Scale Economies, Product Differentiation, and Monopolistic Competition

By Kiminori Matsuyama

(Permanently) Work in Progress

Last Updated: March 17, 2009; 11:34:51 PM

Overview

- Introduction
- Endogenous Technologies in Competitive Models with External Economies of Scale
- Competitive Models with Internal Economies of Scale
- Monopolistic Competitive Models with Internal Economies of Scale

✓ Tradeable Differentiated Goods: Gains from Product Variety and Intra-Industry Trade
✓ Nontradeable Differentiated Goods: Agglomeration Effects,
✓ Costly Trade in Differentiated Goods: Home Market Effect and Economic Geography.

- Extensions; Firm Heterogeneity, Multi-Product, Variable Mark-Up etc. **(Unfinished)**
- Bibliography

Introduction

- Competitive, CRS models of trade stress gains from trade between dissimilar countries. Yet, a bulk of trade is done between similar countries. Aside from those with non-homothetic preferences, models with CRS do not sit well with this observation.

- IRS implies that even (inherently) identical countries can gain from trade, by each specializing in different activities.

- Agglomeration is one of the most salient features of the economic landscape. Yet, it is difficult to explain in a world with CRS, where every country has access to the same technologies.

- Of course, the Ricardian theory allows for technological differences across regions and countries.

- Aside from the geography and climate, however, it is difficult to think of technology differences to be exogenous factors. And if we are to endogenize technology in order to understand how the technological differences across countries emerge and persist, we must confront the problem of how to incorporate IRS in general equilibrium framework.

Part IV look at various ways of introducing IRS in general equilibrium framework.

- ***External Economies of Scale***, where each producer faces CRS but externalities generate IRS at sectoral or economy levels. This is also a useful way of endogenizing technologies within the competitive framework.

- ***Internal Economies of Scale*** within the *competitive* framework.

- ***Internal Economies of Scale*** within the ***Monopolistically Competitive*** framework, where different firms produce ***Differentiated Products***. Within this framework, we consider models with differentiated products that are

✓ **Costlessly Tradeable** to see the *Gains from Trade due to Increased Product Variety,* and *Intra-Industry Trade*
✓ **Nontradeable** to see *Agglomeration Effects*
✓ **Subject to the Iceberg Trade Costs** to see the *Home Market Effect* and *Economic Geography.*

- We also review some recent works that relax some restrictive features of the basic monopolistic competition models (firm heterogeneity, multi-product, and variable mark-up, etc.)

Endogenous Technologies: External Economies & Increasing Returns

External economies are a simple way of introducing sector-wide or economy-wide increasing returns without departing from the competitive framework. Here, I follow mostly Helpman and Krugman (1985; Ch.3).

Let us modify the neoclassical production functions as follows:

Production Function of a sector-*j* firm: $x_j = F^j(v_j;\xi_j)$,

which maps the firm's input, v_j to its output, x_j. Holding ξ_j constant, $F^j(\bullet;\xi_j)$ satisfies all the standard properties of neoclassical productions, including the linear homogeneity.

Since $F^j(\bullet;\xi_j)$ is linear homogeneous, we may define:

Unit Cost Function of a sector-j firm: $C^j(w;\xi_j) \equiv Min_{a_j}\left\{wa_j \middle| F^j(a_j;\xi_j) \geq 1\right\}$

Unit Input Function of a sector-j firm: $a_j(w;\xi_j) \equiv Arg\min_{a_j}\left\{wa_j \middle| F^j(a_j;\xi_j) \geq 1\right\}$

An element, ξ_j, captures the external effects; they are:

- representing all sorts of factors that affect productivity of sector-*j,* such as knowledge and experiences, the shared inputs, etc.
- endogenous, determined in equilibrium, depending on the actions taken by the others.
- beyond the control of each (infinitesimal) firm, hence exogenous from the point of view of each firm.

Example: Let $\xi_j = X_j = \sum x_j$ be the total output and $F^j(v_j;\xi_j) = (\xi_j)^{\lambda_j} F^j(v_j)$, $0 < \lambda_j < 1$.

Then,

$$X_j = \Sigma x_j = F^j(\Sigma v_j;\xi_j) = (X_j)^{\lambda_j} F^j(V_j), \text{ where } V_j = \textstyle\sum v_j.$$

Or $$X_j = \left[F^j(V_j)\right]^{\frac{1}{1-\lambda_j}},$$ which has the homogeneity of degree, $1/(1-\lambda_j) > 1$.

Thus, external economies generate sector-level increasing returns.

Yet, the assumption that each firm takes ξ_j as exogenous enables us to stay within the competitive framework.

What factors enter ξ_j is an important assumption. One may consider a variety of alternatives. That is, ξ_j may contain factors that are

- Country-specific and sector-specific, such as the total output or employment in the country's sector-j, such as X_j. (The most common assumption)
- Sector-specific but not country-specific; German and Japanese automakers migh learn a lot from each other, while German furniture makers might not learn much from German automakers.
- Country-specific but not sector-specific; Japanese consumer electronics industries and automakers might learn from each other.
- Region (bigger than a country)-specific; Europeans might learn more from each other.
- Region (smaller than a country)-specific; Urban externalities might be restricted to a particular metropolitan area.

etc.

Note that these geographical and sectoral scopes are *assumed.*

This could be a major drawback of this approach, as the scope external effects might depend on the environment. For example, government restrictions on trade or factor flows might change the extent to which these external effects are country-specific.

Autarky Equilibrium: In vector notation,

(P=C): $p^A = C(w^A;\xi^A) = w^A A(w^A;\xi^A)$

(RC): $A(w^A,\xi^A)X^A = V^A$

(DC): $X^A = [e_p(p^A)/e(p^A)](p^A X^A) = [e_p(p^A)/e(p^A)](w^A V^A)$

Integrated Equilibrium (IE):

(P=C): $p^I = C(w^I;\xi^I) = w^I A(w^I;\xi^I)$

(RC): $A(w^I,\xi^I)X^I = V^I$

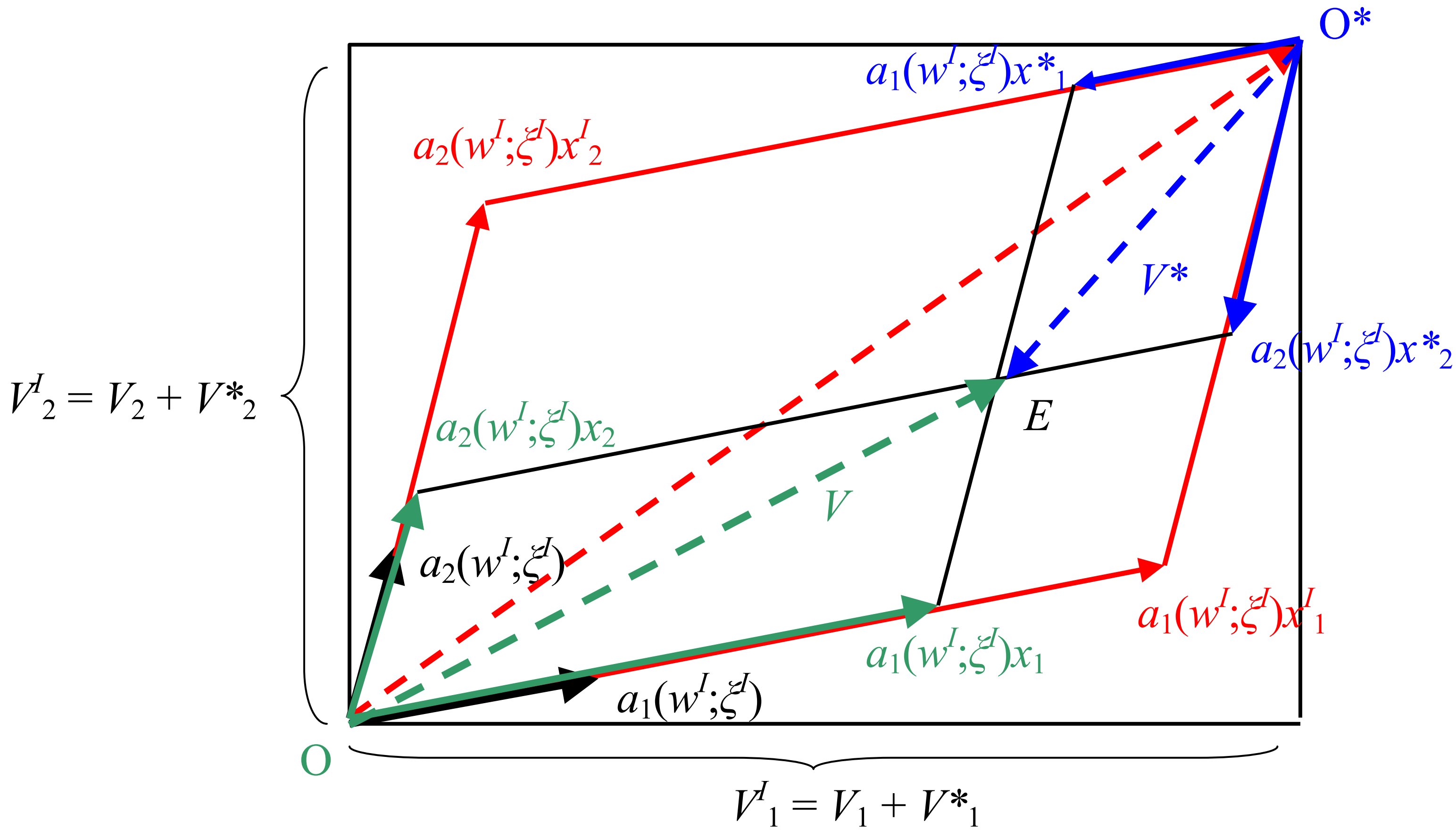
(DC): $X^I = [e_p(p^I)/e(p^I)](p^I X^I) = [e_p(p^I)/e(p^I)](w^I V^I)$

Can we replicate the IE when factors become immobile?

The answer depends critically on the scope of external effects.

Global External Economies of Scale:

When the external effects are entirely global in geographical scope (e.g., Japanese and German engineers can still learn from each other, even though they cannot relocate across borders), the answer is YES, under the same condition in Part 3.



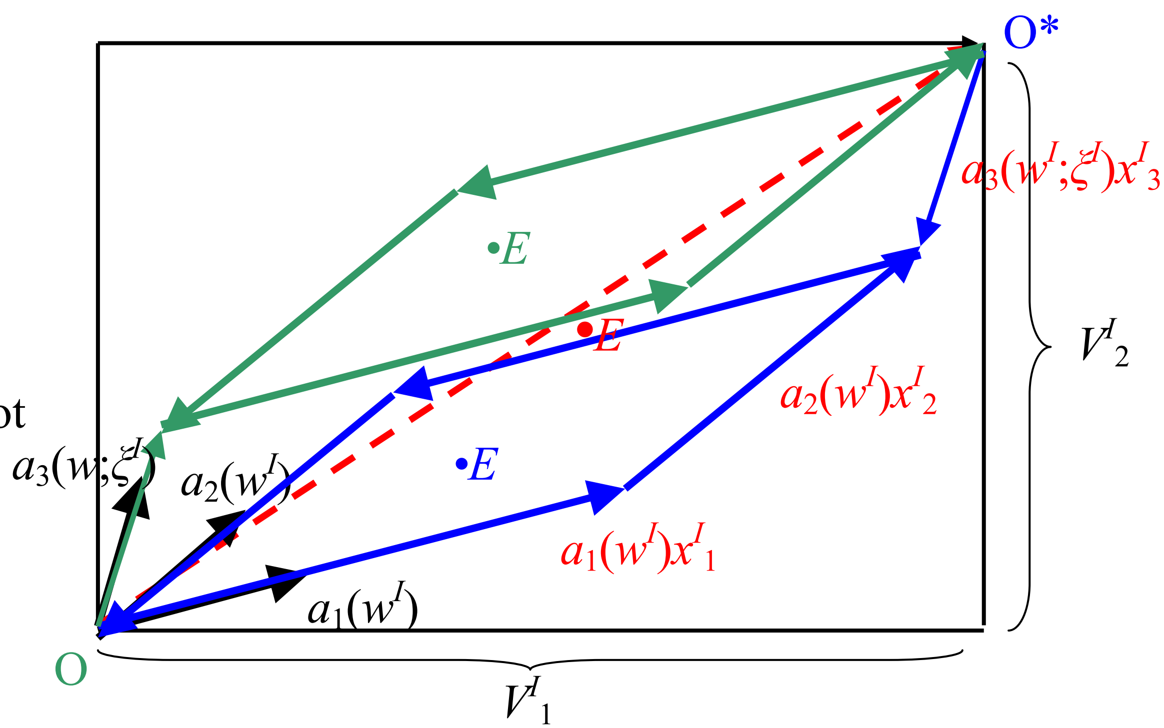
National External Economies of Scale:

What if the external effects are national in geographical scope? To keep things simple, suppose that they are sector-specific. To achieve the same level of productivity as in IE, it is necessary to concentrate each line of productions that are subject to externalities in one country.

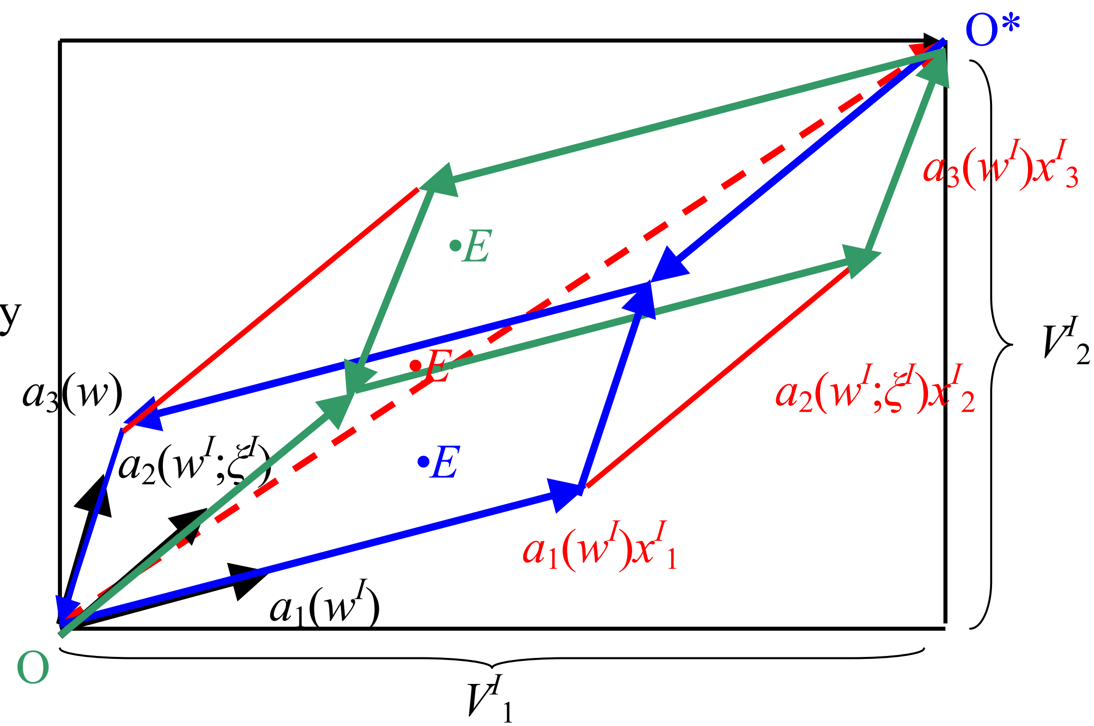
Let N_c = # of CRS sectors; $N_I = N - N_c$ = # of IRS sectors due to external effects.

Example: $M = N_c = 2$; $N = 3$. Only $j = 3$ is subject to country-specific external economies of scale.

- To replicate IE, assign Sector-3 to Home (Foreign) inside the Green (Blue) Parallelogram.
- Outside of these parallelograms, IE cannot be replicated.
- Factor proportion similarity does not ensure FPE, let alone IE. (See the Red Dot, E)



Example: $M = N_c = 2$; $N = 3$. Only $j = 2$ is subject to country-specific external economies of scale.



- To replicate IE, assign Sector-2 to Home (Foreign) inside the Green (Blue) Parallelogram.
- In the overlapping area, sector-2 may be assigned to either country. (See the Red Dot E.)
- Outside of these parallelograms, IE cannot be replicated.

Notes:

- We may hope for replicating IE when $N_c = N - N_I \geq M$, the condition ensuring the IE set has the full dimensionality in the factor space.
- Patterns of trade may be indeterminate even when the IE can be replicated (as depicted by the Red dot, E). But, the factor content of trade is uniquely determined.
- There may be multiple equilibriums, only a few of which replicates IE. Other equilibriums that fail to replicate IE, even though some of them might achieve FPE.
- If national external economies are not sector-specific, there will be additional constraints, as a set of sectors may have to be located in the same country.

Example:

$M = N_c = 1$; $N = 2$. Let $j = 1$ be the IRS sector and j =2 be the CRS sector.

Integrated Equilibrium (IE):

(P=C): $p_1{}^I = C^1(w^I; X_1{}^I)$, $p_2{}^I = C^2(w^I)$

(RC): $L^I = C^1(1, X_1{}^I)X_1{}^I + C^2(1)X_2{}^I$

(DC): $C^2(1)X_2{}^I = \alpha_2(p^I)(w^I L^I)$.

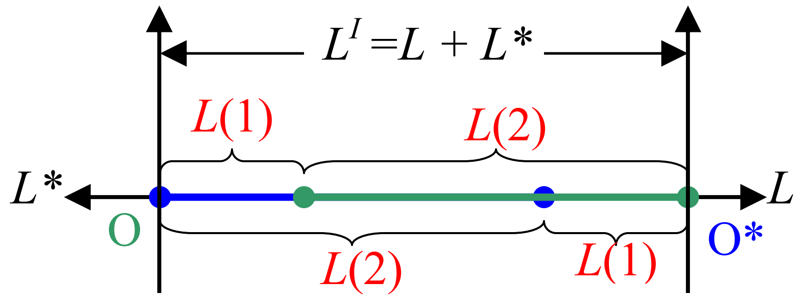
Let $L(1) \equiv C^1(1, X_1{}^I)X_1{}^I$, $L(2) \equiv C^2(1)X_2{}^I$.

Now suppose that the world is divided into Home with L and Foreign with $L^* = L^I - L$.

Can IE be replicated in a Two-Country Equilibrium?

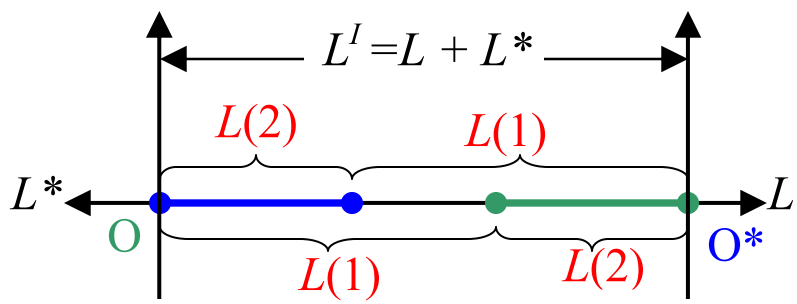
If $L > L(1)$ (the Green Interval), IE can be replicated by assigning Sector-1 to Home.
If $L^* > L(1)$ (the Blue Interval), IE can be replicated by assigning Sector-1 to Foreign.

Case I: $L(1) < L(2)$.



If $L(1) < L,\ L^* < L(2)$,
Sector-1 can be assigned in either country.

Case II: $L(1) > L(2)$.



If $L(1) > L,\ L^* > L(2)$, neither Home nor Foreign is big enough to operate Section-1 at the same scale with IE. Hence, it is not possible to replicate IE.

Two Important Questions:

- What happen when IE cannot be replicated?
- Does the existence of an equilibrium replicating IE guarantee that IE will be replicated? There may be other equilibriums which do not replicate IE.

To address these questions, let us assume:

$$e(p)u = (p_1)^{\alpha}(p_2)^{1-\alpha}(u),\ \ C^1(w;X_1) = w(X_1)^{-1/2}, \text{ and } C^2(w) = w.$$

Exercise: Derive the PPF of the economy endowed with L units of the factor.

Exercise: Show that the Integrated Equilibrium (IE) is characterized by

$$L(1) = \alpha L^I,\ L(2) = (1-\alpha)L^I,\ X_1{}^I = (\alpha L^I)^2,\ X_2{}^I = (1-\alpha)L^I,$$
$$w^I / p_1{}^I = \alpha L^I,\ w^I / p_2{}^I = 1, \text{ and } u^I = (\alpha L^I)^{\alpha}.$$

Note: The presence of scale effects; the welfare is higher with a higher L^I. Thus, IE is clearly better than Autarky. This suggests *potential* gains from trade *even* when the two countries are identical.

Two-Country Trading Equilibrium Condition:

$$p_1 \le w(X_1)^{-1/2},\ p_2 \le w; \qquad p_1 \le w^*(X_1^*)^{-1/2},\ p_2 \le w^*$$
$$L = (X_1)^{1/2} + X_2; \qquad L^I - L = L^* = (X_1^*)^{1/2} + X_2^*$$
$$p_1(X_1 + X_1^*) = \alpha(wL + w^*L^*).$$

There are many types of equilibrium.

We already know the following two types can replicate IE (if the condition is met).

I: If $L > L(1) = \alpha L^I$ → $(X_1, X_2) = ((\alpha L^I)^2, L - \alpha L^I)$; $(X_1^*, X_2^*) = (0, L^*)$;
$w/p_1 = w^*/p_1 = \alpha L^I$; $w/p_2 = w^*/p_2 = 1$.

II: If $L^* > L(1) = \alpha L^I$ → $(X_1, X_2) = (0, L)$; $(X_1^*, X_2^*) = ((\alpha L^I)^2, L^* - \alpha L^I)$;
$w/p_1 = w^*/p_1 = \alpha L^I$; $w/p_2 = w^*/p_2 = 1$.

- Type I & Type II are the mirror images of each other.

There exists another FPE equilibrium, which fails to replicate IE.

III: If $\alpha L^I/2 < L < (1-\alpha/2)L^I$

$$\rightarrow\ (X_1, X_2) = ((\alpha L^I/2)^2, L - (\alpha/2)L^I); \qquad (X_1^*, X_2^*) = ((\alpha L^I/2)^2, L^* - (\alpha/2)L^I)$$

$$w/p_1 = w^*/p_1 = \alpha L^I/2; \qquad w/p_2 = w^*/p_2 = 1.$$

- Worse than IE, as the gains from specialization are not fully exploited.
- A larger country is worse off and a smaller country is better off than in autarky.
- This equilibrium may or may not be stable. See Ethier (1982) on this issue.

There are also non-FPE equilibriums.

IV: If $L < L(1) = \alpha L^I \rightarrow\ (X_1, X_2) = (L^2, 0);\ \ (X_1^*, X_2^*) = (0, L^*)$ with $\dfrac{w}{w^*} = \dfrac{\alpha}{1-\alpha}\dfrac{L^*}{L} > 1$

$$\frac{w}{p_1} = L > \frac{w^*}{p_1} = \frac{1-\alpha}{\alpha}\frac{L^2}{L^*};\ \frac{w}{p_2} = \frac{\alpha}{1-\alpha}\frac{L^*}{L} > \frac{w^*}{p_2} = 1.$$

V: If $L^* < L(1) = \alpha L^I \rightarrow\ (X_1, X_2) = (0, L);\ \ (X_1^*, X_2^*) = (L^{*2}, 0)$ with $\dfrac{w}{w^*} = \dfrac{1-\alpha}{\alpha}\dfrac{L^*}{L} < 1.$

- Type IV & Type V are the mirror images of each other.
- In both cases, a smaller country takes over the IRS sector, even though a larger country should take over the IRS sector to achieve the same level of efficiency with IE.
- The smaller country is better off than in autarky but worse off than in IE. The larger country is worse than in autarky, if the smaller country is sufficiently small. (Graham's argument for protection. See Ethier 1982).
- In these cases, allowing immigration can solve the problem. For example, in Type IV, Foreign workers would migrate to Home until $L = L(1)$.

The above example suggests that

- IRS due to external effects could be a source for potentially huge gains from trade (even among similar countries).
- The gains from trade are not guaranteed. Indeed, some countries may be better off by going back to autarky.
- The multiplicity of equilibrium leaves some rooms for mercantilistic policies.

Sufficient Condition for Gains from (Free) Trade:

In Part 1, we saw the following proof that (Free) Trade is better than Autarky:

$$E(p^F, U^A) \le p^F c^A = p^F x^A \le p^F x^F = E(p^F, U^F) \rightarrow U^A \le U^F.$$

With external effects,

- The first part of this proof should be written as $E(p^F, U^A) \le p^F c^A = p^F x^A(\xi^A)$.
- The second part of the proof should be written as $p^F x^A(\xi^F) \le p^F x^F(\xi^F) = E(p^F, U^F)$.

Therefore, $p^F x^A(\xi^A) \le p^F x^A(\xi^F) \rightarrow U^A \le U^F$.

Sufficient Condition for Gains from (Free) Trade: An Alternative

In Part 3, we saw the following proof that (Free) Trade is better than Autarky:

$$E(p^F, U^A) \le p^F c^A = p^F x^A = w^F A(w^F) x^A \le w^F A(w^A) x^A = w^F V = E(p^F, U^F) \rightarrow U^A \le U^F.$$

With external effects,

- The first part should be written as $E(p^F, U^A) \le p^F c^A = p^F x^A = w^F A(w^F;\xi^F) x^A$.
- The second part should be written as $w^F A(w^A;\xi^A) x^A = w^F V = E(p^F, U^F)$.

Therefore, $w^F A(w^F;\xi^F) x^A \le w^F A(w^A;\xi^A) x^A \rightarrow U^A \le U^F$.

Exercise: Interpret these sufficient conditions.

More on sufficient conditions for gains from trade, see Kemp and Negishi (1970), Helpman and Krugman (1985; Ch.3)

What if there are a large number of sectors with IRS due to external effects?

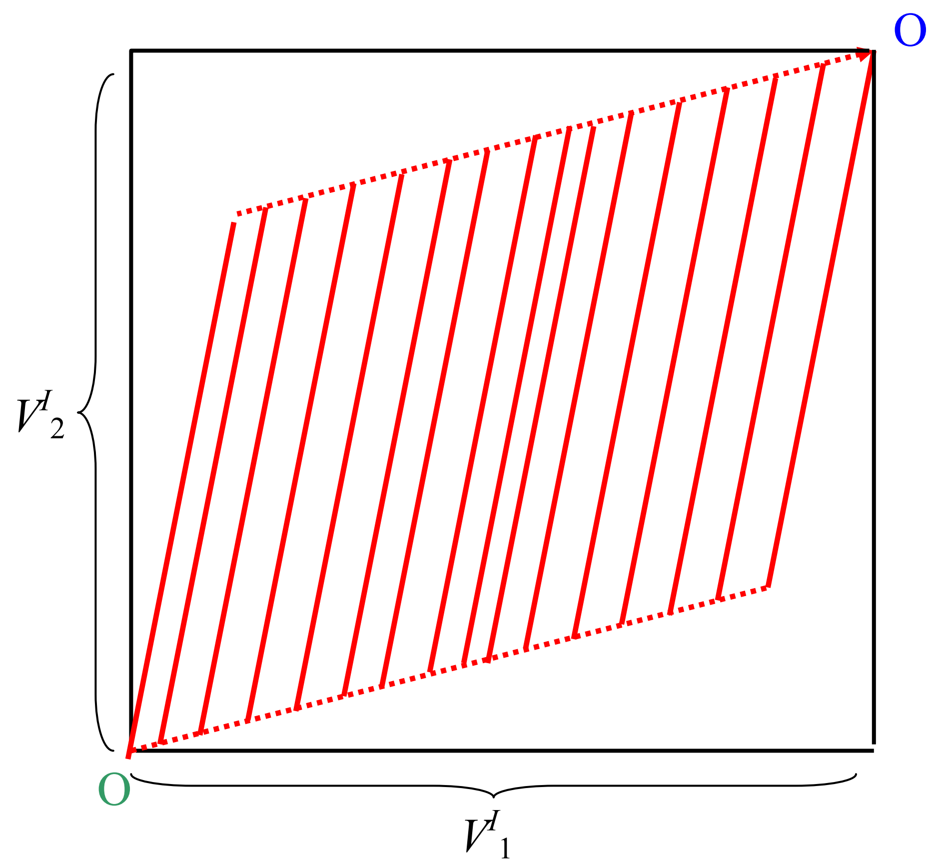
- It was argued earlier that we may hope for replicating IE when $N_c = N - N_I \geq M$, because that ensures that the IE set has the full dimensionality in the factor space.

- This may seem to suggest that the problem is more serious when many sectors are subject to IR due to externalities.

- Perhaps paradoxically, one could argue that this problem is less serious with many such sectors.

Figure illustrates the case where $N_I = 15$ and $N_c = 1 < M = 2$.

- Even though the IE set is measure zero, one could find an equilibrium which approximates IE closely with a large N_I .
- In the limit, $N_I \rightarrow \infty$, IE can be arbitrarily closely replicated.



This argument should be interpreted with great caution, for at least two reasons.

First, the argument assumes that the external economies are sector-specific. If there are some spillovers among a subset of industries, all of these sectors need to be allocated to the same country.

- If we look at fine categories of industries, the sector-specificity assumption becomes less tenable.

- If we try to define a sector broad enough so that the sector-specificity assumption becomes more palatable, we may have to treat the entire production sector as one sector.

Second, the problem of multiplicity becomes worse as N_I becomes large as the following example suggests.

Example: Extend the baseline DFS (1977) Ricardian model to introduce country-specfic and sector-specific external effects as follows:

$$C_z(w, X(z)) = a(z)(X(z))^{-\lambda(z)} w; \quad C_z^*(w^*; X^*(z)) = a^*(z)(X^*(z))^{-\lambda(z)} w^*$$

where $A(z) \equiv a^*(z)/a(z)$ is strictly decreasing in z and $0 \leq \lambda(z) < 1$.

Exercise: Assuming the symmetric Cobb-Douglas preferences, calculate the Autarky Equilibrium. What role does the assumption, $\lambda(z) < 1$, play?

Is the equilibrium allocation uniquely determined when Home and Foreign trade?

- If $\lambda(z)$ = 0, this is the original DFS Ricardian model. Hence, the patterns of trade are uniquely determined.
- If $\lambda(z)$> 0, however small, *any* patterns of trade are possible.

Exercise: Demonstrate the above statement.

Exercise: Study the welfare implications.

Common Justifications for External Effects and Their Limitations

Shared Inputs and Infrastructure:

- Geographical mobility of shared inputs might affect the geographical scope of external effects. If the intermediate inputs are freely tradeable, externalities should be global in scope: Ethier (1979).
- External economies due to certain types of public goods (as well as diseconomies due to congestion externalities) should be local in scope. If there is an optimal size for cities or metropolitan areas, aggregate technologies at the country level might satisfy CRS (with the number of metropolitan areas adjusting endogenously). Then, the country size should not matter: Rauch (1989), Rossi-Hansberg-Wright (2007).

Imperfect Appropriability of Knowledge or Knowledge Spillovers (such as experiences, know-how, etc.)

- If it is through word-of-mouth, spillovers may be local in scope. Or it might depend on the social network. If it is through "reverse engineering", spillovers may be global in scope.
- The static framework is too restrictive. (e.g, knowledge may accumulate through learning-by-doing. In Part V, we will look at some models of dynamic external economies of scale.

All of these suggest that we need to look inside the “Black Box.”

Duration and Puga (2004), in a survey of this issue in urban economics, propose the following classification of the micro-foundations of external economies.

- Sharing
  - Sharing of indivisible goods and facilities
  - Sharing a large variety of available goods and inputs
  - Sharing the gains from individual specialization
  - Sharing risk
- Matching
  - Improving the quality of matches
  - Improving the chance of matching
  - Mitigating the hold-up problems
- Learning
  - Knowledge Creation
  - Knowledge Diffusion, etc.

In the context of international (and interregional) trade, we will focus mostly on models that generate scale economies through sharing a large variety of available goods and inputs.

Competitive Models of Trade with Nonconvex Technologies

A Ricardian Model with Nonconvex Technologies

We modify a Ricardian model by introducing some nonconvexities at the firm-level technologies. In sector-j, any firm can produce x units of good-j using $T_j(x)$ units of labor, where the firm's *average labor requirement,* $a_j(x) \equiv T_j(x)/x$, is U-shaped. That is, it is decreasing for $0 < x < x_j^m$, reaches the bottom at $x = x_j^m$, and is increasing for $x > x_j^m$.

Exercise: Show that this implies the *marginal labor requirement,* $m_j(x) \equiv T_j'(x)$, satisfies $m_j(x) < a_j(x)$ for $x < x_j^m$, $m_j(x) = a_j(x)$ at $x = x_j^m$, and $m_j(x) > a_j(x)$ for $x > x_j^m$.

The average cost curve is downward-sloping at a lower scale, implying nonconvex technology. However, it is upward-sloping at a higher scale. $T_j(x) = f_j + m_j x^{(1+\eta)}$ satisfies this assumption for any $\eta > 0$, but not for $\eta = 0$.

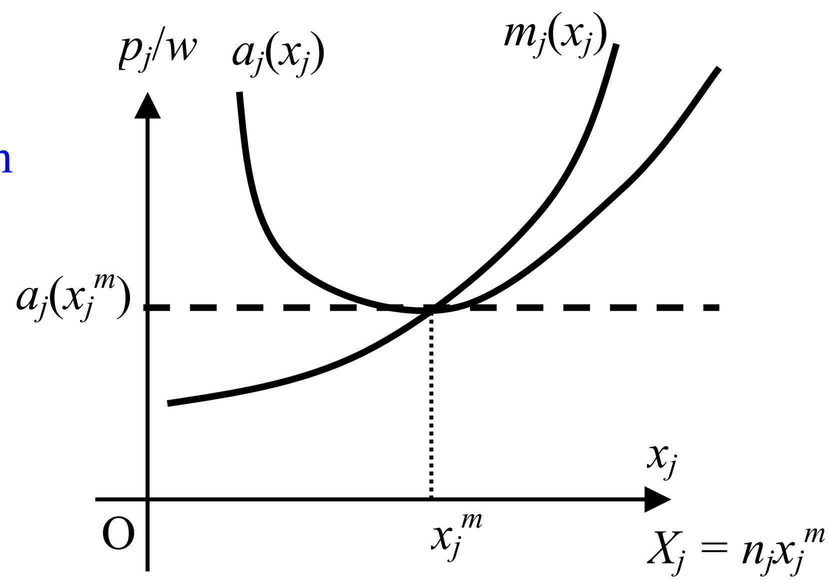
If there are n_j firms in sector-j and each of them produces x_j of good-j, the sector's total output is $X_j = n_j x_j$ and the sector's labor input per output is $n_j T_j(x_j)/X_j = T_j(x_j)/x_j$. This is minimized at $x_j = x_j^m > 0$ and is given by $m_j(x_j^m) = a_j(x_j^m)$.

We now show:

Given p_j and w,

$$p_j \le a_j(x_j^m)w; \quad X_j = n_j x_j^m, \text{ and,} \quad [p_j - a_j(x_j^m)w]X_j = [p_j - a_j(x_j^m)w]n_j = 0,$$

in equilibrium.



Proof:

Profit maximization requires that each active firm in sector j operates at the scale, x_j, satisfying $p_j = wm_j(x_j)$.

- If $x_j < x_j^m$, $p_j = wm_j(x_j) < wa_j(x_j)$, which would mean a negative profit, causing some firms to exit.
- If $x_j > x_j^m$, $p_j = wm_j(x_j) > wa_j(x_j)$, which would mean a positive profit, causing some firms to enter.

The equilibrium thus requires either

- $n_j > 0$ with $x_j = x_j^m$ and $p_j = wa_j(x_j^m)$ so that all active firms make zero profit, or
- $n_j = 0$ with $p_j < wa_j(x_j^m)$, which implies $p_j < wa_j(x)$ for any $x > 0$ so that no firm could make a non-negative profit if enters.

Q.E.D.

Message: This sector, a collection of competitive firms equipped with the nonconvex technology, may be viewed as the CRS sector with its unit labor requirement equal to $a_j = m_j(x_j^m) = a_j(x_j^m)$. The only difference is a matter of interpretation. Here, the sector's output, $X_j = n_j x_j^m$, change only through extensive margin, i.e., by entry-exit of firms, n_j. Indeed, we can normalize $x_j^m = 1$ by choice of the unit, so that $X_j = n_j$.

The same logic can be applied to ***multi-factor models***, such as Heckscher-Ohlin.

- Suppose that any firm can produce x units of good-j at the cost of $C_j(w,x)$, strictly increasing in x with all the properties of the unit cost functions for a given x.
- If it is homothetic in w, $C_j(w,x) = c_j(w)T_j(x)$ and $a_j(x) \equiv T_j(x)/x$, is U-shaped; i.e., it is decreasing for $0 < x < x_j^m$, reaches the bottom at $x = x_j^m$, and is increasing for $x > x_j^m$,
- then sector-j could be viewed as if it has the CRS technology with the unit cost function, $c_j(w)m_j(x_j^m) = c_j(w)a_j(x_j^m)$ and its output is given by $X_j = n_j x_j^m$.

Exercise: Consider how the analysis has to be modified when $C_j(w,x)$ is nonhomothetic? For example, $C_j(w,x) = f_j(w) + m_j(w)x^{(1+\eta)}$ with $f_j(w) \neq m_j(w)$ so that the fixed cost and marginal cost components differ in the factor intensities.

This way, we can incorporate the non-convexity at the firm level technologies into any standard competitive models of trade.

Limitations:

The above logic, however, requires that

- The average cost curve is U-shaped;
- The market for each good is large enough that many firms operate at the bottom of the average cost curve. Or equivalently, the number of goods produced in the economy is small, relatively to the amount of resources (factor endowments) available for production.

When these conditions are violated, we need to depart the competitive paradigm, since price-taking firms cannot operate at the downward-sloping part of the average cost curve.

We now turn to the *monopolistic competitive* paradigm, which allows us to deal with these situations.

Monopolistic Competition and Trade in Differentiated Products:

Chamberlinian (or Large Group) monopolistic competition, formalized by Dixit and Stiglitz (1977), has revolutionalized many fields, particularly,

- Economic Growth and Development
- International and Interregional Trade
- Urban Economics, etc.

For a broady survey, see Matsuyama (1995).

Key Features of the Chamberlian monopolistic competition models:

- The variety of products that can be (potentially) profitably produced in the economy is unlimited, so that no two firms produce the identical (perfectly substitutable) product.
- Because of product differentiation, each firm has some monopoly pricing power over its own product, which makes it possible for each firm to operate at the downward-sloping part of the average cost curve.
- Free entry-exit determines the equilibrium variety of products, and ensures the zero profit.
- Lack of strategic interactions, because of a large number of active firms
- The profit maximization is the valid objective of each firm even with the monopoly power, because the product produced by each firm accounts for a negligible share of the expenditure of its owners.

International Trade in Differentiated Products:

One-Sector One-Factor Model: **Krugman (1979); Krugman (1981; Section I)**

L Homogeneous Households: Each endowed with one unit of labor (the only factor of production) and maximize $U = \int_0^n h(c(z))dz$ subject to $\int_0^n p(z)c(z)dz \le w$.

n:	variety of products available in the market (determined in equilibrium).
z:	an index for a differentiated product
c(z):	the consumption of z
p(z);	the unit price of z, when it is offered
h(•)	h(0) = 0; h'(•) > 0; h''(•) < 0.

F.O.C. $h'(c(z)) = \lambda p(z)$.

- Holding λ constant, this implies that the price elasticity of individual demand is given by $-\dfrac{\partial \log c(z)}{\partial \log p(z)} = -\dfrac{h'(c)}{ch''(c)} \equiv \sigma(c) > 0$, which is assumed to be non-increasing in c (for example, when h is quadratic).
- Strict concavity, h''(•) < 0, means that σ(•) is finite. It also means that $nh(\gamma/n)$ is strictly increasing in n, for any positive constant, γ. → "love-for-variety."

Monopolistically Competitive Firms:

Technology: The producer of z can produce x units of its product with $T(x)$ units of labor, where $T(x)/xT'(x) = a(x)/m(x)$ is *strictly decreasing* in x.
Exercise: Show that $T(x) = f + mx^{(1+\eta)}$ satisfies this assumption for any $\eta \geq 0$, incl. $\eta = 0$.

Profit Maximization:

- Each active firm tries to max $\pi(z) \equiv p(z)x(z) - wT(x(z))$, taking the demand curve given (i.e, holding λ constant).
- This means that the firm perceives that its product accounts for a negligible share in the consumer's expenditure. It also means that no strategic interactions between firms.

Free Entry (and Exit):

- Each firm offers its product if and only if the maximized profit is non-negative.
- When deciding whether to enter or exit, each firm perceives that it has negligible effects.
- Since there are unlimited number of symmetric products that can be introduced, no firm chooses to offer the produce produced by other firms.

Labor Market Equilibrium: $\int_0^n T(x(z))dz = L$.

Closed Economy: The elasticity of the total demand, $x(z) = Lc(z)$, is equal to the elasticity of each household's demand.

Monopoly-Pricing: $p(z)\left(1-\dfrac{1}{\sigma(x(z)/L)}\right) = wm(x(z))$

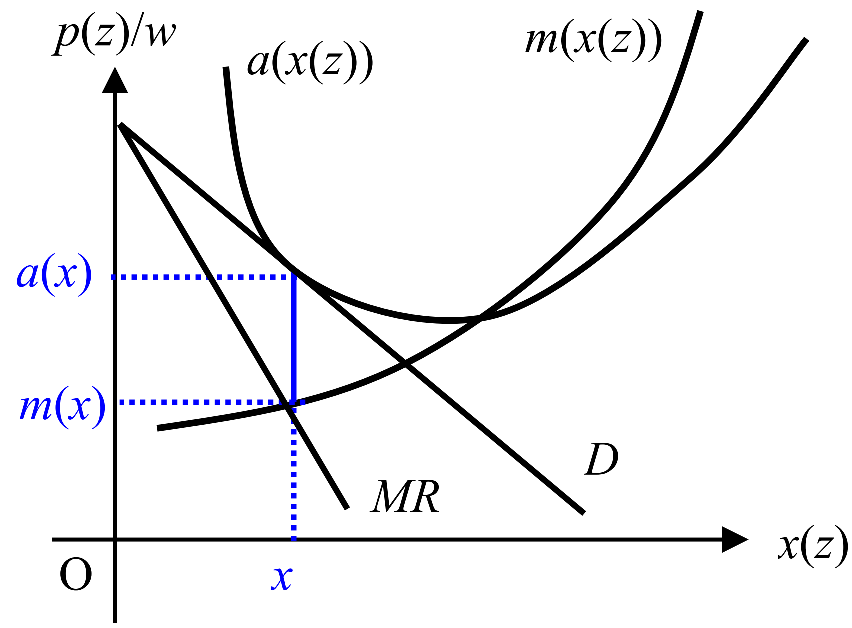
Zero Profit: $p(z) = wa(x(z))$

Combining these two conditions yields

$$1-\frac{1}{\sigma(x(z)/L)} = \frac{m(x(z))}{a(x(z))},$$

which uniquely determines x(z) = x for all z ≤ n, which in turn implies p(z) = p for all z ≤ n, and

$$\frac{p}{w} = a(x) = m(x)\left[1-\frac{1}{\sigma(x/L)}\right]^{-1} > m(x).$$



Labor Market Equilibrium: $\int_0^n T(x(z))dz = L \rightarrow n = \dfrac{L}{T(x)}.$

Dixit-Stiglitz (1977) assumes CES; $h(c) = c^{\theta}$; $0 < \theta < 1$ → $\sigma = \dfrac{1}{1-\theta} > 1$ or $\theta = 1 - \dfrac{1}{\sigma}$.

Hence, recursively,

$$\frac{m(x)}{a(x)} = \theta < 1 \rightarrow \frac{p}{w} = a(x) \rightarrow n = \frac{L}{T(x)}.$$

Note: *x, p/w,* and *n/L* are all independent of *L*.

Welfare: $U = nh\left(\dfrac{x}{L}\right) = \dfrac{L}{T(x)} h\left(\dfrac{x}{L}\right) \equiv V(L) = \dfrac{x^{\theta} L^{1-\theta}}{T(x)}$, increasing in L.

Message: *Aggregate Economies of Scale* due to *Increased Product Variety*
Intuition: A larger economy offers a wider variety of products, as the fixed cost of adding another product can be spread across many households. This enables each household enjoys a higher utility through more variety.

Note: In this case, preferences are homothetic so that the equilibrium would be the same if we had assumed the representative household endowed with L units of labor.

***Exercise:* (Variable Elasticity Case; Krugman 1979)**:
Prove that, if $\sigma(c)$ is strictly decreasing,

$$L\uparrow \Rightarrow n\uparrow, x\uparrow, \frac{p}{w} = a(x)\downarrow, \frac{n}{L}\downarrow, U\uparrow.$$

In this case, *Aggregate Economies of Scale* due to two effects:

- *Increased Product Variety* (as before)

- *Pro-competitive effect:* With more product variety, each household consumes less of each product, which makes different products more substitutable. This leads to more competition, and a lower mark-up. In order to break even, this forces firms to operate at a larger scale, which means a lower average cost, and hence a higher real wage. This also means a saving in the fixed cost (i.e., the product variety per capita declines.)

Note: In this case, preferences are not homothetic. So, the equilibrium would be different if we had assumed the representative household endowed with L units of labor.

World Economy with Home and Foreign, which differ only in L and L^*

Suppose all goods are costlessly tradeable. Let's look for an equilibrium with $w = w^*$. (Are there an equilibrium with $w \neq w^*$? I don't know for the variable elasticity case.)

- As the consumer, each household, regardless of its location, faces the same problem, except the product variety expands to $n + n^*$ under free trade. Hence, it consumes $n + n^*$ variety by c units per variety.
- As the seller, each firm, regardless of its location, faces the same problem, except the market size expands to $L + L^*$ under free trade. Each firm produces $x = c(L+L^*)$.
- Hence, monopoly pricing and zero profit conditions imply

$$\frac{p}{w} = \frac{p^*}{w^*} = a(x) = m(x)\left[1 - \frac{1}{\sigma(x/(L+L^*))}\right]^{-1} \Leftrightarrow \frac{m(x)}{a(x)} = 1 - \frac{1}{\sigma(x/(L+L^*))}.$$

- Labor markets are separate, so that $n = L/T(x)$ and $n^* = L^*/T(x)$.

For the CES case, no change in x, w/p, w^*/p^*, n, and n^*.
For the variable elasticity case, $n + n^*$ goes up, and hence, x goes up after trade. This means that n and n^* both decline. Some firms exit, while the surviving firms expand: ***The pro-competitive effect leads to a rationalization.***

Gains from Trade: $U = U^* = V(L + L^*) > U^A = V(L)$, $U^{*A} = V(L^*)$.

A smaller country gains more from trade, because, in autarky, the consumers in a smaller country would be more restricted in their product choice than those living in a larger country. In the variable elasticity case, there are additional gains through the increased efficiency (and the higher real wage).

Note: The assumption that all the goods are costlessly tradeable is crucial for this result.

Example: Suppose that all goods are nontradable but labor is mobile. Then,

$$L > L^* \rightarrow U = V(L) > U^* = V(L^*) \rightarrow L >> L^*.$$

Likewise,

$$L < L^* \rightarrow U = V(L) < U^* = V(L^*) \rightarrow L << L^*.$$

Households prefer moving to a larger economy, where they could enjoy more product variety. This leads to a geographical concentration or urbanization. Without any offsetting force, all households would move to one location, but the model does not say which location.

We will discuss this issue in more detail later.

Volume of Trade:

Since there are n Home firms, each of which exports $L^*/(L+L^*)$ fraction of its output,

$$\text{Home Export } = n(px)\frac{L^*}{L+L^*} = \frac{LL^*}{L+L^*}\frac{px}{T(x)} = \text{Foreign Export}$$

Since World Income (WI) = $w(L + L^*)$,

$$\frac{WT}{WI} = \frac{2LL^*}{\left(L+L^*\right)^2}\frac{px}{wT(x)} = \frac{2LL^*}{\left(L+L^*\right)^2} = 2s(1-s), \qquad \text{where } s \equiv \frac{L}{L+L^*}.$$

This is maximized when $s = ½$ or $L = L^*$. Or, the Volume of Trade is larger between the countries of similar sizes.

Notes:

- The model predicts trade even among identical countries, unlike competitive models with homogenous products.
- The trade flow is completely indeterminate, as the products enter symmetrically.

Gravity Equation: A Multi-country Extension

Let the world economy consist of many regions (C, D, …). The population of each region is given by L^C, L^D, etc. (A region can be a country, or a group of countries.) Calculate the volume of bilateral trade between two non-overlapping regions, C and D.

Region C produces $n^C = \dfrac{L^C}{T(x)}$ varieties.

Region D consumes $s^D = \dfrac{L^D}{L^W}$ fraction of each.

→ Export from C to D = $n^C s^D px = \dfrac{L^C L^D}{L^W} \dfrac{px}{T(x)}$ = Export from D to C.

→ Bilateral Trade C&D/World Income: $\dfrac{L^C L^D}{\left(L^W\right)^2} \dfrac{2px}{wT(x)} = \dfrac{2L^C L^D}{\left(L^W\right)^2} = 2s^C s^D$

Gravity Equation: The trade volume between two regions is proportional to the product of the masses (measured by population or income) of the two regions.

Note: This equation has no "distance" variables, for which we need to introduce the trade costs in models.

Reinterpreting Dixit-Stiglitz: Gains from Variety in Intermediate Inputs

The Household Sector: supplies L units of labor and consumes the single (nontradeable) final good, taken as a numeraire, by $Y = wL$.

The Final Good Sector: competitive, with the CRS technology, $Y = X \equiv \left\{ \int_0^n [x(z)]^\theta dz \right\}^{\frac{1}{\theta}}$

n: the variety of intermediate inputs offered in equilibrium.
z: an index for a differentiated intermediate input
x(z): the amount of input z used

Cost Minimization: $P \equiv Min\left\{ \int_0^n p(z)x(z)dz \middle| X \geq 1 \right\} = \left\{ \int_0^n [p(z)]^{\theta/(\theta-1)} dz \right\}^{1-1/\theta}$.

Input Demand: $x(z) = X\left[\frac{p(z)}{P}\right]^{\frac{1}{\theta-1}} = PX \frac{[p(z)]^{1/(\theta-1)}}{\int_0^n [p(s)]^{\theta/(\theta-1)} ds}$

The Intermediate Inputs Sector: monopolistically competitive with the total labor requirement, T(x). Each firm sells its own product to the Final Goods sector.

Closed Economy, where the inputs can be sold only domestically:

Monopoly-Pricing + Zero-Profit Conditions: $\frac{p}{w} = a(x) = \frac{m(x)}{\theta}$

Final Goods Sector Zero-Profit Condition: $P = 1 \rightarrow p = n^{(1-\theta)/\theta}$

Labor Market Equilibrium: $n = \frac{L}{T(x)}$

$$\rightarrow w = \frac{p}{a(x)} = \frac{n^{\frac{1-\theta}{\theta}}}{a(x)} = \frac{1}{a(x)}\left[\frac{L}{T(x)}\right]^{\frac{1-\theta}{\theta}}; \qquad Y = wL = \frac{L}{a(x)}\left[\frac{L}{T(x)}\right]^{\frac{1-\theta}{\theta}}$$

GDP per worker is higher in a larger economy. Aggregate Economies of Scale, Again!!

Exercise: How is this measure related to $V(L)$ calculated for the CES earlier?

Exercise: Characterize the World Economy Equilibrium, in which intermediate input producers can sell their products anywhere in the world at zero trade cost.

Exercise: What happens if intermediate inputs are nontradeable but labor is mobile?

Interpreting Gains from Variety as Gains from Specialization

- Ethier (1982) used the production version of Dixit-Stiglitz to show that smaller countries enjoy more productivity gains from trade, as they benefit more from having access to a variety of inputs produced around the world.

- Romer (1990) interpreted this as ***Increasing Returns due to Specialization***. With more specialized inputs available, it is easier for each final goods producer to find the inputs that meet his specific needs. This interpretation, however, suggests heterogeneous final goods producers buying different inputs.

- Weitzman (1994) used a spatial model of monopolistic competition where the buyers are distributed uniformly along the circle. In the standard spatial model, competition becomes more intense with more firms enter. In the Weitzman model, firms choose the product specificity. With more firms competing, each firm makes its product more specific. Under some conditions, his model is isomorphic to the Dixit-Stiglitz model.

- ***Discrete Choice Model:*** Alternatively, for each buyer, the value of each product is randomly drawn from the common distribution. Ex post, the buyers are heterogeneous. Each buyer chooses the product that generates the highest surplus. Under some conditions, this model is isomorphic to the Dixit-Stiglitz model. See Feenstra (2004; Appendix B) or Combes-Mayer-Thisse (2008, Ch.3).

Some Restrictive Features of CES:

Under $U = \int_0^n h(c(z))dz$ with $h(c) = c^{\theta}$,

$$U = nh(w/np) = (w/p)^{\theta} n^{1-\theta} \text{ for } npc = w \qquad \Rightarrow \quad \frac{\partial \log U}{\partial \log n} = 1 - \theta = \frac{1}{\sigma}.$$

Marginal value of variety is thus independent of the existing variety, n, and independent of the real income, w/p. This property is responsible for:

- Mark-up rate is independent of n, (or the lack of pro-competitive effects)
- Homotheticity of preferences (or no income effects on love-for-variety)
- No incentive to price discriminate across markets (so no concern for parallel trade)

Indeed, one could show that homotheticity and CES are closely related:

Exercise: Prove that $U = \int_0^n h(c(z))dz$ is homothetic iff $h(c) = Ac^{\theta}$.

Thus, if $h(c) \neq Ac^{\theta}$, we could discuss (or would have to worry about)

- Variable mark up and pro-competitive effects
- Income effects on love-for-variety
- Possible price discrimination across markets (and related questions of parallel trade)
- Price setting game by firms may have multiple equilibria, or asymmetric equilibria (in spite of symmetry, different firms may set different prices)

The question is then what would be tractable functional forms for $h(c) \neq Ac^{\theta}$. See, e.g., Saint-Paul (2006) and Foellmi-Hepenstick-Zweimüler (2007).

Note: We will later discuss some models with linear-quadratic preferences, but this formulation would eliminate any income effect on demand for differentiated goods.

A Note on the Optimality of the Equilibrium in the CES Case:

Exercise: In the CES case, $U = \int_0^n \left(c(z)\right)^\theta dz$, show that the monopolistic competitive equilibrium is optimal.

The reason for this well-known result is widely misunderstood. Many argue that the equilibrium is optimal, because all the goods are marked-up at the same rate, so that the monopoly pricing does not affect the relative prices. This argument is false. If the argument were correct, the equilibrium allocation would be optimal, even if $h(c) \neq Ac^\theta$, which is not the case. If the argument were correct, the equilibrium allocation would be optimal, as long as every good is sold at the same marked-up rate, even if the pricing rule differs from $p(z) = m(x(z))w/\theta$, which is not the case.

Then, why is the equilibrium is optimal? See Matsuyama (1995; Section 3.E).

Homotheticity without CES:

On the other hand, when applied to the final goods sector, we want to assume CRS technologies, so that homotheticity is required.

How can we have MC models with the CRS technologies (or homothetic preferences) that allow for variable mark-up through diminishing marginal value for variety)?

Of course, one could *assume* that θ is increasing in n, as $X \equiv \left\{\int_0^n [x(z)]^{\Theta(n)} dz\right\}^{\frac{1}{\Theta(n)}}$, but that is too ad-hoc.

Final Goods Sector with CRS: $\int_0^n h\left(\frac{x(z)}{Y}\right) dz = 1$.

Under $h(c) = Ac^{\theta}$, this specification goes back to $Y = X \equiv \left\{\int_0^n [x(z)]^{\theta} dz\right\}^{\frac{1}{\theta}}$.

Again, what would be tractable functional forms for $h(c) \neq Ac^{\theta}$?

See Behrens-Murata (2007) and Barde (2008) on this issue.

***Multi-Sector, Multi-Factor Case: Heckscher-Ohlin-Chamberlian Model*; Based mostly on Helpman and Krugman (1985; Chs.7-8)**

Household Sector: supplies its factor endowment, *V,* inelastically and consumes the single (nontradeable) final good, taken as a *numeraire*, by $Y = wV$.

Competitive *Y*-Sector: CRS technologies, $Y = F(X_1, \ldots, X_J)$, where F is linear homogeneous and

$$X_j \equiv \left\{ \int_0^{n_j} \left[x(z_j) \right]^{\theta_j} dz_j \right\}^{\frac{1}{\theta_j}}$$

is a composite of differentiated inputs of type-*j,* with $0 < \theta_j = 1 - 1/\sigma_j \leq 1$. (When $\theta_j = 1$ or $\sigma_j = \infty$, they are perfect substitutes.)

Two-Stage Cost Minimization:

1st: $\dfrac{x(z_j)}{X_j} = \left[\dfrac{p(z_j)}{P_j} \right]^{\frac{1}{\theta_j - 1}} = \left[\dfrac{p(z_j)}{P_j} \right]^{-\sigma_j}$ with $P_j \equiv \left\{ \int_0^{n_j} \left[p(z_j) \right]^{1-\sigma_j} dz \right\}^{\frac{1}{1-\sigma_j}}$

2nd: $\dfrac{X_j}{Y} = \dfrac{\partial C^Y}{\partial P_j}$ where $C^Y(P) \equiv Min \left\{ \sum_{j=1}^{J} P_j X_j \middle| F(X_1, \ldots X_J) \geq 1 \right\}$.

Intermediate Inputs Sectors:

***X_j*-sector:** monopolistically competitive with the total cost, $C^j(w,x_j)$, with the properties:
- $C^j(w_j,x_j)$ is strictly increasing in x_j.
- $x_jC^j{}_x(w,x_j)/C^j(w,x_j)$ is strictly increasing in x_j with the range that includes θ_j.

$$\left.\begin{array}{ll}\textbf{Monopoly Pricing:} & C^j{}_x(w,x(z_j)) = \theta_j p(z_j) \\ \textbf{Zero-profit:} & C^j(w,x(z_j)) = p(z_j)x(z_j)\end{array}\right\} \rightarrow C^j{}_x(w,x_j)x_j/C^j(w,x_j) = \theta_j \text{ for all } z_j \leq n_j.$$

In the homothetic case, $C^j(w,x_j) = c_j(w)T_j(x_j)$, this condition becomes

$$T_j'(x_j)x_j/T_j(x_j) = m_j(x_j)/a_j(x_j) = \theta_j \text{ for all } z_j \leq n_j,$$

which uniquely determines $x_j = x(z_j)$ for all $z_j \leq n_j$ independent of w.

Hence, $p(z_j) = p_j = c_j(w)a_j(x_j)$ for all $z_j \leq n_j$.

Note:
For $\theta_j = 1$, this means the marginal cost pricing and each firm operates at the bottom of its average cost curve, where $p_j = c_j(w)a_j(x^m{}_j) = c_j(w)a_j(x^m{}_j)$.

Closed Economy (Autarky or Integrated) Equilibrium Conditions:

Monopoly Pricing in X_j-sector: $m_j(x_j)/a_j(x_j) = \theta_j$

Zero Profit in X_j-sector: $p_j = c_j(w)a_j(x_j)$

Zero Profit in Y-sector: $C^Y(P) = 1$

Demand for Inputs by Y-sector: $X_j = \frac{\partial C^Y}{\partial P_j}(P)Y = \frac{\partial C^Y}{\partial p_j}(P)wV$

Factor Market Equilibrium: $\sum_{j=1}^J C_w^j(w, x_j)n_j = \sum_{j=1}^J c_{jw}(w)a_j(x_j)x_j n_j = V$

with $X_j = x_j(n_j)^{1/\theta_j}$ and $P_j = p_j(n_j)^{(\theta_j - 1)/\theta_j}$.

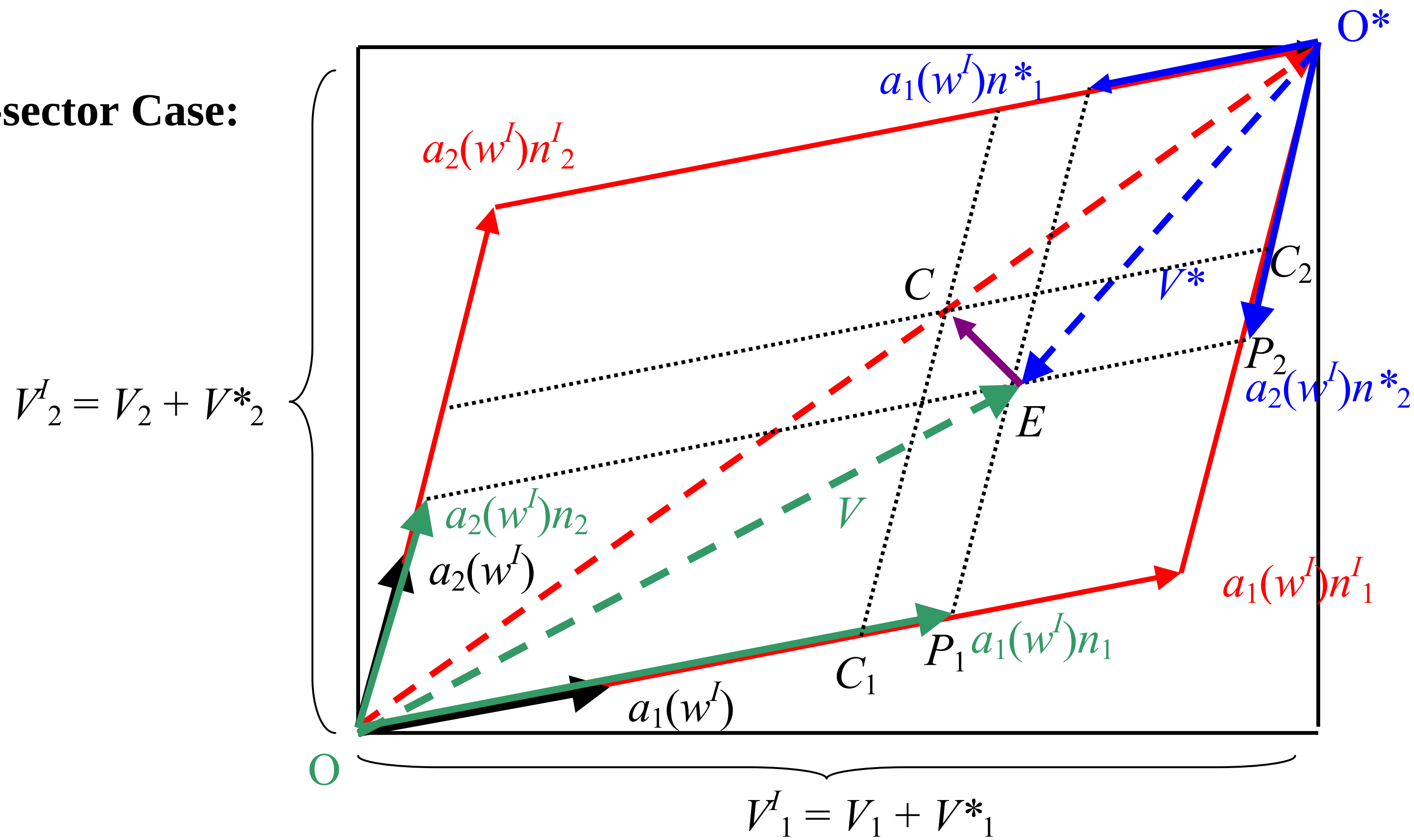
Notes:

- No profit means that $Y = wV$ in equilibrium.
- Since x_j is determined solely by $m_j(x_j)/a_j(x_j) = \theta_j$, we may normalize it to $x_j = 1$ and let $a_j(1)c_{jw}(w) \equiv a_j(w)$, which is the factor demand by each firm producing a type-j input.

Trading Equilibrium with FPE:

FPE is achieved when the factor allocation in IE, $\sum_{j=1}^{J} a_j(w^I)n_j^I = V^I = V + V^*$, can be replicated by $\sum_{j=1}^{J} a_j(w^I)n_j = V$ and $\sum_{j=1}^{J} a_j(w^I)n_j^* = V^*$ satisfying $0 \le n_j = n_j^I - n_j^* \le n_j^I$.

2-factor, 2-sector Case:



Volume of Trade in the Heckscher-Ohlin Model

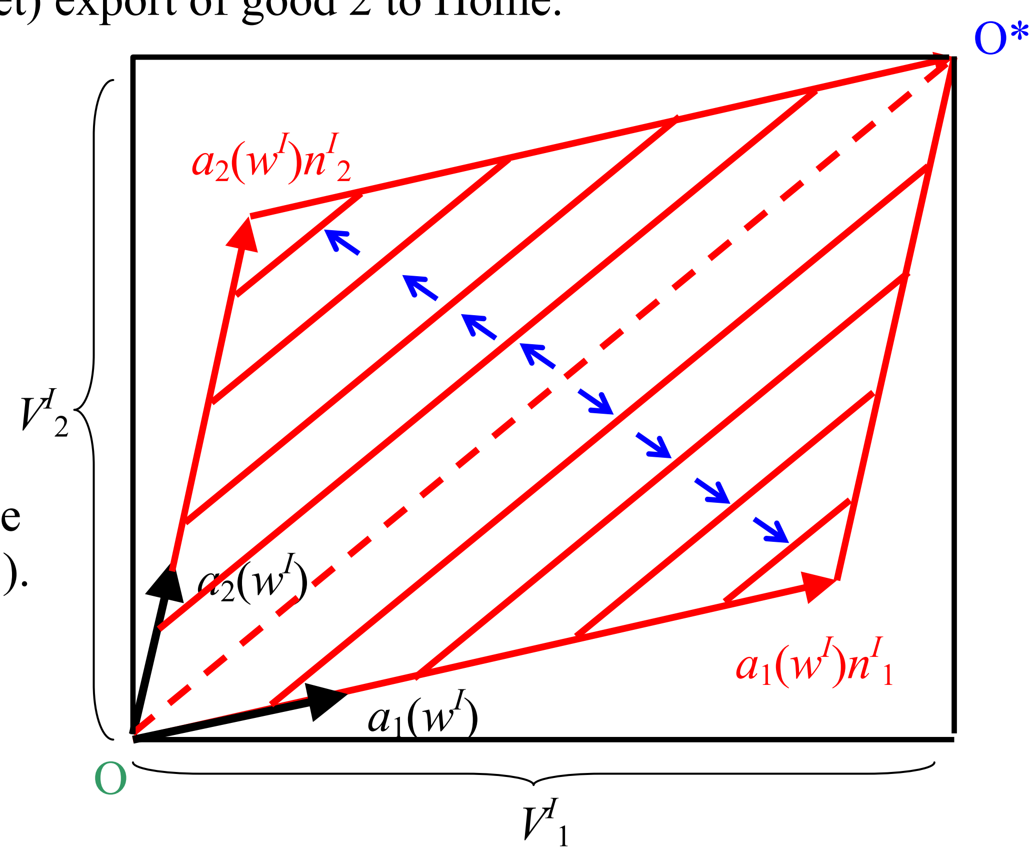
If $\theta_1 = \theta_2 = 1$, both sectors produce homogenous goods competitively. This is nothing but the Heckscher-Ohlin model.

- $\overrightarrow{C_1 P_1}$ represents the Home gross (and net) export of good 1 to Foreign.
- $\overrightarrow{C_2 P_2}$ represents the Foreign gross (and net) export of good 2 to Home.

Within the FPE set, Volume of Trade is

- zero along the diagonal.
- constant along a line parallel to the diagonal.
- greater farther away from the diagonal.

Exercise: Prove the above statement. For the answer, see Helpman-Krugman (1985, Ch.8).



Volume of Trade when All Goods are Differentiated.
If $\theta_{1,}\ \theta_2 < 1$, both sectors are monopolistically competitive and produce differentiated products. In this case, there are *two-way* flows of both type-1 and type-2 goods.

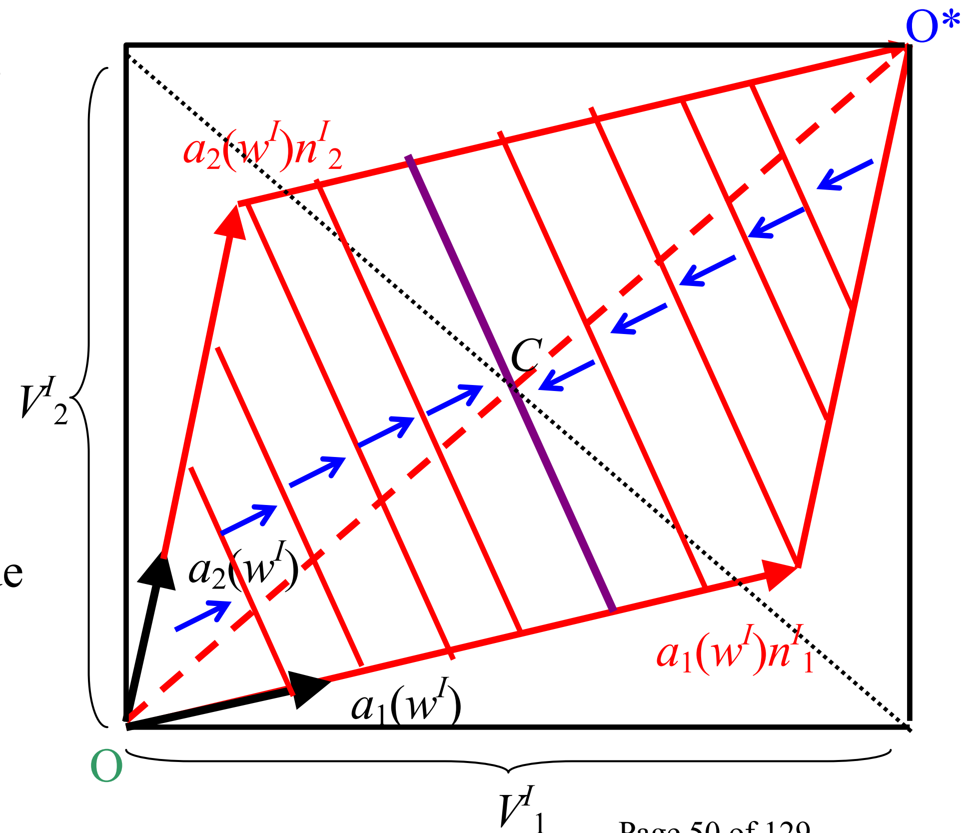
- $\overrightarrow{C_1P_1}$ represents the Home *net* (but not gross) export of type-1 goods to Foreign.
- $\overrightarrow{C_2P_2}$ represents the Foreign *net* (but not gross) export of type-2 goods to Home.

Volume of Gross Trade Flows is proportional to the product of the Home and Foreign incomes (the Gravity Equation).
Inside the FPE set, Volume of Gross Trade is

- Maximized along the Purple Line, passing through the center of the box, C, with the slope equal to the relative factor price.
- constant along a line parallel to the Purple Line.
- Smaller farther away from the Purple Line.

See Helpman-Krugman (1985, Ch.8).

Exercise: Show that the volume of gross trade is proportional to the product of the Home income and Foreign income.



Intra-Industry versus Inter-Industry Trade

If $\theta_1 = 1 > \theta_2$, sector-1 is competitive, producing a homogenous good; sector-2 is monopolistically competitive, producing differentiated goods. In this case, there are *two-way* flows of type-2 goods, but not type-1 goods.

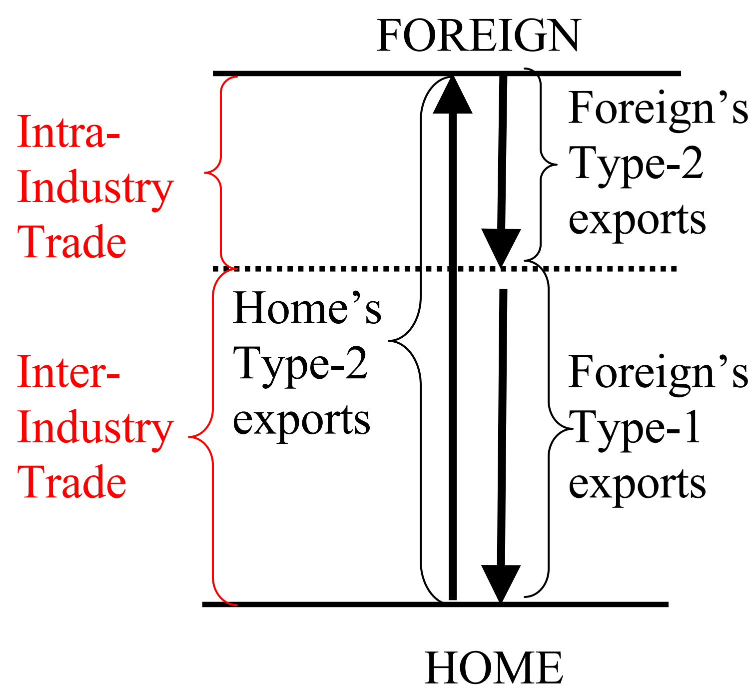
- $\overrightarrow{C_1 P_1}$ represents the Home *gross and net* export of type-1 goods to Foreign.
- $\overrightarrow{C_2 P_2}$ represents the Foreign *net* (but not gross) export of type-2 goods to Home.

The share of intra-industry in the total trade would be larger when the two countries are more similar in the factor proportions.

See Helpman-Krugman (1985, Ch.8) for

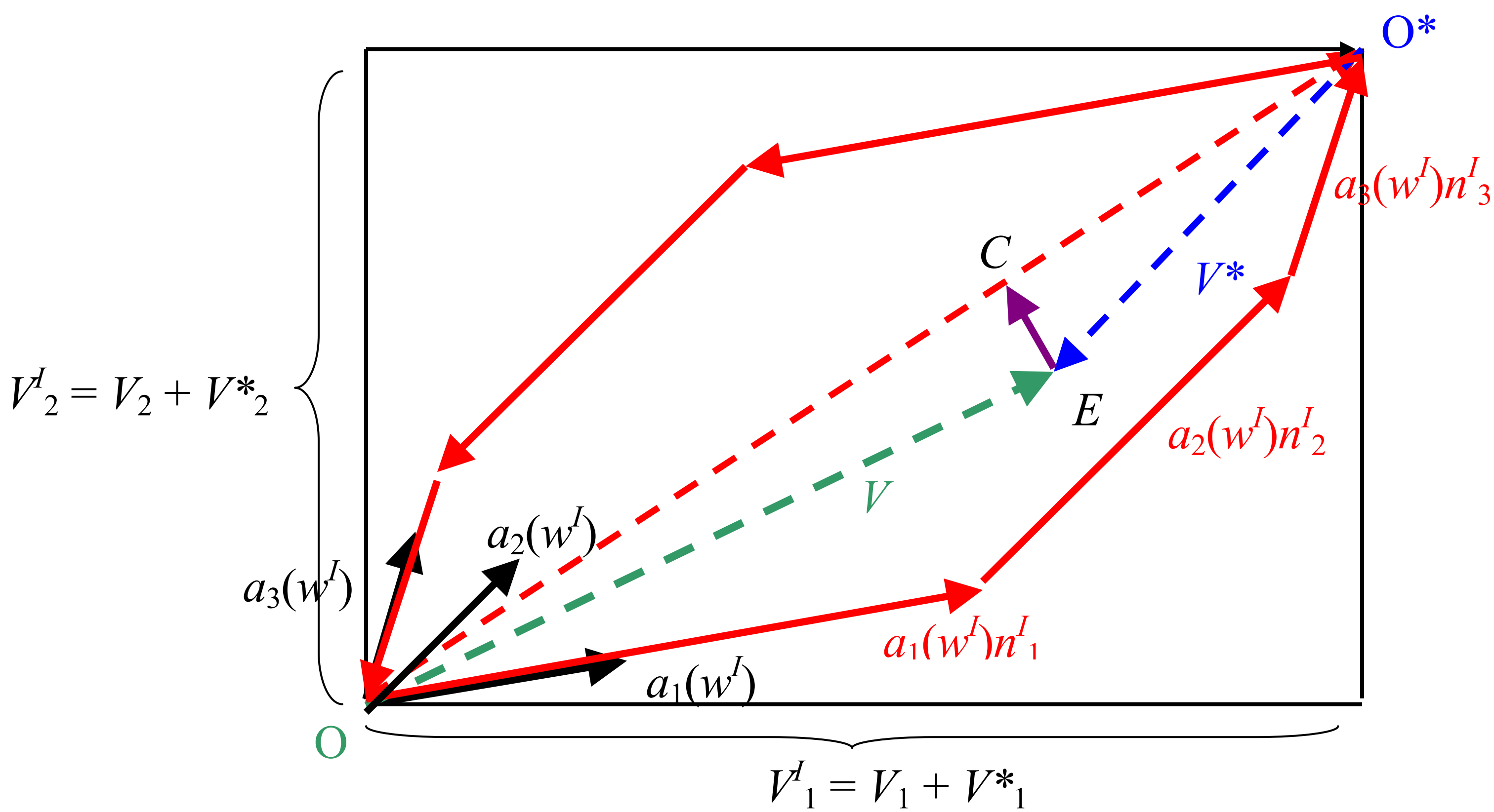
- the isocurves for the volume of gross trade
- the isocurves for the share of intraindustry trade in the total trade

inside the FPE parallelogram.



Notes:

- Obviously, these predictions would not survive for the case of $M < N$, which makes the equilibrium variety of each type of inputs produced in each country indeterminate, as in a 2-factor, 3-Sector Case shown below.
- However, the ***Factor content of Net trade*** would survive, under the assumption of the identical homothetic preferences.



Welfare and Distributional Implications:

In a Hechscher-Ohlin world of homogenous goods,
- Trade takes place when countries differ in their factor proportions.
- Trade has strong distributional consequences (i.e., Stolper-Samuelson), as it creates both winners and losers within each country through relative factor price changes.

In a world where all goods are differentiated,
- Trade takes place even among countries with the same factor proportions without distributional consequences.
- Everyone could gain from increased product variety.

This suggests that
- Even when countries differ in their factor proportions, gains from increased product variety might dominate the Stopler-Samuelson effect to make trade Pareto-improving.
- Trade liberalization among countries whose factor proportions are not too dissimilar more politically acceptable.

Krugman's* (1981) *Example:

Two Factors (j = 1, 2); Two Types of Households (j = 1, 2): Each type-j household supplies one unit of factor-j inelastically, consumes the single (nontradeable) final good, taken as a *numeraire*, by w_j.

Two Countries: Home and Foreign(*): For $0 < S < 1$,

	Home	Foreign	World
# of Type-1 households	$(1-S/2)V$	$(S/2)V$	V
# of Type-2 households	$(S/2)V$	$(1-S/2)V$	V

Competitive *Y*-Sector: CRS technologies given by

$$Y = \sqrt{X_1 X_2} = \left\{\int_0^{n_1} [x(z_1)]^{\theta} dz_1\right\}^{\frac{1}{2\theta}} \left\{\int_0^{n_2} [x(z_2)]^{\theta} dz_2\right\}^{\frac{1}{2\theta}}$$

where $0 < \theta_1 = \theta_2 = \theta < 1$ (or $1 < \sigma_1 = \sigma_2 = \sigma < \infty$).

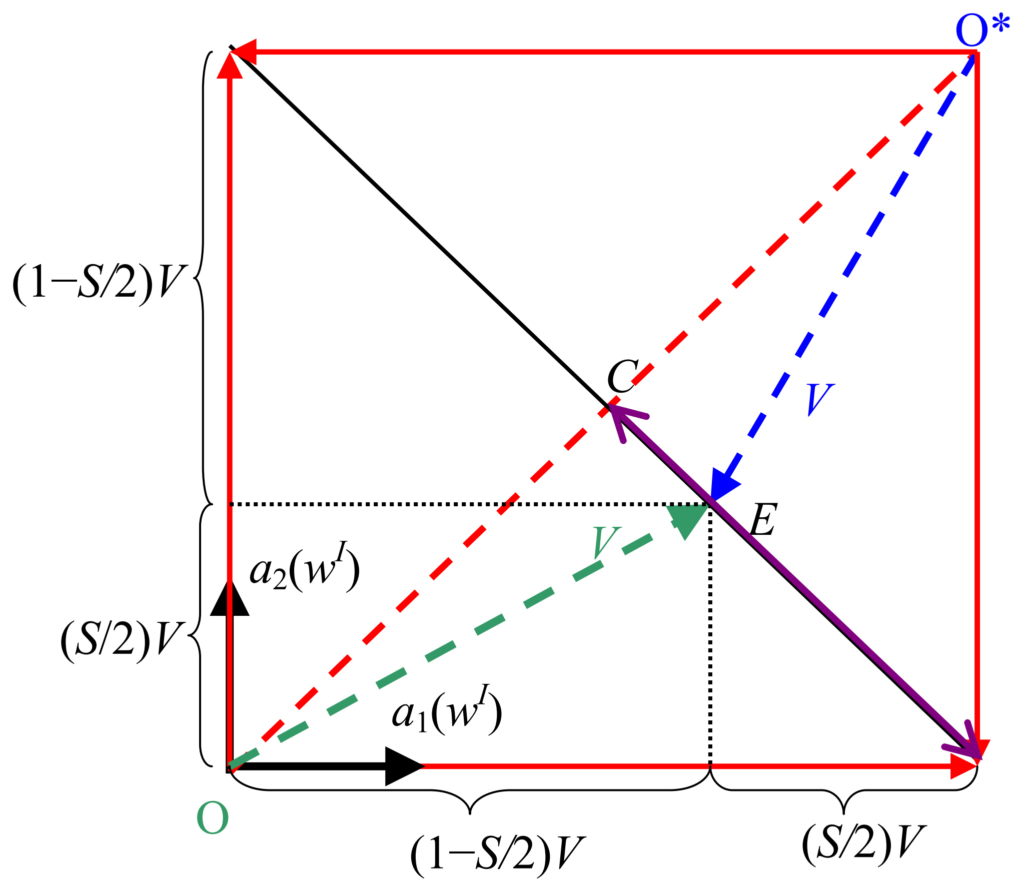
Two Intermediate Inputs Sectors: X_j-sector (j =1,2) is monopolistically competitive with the total cost, $C^j(w,x_j) = w_j T(x_j)$, where $T(x_j)/T'(x_j)x_j = a(x_j)/m(x_j)$ is strictly decreasing in x_j with the range including θ.

Notes:

- Two-types of intermediate inputs enter symmetrically in the final goods production.
- Each factor is specific; Factor-*j* is used only in production of Type-*j* intermediate inputs. ➔ The entire square box is the FPE set.
- The two countries are the mirror images of each other. ➔ The endowment point, E, must be located in the Purple Line, with S representing the similarity of the factor proportions.

Exercises:

- Given $0 < S < 1$, derive the variety of type-j inputs produced in each country both before and after trade.
- Calculate the utility change for each type of households in each country, caused by trade.
- Show that, for each S, a sufficiently small θ makes trade Pareto-superior to Autarky.
- Show that, for each θ, a sufficiently large S makes trade Pareto-superior to Autarky.



Other Notable Contributions:

- Romalis (2004) looked at the case where FPE fails (due to the trade cost) with a continuum of sectors, effectively introducing the MC structure in the DFS (1980) Heckscher-Ohlin Model with Trade Costs. As seen later, trade costs often change the nature of equilibrium drastically in a MC model. This does not happen in his model because he imposes the strong symmetry across countries.

- Markusen (1988) introduces Stone-Geary preferences in the Heckscher-Ohlin-Chamberlian setup. Each household supplies one unit of labor, but North households have more capital than South households. With the Engel's law, the North consume a larger fraction of their income on M-goods (K-intensive, differentiated with CES, and monopolistically competitive) than A-good (L-intensive, homogeneous, competitive). This explains why we observe more trade among developed countries.

- Foellmi-Hepenstrick-Zweimüller (2007) dropped the CES aggregator to introduce the income effects on "love-for-variety" and to show more trade when countries have similar per capita income. They also discussed variable mark-up and how firms might price-discriminate across countries with different per capita income (with or without threat of parallel trade).

Nontradeable Differentiated Goods with Factor (Geographical) Mobility

It has been assumed thus far that the factor is geographically immobile and that all differentiated products are costlessly tradeable. We now reverse this assumption.

A Simple Model of Agglomeration through Labor Migration

Two (Inherently Identical) Regions; East & West

L **Homogeneous Households**: $L^E + L^W = L$. Each endowed with one unit of nontradeable labor (the only factor of production) and consume nontradeable services to maximize:

$$U^k = \int_0^{n^k} h(c(z))dz \text{ subject to } \int_0^{n^k} p(z)c(z)dz \le w^k; \quad (k = E \text{ or } W).$$

Nontradeable Services Sector; monopolistically competitive. Total labor requirement is $T(x)$, where $T(x)/xT'(x) = a(x)/m(x)$ is *strictly decreasing* in x.

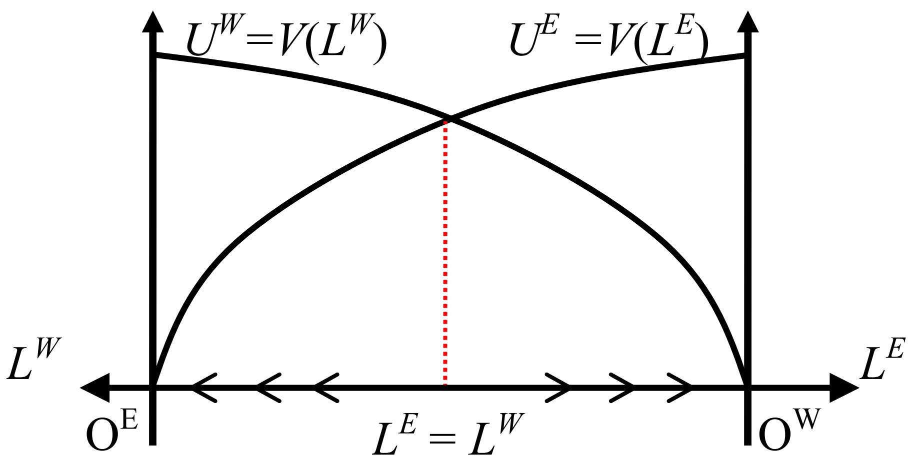
Labor Market Equilibriums: $\int_0^{n^k} T(x(z))dz = L^k; \quad (k = E \text{ or } W).$

Exercise: Show that **Standard-of-Living (Indirect Utility)** increases with its population size: $U^E = V(L^E)$ and $U^W = V(L^W)$.

Household Migration: They would move to the region offering better standard-of-living, taking the current distribution of the population given.

Figure shows that:

- The 50/50 division is unstable.
- The entire population agglomerates into one region.
- Agglomeration is the best outcome.



Notes:

- It is assumed that, when they move, the households move both their residence *and* their work. That is, the factor and its owner are inseparable. Of course, one could think of cases where the factor moves, but not the owners (certain types of capital or e-commute) or where the factor stays, but the owners would move (e.g., the absentee landlord). These distinctions are important whenever there are trade costs.
- Instead of the myopic migration dynamics, one could develop more sophisticate migration dynamics, where each household decides where to move based on their expectations of what other households do in the future. See Matsuyama (1991) and Matsuyama and Takahashi (1998).

In the model above,

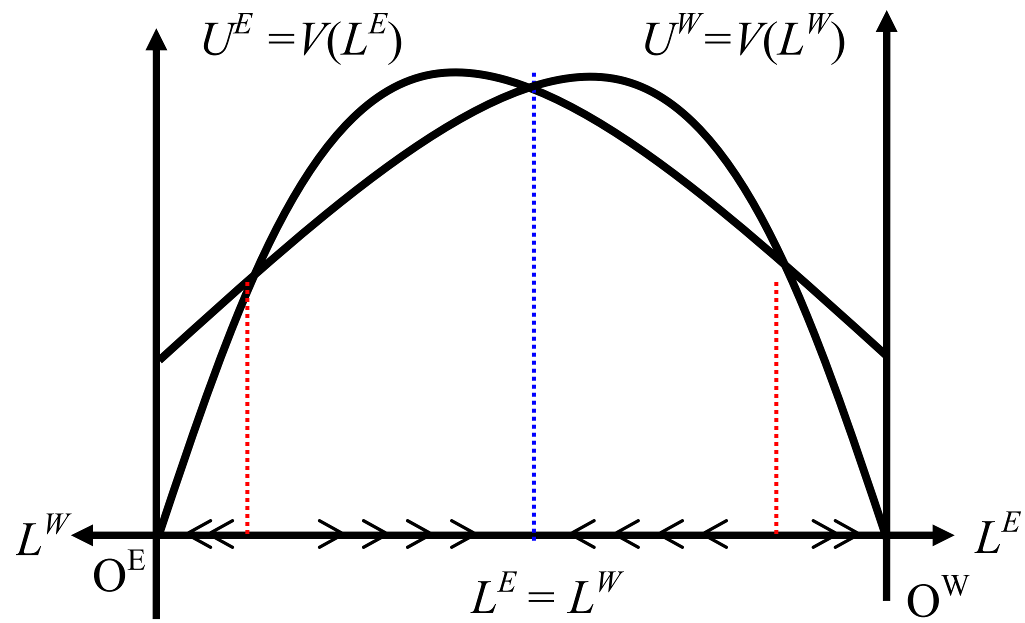
- Agglomeration is the only stable outcome, although we cannot say to which region the the economy agglomerates.
- Agglomeration is the desirable outcome. Of course, if the two regions are not inherently identical, it is possible that the economy ends up agglomerating to the wrong region. For example, imagine that the West Coast is more pleasant place to live than the East Coast, and for historical reasons, the entire population may be stuck to the East Coast. However, there is nothing wrong with the agglomeration *per se*.

This model contains only ***"centripetal force,"*** which keeps the population together, but not ***"centrifugal force,"*** which would keep the population spread. An interesting model of agglomeration would need both forces.

***A Simple Model of Urban Economics with Agglomeration Economies & Diseconomies*:**

In urban economics, concerned with the problems of mega-cities such as Mexico City, Sao Paulo, Tokyo, etc., it is common to add some agglomeration *diseconomies*, such as *Congestion Externalities* to make the standard-of-living of a metropolitan area, a hump-shaped function of its population size.

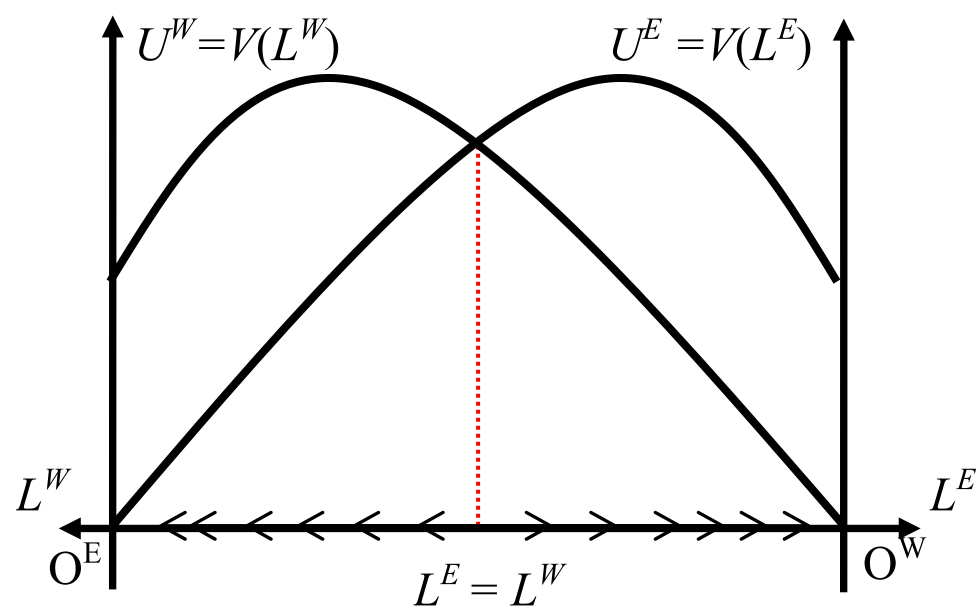
For example, in a two-city model,



This Figure shows:
- Three stable outcomes (and two unstable ones): 2 complete concentrations & the 50/50 division.
- The 50/50 division is the most desirable among the stable outcomes.

suggesting possible Pareto-improving government interventions.

However, this Figure shows:



- The only stable outcomes are complete concentrations.
- These stable outcomes are Pareto-dominated by the (unstable) 50/50 division.

suggesting some fundamental difficulties of avoiding the emergence of mega-cities.

In these models of Urban Economics,

- There is the optimal city size, and the bad things happen when the city becomes bigger than its optimal city.
- Agglomeration creates bad outcomes, because it causes over-crowding. There is nothing wrong with the agglomeration *per se.*
- To put it differently, doubling the population size of the entire economy would create the same problem as agglomeration in this model.
- The problem could be eliminated by creating a new metropolitan area (up to the integer constraint). For example, if the optimal size of

In Regional Economics, on the other hand, we are more concerned with the question of *Uneven Regional Growth and Development,* such as

- The US tendency to become bi-coastal, leaving the fertile heartland relatively empty.
- Disparity between Northern and Southern Italy *(Mezzogiorno),* between the Pacific belt and the rest of Japan, between South and Northeast regions of Brasil, or between West and East Germany, etc.

If there are any problems with these examples of *Regional Unbalance*, congestion externalities would not be of the first order importance.

Inefficiency of Unbalanced Regional Growth; Matsuyama-Takahashi (1998)

We now turn to a model where *Regional Unbalance* created by agglomeration economies could lead to inefficiency in the presence of *Interregional Trade*, through the *Terms of Trade* effect.

Key Features: A hybrid of Dixit-Stiglitz and Ricardian, with two competing forces:

- *Centripetal Force*: due to the consumer's desire to have access to a wider range of differentiated nontradeable services available in the larger region
- *Centrifugal Force*: Each region has absolute advantage in something, which gives the reason for the population to spread across different regions. If the population share in one region declines, the ToT would move in its favor to bring the population back.

Two (Inherently Identical) Regions; East & West

Two Tradeable Goods (E & W): Competitive, Ricardian

Unit Labor Requirement	East	West
Good E	1	$\Omega > 1$
Good W	$\Omega > 1$	1

Thus, East (West) has comparative *and* absolute advantages in Good *E* (*W*).

Nontradeable Services Sector; monopolistically competitive. Total labor requirement is $T(x)$, where $T(x)/xT'(x) = a(x)/m(x)$ is *strictly decreasing* in x.

***L* Homogeneous Households**: $L^E + L^W = L$. Each household supplies one unit of nontradeable labor. Households in Region $k = (E$ or $W)$ maximize:

$$U^k = \left(u(c_E^k, c_W^k)\right)^{\mu}\left(C_N^k\right)^{1-\mu} \equiv \left(u(c_E^k, c_W^k)\right)^{\mu}\left(\int_0^{n^k}\left(c(z)\right)^{\theta} dz\right)^{\frac{1-\mu}{\theta}},$$

where $u(c_E^k, c_W^k)$ is symmetric, and linear homogeneous.

Labor Market Equilibriums:

$$X_E^E + \Omega X_W^E + \int_0^{n^E} T(x(z))dz = L^E;\ X_W^W + \Omega X_E^W + \int_0^{n^W} T(x(z))dz = L^W$$

X_j^k: the output of Good j in Region k.

First, we solve for equilibrium with the fixed population distribution, and calculate the Standard-of-living indices for each region. Then, we will let households move.

***Nontradeable Services*:** In each region, this sector receives $1-\mu$ fraction of the region's income. Hence,

$$n^k = \frac{(1-\mu)L^k}{T(x)}, \text{ where } x \text{ is determined uniquely by } \frac{xT'(x)}{T(x)} = \theta.$$

Relative Demand of Tradeables:

Let $e(p_E, p_W)$ be the unit expenditure function for $u(c_E^k, c_W^k)$, where $p_E(p_W)$ is the price of Good E (Good W). Then,

$$\frac{D_E}{D_W} = \frac{e_E(p_E, p_W)}{e_W(p_E, p_W)} \equiv f\left(\frac{p_E}{p_W}\right) \quad \rightarrow \quad \frac{p_E}{p_W} = \Psi\left(\frac{D_E}{D_W}\right), \Psi' < 0.$$

Patterns of Intraregional Trade:

If $\frac{1}{\Omega} < \frac{p_E}{p_W} < \Omega$, each region specializes. $\rightarrow$ $X_E^E = \mu L^E$, $X_W^W = \mu L^W$ $\rightarrow$ $\frac{p_E}{p_W} = \Psi\left(\frac{L^E}{L^W}\right)$.

If $\frac{p_E}{p_W} = \Omega \leq \Psi\left(\frac{L^E}{L^W}\right)$, West produces both. If $\frac{p_E}{p_W} = \frac{1}{\Omega} \geq \Psi\left(\frac{L^E}{L^W}\right)$, East produces both.

Standard-of-Living Indices:

In absolute terms: $$U^k = K\left(\frac{p_k}{e(p_E, p_W)}\right)^{\mu}\left(n^k\right)^{\frac{1-\mu}{\sigma-1}} = K'\left(\frac{p_k}{e(p_E, p_W)}\right)^{\mu}\left(L^k\right)^{\frac{1-\mu}{\sigma-1}}$$

In relative terms: $$\frac{U^E}{U^W} = \left(\frac{p_E}{p_W}\right)^{\mu}\left(\frac{n^E}{n^W}\right)^{\frac{1-\mu}{\sigma-1}} = \left(\frac{p_E}{p_W}\right)^{\mu}\left(\frac{L^E}{L^W}\right)^{\frac{1-\mu}{\sigma-1}}.$$

1st Term; Terms of Trade Effect, *negatively* related to the relative population size when both regions specialize, otherwise constant.

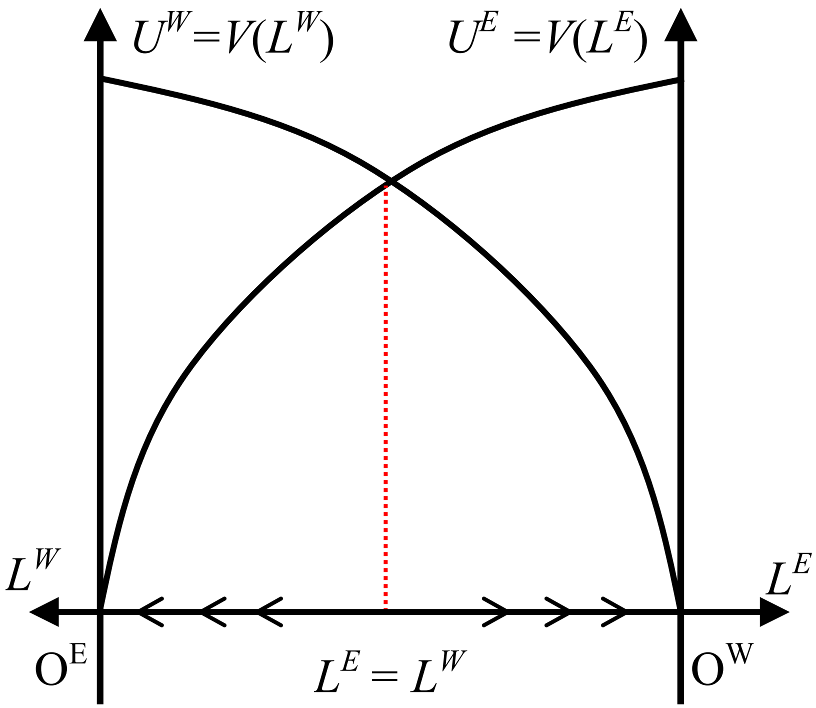
2nd Term: Variety Effect, *positively* related to the relative population size.

This model is truly a hybrid of

- the first Dixit-Stiglitz model of agglomeration (if μ = 0).
- the Ricadian model of (interregional) trade (if μ = 1).

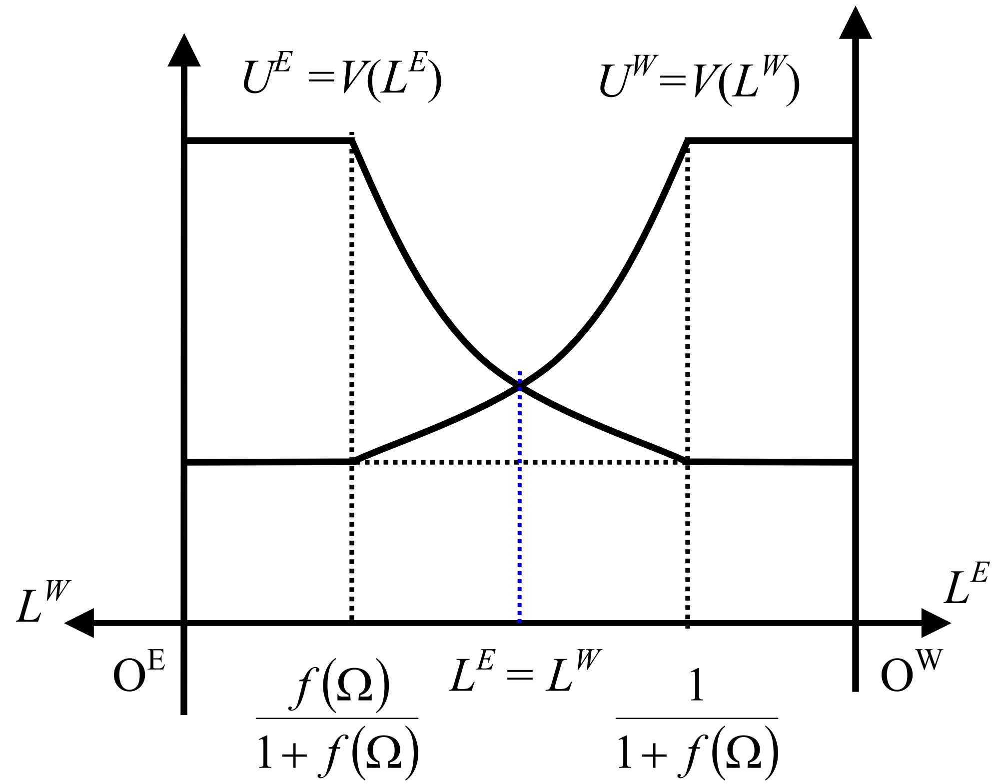
Dixit-Stiglitz Agglomeration (μ = 0)

Centrifugal Force



Ricardian Interregional Trade (μ= 1)

Centripetal Force



Exercise: Now, let the households move to the region with higher standard-of-living.

i) Show that the two complete concentration outcomes, $L^E = L$; $L^W = 0$, or $L^E = 0$; $L^W = L$, are stable, whenever $\mu < 1$ and $\Omega < \infty$.

ii) Characterize the stability of the 50/50 division, $L^E = L^W = L/2$, for three different cases:
1) $u(c_E^k, c_W^k)$ is symmetric Cobb-Douglas;
2) $u(c_E^k, c_W^k)$ is symmetric CES;
3) $u(c_E^k, c_W^k)$ is for a general symmetric linear homogeneous function.

iii) For these three cases, identify the conditions under which
a) The 50/50 division is stable but Pareto-dominated by the complete concentration outcomes (desirable uneven regional growth may fail to occur).
b) The 50/50 division is unstable and Pareto dominates the complete concentration outcomes (undesirable uneven regional growth is inevitable).

iv) Show also that your answer is independent of L in all cases.

In these models above, we have made the simplifying assumptions of:

- No Immobile Factor
- Homogeneous Households

If we allow for:

- Immobile Factors (such as land);
- Heterogeneous Households; Some may want access to a wide range of nontradeable services (e.g., restaurants and theaters), while living in a small apartment; Others may want to live in a big house, while having limited access to nontradeable services;

we expect different households sort themselves out in different locations.

Instead, we explore a similar mechanism in a model of international trade, with

- Immobile households supplying immobile factor (labor).
- Nontradeable intermediate inputs (local support industries).
- Tradeable consumer goods industries, which differ in their intensity.

Then, different consumer goods industries sort themselves out in different countries.

Endogenous Comparative Advantage and Endogenous Inequality of Nations: based on Matsuyama (1995; 1996)

L Households: each supplies one unit of labor (the only primary factor of production, and nontradeable) with the expenditure function:

$$e(P)U = (P_1)^{\beta_1}(P_2)^{\beta_2}U,$$

P_j: the price of consumer good-j (= 1 or 2)
β_j: the consumer good-j's share in the expenditure ($\beta_1 + \beta_2 = 1$)

Tradeable Consumer Goods Sectors: j = 1 & 2.

- ***Competitive:***
- Sector-j produces consumer good-j with labor and a composite of the nontradeable differentiated inputs, using Cobb-Douglas technologies with the unit cost function:

$$C_j = (w)^{1-\alpha_j}(P_N)^{\alpha_j} = (w)^{1-\alpha_j}\left\{\int_0^n [p(z)]^{1-\sigma} dz\right\}^{\frac{\alpha_j}{1-\sigma}}$$

w: the wage rate
$p(z)$: the price of a differentiated input, z, consumer good-j (= 1 or 2)
P_N: the price index of the nontradeable differentiated inputs
α_j: the share of differentiated inputs in Sector-j, with $\alpha_1 < \alpha_2$.

Nontradeable Differentiated Inputs (Producer Service) Sector:

- ***Monopolistically Competitive***
- Each firm produces its variety with the total labor requirement, $T(x) = F + mx$.

  F: Fixed (or Entry) Cost

  m: Marginal Labor Requirement: Let us normalize it to $m = 1-1/\sigma$.

Monopoly-Pricing: $p(z)(1-1/\sigma) = mw \rightarrow p = p(z) = w$ for all $z \le n$.

$\rightarrow x = x(z)$ for all $z \le n$.

Profit: $\pi(z)/w = [px - wT(x)]/w = x - mx - F = x/\sigma - F$

Free Entry: $x \le \sigma F$; $n \ge 0$.

Relative Cost of Producing Tradeable Consumer Goods ($j = 1$ & 2):

$$\frac{C_1}{C_2} = \left(\frac{w}{P_N}\right)^{(\alpha_2-\alpha_1)} = (n)^{(\alpha_2-\alpha_1)/(\sigma-1)}$$

Note: This is like the Ricardian comparative advantage. The country with the more developed local support industry (a higher n) has comparative advantage in Sector-2. However, the size of the local support industry, n, is endogenous variable, here.

Autarky Equilibrium:

The economy produces both consumer goods → $P_j = C_j$ (j = 1 & 2)

- The representative consumer spends β_j of the income, Y, on Sector-j.
- Sector-j spends α_j of its revenue on the nontradeble differentiated inputs.

Nontradeable Inputs Market: $nwx = \alpha^A Y$, where $\alpha^A \equiv \alpha_1\beta_1 + \alpha_2\beta_2$.

- Sector-j spends $1 - \alpha_j$ of its revenue on labor.
- Each nontradeable differentiated input firm spends $wT(x)$ on labor.

Labor Market: $wL = (1 - \alpha^A)Y + nw[(1-1/\sigma)x + F]$

Solving the two market clearing conditions for

$$\frac{Y}{w} = \left(\frac{\sigma}{\sigma - \alpha^A}\right)(L - nF); \qquad \frac{x}{\sigma} = \left(\frac{\alpha^A}{\sigma - \alpha^A}\right)\left(\frac{L}{n} - F\right).$$

Free Entry Condition:

The profit is monotonically decreasing in n. Thus, Free Entry uniquely determines n^A.

$$\frac{\pi}{w} = \frac{x}{\sigma} - F = \left(\frac{\alpha^A}{\sigma - \alpha^A}\right)\left(\frac{L}{n} - F\right) - F = 0 \;\rightarrow n^A = \frac{\alpha^A L}{\sigma F}.$$

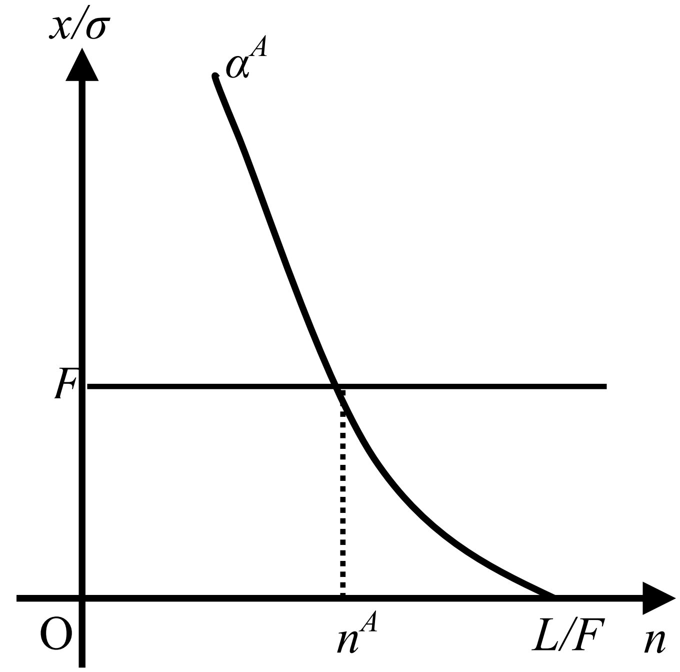
From this, we can show that, in autarky,

$$Y^A = w^A L$$

$$P_j^A = C_j^A = w^A \left[\frac{\alpha^A L}{\sigma F}\right]^{\frac{\alpha_j}{1-\sigma}}$$

$$q^A \equiv \left(\frac{P_1}{P_2}\right)^A = \left(\frac{C_1}{C_2}\right)^A = \left[\frac{\alpha^A L}{\sigma F}\right]^{\frac{\alpha_2 - \alpha_1}{\sigma - 1}}$$

$$U^A = \left(P_1^A\right)^{-\beta_1}\left(P_2^A\right)^{-\beta_2} w^A = \left[\frac{\alpha^A L}{\sigma F}\right]^{\frac{\alpha^A}{\sigma-1}}.$$



Equilibrium for a Small Open Economy (with $q \equiv P_1/P_2$ given)

Recall that C_1/C_2 is strictly increasing in n**.** Let $N(q) \equiv (q)^{(\sigma-1)/(\alpha_2-\alpha_1)}$. Then,

Case 1: $n < N(q) \rightarrow C_1/C_2 < q \equiv P_1/P_2$.

Case 2: $n > N(q) \rightarrow C_1/C_2 > q \equiv P_1/P_2$.

In Case j, the economy specializes in Good-j.

Let us look for an equilibrium in each case. If we denote the output of Sector-j in Case-k by $(Q_j)^k$, $(Q_j)^k = 0$ for $j \neq k$, so let's keep it simple by letting $(Q_j)^j = Q_j$.

Nontradeable Inputs Market: $nwx = \alpha_j P_j Q_j$.

Labor Market: $wL = (1 - \alpha_j)P_jQ_j + nw[(1-1/\sigma)x + F]$

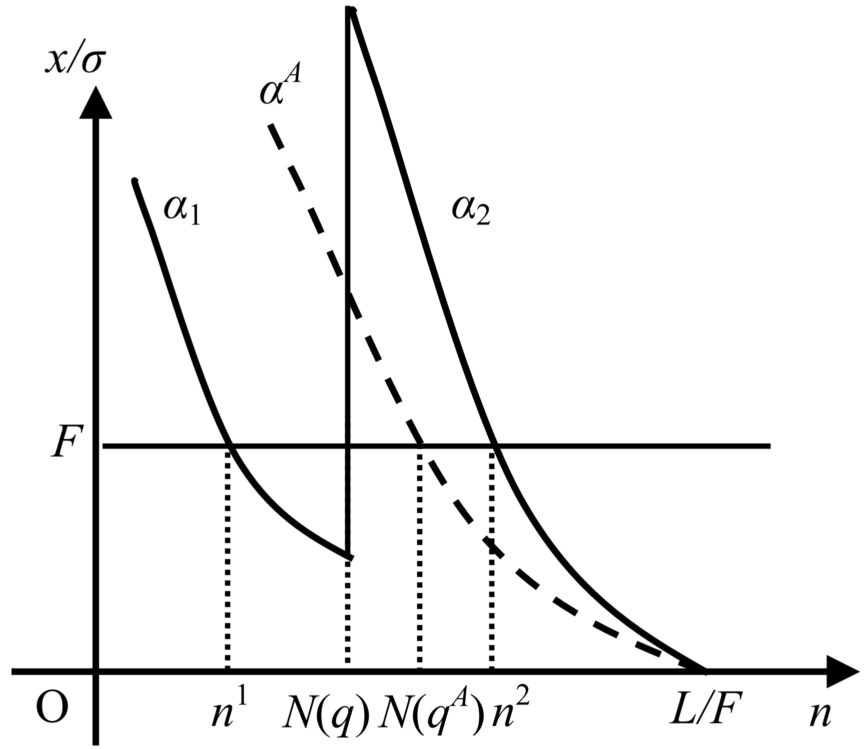
From these two conditions, we have

$$\frac{P_j Q_j}{w} = \left(\frac{\sigma}{\sigma - \alpha_j}\right)(L - nF); \qquad \frac{x}{\sigma} = \left(\frac{\alpha_j}{\sigma - \alpha_j}\right)\left(\frac{L}{n} - F\right).$$

Free Entry Condition:

- Note that $\alpha_1 < \alpha^A \equiv \alpha_1\beta_1 + \alpha_2\beta_2 < \alpha_2$.
- The profit is no longer monotone in n. It jumps at $n = N(q)$, because the economy's comparative advantage shifts to Sector-2, which requires the differentiated inputs more.

$$\frac{\pi}{w} = \begin{cases} \left(\dfrac{\alpha_1}{\sigma-\alpha_1}\right)\left(\dfrac{L}{n}-F\right)-F & \text{if } n < N(q) \\ \\ \left(\dfrac{\alpha_2}{\sigma-\alpha_2}\right)\left(\dfrac{L}{n}-F\right)-F & \text{if } n > N(q). \end{cases}$$



If $n^1 \equiv \dfrac{\alpha_1 L}{\sigma F} < N(q)$, this is an equilibrium in Case 1.

If $n^2 \equiv \dfrac{\alpha_2 L}{\sigma F} > N(q)$, this is an equilibrium in Case 2.

Multiple Equilibriums if $n^1 \equiv \dfrac{\alpha_1 L}{\sigma F} < N(q) < n^2 \equiv \dfrac{\alpha_2 L}{\sigma F}$

Intuition behind Multiple Equilibriums: Two-way causality between the size of the local support industry and the patterns of trade:

Furthermore, the above condition may also be rewritten as

$$\Leftrightarrow \quad \left[\frac{\alpha_1 L}{\sigma F}\right]^{\frac{\alpha_2-\alpha_1}{\sigma-1}} \equiv q^1 < q < q^2 \equiv \left[\frac{\alpha_2 L}{\sigma F}\right]^{\frac{\alpha_2-\alpha_1}{\sigma-1}}$$

$$\Leftrightarrow \quad \left[\frac{\alpha_1}{\alpha^A}\right]^{\frac{\alpha_2-\alpha_1}{\sigma-1}} = \frac{q^1}{q^A} < \frac{q}{q^A} < \frac{q^2}{q^A} = \left[\frac{\alpha_2}{\alpha^A}\right]^{\frac{\alpha_2-\alpha_1}{\sigma-1}}.$$

LHS is less than one and RHS is greater than one. Thus, free trade would create multiple equilibriums, even though it may not change the relative price of the two tradeable goods.

Intuition: In response to a change in *n,*

- In autarky the relative price would have to adjust so that both goods are produced.
- In a small open economy case, the relative price would not adjust, and hence the economy could abruptly change its patterns of production, which makes non-specialization unstable.

Furthermore, we can show:

$$P_j(Q_j)^j = Y^j = w^j L; \quad w^j = P_j\left[\frac{\alpha_j L}{\sigma F}\right]^{\frac{\alpha_j}{\sigma-1}}; \quad (Q_j)^j = \frac{w^j L}{P_j} = L\left[\frac{\alpha_j L}{\sigma F}\right]^{\frac{\alpha_j}{\sigma-1}}$$

$$q^j \equiv \left(\frac{C_1}{C_2}\right)^j = \left[\frac{\alpha^j L}{\sigma F}\right]^{\frac{\alpha_2-\alpha_1}{\sigma-1}}; \ \frac{U^1}{U^A} = \left[\frac{q}{q^A}\right]^{\beta_2}\left[\frac{\alpha_1}{\alpha^A}\right]^{\frac{\alpha_1}{\sigma-1}}; \ \frac{U^2}{U^A} = \left[\frac{q}{q^A}\right]^{-\beta_1}\left[\frac{\alpha_2}{\alpha^A}\right]^{\frac{\alpha_2}{\sigma-1}}.$$

Under the condition ensuring multiple equilibriums,

$$\left[\frac{\alpha_1}{\alpha_2}\right]^{\frac{\alpha_2}{\sigma-1}} < \frac{w^1}{w^2} = \frac{U^1}{U^2} < \left[\frac{\alpha_1}{\alpha_2}\right]^{\frac{\alpha_1}{\sigma-1}} < 1;$$

- The country is better off at the Case-2 equilibrium than in the Case-1 equilibrium.

$$\left[\frac{\alpha_1}{\alpha^A}\right]^{\frac{\alpha^A}{\sigma-1}} < \frac{U^1}{U^A} < \left[\frac{\alpha^2}{\alpha^A}\right]^{\frac{\alpha^A-\alpha_1}{\sigma-1}}\left[\frac{\alpha_1}{\alpha^A}\right]^{\frac{\alpha_1}{\sigma-1}}; \quad 1 < \left[\frac{\alpha_2}{\alpha^A}\right]^{\frac{\alpha^A}{\sigma-1}} < \frac{U^2}{U^A} < \left[\frac{\alpha_1}{\alpha^A}\right]^{\frac{\alpha^A-\alpha_2}{\sigma-1}}\left[\frac{\alpha_2}{\alpha^A}\right]^{\frac{\alpha_2}{\sigma-1}},$$

- The country may lose from trade at the Case-1 equilibrium (unless $q >> q^A$).
- The country unambiguously gains from trade at the Case-2 equilibrium.

For related work, see Rodriguez-Clare (1996) and Rodrik (1996).

World Economy with a Continuum of (Inherently) Identical Countries

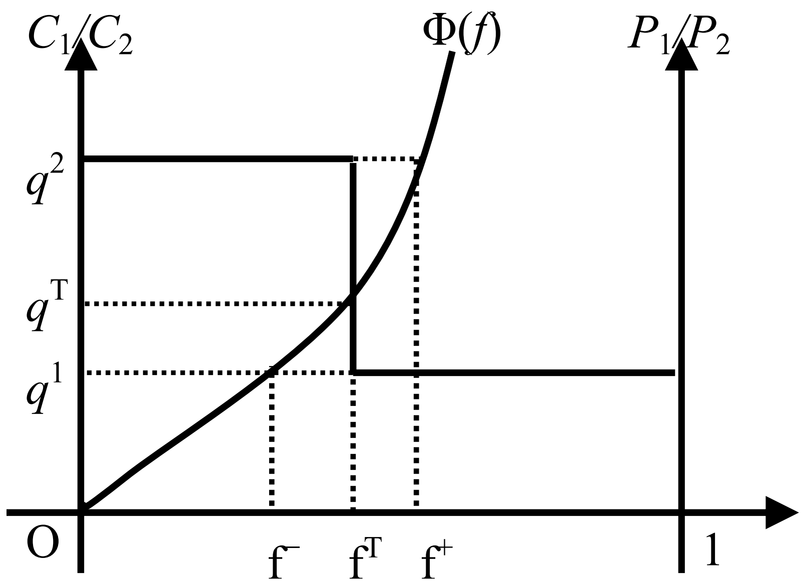
Suppose that

- a fraction $1 - f$ of the countries is in the Case-1 equilibrium ($n = n^1$).
  Their relative cost of producing Good 1 is q^1; each produces Good 1 by $(Q_1)^1$.
- a fraction f is in the Case-2 equilibrium ($n = n^2$).
  Their relative cost of producing Good 1 is q^2; each produces Good 1 by $(Q_2)^2$.

Figure shows:

- The relative cost curve as the step-function by arranging the countries along the horizontal axis, starting from those in the Case-2 equilibrium.
- Since the relative demand for the two tradeables is $D_1/D_2 = (\beta_1/\beta_2)/q$,

$$q = \frac{\beta_1 D_2}{\beta_2 D_1} = \frac{\beta_1 (Q_2)^2}{\beta_2 (Q_1)^1}\left(\frac{f}{1-f}\right) \equiv \Phi(f)$$



There are a continuum of world equilibriums, $f^{\mathrm{T}} \in (f^{-}, f^{+})$.

However, f^{T} is bounded away from zero and from one.   This means that

In *any* equilibrium, some countries specialize in Sector-1 and the other specialize in Sector-2, despite that these countries are inherently identical.

- Symmetry-Breaking; The World Economy is divided into two groups through endogenous comparative advantage
- Endogenous Inequality; inequality is an inevitable feature of the International Economic Order.

Exercise: Extend the above model by adding another competitive sector, which produces nontradeable services, whose unit cost function is given by $C_3 = (w)^{1-\alpha_3}(P_N)^{\alpha_3}$. The consumer's preferences are $e(P)U = (P_1)^{\beta_1}(P_2)^{\beta_2}(P_3)^{\beta_3}U$, with $\beta_1 + \beta_2 + \beta_3 = 1$.

i) Characterize a set of the world trade equilibriums.
ii) For each equilibrium, show the cross-country variations of the CPI under three alternative assumptions; 1) $\alpha_3 < \alpha_1 < \alpha_2$; 2) $\alpha_1 < \alpha_3 < \alpha_2$; and 3) $\alpha_1 < \alpha_2 < \alpha_3$.

Iceberg Costs for Trading Differentiated Products

- We just saw that differentiated goods have very different implications, depending on whether they are assumed to be tradeable at zero cost or nontradeable.
- This is parallel to what we had seen earlier; External economies have very different implications, depending on whether they are global or national in geographical scope.
- Indeed, one might view a model with product differentiation as offering a microfoundation for external economies.
- One advantage of having such a microfoundation is that it is straightforward to look at intermediate cases by assuming that differentiated goods are tradeable at positive costs.
- It turns out that such models yield some new insights, which cannot be captured by the two extreme cases of "tradeable at zero cost" and "nontradeable."

Key Ideas:

- In the presence of scale economies, each firm wants to produce its good only in one country, from which it exports to other countries.
- Other things being equal, the firm wants to be in the largest market to minimize the trade cost.

Note: The iceberg trade costs are also used to introduce the "distance" in the Gravity Equation. See Feenstra (2004, pp.152-162) and Combes-Mayer-Thisse (2008, Ch.5)

One-Sector, One-Factor Model: Krugman (1980)

One Nontradeable Factor: Labor

Two Countries: Home and Foreign

L^H (L^F) **Households**: each supplies one units of labor with the preferences:

$$U^H = C^H = \left\{\int_{\Omega}\left[c^H(z)\right]^{\theta} dz\right\}^{\frac{1}{\theta}}; \qquad U^F = C^F = \left\{\int_{\Omega}\left[c^F(z)\right]^{\theta} dz\right\}^{\frac{1}{\theta}}$$

$\Omega = \Omega^H + \Omega^F$, where Ω^C is the equilibrium set of goods produced in $C = H$ or F.

Household Demand: $$c^D(z) = \left(\frac{p^D(z)}{P^D}\right)^{-\sigma} C^D = \left(\frac{p^D(z)}{P^D}\right)^{-\sigma} \frac{w^D}{P^D} \quad (D = H \text{ or } F)$$

$p^D(z)$; $z \in \Omega$: the price of good z in $D = H$ or F.

$$P^D \equiv \left\{\int_{\Omega}\left[p^D(z)\right]^{1-\sigma} dz\right\}^{\frac{1}{1-\sigma}}$$: the price index in $D = H$ or F.

Technology: Differentiated goods are produced with the total factor requirement, $T(x)$: $xT'(x)/T(x)$ is strictly increasing in x with the range including θ.

Iceberg Trade Costs: when shipped from C to D ($\neq C$), only a fraction $1/\tau < 1$ arrives. No Trade Cost when shipped domestically.

For $z \in \Omega^C$: to consume $c^D(z)$ in D ($\neq C$), $\tau c^D(z)$ must be ordered → $p^D(z) = p^C(z)\tau$.

Total Demand for Good $z \in \Omega^C$: If its producer charges $p(z)$ at its domestic market,

$$x(z) = c^C(z)L^C + \tau c^D(z)L^D = \frac{(p(z))^{-\sigma} w^C L^C}{(P^C)^{1-\sigma}} + \frac{\tau(\tau p(z))^{-\sigma} w^D L^D}{(P^D)^{1-\sigma}}$$

$$= \left[\frac{w^C L^C}{(P^C)^{1-\sigma}} + \frac{\rho w^D L^D}{(P^D)^{1-\sigma}}\right](p(z))^{-\sigma}$$

where $0 \leq \rho \equiv (\tau)^{1-\sigma} < 1$.

Notes:

- The elasticity of the domestic and export demands are both equal to σ. This means no incentive for price discrimination even if a firm could charge different prices for different destinations.
- This is due to the combination of the iceberg and CES. Without these assumptions, the firms want to set the different prices in different countries.

Monopoly Pricing: $p(z)(1-1/\sigma) = w^C T'(x(z))$. $z \in \Omega^C$:

Zero profit/Free Entry: $p(z)x(z) = w^C T(x(z))$. $z \in \Omega^C$

$\rightarrow$ $x(z) = x$ for all $z \in \Omega$, where $xT'(x)/T(x) = m(x)/a(x) = 1-1/\sigma$.

$\rightarrow$ $p(z) = \dfrac{w^C T(x)}{x} \equiv p^C$ for all $z \in \Omega^C$.

$$\rightarrow \quad \left(P^D\right)^{1-\sigma} = n^C\left(\tau p^C\right)^{1-\sigma} + n^D\left(p^D\right)^{1-\sigma} = \rho n^C\left(p^C\right)^{1-\sigma} + n^D\left(p^D\right)^{1-\sigma}$$

Hence, the total demand must satisfy:

$$x = \left[\frac{w^C L^C}{(P^C)^{1-\sigma}} + \frac{\rho w^D L^D}{(P^D)^{1-\sigma}}\right]\left(p^C\right)^{-\sigma}$$

$$\rightarrow \quad T(x) = \left[\frac{p^C L^C}{n^C (p^C)^{1-\sigma} + \rho n^D (p^D)^{1-\sigma}} + \frac{\rho p^D L^D}{\rho n^C (p^C)^{1-\sigma} + n^D (p^D)^{1-\sigma}}\right]\left(p^C\right)^{-\sigma}.$$

$$= \frac{L^C}{n^C + \rho n^D (p^D / p^C)^{1-\sigma}} + \frac{\rho (p^D / p^C) L^D}{\rho n^C + n^D (p^D / p^C)^{1-\sigma}}.$$

Since the labor market condition is $n^C = L^C / T(x)$,

$$1 = \frac{L^C}{L^C + \rho L^D (p^D / p^C)^{1-\sigma}} + \frac{\rho (p^D / p^C) L^D}{\rho L^C + L^D (p^D / p^C)^{1-\sigma}} = \frac{1}{1 + \rho\lambda(\omega)^{1-\sigma}} + \frac{\rho\omega\lambda}{\rho + \lambda(\omega)^{1-\sigma}},$$

which defines a mapping, $\omega = \Omega(\lambda)$, where $\omega \equiv p^C / p^D = w^C / w^D$ and $\lambda \equiv L^C / L^D$.

- $\rho \equiv (\tau)^{1-\sigma} = 1 \;\rightarrow\; \omega \equiv p^C / p^D = w^C / w^D = 1$ for any $\lambda \equiv L^C / L^D$;
- $\rho \equiv (\tau)^{1-\sigma} < 1 \;\rightarrow\; \omega \equiv p^C / p^D = w^C / w^D > 1$ if and only if $\lambda \equiv L^C / L^D > 1$.

Intuition: Other things being equal, a firm wants to be in the larger market to minimize the trade costs. To make the firms in both countries break even, the wage rate must be higher in the larger market.

Note: L represents not only the market size but also labor supply. The effect of more labor supply, however, is absorbed by having more firms, as $n^C = L^C / T(x)$.

Exercise: Evaluate the welfare, $U^H = w^H / P^H$ and $U^F = w^F / P^F$.
Exercise: How would you modify the analysis if the trade cost depends on the aggregate volume of trade? Or if the trade cost for each product depends on its export trade?

Home Market Effect:

In the above model, the firm's desire to locate in the larger market is entirely offset by the change in the factor prices, eliminating any quantity effect.

One-sector, one-factor framework is too restrictive for generating interesting effects on the product variety or the number of firms.

We now consider three different ways of obtaining the so-called "Home Market Effect." That is, the larger country produces disproportionately more differentiated goods and become the net exporter of these goods.

- 1^{st} model introduces another sector from which the differentiated sector can absorb additional resources.

- 2^{nd} model has two-types of differentiated goods to generate a composition effect.

- 3^{rd} model introduces in the 1^{st} model another factor that is mobile across countries.

Two-Sector, One Factor Case: The 1st Model of the Home Market Effect

Consider a variant of the above model by adding another sector, which is competitive and converts one unit of labor to one unit of the homogenous outside good, which is tradeable at zero cost.

Cobb-Douglas Preferences: with α being the share of differentiated goods and $1 - \alpha$ being the share of the outside good. Thus,

Budget Constraints: $w^H = (p_o)^{1-\alpha}\left(P^H\right)^{\alpha} U^H$; $\qquad w^F = (p_o)^{1-\alpha}\left(P^F\right)^{\alpha} U^F$.

Then,

$$T(x) = \frac{\alpha L^C}{n^C + \rho n^D (p^D / p^C)^{1-\sigma}} + \frac{\rho (p^D / p^C) \alpha L^D}{\rho n^C + n^D (p^D / p^C)^{1-\sigma}}$$

Let α be so small that both Home and Foreign produce the outside good. This equalizes the wage rate, $w^H = w^F$ and $p^H = p^F$.. Thus,

$$\frac{L^H}{n^H + \rho n^F} + \frac{\rho L^F}{\rho n^H + n^F} = \frac{T(x)}{\alpha} = \frac{L^F}{n^F + \rho n^H} + \frac{\rho L^H}{\rho n^F + n^H},$$

This is derived under the assumption, $0 < n^C T(x) < L^C = X_0^C + n^C T(x)$ for $C = H$ and F.

This can be solved for

$$n^H = \frac{\alpha(L^H - \rho L^F)}{(1-\rho)T(x)} \quad ; \qquad n^F = \frac{\alpha(L^F - \rho L^H)}{(1-\rho)T(x)}$$

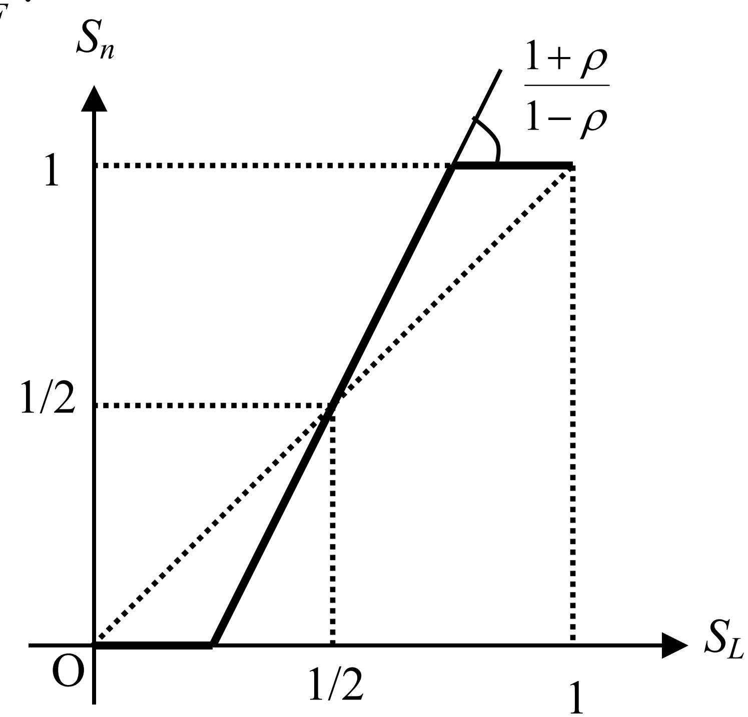
Or

$$S_n \equiv \frac{n^H}{n^H + n^F} = \frac{(1+\rho)S_L - \rho}{1-\rho}, \text{ where } S_L \equiv \frac{L^H}{L^H + L^F}.$$

Home Market (Magnification) Effect:

The slope, $(1+\rho)/(1-\rho)$, is greater than one; the larger country has a disproportionately large share of the differentiated goods firms.

Intuition: There is an incentive for these firms to be closer to the larger market. Here, instead of driving up the wage rate in the larger country, the differentiated goods sector attracts labor from the outside good sector.

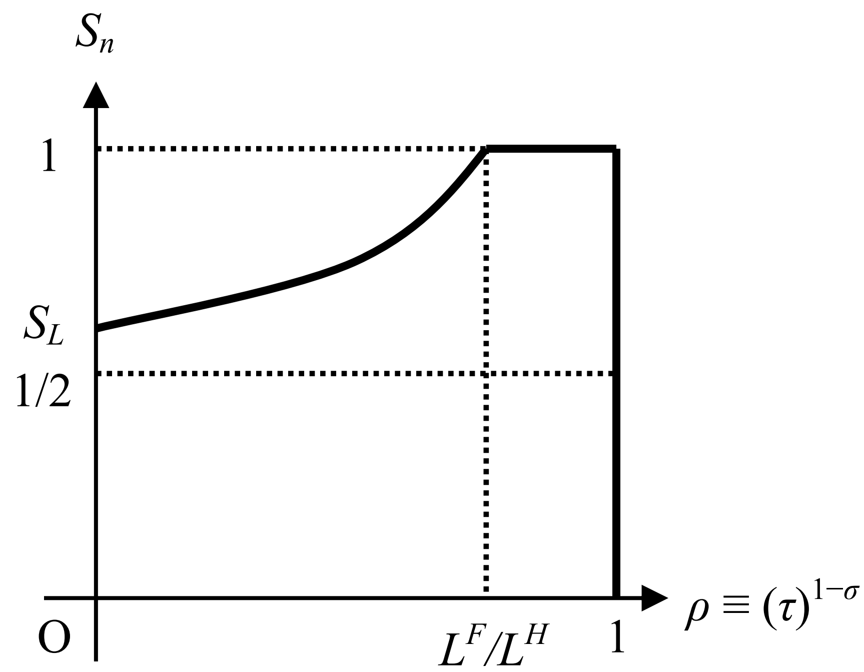


Note:

- Home Market Effect is *larger* if the trade costs are *smaller* (i.e. with a *larger* ρ).
- This may seem surprising at first, since we need positive trade costs to generate the Home Market Effect.
- One may think that the model exhibits discontinuity at $\rho = 1$ (or $\tau = 1$), i.e., when the trade costs disappear.

Actually, this makes perfect sense.

- As the trade costs get smaller, the cost of being away from the smaller market becomes smaller.
- No discontinuity in the sense that, at $\rho = 1$, the distribution of the firms is indeterminate. Technically speaking, S_n, as a function (correspondence) of ρ, is still upper-hemi continuous at $\rho = 1$.
- Indeterminacy occurs at $\rho = 1$, because # of tradeable sectors = 2 > 1 = # of nontradeable factor and FPE holds.



$$\rho \equiv (\tau)^{1-\sigma}$$

Welfares:

$$\frac{U^H}{U^F}=\left(\frac{n^H+\rho n^F}{n^F+\rho n^H}\right)^{\frac{\alpha}{\sigma-1}}>1 \;\Leftrightarrow \frac{n^H+\rho n^F}{n^F+\rho n^H}>1 \Leftrightarrow n^H>n^F \Leftrightarrow L^H>L^F$$

Two-Sector, One Factor Case: Second Model of the Home Market Effect

Two Types of Differentiated Goods: indexed by z_j (j = 1 or 2) .

Two Types of Households: j = 1 or 2.

$$U_j^H = C_j^H = \left\{\int_{\Omega_j}\left[c^H(z_j)\right]^\theta dz_j\right\}^{\frac{1}{\theta}}; \qquad U_j^F = C_j^F = \left\{\int_{\Omega_j}\left[c^F(z_j)\right]^\theta dz_j\right\}^{\frac{1}{\theta}}$$

$\Omega_j = \Omega_j^H + \Omega_j^F$, where Ω_j^C is the equilibrium set of Type-j goods produced in $C = H$ or F.

Two Countries: Home and Foreign(*): For $0 < \beta < 1$,

	Home	Foreign	World
# of Type-1 households	$(1+\beta)L/2$	$(1-\beta)L/2$	L
# of Type-2 households	$(1-\beta)L/2$	$(1+\beta)L/2$	L
Total Labor Supply	L	L	

Notes:

- The two countries are the mirror images of each other.
- Type-j households consume only type-j products. The parameter, β, captures the taste differences across countries, due to the differences in the distribution of preferences.
- Otherwise, the model is identical with the one-sector, one-factor case.

Exercises:

- Derive the equilibrium product varieties of each type produced in each country.
- Derive the net flow of each type of differentiated goods between the two countries.
- Conduct the comparative statics with respect to changes in β, τ, σ, and give the intuition for each result.

Notes:

- The prediction, a country becomes an *exporter* of those goods for which the country has the larger domestic market than other countries is in stark contract with the prediction of the models with an exogenous set of products.
- Suppose that the countries differ only in their (distributions) of preferences. In particular, technologies are identical everywhere. In a competitive model with an exogenous set of products, a country becomes an *importer* of the goods for which it has the larger domestic market than other countries.
- Thus, this prediction offers a way of separating the two models empirically. See, e.g, Hanson and Xiang (2004), Behrens-Lamorgese-Ottaviano-Tabuchi (2007). Head and Mayer (2004) offers a survey.

Two-Sector, Two-Factor Case: 3rd Model of the Home Market Effect

Let us extend the 1st model of the Home Market Effect by adding another (mobile) factor.

Two Countries: Home and Foreign

Two Factors: Labor (L; Immobile) and Capital (K: Mobile)

L^H (L^F) **Households**: each supplies one unit of labor and k units of capital with **Cobb-Douglas Preferences:** with α being the share of differentiated goods and $1 - \alpha$ being the share of the outside good. Thus,

Budget Constraints: $w^D + r^D k = (p_o)^{1-\alpha}\left(P^D\right)^{\alpha} U^D$;

$p^D(z)$; $z \in \Omega$: the price of good z in $D = H$ or F.

$P^D \equiv \left\{\int_\Omega \left[p^D(z)\right]^{1-\sigma} dz\right\}^{\frac{1}{1-\sigma}}$: the price index in $D = H$ or F.

$\Omega = \Omega^H + \Omega^F$, where Ω^C is the equilibrium set of goods produced in $C = H$ or F.

Household Demand: $c^D(z) = \left(\dfrac{p^D(z)}{P^D}\right)^{-\sigma} \dfrac{w^D + r^D k}{P^D} L^D \quad (D = H \text{ or } F)$

Technology:

- Producing the outside good requires one unit of labor.
- Producing each differentiated good requires f units of capital and m units of labor per unit of the output. Hence, the total cost in C is equal to: $TC^C(x) = r^C f + w^C mx$

Trade Costs:

- The outside good: no trade cost, hence $w^H = w^F = p_o{}^H = p_o{}^F = 1$, if both countries produce it.
- The differentiated goods, when shipped from C to D $(\neq C)$, only a fraction $1/\tau < 1$ arrives. No Trade Cost when shipped domestically.

Free Capital Mobility:

- Equalization of the rental rate, $r^H = r^F = r$
- Capital Resource Constraint: $f(n^H + n^F) = k(L^H + L^F)$

Note:

- Even though capital moves freely, the households don't.
- When some capital moves from Home to Foreign, the rents earned by these capitals are repatriated to Home, and the Home households spend it at Home.
- The aggregate income of each country is $(w^D + r^D k)L^D = (1 + r k)L^D$ and the relative market size, L^H/L^F, is exogenously given.

Exercise:

- Derive the equilibrium product varieties produced in each country.
- Derive the net flow of each type of differentiated goods between the two countries.
- Conduct the comparative statics with respect to changes in L^H/L^F, τ, σ, and give the intuition for each result.

All three models show the Home Market Effect in two-country settings, i.e., the country with the large market for differentiated goods produces a disproportionately wider range of differentiated goods.

The next model examines this mechanism in a multi-country setting. This allows us to ask if there are

- any incentive for firms to be "near " the big market, as opposed to "in" the big market?
- any advantages for regions to be *centrally* located?

A Model of Geographical Advantage

Let us extend the 1st model of the Home Market Effect to a multi-country setting.

One Nontradeable Factor: Labor

R* Regions; *r* = 1, 2, ... , *R

***L^r* Households:** each supplies one unit of labor with the budget constraint:

$$w^r = (p_o^r)^{1-\alpha}(P^r)^{\alpha}U^r \qquad \text{where } P^r \equiv \left\{\int_{\Omega}\left[p^r(z)\right]^{1-\sigma}dz\right\}^{\frac{1}{1-\sigma}}$$

Ω: The range of differentiated goods produced in equilibrium;
$p^r(z) \in \Omega$: the price of good z in Region r;
P^r: the price index for differentiated goods in Region r.

Individual Demand:

$$c^r(z) = \left(\frac{p^r(z)}{P^r}\right)^{-\sigma}\frac{\alpha w^r}{P^r}; \qquad z \in \Omega; \quad r = 1, 2, \ldots, R.$$

Technology:

- **Homogeneous Outside Good;** competitive, CRS (one-to-one), and zero trade cost, the law of one price. By letting a numeraire, $p_o^r = 1$ for all $r = 1, 2, \ldots, R$.

  $w^r \geq 1$ for all r ; $w^r = 1$ if Region r produces the outside good.
- **Differentiated Goods**: monopolistically competitive, IRS; Total Factor Requirement, $T(x)$: $xT'(x)/T(x)$ is strictly increasing in x, $\Omega = \sum_j \Omega^r$, where Ω^r is the set of manufactures made in Region r.

Iceberg Trade Costs: from Region c to d, only a fraction $1/\tau_{cd}$ of the good shipped arrives. → $p^d(z) = p^c(z)\tau_{cd}$.

For simplicity, assume $\tau_{jj}= 1$ and $\tau_{ji}= \tau_{ij}$.

Total Demand for a good produced in Region c, if its producer charges $p(z)$:

$$x(z) = \sum_d \tau_{cd} c^d(z) = \sum_d \frac{\tau_{cd}(p(z)\tau_{cd})^{-\sigma} \alpha w^d L^d}{(P^d)^{1-\sigma}} = \alpha\left[\sum_d \frac{\rho_{cd} w^d L^d}{(P^d)^{1-\sigma}}\right](p(z))^{-\sigma}$$

where $0 \leq \rho_{cd} \equiv (\tau_{cd})^{1-\sigma} \leq 1$ is the proximity between c and d. The assumptions, $\tau_{cc} = 1$ and $\tau_{dc}= \tau_{cd}$ imply $\rho_{cc}= 1$ and $\rho_{dc}=\rho_{cd}$.

Again, the combination of Monopoly Pricing, $p(z)(1-1/\sigma) = w^r T'(x(z))$ and Zero profit/Free Entry, $p(z)x(z) = w^r T(x(z))$ yields

$$x(z) = x\,,\quad p(z) = \frac{T(x)}{x} w^c = p^c; \qquad z \in \Omega^r \qquad \text{if } n^r > 0.$$

$$x(z) < x\,; \qquad\qquad z \in \Omega^r \qquad \text{if } n^r = 0.$$

Equilibrium Conditions:

$$\alpha\left[\sum_d \frac{\rho_{cd} w^d L^d}{\sum_k n^k (w^k)^{1-\sigma} \rho_{kd}}\right](w^c)^{-\sigma} \le T(x) \quad \text{for } c = 1, 2, \ldots, R.$$

$$n^c \ge 0 \qquad\qquad \text{for } c = 1, 2, \ldots, R.$$

$$w^c \ge 1 \qquad\qquad \text{for } c = 1, 2, \ldots, R.$$

$$T(x)n^c \le L^c \qquad\qquad \text{for } c = 1, 2, \ldots, R.$$

For a sufficiently small α, all the regions produce the outside good and $w^r = 1$. Then,

$$\alpha\left[\sum_d \frac{\rho_{cd} L^d}{\sum_k n^k \rho_{kd}}\right] \leq T(x) \text{ and } 0 \leq N^c \qquad \text{for } c = 1, 2, \ldots, R.$$

$$T(x)n^c < L^c \qquad \text{for } c = 1, 2, \ldots, R.$$

Since $\alpha \sum_c L^c = T(x) \sum_k N^k$,

$$\sum_d \frac{\rho_{cdi} v^d}{\sum_k n^k \rho_{kd}} \leq 1 \text{ and } n^c \geq 0 \qquad \text{for } c = 1, 2, \ldots, R,$$

$$\alpha n^c < v^c \qquad \text{for } c = 1, 2, \ldots, R,$$

where $v^c \equiv \dfrac{L^c}{\sum_k L^k}$ and $n^c \equiv \dfrac{N^c}{\sum_k N^k}$.

The task is to solve for a nonnegative vector, $n = [\, n^r \,]$, $\sum_r n^r = 1$, satisfying eq. (10) for a given positive vector $v = [\, v_j^r]$, $\sum_r v^r = 1$, and a given nonnegative symmetric matrix, $M = [\rho_{ij}\,]$, $0 \leq \rho_{ij} \leq 1$ for all i and j, with 1's on the diagonal.

Three Benchmarks:

A: *Autarky*: $\rho_{ij} = 0$ $(i \neq j)$. (M is an identity matrix.) → $n = [\, n^r \,] = [\, v^r \,] = v$.

B: *World without Trade Costs:* $\rho_{ij} = 1$ for all i and j. → Any nonnegative vector, $n = [\, n^r \,]$, $\sum_r n^r = 1$, satisfies the equilibrium condition. The location does not matter! (The indeterminacy is due to FPE.)

C: *World with Symmetric Regions*:

- Resources are evenly distributed: $v = [\, v^r \,] = [\, 1/R \,]$
- All regions are symmetrically located in that $\sum_i \rho_{ij}$ is independent of j. (M is a positive scalar multiple of a symmetric, stochastic matrix.)

  → $n = [\, n^r \,] = [\, 1/R \,]$.

Resource Size Effects

Example 1: $\rho_{ij} = \rho < 1$ ($i \neq j$), implying symmetrically located regions.

$$\rightarrow [\, n_j \,] = \left[\frac{\rho R}{1-\rho}\left(v_j - \frac{1}{R} \right) + v_j \right], \text{(ignoring the non-negativity constraint.)}$$

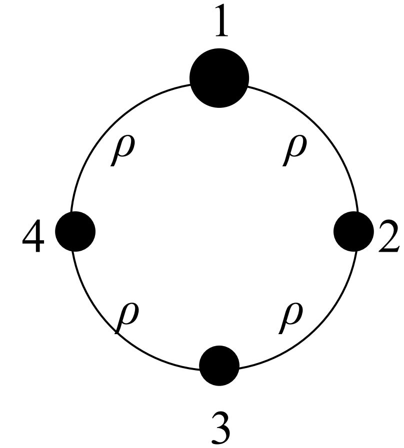
Home Market Effect (A coefficient on v_j is larger than one.)

- A large market serves as a base for exports.
- The smaller the trade cost, the bigger the effect.

Intuition: With trade costs, more firms would prefer locating in a larger market. Of course, these firms have to export to other (smaller) markets, but this cost becomes smaller, as trade costs decline (but not eliminated).

The proximity to the larger market implies a larger industrial base? No, as the following example demonstrates.

Example 2: $R = 4$, $v = \frac{1}{4}\begin{bmatrix} 1+3\gamma \\ 1-\gamma \\ 1-\gamma \\ 1-\gamma \end{bmatrix}$ for $0 < \gamma < 1$, $M = \begin{bmatrix} 1 & \rho & \rho^2 & \rho \\ & 1 & \rho & \rho^2 \\ & & 1 & \rho \\ & & & 1 \end{bmatrix}$



- Four regions on the circle; each region has two neighbors.
- Shipping goods to a nonneighbor is more costly, $\rho^2 < \rho < 1$.
- Regions 2 & 4 are next to the bigger Region 1, but not Region 3.

→ For a small ρ, $n = v + \frac{\rho\gamma}{(1-\rho)^2}\begin{bmatrix} 2-\rho \\ -1 \\ \rho \\ -1 \end{bmatrix}$.

A higher ρ makes 1 & 3 bigger, and 2 & 4 smaller.

- Regions 2 and 4 are in the "shadow" of the big neighbor, Region 1.
- Region 3 emerges as a regional center, simply because it does not have a big neighbor.

Can Japan's emergence as an industrial power in the late 19th century be partially attributed to the fact that it was far away from the industrial centers of Europe and the United States?

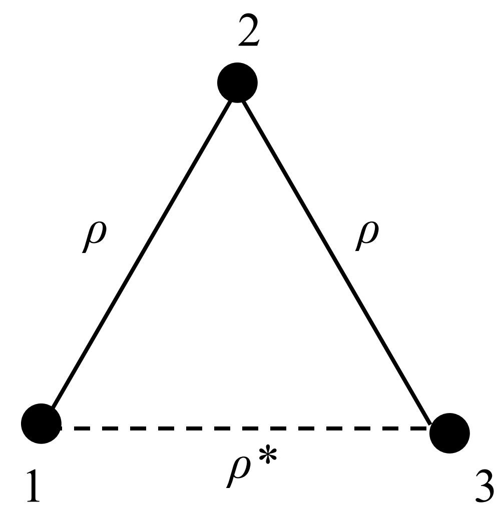
Exercise: Show that, once ρ is high enough to make the industries disappear from 2 & 4, a further increase in ρ makes Region 1 bigger at the expense of Region 3.

Example 3: $R = 3$, $v = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}$, and $M = \begin{bmatrix}1 & \rho & \rho' \\ & 1 & \rho \\ & & 1\end{bmatrix}$, where $\rho' \equiv Max\{\rho^2, \rho^*\}$.

If $\rho^2 \geq \rho^*$, the indirect route is used between Regions 1 and 3.

$$\rightarrow \quad n = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{1}{3}\left(\frac{\rho}{1-\rho}\right)^2 \begin{bmatrix}-1\\2\\-1\end{bmatrix} \quad \text{if } 0 \leq \rho < \frac{1}{2};$$

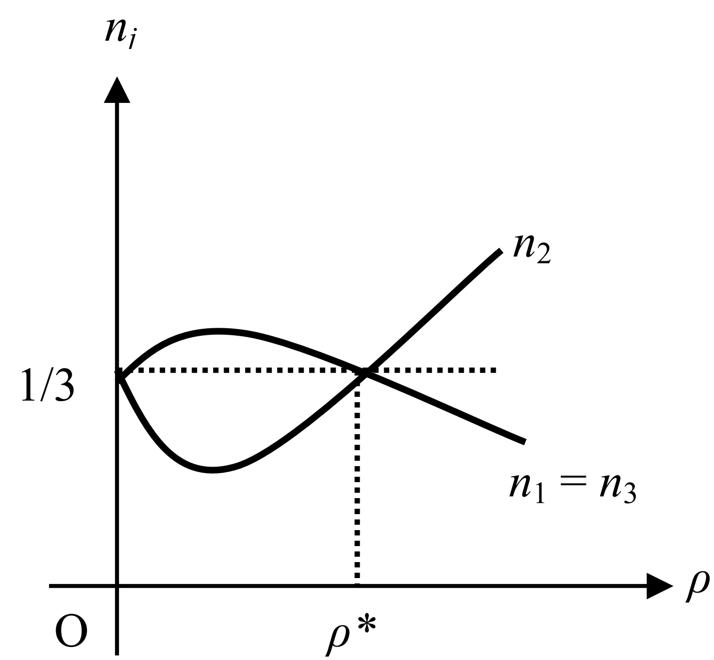
$$n = \begin{bmatrix}0\\1\\0\end{bmatrix} \quad \text{if } \frac{1}{2} \leq \rho < 1.$$



- Region 2 attracts more firms due to its central location.
- A higher ρ magnifies Region 2's geographical advantage.

If $\rho^2 < \rho^*$, the direct route is used between Regions 1 and 3. For the parameter ranges ensuring that each region attracts positive numbers of firms,

$$\rightarrow n = \frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} + \frac{1}{3}\frac{\rho(\rho^* - \rho)}{(1-\rho)(1+\rho^* - 2\rho)}\begin{bmatrix}1\\-2\\1\end{bmatrix}.$$



- Note that $n_j = 1/3$ for $\rho = 0$ and $\rho = \rho^*$. (Why?)
- A higher ρ^* increases the share of Region 1 & 3 monotonically. (Think of the European integration. Regions 1 & 3 are two European countries; Region 2 is ROW.)

On the other hand,

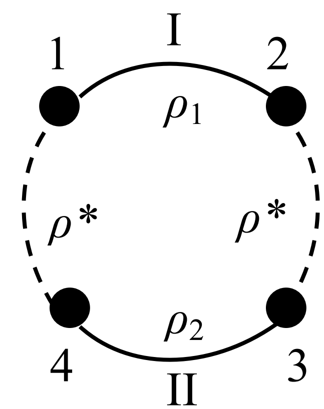
- A change in ρ has non-monotonic effects.

For a small ρ, a higher ρ reduces the share of Region 2. For a high ρ, a higher ρ increases the share of Region 2.

Why Non-monotonic? Two competing forces

- A higher ρ makes Region 2 attractive as a home base from which to export products.
- A higher ρ makes it easier to export to Region 2, reducing the need to locate there.

Internal Trade Costs and Distribution across Superregions



Example 4: $R=4$, $v=\frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$, $M=\begin{bmatrix}1 & \rho_1 & \rho'\rho^* & \rho^* \\ & 1 & \rho^* & \rho'\rho^* \\ & & 1 & \rho_2 \\ & & & 1\end{bmatrix}$,

$\rho' \equiv Max\{\rho_1, \rho_2\}$, $(\rho^*)^2 < Min\{\rho_2/\rho_1, \rho_1/\rho_2\}$.

- Regions 1 & 2 form Superregion I, whose internal cost of trade is captured by ρ_1.
- Regions 3 & 4 form Superregion II, whose internal cost of trade is captured by ρ_2.
- Superregions are linked via two external routes, one connecting Regions 1 & 4, the other Regions 2 & 3. The proximity between the two Superregions is ρ^*.
- When goods are shipped between Regions 1 and 3, or between Regions 2 and 4, the route connecting Regions 1 & 2 are used when $\rho_1 > \rho_2$, while the route connecting Regions 3 & 4 are used when $\rho_1 < \rho_2$.
- The condition, $(\rho^*)^2 < Min\{\rho_2/\rho_1, \rho_1/\rho_2\}$, by making the cost across the superregions high, ensures that the internal trade always takes place directly.

$$\rightarrow \quad n = \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \frac{1}{4}\frac{\rho^*(\rho_1-\rho_2)}{(1-\rho^*)(1+\rho_2-\rho^*(1+\rho_1))}\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix} \quad \text{if } \rho_1 > \rho_2 > \rho_1(\rho^*)^2,$$

$$n = \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \frac{1}{4}\frac{\rho^*(\rho_1-\rho_2)}{(1-\rho^*)(1+\rho_1-\rho^*(1+\rho_2))}\begin{bmatrix}1\\1\\-1\\-1\end{bmatrix} \quad \text{if } \rho_2 > \rho_1 > \rho_2(\rho^*)^2$$

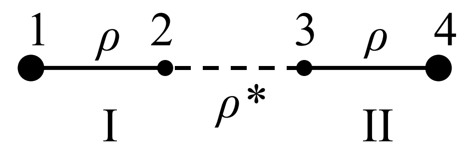
- The share of Superregion I, (Region 1&2) is increasing in ρ_1, and decreasing in ρ_2.
- Superregion I accounts for a larger share if $\rho_1 > \rho_2$.
- A reduction in the cost of trade across superregions, a higher ρ^*, amplifies the existing bias in distribution.

Example: **“**World According to McNeill”

- Superregion I = the Atlantic zone; Superregion II = the Mediterranean zone
- External routes = transalpine passes; Internal routes = the Atlantic & Mediterranean.
- For centuries, the Mediterranean zone enjoyed its “nautical advantage.”
- When the improvements in ship design and navigation made travel on the Atlantic waters as safe as on the Mediterranean, Southern Europe lost its advantage and the center of Europe shifted toward north.

External Trade Costs and Internal Distributions within Superregions

Example 5: $R = 4$, $v = \frac{1}{4}\begin{bmatrix} 1+\gamma \\ 1-\gamma \\ 1-\gamma \\ 1+\gamma \end{bmatrix}$ for $0 < \gamma < 1$,



$$M = \begin{bmatrix} 1 & \rho & \rho\rho^* & \rho^2\rho^* \\ & 1 & \rho^* & \rho\rho^* \\ & & 1 & \rho \\ & & & 1 \end{bmatrix}.$$

- Regions 1 and 2 form a superregion, so do Regions 3 and 4.
- Internal trade cost is captured by ρ*;* external trade cost by ρ^*.
- Regions 1 and 4 are in the interior, while Regions 2 and 3 are on the border.
- The interior regions have larger home markets. A higher γ favors the interior regions at the expense of the border regions.
- The border regions have better access to the other superregion. A higher ρ^* favors the border regions at the expense of the interior regions.

$$\rightarrow \quad n = v + \frac{\rho\{(2-\rho\rho^* + \rho^*)\gamma - (1+\rho)\rho^*\}}{4(1-\rho)(1-\rho\rho^*)}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} = \frac{1}{4}\begin{bmatrix}1\\1\\1\\1\end{bmatrix} + \frac{(1+\rho)(\gamma-\rho\rho^*)}{4(1-\rho)(1-\rho\rho^*)}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}$$

A decline in ρ^* is against the border regions; A higher ρ^* favors the border regions;

- *the Turkish expansion to the Balkan and the North Africa might have contributed to a relative decline of the Southern Europe.*
- *After WWII, the loss of the colonial markets in Asia, as well as a reduction in trans-Pacific transport costs, shifted the industrial center of Japan from the West (i.e., kitakyushu and hanshin areas) to the East (chukyo and keihin areas).*
- *The European integration favored the border regions such as Baden-Württemberg, Rhone-Alpes, Catalunya, and Lombardia (the so-called Four Motors of Europe).*

The effect of ρ is more subtle.

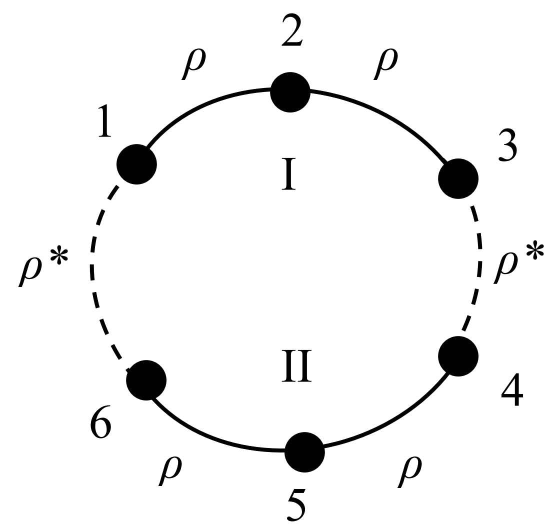
- If $\gamma > \rho^*$, a higher ρ magnifies the advantage of the internal regions
- If $\gamma < \rho^*/(2+\rho^*)$, it magnifies the advantage of the border regions.
- If $\rho^*/(2+\rho^*) < \gamma < \rho^*$, it first causes a shift towards the internal regions, and then a shift towards the border regions.

Exercise: Consider the next two examples, where are actually easier to solve due to the greater symmetry, and interpret the results.

Example 6:

$$R = 6,\ v = \frac{1}{6}\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix},\ M = \begin{bmatrix}1 & \rho & \rho^2 & \rho^2\rho^* & \rho\rho^* & \rho^* \\ & 1 & \rho & \rho\rho^* & \rho^2\rho^* & \rho\rho^* \\ & & 1 & \rho^* & \rho\rho^* & \rho^2\rho^* \\ & & & 1 & \rho & \rho^2 \\ & & & & 1 & \rho \\ & & & & & 1\end{bmatrix}.$$

$$\rightarrow \quad n = \frac{1}{6}\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix} + \frac{\rho(\rho^* - \rho)}{6(1-\rho)^2(1-\rho\rho^*)}\begin{bmatrix}1\\-2\\1\\1\\-2\\1\end{bmatrix}.$$



A Model of Hierarchical System: Subregions, Regions, and Superregions

Example 7: INSERT FIGURE $R = 8$,

$$v = \frac{1}{8}\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}, M = \begin{bmatrix}
1 & \rho & \rho\rho_1^* & \rho^2\rho_1^* & \rho^2\rho_1^*\rho_2^* & \rho\rho_1^*\rho_2^* & \rho\rho_2^* & \rho_2^* \\
 & 1 & \rho_1^* & \rho\rho_1^* & \rho\rho_1^*\rho_2^* & \rho^2\rho_1^*\rho_2^* & \rho^2\rho_2^* & \rho\rho_2^* \\
 & & 1 & \rho & \rho\rho_2^* & \rho^2\rho_2^* & \rho^2\rho * \rho_2^* & \rho\rho_1^*\rho_2^* \\
 & & & 1 & \rho_2^* & \rho\rho_2^* & \rho\rho_1^*\rho_2^* & \rho^2\rho_1^*\rho_2^* \\
 & & & & 1 & \rho & \rho\rho_1^* & \rho^2\rho_1^* \\
 & & & & & 1 & \rho_1^* & \rho\rho_1^* \\
 & & & & & & 1 & \rho \\
 & & & & & & & 1
\end{bmatrix}$$

$$\rightarrow \quad n = \frac{1}{8}\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix} + \frac{1}{8}\frac{\rho(\rho_2^* - \rho_1^*)}{(1-\rho\rho_1^*)(1-\rho\rho_2^*)}\begin{bmatrix}1\\-1\\-1\\1\\1\\-1\\-1\\1\end{bmatrix}$$

Example 8: INSERT FIGURE

$$R = 6,\ v = \frac{1}{6}\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix},\ \text{and } M = \begin{bmatrix}1 & \rho^* & \rho^* & \rho & \rho\rho^* & \rho\rho^* \\ & 1 & \rho^* & \rho\rho^* & \rho & \rho\rho^* \\ & & 1 & \rho\rho^* & \rho\rho^* & \rho \\ & & & 1 & \rho^2\rho^* & \rho^2\rho^* \\ & & & & 1 & \rho^2\rho^* \\ & & & & & 1\end{bmatrix}$$

$$\rightarrow \quad n = \frac{1}{6}\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix} + \frac{\rho\rho^*(1+\rho)}{3(1-\rho)(1-2\rho\rho^*)}\begin{bmatrix}1\\1\\1\\-1\\-1\\-1\end{bmatrix} \quad \text{or } n = \frac{1}{3}\begin{bmatrix}1\\1\\1\\0\\0\\0\end{bmatrix}$$

Agglomeration and Economic Geography:

In the Home Market Effect models or in the Geographical Advantage model, the market size of each region or country is *exogenous*. What if the market size is *endogenous*?

An increase in the region's market size → more firms choose the region as the home base → The region becomes a more desirable place for the consumers and firms to relocate → Further increase in the region's market size.

With such ***Positive Feedback*** mechanism, or circular or ***Cumulative Causation***,

- Possibility of ***Multiple Equilibria***
- ***Symmetry-Breaking***: Among many inherently similar locations or regions, only a few of them might develop into industrial centers through such agglomeration.
- ***Hysteresis*** *or* ***Path-dependence:*** even a small temporary shock (or a historical accident) could have long lasting effects.

Two (Complementary) Approaches:

- ***Labor Migration:*** *Workers want to move to the big market, where more goods and services are available at cheaper prices.*
- ***Vertical Linkages:*** *Firms want to move to the big market, not only to be near the customers but also to be near the suppliers.*

Cumulative Causation, Symmetry-Breaking, and Path-Dependence: Labor Migration Model ; **Krugman (1991); Fujita-Krugman-Venables (1999, Ch. 5); Combes-Mayer-Thisse (2008, Ch.6)**

The model is similar to the 3rd model of the Home Market Effect in that it has two factors; one mobile and one immobile.

Two Types of Goods (Sectors): Agriculture and **Manufacturing**

- A-good: homogenous, competitively supplied. CRS, converting a unit of farmer's labor into a unit of the output; zero trade cost. *numeraire*
- M-goods: differentiated, CES aggregate; monopolistic competition. Total requirement of worker's labor is $T(x)$ units of worker's labor, subject to IRS; iceberg trade cost.

Two Types of Households: (Immobile) Farmers and **(Mobile) Workers**

- Each farmer (worker) supplies one unit of specific labor to the A-sector (M-sector).
- Common Cobb-Douglas Preferences (M-goods' share = α; A-good's share = $1-\alpha$).

Two Regions: East and West

	West	East	Country
# of (Immobile) Farmers	$(1-\mu)/2$	$(1-\mu)/2$	$1-\mu$
# of (Mobile) Workers	$(\lambda^W\mu)$	$(\lambda^E)\mu$	μ

Notes:

- Farmers are immobile across regions, but their products are tradeable at zero cost, hence their wages are equalized at one in both regions.
- Workers are mobile across regions, so that λ^W and $\lambda^E = 1 - \lambda^W$ are endogenous. But, we first solve for an equilibrium for a given distribution. One of the equilibrium conditions imposed is free entry of M-firms. Then, we would let workers move to the region that offers the higher utility to find the equilibrium distribution of workers.
- Effectively, this assumes that firms entry-exist process takes place a lot faster than the moving process of workers across regions.

Total Demand for Good z $\in \Omega^C$: If its producer charges $p(z)$ at its domestic market, With the worker's wages in the two regions, w^C

$$x(z) = \frac{(p(z))^{-\sigma}}{(P^C)^{1-\sigma}}\alpha\left(\frac{1-\mu}{2} + \lambda^C \mu w^C\right) + \frac{\tau(\tau p(z))^{-\sigma}}{(P^D)^{1-\sigma}}\alpha\left(\frac{1-\mu}{2} + \lambda^D \mu w^D\right)$$

$$= \alpha\left[\frac{1}{(P^C)^{1-\sigma}}\left(\frac{1-\mu}{2} + \lambda^C \mu w^C\right) + \frac{\rho}{(P^D)^{1-\sigma}}\left(\frac{1-\mu}{2} + \lambda^D \mu w^D\right)\right](p(z))^{-\sigma}.$$

Note: When workers move, not only their factor endowments move, but also their consumption bases move as well. (unlike from the 3rd model of the Home Market Effect.)

Again, the mark-up is constant, and monopoly pricing and free entry ensure:

$$x(z) = x \quad \text{for all } z \in \Omega, \quad \text{where } xT'(x)/T(x) = m(x)/a(x) = 1-1/\sigma.$$

$$p(z) = \frac{w^C T(x)}{x} \equiv p^C \text{ for all } z \in \Omega^C.$$

$$\left(P^D\right)^{1-\sigma} = n^C\left(\tau p^C\right)^{1-\sigma} + n^D\left(p^D\right)^{1-\sigma} = \rho n^C\left(p^C\right)^{1-\sigma} + n^D\left(p^D\right)^{1-\sigma}$$

$$n^C = \frac{\lambda^C \mu}{T(x)}.$$

Hence,

$$x = \mu\left[\frac{\left(\frac{1-\mu}{2x} + \lambda^C \mu w^C\right)}{\rho n^D (p^D)^{1-\sigma} + n^C (p^C)^{1-\sigma}} + \frac{\rho\left(\frac{1-\mu}{2x} + \lambda^D \mu w^D\right)}{\rho n^C (p^C)^{1-\sigma} + n^D (p^D)^{1-\sigma}}\right]\left(p^C\right)^{-\sigma}$$

Some Normalizations: We may choose:

- the unit of each differentiated good so that $x = \mu$.
- The unit of Workers so that $T(x) = T(\mu) = \mu$, which ensures $w^C = p^C$ and $n^C = \lambda^C$
- The unit of Farmers so that $\mu = \alpha$. This ensures that the Worker's wage would also become one in equilibrium, the farmer's wage, simplifying the algebra. Essentially, we

define the unit of farmers by the number of farmers who would earn the same income with one worker.

Then,

$$w^C = \frac{\left(\frac{1-\mu}{2} + \lambda^C \mu w^C\right)}{\rho\lambda^D (w^D / w^C)^{1-\sigma} + \lambda^C} + \frac{\rho\left(\frac{1-\mu}{2} + \lambda^D \mu w^D\right)}{\rho\lambda^C + \lambda^D (w^D / w^C)^{1-\sigma}} \qquad (C,D) = (W,E) \text{ or } (E,W).$$

Given $\lambda = \lambda^W = 1-\lambda^E$, these two equations determine w^W and w^E.

Worker's Welfare:

$$U^D = \frac{w^D}{\left(P^D\right)^{\mu}} = \frac{w^D}{\left[\rho n^C\left(p^C\right)^{1-\sigma} + n^D\left(p^D\right)^{1-\sigma}\right]^{\frac{\mu}{1-\sigma}}} = \frac{w^D}{\left[\rho\lambda^C\left(w^C\right)^{1-\sigma} + \lambda^D\left(w^D\right)^{1-\sigma}\right]^{\frac{\mu}{1-\sigma}}}.$$

It is easy to verify that

- $\rho \equiv (\tau)^{1-\sigma} = 1 \quad \rightarrow w^W = w^E = 1$ for any $\lambda = \lambda^W = 1-\lambda^E \rightarrow U^W = U^E = 1$.
- $\lambda = \lambda^W = 1-\lambda^E = 1/2 \rightarrow w^W = w^E = 1$ for any $\rho \equiv (\tau)^{1-\sigma} \rightarrow U^W = U^E = \left[\frac{1+\rho}{2}\right]^{\frac{\mu}{\sigma-1}}$.

Two Questions to Ask:

Question #1 (Sustainability): Is λ = 1* stable? *When is $U^W > U^E$ at λ = 1?

$\lambda = \lambda^W = 1 - \lambda^E = 1$ → $w^W = U^W = 1$ → $(w^E)^\sigma = \rho\left(\frac{1+\mu}{2}\right) + \frac{1}{\rho}\left(\frac{1-\mu}{2}\right)$ and

$$U^E = \rho^{\frac{\mu}{\sigma-1}} w^E = \rho^{\frac{\mu}{\sigma-1}}\left[\rho\left(\frac{1+\mu}{2}\right) + \frac{1}{\rho}\left(\frac{1-\mu}{2}\right)\right]^{\frac{1}{\sigma}} = \frac{1}{\tau^\mu}\left[\tau^{1-\sigma}\left(\frac{1+\mu}{2}\right) + \tau^{\sigma-1}\left(\frac{1-\mu}{2}\right)\right]^{\frac{1}{\sigma}}$$

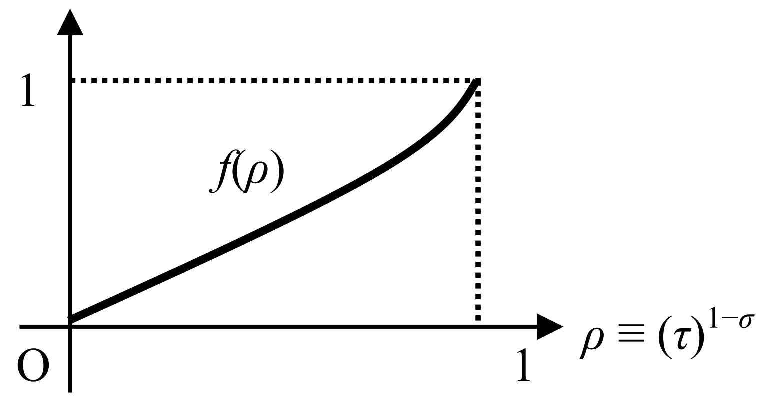
Intuition:

- 1st term: the cost of living disadvantage: the workers moving to the East must pay extra for all M-goods coming from the West.
- The bracket (to the power of 1/σ): the wage rate that the firms in the East is able to pay.
- 1st term in the bracket: the effective market size of the West for the firms in the East; Multiplied by $\rho < 1$, due to the disadvantage of being away.
- 2nd term in the bracket: the effective market size of the East for the firms in the East; Multiplied by $1/\rho > 1$, due to the advantage of being close.

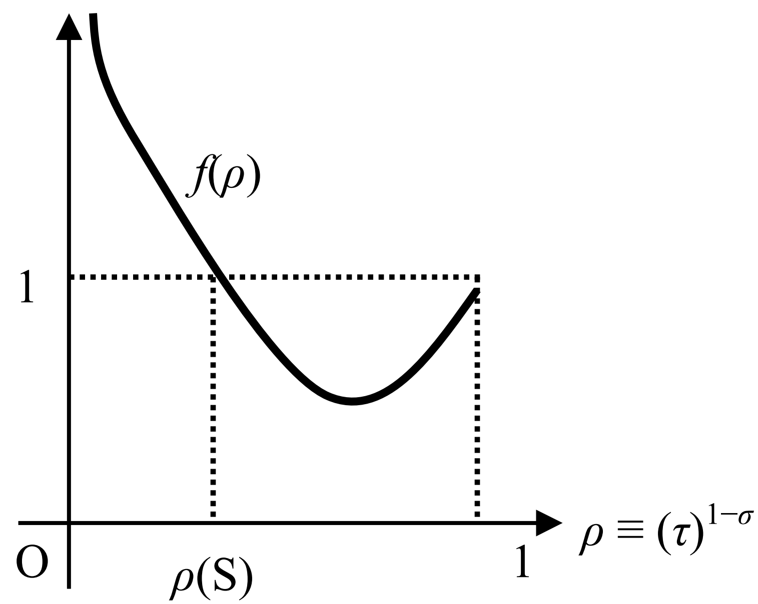
We can further rewrite this expression to:

$$(U^E)^\sigma = f(\rho) \equiv \rho^{\frac{\mu+\theta}{\theta}}\left(\frac{1+\mu}{2}\right) + \rho^{\frac{\mu-\theta}{\theta}}\left(\frac{1-\mu}{2}\right).$$

If $1-1/\sigma \equiv \theta < \mu$, $f(\rho) < 1$ for all $\rho < 1$.
In this case, $\lambda = 1$ ($\lambda = 0$) is *always* stable. The high share of M-sector, and high differentiation of their products create such a strong "centripetal" force of agglomeration, that, once one region attracts all firms, no firm will escape from it, like a "black hole."



If $1-1/\sigma \equiv \theta > \mu$,
$f(\rho)>1$ for $\rho<\rho(S)$; $f(\rho)<1$ for $\rho(S)<\rho< 1$.
In this case, $\lambda = 1$ ($\lambda = 0$) is stable only when the trade cost is sufficiently small (but positive). Fujita-Krugman-Venables (1999) called this condition, $\theta > \mu$, "No Black Hole Condition."



In what follows, we will assume this is the case.

Question#2 (Symmetry-Breaking): When is $\lambda = \lambda^W = 1 - \lambda^E = 1/2$ unstable? $\Leftrightarrow$ When is U^W/U^E increasing in $\lambda^W/\lambda^E = 1$?

To answer this, we need to differentiate

$$\frac{U^W}{U^E} = \frac{w^W}{w^E}\left[\frac{1+\rho\left(\lambda^W/\lambda^E\right)\left(w^W/w^E\right)^{1-\sigma}}{\rho+\left(\lambda^W/\lambda^E\right)\left(w^W/w^E\right)^{1-\sigma}}\right]^{\frac{\mu}{1-\sigma}}$$

and evaluate it at $\lambda^W = 1-\lambda^E = 1/2$ and $w^W = w^E = 1$.

The answer turns out:

- With "the No Black Hole Condition," $\theta > \mu$, it is unstable for $0 < \rho(\mathrm{B}) < \rho < 1$.
- If $\theta < \mu$, it is always unstable.

Exercise: Prove the above.

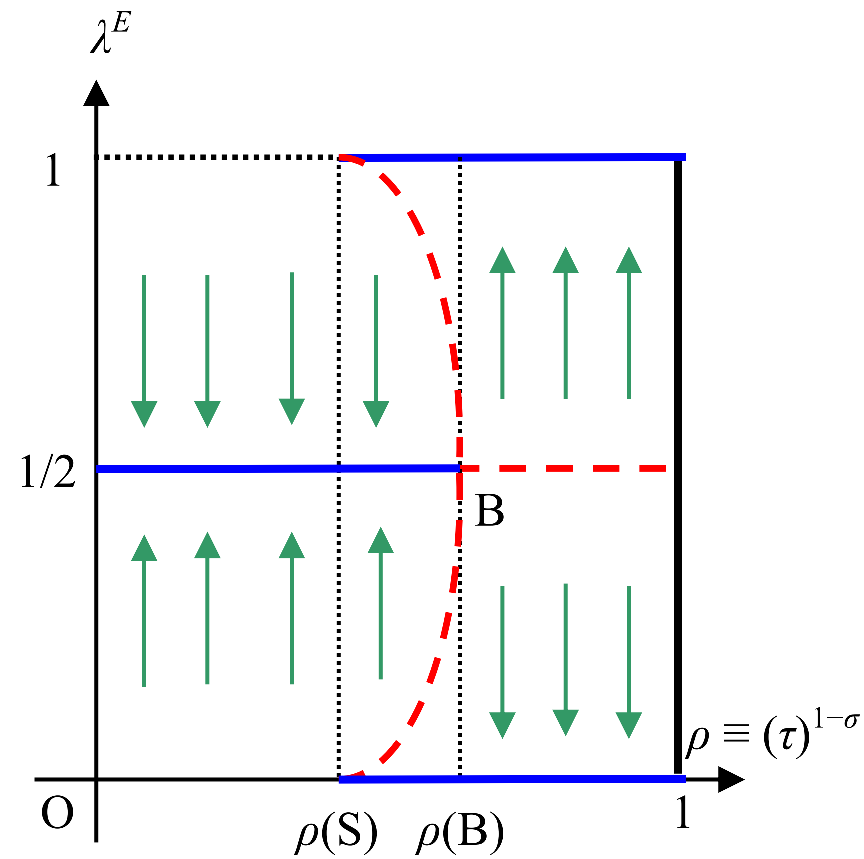
Exercise: Show that, if $\theta > \mu$,

- $0 < \rho(\mathrm{S}) < \rho(\mathrm{B}) < 1$
- Both $\rho(\mathrm{S})$ and $\rho(\mathrm{B})$ are increasing in θ, and decreasing in μ.

Bifurcation Diagram:

Green arrows indicate the incentive to migrate.
Blue solid lines are stable equilibriums.
Red dashed line/curves are unstable equilibriums.

- $\rho < \rho(\text{S})$ → $\lambda^E = 1-\lambda^W = 1/2$ is stable. → No agglomeration.
- $\rho(\text{S}) < \rho < \rho(\text{B})$ → $\lambda^E = 0$, $\lambda^E = ½$, & $\lambda^E = 1$ are stable. →Agglomeration is sustainable.
- $\rho(\text{B}) < \rho < 1$ → $\lambda^E = 0$ & $\lambda^E = 1$ are stable. → Agglomeration is inevitable.
- $\rho = 1$ → λ^E is indeterminate.



Thought Experiment:

Initially, the trade cost is high and the M-sector is evenly divided between the two regions. Then, the trade cost declines gradually. When it reaches the break point, B, one region suddenly emerges as the Industrial Core and the other region becomes Agricultural Periphery.

Adding the Trade Cost for the A-Good:

- We have thus far assumed no trade cost for the A-good, so that
  - ➢The law of one price holds for the A-good, the numeraire.
  - ➢The A-good can be used to "settle" the trade balance across the regions when one becomes the core and exports M-goods to the periphery.
- If the A-good is interpreted as a composite of agricultural goods or some homogeneous goods that are traded in organized exchanges, we cannot ignore traded costs.
- However, adding the trade cost mechanically on the A-good in the previous model would be silly, because many other simplifying assumptions such as
  - ➢Both regions produce the A-goods that are perfectly substitutable;
  - ➢Supply of the A-good is completely inelastic in each region;

  would no longer be innocuous in the presence of the trade cost.
- Fujita-Krugman-Venables (1999, Ch.7) instead examined the effects of the trade cost in the A-sector by a model modified a la Matsuyama-Takahashi (1998).

Two Goods (E & W) in the A-sector; Imperfect Substitutes (e.g., Beef & Fish, Grapes & Grains or Rice & Wheat)

Regional Specialization: A-workers in East (West) produce only E-good (W-good).

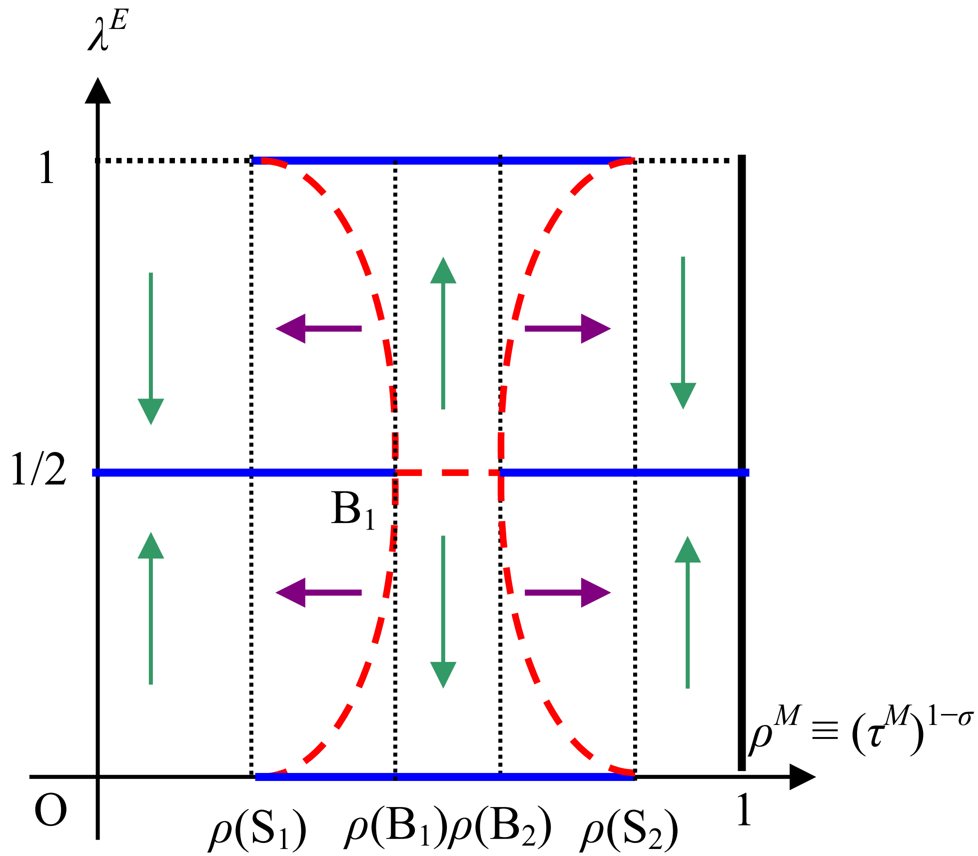
Preferences: $U^k = \left(u(c_E^k, c_W^k)\right)^{1-\alpha}\left(C_M^k\right)^{\alpha} \equiv \left(u(c_E^k, c_W^k)\right)^{1-\alpha}\left(\int_0^{n^k}(c_M(z))^{\theta}\,dz\right)^{\frac{\alpha}{\theta}}$,

where $u(c_E^k, c_W^k)$ is a symmetric CES.

Iceberg Trade Costs; $\tau^A > 1$ and $\tau^M > 1$.

Figure shows the bifurcation diagram for a given $\tau^A > 1$. Purple arrows indicate the effects of a decline in τ^A.

Experiment-1: A Decline in τ^M;
Initially, the M-sector is evenly divided. When it reaches the break point, B_1, one region suddenly emerges as the Core, leaving the other the Periphery. But, a further decline leads to an unraveling of the Core-Periphery patterns. Kuznets curve for the Regional Inequality.



Experiment-2: A Decline in τ^A may cause symmetry-breaking, leading to the emergence of the Core-Periphery Patterns.

Cumulation Causation, Symmetry-Breaking, Path-Dependence: Vertical Linkages: (Unfinished)

Krugman-Venables (1995)
Puga-Venables (1996)
Puga (1999)
Combes-Mayer-Thisse (2008, Ch.7)

Monopolistic Competition Models with Heterogeneous Firms (Unfinished)

Motivations (Unfinished)

Melitz (2003) Model: (Unfinished)

Technology: Firms are *ex ante* identical, but *ex post* heterogeneous.

- A sunk cost of entry, f_E (in labor) to enter. Upon entry, the firm learns its marginal labor requirement, m, drawn from the distribution, $G(m)$. Then, it decides whether to exit or produce. If it decides to produce, it also decides whether to export.
- A fixed cost of production, f_D, if it chooses to produce a positive amount.
- A sunk cost of export, f_X, as well as the iceberg cost, τ_{cd}, if it chooses to export.

The structure is otherwise identical with Krugman (1980).

Sale to Country d:

$$\tau_{cd} c^d(z) = \tau_{cd} \left(\frac{p^d(z)}{P^d} \right)^{-\sigma} \frac{Y^d}{P^d} = \frac{\rho_{cd} Y^d}{\left(P^d\right)^{1-\sigma}} (p(z))^{-\sigma} = B^{cd} [p(z)]^{-\sigma}$$

Monopoly Pricing: $p^c(m) = w^c m/\theta$

Price depends not only on w^c but also on the realized marginal cost, m.

Revenue from Selling to d: $R^{cd}(m) = B^{cd}[p^c(m)]^{1-\sigma} = B^{cd}[w^c m/\theta]^{1-\sigma}$

Gross Profit from Selling to d: $\pi^{cd}(m) = (B^{cd}/\sigma)[w^c m/\theta]^{1-\sigma}$, increasing in $(m)^{1-\sigma}$.

Closed Economy Case ($f_X = \infty$): Let $w^c = 1$ (the numeraire) and drop the superscripts so that $p(m) = m/\theta$, $B = L/(P)^{1-\sigma}$, $\pi(m) = (B/\sigma)[m/\theta]^{1-\sigma} = (L/\sigma)[m/P\theta]^{1-\sigma}$ etc.

Firm's Exit Rule: They exit if $m > m^*$ and stay if $m \le m^*$, where

$$\pi(m^*) = \frac{L}{\sigma}\left(\frac{m^*}{P\theta}\right)^{1-\sigma} = f_D$$

Distribution of m among active firms: $G^*(m) \equiv \dfrac{G(m)}{G(m^*)}$.

Price Index: $P^{1-\sigma} = N\int_0^{m^*} p(m)^{1-\sigma}\,dG^*(m) = \dfrac{N}{\theta}\int_0^{m^*}(m)^{1-\sigma}\,dG^*(m) = \dfrac{N}{\theta}(\tilde{m})^{1-\sigma}$

$\tilde{m} \equiv \left[\int_0^{m^*} m^{1-\sigma}\,dG^*(m)\right]^{\frac{1}{1-\sigma}}$: the marginal cost index among the active.

Average Net Profit among active firms:

$$\tilde{\pi} \equiv \int_0^{m^*}\pi(m)\,dG^*(m) - f_D = f_D\left[\int_0^{m^*}\frac{\pi(m)}{\pi(m^*)}\,dG^*(m) - 1\right] = f_D\left[\int_0^{m^*}\left(\frac{m}{m^*}\right)^{1-\sigma} dG^*(m) - 1\right]$$

or

$$\tilde{\pi} = f_D\left[\left(\frac{\tilde{m}}{m^*}\right)^{1-\sigma} - 1\right]$$

Or $\tilde{\pi} \equiv \int_0^{m^*} \pi(m) dG^*(m) - f_D = \frac{L}{\sigma(P\theta)^{1-\sigma}} \int_0^{m^*} m^{1-\sigma} dG^*(m) - f_D$

$$= \frac{L}{\sigma}\left(\frac{\tilde{m}}{P\theta}\right)^{1-\sigma} - f_D = \frac{L}{\sigma\theta N}(\theta)^{\sigma-1} - f_D$$

Free Entry (FE) Condition: $\tilde{\pi} G(m^*) = f_E$

Impact of Trade:

Intensive versus Extensive Margin: **Chaney (2006)** ***(Unfinished)***

Adding the Heckscher-Ohlin Elements: **Bernard-Redding-Schott (2007)** ***(Unfinished)***

Linear-Quadratic Utility: An Alternative to CES (Unfinished)

Ottaviano-Tabuchi-Thisse (2002)
Melitz-Ottaviano (2008)
Combes-Mayer-Thisse (2008, Ch.8)

Monopolistic Competition with Multi-product Firms (Unfinished)

Some Oligopoly Models of Trade (Unfinished)

Brander (1981)
Brander-Krugman (1983)
Ben-Zvi and Helpman (1992)
Eaton-Kortum-Kramarz
Bernard-Eaton-Jensen-Kortum
Neary (2007)

Bibliography (Incomplete): (*) indicate essential readings.

Behrens, Lamorgese, Ottaviano, and Tabuchi, 2007.
Chaney, “Distorted Gravity,”
*Combes, Mayer, and Thisse, Economic Geography: The Integration of Regions and Nations, Princeton University Press, 2008.
Duranton and Puga, “The Microfoundations….” Handbook of Regional and Urban Economics.
Ethier, “International Decreasing Costs and World Trade,” JIE 1979.
Either, “Decreasing Cost in International Trade and Frank Graham’s Argument for Protection,” Ecta 1982
Ethier, AER, 1982.
Feenstra, Ch. 5
Foellmi-Hepenstrick-Zweimüller, Oct 2007.
*Fujita, Krugman, and Venables. The Spatial Economy, Cambridge: MIT Press, 1999
Hanson and Xiang, “Home-Market Effect and Bilateral Trade Patterns,” AER Sep 2004
Head and Mayer, Handbook of Regional and Urban Economics
*Helpman and Krugman, Market Structure and Foreign Trade 1985.
Krugman, , Rethinking International Trade Ch. 1; “Increasing Returns, Monopolistic Competition, ….” JIE 1979
Krugman, Rethinking International Trade Ch. 2; “Scale Economies, Product Differentiation, …. ,“ AER 1980
Krugman, Rethinking International Trade Ch. 3; “Intraindustry Specialization and Gains from Trade,” JPE 1981
Krugman, "Increasing Returns and Economic Geography," JPE June 1991
Krugman and Venables, "Globalization and the Inequality of Nations," QJE, 1995
Markusen, “Explaining the Volume of Trade: An Eclectic Approach" AER Dec 1986
Matsuyama, "Complementarities & cumulative processes in models of monopolistic competition,"JEL June 1995
*Matsuyama, "Why Are there rich and poor countries?: Symmetry-breaking in the world economy," JJIE 1996
*Matsuyama, "Geography of the World Economy." Fukuzawa Lecture, Far Eastern Econometric Society Meeting
*Matsuyama and Takahashi, "Self-Defeating Regional Agglomeration," REStud April 1998
*Melitz, “The Impact of Trade on Intraindustry reallocations and aggregate industry productivity,” Ecma, Nov 2003.
Melitz-Ottaviano, “Market Size, Trade, and Productivity,” REStud January 2008
Puga and Venables, “The Spread of Industry: Spatial Agglomeration in Economic Development,” JJIE 1996.
Puga, “The Rise and Fall of Regional Inequalities,” EER,
Rauch, JIE
Romalis, “Factor Proportions and Structure of Commodity Trade,” AER 2004