Selection and Sorting of Heterogeneous Firms Through Competitive Pressures

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Last Updated: 2022-04-05; 12:15:59 PM

Hitotsubashi Conference on International Trade and FDI
December 11-12, 2021
Competitive Pressures on Heterogeneous Firms

How do competitive pressures affect selection of firms with different productivity? Or sorting across different markets?

- Melitz (2003): monopolistic competition (MC) with heterogeneous firms under CES Demand System (DS)
  - MC firms sell their products at an exogenous & common markup rate, unresponsive to competitive pressures
  - Market size: no effect on distribution of firm types and on their behaviors; All adjustments at the extensive margin.
- Melitz-Ottaviano (2008) depart from CES using Linear DS + the outside competitive sector

We depart from CES using H.S.A. (Homothetic with a Single Aggregator) DS with gross substitutes

- Homothetic (unlike the linear DS and most other commonly-used non-CES DSs)
  - a single measure of market size; the demand composition does not matter.
  - isolate the effect of endogenous markup rate from nonhomotheticity
  - straightforward to use it as a building block for multi-sector GE models
- Nonparametric and flexible (unlike CES and translog, which are special cases)
  - can be used to perform robustness-check for CES and translog
  - allow for (but no need to impose) the choke price, Marshall’s 2nd law as well as what we call the 3rd law
- Tractable due to Single Aggregator (unlike Kimball, which needs two aggregators), a sufficient statistic for competitive pressures, which acts like a magnifier of firm heterogeneity. A simple diagram for
  - proving the existence & the uniqueness of free-entry equilibrium with firm heterogeneity
  - conducting most comparative statics without parametric restrictions on the demand or productivity distribution.
    - e.g., no need to assume zero overhead cost (unlike MO and ACDR)
- Defined by the market share function, for which data is readily available and easily identifiable.
Symmetric H.S.A. (Homothetic with a Single Aggregator) with Gross Substitutes
Here we consider a continuum of varieties \((\omega \in \Omega)\), gross substitutes, and symmetry (Our 2017 paper for a general analysis)

Market Share of \(\omega \in \Omega\) depends solely on its single relative price (= its own price/the common price aggregator)

\[
\frac{p_{\omega} x_{\omega}}{p_x} = \frac{\partial \ln P(p)}{\partial \ln p_\omega} = s \left( \frac{p_\omega}{A(p)} \right),
\]

where \(\int_{\Omega} s \left( \frac{p_\omega}{A(p)} \right) d\omega \equiv 1\).

- \(s: \mathbb{R}_{++} \to \mathbb{R}_{+}\): the market share function, decreasing in the relative price for \(s(z) > 0\) with \(\lim_{z \to \bar{z}} s(z) = 0\).
- If \(\bar{z} \equiv \inf\{z > 0 | s(z) = 0\} < \infty\), \(\bar{z} A(p)\) is the choke price.

- \(A(p)\): the common price aggregator defined implicitly by the adding-up constraint \(\int_{\Omega} s(p_\omega/A) d\omega \equiv 1\).

By construction, market shares add up to one; \(A(p)\) linear homogenous in \(p\) for a fixed \(\Omega\). A larger \(\Omega\) reduces \(A(p)\).

CES if \(s(z) = \gamma z^{1-\sigma}, (\sigma > 1)\); translog cost if \(s(z) = -\gamma \ln \left( \frac{z}{\bar{z}} \right)\); CoPaTh if \(s(z) = \gamma \left[ 1 - \left( \frac{z}{\bar{z}} \right)^{1-\rho} \right]^{1-\rho}, (0 < \rho < 1)\).

Unit Cost Function: \(P(p) \propto A(p) \exp \left\{ -\int_{\Omega} \left[ \int_{p_\omega/A(p)} s(\xi) d\xi \right] d\omega \right\}\)

Note: Our 2017 paper proved that \(P(p)\) is quasi-concave and that \(P(p)/A(p) \neq c\) for any \(c > 0\), unless CES

\(A(p)\), the inverse measure of competitive pressures, fully captures cross price effects in the demand system

\(P(p)\), the inverse measure of TFP, captures the productivity consequences of price changes
Monopolistic Competition under H.S.A.: Pricing

Pricing (Lerner) Formula

\[
p_\omega \left[ 1 - \frac{1}{\zeta(p_\omega/A)} \right] = \psi_\omega \Rightarrow \frac{p_\omega}{A} \left[ 1 - \frac{1}{\zeta(p_\omega/A)} \right] = \frac{\psi_\omega}{A},
\]

\[
\zeta(z) \equiv 1 - \frac{d \ln s(z)}{d \ln z} \equiv 1 - \varepsilon_s(z) > 1, \quad \text{for } z \in (0, \bar{z}); \quad \lim_{z \to \bar{z}} \zeta(z) = -\lim_{z \to \bar{z}} \varepsilon_s(z) = \infty, \text{if } \bar{z} < \infty.
\]

\(\psi_\omega\): firm-specific marginal cost (in labor, the numeraire)

\(A = A(p)\): the inverse measure of competitive pressures, common across firms, a sufficient statistic.

Under mild regularity conditions, firms with the same \(\psi\) set the same price, so that \(p_\omega = p_\psi\) and

**Relative price**

\[
\frac{p_\psi}{A} = z_\psi \equiv Z \left( \frac{\psi}{A} \right), \quad \text{an increasing function of } \psi/A, \text{the normalized cost, only.}
\]

**Price elasticity**

\[
\zeta \left( Z \left( \frac{\psi}{A} \right) \right) \equiv \sigma \left( \frac{\psi}{A} \right) > 1
\]

**Markup rate**

\[
\mu_\psi \equiv \frac{p_\psi}{\psi} = \frac{\sigma(\psi/A)}{\sigma(\psi/A) - 1} \equiv \mu \left( \frac{\psi}{A} \right) > 1
\]

**Pass-through rate**

\[
\rho_\psi \equiv \frac{\partial \ln p_\psi}{\partial \ln \psi} = \frac{d \ln Z(\psi/A)}{d \ln (\psi/A)} \equiv \varepsilon_z \left( \frac{\psi}{A} \right) \equiv \rho \left( \frac{\psi}{A} \right) = 1 + \varepsilon_\mu \left( \frac{\psi}{A} \right)
\]

are all functions of \(\psi/A\) only, continuously differentiable.

More competitive pressures, a lower \(A\), act like a magnifier of firm heterogeneity.
Monopolistic Competition under H.S.A.: Revenue, Profit, & Employment

Revenue

\[ R_\psi = s(z_\psi)L = s \left( \frac{Z(\psi)}{A} \right) L \equiv r \left( \frac{\psi}{A} \right) L \quad \Rightarrow \quad \varepsilon_r \left( \frac{\psi}{A} \right) = - \left[ \sigma \left( \frac{\psi}{A} \right) - 1 \right] \rho \left( \frac{\psi}{A} \right) < 0 \]

(Gross) Profit

\[ \Pi_\psi = \frac{s(z_\psi)L}{\kappa(z_\psi)} = \frac{r(\psi/A)}{\sigma(\psi/A)} L \equiv \pi \left( \frac{\psi}{A} \right) L \quad \Rightarrow \quad \varepsilon_\pi \left( \frac{\psi}{A} \right) = 1 - \sigma \left( \frac{\psi}{A} \right) < 0 \]

(Variable) Employment

\[ \psi x_\psi = R_\psi - \Pi_\psi = \frac{r(\psi/A)}{\mu(\psi/A)} L \equiv \ell \left( \frac{\psi}{A} \right) L \quad \Rightarrow \quad \varepsilon_\ell \left( \frac{\psi}{A} \right) = 1 - \sigma \left( \frac{\psi}{A} \right) \rho \left( \frac{\psi}{A} \right) \leq 0 \]

- Revenue \( r(\psi/A)L \), profit \( \pi(\psi/A)L \), employment \( \ell(\psi/A)L \) all functions of \( \psi/A \), multiplied by market size \( L \), continuously differentiable under mild regularity conditions.
- Market size affects the relative profit, revenue, and employment across firms only through its effects on \( A \).
- Both revenue \( r(\psi/A)L \) and profit \( \pi(\psi/A)L \) are always strictly decreasing in \( \psi/A \).
- Employment \( \ell(\psi/A)L \) may be nonmonotonic in \( \psi/A \).
  - If the markup rate declines with \( \psi/A \), employment cannot decline as fast as the revenue.
  - If the markup rate declines faster than the revenue, the employment is increasing in \( \psi/A \).

Again, more competitive pressures, a lower \( A \), act like a magnifier of firm heterogeneity.
**General Equilibrium: Existence and Uniqueness**

As in Melitz, firms pay the entry cost $F_e > 0$ to draw $\psi \sim G(\psi)$; cdf with the support, $(\underline{\psi}, \bar{\psi}) \subset (0, \infty)$, and pay the overhead $F > 0$ to stay & produce.

**Cutoff Rule:** stay if $\psi < \psi_c$; exit if $\psi > \psi_c$, where

$$\pi \left( \frac{\psi_c}{A} \right) L = F$$

positively-sloped $A \downarrow$ (more competitive pressures) $\Rightarrow \psi_c \downarrow$ (tougher selection)

**Free Entry Condition:**

$$F_e = \int_{\underline{\psi}}^{\psi_c} \left[ \pi \left( \frac{\psi}{A} \right) L - F \right] dG(\psi)$$

negative-sloped. $A \downarrow$ (more competitive pressures) and $\psi_c \downarrow$ (tougher selection) both make entry less attractive.

$$A = A(p) \text{ and } \psi_c: \text{ uniquely determined, respond continuously to } F_e/L \text{ & } F/L \text{ under mild regularity conditions.}$$

(This proof of unique existence applies also to the Melitz model under CES.)

With $A$ and $\psi_c$ determined, **the adding-up constraint** pins down masses of entrants, $M$ and of active firms, $MG(\psi_c)$. 

Cross-Sectional Implications of Marshall’s 2nd Law

(A2): $\zeta(z_\psi) \equiv p_\psi / A = Z(\psi / A)$

- **Price elasticity** $\zeta(Z(\psi / A)) \equiv \sigma(\psi / A)$ increasing in $\psi / A$;
  high-$\psi$ firms operate at more elastic parts of demand curve.

  o **Markup Rate**, $\mu(\psi / A)$, decreasing in $\psi / A \Leftrightarrow \varepsilon_\mu(\psi / A) < 0$
  high-$\psi$ firms charge lower markup rates.

  o **Incomplete Pass-Through**: The pass-through rate, $\rho(\psi / A) = 1 + \varepsilon_\mu(\psi / A) < 1$.

- **Procompetitive effect of entry/Strategic complementarity in pricing**, $\frac{\partial \ln p_\psi}{\partial \ln A} = 1 - \rho(\psi / A) > 0$.
  Firms set the price lower under more competitive pressures ($A = A(p) \downarrow$), due to either a larger $\Omega$ and/or a lower $p$.

- **Profit**, $\pi(\psi / A)L$, always decreasing, strictly log-supermodular in $\psi$ and $A$.
  $A \downarrow \rightarrow$ a proportionately larger decline in profit for high-$\psi$ firms $\rightarrow$ Larger dispersion of profit

  ✓ $f(\psi / A)$ is (strictly) log-super(sub)modular in $\psi & A \Leftrightarrow \varepsilon_f(\frac{\psi}{A}) \equiv \frac{d \ln f(\psi / A)}{d \ln(\psi / A)}$ is (strictly) decreasing (increasing) in $\psi / A$. 
Cross-Sectional Implications of Marshall’s 3rd Law

In addition to A2, if we further assume, with some empirical support,

\( \rho(\psi/A) = 1 + \varepsilon_\mu(\psi/A) \) is weak(strictly) increasing--we call it **Weak (Strong) Marshall’s 3rd Law**. Under translog, \( \rho(\psi/A) \) is strictly decreasing, violating A3

- **Markup rate**, \( \mu(\psi/A) \), decreasing under A2, **log-submodular** in \( \psi \& A \) under A3. For strong A3, it is strict and \( A \downarrow \Rightarrow \) a proportionately smaller decline in markup rate for high-\( \psi \) firms \( \Rightarrow \) smaller dispersion of markup rate

- **Revenue**, \( r(\psi/A)L \), always decreasing, **strictly log-supermodular** in \( \psi \& A \) under weak A3
  \( A \downarrow \Rightarrow \) a proportionately larger decline in revenue for high-\( \psi \) firms \( \Rightarrow \) Larger dispersion of revenue

- **Employment**, \( \ell(\psi/A)L = \frac{r(\psi/A)}{\mu(\psi/A)} L \), **hump-shaped** in \( \psi/A \), **strictly log-supermodular** in \( \psi \& A \) under weak A3
  Employment is increasing in \( \psi \) across all active firms with a large enough overhead/market size ratio.
  \( A \downarrow \Rightarrow \) Employment up for the most productive firms.

- **Pass-through rate**, \( \rho(\psi/A) \), **strictly log-submodular** in \( \psi \& A \) for a small enough \( \bar{Z} \) under strong A3
  \( A \downarrow \Rightarrow \) a proportionately smaller increase in the pass-through rate for low-\( \psi \) firms among the active.
Cross-Sectional Implications of More Competitive Pressures ($A \downarrow$)

**Profit Function:** $\Pi_{\psi} = \pi(\psi / A)L$

- *always* decreasing in $\psi$
- strictly log-supermodular *under A2*
- $A \downarrow$ with $L$ fixed shifts down with a steeper slope at each $\psi$;  
- $A \downarrow$ due to $L \uparrow$, a parallel shift up, a single-crossing.

**Markup Rate Function:** $\mu_{\psi} = \mu(\psi / A) > 1$

- *decreasing* in $\psi$ *under A2*
- weakly log-submodular *under Weak A3*
- strictly log-submodular *under Strong A3*
- $A \downarrow$ shifts down with a flatter slope at each $\psi$

\[
\ln \Pi_{\psi} = \ln \pi \left( \frac{\psi}{A} \right) + \ln L
\]

\[
\ln \mu_{\psi} = \ln \mu \left( \frac{\psi}{A} \right) > 0
\]

✓ With $\ln \psi$ in the horizontal axis, $A \downarrow$ causes a parallel leftward shift of the graphs in these figures.
✓ $f(\psi / A)$ is strictly log-super(sub)modular in $\psi$ & $A \Leftrightarrow \ln f(\psi / A)$ is (strictly) concave(convex) in $\ln(\psi / A)$.

Under Weak A3, $R_{\psi} = r(\psi / A)L$, strictly log-supermodular and shares similar properties with $\pi(\psi / A)L$. 
Employment Function: \( \ell(\psi/A)L = r(\psi/A)L/\mu(\psi/A) \)
- Hump-shaped in \( \psi \) under A2 and weak A3.
  \( A \downarrow \) shifts up (down) for a low (high) \( \psi \) with \( A \downarrow \)
- Strictly log-supermodular under weak A3
  for \( A \downarrow \) with a fixed \( L \); for \( A \downarrow \) caused by \( L \uparrow \)
  Single-crossing even with a fixed \( L \)

Pass-Through Rate Function: \( \rho_\psi = \rho(\psi/A) \)
- \( \rho(\psi/A) < 1 \) under A2, hence it cannot be strictly log-submodular for a higher range of \( \psi/A \)
- Increasing in \( \psi \) under Strong A3
- Strictly log-submodular for a lower range of \( \psi/A \) under A2 and Strong A3 \( \Rightarrow A \downarrow \) shifts up with a steeper slope at each \( \psi \) with a small enough \( \bar{z} \).

In summary, more competitive pressures (\( A \downarrow \))
- \( \mu(\psi/A) \downarrow \) under A2 & \( \rho(\psi/A) \uparrow \) under strong A3
- Profit, Revenue, Employment become more concentrated among the most productive.
GE Comparative Statics Implications: Selection (in a single-market setting)

Effects of $F_e \downarrow$

Effects of $L \uparrow$ if $\sigma'(\cdot) > 0$ (i.e., A2)

Effects of $F \downarrow$ if $\ell'(\cdot) > 0$

- $L \uparrow$ under A2: the profit up for low-$\psi$ and down for high-$\psi$. (Similarly on the revenue under A2 and the weak A3)
- All 3 cases lead to $\psi_c \downarrow$ & $A \downarrow$, creating a non-trivial composition effect
  - Under A2, $A \downarrow$ causes $\mu(\psi/A) \downarrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with lower $\mu(\psi/A)$ drop out.
  - Under strong A3, $A \downarrow$ causes $\rho(\psi/A) \uparrow$ for each $\psi$, but $\psi_c \downarrow$ means high-$\psi$ firms with higher $\rho(\psi/A)$ drop out.

**The average markup (or pass-through) rate can go either way, with $F_e \downarrow$ + Pareto-productivity a knife-edge case**

**More competition, which causes more concentration, may result in the rise of markup.**

- The effects on $M$ & $MG(\psi_c)$ depend on whether $E_G(\psi) \equiv \psi g(\psi)/G(\psi)$ is decreasing, constant, or increasing.
GE Implications: Sorting (in a multi-market setting)

More competitive pressures in larger markets:

\[ L_1 > L_2 > \ldots > L_j > 0 \Rightarrow 0 < A_1 < A_2 < \ldots < A_J < \infty \]

Under A2, more efficient firms sort themselves into larger markets
Firms \( \psi \in (\psi_{j-1}, \psi_j) \) entering market- \( j \)

Markup Rate across markets under A2

Pass-Through Rate across markets under strong A3

The Composition Effect: examples with Pareto-productivity such that

- The average markup rates \( \text{higher} \) (the average pass-through rates \( \text{lower} \) under Strong A3) in larger (more competitive) markets
- A decline in \( F_e \) causes uniform declines in \( \psi_j \& A_j \) with the average markup/pass-through rates unchanged.

A caution against testing A2/A3 by comparing the average markup/pass-through rates in cross-section of cities.
(Highly Selective) Literature Review

**H.S.A. Demand System:** Matsuyama-Ushchev (2017)

**MC with Heterogeneous Firms:** Melitz (2003) and many others: Melitz-Redding (2015) for a survey

**MC under non-CES demand systems:** Thisse-Ushchev (2018) for a survey

- **Nonhomothetic non-CES:**
  - $U = \int_\Omega u(x_\omega)d\omega$: Dixit-Stiglitz (77), Behrens-Murata (07), ZKPT (12), Mrázová-Neary(17), Dhingra-Morrow (19); ACDR (19)
  - **Linear-demand system:** Ottaviano-Tabuchi-Thisse (2002)

- **Homothetic non-CES:** Feenstra (2003), Kimball (1995), Matsuyama-Ushchev (2020a, 2020b, 2022)

**Empirical Evidence:**

- **The 2nd Law:** DeLoecker-Goldberg (14), Burstein-Gopinath (14); **The 3rd Law:** Berman et.al.(12), Amiti et.al.(19), **Market Size Effects:** Campbell-Hopenhayn(05); **Rise of markup:** Autor et.al.(20), DeLoecker et.al.(20)

**Selection of Heterogeneous Firms through Competitive Pressures**
Melitz-Ottaviano (2008), Baqee-Fahri-Sangani (2021)

**Sorting of Heterogeneous Firms Across Markets in General Equilibrium**

**Sorting of Heterogeneous Firms Across Markets in Reduced Form/Partial Equilibrium**

**Log-Super(Sub)modularity:** Costinot (2009), Costinot-Vogel (2015)