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Lecture #7: Exchange Rate Determination in the Long Run

The Simple Monetary Approach to the Exchange Rate. This approach is called ‘simple’ because it adopts PPP. It determines the endogenous variables $R_{\$}$, E , P_{US} using the following relations:

$$\begin{aligned}\text{UIP} &: R_{\$} = R_{DM} + \frac{E^e - E}{E} \\ \text{Money Market} &: \frac{M}{P_{US}} = L(R_{\$}, Y). \\ \text{PPP} &: \frac{P_G E}{P_{US}} = 1.\end{aligned}$$

The idea is that every equation is satisfied at every date in the long run.

1. Equilibrium. In the above relationships, the superscript, e , on E means the value of the exchange rate, ‘later’. Under the UIP, the differential between nominal interest rates in the US and abroad depends on the current interest rate and the interest rate expected to prevail later, when the interest rate payments are made.¹ For example, if the interest rate pays off in three months’ time, then E^e refers to the interest rate three months in the future. We could think of applying the e superscript to other variables too. For example, M^e means the money supply three months later, in contrast with the money supply today, M . The money growth rate over the next three months is expected to be $(M^e - M)/M$. We can also think of applying the superscript, e , to the price level, so that P_{US}^e denotes the US price level in three months. The rate of inflation expected over the next three months is written $\pi^{US} = (P_{US}^e - P_{US})/P_{US}$. This superscript convention could be pushed even further. For example, we could let $E^{e,e}$ mean $(E^e)^e$, the value of the exchange rate *six* months later.

Now, the variables of the model are assumed to satisfy the above equations at *every* date in the long run. In particular, they should satisfy the equations three months from now:

$$\text{UIP} : R_{\$}^e = R_{DM}^e + \frac{E^{e,e} - E^e}{E^e}$$

¹You should make sure you understand why this is so.

$$\begin{aligned}\text{Money Market} & : \frac{M^e}{P_{US}^e} = L(R_{\$}^e, Y). \\ \text{PPP} & : \frac{P_G^e E^e}{P_{US}^e} = 1.\end{aligned}$$

We will always assume that interest rates are constant in the long run. That is, $R_{\$} = R_{\e , and $R_{DM}^e = R_{DM}$. In addition, for the most part we will assume that Y is constant, i.e., $Y^e = Y$. We will assume that if the other exogenous variables, M , P_G , are changing, then their rate of change is constant in the long run. That is, $(M^e - M)/M$ and $\pi_G = (P_G^e - P_G)/P_G$ are constant.

2. Properties of Equilibrium. Given the assumptions just stated, it is easy to see that UIP implies the rate of change in E , $(E^e - E)/E$, must be constant. The PPP equation then implies

$$(\text{PPP}): \frac{E^e - E}{E} = \pi_{US} - \pi_G,$$

where $\pi_{US} = (P_G^e - P_G)/P_G$.² Combining this with UIP, we get:

$$(\text{PPP and UIP}): R_{\$} = R_{DM} + \pi_{US} - \pi_G.$$

This relationship shows that our framework implies the Fisher effect: a rise in π_{US} translates one-for-one into a rise in $R_{\$}$, assuming the foreign variables, R_{DM} and π_G , do not change. Rearranging the above equation, we obtain:

$$R_{\$} - \pi_{US} = R_{DM} - \pi_G,$$

so that the real rate of interest in the US and other countries must be the same, in the long run.

The money market equation helps us to determine the US inflation rate. In particular,

$$\pi_{US} = \% \Delta M,$$

since $L(R_{\$}, Y)$ is constant under our assumptions. According to this expression, if a 5% inflation is desired in the long run, then to achieve that target money growth must be 5% too.

²A simple principle was applied here. Let $\% \Delta x$ denote the percent change in x , i.e., $100(x^e - x)/x$. Then, $xy/z = q$ implies, approximately, that $\% \Delta x + \% \Delta y - \% \Delta z = \% \Delta q$. In the discussion in the text, $x = E$, $y = P_G$, $z = P_{US}$, and $q = 1$.

It is instructive to temporarily drop the assumption that Y is constant. Also, write the money demand equation as $L(R_{\$}, Y) = f(R_{\$})Y^{\gamma}$. Then it is easy to confirm:

$$\pi_{US} = \% \Delta M - \gamma \% \Delta Y.$$

That is, to know what sort of money growth is required to hit a given long-run inflation target, one must have an idea about the economy's long run growth rate.

3. Experiments.

- (a) One time, permanent increase (jump) in M . The variables to be determined are, P_{US} , $R_{\$}$, E . Conjecture that $R_{\$}$ does not change (momentarily, this conjecture will be verified.) Given this conjecture, L does not respond to the jump in M , so that for the money demand equation to be satisfied it is necessary that P_{US} jumps by the same percent as the jump in M , so that M/P_{US} remains unchanged. Given the jump in P_{US} , PPP indicates that E and E^e must jump equiproportionally to M too. Suppose the jump in M was x percent, so that the new M and P_{US} are $(1+x)M$ and $(1+x)P_{US}$, respectively. Also, the new E^e and E are, respectively, $(1+x)E^e$ and $(1+x)E$. With this change in the exchange rate, its rate of change does not change in this experiment (verify this by substituting the new E^e and E into the rate of change formula). As a result, the UIP relation can continue to be satisfied at the old $R_{\$}$. This verifies our conjecture that $R_{\$}$ does not change.
- (b) Increase in money growth. Suppose an unexpected change in the rate of money growth occurs in period t_0 . The money stock follows the path in the curve in Figure 15-1 (a), on page 397 of KO. Its growth rate is assumed to be some (unspecified) number π before the change. At date t_0 , its growth rate becomes $\pi + \Delta\pi$, where $\Delta\pi$ is the notation used to designate the change in the money growth rate.³ It is important to understand the nature of this experiment, which is very different from the one just discussed, where the money stock took a permanent *jump* at t_0 . Here, the value of the money stock does not suddenly change at any point in time (see Figure (a) again). For example, the event at t_0 is not that M jumps, only that its growth rate changes. Our objective now is to figure out the impact of this change on the three variables: P_{US} , $R_{\$}$, E . We also want to know how their growth rates are affected.

³Example: if π is .08 and $\Delta\pi$ is .01, then the money growth rate goes from 8 percent to 9 percent.

- i. Inflation jumps from π_{US} to $\pi_{US} + \Delta\pi$. Why? We know (this will be confirmed momentarily) that whatever happens to $R_{\$}$ in the instant, t_0 , it is constant from then on. This means that money demand is constant after instant t_0 . But, if money demand is constant, then the ratio, M/P_{US} must be constant too. This means that, after t_0 , P_{US} must be growing at the same rate as the new growth rate of M .
- ii. The interest rate, $R_{\$}$, jumps at t_0 because of the Fisher effect.
- iii. Real money. This drops at t_0 because of the rise in $R_{\$}$.
- iv. The price level. We just showed that the growth rate of P_{US} (i.e., the inflation rate) jumps at t_0 . But, what does the price *level* do? Does it jump, or does it behave more like the money stock itself, which was assumed not to jump at t_0 ? The answer is that P_{US} must jump at t_0 . This is the only way that M/P_{US} can drop, given that M does not drop. This explains the price path depicted in Figure 15-1 (c) on page 397.
- v. The effect of all this on the exchange rate can be determined from PPP. First, since P_{US} jumps at t_0 , then E must too, in the same proportion. Second, since the growth rate of P_{US} jumps by $\Delta\pi$, the growth rate of E must jump by the same amount, according to *PPP*.

In sum, the increased money growth induces an equal increase in inflation and in the rate of depreciation of the currency. It also induces an immediate jump in the price level and immediate depreciation of the currency.