

Professor Christiano
C-11, Winter 1999

Notes to Answers to Final Exam

1. Suppose the production function has the following form:

$$f(K, L) = K^{\frac{1}{3}}L^{\frac{2}{3}}.$$

- (a) Under the neoclassical theory of income distribution, the real wage and the real rent on capital are equated to the marginal productivity of labor and capital, respectively. Explain the role in this of the assumption of perfect competition.

Answer: profits are maximized when marginal productivity equals the real wage (or real rental rate) when the real wage, i.e., W and P , are taken as given by the firm (or, r in the case of capital). These variables are taken as given only under perfect competition.

- (b) Under the neoclassical theory of income distribution, show that the real income of workers plus the real income of owners of capital equals total output.

Answer: should write $(W/P)L + (r/P)K$, equate real factor prices to marginal productivities and then show that the sum is $K^{\frac{1}{3}}L^{\frac{2}{3}}$.

- (c) Under the neoclassical theory of income distribution, what is the share of total income going to labor? Carefully explain your result.

Answer: $2/3$, should be derived as $(W/P)L/Y = 2/3$, after equating W/P to marginal product of labor.

- (d) Suppose there is a sudden increase in the population. (Examples include the influx of Portuguese nationals into Portugal when it gave up its colonies, or the influx of Russians into Israel.) What happens to the real wage rate? What happens to the real rent on capital?

Answer: the real wage falls and the real rental rate on capital rises. That can be seen from the marginal productivity formulas: $MPL = (2/3)(K/L)^{1/3}$.

2. Some have proposed that we ought to have a balanced budget amendment. Explain the pros and cons of this proposition.

Answer: con - this restricts the government's ability to deal with adverse shocks. pro - some argue that because of the nature of the political system, there is a type of time consistency problem 'today we spend, tomorrow we tax'. This threatens to put us into a 'spend, spend, spend...' situation. The balanced budget amendment is supposed to shift us into a better situation.

3. Following is the neoclassical (or, Solow) growth model:

$$\begin{aligned} Y &= K^\alpha L^{1-\alpha}, \quad 0 < \alpha < 1, \\ \text{Household Saving} &= s\text{Income} \\ \Delta k &= i - \delta k, \end{aligned}$$

where $0 < \delta < 1$ is the depreciation rate on capital, and i is per capita investment. The economy starts in a long-run equilibrium, with no population growth and no technology growth.

- (a) Suppose the saving rate increases. Explain carefully, what happens over time. Where is the new long-run equilibrium?

Answer: same analysis as in class. The rise in saving rate initially causes investment to exceed depreciation, so the stock of capital begins to rise. Eventually, you converge to a new, higher equilibrium. This answer should be accompanied by graphs.

- (b) Suppose there is a jump in the population, say because of a huge wave of immigration. Suppose the new immigrants have the same saving rate as the people who were there initially. Explain carefully what happens to the amount of capital per person and output per person over time.

Answer: this analysis is formally identical to the case where a bunch of capital is destroyed. If the amount of capital per capita in the old, long run equilibrium, is $k = 2$ and the population doubles, then k drops suddenly to $k = 1$. There is no change in the saving rate, so the economy begins a slow climb - during which there is lots of growth - back up to where it was before. You have

to make an assumption about the saving rate of the immigrants, because that will impact on the post-immigration saving rate, and therefore, on the post-immigration long run equilibrium.

4. Consider the AD-AS model:

$$\begin{aligned}
 C^d &= c_0 + c_1(Y - T), \quad 0 < c_1 < 1. \\
 I^d &= \bar{I} - bi \\
 M^d &= PYL(i) \\
 L(i) &= \bar{L} - c_2i \\
 G^d &= \bar{G} \\
 T &= \bar{T} \\
 M^s &= \bar{M} \\
 \text{Pricing} &: P = (1 + \mu)W \\
 \text{Wage setting} &: W = P^e F(u, z),
 \end{aligned}$$

where \bar{T} , \bar{G} , \bar{L} , \bar{M} , \bar{I} , c_0 , c_1 , z , μ are exogenous variables, determined outside the AS-AD model. Also, u denotes the unemployment rate, which you may relate to Y as follows: $u = 1 - Y$. The remaining parts of the question present several different time patterns for the endogenous variables. Provide a well-reasoned guess about what exogenous variable is responsible.

(a) The price level rises and output falls. The interest rate rises.

Answer: that P and Y go in the same direction indicates that the supply curve caused the shift. This could reflect a move in z or μ . The rise in the interest rate is simply what one would expect, as the rise in the price level emanating from the supply side shifts the LM curve to the left. To figure out if it was z or μ , one would need to determine if there had been changes in social attitudes or in the nature of unemployment insurance (in the case of z) or changes in the competitive structure of the economy or in the price of some other factor of production (in the case of μ).

(b) The following variables all increase: P, I, i, Y, C .

Answer: That P and Y move in the same direction indicates it was an AD curve shift. That i and Y both increase suggest it was

an *IS* curve shift. That *I* moves up means it *must* be a rise in the investment function. Any other shifter for the *IS* curve would have caused investment to drop.

- (c) The following variables all increase: P, i, Y, C , but I falls.

Answer: the rise in P and Y indicates it was an *AD* curve shift. The rise in i and Y suggest it was an *IS* curve shift. But, several things could account for the rightward *IS* shift: a fall in T (get data on tax payments); a rise in G (get data on government expenditures); a rise in c_0 (get data on consumption, and see if it went up by more than can be accounted for simply by the rise in Y , holding c_0 and c_1 fixed.)

- (d) The following variables increase: P, Y, C, I , but i falls.

Answer: The rise in both P and Y indicates that the *AD* curve shifted right. The fact that Y moved up, but i fell suggests that the *LM* curve shifted right. This could have happened because M increase (get data on the money supply) or \bar{L} went down (get data on M/P and i and Y , and see if the relationship appears unusual).

5. Some have argued that the government should undertake policies that would make the society's saving rate equal to the 'Golden Rule' saving rate: the one which results in the highest long run equilibrium level of consumption.

- (a) What sort of things could the government do to influence the saving rate?

Answer: the government could provide tax incentives for people to save. Or, the government could in effect do the saving itself. It could tax people a lot and then lend the proceeds into financial markets, where it would be absorbed by investment, or it could just spend the money itself on investment projects.

- (b) Suppose the economy is in a long run equilibrium with a saving rate that is lower than the Golden Rule saving rate. It is proposed that the society's saving rate should be moved up to the Golden Rule rate. Explain carefully the case for and the case against this proposal.

Answer: Implementing the higher saving rate will result in a period of reduced consumption, followed by an eternity of higher consumption. The argument against this is that only one generation suffers, while countless generations benefit. Anyone would say this is the right thing, if they were asked *before* they knew which generation they would belong to. In practice, things don't work this way, however. The proposition is being considered after the current generation knows who it is, and they may not like it, despite the knowledge that future generations will benefit. After all, what did future generations ever do for the current generation?