Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

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Introduction

Competitive Pressures on Heterogeneous Firms

Main Questions: How do an increase in ***competitive pressures***, caused by entry of new firms, due to lower ***entry cost*** or larger ***market size***, affect firms with different productivity.

- Selection of firms
- Distribution of firm size (in revenue, profit and employment), Distribution of markup and pass-through rates, etc.
- Sorting of firms across markets with different market sizes

Existing Monopolistic Competition Models with Heterogenous Firms

- Melitz (2003): under **CES** Demand System (DS)
    - MC firms sell their products at an exogenous & common markup rate, *unresponsive to competitive pressures*
    - Market size: no effect on distribution of firm types nor their behaviors; All adjustments at *the extensive margin*.

  Inconsistent with some evidence for
    - An increase in the production cost leads to less than proportional increase in the price (the pass-through rate < 1)
    - More productive firms have higher markup rates
    - More productive firms have lower pass-through rates
- Melitz-Ottaviano (2008) departs from CES using **Linear DS + the outside competitive sector**, which comes with its own restrictions.

This Paper: Melitz under **H.S.A.** Demand System as a theoretical framework to study the implications of departing from CES in the direction consistent with the evidence on the impact of competitive pressures on heterogeneous firms.

Symmetric H.S.A. (Homothetic with a Single Aggregator) DS with Gross Substitutes

Think of a competitive final goods industry generating demand for a continuum of **intermediate inputs** $\omega \in \Omega$, with **CRS production function:** $X = X(\mathbf{x});\ \mathbf{x} = \{x_\omega;\ \omega \in \Omega\} \Leftrightarrow$ **Unit cost function,** $P = P(\mathbf{p});\ \mathbf{p} = \{p_\omega;\ \omega \in \Omega\}$.

Market share of ω depends *solely* on a single variable, its own price normalized by the *common* price aggregator

$$s_\omega \equiv \frac{p_\omega x_\omega}{\mathbf{px}} = \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right), \qquad \text{where} \qquad \int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

- $s: \mathbb{R}_{++} \to \mathbb{R}_+$: **the market share function**, decreasing in the **normalized price** for $s(z) > 0$ with $\lim_{z\to\bar{z}} s(z) = 0$.
  - If $\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty$, $\bar{z}A(\mathbf{p})$ is the **choke price**.
- $A = A(\mathbf{p})$: the **common price aggregator** defined implicitly by **the adding-up constraint** $\int_\Omega s(p_\omega/A)d\omega \equiv 1$.
  $A(\mathbf{p})$ linear homogenous in $\mathbf{p}$ for a fixed Ω. A larger Ω reduces $A(\mathbf{p})$.

	CES	$s(z) = \gamma z^{1-\sigma};$	$\sigma > 1$
Special Cases	**Translog Cost Function**	$s(z) = \gamma \max\{-\ln(z/\bar{z}), 0\};$	$\bar{z} < \infty$
	Constant Pass Through (CoPaTh)	$s(z) = \gamma \max\left\{\left[\sigma + (1-\sigma)z^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}, 0\right\}$	$0 < \rho < 1$

As $\rho \nearrow 1$, CoPaTh converges to CES with $\bar{z}(\rho) \equiv (\sigma/(\sigma-1))^{\frac{\rho}{1-\rho}} \to \infty$.

$P(\mathbf{p})$ vs. $A(\mathbf{p})$

Definition: $$s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = s\left(\frac{p_\omega}{A(\mathbf{p})}\right) = s(z_\omega) \quad \text{where} \quad \int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega \equiv 1.$$

By differentiating the adding-up constraint, $$\frac{\partial \ln A(\mathbf{p})}{\partial \ln p_\omega} = \frac{[\zeta(z_\omega) - 1]s(z_\omega)}{\int_\Omega [\zeta(z_{\omega'}) - 1]s(z_{\omega'})d\omega'};$$

where

Price Elasticity Function: $$\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \mathcal{E}_s(z) > 1 \text{ for } z \in (0, \bar{z}); \quad \lim_{z \to \bar{z}} \zeta(z) = \infty, \text{if } \bar{z} < \infty.$$

By integrating the definition, $$\frac{A(\mathbf{p})}{P(\mathbf{p})} = c \exp\left[\int_\Omega s\left(\frac{p_\omega}{A(\mathbf{p})}\right) \Phi\left(\frac{p_\omega}{A(\mathbf{p})}\right) d\omega\right], \quad \text{where} \quad \Phi(z) \equiv \frac{1}{s(z)} \int_z^{\bar{z}} \frac{s(\xi)}{\xi} d\xi$$

Note: $A(\mathbf{p})/P(\mathbf{p})$ is not constant, unless CES $\Leftrightarrow \zeta(z) = \sigma \Leftrightarrow s(z) = \gamma z^{1-\sigma} \Leftrightarrow \Phi(z) = 1/(\sigma - 1)$.

- ✓ $A(\mathbf{p})$, the inverse measure of *competitive pressures*, captures *cross price effects* in the demand system.
- ✓ $P(\mathbf{p})$, the inverse measure of TFP, captures the *productivity consequences* of price changes.
- ✓ $\Phi(z)$, the measure of "love for variety."

Note: Our 2017 paper proved the integrability = the quasi-concavity of $P(\mathbf{p})$, iff $\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} > 0$.

Why H.S.A.

- **Homothetic** (unlike the linear DS and most other commonly used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block in multi-sector models with any upper-tier (incl. nonhomothetic) DS

- **Nonparametric** and **flexible** (unlike **CES** and **translog**, which are special cases)
  - can be used to perform robustness-check for CES
  - allow for (but no need to impose)
    - ✓ the choke price,
    - ✓ **Marshall's 2nd law** (Price elasticity is increasing in price) → more productive firms have higher markup rates
    - ✓ *what we call* **the 3rd law** (the rate of increase in the price elasticity is decreasing in price) → more productive firms have lower pass-through rates.

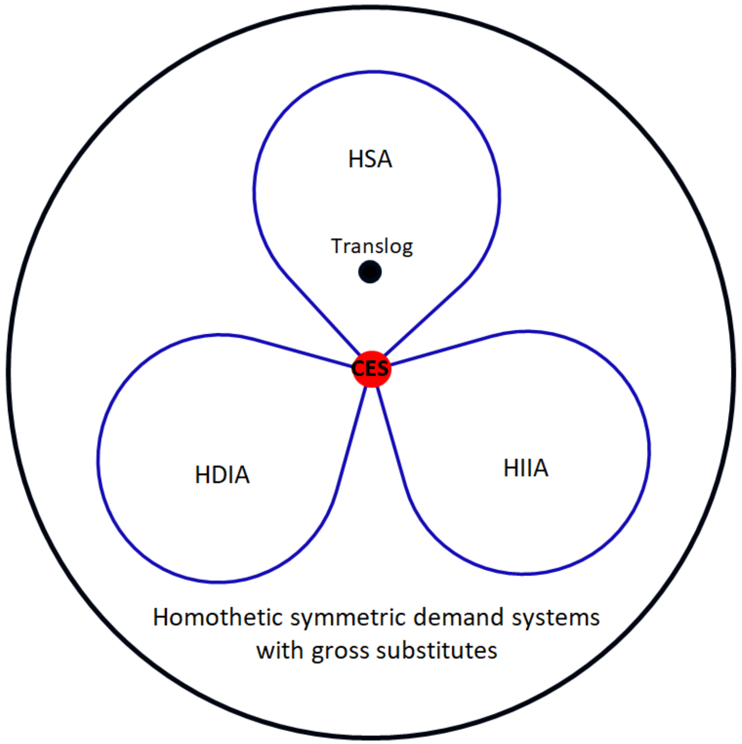
- **Tractable** due to **Single Aggregator** (unlike **Kimball**, which needs two aggregators), a *sufficient statistic* for competitive pressures, which acts like a *magnifier of firm heterogeneity*
  - the existence & uniqueness of free-entry equilibrium with firm heterogeneity straightforward
  - simple to conduct most comparative statics without *parametric* restrictions on demand or productivity distribution.
  - no need to assume zero overhead cost (unlike MO and ACDR)

- Defined by **the market share function,** for which data is readily available and easily identifiable.

Three Classes of Homothetic Demand Systems: Matsuyama-Ushchev (2017)

Here we consider a **continuum** of varieties ($\omega \in \Omega$), **gross substitutes**, and **symmetry**

CES	$s_\omega \equiv \frac{\partial \ln P(\mathbf{p})}{\partial \ln p_\omega} = f\left(\frac{p_\omega}{P(\mathbf{p})}\right) \propto \left(\frac{p_\omega}{P(\mathbf{p})}\right)^{1-\sigma}$	
H.S.A. (Homotheticity with a Single Aggregator)	$s_\omega = s\left(\frac{p_\omega}{A(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{A(\mathbf{p})} \neq c$, unless CES
HDIA (Homotheticity with Direct Implicit Additivity) Kimball is a special case:	$s_\omega = \frac{p_\omega}{P(\mathbf{p})}(\phi')^{-1}\left(\frac{p_\omega}{B(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{B(\mathbf{p})} \neq c$, unless CES
HIIA (Homotheticity with Indirect Implicit Additivity)	$s_\omega = \frac{p_\omega}{C(\mathbf{p})}\theta'\left(\frac{p_\omega}{P(\mathbf{p})}\right)$,	$\frac{P(\mathbf{p})}{C(\mathbf{p})} \neq c$, unless CES



The 3 classes are pairwise disjoint with the sole exception of CES.

Under HDIA and HIIA, unlike HSA

- Two aggregators are necessary.
- The free-entry equilibrium may not exist, or if it exists, may not be unique, unless we impose some strong restrictions on both productivity distributions and the price elasticity functions.

(Highly Selective) Literature Review

Non-CES Demand Systems: Matsuyama (2023) for a survey; **H.S.A. Demand System:** Matsuyama-Ushchev (2017)

MC with Heterogeneous Firms: Melitz (2003) and many others: Melitz-Redding (2015) for a survey

MC under non-CES demand systems: Thisse-Ushchev (2018) for a survey

- *Nonhomothetic non-CES:*
  - $U = \int_\Omega u(x_\omega)d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
  - *Linear-demand system:* Ottaviano-Tabuchi-Thisse (2002)
- *Homothetic non-CES:* Feenstra (2003), Kimball (1995)
- *H.S.A.* Matsuyama-Ushchev (2020a,b, 2022), Kasahara-Sugita (2020), Grossman-Helpman-Lhuiller (2021), Fujiwara-Matsuyama (2022)

Empirical Evidence: *The 2nd Law:* DeLoecker-Goldberg (14), Burstein-Gopinath (14); *The 3rd Law:* Berman et.al.(12), Amiti et.al.(19), *Market Size Effects:* Campbell-Hopenhayn(05); *Rise of markup:* Autor et.al.(20), DeLoecker et.al.(20)

Selection of Heterogeneous Firms through Competitive Pressures Melitz-Ottaviano (2008), Baqaee-Fahri-Sangani (2021)

Sorting of Heterogeneous Firms Across Markets:

- *Reduced Form/Partial Equilibrium***;** Mrázová-Neary (2019), Nocke (2006)
- *General Equilibrium***:** Baldwin-Okubo (2006), Behrens-Duranton-RobertNicoud (2014), Davis-Dingel (2019), Gaubert (2018), Kokovin et.al. (2022)

Log-Super(Sub)modularity: Costinot (2009), Costinot-Vogel (2015)

Structure of the Talk

- Introduction
- Monopolistic Competition under H.S.A.
- Selection of Heterogenous Firms: A Single Market Setting
    - Existence and Uniqueness
    - Cross-Sectional Implications under the 2^{nd} & 3rd Laws
    - Comparative Statics: General Equilibrium Effects
- Sorting of Heterogenous Firms: A Multi-Market Setting
- Appendix: Some Parametric Families of H.S.A.

Monopolistic Competition under H.S.A.

Pricing: Markup and Pass-Through Rates

Lerner Pricing Formula

$$p_\omega\left[1-\frac{1}{\zeta(p_\omega/A)}\right]=\psi_\omega \Rightarrow \frac{p_\omega}{A}\left[1-\frac{1}{\zeta(p_\omega/A)}\right]=\frac{\psi_\omega}{A},$$

ψ_ω: *firm-specific* marginal cost (in labor, the numeraire)
Under the mild regularity condition, LHS is monotone → firms with the same ψ set the same price → $p_\omega = p_\psi$.

Normalized price $\frac{p_\psi}{A}=z_\psi\equiv Z\left(\frac{\psi}{A}\right),$ an increasing function of ψ/A, the *normalized cost*, only.

Price elasticity

$$\zeta\left(Z\left(\frac{\psi}{A}\right)\right)\equiv\sigma\left(\frac{\psi}{A}\right)>1$$

Markup rate

$$\mu_\psi\equiv\frac{p_\psi}{\psi}=\frac{\sigma(\psi/A)}{\sigma(\psi/A)-1}\equiv\mu\left(\frac{\psi}{A}\right)>1$$

Pass-through rate

$$\rho_\psi\equiv\frac{\partial\ln p_\psi}{\partial\ln\psi}=\frac{d\ln Z(\psi/A)}{d\ln(\psi/A)}\equiv\mathcal{E}_Z\left(\frac{\psi}{A}\right)\equiv\rho\left(\frac{\psi}{A}\right)=1+\mathcal{E}_\mu\left(\frac{\psi}{A}\right)$$

are all functions of ψ/A *only,* continuously differentiable.

- Market size $L=\mathbf{px}$ affects the pricing behaviors of firms only through its effects on A.
- More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

unless CES, where $\sigma(\cdot)=\sigma;\ \mu(\cdot)=\mu;\ \rho(\cdot)=1,$

Revenue, Profit, & Employment

Revenue $$R_\psi = s(z_\psi)L = s\left(Z\left(\frac{\psi}{A}\right)\right)L \equiv r\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_r\left(\frac{\psi}{A}\right) = -\left[\sigma\left(\frac{\psi}{A}\right) - 1\right]\rho\left(\frac{\psi}{A}\right) < 0$$

(Gross) Profit $$\Pi_\psi = \frac{s(z_\psi)}{\zeta(z_\psi)}L = \frac{r(\psi/A)}{\sigma(\psi/A)}L \equiv \pi\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right) < 0$$

(Variable) Employment $$\psi x_\psi = R_\psi - \Pi_\psi = \frac{r(\psi/A)}{\mu(\psi/A)}L \equiv \ell\left(\frac{\psi}{A}\right)L \quad \Longrightarrow \quad \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1 - \sigma\left(\frac{\psi}{A}\right)\rho\left(\frac{\psi}{A}\right) \lesseqgtr 0$$

- Revenue $r(\psi/A)L$, profit $\pi(\psi/A)L$, employment $\ell(\psi/A)L$ all functions of ψ/A, multiplied by **market size** L, continuously differentiable under mild regularity conditions.
- Their elasticities $\mathcal{E}_r(\cdot), \mathcal{E}_\pi(\cdot)$ and $\mathcal{E}_\ell(\cdot)$ depend solely on $\sigma(\cdot)$ and $\rho(\cdot)$.

  More competitive pressures, a lower A, act like a magnifier of firm heterogeneity.

  Market size affects the distribution of the profit, revenue and employment across firms only via its effects on A.

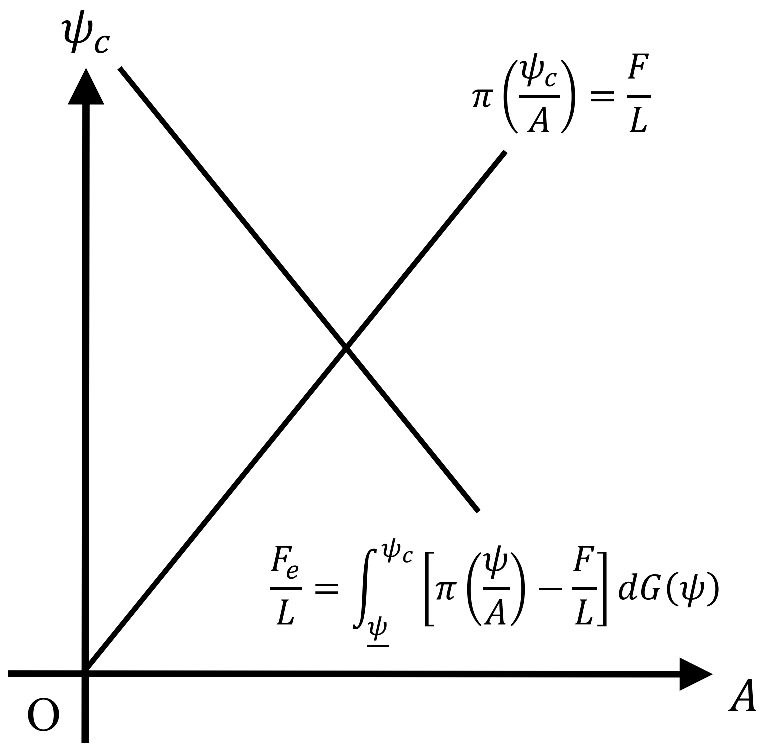
unless CES, where $\mathcal{E}_r(\cdot) = \mathcal{E}_\pi(\cdot) = \mathcal{E}_\ell(\cdot) = 1 - \sigma < 0$,

- Both revenue $r(\psi/A)L$ and profit $\pi(\psi/A)L$ are always strictly decreasing in ψ/A.
- Employment $\ell(\psi/A)L$ may be nonmonotonic in ψ/A.
  - If the markup rate declines with ψ/A, employment cannot decline as fast as the revenue.
  - If the markup rate declines faster than the revenue, the employment is *increasing* in ψ/A.

Selection of Heterogenous Firms: A Single-Market Setting

General Equilibrium: Existence and Uniqueness

As in the closed economy Melitz, Market size = total labor supply is $L > 0$
Ex-ante homogeneous firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$, cdf whose support, $(\underline{\psi}, \bar{\psi}) \subset (0, \infty)$,
After learning its ψ, decidewhether to pay the overhead $F > 0$ to stay & produce.



Cutoff Rule: stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\pi\left(\frac{\psi_c}{A}\right) L = F$$

positive-sloped $A \downarrow$ (more competitive pressures) $\Longrightarrow \psi_c \downarrow$ (tougher selection)

Free Entry Condition:

$$F_e = \int_{\underline{\psi}}^{\psi_c} \left[\pi\left(\frac{\psi}{A}\right) L - F\right] dG(\psi)$$

negative-sloped. $A \downarrow$ (more competitive pressures) and $\psi_c \downarrow$ (tougher selection) both make entry less attractive.

$A = A(\mathbf{p})$ and ψ_c: uniquely determined, respond continuously to F_e/L & F/L under mild regularity conditions.
(This proof of unique existence applies also to the Melitz model under CES.)

Equilibrium Mass of Firms under H.S.A. With A & ψ_c determined, from the adding-up constraint,

Mass of active firms
= the measure of Ω.

$$MG(\psi_c) = \left[\int_{\underline{\psi}}^{\psi_c} r\left(\frac{\psi}{A}\right)\frac{dG(\psi)}{G(\psi_c)}\right]^{-1} = \left[\int_{\underline{\xi}}^{1} r\left(\frac{\psi_c}{A}\xi\right) d\tilde{G}(\xi;\psi_c)\right]^{-1} > 0$$

where $\tilde{G}(\xi;\psi_c) \equiv \frac{G(\psi_c\xi)}{G(\psi_c)}$ is the cdf of $\xi \equiv \psi/\psi_c$, conditional on $\underline{\xi} \equiv \underline{\psi}/\psi_c < \xi \leq 1$.

Lemma 1: $\mathcal{E}_g'(\psi) < 0 \Longrightarrow \mathcal{E}_G'(\psi) < 0$ generally; $\mathcal{E}_g'(\psi) \geq 0 \Longrightarrow \mathcal{E}_G'(\psi) \geq 0$, with some additional conditions.
Lemma 2: A lower ψ_c (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the right (left) in the MLR ordering if $\mathcal{E}_g'(\psi) < (>)0$ and in the FSD ordering if $\mathcal{E}_G'(\psi) < (>)0$.

$\tilde{G}(\xi;\psi_c)$ is independent of ψ_c if $\mathcal{E}_g(\psi)$ & $\mathcal{E}_G(\psi)$ are constant $\Leftrightarrow$ $G(\psi) = (\psi/\bar{\psi})^{\kappa}$ $\Leftrightarrow$ Pareto-productivity

A lower ψ_c (tougher selection) shifts $\tilde{G}(\xi;\psi_c)$ to the right if Fréchet, Weibull, or Lognormal.

Equilibrium TFP under H.S.A.

$$\ln\left(\frac{X}{L}\right) = \ln\left(\frac{1}{P}\right) = \ln\left(\frac{c}{A}\right) + \frac{\int_{\underline{\psi}}^{\psi_c} \Phi\big(Z(\psi/A)\big)\, r(\psi/A) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} r(\psi/A) dG(\psi)} = \ln\left(\frac{c}{A}\right) + \frac{\int_{\underline{\xi}}^{1} \Phi\left(Z\left(\frac{\psi_c}{A}\xi\right)\right) r\left(\frac{\psi_c}{A}\xi\right) d\tilde{G}(\xi;\psi_c)}{\int_{\underline{\xi}}^{1} r\left(\frac{\psi_c}{A}\xi\right) d\tilde{G}(\xi;\psi_c)}.$$

Equilibrium can be solved recursively under H.S.A.!!!
Under HDIA/HIIA, the 3 variables, ψ_c & the two aggregates, need to be solved for simultaneously.

Revisiting Melitz (2003) under CES: $s(z) = \gamma z^{1-\sigma}$

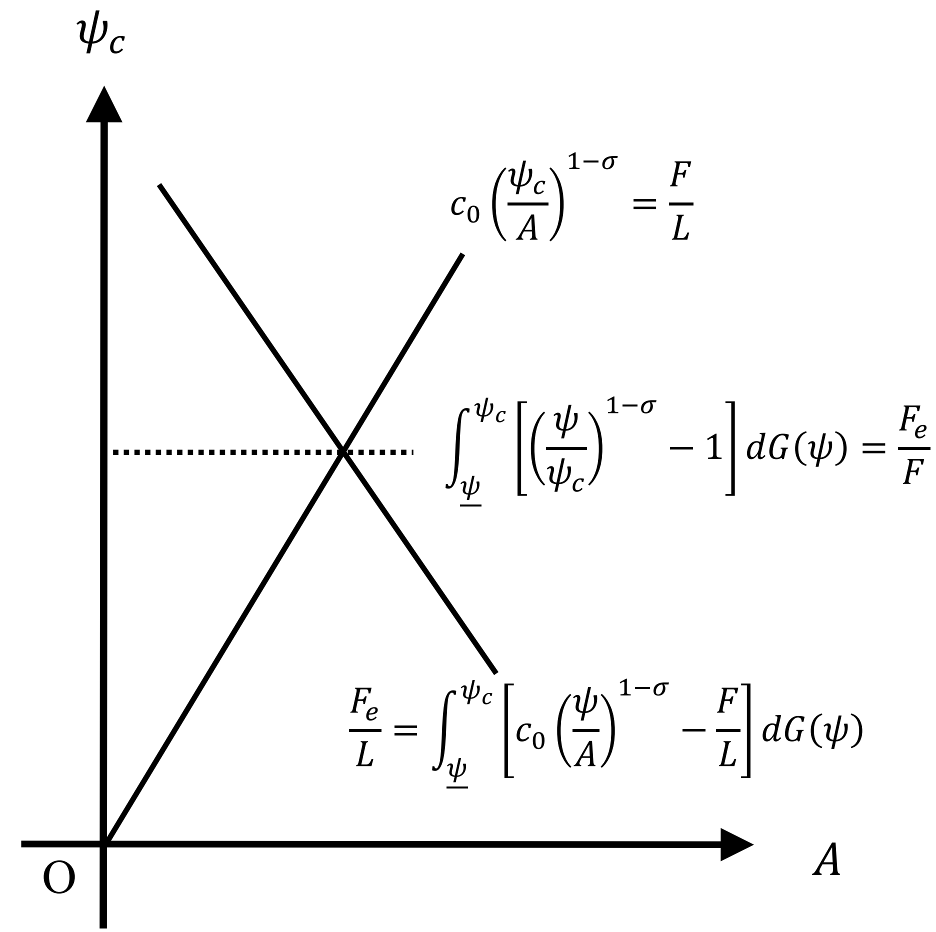
Pricing:
$$\mu\left(\frac{\psi}{A}\right) = \frac{\sigma}{\sigma-1} > 1 \Rightarrow \rho\left(\frac{\psi}{A}\right) = 1$$

$$\Rightarrow \mathcal{E}_r\left(\frac{\psi}{A}\right) = \mathcal{E}_\pi\left(\frac{\psi}{A}\right) = \mathcal{E}_\ell\left(\frac{\psi}{A}\right) = 1-\sigma < 0.$$

Cutoff Rule:
$$c_0 L\left(\frac{\psi_c}{A}\right)^{1-\sigma} = F,$$

Free Entry Condition:
$$\int_{\underline{\psi}}^{\psi_c}\left[c_0 L\left(\frac{\psi}{A}\right)^{1-\sigma} - F\right] dG(\psi) = F_e,$$

with $c_0 > 0$. As L changes, the intersection moves along
$$\int_{\underline{\psi}}^{\psi_c}\left[\left(\frac{\psi}{\psi_c}\right)^{1-\sigma} - 1\right] dG(\psi) = \frac{F_e}{F}$$



Proposition 1: Under CES,
- $L \uparrow$ keeps ψ_c unaffected; increases both M and $MG(\psi_c)$ *proportionately;*
- $F_e \downarrow$ reduces ψ_c; increases M; increases (decreases) $MG(\psi_c)$ if $\mathcal{E}'_G(\psi) < (>) 0$; $MG(\psi_c)$ unaffected under Pareto.
- $F \downarrow$ increases ψ_c; increases $MG(\psi_c)$; increases (decreases) M if $\mathcal{E}'_G(\psi) < (>) 0$; M unaffected under Pareto.

Cross-Sectional Implications under 2nd & 3rd Laws

Marshall's 2nd Law: Cross-Sectional Implications (Proposition 2)

(A2): $\zeta(z_\psi)$ is increasing in $z_\psi \equiv p_\psi/A = Z(\psi/A)$

- **Price elasticity** $\zeta(Z(\psi/A)) \equiv \sigma(\psi/A)$ increasing in ψ/A;
  high-ψ firms operate at more elastic parts of demand curve.

  ○ **Markup Rate**, $\mu(\psi/A)$, decreasing in $\psi/A \Leftrightarrow \mathcal{E}_\mu(\psi/A) < 0$
  high-ψ firms charge lower markup rates.
  ○ **Incomplete Pass-Through:** The pass-through rate, $\rho(\psi/A) = 1 + \mathcal{E}_\mu(\psi/A) < 1$.

- **Procompetitive effect of entry/Strategic complementarity in pricing,** $\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho(\psi/A) > 0$.
  Firms set the price lower under more competitive pressures ($A = A(\mathbf{p}) \downarrow$), due to either a larger Ω and/or a lower $\mathbf{p}$

Lemma 5: $f(\psi/A)$ *log-super(sub)modular* in ψ & $A \Leftrightarrow \mathcal{E}_f'(\cdot) < (>)0 \Leftrightarrow \ln f\left(e^{\ln(\psi/A)}\right)$ concave (convex) in $\ln(\psi/A)$

- **Profit,** $\pi(\psi/A)L$, always decreasing, **strictly log-supermodular** in ψ and A.
  $A \downarrow \rightarrow$ a proportionately larger decline in profit for high-ψ firms → Larger dispersion of profit

3rd Law: Cross-Sectional Implications (Propositions 3, 4, and 5)

In addition to A2, if we further assume, with some empirical support, e.g. Berman et.al.(2012), Amiti et.al.(2019),

(A3): $\rho(\psi/A) = 1 + \mathcal{E}_{\mu}(\psi/A)$ is weak(strict)ly increasing--we call it **Weak (Strong) 3rd Law**.
Under translog, $\rho(\psi/A)$ is strictly decreasing, violating A3

- **Markup rate**, $\mu(\psi/A)$, decreasing under A2, **log-submodular** in ψ & A under A3. For strong A3, it is strict and $A \downarrow$ → a proportionately smaller decline in markup rate for high-ψ firms → smaller dispersion of markup rate

- **Revenue**, $r(\psi/A)L$, always decreasing, **strictly log-supermodular** in ψ & A under *weak A3*
$A \downarrow$ → a proportionately larger decline in revenue for high-ψ firms → Larger dispersion of revenue

- **Employment,** $\ell(\psi/A)L = \frac{r(\psi/A)}{\mu(\psi/A)}L$, *hump-shaped* in ψ/A, **strictly log-supermodular** in ψ & A under *weak A3*
Employment is increasing in ψ across all active firms with a large enough overhead/market size ratio.
$A \downarrow$ → Employment up for the most productive firms.

- **Pass-through rate,** $\rho(\psi/A)$, **strictly log-submodular** in ψ & A for a small enough $\bar{z}$ under strong A3
$A \downarrow$ → a proportionately smaller increase in the pass-through rate for low-ψ firms among the active.

Cross-Sectional Implications of More Competitive Pressures, $A\downarrow$: A Graphic Representation

Profit(Revenue) Function: $\Pi_\psi = \pi(\psi/A)L$; $R_\psi = r(\psi/A)L$,

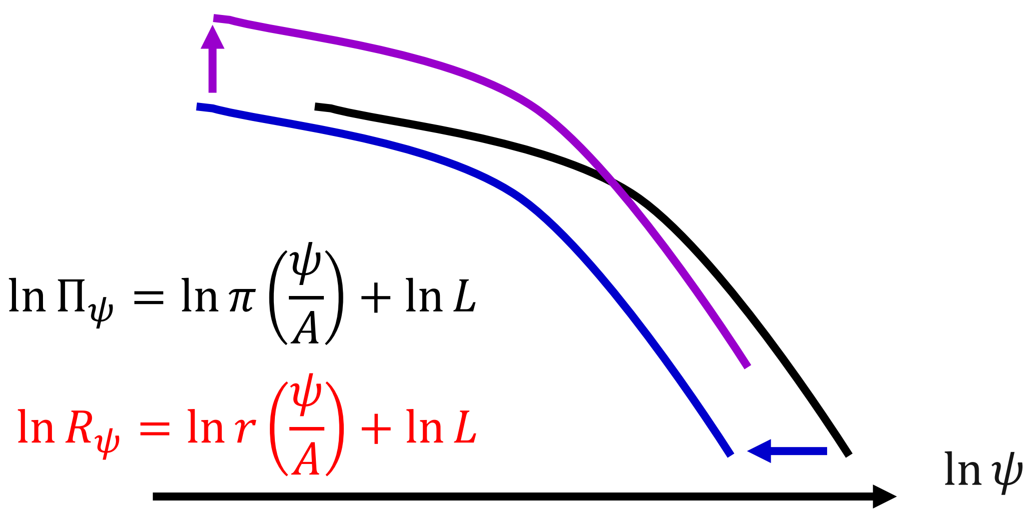
- *always* decreasing in ψ
- strictly log-supermodular *under A2 (Weak A3)*

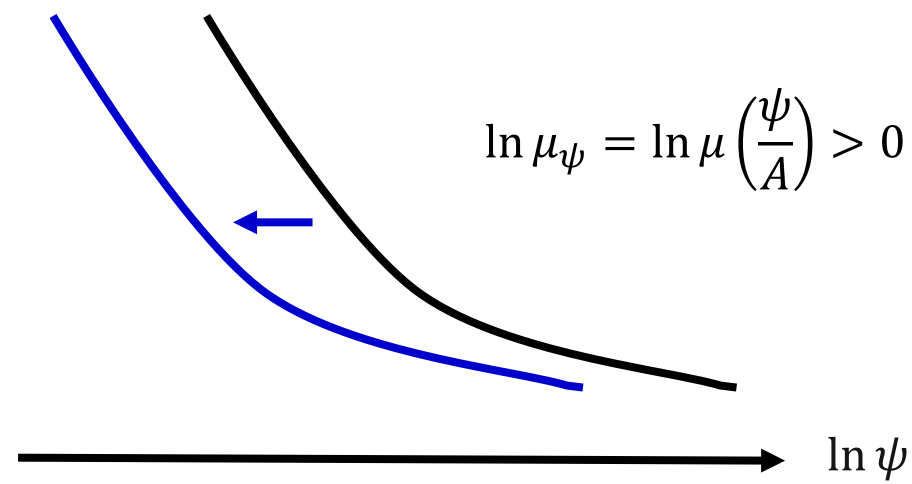
→ $A\downarrow$ with L fixed shifts down with a steeper slope at each ψ;
→ $A\downarrow$ due to $L\uparrow$, a parallel shift up, a single-crossing.

Markup Rate Function: $\mu_\psi = \mu(\psi/A) > 1$

- decreasing in ψ *under A2*
- weakly log-submodular *under Weak A3*
- strictly log-submodular *under Strong A3*

→ $A\downarrow$ shifts down with a flatter slope at each ψ





✓ With $\ln\psi$ in the horizontal axis, $A\downarrow$ causes a parallel leftward shift of the graphs in these figures.
✓ $f(\psi/A)$ is strictly log-super(sub)modular in ψ & $A \Leftrightarrow \ln f(\psi/A)$ is (strictly) concave(convex) in $\ln(\psi/A)$.

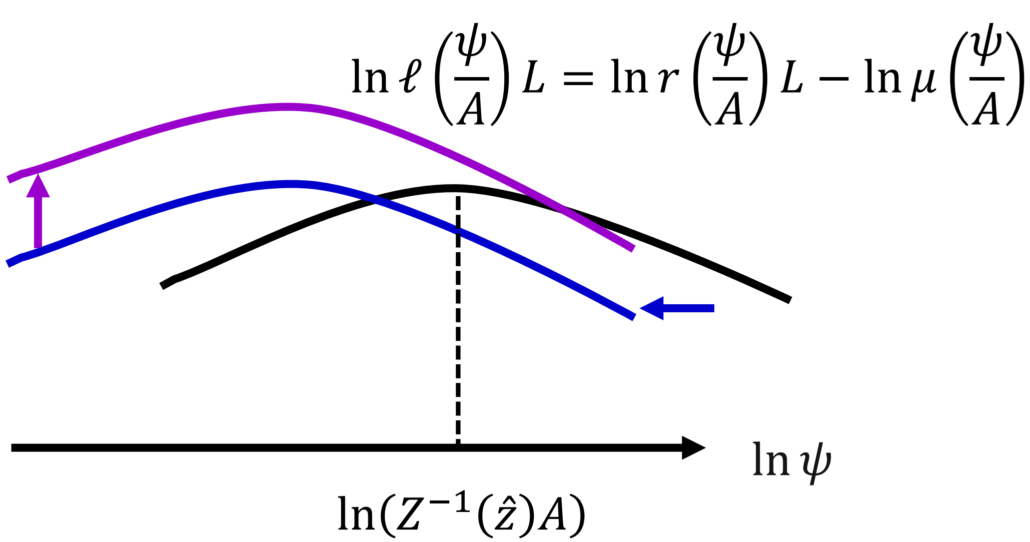
Employment Function: $\ell(\psi/A)L = r(\psi/A)L/\mu(\psi/A)$

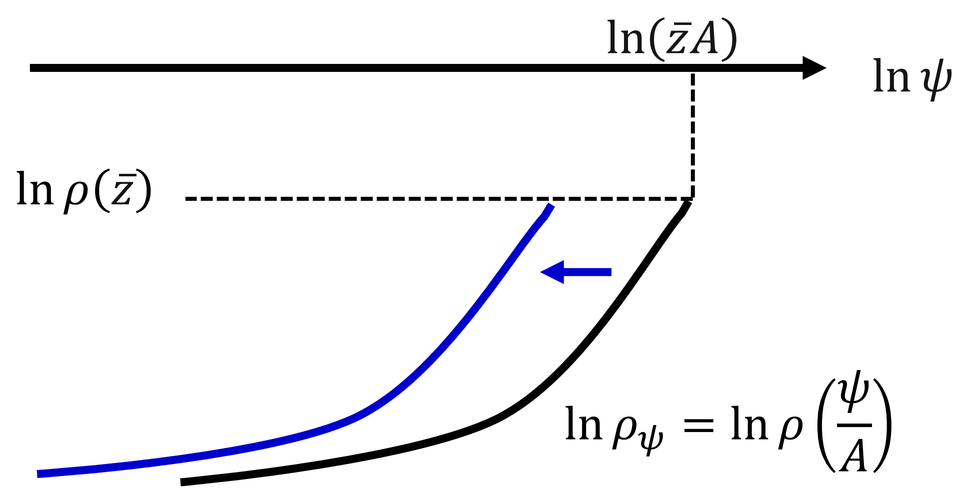
- *Hump-shaped* in ψ *under A2 and weak A3.*
  $\rightarrow A\downarrow$ shifts up (down) for a low (high) ψ with $A\downarrow$
- Strictly log-supermodular *under weak A3*
  for $A\downarrow$ with a fixed L; for $A\downarrow$ caused by $L\uparrow$

Single-crossing even with a fixed L

Pass-Through Rate Function: $\rho_\psi = \rho(\psi/A)$

- $\rho(\psi/A) < 1$ *under A2, hence it cannot be s*trictly log-submodular for a higher range of ψ/A
- Increasing in ψ *under Strong A3*
- Strictly log-submodular for a lower range of ψ/A *under A2 and Strong A3* $\Rightarrow A\downarrow$ shifts up with a steeper slope at each ψ *with a small enough* $\bar{z}$*.*





In summary, more competitive pressures ($A\downarrow$)

- $\mu(\psi/A)\downarrow$ under A2 & $\rho(\psi/A)\uparrow$ under strong A3
- Profit, Revenue, Employment become more concentrated among the most productive.

Comparative Statics: General Equilibrium Effects

General Equilibrium Effects of F_e, L, and F on A and ψ_c

Proposition 6:

$$\begin{bmatrix} \dfrac{dA}{A} \\ \dfrac{d\psi_c}{\psi_c} \end{bmatrix} = \frac{1}{\left[\mathbb{E}_\sigma\left(\underline{\psi}, \psi_c\right) - 1\right]} \begin{bmatrix} 1 - f_x & f_x \\ 1 - f_x & f_x - \delta \end{bmatrix} \begin{bmatrix} \dfrac{d(F_e/L)}{(F_e/L)} \\ \dfrac{d(F/L)}{(F/L)} \end{bmatrix}$$

where

$$\mathbb{E}_\sigma\left(\underline{\psi}, \psi_c\right) \equiv \frac{\int_{\underline{\psi}}^{\psi_c} \sigma(\psi/A)\pi(\psi/A)\, dG(\psi)/G(\psi_c)}{\int_{\underline{\psi}}^{\psi_c} \pi(\psi/A)\, dG(\psi)/G(\psi_c)} = 1 + \frac{\int_{\underline{\psi}}^{\psi_c} \ell(\psi/A)\, dG(\psi)/G(\psi_c)}{\int_{\underline{\psi}}^{\psi_c} \pi(\psi/A)\, dG(\psi)/G(\psi_c)} > 1$$

The profit-weighted average of $\sigma(\psi/A)$ among the active firms.

$$f_x \equiv \frac{FG(\psi_c)}{F_e + FG(\psi_c)} = \frac{FG(\psi_c)}{L\int_{\underline{\psi}}^{\psi_c} \pi(\psi/A) dG(\psi)} = \frac{\pi(\psi_c/A)}{\int_{\underline{\psi}}^{\psi_c} \pi(\psi/A)\, dG(\psi)/G(\psi_c)} < 1$$

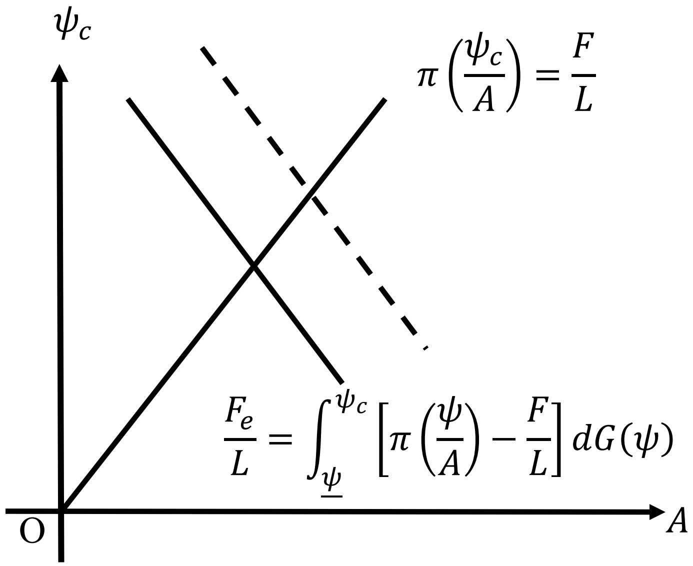
The share of the overhead in the total fixed cost = the profit at the cut-off relative to the average profit among the active firms.

$$\delta \equiv \frac{\mathbb{E}_\sigma\left(\underline{\psi}, \psi_c\right) - 1}{\sigma(\psi_c/A) - 1} = \frac{\pi(\psi_c/A)}{\ell(\psi_c/A)} \frac{\int_{\underline{\psi}}^{\psi_c} \ell(\psi/A)\, dG(\psi)/G(\psi_c)}{\int_{\underline{\psi}}^{\psi_c} \pi(\psi/A)\, dG(\psi)/G(\psi_c)} = f_x \frac{\int_{\underline{\psi}}^{\psi_c} \ell(\psi/A)\, dG(\psi)/G(\psi_c)}{\ell(\psi_c/A)} > 0.$$

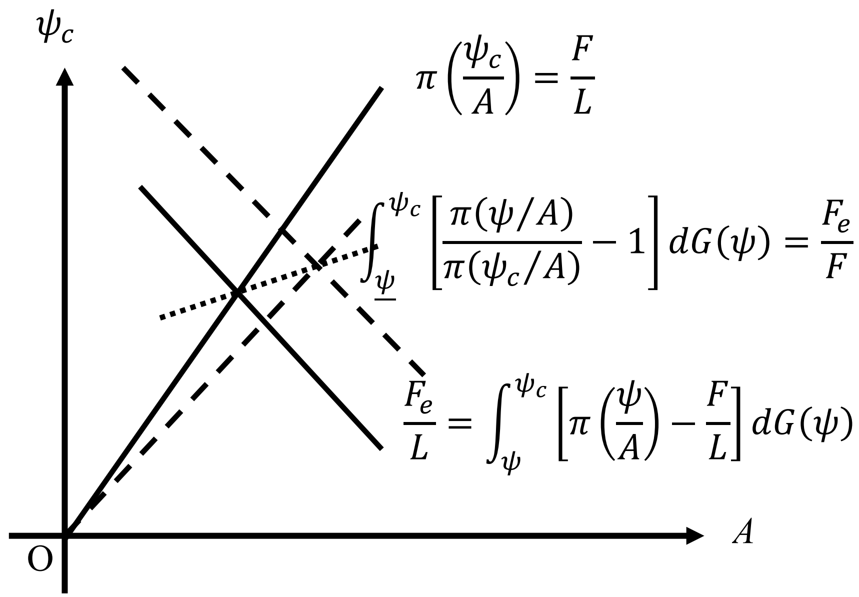
The profit/employment ratio at the cut-off to the average profit/the average employment ratio among the active firms.

Proposition 6: A Graphic Representation

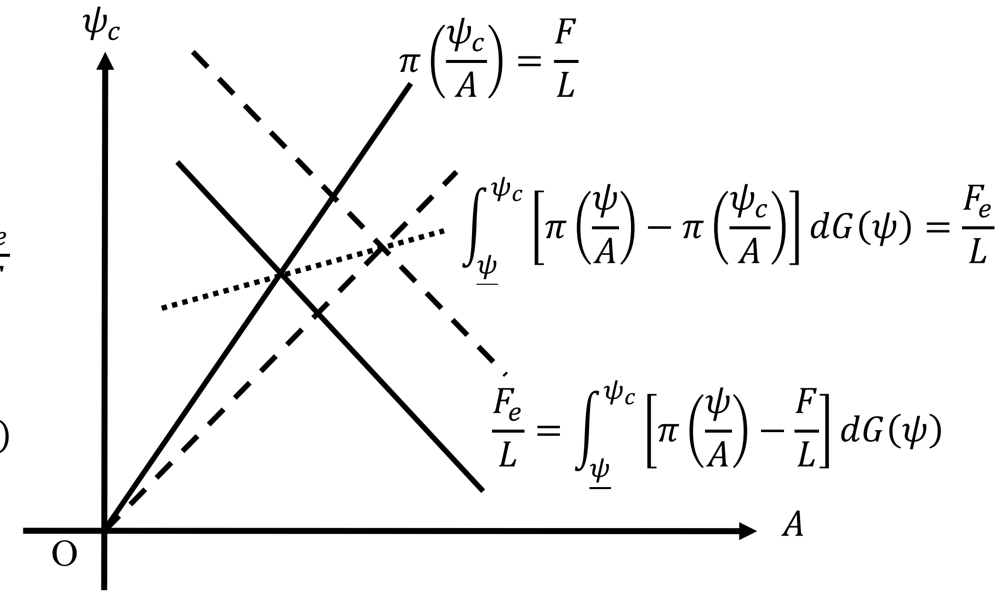
Effects of $F_e \downarrow$



Effects of $L \uparrow$ if $\sigma'(\cdot) > 0$ (i.e., A2)



Effects of $F \downarrow$ if $\ell'(\cdot) > 0$



In all three cases, more competitive pressures imply tougher selection.

$$\left.\begin{matrix} F_e \downarrow \\ L \uparrow \\ F \downarrow \end{matrix}\right\} \Longrightarrow A \downarrow \Longrightarrow \psi_c \downarrow$$

Proposition 7: Market Size Effect on Profit and Revenue Distributions

7a: Under **A2,** there exists a unique $\psi_0 \in (\underline{\psi}, \psi_c)$ such that $\sigma\left(\frac{\psi_0}{A}\right) = \mathbb{E}_\sigma\left(\underline{\psi}, \psi_c\right)$ with

$$\frac{d \ln \Pi_\psi}{d \ln L} > 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) < \mathbb{E}_\sigma\left(\underline{\psi}, \psi_c\right) \text{ for } \psi \in \left(\underline{\psi}, \psi_0\right),$$

and

$$\frac{d \ln \Pi_\psi}{d \ln L} < 0 \Leftrightarrow \sigma\left(\frac{\psi}{A}\right) > \mathbb{E}_\sigma\left(\underline{\psi}, \psi_c\right) \text{ for } \psi \in (\psi_0, \psi_c).$$

7b: Under **A2 and the weak A3,** there exists $\psi_1 > \psi_0$, such that

$$\frac{d \ln R_\psi}{d \ln L} > 0 \text{ for } \psi \in \left(\underline{\psi}, \psi_1\right).$$

Furthermore, $\psi_1 \in (\psi_0, \psi_c)$ and

$$\frac{d \ln R_\psi}{d \ln L} < 0 \text{ for } \psi \in (\psi_1, \psi_c),$$

for a sufficiently small F.

In short, more productive firms expand, while less productive firms shrink.

More on GE Effects: Propositions 8 and 9

Average Markup and Pass-Through Rates: Composition Effect of $\psi_c \downarrow$ & $A \downarrow$

- Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each ψ, but $\psi_c \downarrow$ means high-ψ firms with lower $\mu(\psi/A)$ drop out.
- Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each ψ, but $\psi_c \downarrow$ means high-ψ firms with higher $\rho(\psi/A)$ drop out.

Prop.8: *Assume* $\underline{\psi} = 0$. *Consider a shock, such that* ψ_c/A *remains constant. For any weighting function* $w(\psi/A)$, *the weighted average of monotone* $f(\psi/A)$, $I \equiv \frac{\int_{\underline{\psi}}^{\psi_c} f(\psi/A) w(\psi/A) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} w(\psi/A) dG(\psi)}$ *satisfies* $sgn\left\{\frac{dI}{dA}\right\} = sgn\{f'(\cdot)\mathcal{E}_g'(\cdot)\}$.

The average markup (or pass-through) rate can go either way, with $F_e \downarrow$ + Pareto-productivity a knife-edge case

More competition, which causes more concentration, may result in the rise of markup.

Impact on TFP/Unit Cost: Since $\ln\left(\frac{A}{P}\right) + const. = \frac{\int_{\underline{\psi}}^{\psi_c} \Phi(Z(\psi/A)) r(\psi/A) dG(\psi)}{\int_{\underline{\psi}}^{\psi_c} r(\psi/A) dG(\psi)}$ and $\zeta'(\cdot) \gtreqless 0 \implies \Phi'(\cdot) \lesseqgtr 0$,

Corollary of Prop.8: *If* $\underline{\psi} = 0$ *and* $\zeta'(\cdot)\mathcal{E}_g'(\cdot) \geq 0$, $F_e \downarrow \implies dP/P \leq dA/A < 0$ with the equality iff $\zeta'(\cdot)\mathcal{E}_g'(\cdot) = 0$.

Impact on Masses of Firms: Proposition 9

The effects on M & $MG(\psi_c)$ depend on whether $\mathcal{E}_G(\psi) \equiv \psi g(\psi)/G(\psi)$ is decreasing, constant, or increasing.

Sorting of Heterogenous Firms: A Multi-Market Setting

Sorting: GE Implications in a Multi-Market Setting

Many markets of different size. Firms, after learning their ψ, choose which market to enter.

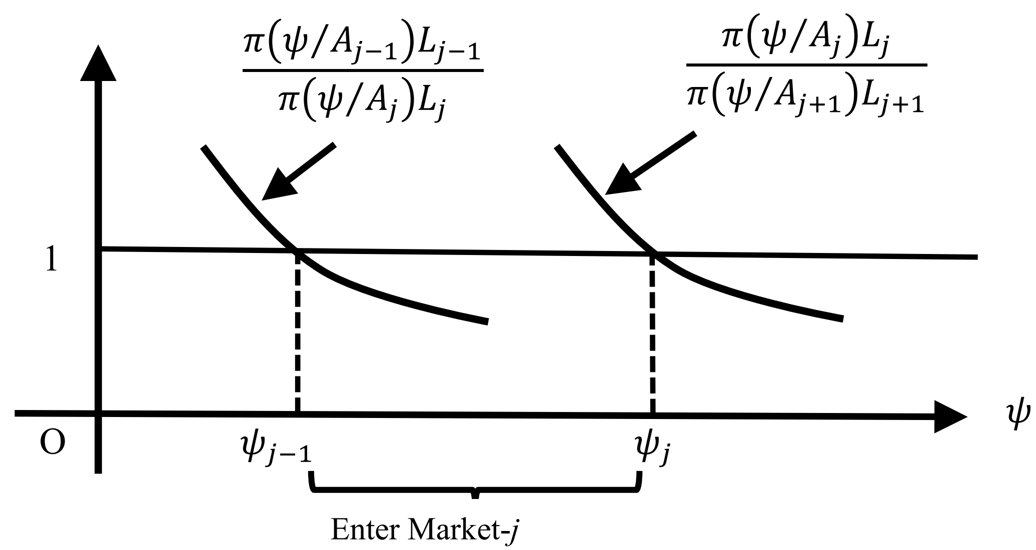
Proposition 10: Assortative Matching

More competitive pressures in larger markets:

$$L_1 > L_2 > \cdots > L_J > 0 \;\Rightarrow\; 0 < A_1 < A_2 < \cdots < A_J < \infty$$

Under A2, more efficient firms sort themselves into larger markets: Firms $\psi \in (\psi_{j-1}, \psi_j)$ entering market-j, where

$$0 \leq \underline{\psi} = \psi_0 < \psi_1 < \psi_2 < \cdots < \psi_J < \overline{\psi} \leq \infty.$$

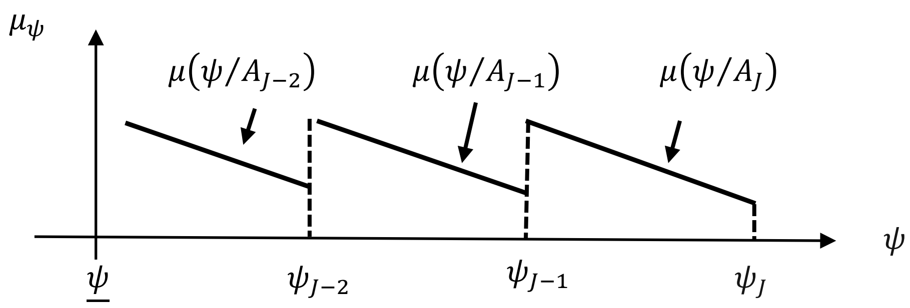


Sorting: GE Implications in a Multi-Market Setting

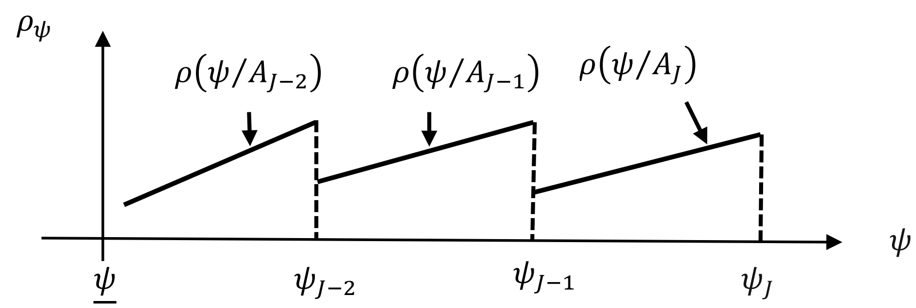
Proposition 11: The Composition Effect: *Examples* with Pareto-productivity such that
- The average markup rates *higher* (the average pass-through rates *lower* under Strong A3) in larger (more competitive) markets

A decline in F_e causes uniform declines in ψ_j & A_j with the average markup/pass-through rates unchanged.

Markup Rate across markets under A2



Pass-Through Rate across markets under strong A3



A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.

Three Parametric Families of H.S.A. (Appendix D)

Generalized Translog For $\eta > 0, \sigma > 1$	$s(z) = \gamma\left(-\frac{\sigma-1}{\eta}\ln\left(\frac{z}{\bar{z}}\right)\right)^{\eta}$; $z < \bar{z} \equiv \beta e^{\frac{\eta}{\sigma-1}}$	$1 - \frac{1}{\zeta(z)} = \frac{\eta}{\eta - \ln\left(\frac{z}{\bar{z}}\right)} \Rightarrow \begin{matrix}\mathcal{E}_\mu(\cdot) < 0 \\ \mathcal{E}'_\mu(\cdot) = \rho'(\cdot) < 0\end{matrix}$ satisfying **A2;** violating **A3.**

Translog is the special case where $\eta = 1$. CES is the limit case, as $\eta \to \infty$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Constant Pass-Through (CoPaTh) For $0 < \rho < 1, \sigma > 1$	$s(z) = \gamma\sigma^{\frac{\rho}{1-\rho}}\left[1 - \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}}\right]^{\frac{\rho}{1-\rho}}$; $\bar{z} \equiv \beta\left(\frac{\sigma}{\sigma-1}\right)^{\frac{\rho}{1-\rho}}$	$1 - \frac{1}{\zeta(z)} = \left(\frac{z}{\bar{z}}\right)^{\frac{1-\rho}{\rho}} \Rightarrow \begin{matrix}\mathcal{E}_\mu(\cdot) < 0 \\ \mathcal{E}'_\mu(\cdot) = \rho'(\cdot) = 0\end{matrix};$ satisfying **A2** & weak **A3;** violating strong **A3**

CES is the limit case, as $\rho \to 1$, while holding $\beta > 0$ and $\sigma > 1$ fixed.

Power Elasticity of Markup Rate (Fréchet Inverse Markup Rate) For $\kappa \geq 0$ and $\lambda > 0$	$s(z) = \exp\left[\int_{z_0}^{z} \frac{c}{c - \exp\left[-\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[\frac{\kappa\xi^{-\lambda}}{\lambda}\right]}\frac{d\xi}{\xi}\right]$	$1 - \frac{1}{\zeta(z)} = c\exp\left[\frac{\kappa\bar{z}^{-\lambda}}{\lambda}\right]\exp\left[-\frac{\kappa z^{-\lambda}}{\lambda}\right]$ $\Rightarrow \mathcal{E}_\mu(\cdot) < 0; \mathcal{E}'_\mu(\cdot) = \rho'(\cdot) > 0$ satisfying **A2** and strong **A3** for $\kappa > 0$ and $\lambda > 0$.

CES for $\kappa = 0$; $\bar{z} = \infty$; $c = 1 - \frac{1}{\sigma}$; CoPaTh for $\bar{z} < \infty$; $c = 1$; $\kappa = \frac{1-\rho}{\rho} > 0$, and $\lambda \to 0$.