Economic Growth & Development: Part 5
Growth in an Interdependent World

By Kiminori Matsuyama

Updated on 2011-04-22, 5:52:43 PM

Overview:

- International Technology Diffusion
- Inappropriate Technologies
- International Spillovers of IPR Protection
- Trade and Factor Accumulation: Growth Convergence
- Trade and Factor Accumulation: Growth Persistence
- Trade, Learning-By-Doing and Endogenous Comparative Advantage
- Bounded Learning-By-Doing: Unbounded Growth and Leapfrogging

Motivation:

- So far, we have studied growth processes of a closed economy. We now turn to models of growth with many countries.

- It is important to understand how growth and development process of different countries interact each other.

➢ Much of growth theory is developed under the closed economy assumption, and its implications are tested with the cross-country data, under the implicit assumption that each country is in autarky.
➢ In reality, of course, no country is in autarky. The only closed economy is the world economy as a whole.
➢ The world economy models with many interdependent countries might reveal cross-country implications that are very different from the world economy consisting of many isolated autarky economies.

A Simple Technology Diffusion Model (Acemoglu Ch.18.3.1):

The baseline lab-equipment model (Acemoglu Ch.13), except

- The exogenously growing technology frontier
- R&D adopts the technology to make new products available as follows:

R&D Sector (Adoption): $\dot{N}(t) = \eta\left(\frac{\tilde{N}(t)}{N(t)}\right)^{\phi} Z(t) = \frac{\eta Z(t)}{(\mu(t))^{\phi}}$, with $\mu(t) \equiv N(t)/\tilde{N}(t) < 1$

$\tilde{N}(t)$; Technology frontier, growing *exogenously* as $\tilde{N}(t) = \tilde{N}(0)\exp(gt)$ for all t.

$N(t)$: The number of products available for use

$\mu(t) \equiv N(t)/\tilde{N}(t) < 1$

Interpretation: A country is trying to catch up with the technology frontier.

- R&D productivity is $\eta(\mu(t))^{-\phi}$, inversely related to $\mu(t)$. Thus, the more backward the country is, the easier it is to improve its technology, because there are more to learn and more to adopt.

Free Entry to R&D: $\dot{N}(t) > 0$ implies

$$\eta\left(\frac{\tilde{N}(t)}{N(t)}\right)^{\phi} V(t) = 1 \quad \text{or } V(t) = \frac{(\mu(t))^{\phi}}{\eta}.$$

As before, $x(v,t) = L$ and $\pi(v,t) = \beta L$ for all v and t.

$$\Rightarrow Y(t) = \frac{L}{1-\beta} N(t); \quad X(t) = (1-\beta)LN(t); \qquad w(t) = \frac{\beta}{1-\beta} N(t).$$

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t)-\rho}{\theta}$$

$$Y(t) = C(t) + X(t) + Z(t).$$

Let us look for the BGP, where $r(t) = r^*$. Then,

$$V^* = \frac{\pi}{r^*} = \frac{\beta L}{r^*} = \frac{(\mu^*)^{\phi}}{\eta} \quad \Rightarrow \mu^* = \left(\frac{\eta\beta L}{r^*}\right)^{1/\phi} = \frac{N(t)}{\tilde{N}(t)} \quad \Rightarrow \frac{\dot{N}(t)}{N(t)} = g\,.$$

From the aggregate resource constraint, $Y(t) = C(t) + X(t) + Z(t)$,

$$\frac{L}{1-\beta} = \frac{C(t)}{N(t)} + (1-\beta)L + \frac{(\mu^*)^{\phi}}{\eta}\frac{\dot{N}(t)}{N(t)} = \frac{C(t)}{N(t)} + (1-\beta)L + \frac{g(\mu^*)^{\phi}}{\eta}$$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{r^* - \rho}{\theta} = g \quad \Rightarrow \mu^* = \left(\frac{\eta\beta L}{\theta g + \rho}\right)^{1/\phi} \quad \Rightarrow \frac{Y(t)}{L} = \frac{\mu^*}{1-\beta}\tilde{N}(t)\,.$$

Notes:

- The growth rate is exogenous and independent of the scale (L), tastes (θ, ρ), and efficiency (η) parameters. No *growth* effects. However, they affect μ^*, so that they have the *level* effects.
- The existence of the BGP requires that $g < \theta g + \rho = r^*$ or $(1-\theta)g < \rho$.

Now,

- Imagine that there are J-countries, indexed by j = 1, 2, …, J.
- The countries differ in the scale (L) and efficiency (η) parameters, but not in the taste (θ, ρ) or any other parameters.
- Each country independently tries to adopt the world technology frontier.

Then, in steady state, all countries grow at the same rate, and the distribution of the per capita income is described by:

$$\mu_j{}^* = \left(\frac{\eta_j \beta L_j}{\theta g + \rho}\right)^{1/\phi} \equiv \mu_j{}^*(g) \qquad (j = 1, 2, \ldots, J).$$

In this model,

- the cross-country difference in efficiency leads to cross-country differences in the income levels; but not in the steady state growth rates.
- This is because of the built-in stabilizing effect in the R&D technology, which makes it easier for the more backward country to improve its technology.

International Technology Diffusion and Endogenous Growth (Acemoglu Ch.18.3.2):

In the above model,

- No interdependence across countries. Countries do not learn from the R&D experiences from the rest of the world.
- World technology frontier is exogenous.

Now modify the model so that each country contributes to the world stock of knowledge, as:

$$\tilde{N}(t) \equiv G(N_1(t), N_2(t), ..., N_J(t))$$

where G is linear homogeneous, and increasing in each argument. In steady state,

$$G(g) \equiv G(\mu_1 *(g), \mu_2 *(g), ..., \mu_J *(g)) = 1.$$

where $G(\bullet)$ is a decreasing function. If $G(0) < 1$, its solution is positive. And, if it satisfies $(1-\theta)g < \rho$, it is the BGP.

Notes: The (common) growth rate is endogenous.

- A higher L_j or η_j contributes to an increase in the common growth rate.
- Cross-country differences in L or η show up in income levels, not in the growth rates.

Inappropriate Technologies (Ch.18.4):

International Spillovers of IPR Protection:

Trade and Factor Accumulation:
One-Page Refresher on One-Sector, Closed Economy, Growth Models

Resource Constraint: $\dot{K}_t = R(Y_t - C_t) - \delta K_t = sRY_t - \delta K_t$,

s: Aggregate saving rate
R: Productivity of investment technologies (or the inverse measure of the investment distortions)

Harrod-Domar (AK) Model: $Y_t = F(K_t) = ZK_t$; *Z:* Total Factor Productivity (TFP)

$$\Rightarrow \dot{K}_t = sRZK_t - \delta K_t \qquad \Rightarrow g_t = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sRZ - \delta .$$

Message: *With the linear accumulation technology, the saving rate, s, the investment productivity, R, and production productivity Z have the long-run growth effect.*

Solow (Neoclassical) Model: $Y/K = ZF(K)/K$ is decreasing in K and $\lim_{K\to\infty}F(K)/K = 0$.

$$\Rightarrow \frac{\dot{K}_t}{K_t} = \frac{sRZF(K_t)}{K_t} - \delta \to \frac{sRZF(K_\infty)}{K_\infty} - \delta \equiv 0, \ \Rightarrow g_\infty = 0 \text{ at } Y_\infty = ZF(K_\infty).$$

Message: *With the accumulation technology subject to diminishing returns, s, R, and Z, change* K_∞ and Y_∞ *but not* g_∞*. They have "level" effects, but no long run "growth" effect.*

Trade and Factor Accumulation: Growth Convergence, inspired by Acemoglu-Ventura (2002); see also Acemoglu (Ch.19.4).

We introduce factor accumulation in Dornbusch-Fischer-Samuelson (1977), which developed a Ricardian Model with a continuum of goods.

Two Countries: Home and Foreign(*)

One Primary Factor of Production (Capital): K_t and K_t^* (the factor prices, r_t and r_t^*)
Reproducible & accumulative, but nontradeable.

A Continuum of (Tradeable) Intermediate Input Sectors: $z \in [0,1]$.

Unit Capital Requirements: $a(z)$, $a^*(z)$,

The sectors are indexed so that $A(z) \equiv a^*(z)/a(z)$ is strictly decreasing in $z \in [0,1]$. That is, Home has comparative advantage in lower-indexed sectors.

Resource Constraints:

$$\int_0^1 a(z)q_t(z)dz = K_t; \quad \int_0^1 a^*(z)q_t^*(z)dz = K_t^*.$$

$q_t(z)$ & $q_t^*(z)$: input z produced at Home and Foreign

(Nontradeable) Final Good Sector: Cobb-Douglas Technologies:

$$\log Y_t = \int_0^1 \log(c_t(z))dz; \quad \log Y_t^* = \int_0^1 \log(c_t^*(z))dz;$$

$c_t(z)$ & $c_t^*(z)$: input z used in the final good production at Home and Foreign

Equivalently, the unit cost functions are given by:

$$\exp[\int_0^1 \log(p_t(z))dz]; \qquad \exp[\int_0^1 \log(p_t^*(z))dz];$$

$p_t(z)$ & $p_t^*(z)$: the prices of input z at Home and Foreign.

Capital Accumulation:

- Final Goods may be consumed or invested to accumulate capital.
- Representative agents consume a constant fraction of the final good.

$$\dot{K}_t = R[Y_t - C_t] = sRY_t; \quad \dot{K}_t^* = R^*[Y_t^* - C_t^*] = s^*R^*Y_t^*$$

where

s & *s**: the saving rates
R & *R**: the efficiencies of the investment technologies (or the inverse measure of the investment distortions).

Notes:

- This is the Ricardian model of trade with a continuum of goods, developed by Dornbusch-Fischer-Samuelson (1977), except that the single primary factor is interpreted as "capital," which is reproducible and accumulative.
- No capital flows; no international lending and borrowing. Trade must balance each period.

Home Autarky Equilibrium: $c_t(z) = q_t(z)$ for all $z \in [0,1]$, all t.

(Price = Cost): $p_t(z) = a(z)r_t$ for all $z \in [0,1]$, all t. With the final good as the *numeraire,*

$$1 = \exp[\int_0^1 \log(p_t(z))dz] = \exp[\int_0^1 \log(a(z)r_t)dz] \quad \Rightarrow r_t = \exp[-\int_0^1 \log(a(z))dz] \equiv Z$$

(Factor Market Equilibrium): Since $p_t(z)c_t(z) = Y_t$ for all $z \in [0,1]$ and all t,

$$K_t = \int_0^1 a(z)c_t(z)dz = Y_t \int_0^1 a(z)/p_t(z)dz = Y_t/r_t = Y_t/Z$$

$$\Rightarrow \frac{\dot{Y}_t}{Y_t} = \frac{\dot{K}_t}{K_t} = sR\frac{Y_t}{K_t} = sRZ \equiv g^A.$$

Foreign Autarky Equilibrium: Likewise,

$$\frac{Y_t^*}{K_t^*} = r_t^* = \exp[-\int_0^1 \log(a^*(z))dz] \equiv Z^*; \quad \frac{\dot{Y}_t^*}{Y_t^*} = \frac{\dot{K}_t^*}{K_t^*} = s^*R^*Z^* \equiv g^{*A},$$

Notes:

- Z and Z^* are TFPs of the two countries *in autarky*.
- Both countries grow as in the Harrod-Domar (*AK*) model, *in autarky*.

Trade Equilibrium (without any trade cost):

From Terms of Trade to Patterns of Trade

- Given ToT, r_t / r_t^*, the relative cost of input z, $(a^*(z)r_t^*)/(a(z)r_t) = A(z)(r_t^* / r_t)$ is strictly decreasing in z.
- The final goods sectors purchase inputs from the cheapest suppliers. Hence,

$$p_t(z) = \begin{cases} a(z)r_t < a^*(z)r_t^* & \text{for } z \in [0, m_t), \\ a^*(z)r_t^* < a(z)r_t & \text{for } z \in (m_t, 1], \end{cases}$$

where m_t is the *marginal sector*, satisfying

$$\text{(PT):} \quad \frac{r_t}{r_t^*} = A(m_t) \equiv \frac{a^*(m_t)}{a(m_t)},$$

which determines the Patterns of Trade (PT), given ToT, r_t / r_t^*:

- All inputs $z \in [0, m_t)$ are produced and exported by Home;
- All inputs $z \in (m_t, 1]$ are produced and exported by Foreign.

From Patterns of Trade to Terms of Trade:

Since the Home produces the inputs in [0, m_t) for the entire world,

$$r_t K_t = m_t (Y_t + Y_t^*) = m_t (r_t K_t + r_t^* K_t^*),$$

or Home Imports = $(1-m_t) r_t K_t = m_t r_t^* K_t^*$ = Foreign Imports

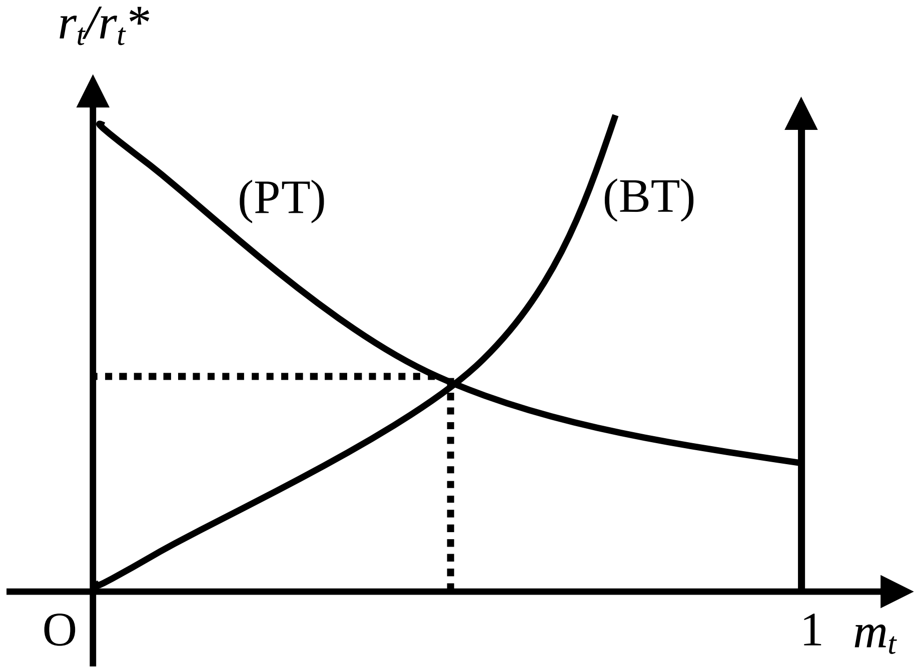
because the Foreign imports the inputs in [0, m_t) from Home and Home imports those in (m_t,1] from Foreign. This Balance of Trade condition (BT) can be rewritten to:

(BT) $$\frac{r_t}{r_t^*} = \frac{m_t}{1-m_t} \frac{K_t^*}{K_t}.$$

Joint Determination of Terms of Trade & Patterns of Trade: A Graphic Illustration

(BT) & (PT) jointly determines m_t and r_t / r_t^* as a function of K_t / K_t^*:

(BT)+(PT): $\frac{m_t}{1-m_t}\frac{K_t^*}{K_t} = \frac{r_t}{r_t^*} = A(m_t)$,



In fact, we can also determine r_t & r_t^*, as follows:

Calculating TFPs under Free Trade: (Static) Gains from Trade:

Taking the final good as the *numeraire*,

$$0 = \int_0^1 \log(p_t(z))dz = \int_0^{m_t} \log(a(z)r_t)dz + \int_{m_t}^1 \log(a^*(z)r_t^*)dz.$$

$$= \int_0^1 \log(a(z)r_t)dz + \int_{m_t}^1 \log\left(\frac{A(z)}{A(m_t)}\right)dz$$

$$\Rightarrow \log(r_t) = -\int_0^1 \log(a(z))dz - \int_{m_t}^1 \log\left(\frac{A(z)}{A(m_t)}\right)dz = \log Z + \int_{m_t}^1 \log\left(\frac{A(m_t)}{A(z)}\right)dz$$

$$\Rightarrow r_t = Z\exp\left[\int_{m_t}^1 \log\left(\frac{A(m_t)}{A(z)}\right)dz\right] \equiv ZW(m_t),$$

where $W(\bullet)$ is strictly decreasing and $ZW(m_t) > ZW(1) = Z$.

Likewise, $r_t^* = Z^*\exp\left[\int_0^{m_t} \log\left(\frac{A(z)}{A(m_t)}\right)dz\right] \equiv Z^*W^*(m_t) > Z^*W^*(0) = Z^*,$

where $W^*(\bullet)$ is strictly increasing and $Z^*W^*(m_t) > Z^*W^*(0) = Z^*$.

Note: TFPs (& the factor prices) rise in both countries; the (static) gains from trade!

Capital Accumulation in Trade Equilibrium:

The above equation in turn determines capital accumulation as follows:

$$\frac{\dot{K}_t}{K_t} = sRr_t = sRZW(m_t); \quad \frac{\dot{K}_t^*}{K_t^*} = s^* R^* r_t^* = s^* R^* Z^* W^*(m_t).$$

$$\frac{(K_t \dot{/} K_t^*)}{K_t / K_t^*} = \frac{\dot{K}_t}{K_t} - \frac{\dot{K}_t^*}{K_t^*} = sRZW(m_t) - s^* R^* Z^* W^*(m_t) \equiv \Psi(K_t / K_t^*)$$

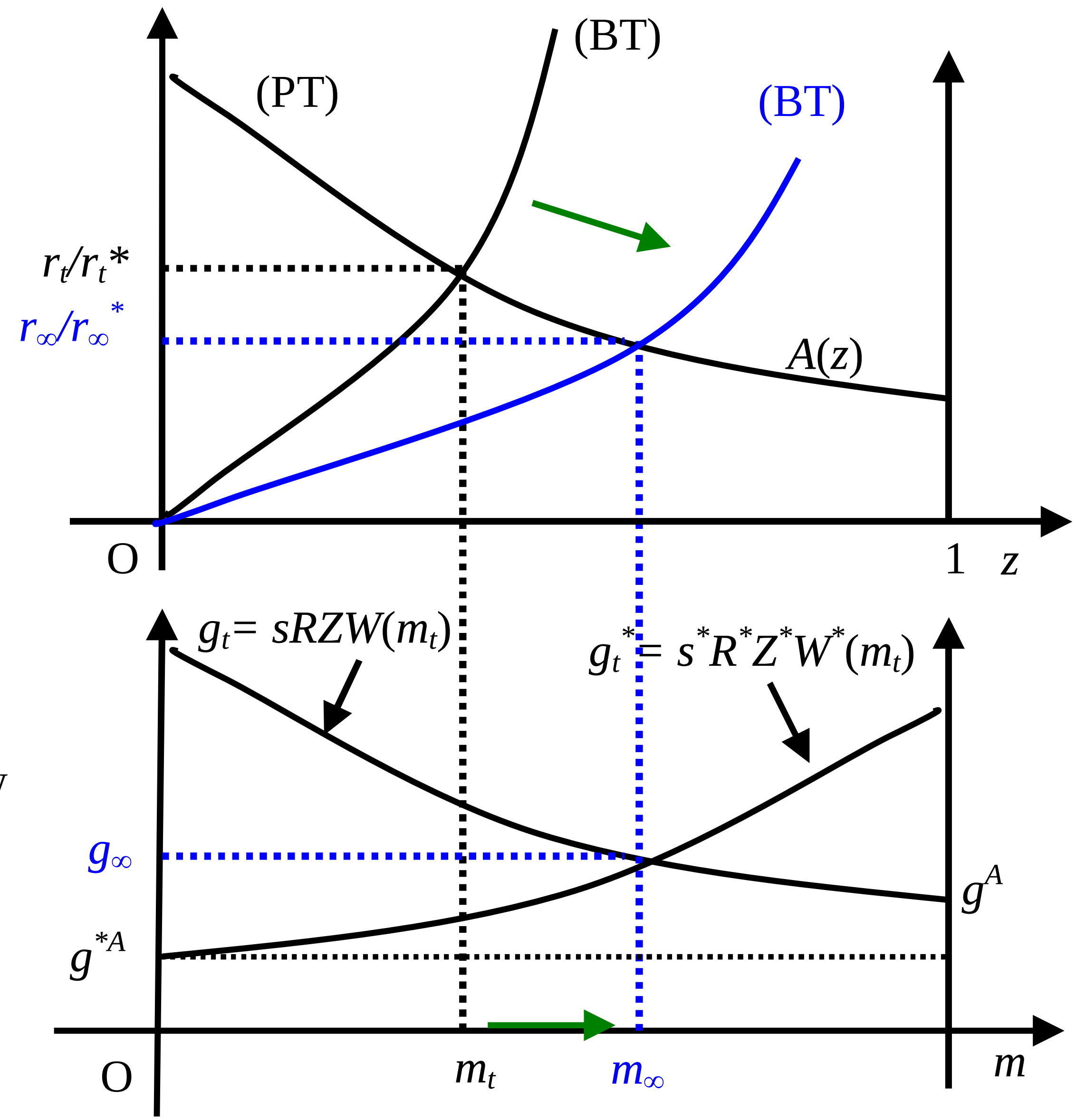
Steady State:

$$\frac{m_\infty}{1-m_\infty}\left(\frac{K^*}{K}\right)_\infty = \frac{r_\infty}{r_\infty^*} = A(m_\infty); \quad sRZW(m_\infty) = g_\infty = s^* R^* Z^* W^*(m_\infty).$$

A Graphic Illustration:

Suppose $K_t / K_t^* < K_\infty / K_\infty^*$.

- The equilibrium in t is given by the intersection of the two black curves, (PT) & (BT) in the upper panel.
- The lower panel shows that, at this equilibrium, both countries grow faster than in autarky, and that Home accumulates capital faster than Foreign.
- This shifts the (BT) curve to the right, as shown by the Green Arrow in the upper panel.
- This process continues until the world economy reaches the steady state, depicted in Blue.



Thus, when they trade with each other,

- Both countries grow faster than in autarky: $g_t > g^A$; $g_t^* > g^{*A}$.

- In steady state, they grow at the common rate; $g_\infty = g_\infty^*$.

- A higher s, a higher R, and a higher Z lead to a higher m_t, and a deterioration of the Home ToT. They increase the common steady state growth rate.

- Home's share in the world income is equal to m_t. Thus, a change in the parameters, s, R, and Z, has only the (relative) level effects, but no (relative) growth effects in the long run, somewhat similar to the Solow (Neoclassical) growth model.

- Note that r_t / r_t^* is both ToT and the ratio of the TFPs in the two countries.

Basic Message:

In spite of the linear accumulation technology, the endogeneity of the ToT and the patterns of trade introduce *de facto* diminishing returns in the growth process of each country. → growth convergence across countries.

Note: The above model differs from Acemoglu & Ventura (2002) in that

- AV solve for the intertemporal optimization problem by the representative agent, instead of the exogenous saving.
- AV assume that the final goods production is the Cobb-Douglas composite of the nontradeable capital and the CES aggregate of the tradeable intermediates, with the elasticity of substitution, $\varepsilon > 1$. Here, it is the Cobb-Douglas aggregate of the tradeable intermediates only.
- AV considered an arbitrary number of countries under the Armington assumption (hence, the patterns of trade do not respond to ToT changes).

But, the message is essentially the same.

Exercise: Instead of the exogenous saving rate, let us assume that the Home representative agent chooses the consumption path to maximize:

$$\int_0^{\infty} \log(C_t)\exp(-\rho t)dt \text{ subject to } \dot{K}_t = R[Y_t - C_t] = R[r_t K_t - C_t].$$

1) Show that $C_t = \rho K_t / R$, and hence $\dot{K}_t / K_t = Rr_t - \rho$.
2) Suppose that the Foreign represent agent solves the same problem with the discount rate, ρ^*. Analyze the world economy equilibrium, for a given (ρ, R, Z) and (ρ^*, R^*, Z^*).

Exercise: Redo the above analysis for the final goods production technologies given by:

$$\log Y_t = (1-\tau)\log(n_t) + \tau\int_0^1 \log(c_t(z))dz\,;$$
$$\log Y_t^* = (1-\tau)\log(n_t^*) + \tau\int_0^1 \log(c_t^*(z))dz\,,$$

where n and n^* are the nontradeable intermediates, one unit of which is produced with one unit of capital.

Exercise: Read Acemoglu and Ventura (2002) or Acemoglu (Ch.19.4). Explain why AV assumed CES with $\varepsilon > 1$, instead of the Cobb-Doulgas technologies.

Trade and Factor Accumulation: Growth Persistence (Acemoglu, Ch.19.3)

We use Two-Sector, Two-Factor (Heckscher-Ohlin) Model of Trade.

- In this model, countries share the same technologies, but they differ in the relative factor endowments.
- Each country becomes an exporter of the good whose production requires more intensive use of the factor that is relatively abundant in that country.

A Small Open Economy: normalize the prices of all tradeables to one.

Two Primary (Nontradeable) Factors of Production:

Capital: K_t (the factor price, r_t) is reproducible and cumulative.
Labor: L_t (the factor price, w_t) grows exogenously at the rate, n.

Two Competitive (Tradeable) Intermediate Inputs Sectors: No Factor Intersity Reversal (i.e., the factor intensities of the two sectors never get reverse when the factor prices change; e.g., Cobb-Douglas technologies case)

One (Nontradeable) Final Good Sector: consumed or invested to accumulate capital

Technologies: All CRS

		Production Functions	Unit Cost Functions
Intermediate Inputs Sectors	*K*-Intensive	$Y_t^K = F^K(K_t^K, L_t^K)$	$c^K(r_t, w_t) \geq p_t^K = 1$
	L-Intensive	$Y_t^L = F^L(K_t^L, L_t^L)$	$c^L(r_t, w_t) \geq p_t^L = 1$
Final Goods Sector		$Y_t = F(X_t^K, X_t^L)$	$c(p_t^K, p_t^L) = c(1,1) = 1$

Note: The price of the final good is c(1,1), which is set to one by normalization.

Capital Accumulation:

$$\dot{K}_t = R[Y_t - C_t] - \delta K_t = sRY_t - \delta K_t$$

Note:

- This model differs from the standard Heckscher-Ohlin model in that one of the two factors is reproducible and accumulative.
- No capital flows, and no international lending and borrowing across countries; Trade must balance each period.

In Autarky: The country produces all of the intermediate inputs, $Y_t^K = X_t^K$ and $Y_t^L = X_t^L$. Hence, the aggregate production function is:

$$F^A(K,L) \equiv \underset{K^K,K^L,L^K,L^L \geq 0}{Max} \left\{ F\left(F^K(K^K, L^K), F^L(K^L, L^L)\right) \middle| K^K + K^L \leq K; L^K + L^L \leq L \right\}.$$

$$\Rightarrow \dot{K}_t = sRF^A(K_t, L_t) - \delta K_t \quad \Rightarrow \dot{k}_t = sRf^A(k_t) - (n+\delta)k_t, \text{ where } f^A(k) \equiv F^A(k,1).$$

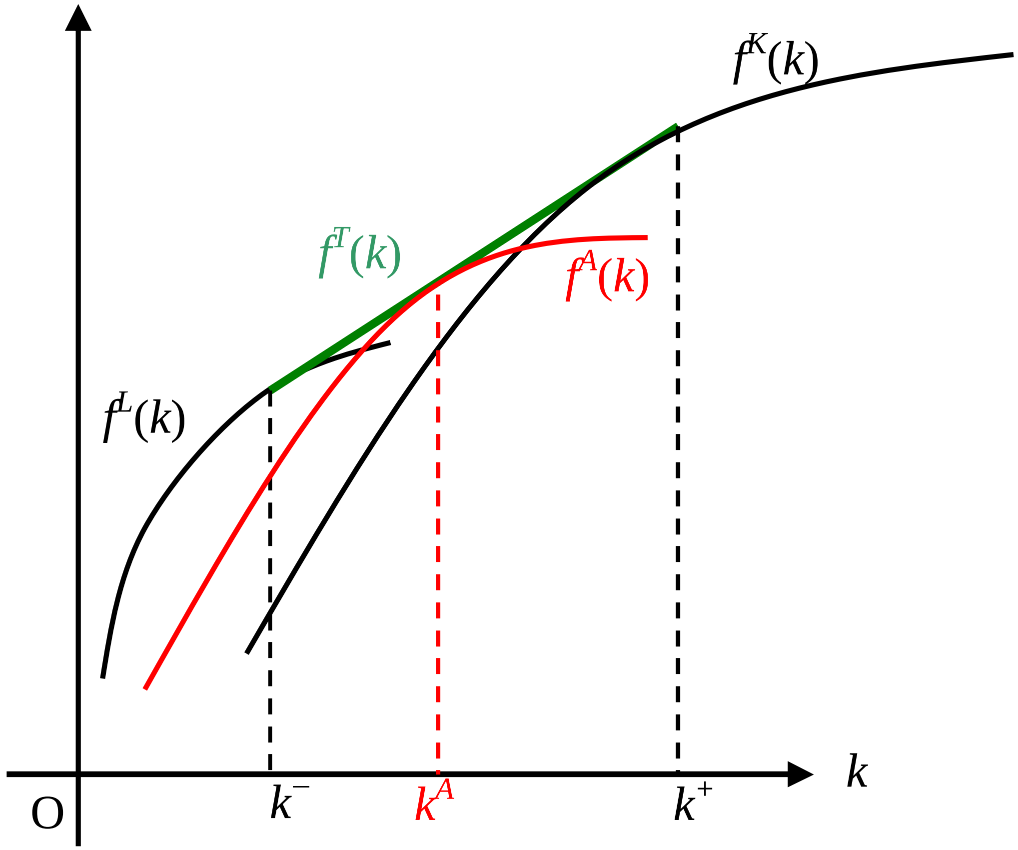
In Trade: The country trade the intermediate inputs, with the balance of trade condition, $Y_t^K + Y_t^L = X_t^K + X_t^L$. Hence, the aggregate production function is:

$$F^T(K,L) \equiv$$
$$\underset{K^K,K^L,L^K,L^L \geq 0}{Max} \left\{ F\left(X^K, X^L\right) \middle| F^K(K^K, L^K) + F^L(K^L, L^L) = X^K + X^L; K^K + K^L \leq K, L^K + L^L \leq L \right\}$$

$$\Rightarrow \dot{K}_t = sRF^T(K_t, L_t) - \delta K_t \quad \Rightarrow \dot{k}_t = sRf^T(k_t) - (n+\delta)k_t, \text{ where } f^T(k) \equiv F^T(k,1).$$

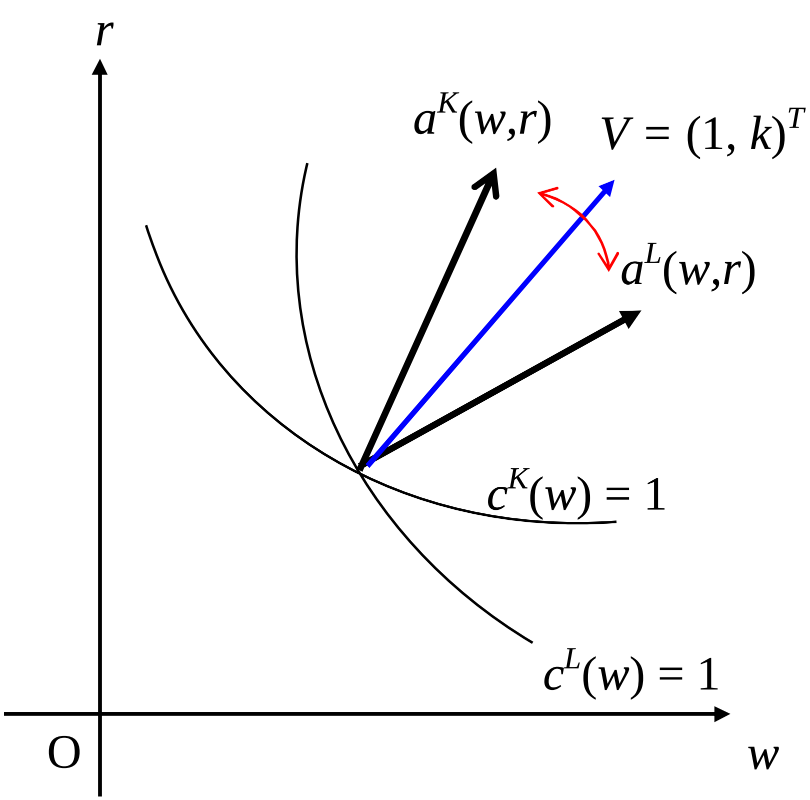
Aggregate Production Functions under Autarky and under Free Trade:

- $f^T(k)$ is obtained as the upper envelope of $f^K(k)$ & $f^L(k)$, which gives **its flat segment**.
- $f^T(k)$ dominates $f^A(k)$, except one point where this country's net trade is zero. (Again, gains from trade!)

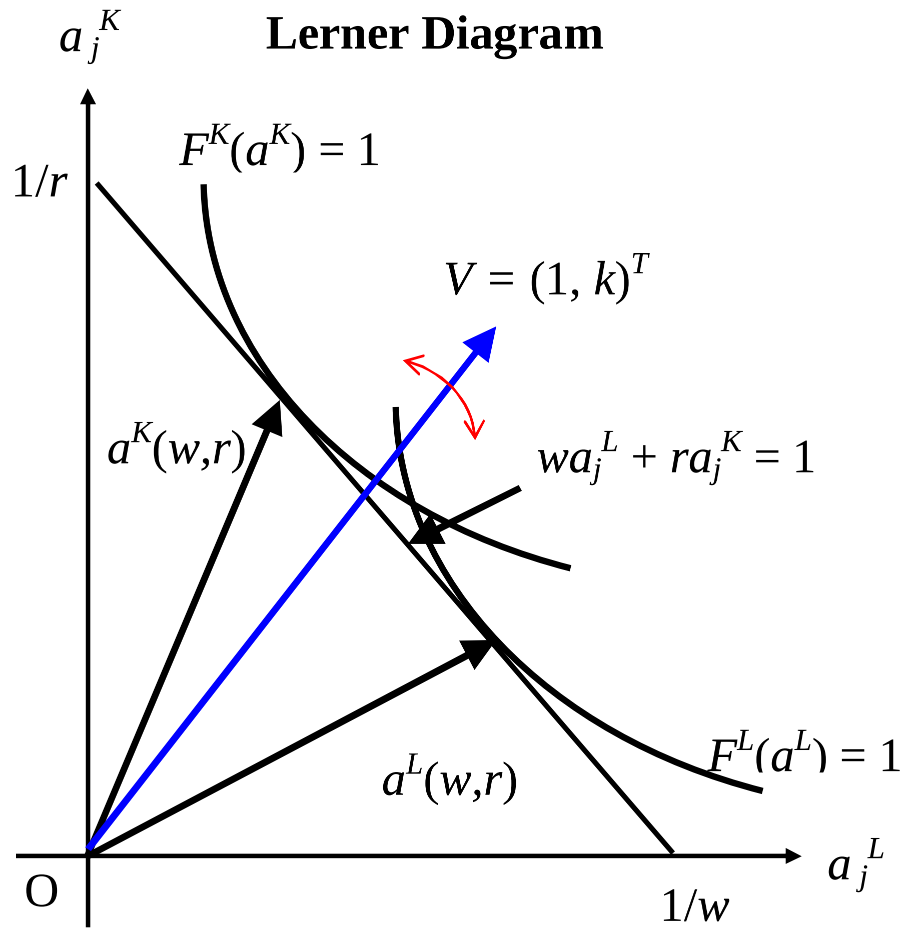


For those who studied International Trade, the following diagrams might be useful understanding why the return of return is independent of the capital-labor ratio when the country produces in both sectors.

Mussa Diagram



Lerner Diagram



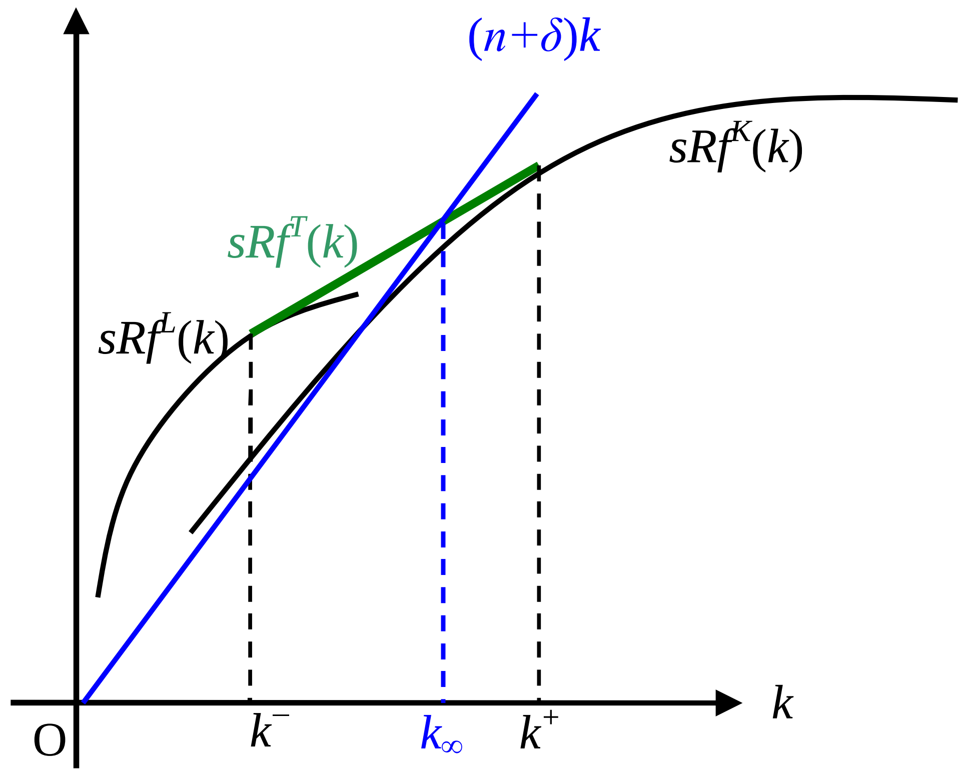
Solow Growth Diagram under Trade;

For $k_t < k^-$, the economy specializes in the labor intensive input.
For $k_t > k^+$, the economy specializes in the capital-intensive input.

For $k^- < k_t < k^+$, the economy produces both inputs. The factor prices are insensitive to the factor-ratio, hence, no diminishing returns (as indicated by the Green Envelope).

If the Blue Line intersects with the Green Segment, the economy produces both inputs in the steady state, k_∞.

Furthermore,



Long-Run Rybczynski Theorem:

An increase in $sR/(n+\delta)$ increases k_∞, hence, the economy produces more capital-intensive inputs and less labor-intensive inputs.

From this, it is straightforward to show that:

Long-Run Heckscher-Ohlin Theorem:

Consider two small open countries, Home & Foreign, which differ only in the parameters, s (s^*), R (R^*), and n (n^*). If both countries are diversified in the steady state, Home exports the capital intensive inputs and Foreign exports the labor intensive inputs, if and only if $sR/(n+\delta) > s^*R^*/(n^*+\delta)$.

Notes:

- The assumption that the two countries are both "small" is not necessary. Even in the two-country model, where Home & Foreign are both large, the above result holds as long as the two countries are both diversified in steady state. This condition, both countries are diversified, holds if the differences between $sR/(n+\delta)$ and $s^*R^*/(n^*+\delta)$ are sufficiently small.

- Findlay (1970) obtained this result for the case with two tradeable consumption goods and one nontradeable investment good, under the assumption that the factor intensity of the investment goods sector lies between the factor intensities of the two consumption goods sector.

- Stiglitz (1970) looked at the case with one consumption good and one investment good, both of which are tradeable and studied the likelihood of the economy staying in the diversification cone in steady state. The result depends, among other things, on the relative intensity of the two goods sectors.
- Matsuyama (1988) showed that, in the OLG, life-cycle saving framework, the aggregate saving depends on the population growth rate (as it changes the ratio of the young saver and the old retirees), and restated the Findlay result in the terms of the discount rate and the population growth rate.

From the perspective of growth theory, more important is that cross-country differences in capital-labor ratio does not imply any cross-country differences in the rate of return.

- Ventura (1997) studied this setup with optimal savings by the representative agents, without assuming that the economy is diversified in steady state. The economy still passes through the diversified region during transition. He used the model to address, the question of the East Asian Miracle; why these countries managed to grow so fast for so long. He argued that the export-led growth, accompanied by structural transformation from labor-intensive to capital-intensive industries might delay the standard capital deepening effect.

- Acemoglu (Ch.19.4) assumes that the K-intensive input uses K only, and the L-intensive input uses L only. This extreme intensity assumption implies that countries are always diversified, and that the factor prices are uniquely pinned down by international trade in inputs. This means that the factor prices are equalized across countries, regardless of their factor supplies (the factor price insensitivity), and the above result follows immediately. (The small country assumption is not essential for this result.)
- Those who studied international trade should know that the factor price insensitivity is not a robust result. (For example, it breaks down if another factor is introduced in the above setting.) But, the idea that the factor prices depend *less* on the local factor availability in a trading world is compelling. So, it would be interesting questions for future research to see how much of the above result can be intended to general factor proportion models with trade costs.

Exercise:

Let $Y_t^K = F^K(K_t^K, L_t^K) = A^K(K_t^K)^\alpha (L_t^K)^{1-\alpha}$, $Y_t^L = F^L(K_t^L, L_t^L) = A^L(K_t^L)^\beta (L_t^L)^{1-\beta}$, and $Y_t = F(X_t^K, X_t^L) = A(X_t^K)^\gamma (X_t^L)^{1-\gamma}$ with $1 \geq \alpha > \beta \geq 0$; $0 < \gamma < 1$. Show that

- $f^T(k)$ is obtained as the upper envelope of $f^K(k)$ and $f^L(k)$, which gives its flat segment.
- $f^T(k)$ generally dominates $f^A(k)$, except one point where this country's net trade is zero.

Trade, Learning-By-Doing, and Endogenous Comparative Advantage

- So far, we assume that trade does not have any direct effects on productivity. We now turn to models where trade affects growth performances of different countries through its effect on productivity of different sectors.

- The key mechanism is that trade shifts the resources from import-competing sectors to export-competing sectors. To the extent that these reallocation affect productivity growth in each sector, this could affect overall growth performance of each country.

- To keep it simple, assume that productivity growth in each sector takes place through learning-by-doing (LBD) spillovers. That is,
  - Productivity improves as a result of learning from production experiences
  - Such learning is external to firms that generate, spill over to other firms

- LBD can be classified according to the scopes of spillovers
  - *Geographical:* local (region- or country-specific) or global in scope
  - *Sectoral:* specific to the industry where it is acquired or spillovers to other industries

In addition,

- The potential for productivity growth through LBD may be *unbounded* or *bounded*.

Unbounded, Country-Specific, Sector-Specific LBD

Two Countries: Home (L) and Foreign (L^*)

Two (Tradeable) Intermediate Input Sectors: Labor is the sole input, and its productivity is given by:

$$A_j(t)/a_j \quad (j = 1, 2) \qquad A_j^*(t)/a_j^* \quad (j = 1, 2)$$

a_j , a_j^*: time-invariant, reflecting the inherent advantages,
$A_j(t)$, $A_j^*(t)$: time-variant, reflecting the learning components.

(Nontradeable) Final Good Sector: Cobb-Douglas Technologies:

$$Y = \left(\frac{C_1}{\alpha}\right)^{\alpha}\left(\frac{C_2}{1-\alpha}\right)^{1-\alpha}; \quad Y^* = \left(\frac{C_1^*}{\alpha}\right)^{\alpha}\left(\frac{C_2^*}{1-\alpha}\right)^{1-\alpha}$$

where C_j (C_j^*): input-j (j = 1, 2) used in the Home (Foreign) final good production. The cost functions are $(p_1)^{\alpha}(p_2)^{1-\alpha}$ and $(p_1^*)^{\alpha}(p_2^*)^{1-\alpha}$ where p_j (p_j^*) is the price of input-j.

Learning-By-Doing (Knowledge) Spillovers:
One may write that labor productivity in each sector changes with the experiences accumulated in different sectors and different countries, as follows:

$$A_j = A_j(Q_1, Q_2, Q_1^*, Q_2^*); \quad A_j^* = A_j^*(Q_1, Q_2, Q_1^*, Q_2^*)$$

where $Q_j (Q_j^*)$ is the experience in Home (Foreign) sector-j.

Here, let $Q_j(t) = \int_0^t X_j(s)ds + Q_j(0)$ and $Q_j^*(t) = \int_0^t X_j^*(s)ds + Q_j^*(0)$ and

$$A_j(t) = \delta_j Q_j(t), \quad A_j^*(t) = \delta_j Q_j^*(t) \qquad \text{(j = 1, 2)}$$

where $\delta_j \geq 0$ is the learning speed in sector-j. By letting "•" denote the time derivative,

$$\dot{A}_j(t) = \delta_j X_j(t) \quad \text{and} \quad \dot{A}_j^*(t) = \delta_j X_j^*(t). \qquad \text{(j = 1, 2)}$$

Each sector improves its productivity (or accumulates its knowledge capital) at the rate proportional to its own production.

Notes:

- No sector learns from the production in the other sector. Learning is (completely) *sector-specific*.

- No country learns from the production of the other country. Learning is (completely) *country-specific*.

- Productivity improvement is not subject to diminishing returns; Learning is *unbounded*.

- It is also assumed that learning capacity, δ_j, might differ across sectors, but not across countries.

- Furthermore, it is assumed that these learning effects are external to competitive producers that generate them. Thus, the firms do not take into account these learning effects when making production decisions.

- This allows us to solve for the dynamics by first solving for the static equilibrium at each t, holding the labor productivity as given, and look at the sequence of the static equilibriums, as the labor productivity evolves over time.

To keep it simple, let us $a_j = a_j^* = 1$ for j = 1 and 2.

Autarky Case:

Statics: Each period, the Home autarky static equilibrium is characterized by

$$X_1 = C_1 = \alpha A_1 L; \quad X_2 = C_2 = (1-\alpha) A_2 L; \quad Y = (A_1)^\alpha (A_2)^{1-\alpha} L.$$

Furthermore, by taking the final good as the numeraire, $w = (A_1)^\alpha (A_2)^{1-\alpha}$.
Likewise for Foreign.

Exercise: Show the above.

Dynamics: from $\dot{A}_j(t) = \delta_j X_j(t)$, $\dot{A}_1(t)/A_1(t) = \delta_1 \alpha L$, $\dot{A}_2(t)/A_2(t) = \delta_2 (1-\alpha) L$, and

$$g_Y \equiv \frac{\dot{Y}(t)}{Y(t)} = g_w \equiv \frac{\dot{w}(t)}{w(t)} = [\delta_1 (\alpha)^2 + \delta_2 (1-\alpha)^2] L$$

Likewise, $g_{Y*} \equiv \frac{\dot{Y}^*(t)}{Y^*(t)} = g_{w*} \equiv \frac{\dot{w}^*(t)}{w^*(t)} = [\delta_1 (\alpha)^2 + \delta_2 (1-\alpha)^2] L^*$

Note: The larger country grows faster.

Free Trade Equilibrium: Suppose that labor productivities at time t are such that

$$\frac{A_1(t)}{A_1^*(t)} > \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2(t)}{A_2^*(t)}.$$

Then, in the static free trade equilibrium, Home specializes in Input 1 and Foreign specializes in Input 2:

$$X_1 = A_1 L;\; X_2 = 0; \qquad X_1^* = 0; \qquad X_2^* = A_2^* L^*$$

$$\frac{w}{A_1} = p_1 < \frac{w^*}{A_1^*}; \qquad \frac{w}{A_2} > p_2 = \frac{w^*}{A_2^*}; \;\; \frac{A_1}{A_1^*} > \frac{w}{w^*} = \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2}{A_2^*}$$

and

$$Y = wL = \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}(A_1 L)^{\alpha}\left(A_2^* L^*\right)^{1-\alpha};\; Y^* = w^* L^* = \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}(A_1 L)^{\alpha}\left(A_2^* L^*\right)^{1-\alpha}$$

Again, the final good is chosen as the numeraire, which is permissible because, even though the final good is nontradable, its price is the same in the two countries under free trade.

Exercise: Show the above. Show that the outputs of the final good go up in both countries by moving from Autarky to Free Trade.

Dynamics: From $X_1 = A_1 L$; $X_2 = 0$; $X_1^* = 0$; and $X_2^* = A_2^* L^*$,

$$\frac{\dot{A}_1(t)}{A_1(t)} = \delta_1 L; \quad \frac{\dot{A}_2(t)}{A_2(t)} = 0; \quad \frac{\dot{A}_1^*(t)}{A_1^*(t)} = 0; \quad \frac{\dot{A}_2^*(t)}{A_2^*(t)} = \delta_2 L^*.$$

This implies that the initial patterns of trade are sustained forever:

$$\frac{A_1(s)}{A_1^*(s)} = \frac{A_1(t)}{A_1^*(t)} e^{\delta_1 L(s-t)} > \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2(s)}{A_2^*(s)} = \frac{A_2(t)}{A_2^*(t)} e^{-\delta_2 L^*(s-t)} \text{ for all } s > t.$$

Furthermore,

$$g_Y = g_w = g_{Y*} = g_{w*} = \alpha\delta_1 L + (1-\alpha)\delta_2 L^*.$$

Notes:

- More generally, Home (Foreign) specializes in 1 (2) *forever* if the initial patterns are

$$\frac{A_1(0)/a_1}{A_1^*(0)/a_1^*} > \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{A_2(0)/a_2}{A_2^*(0)a_2^*}.$$

  Thus, the initial condition, or "history," matters. This could occur even when Foreign (Home) has the *natural* comparative advantage in sector 1 (2), $a_1^* / a_1 < a_2^* / a_2$.

- Both countries grow at the same rate under Free Trade. Cobb-Douglas is essential for this result, as it implies that the output growth in each sector is exactly offset by the terms of trade change. For the CES case, see the next example adopted from Lucas (1988; section 5) or Acemoglu (Ch.19.7).

- When the two countries are of the equal size, the (identical) growth rate goes up under Free Trade. When the two countries differ in size, the growth rate always goes up for the smaller country, but may goes down for the larger country if it ends up specializing in Sector 1, for the case, $\alpha\delta_1 < (1-\alpha)\delta_2$, or in Sector 2, for the case, $\alpha\delta_1 > (1-\alpha)\delta_2$.

Exercise: Demonstrate the above.

Exercise: Extend the above analysis for the case where the final goods use more than two tradeable intermediate inputs with Cobb-Douglas.

Case of the CES final goods production: adopted from Lucas (1988, sec.5).

- CES final goods production, with σ is the elasticity of substitution between Input 1 & 2.
- A continuum of countries of the same size, indexed by c(t) ∈ [0,1] so that $A_1^c(t)/A_2^c(t)$ is strictly decreasing in c(t) ∈ [0,1].
- All countries in c(t) ∈ [0, m(t)) specialize in input 1; all countries in c(t) ∈ (m(t), 1] specialize in 2, where m(t) is the marginal country at t.
- All countries specializing in input-j improve its labor productivity in sector j at the rate equal to $\dot{A}_j^c(t)/A_j^c(t) = \delta_j$. The output also grows at the same rate.

Exercises:

- Show that c(t) is time-invariant; there is no need to change the country indices over time.
- Under which condition, m(t) is also time-invariant?
- Compare the output growth of different countries.

Unbounded LBD that is Sector-Specific, but only partially Country-Specific:

Krugman's (1987) Model with Two Countries and a Continuum of Goods:

- Let us modify the DFS (1977) model with the following LBD.

$$\dot{A}_z(t) = \delta[X_z(t) + \rho X_z^*(t)]; \qquad \dot{A}_z^*(t) = \delta[\rho X_z(t) + X_z^*(t)], \quad z \in [0,1].$$

where $A_z(t)$ and $A_z^*(t)$ are the Home and Foreign Labor Productivity in Sector z at time t. LBD is confined to each sector. With $0 < \rho < 1$, there are some cross-country spillovers.

- As usual, index the sectors so that the Home relative labor productivity, $A_z(t)/A_z^*(t)$, is decreasing in $z \in [0,1]$, so that Patterns of Trade (PT) is given by:

$$\frac{A_z(t)}{A_z^*(t)} > \frac{w(t)}{w^*(t)} \quad \text{for } z \in [0, m(t)); \qquad \frac{A_z(t)}{A_z^*(t)} < \frac{w(t)}{w^*(t)} \quad \text{for } z \in (m(t), 1];$$

with $m(t)$ is the marginal sector; $\frac{A_m(t)}{A_m^*(t)} = \frac{w(t)}{w^*(t)}$.

- Again, the Balance of Trade (BT) condition is given by $wL = m(wL+w^*L^*)$ so that

(BT)+(PT): $$\frac{A_m(t)}{A_m^*(t)} = \frac{w(t)}{w^*(t)} = \frac{m(t)}{1-m(t)}\frac{L^*}{L}.$$

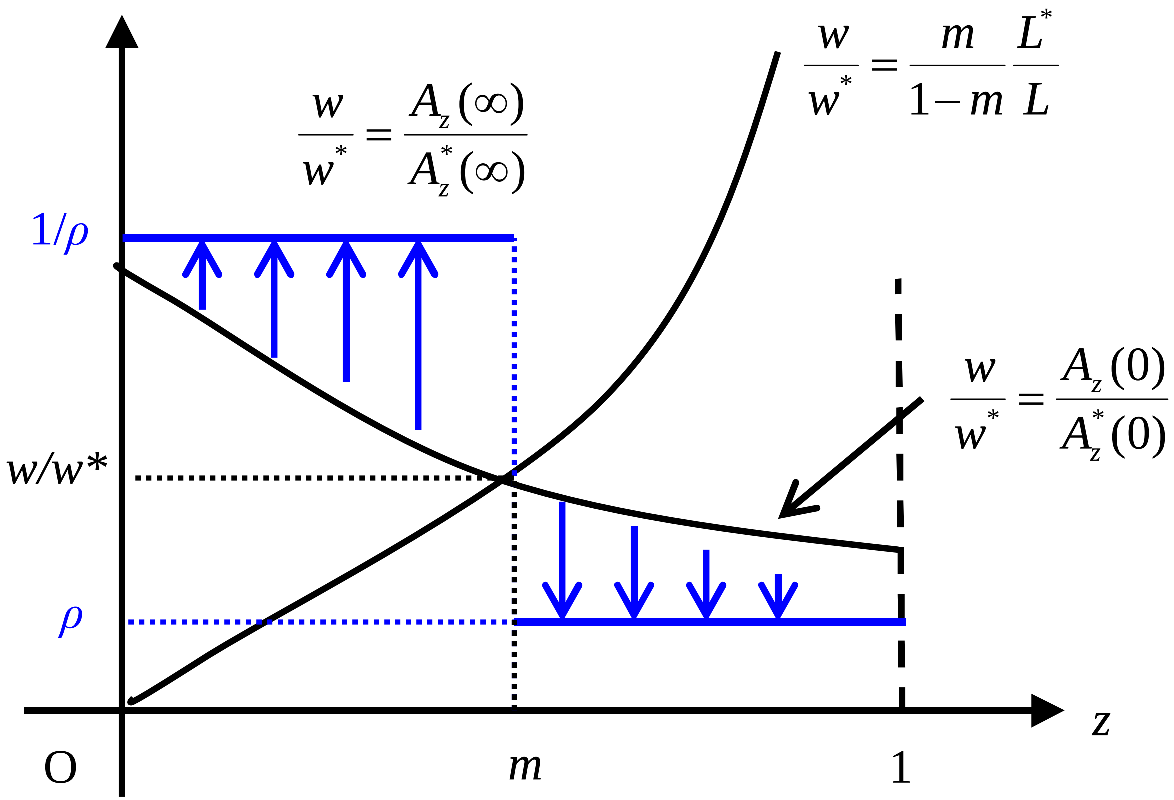
- Since only Home produces $z \in [0,m(t))$ and only Foreign produces $(m(t), 1]$,

$$\left(\dot{\frac{A_z}{A_z^*}}\right) = \delta L_z\left(1-\rho\frac{A_z}{A_z^*}\right)\left(\frac{A_z}{A_z^*}\right) \quad \text{for } z \in [0,m);$$

$$\left(\dot{\frac{A_z}{A_z^*}}\right) = \delta L_z^*\left(\rho-\frac{A_z}{A_z^*}\right) \quad \text{for } z \in (m,1].$$

- If $\rho < w/w^* = A_m(0)/A_m^*(0) < 1/\rho$, $m(t)$ will never change, and the relative labor productivity converges to:

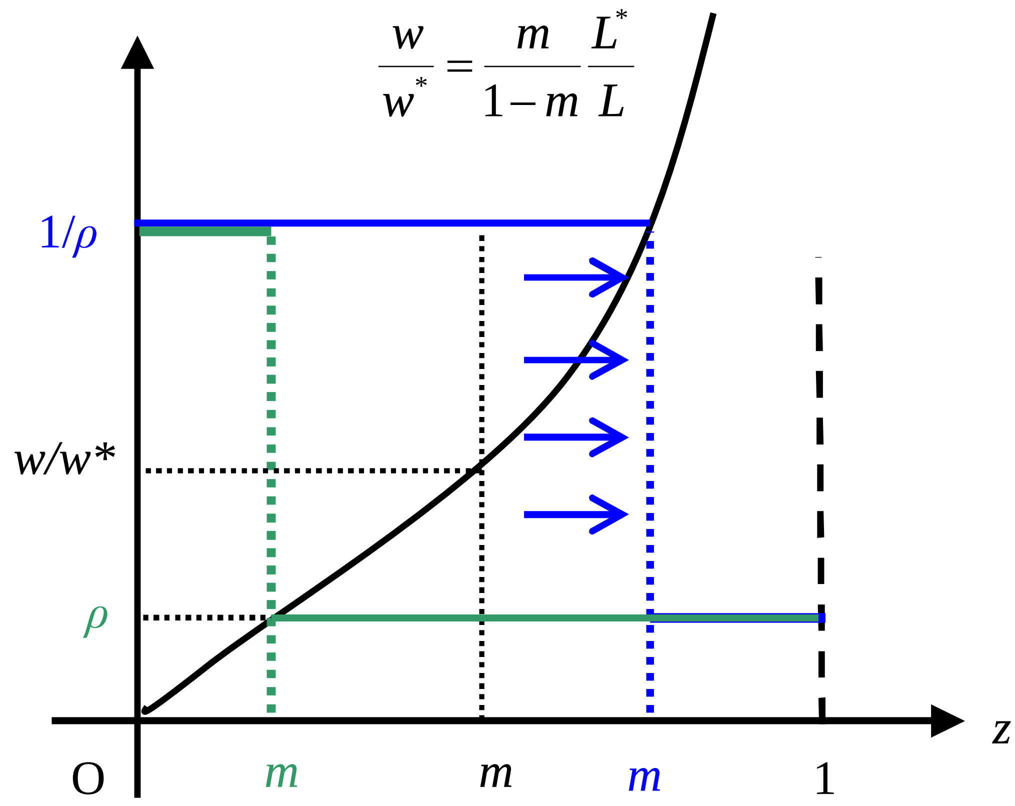
$$\frac{A_z(t)}{A_z^*(t)} \to \frac{1}{\rho} \text{ for } z \in [0,m); \qquad \frac{A_z(t)}{A_z^*(t)} \to \rho \text{ for } z \in (m,1].$$



$\frac{w}{w^*} = \frac{m}{1-m}\frac{L^*}{L}$
$\frac{w}{w^*} = \frac{A_z(\infty)}{A_z^*(\infty)}$
$1/\rho$
w/w^*
ρ
$\frac{w}{w^*} = \frac{A_z(0)}{A_z^*(0)}$
O
m
1
z

Notice ***a multiplicity of steady states***, depending on the initial condition. ***History matters.*** If the initial patterns of trade are such that $m \in [m, m]$, the steady state ToT is determined by $mL^*/(1-m)L$. The steady state Home welfare would be higher with a higher m.

- Home has an incentive to subsidize (temporarily) a few sectors slightly above the marginal sector, so that they can gain experiences and take over.
- This expands the range of goods it produces, increases the demand for its labor, and improves its ToT.
- This incentive to slice off a few sectors near the margin at a time continues until the ToT reaches to the upper limit. (Krugman called this "**narrow moving band**.")
- Obviously, Foreign has the same incentive to do so.



Bounded Learning-By-Doing: Unbounded Growth and Leapfrogging

- We have so far assumed that unbounded productivity growth is possible through LBD.

- Empirical evidence from any particular manufacturing activity suggests strong diminishing returns to LBD.

- Even if we interpret LBD more broadly as a reduced form way of modeling the causality from the industry size to its productivity gains, it is hard to imagine how productivity could grow unbounded in any particular industry.

- However, even if productivity growth is bounded in each industry, the economy may be able to sustain long run growth if experiences in one industry helps the economy to move into more sophisticated, higher value-added activities.

- At the same time, bounded learning also suggests the possibility that having expertise in certain areas, the economy may fail to move into new, more promising activities.

- To see this, let us imagine that the M-sector consists of many industries.

***Learning-By-Doing Model with Many Industries* (j = 1, 2,…,J)**

$X_t^j = A^j(Q_t^j)L_t^j$, where X_t^j: Output in Industry j; L_t^j: Employment in Industry *j*.

$A^j(Q_t^j)$; Labor Productivity, increasing in Q_t^j, the cumulative experience in *j*.

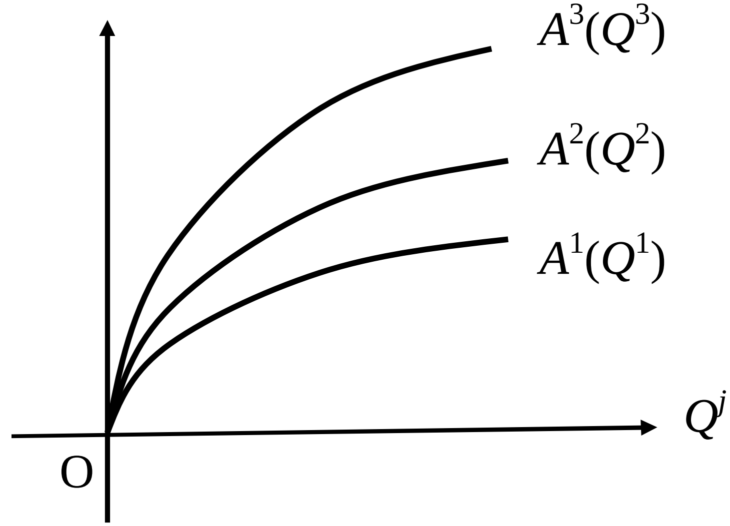
The state space is J-dimensional; $\mathbf{Q_t} \equiv [Q_t^j]$.

All manufacturing goods, *j* = 1, 2, …, *J*, are ***Perfect Substitutes***. Or, the prices are exogenously determined in the world market and normalized to one.

$$X_t^M = \sum_{j=1}^{J} X_t^j = \sum_{j=1}^{J} A^j(Q_t^j)L_t^j.$$

$$\Rightarrow X_t^j = A^j(Q_t^j)L_t^j > 0 \quad \text{only if } w_t = A^j(Q_t^j) = Max_k\{A^k(Q_t^k)\}.$$

- $A^j(q) = \lambda^{j-1}A(q)$, with $\lambda > 1$;
  $A(q)$ is strictly increasing and $A(0) = 0$.



Note: Higher-indexed goods are *potentially* more productive. Or we may think higher-indexed goods are of higher quality, as follows:

$$X_t^M = \sum_{j=1}^{J} \lambda^{j-1} X_t^j \text{ with } A^j(q) = A(q).$$

- *No Inter-industry spillovers*; each industry learns only from its own production.

$$\dot{Q}_t^j = \delta_j \left(L_t^j - Q_t^j\right) \text{ or } Q_t^j = Q_0^j \exp(-\delta_j t) + \int_0^t Q_0^j \exp[\delta_j (s-t)] ds$$

Note: Here, knowledge is assumed to depreciate. That is, you could also “forget by not-doing.” This prevents Q and A(Q) from growing forever. Alternatively, we could have assumed A(Q) is bounded above, in which we could let Q grow unbounded. Notice that the depreciating rate is set equal to the learning speed. No loss of generality, here. *Why?*

- Furthermore, suppose, for simplicity, the total manufacturing employment is fixed at L^M.

$$L^M = \sum_{j=1}^{J} L_t^j .$$

Then, it is easy to see that there are J-stable steady states: $\mathbf{Q}$ = (0, 0, …, L^M, 0, …, 0).

Exercise: Why do I need the assumption, $A(0) = 0$? How would you change the specification if you want to keep the same conclusion with $A(0) > 0$?

Now, let us see what might happen if there are some inter-industry learning spillovers?

Unbounded Growth through Bounded Learning

Suppose J = 2 and

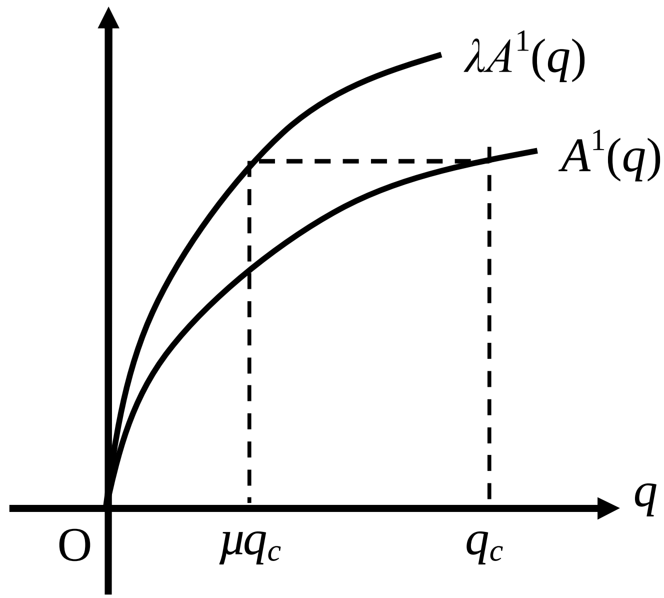
$$X_t^1 = A(Q_t^1)L_t^1;$$

$$X_t^2 = \lambda A(\mu Q_t^1 + Q_t^2)L_t^2$$

with $\mu < 1 < \lambda$. Assume further:

- $\lambda A(\mu q)/A(q)$ is strictly increasing in q.
- $\lambda A(\mu q_c)/A(q_c) = 1$.

e.g., $A(q) = q/(q+\alpha)$ with $\lambda\mu < 1 < \lambda$.
q_c is decreasing in λ and in μ.



Starting from the initial condition, $0 < Q_t^1 < q_c$ and $Q_t^2 = 0$,

For $L^M < q_c$, the economy is trapped in Industry 1.
For $L^M > q_c$, the economy makes a successful transition from Industry 1 to 2.

A larger L^M, a larger λ and a larger μ can the successful transition more likely.

This idea can be extended for the case of an infinite number of industries:

$$L^M = \sum L_t^j = \sum \frac{X_t^j}{\lambda^j A(\mu Q_t^{j-1} + Q_t^j)}$$

For $L^M < q_c$, the economy is trapped in Industry 1.
For $L^M > q_c$, the economy makes a successive transition from 1 to 2, to 3, to 4, …., and its labor productivity grows indefinitely.

- Stokey (1988) illustrated this idea in a closed economy model with a continuum of goods, where the goods are *imperfect* substitutes. At any time in point, the economy produces a finite range of goods, and this range moves up over time.

- Young (1991) explored this idea in a two-country Ricardian model with a continuum of goods, where the goods are *imperfect* substitutes (but horizontally differentiated). Two countries produce (non-overlapping) finite ranges of goods. The country with more experienced grow faster, while the other country might be worse off under Free Trade.

- For more on this issue, see Lucas (1993) "Making a Miracle" paper.

Bounded Learning and Leapfrogging: based on Bresiz-Krugman-Tsiddon (1993)

- Many trade models with LBD imply that a technologically more advanced country has advantage in achieving higher growth, often at the expense of countries behind.
- But, we have observed changes in the technological leader, from the Italian city states, to the Dutch Republic, to Great Britain and to the US.
- The following model, adopted from Bresiz et. al. (1993), suggests that this may be a natural consequence of the bounded LBD.

Two Countries: Home and Foreign, the equal size, $L = L^*$. It is assumed that $L = L^*$ is sufficiently large.

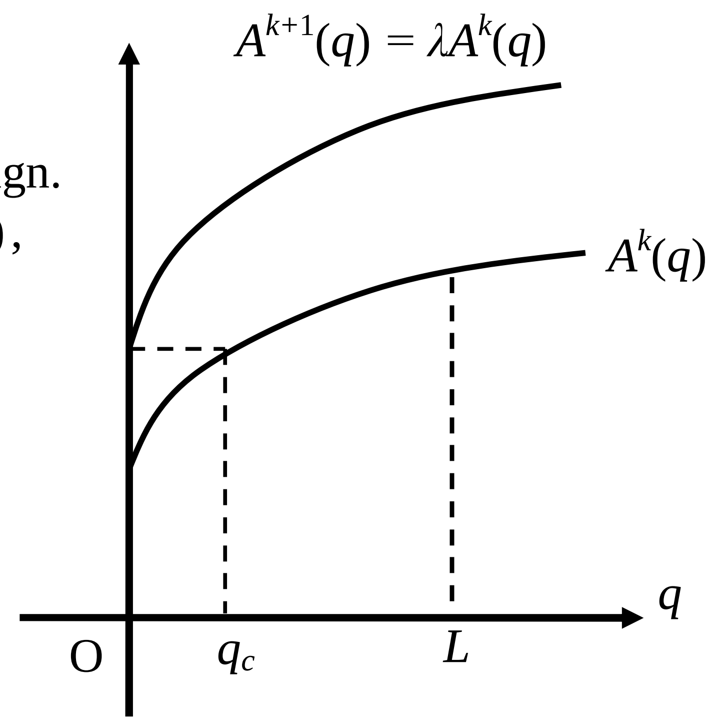
Two (Tradeable) Sectors: Agriculture & Manufacturing.

- Agriculture produces a homogenous good. We index it by 0.
- Manufacturing consists of many (perfectly substitutable) goods, indexed by j = 1, 2, …, We assume that only the first *k* goods are initially available and study the impact of an exogenous arrival of the (k+1)-th generation of the good.

Preferences: $\left(C_t^0\right)^{1-\beta}\left(C_t^M\right)^{\beta} = \left(C_t^0\right)^{1-\beta}\left(\sum_j C_t^j\right)^{\beta}$.

Technologies:

- Agricultural labor productivity is one at Home & Foreign.
- Labor productivity of Industry *j* is $A^j(Q_t^j) = \lambda^{j-1}A(Q_t^j)$, with $\lambda > 1$, where $A(\bullet)$ is strictly increasing but bounded and satisfies:



$$\frac{A(L)}{A(0)} > \frac{\beta}{1-\beta} > \frac{A(L)}{\lambda A(0)} > 1$$

and

$$\dot{Q}_t^j = \delta_j(L_t^j - Q_t^j), \text{ with } Q_t^j \leq L$$

Note: The above inequalities requires $A(0) > 0$. The assumption that $A(\bullet)$ is bounded is not necessary, but, since L is assumed to be sufficiently large, this makes it easier to satisfy the above inequalities.

Let us now demonstrate that, under these assumptions,

❖ ***Once Home specializes in the k-th M-good, and Foreign specializes in A-good, it is a steady state.***

1) As long as Home specializes in (any) M-good and Foreign specializes in A-good, the Balanced Trade condition also requires that $\beta w_t^* L^* = (1-\beta) w_t L$, or $\frac{w_t}{w_t^*} = \frac{\beta}{1-\beta}$.

2) The condition that Home specializes in k and Foreign specializes in 0 at T_0 is given by

$$\frac{\lambda^{k-1} A(Q_{T_0}^k)}{Max_{1\le j\le k}\{\lambda^{j-1} A(Q_{T_0}^{*j})\}} > \frac{w_{T_0}}{w_{T_0}^*} = \frac{\beta}{1-\beta} > 1.$$

3) Furthermore, since $X_t^j = 0$ for all $1 \le j < k$, $X_t^k > 0$, and $X_t^{*j} = 0$ for all $1 \le j \le k$,

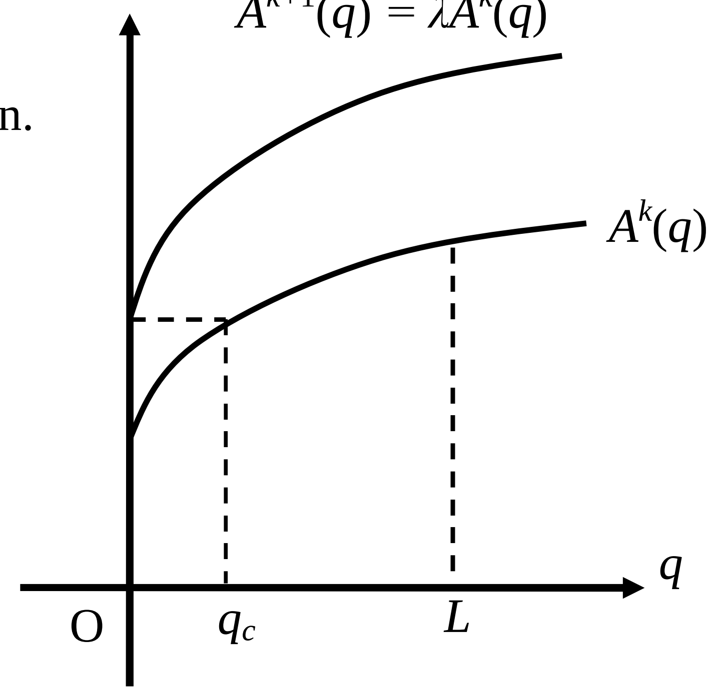
$\dot{Q}_t^j \le 0$ for all $1 \le j < k$, $\dot{Q}_t^k \ge 0$, and $\dot{Q}_t^{*j} \le 0$ for all $1 \le j \le k$

$$\Rightarrow \frac{\lambda^{k-1} A(Q_t^k)}{Max_{1\le j\le k}\{\lambda^{j-1} A(Q_t^{*j})\}} > \frac{w_t}{w_t^*} = \frac{\beta}{1-\beta} > 1 \text{ for all } t \ge T_0.$$

Thus, Home specializes in k and Foreign specializes in 0 for all $t \ge T_0$.

Suppose that the world economy has been in this situation long enough that Q_t^k has grown sufficiently close to L and Q_t^{*k} has shrunk sufficiently close to zero. Then, a new generation of the M-good, k+1, arrives at T_1. What happens?

1) Labor productivity for the new good starts at $A^{k+1}(0) = A^k(q_c)$, where $0 < q_c < L$. Thus, $A^k(Q_t^k) > A^{k+1}(0) > A^k(Q_t^{*k})$.



The k-th is more productive than the (k+1)-th at Home.
The (k+1)-th is more productive than the k-th at Foreign.

2) $\dfrac{\beta}{1-\beta} > \dfrac{A^k(Q_t^k)}{A^{k+1}(0)} \approx \dfrac{A(L)}{\lambda A(0)} > 1$ implies that the (k+1)-th industry at Foreign, whose wage is lower, can compete with the k-th industry at Home. Thus, from T_1 on, Foreign starts producing both 0 and k+1, while Home continues to specialize in k, and the relative wage satisfies

$$\frac{\beta}{1-\beta} > \frac{A(Q_t^k)}{\lambda A(Q_t^{*k+1})} = \frac{w_t}{w_t^*} > 1.$$

3) As Foreign improves its productivity in k+1 (faster than Home does in k), both countries become equally efficiency at T_2. After T_2, the patterns of comparative advantage are reversed. Foreign has CA in manufacturing, and starts specializing k+1, while Home has CA in Agriculture, and produces both 0 and k, with

$$1 > \frac{A(Q_t^k)}{\lambda A(Q_t^{*k+1})} = \frac{w_t}{w_t^*} > \frac{1-\beta}{\beta}.$$

4) This could continue until the relative manufacturing efficiency reaches to the point where Home stops producing k and starting specializing in 0, with

$$1 > \frac{A(Q_t^k)}{\lambda A(Q_t^{*k+1})} = \frac{w_t}{w_t^*} = \frac{1-\beta}{\beta}.$$

5) This situation continues as a new steady state, at least until the (k+2)-th generation of the good arrives.

In summary:

The lagging country, due to its *lack* of expertise in the existing technology, has a *comparative* advantage in the new technology, making it possible to take over the technology leader.