Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

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Last Updated: 2022-06-24; 7:37:43 PM

Princeton IES Summer Trade Workshop
June 13-16, 2022
Competitive Pressures on Heterogeneous Firms

How do competitive pressures affect selection of firms with different productivity? Or sorting across different markets?

- Melitz (2003): monopolistic competition (MC) with heterogeneous firms under CES Demand System (DS)
  - MC firms sell their products at an exogenous & common markup rate, \textit{unresponsive to competitive pressures}
  - Market size: no effect on distribution of firm types and on their behaviors; All adjustments at the extensive margin.
- Melitz-Ottaviano (2008) depart from CES using \textit{Linear DS + the outside competitive sector}

We depart from CES using \textbf{H.S.A. (Homothetic with a Single Aggregator)} DS with gross substitutes

- \textbf{Homothetic} (unlike the linear DS and most other commonly-used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block in multi-sector models with any upper-tier (incl. nonhomothetic) DS
- \textbf{Nonparametric} and \textbf{flexible} (unlike CES and \textit{translog}, which are special cases)
  - can be used to perform robustness-check for CES
  - allow for (but no need to impose) the choke price, Marshall’s 2\textsuperscript{nd} law as well as \textit{what we call} the 3\textsuperscript{rd} law
- \textbf{Tractable} due to \textbf{Single Aggregator} (unlike \textit{Kimball}, which needs two aggregators), a \textit{sufficient statistic} for competitive pressures, which acts like a \textit{magnifier of firm heterogeneity}
  - the existence & uniqueness of free-entry equilibrium with firm heterogeneity straightforward
  - simple to conduct most comparative statics without \textit{parametric} restrictions on demand or productivity distribution.
  - no need to assume zero overhead cost (unlike MO and ACDR)
- Defined by \textbf{the market share function}, for which data is readily available and easily identifiable.
Symmetric H.S.A. (Homothetic with a Single Aggregator) with Gross Substitutes
Here we consider a continuum of varieties ($\omega \in \Omega$), gross substitutes, and symmetry (Our 2017 paper for a general analysis)

Market Share of $\omega \in \Omega$ depends solely on its single relative price (= its own price/the common price aggregator)

$$s_\omega \equiv \frac{p_\omega x_\omega}{px} = \frac{\partial \ln P(p)}{\partial \ln p_\omega} = s \left( \frac{p_\omega}{A(p)} \right),$$
where

$$\int_\Omega s \left( \frac{p_\omega}{A(p)} \right) d\omega \equiv 1.$$

• $s: \mathbb{R}^+ \to \mathbb{R}^+$: the market share function, decreasing in the relative price for $s(z) > 0$ with $\lim_{z \to \bar{z}} s(z) = 0$.
  • If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(p)$ is the choke price.

• $A(p)$: the common price aggregator defined implicitly by the adding-up constraint $\int_\Omega s(p_\omega/A)d\omega \equiv 1$.
  By construction, market shares add up to one; $A(p)$ linear homogenous in $p$ for a fixed $\Omega$. A larger $\Omega$ reduces $A(p)$.

  CES
  $$s(z) = \gamma z^{1-\sigma}; \quad \sigma > 1$$

Special Cases
  Translog Cost
  $$s(z) = -\gamma \ln(z/\bar{z})$$

  Constant Pass-Through (CoPaTh)
  $$s(z) = \gamma \left[ 1 - \left( \frac{z}{\bar{z}} \right)^{1-\rho} \right]^{\frac{\rho}{1-\rho}}; \quad 0 < \rho < 1$$
$P(p)$ vs. $A(p)$

$$s_\omega \equiv \frac{\partial \ln P(p)}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(p)}\right),$$

where

$$\int_\Omega s\left(\frac{p_\omega}{A(p)}\right) d\omega \equiv 1.$$

$$\Rightarrow P(p) \propto A(p) \exp\left\{ - \int_\Omega \int_{p_\omega/A(p)}^z \frac{s(\xi)}{\xi} d\xi d\omega \right\}$$

$$\Rightarrow \frac{\partial \ln A(p)}{\partial \ln p_\omega} = \frac{[\zeta(z_\omega) - 1]s(z_\omega)}{\int_\Omega [\zeta(z_\omega') - 1]s(z_\omega') d\omega'}; \quad \frac{\partial \ln P(p)}{\partial \ln p_\omega} = s(z_\omega),$$

where $\zeta(z)$ is the price elasticity function, defined by

$$\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \varepsilon_s(z) > 1, \quad \text{for } z \in (0, \bar{z}); \lim_{z \to \bar{z}} \zeta(z) = - \lim_{z \to \bar{z}} \varepsilon_s(z) = \infty, \text{if } \bar{z} < \infty.$$

Note: $P(p)/A(p) \neq c$ for any $c > 0$, unless CES

✓ $A(p)$, the inverse measure of competitive pressures, captures cross price effects in the demand system

✓ $P(p)$, the inverse measure of TFP, captures the productivity consequences of price changes

Note: Our 2017 paper proved the integrability = the quasi-concavity of $P(p)$, iff $\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} > 0.$
Three Classes of Homothetic Demand Systems: Matsuyama-Ushchev (2017)

<table>
<thead>
<tr>
<th>Class</th>
<th>Demand Function</th>
<th>Conditions</th>
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<tbody>
<tr>
<td><strong>CES</strong></td>
<td>( s_\omega = f \left( \frac{p_\omega}{P(p)} \right) )</td>
<td></td>
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<tr>
<td><strong>H.S.A.</strong> (Homotheticity with a Single Aggregator)</td>
<td>( s_\omega = s \left( \frac{p_\omega}{A(p)} \right) ), ( \frac{P(p)}{A(p)} \neq c ), unless CES</td>
<td></td>
</tr>
<tr>
<td><strong>HDIA</strong> (Homotheticity with Direct Implicit Additivity)</td>
<td>( s_\omega = \frac{p_\omega}{P(p)} \left( \phi' \right)^{-1} \left( \frac{p_\omega}{B(p)} \right) ), ( \frac{P(p)}{B(p)} \neq c ), unless CES</td>
<td></td>
</tr>
<tr>
<td><strong>HIIA</strong> (Homotheticity with Indirect Implicit Additivity)</td>
<td>( s_\omega = \frac{p_\omega}{C(p)} \left( \frac{P(p)}{P(p)} \right)^{\theta'} ), ( \frac{P(p)}{C(p)} \neq c ), unless CES</td>
<td></td>
</tr>
</tbody>
</table>

The 3 classes are pairwise disjoint with the sole exception of CES.

Under HDIA and HIIA,
- Two aggregators are necessary.
- The free-entry equilibrium may not exist, or if it exists, may not be unique, unless we impose some strong restrictions on both productivity distributions and the price elasticity functions.
Monopolistic Competition under H.S.A.: Pricing

Pricing (Lerner) Formula

\[ p_\omega \left[ 1 - \frac{1}{\zeta(p_\omega / A)} \right] = \psi_\omega \implies p_\omega \left[ 1 - \frac{1}{\zeta(p_\omega / A)} \right] = \frac{\psi_\omega}{A} \]

\( \psi_\omega \): firm-specific marginal cost (in labor, the numeraire)

Under mild regularity conditions (A1), firms with the same \( \psi \) set the same price, so that \( p_\omega = p_\psi \)

Relative price

\[ \frac{p_\psi}{A} = z_\psi \equiv Z\left(\frac{\psi}{A}\right) \text{, an increasing function of } \frac{\psi}{A}, \text{ the normalized cost, only.} \]

Price elasticity

\[ \zeta\left(Z\left(\frac{\psi}{A}\right)\right) \equiv \sigma\left(\frac{\psi}{A}\right) > 1 \]

Markup rate

\[ \mu_\psi \equiv \frac{p_\psi}{\psi} = \frac{\sigma(\psi/A)}{\sigma(\psi/A) - 1} \equiv \mu \left(\frac{\psi}{A}\right) > 1 \]

Pass-through rate

\[ \rho_\psi \equiv \frac{\partial \ln p_\psi}{\partial \ln \psi} = \frac{\partial \ln Z(\psi/A)}{\partial \ln (\psi/A)} \equiv \varepsilon_Z \left(\frac{\psi}{A}\right) \equiv \rho \left(\frac{\psi}{A}\right) = 1 + \varepsilon_\mu \left(\frac{\psi}{A}\right) \]

are all functions of \( \psi/A \) only, continuously differentiable.

More competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
### Monopolistic Competition under H.S.A.: Revenue, Profit, & Employment

**Revenue**

\[ R_\psi = s(z_\psi)L = s\left(Z \left(\frac{\psi}{A}\right)\right)L \equiv r\left(\frac{\psi}{A}\right)L \quad \Rightarrow \quad \mathcal{E}_r\left(\frac{\psi}{A}\right) = -\left[\sigma\left(\frac{\psi}{A}\right) - 1\right] \rho\left(\frac{\psi}{A}\right) < 0 \]

**Gross) Profit**

\[ \Pi_\psi = \frac{s(z_\psi)}{\zeta(z_\psi)}L = \frac{r(\psi/A)}{\sigma(\psi/A)}L \equiv \pi\left(\frac{\psi}{A}\right)L \quad \Rightarrow \quad \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) < 0 \]

**Variable) Employment**

\[ \psi x_\psi = R_\psi - \Pi_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}L \equiv \ell\left(\frac{\psi}{A}\right)L \quad \Rightarrow \quad \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) \rho\left(\frac{\psi}{A}\right) \lesssim 0 \]

- Revenue \( r(\psi/A)L \), profit \( \pi(\psi/A)L \), employment \( \ell(\psi/A)L \) all functions of \( \psi/A \), multiplied by market size \( L \), continuously differentiable under mild regularity conditions.

  Market size affects the relative profit, revenue, and employment across firms only through its effects on \( A \).

- Both revenue \( r(\psi/A)L \) and profit \( \pi(\psi/A)L \) are always strictly decreasing in \( \psi/A \).

- Employment \( \ell(\psi/A)L \) may be nonmonotonic in \( \psi/A \).
  - If the markup rate declines with \( \psi/A \), employment cannot decline as fast as the revenue.
  - If the markup rate declines faster than the revenue, the employment is increasing in \( \psi/A \).

Again, more competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
**General Equilibrium: Existence and Uniqueness**

As in Melitz, firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$; cdf with the support, $\left(\underline{\psi}, \overline{\psi}\right) \subset (0, \infty)$, and pay the overhead $F > 0$ to stay & produce.

**Cutoff Rule:** stay iff $\psi < \psi_c$; exit if $\psi > \psi_c$, where
\[
\pi \left( \frac{\psi_c}{A} \right) L = F
\]
positively-sloped $A \downarrow$ (more competitive pressures) $\Rightarrow \psi_c \downarrow$ (tougher selection)

**Free Entry Condition:**
\[
F_e = \int_{\underline{\psi}}^{\psi_c} \left[ \pi \left( \frac{\psi}{A} \right) L - F \right] dG(\psi)
\]
negative-sloped. $A \downarrow$ (more competitive pressures) and $\psi_c \downarrow$ (tougher selection) both make entry less attractive.

$A = A(p)$ and $\psi_c$: uniquely determined, respond continuously to $F_e/L$ & $F/L$ under mild regularity conditions.

(This proof of unique existence applies also to the Melitz model under CES.)
Equilibrium Masses of the Firms under H.S.A.
With $A$ and $\psi_c$ determined, the adding-up constraint pins down masses of active firms, $MG(\psi_c)$.

Mass of active firms
= the measure of $\Omega$

\[
MG(\psi_c) = \left[ \int_{\psi}^{\psi_c} r \left( \frac{\psi}{A} \right) \frac{dG(\psi)}{G(\psi_c)} \right]^{-1} = \left[ \int_0^1 r \left( \pi^{-1} \frac{F}{L} \right) \xi d\tilde{G}(\xi; \psi_c) \right]^{-1} > 0
\]

where $\xi \equiv \psi / \psi_c$ and

\[
\tilde{G}(\xi; \psi_c) \equiv \frac{G(\psi_c \xi)}{G(\psi_c)}
\]

is the cdf of $\xi$, conditional on $\xi \leq 1$.

**Lemma 1:** $E'_g(\psi) < 0 \Rightarrow E'_G(\psi) < 0$ generally; $E'_g(\psi) \geq 0 \Rightarrow E'_G(\psi) \geq 0$, with some additional conditions.

**Lemma 2:** A lower $\psi_c$ (tougher selection) shifts $\tilde{G}(\xi; \psi_c)$ to the right (left)

- o in the MLR ordering if $E'_g(\psi) < (>)0$.
- o in the FSD ordering if $E'_g(\psi) < (>)0$.

$\tilde{G}(\xi; \psi_c)$ is independent of $\psi_c$ if $E_g(\psi) \& E_G(\psi)$ are constant, i.e., power-distributed cost $\Leftrightarrow$ Pareto-productivity

A lower $\psi_c$ (tougher selection) shifts $\tilde{G}(\xi; \psi_c)$ to the right if Fréchet, Weibull, or Lognormal.

**Note:** The equilibrium under H.S.A. can be solved recursively. Under HDIA/HIIA, the three variables, $\psi_c$ and the two aggregates, need to be solved for simultaneously.
Revisiting Melitz (2003) under CES: $s(z) = \gamma z^{1-\sigma}$

Pricing:
\[
\mu \left( \frac{\psi}{A} \right) = \frac{\sigma}{\sigma - 1} > 1 \Rightarrow \rho \left( \frac{\psi}{A} \right) = 1
\]
\[
\Rightarrow \varepsilon_r \left( \frac{\psi}{A} \right) = \varepsilon_\pi \left( \frac{\psi}{A} \right) = \varepsilon_\ell \left( \frac{\psi}{A} \right) = 1 - \sigma < 0.
\]

Cutoff Rule:
\[
c_0 L \left( \frac{\psi_c}{A} \right)^{1-\sigma} = F,
\]

Free Entry Condition:
\[
\int_{\psi_c}^{\psi} \left[ c_0 L \left( \frac{\psi}{A} \right)^{1-\sigma} - F \right] dG(\psi) = F_e,
\]

with $c_0 > 0$. As $L$ changes, the intersection moves along
\[
\int_{\psi_c}^{\psi} \left[ \left( \frac{\psi}{\psi_c} \right)^{1-\sigma} - 1 \right] dG(\psi) = \frac{F_e}{F}.
\]

**Proposition 1:** Under CES,
- $L \uparrow$ keeps $\psi_c$ unaffected; increases both $M$ and $MG(\psi_c)$ *proportionately*;
- $F_e \downarrow$ reduces $\psi_c$; increases $M$; increases (decreases) $MG(\psi_c)$ if $\varepsilon_G'(<)>0$;
- $F \downarrow$ increases $\psi_c$; increases $MG(\psi_c)$; increases (decreases) $M$ if $\varepsilon_G'(<)>0$.
Marshall’s 2nd Law: Cross-Sectional Implications (Proposition 2)

(A2): $\zeta(z_\psi)$ is increasing in $z_\psi \equiv p_\psi/A = Z(\psi/A)$

- **Price elasticity** $\zeta(Z(\psi/A)) \equiv \sigma(\psi/A)$ increasing in $\psi/A$;
  high-$\psi$ firms operate at more elastic parts of demand curve.

  - **Markup Rate**, $\mu(\psi/A)$, decreasing in $\psi/A \iff \mathcal{E}_\mu(\psi/A) < 0$
    high-$\psi$ firms charge lower markup rates.
  - **Incomplete Pass-Through**: The pass-through rate, $\rho(\psi/A) = 1 + \mathcal{E}_\mu(\psi/A) < 1$.

- **Procompetitive effect of entry/Strategic complementarity in pricing**, $\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho(\psi/A) > 0$.
  Firms set the price lower under more competitive pressures ($A = A(p) \downarrow$), due to either a larger $\Omega$ and/or a lower $p$

**Lemma 5**: $f(\psi/A)$ log-super(sub)modular in $\psi$ & $A \iff \mathcal{E}_f'() < (>)0 \iff \ln f(e^{\ln(\psi/A)})$ concave (convex) in $\ln(\psi/A)$

- **Profit**, $\pi(\psi/A)L$, always decreasing, strictly log-supermodular in $\psi$ and $A$.
  $A \downarrow \rightarrow$ a proportionately larger decline in profit for high-$\psi$ firms $\rightarrow$ Larger dispersion of profit
3rd Law: Cross-Sectional Implications (Propositions 3, 4, and 5)

In addition to A2, if we further assume, with some empirical support, e.g. Berman et.al.(2012), Amiti et.al.(2019),

\[ \rho(\psi/A) = 1 + \varepsilon_\mu(\psi/A) \]

is weak(strictly)ly increasing--we call it **Weak (Strong) 3rd Law**.

Under translog, \( \rho(\psi/A) \) is strictly decreasing, violating A3

- **Markup rate**, \( \mu(\psi/A) \), decreasing under A2, **log-submodular** in \( \psi \) & \( A \) under A3. For strong A3, it is strict and \( A \downarrow \) \( \rightarrow \) a proportionately smaller decline in markup rate for high-\( \psi \) firms \( \rightarrow \) smaller dispersion of markup rate

- **Revenue**, \( r(\psi/A)L \), always decreasing, **strictly log-supermodular** in \( \psi \) & \( A \) under **weak A3**
  
  \( A \downarrow \) \( \rightarrow \) a proportionately larger decline in revenue for high-\( \psi \) firms \( \rightarrow \) Larger dispersion of revenue

- **Employment**, \( \ell(\psi/A)L = \frac{r(\psi/A)}{\mu(\psi/A)}L \), **hump-shaped** in \( \psi/A \), **strictly log-supermodular** in \( \psi \) & \( A \) under **weak A3**
  
  Employment is increasing in \( \psi \) across all active firms with a large enough overhead/market size ratio.

  \( A \downarrow \) Employment up for the most productive firms.

- **Pass-through rate**, \( \rho(\psi/A) \), **strictly log-submodular** in \( \psi \) & \( A \) for a small enough \( \bar{Z} \) under strong A3
  
  \( A \downarrow \) \( \rightarrow \) a proportionately smaller increase in the pass-through rate for low-\( \psi \) firms among the active.
Cross-Sectional Implications of More Competitive Pressures ($A \downarrow$)

**Profit Function:** $\Pi_\psi = \pi(\psi / A)L$
- always decreasing in $\psi$
- strictly log-supermodular under $A2$
- $A \downarrow$ with $L$ fixed shifts down with a steeper slope at each $\psi$;
- $A \downarrow$ due to $L \uparrow$, a parallel shift up, a single-crossing.

**Markup Rate Function:** $\mu_\psi = \mu(\psi / A) > 1$
- decreasing in $\psi$ under $A2$
- weakly log-submodular under Weak $A3$
- strictly log-submodular under Strong $A3$
- $A \downarrow$ shifts down with a flatter slope at each $\psi$

With $\ln \psi$ in the horizontal axis, $A \downarrow$ causes a parallel leftward shift of the graphs in these figures.

$f(\psi / A)$ is strictly log-super(sub)modular in $\psi$ & $A \iff \ln f(\psi / A)$ is (strictly) concave( convex) in $\ln(\psi / A)$.

Under Weak A3, $R_\psi = r(\psi / A)L$, strictly log-supermodular and shares similar properties with $\pi(\psi / A)L$. 
**Employment Function:** \( \ell(\psi/A)L = r(\psi/A)L/\mu(\psi/A) \)
- Hump-shaped in \( \psi \) under \( A2 \) and weak \( A3 \).
  - \( A \downarrow \) shifts up (down) for a low (high) \( \psi \) with \( A \downarrow \)
- Strictly log-supermodular under weak \( A3 \)
  - for \( A \downarrow \) with a fixed \( L \); for \( A \downarrow \) caused by \( L \uparrow \)
- Single-crossing even with a fixed \( L \)

**Pass-Through Rate Function:** \( \rho_\psi = \rho(\psi/A) \)
- \( \rho(\psi/A) < 1 \) under \( A2 \), hence it cannot be strictly log-supermodular for a higher range of \( \psi/A \)
- Increasing in \( \psi \) under Strong \( A3 \)
- Strictly log-submodular for a lower range of \( \psi/A \) under \( A2 \) and Strong \( A3 \) \( \Rightarrow A \downarrow \) shifts up with a steeper slope at each \( \psi \) with a small enough \( \tilde{z} \).

In summary, more competitive pressures \( (A \downarrow) \)
- \( \mu(\psi/A) \downarrow \) under \( A2 \) & \( \rho(\psi/A) \uparrow \) under strong \( A3 \)
- Profit, Revenue, Employment become more concentrated among the most productive.
GE Comparative Statics: Selection (in a single-market setting) Propositions 6, 7, 8, & 9.

Effects of $F_e \downarrow$

Effects of $L \uparrow$ if $\sigma'(\cdot) > 0$ (i.e., A2)

Effects of $F \downarrow$ if $\ell'(\cdot) > 0$

Prop.7: $L \uparrow$ under A2: profit up for low-$\psi$ & down for high-$\psi$. (Similarly on the revenue under A2 and the weak A3)

Prop.8: All 3 cases lead to $\psi_c \downarrow$ & $A \downarrow$, creating a non-trivial composition effect

- Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with lower $\mu(\psi/A)$ drop out.

- Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with higher $\rho(\psi/A)$ drop out.

The average markup (or pass-through) rate can go either way, with $F_e \downarrow +$ Pareto-productivity a knife-edge case

More competition, which causes more concentration, may result in the rise of markup.

Prop.9: The effects on $M$ & $MG(\psi_c)$ depend on whether $E_G(\psi) \equiv \psi g(\psi)/G(\psi)$ is decreasing, constant, or increasing.
GE Implications: Sorting (in a multi-market setting);

**Proposition 10: Assortative Matching**

More competitive pressures in larger markets:

\[ L_1 > L_2 > \cdots > L_j > 0 \Rightarrow 0 < A_1 < A_2 < \cdots < A_j < \infty \]

Under A2, more efficient firms sort themselves into larger markets

Firms \( \psi \in (\psi_{j-1}, \psi_j) \) entering market- \( j \)

**Markup Rate across markets under A2**

**Pass-Through Rate across markets under strong A3**

**Proposition 11: The Composition Effect:** examples with Pareto-productivity such that

- The average markup rates higher (the average pass-through rates lower under Strong A3) in larger (more competitive) markets
- A decline in \( F_e \) causes uniform declines in \( \psi_j \) & \( A_j \) with the average markup/pass-through rates unchanged.

A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.
### Three Parametric Families of H.S.A. (Appendix D)

<table>
<thead>
<tr>
<th>Generalized Translog</th>
<th>[ s(z) = \gamma \left( -\frac{\sigma - 1}{\eta} \ln \left( \frac{z}{\bar{z}} \right) \right)^\eta ; z &lt; \bar{z} \equiv \beta e^{\sigma - 1} ]</th>
<th>[ 1 - \frac{1}{\zeta(z)} = \frac{\eta}{\eta - \ln \left( \frac{z}{\bar{z}} \right)} \Rightarrow \mathcal{E}<em>\mu(\cdot) &lt; 0; \mathcal{E}</em>\mu'(\cdot) &lt; 0; ] satisfying A2; violating A3.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Translog is the special case where ( \eta = 1 ). CES is the limit case, as ( \eta \to \infty ), while holding ( \beta &gt; 0 ) and ( \sigma &gt; 1 ) fixed.</td>
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<thead>
<tr>
<th>Constant Pass-Through (CoPaTh)</th>
<th>[ s(z) = \gamma \sigma^{1-\rho} \left[ 1 - \left( \frac{z}{\bar{z}} \right)^{\frac{1-\rho}{\rho}} \right]^{\frac{\rho}{1-\rho}} ; \bar{z} \equiv \beta \left( \frac{\sigma}{\sigma - 1} \right)^{\frac{\rho}{1-\rho}} ]</th>
<th>[ 1 - \frac{1}{\zeta(z)} = \left( \frac{z}{\bar{z}} \right)^{\frac{1-\rho}{\rho}} \Rightarrow \mathcal{E}<em>\mu(\cdot) &lt; 0; \mathcal{E}</em>\mu'(\cdot) = 0 ] satisfying A2 &amp; weak A3; violating strong A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES is the limit case, as ( \rho \to 1 ), while holding ( \beta &gt; 0 ) and ( \sigma &gt; 1 ) fixed.</td>
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<tr>
<th>Power Elasticity of Markup Rate (Frechet Inverse Markup Rate)</th>
<th>[ s(z) = \exp \left[ \int_{z_0}^{z} \frac{c}{c - \exp \left[ -\frac{\kappa \xi^{-\lambda}}{\lambda} \right] \exp \left[ \frac{\kappa \xi^{-\lambda}}{\lambda} \frac{\xi}{\lambda} \right] \xi} \right] ]</th>
<th>[ 1 - \frac{1}{\zeta(z)} = c \exp \left[ \frac{\kappa \xi^{-\lambda}}{\lambda} \right] \exp \left[ -\frac{\kappa \xi^{-\lambda}}{\lambda} \right] \Rightarrow \mathcal{E}<em>\mu(\cdot) &lt; 0; \mathcal{E}</em>\mu'(\cdot) &gt; 0 ] satisfying A2 and strong A3 for ( \kappa &gt; 0 ) and ( \lambda &gt; 0 ).</th>
</tr>
</thead>
<tbody>
<tr>
<td>CES for ( \kappa = 0 ); ( \bar{z} = \infty ); ( c = 1 - \frac{1}{\sigma} ); CoPaTh for ( \bar{z} &lt; \infty ); ( c = 1; \kappa = \frac{1-\rho}{\rho} &gt; 0 ), and ( \lambda \to 0 ).</td>
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</tbody>
</table>
(Highly Selective) Literature Review

H.S.A. Demand System: Matsuyama-Ushchev (2017)

MC with Heterogeneous Firms: Melitz (2003) and many others: Melitz-Redding (2015) for a survey

MC under non-CES demand systems: Thisse-Ushchev (2018) for a survey
- Nonhomothetic non-CES:
  - $U = \int_\Omega u(x_\omega) d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)


Selection of Heterogeneous Firms through Competitive Pressures Melitz-Ottaviano (2008), Baqee-Fahri-Sangani (2021)

Sorting of Heterogeneous Firms Across Markets:
- Reduced Form/Partial Equilibrium; Mrázová-Neary (2019), Nocke (2006)

Log-Super(Sub)modularity: Costinot (2009), Costinot-Vogel (2015)