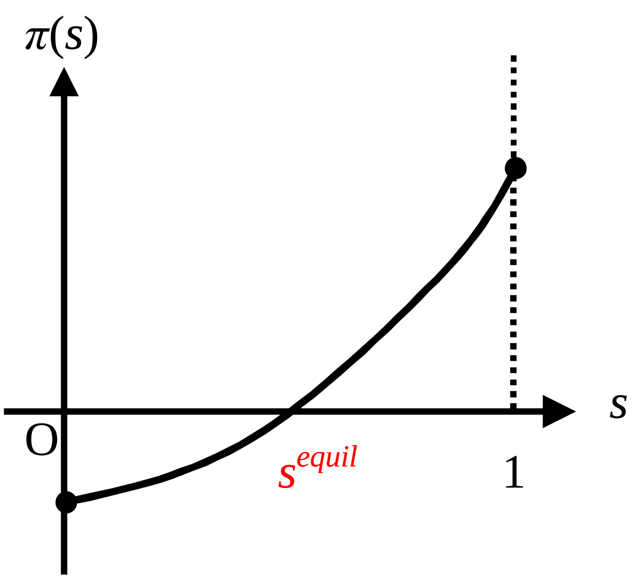
or

$$\pi(s) = \frac{\mu[\alpha(L - T) + G] - (1 - \mu s)F}{1 - \alpha\mu s}$$

Strategic Complementarities in Technology Adoption: $\pi(s)$ is increasing in s. Each industry is more inclined to adopt the modern technology if more industries invest in it.

Figure shows $\pi(s)$ for $\frac{1-\mu}{\mu} < \frac{\alpha(L-T)+G}{F} < \frac{1}{\mu}$.

- 1^{st} inequality → $\pi(1) > 0$; $m = 1$ is an equilibrium.
- 2^{nd} inequality → $\pi(0) < 0$; $m = 0$ is an equilibrium.



Exercises:

- Compare welfares for $m = 0$ and $m = 1$.
- What are the effects of the government policies?

Modeling Product Development in Dixit-Stiglitz Monopolistic Competition; taken from Matsuyama (JEL 1995; Section 3 & 4); see also Acemoglu (Ch.12.4);

For a theory of product development, we need a model in which a set of products offered in the market is endogenously determined. Monopolistic competition offers such a framework.

Household Sector: endowed with L units of labor, only (primary) factor of production; numeraire

Final Good Sector: Competitive with CRS as $Y = C = F(X, H)$

H; direct labor input

$X = \left[\int_0^n [x(z)]^{1-1/\sigma} dz\right]^{\frac{\sigma}{\sigma-1}}$ ($\sigma > 1$); the composite of specialized intermediate inputs (machines, services, etc.); these inputs are in turn produced by labor. In this sense, they are "intermediate" inputs.

n: available variety in the marketplace (determined in equilibrium and taken as given by the final good sector)

They use the inputs to minimize $H + \int_0^n p(z)x(z)dz$ s.t. $F(X,H) \geq Y$

First step: $P \equiv Min\left\{ \int_0^n p(z)x(z)dz \middle| X \geq 1 \right\}$

Second step: $H + PX$ s.t. $F(X,H) \geq Y$

→ $x(z) = \left[\frac{p(z)}{P} \right]^{-\sigma} X$, where $P \equiv \left[\int_0^n [p(z)]^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$

$$P = \frac{F_x(X,H)}{F_H(X,H)} = \frac{F_x(X/H,1)}{F_H(X/H,1)} \Leftrightarrow \frac{H}{X} = \Phi(P), \text{ where } \Phi'(P) > 0$$

Notes:

$P \equiv \left[\int_0^n [p(z)]^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}$ is the price index for the composite of specialized intermediate inputs. If $p(z) = p$ for all z, $P = p(n)^{\frac{1}{1-\sigma}}$, which is decreasing in n. Productivity gains from variety. (Romer (1990) called it "gains from specialization.")

Intermediate Goods Sector; monopolistically competitive;

- Each firm is a sole supplier of its unique variety. (In this model, new goods are offered only as new firms enter the market.)
- IRS Technology: to supply x(z) units requires $f + ax$(z) units of labor

a: Variable (marginal) cost
f: Fixed cost (Start-up costs, development cost, etc.)

Monopoly-pricing: As the demand for each z can be written as $x(z) = \Omega\left[p(z)\right]^{-\sigma}$, each supplier set its price such that

$$p(z)\left(1 - \frac{1}{\sigma}\right) = a \qquad \text{or} \qquad p(z) = p \equiv \frac{\sigma a}{\sigma - 1}$$

Monopoly Profit: $\pi = px - ax - f = \dfrac{px}{\sigma} - f$

Labor Market Clearing: $L = H + \int_0^n \left[f + ax(z)\right] dz = H + nf + nax.$

Entry (Product Development) Game:

Calculate $\pi(n)$, the monopoly profit earned by each firm for a fixed number of active firms, n. This is an incentive to enter the market (and offer a new product).

$$L = H + nf + nax = X\Phi(P) + nf + nax = \Phi(P)(n)^{\frac{\sigma}{\sigma-1}}x + nf + nax$$

$$\Rightarrow \quad \frac{L}{n} - f = x\left[\Phi(P)(n)^{\frac{1}{\sigma-1}} + a\right] = px\left[\Phi(P)/P + 1 - 1/\sigma\right]$$

$$\Rightarrow \quad \pi(n) = \frac{px}{\sigma} - f = \frac{L/n - f}{\sigma\Phi(P)/P + \sigma - 1} - f \text{, where } P = p(n)^{\frac{1}{1-\sigma}}.$$

If $\pi(n) > 0$, more firms want to enter. If $\pi(n) < 0$, firms want to exit.

In equilibrium, $\pi(n) \le 0$; $n \ge 0$, & $n\pi(n) = 0$.

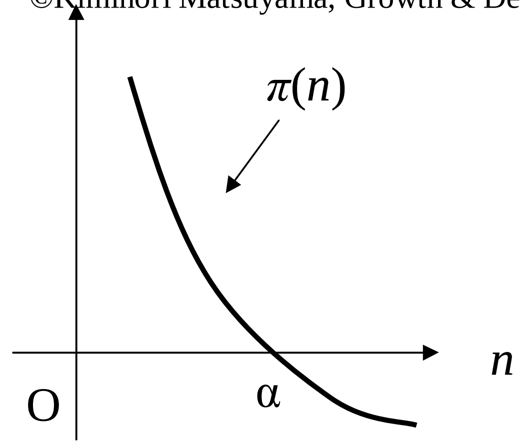
Is $\pi(n)$ increasing or decreasing in n?

Case I: $F(X,H) = AX^{\alpha}H^{1-\alpha}$

$$\Phi(P) = \frac{1-\alpha}{\alpha}P$$

$$\rightarrow \pi(n) = \frac{1}{\sigma-\alpha}\left[\frac{\alpha L}{n} - \sigma f\right]$$

$$\rightarrow n^e = \frac{\alpha L}{\sigma f}.$$



Notes:

- $\pi(n)$ is decreasing in *n*. Entries (and product development) are **strategic substitutes**. Different products compete with each other.
- Dixit-Stiglitz imposed the restriction on $F(X,H)$ to ensure the unique equilibrium.
- With the fixed cost of entry (or product development), how many products (or firms) can be accommodated in equilibrium depends on the market size, αL.

Calculating Equilibrium Output:

$$Y = F(X,H) = AX^{\alpha}H^{1-\alpha} = A\left(\frac{H}{X}\right)^{1-\alpha} X = A(\Phi(P))^{1-\alpha} X = A(\alpha^{-1}-1)^{1-\alpha} P^{1-\alpha} X$$

$$= A(\alpha^{-1}-1)^{1-\alpha}\left(pn^{\frac{1}{1-\sigma}}\right)^{1-\alpha} x(n)^{\frac{\sigma}{\sigma-1}} = A(\sigma f)\,p^{-\alpha}(\alpha^{-1}-1)^{1-\alpha}(n)^{1+\frac{\alpha}{\sigma-1}}$$

$$= A(\sigma f)(1-\sigma^{-1})^{\alpha} a^{-\alpha}(\alpha^{-1}-1)^{1-\alpha}\left(\frac{\alpha L}{\sigma f}\right)^{1+\frac{\alpha}{\sigma-1}} \Rightarrow \quad Y \propto (L)^{1+\frac{\alpha}{\sigma-1}} \quad \text{or} \quad \frac{Y}{L} \propto (L)^{\frac{\alpha}{\sigma-1}}.$$

That is, TFP of this economy (and per capita output) is higher if the economy is larger. Why?

A larger economy can accommodate more specialized firms. Hence, gains from specialization lead to aggregate increasing returns.

Optimality of Equilibrium: Two separate effects of monopoly working in the opposite direction. First, it creates the pricing distortion. Second, it compensates the firms for introducing new products. (A firm cannot fully appropriate social surplus generated by its product.) It turns out that, for the case, $\alpha = 1$, these two effects exactly offset each other and the equilibrium allocation is optimal. (But, this is not a robust result.)

Case II: *Any* convex combination of $AX^{\alpha}H^{1-\alpha}$ and $BX^{\beta}H^{1-\beta}$ $(0 \le \alpha < \beta \le 1)$.

$$F(X,H) \equiv Max\left\{A(X_{\alpha})^{\alpha}(H_{\alpha})^{1-\alpha} + B(X_{\beta})^{\beta}(H_{\beta})^{1-\beta}\right\}$$

subject to $X_{\alpha} + X_{\beta} \le X; H_{\alpha} + H_{\beta} \le H; X_{\alpha} \ge 0; X_{\beta} \ge 0; H_{\alpha} \ge 0; H_{\beta} \ge 0$.

The final goods sector use **α-technology** instead of **β-technology**, if

Cost of **α-technology** $= \dfrac{P^{\alpha}}{A(\alpha)^{\alpha}(1-\alpha)^{1-\alpha}} < \dfrac{P^{\beta}}{B(\beta)^{\beta}(1-\beta)^{1-\beta}}$ = Cost of **β-technology**:

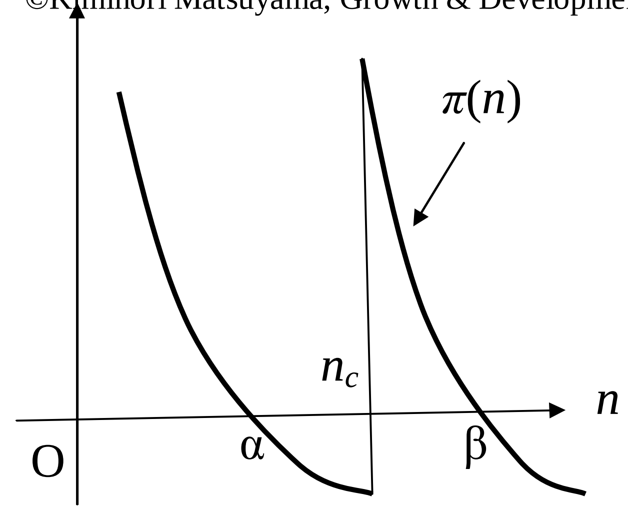
➔ $P^{\alpha-\beta} < \dfrac{A(\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{B(\beta)^{\beta}(1-\beta)^{1-\beta}}$

➔ $(n)^{\frac{\beta-\alpha}{\sigma-1}} < \dfrac{A(\alpha)^{\alpha}(1-\alpha)^{1-\alpha}}{B(\beta)^{\beta}(1-\beta)^{1-\beta}} \equiv (n_c)^{\frac{\beta-\alpha}{\sigma-1}}$

When $n < n_c$, the availability of specialized inputs is limited so they use **α-technology,** which is less dependent on such inputs. When $n > n_c$, they use **β-technology.**

$$\pi(n) = \frac{1}{\sigma-\alpha}\left[\frac{\alpha L}{n} - \sigma f\right] \qquad \text{if } n < n_c$$

$$\pi(n) = \frac{1}{\sigma-\beta}\left[\frac{\beta L}{n} - \sigma f\right] \qquad \text{if } n > n_c$$



Entries (product development) are **strategic complements** in some range.

Why? Two-way causality between the market size and specialization:
➢Smith (1776): "Division of labor is limited by the extent of the market."
➢Young (EJ 1928); "The extent of the market is also limited by the division of labor."

Multiple equilibria iff $\dfrac{\alpha L}{\sigma f} < n_c < \dfrac{\beta L}{\sigma f}$.

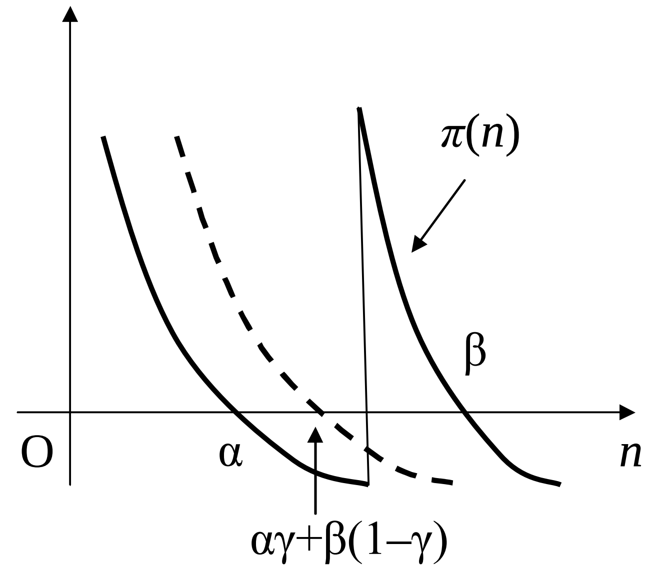
You can prove that TFP (and per capita output) is lower in the **α-equilibrium** than the **β-equilibrium.** (Prove it!)

Inequality in the Global Economy; Matsuyama (JEL 1995; JJIE 1996)

Two consumption goods, **α-good** & **β-good**, with the preferences, $(C_\alpha)^\gamma(C_\beta)^{1-\gamma}$.
α-good is produced by $AX^{\alpha}H^{1-\alpha}$ and β-good is produced by $BX^{\beta}H^{1-\beta}$.

Autarky: like Case I; a single good produced with Cobb-Douglas with αγ+β(1–γ).

Small Open Economy (α-good, and **β-good** are traded at an exogenous relative price with nontraded inputs):
like Case II,
The economy specializes (but we can't say which!)



World Economy: (the relative price of **α-good**, and **β-good** is now endogenous) Both goods need to be produced somewhere in the world. Hence,
Some economies must produce α-good.
Some economies must produce β-good.

Local **Positive Feedback &** ***Global*** **Resource Constraints;** World as "System;" regional economies its "Components"

➔ Endogenous Inequality and Patterns of specialization