

Notes on Financial Frictions Under Asymmetric Information and Costly State Verification

by

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Incorporating Financial Frictions into a Business Cycle Model

- General idea:
 - Standard model assumes borrowers and lenders are the same people..no conflict of interest
 - Financial friction models suppose borrowers and lenders are different people, with conflicting interests
 - Financial frictions: features of the relationship between borrowers and lenders adopted to mitigate conflict of interest.

Discussion of Financial Frictions

- Simple model to illustrate the basic costly state verification (csv) model.
 - Original analysis of Townsend (1978), Bernanke-Gertler.
- Integrating the csv model into a full-blown dsge model.
 - Follows the lead of Bernanke, Gertler and Gilchrist (1999).
 - Empirical analysis of Christiano, Motto and Rostagno (2003,2012).

Simple Model

- There are entrepreneurs with all different levels of wealth, N .
 - Entrepreneur have different levels of wealth because they experienced different idiosyncratic shocks in the past.
- For each value of N , there are many entrepreneurs.
- In what follows, we will consider the interaction between entrepreneurs with a specific amount of N with competitive banks.
- Later, will consider the whole population of entrepreneurs, with every possible level of N .

Simple Model, cont'd

- Each entrepreneur has access to a project with rate of return,

$$(1 + R^k)\omega$$

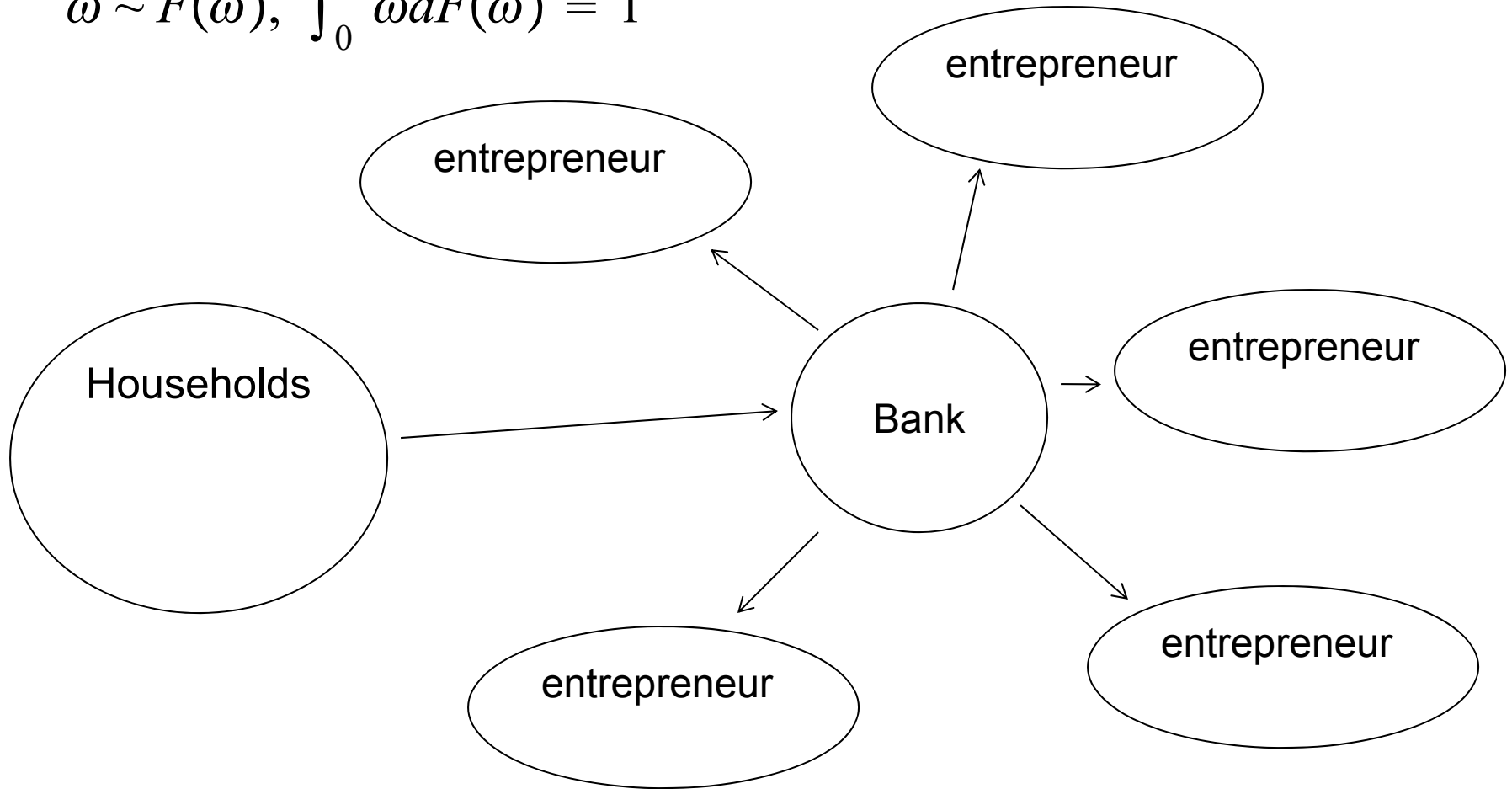
- Here, ω is a unit mean, idiosyncratic shock experienced by the individual entrepreneur after the project has been started,

$$\int_0^{\infty} \omega dF(\omega) = 1$$

- The shock, ω , is privately observed by the entrepreneur.
- F is lognormal cumulative distribution function.

Banks, Households, Entrepreneurs

$$\omega \sim F(\omega), \int_0^\infty \omega dF(\omega) = 1$$



Standard debt contract

- Entrepreneur receives a contract from a bank, which specifies a rate of interest, Z , and a loan amount, B .
 - If entrepreneur cannot make the interest payments, the bank pays a monitoring cost and takes everything.

- Total assets acquired by the entrepreneur:

$$\overbrace{A}^{\text{total assets}} = \overbrace{N}^{\text{net worth}} + \overbrace{B}^{\text{loans}}$$

- Entrepreneur who experiences sufficiently bad luck, $\omega \leq \bar{\omega}$, loses everything.

- Cutoff, $\bar{\omega}$

gross rate of return experience by entrepreneur with 'luck', $\bar{\omega}$ total assets

$$\overbrace{(1 + R^k)\bar{\omega}} \quad \times \quad \overbrace{A}$$

interest and principle owed by the entrepreneur

$$= \overbrace{ZB}$$

$$(1 + R^k)\bar{\omega}A = ZB \rightarrow$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{\frac{B}{N}}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{\overbrace{\frac{A}{N}}^{\text{leverage} = L} - 1}{\frac{A}{N}} = \frac{Z}{(1+R^k)} \frac{L-1}{L}$$

- Cutoff higher with:

- higher leverage, L
- higher $Z/(1 + R^k)$

- Expected return to entrepreneur from operating risky technology, over return from depositing net worth in bank:

$$\frac{\int_{\bar{\omega}}^{\infty} [(1+R^k)\omega A - ZB] dF(\omega)}{N(1+R)}$$

Expected payoff for entrepreneur

For lower values of ω , entrepreneur receives nothing 'limited liability'.

gain from depositing funds in bank ('opportunity cost of funds')

- Rewriting entrepreneur's rate of return:

$$\frac{\int_{\bar{\omega}}^{\infty} [(1 + R^k)\omega A - ZB] dF(\omega)}{N(1 + R)} = \frac{\int_{\bar{\omega}}^{\infty} [(1 + R^k)\omega A - (1 + R^k)\bar{\omega}A] dF(\omega)}{N(1 + R)}$$

$$= \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \left(\frac{1 + R^k}{1 + R} \right) L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{L-1}{L}$$

Gets smaller with L

Larger with L

- Rewriting entrepreneur's rate of return:

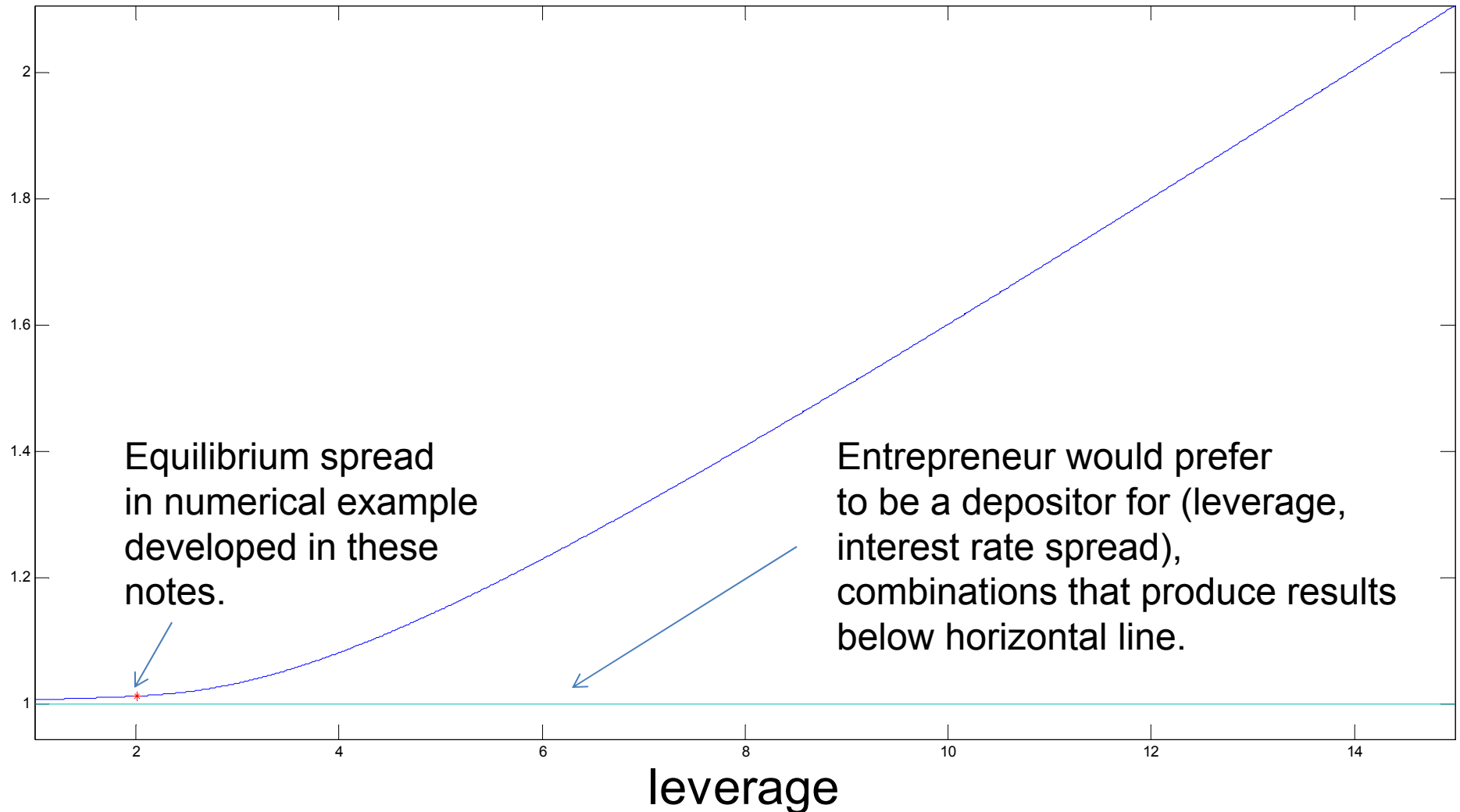
$$\frac{\int_{\bar{\omega}}^{\infty} [(1 + R^k)\omega A - ZB] dF(\omega)}{N(1 + R)} = \frac{\int_{\bar{\omega}}^{\infty} [(1 + R^k)\omega A - (1 + R^k)\bar{\omega}A] dF(\omega)}{N(1 + R)}$$

$$= \int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \left(\frac{1 + R^k}{1 + R} \right) L$$

$$\bar{\omega} = \frac{Z}{(1+R^k)} \frac{L-1}{L} \rightarrow_{L \rightarrow \infty} \frac{Z}{(1+R^k)}$$

- Entrepreneur's return unbounded above
 - Risk neutral entrepreneur would always want to borrow an infinite amount (infinite leverage).

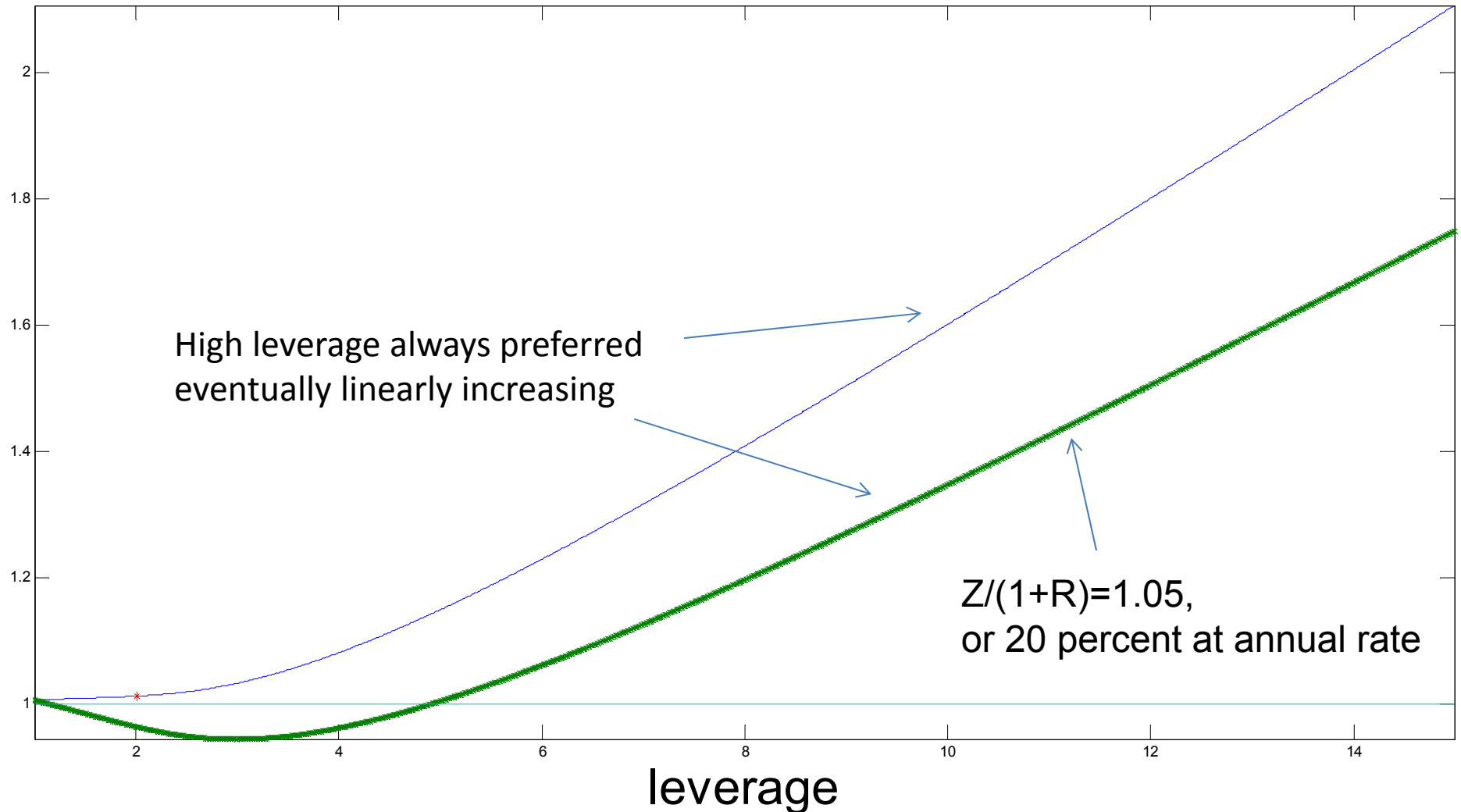
Expected entrepreneurial return, over opportunity cost, $N(1+R)$



Interest rate spread, $Z/(1+R)$, = 1.0016, or 0.63 percent at annual rate $\sigma = 0.26$

Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

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Return spread, $(1+R^k)/(1+R)$, = 1.0073, or 2.90 percent at annual rate

- If given a fixed interest rate, entrepreneur with risk neutral preferences would borrow an unbounded amount.
- In equilibrium, bank can't lend an infinite amount.
- This is why a loan contract must specify *both* an interest rate, Z , and a loan amount, B .

Simplified Representation of Entrepreneur Utility

- Utility:

$$\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1 + R^k}{1 + R} L$$

$$= [1 - \Gamma(\bar{\omega})] \frac{1 + R^k}{1 + R} L$$

- Where

$$\Gamma(\bar{\omega}) \equiv \bar{\omega}(1 - F(\bar{\omega})) + G(\bar{\omega})$$

$$G(\bar{\omega}) \equiv \int_0^{\bar{\omega}} \omega dF(\omega)$$

Share of gross entrepreneurial earnings kept by entrepreneur

- Easy to show: $0 \leq \Gamma(\bar{\omega}) \leq 1$

$$\Gamma'(\bar{\omega}) = 1 - F(\bar{\omega}) > 0, \Gamma''(\bar{\omega}) < 0$$

$$\lim_{\bar{\omega} \rightarrow 0} \Gamma(\bar{\omega}) = 0, \lim_{\bar{\omega} \rightarrow \infty} \Gamma(\bar{\omega}) = 0$$

$$\lim_{\bar{\omega} \rightarrow 0} G(\bar{\omega}) = 0, \lim_{\bar{\omega} \rightarrow \infty} G(\bar{\omega}) = 1$$

Banks

- Source of funds from households, at fixed rate, R
- Bank borrows B units of currency, lends proceeds to entrepreneurs.
- Provides entrepreneurs with standard debt contract, (Z, B)

Banks, cont'd

- Monitoring cost for bankrupt entrepreneur

with $\omega < \bar{\omega}$

Bankruptcy cost parameter

$$\mu(1 + R^k)\omega A$$

- Bank zero profit condition

fraction of entrepreneurs with $\omega > \bar{\omega}$

quantity paid by each entrepreneur with $\omega > \bar{\omega}$

$$\overbrace{[1 - F(\bar{\omega})]}$$

$$\overbrace{ZB}$$

quantity recovered by bank from each bankrupt entrepreneur

$$+ \overbrace{(1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) (1 + R^k) A}$$

amount owed to households by bank

$$= \overbrace{(1 + R)B}$$

Banks, cont'd

- Zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

$$\frac{[1 - F(\bar{\omega})]ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A}{B} = (1 + R)$$

The risk free interest rate here is equated to the ‘average return on entrepreneurial projects’.

This is a source of inefficiency in the model. A benevolent planner would prefer that the market price savers correspond to the *marginal* return on projects (Christiano-Ikeda).

Banks, cont'd

- Simplifying zero profit condition:

$$[1 - F(\bar{\omega})]ZB + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$



$$[1 - F(\bar{\omega})]\bar{\omega}(1 + R^k)A + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)(1 + R^k)A = (1 + R)B$$

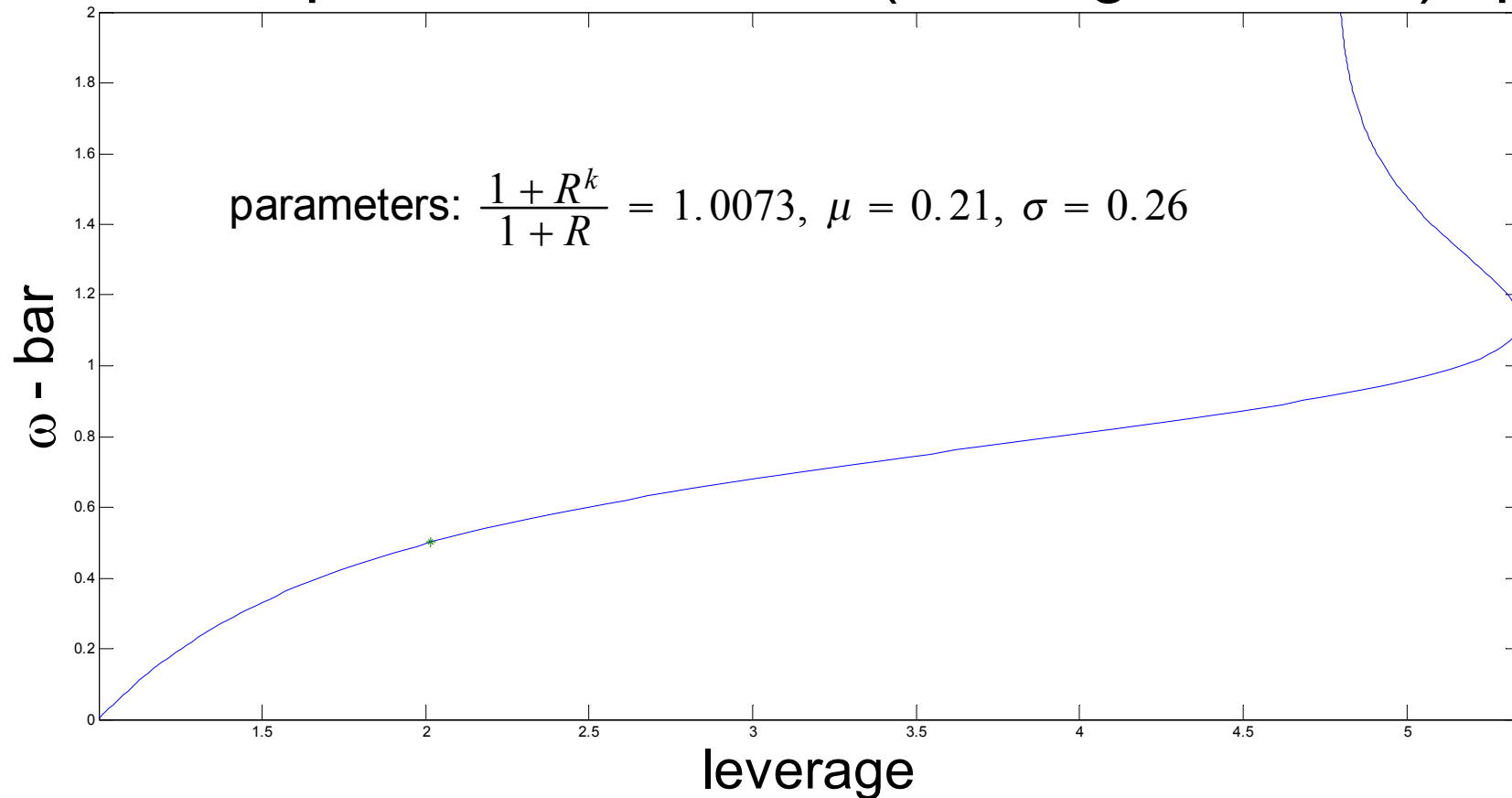
share of gross return, $(1+R^k)A$, (net of monitoring costs) given to bank

$$\overbrace{\left([1 - F(\bar{\omega})]\bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) \right)} \quad (1 + R^k)A = (1 + R)B$$

$$\begin{aligned} [1 - F(\bar{\omega})]\bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega) &= \frac{1 + R}{1 + R^k} \frac{B/N}{A/N} \\ &= \frac{1 + R}{1 + R^k} \frac{L - 1}{L} \end{aligned}$$

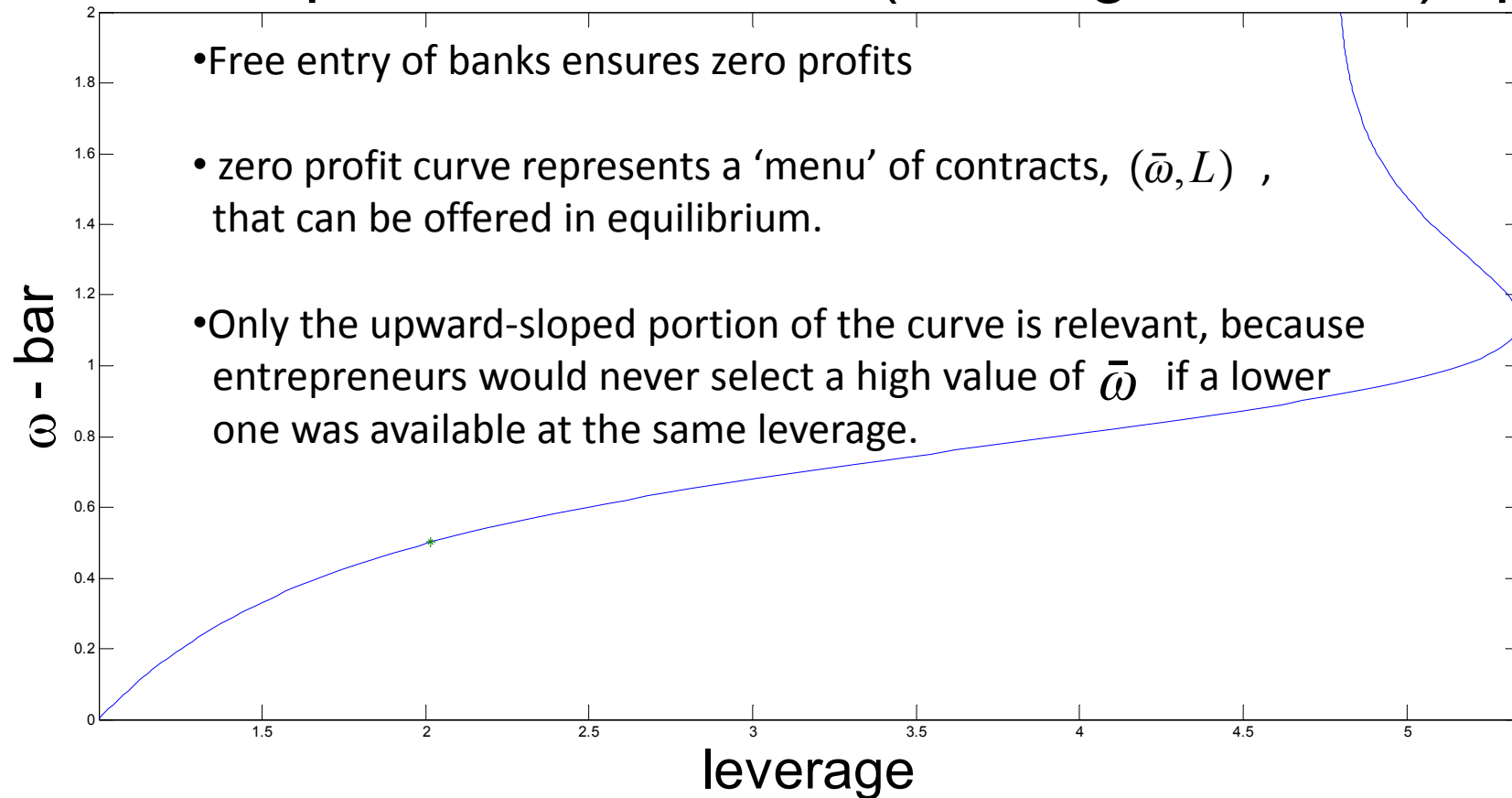
Expressed naturally in terms of $(\bar{\omega}, L)$

Bank zero profit condition, in (leverage, $\bar{\omega}$) space



Our value of $\frac{1+R^k}{1+R}$, 290 basis points at an annual rate, is a little higher than the 200 basis point value adopted in BGG (1999, p. 1368); the value of μ is higher than the one adopted by BGG, but within the range, 0.20-0.36 defended by Carlstrom and Fuerst (AER, 1997) as empirically relevant; the value of $Var(\log \omega)$ is nearly the same as the 0.28 value assumed by BGG (1999,p.1368).

Bank zero profit condition, in (leverage, $\bar{\omega}$) space



Expressing Zero Profit Condition In Terms of New Notation

share of entrepreneurial profits (net of monitoring costs) given to bank

$$\overbrace{(1 - F(\bar{\omega}))\bar{\omega} + (1 - \mu) \int_0^{\bar{\omega}} \omega dF(\omega)} = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}$$

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = \frac{1 + R}{1 + R^k} \frac{L - 1}{L}$$

$$L = \frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

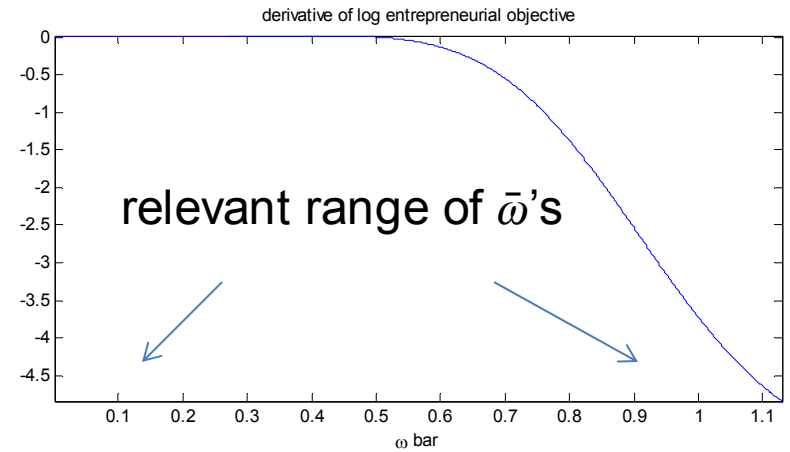
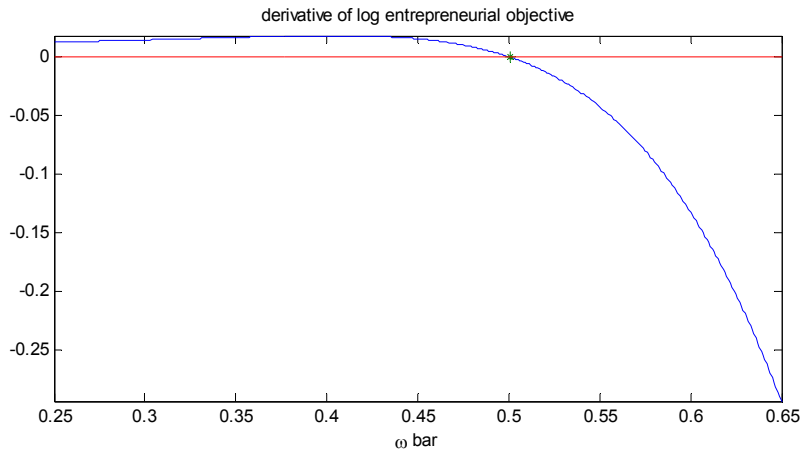
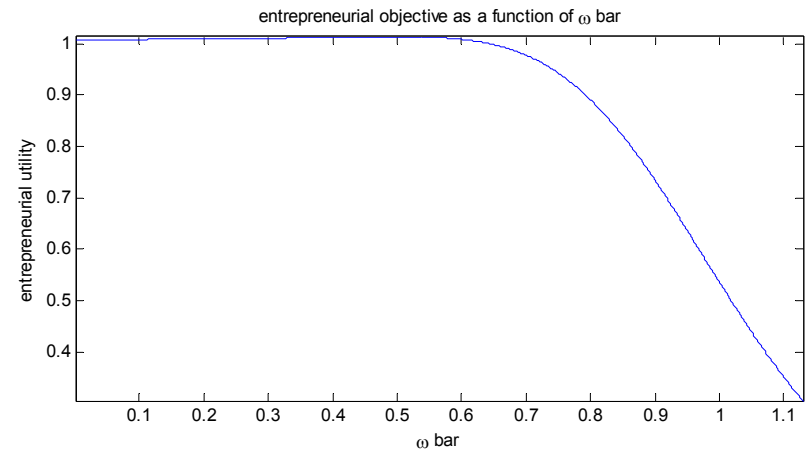
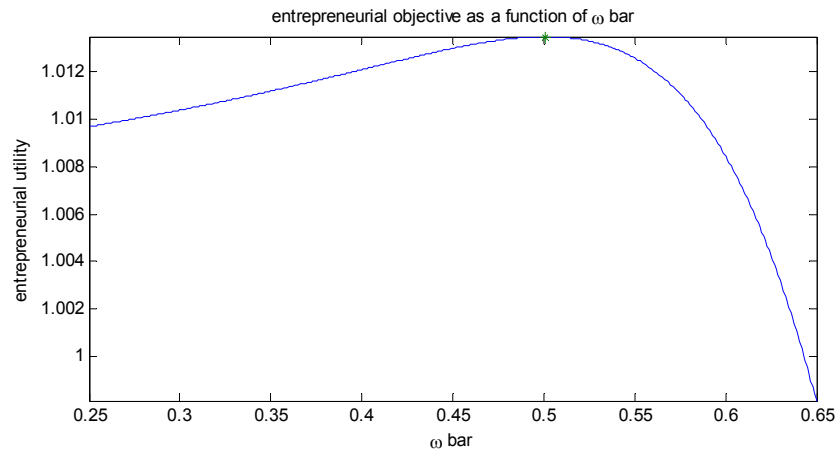
Equilibrium Contract

- Entrepreneur selects the contract is optimal, given the available menu of contracts.
- The solution to the entrepreneur problem is the $\bar{\omega}$ that maximizes, over the relevant domain (i.e., $\bar{\omega} \in [0, 1.13]$ in the example):

$$\log \left\{ \overbrace{\int_{\bar{\omega}}^{\infty} [\omega - \bar{\omega}] dF(\omega) \frac{1+R^k}{1+R}}^{\text{profits, per unit of leverage, earned by entrepreneur, given } \bar{\omega}} \times \overbrace{\frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}}^{\text{leverage offered by bank, conditional on } \bar{\omega}} \right\}$$

$$= \log \underbrace{\left[1 - \Gamma(\bar{\omega}) \right]}_{\text{higher } \bar{\omega} \text{ drives share of profits to entrepreneur down (bad!)}} + \log \frac{1+R^k}{1+R} \overbrace{\left(-\log \left(1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})] \right) \right)}^{\text{higher } \bar{\omega} \text{ drives leverage up (good!)}}$$

Entrepreneur Objective



Computing the Equilibrium Contract

- Solve first order optimality condition uniquely for the cutoff, $\bar{\omega}$:

$$\frac{\overbrace{1 - F(\bar{\omega})}^{\text{elasticity of entrepreneur's expected return w.r.t. } \bar{\omega}}}{1 - \Gamma(\bar{\omega})} = \frac{\overbrace{\frac{1+R^k}{1+R} [1 - F(\bar{\omega}) - \mu\bar{\omega}F'(\bar{\omega})]}^{\text{elasticity of leverage w.r.t. } \bar{\omega}}}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

- Given the cutoff, solve for leverage:

$$L = \frac{1}{1 - \frac{1+R^k}{1+R} [\Gamma(\bar{\omega}) - \mu G(\bar{\omega})]}$$

- Given leverage and cutoff, solve for risk spread:

$$\text{risk spread} \equiv \frac{Z}{1+R} = \frac{1+R^k}{1+R} \bar{\omega} \frac{L}{L-1}$$

Result

- Leverage, L , and entrepreneurial rate of interest, Z , **not a function of net worth, N .**
- Quantity of loans proportional to net worth:

$$L = \frac{A}{N} = \frac{N+B}{N} = 1 + \frac{B}{N}$$

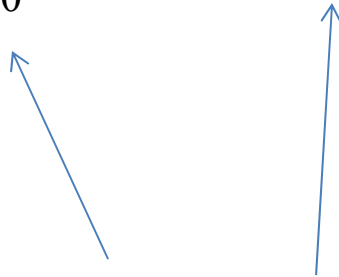
$$B = (L - 1)N$$

- To compute L , $Z/(1+R)$, must make assumptions about F and parameters.

$$\frac{1 + R^k}{1 + R}, \mu, F$$

Formulas Needed to do the Computations

- Need: $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega dF(\omega), F'(\omega)$



Can get these from the pdf and the cdf of the standard normal distribution.

These are available in most computational software, like MATLAB.

Also, they have simple analytic representations.

Numerical Example

- Parameters: Percent of average product of entrepreneurial Projects, absorbed by monitoring costs: 0.06%

$$\frac{1 + R^k}{1 + R} = 1.0073, \sigma = 0.26, \mu = 0.21$$

- (Micro) equilibrium quantities:

$\overline{\omega} = 0.50$, $\Gamma(\overline{\omega}) = 0.5008$, $F(\overline{\omega}) = 0.0056$, $G(\overline{\omega}) = 0.0026$,

$L = 2.02$, $\frac{Z}{R} = 1.0015$, $[1 - \Gamma(\overline{\omega})] \frac{1 + R^k}{1 + R} L = 1.0135 > 1$

cutoff ω fraction of gross entrepreneurial earnings going to lender bankruptcy rate: 0.56% average ω among bankrupt entrepreneurs
 leverage interest rate spread 0.62 (APR) avg earnings of entrepreneur, divided by opportunity cost

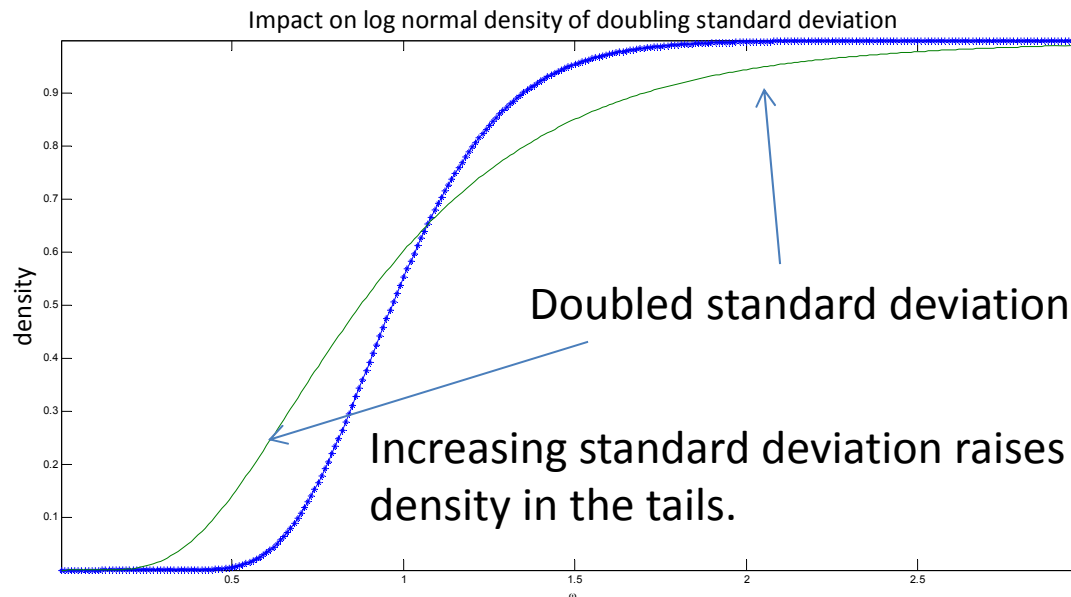
- Note: on average, entrepreneur better off leveraging net worth and investing in project, rather than depositing net worth in bank.

Effect of Increase in Risk, σ

- Keep

$$\int_0^{\infty} \omega dF(\omega) = 1$$

- But, double standard deviation of Normal underlying F .



Jump in Risk

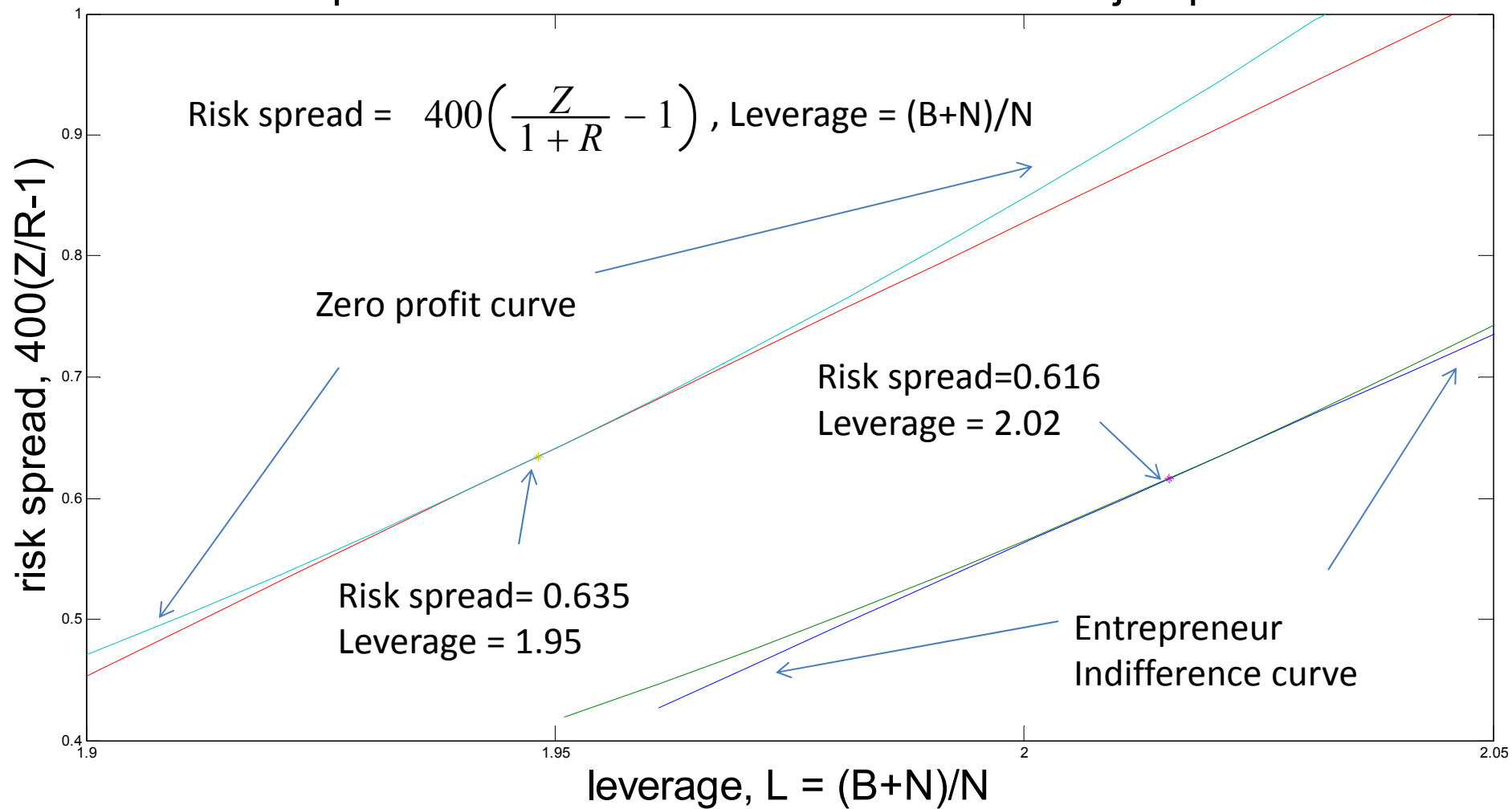
- σ replaced by $\sigma \times 3$

$$\begin{array}{ccccccc}
 \text{cutoff } \omega & \text{fraction of gross entrepreneurial earnings going to lender} & \text{bankruptcy rate: 1.08\%} & \text{average } \omega \text{ among bankrupt entrepreneurs} & & & \\
 \overbrace{\bar{\omega} = 0.12,} & \overbrace{\Gamma(\bar{\omega}) = 0.12} & , \overbrace{F(\bar{\omega}) = 0.0108,} & \overbrace{G(\bar{\omega}) = 0.0011} & , & & \\
 & \text{leverage} & \text{interest rate spread} & \text{1.66 (APR)} & \text{avg earnings of entrepreneur, per unit of net worth} & & \\
 & \overbrace{L = 1.1418,} & \overbrace{\frac{Z}{R}} & = \overbrace{1.0041,} & \overbrace{[1 - \Gamma(\bar{\omega})] \frac{1+R^k}{1+R} L = 1.0080} & > 1 &
 \end{array}$$

- Comparison with benchmark:

$$\begin{array}{ccccccc}
 \text{cutoff } \omega & \text{fraction of gross entrepreneurial earnings going to lender} & \text{bankruptcy rate: 0.56\%} & \text{average } \omega \text{ among bankrupt entrepreneurs} & & & \\
 \overbrace{\bar{\omega} = 0.50,} & \overbrace{\Gamma(\bar{\omega}) = 0.5008} & , \overbrace{F(\bar{\omega}) = 0.0056,} & \overbrace{G(\bar{\omega}) = 0.0026} & , & & \\
 & \text{leverage} & \text{interest rate spread} & \text{0.62 (APR)} & \text{avg earnings of entrepreneur, divided by opportunity cost} & & \\
 & \overbrace{L = 2.02,} & \overbrace{\frac{Z}{R}} & = \overbrace{1.0015,} & \overbrace{[1 - \Gamma(\bar{\omega})] \frac{1+R^k}{1+R} L = 1.0135} & > 1 &
 \end{array}$$

Impact on standard debt contract of a 5% jump in σ



Incorporating CSV Financial Frictions into Neoclassical Model

- Bernanke, Gertler and Gilchrist, 1999
Handbook of Macroeconomics Chapter.
- Outline
 - Broad overview of model.
 - Details of entrepreneur
 - Must take into account the heterogeneity of entrepreneurs by net worth, N .
 - Describe relationship of entrepreneur to household
 - Describe household and general equilibrium.

Standard, Neoclassical Model

$$\max_{\{c_t, B_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = \log(c_t)$$

subject to:

$$c_t + B_{t+1} + K_{t+1} - (1 - \delta)K_t \leq w_t l_t + r_t K_t + (1 + R_{t-1})B_t$$

$$0 \leq l_t \leq 1$$

Optimization:

$$u'(c_t) = \beta u'(c_{t+1})(r_{t+1} + 1 - \delta),$$

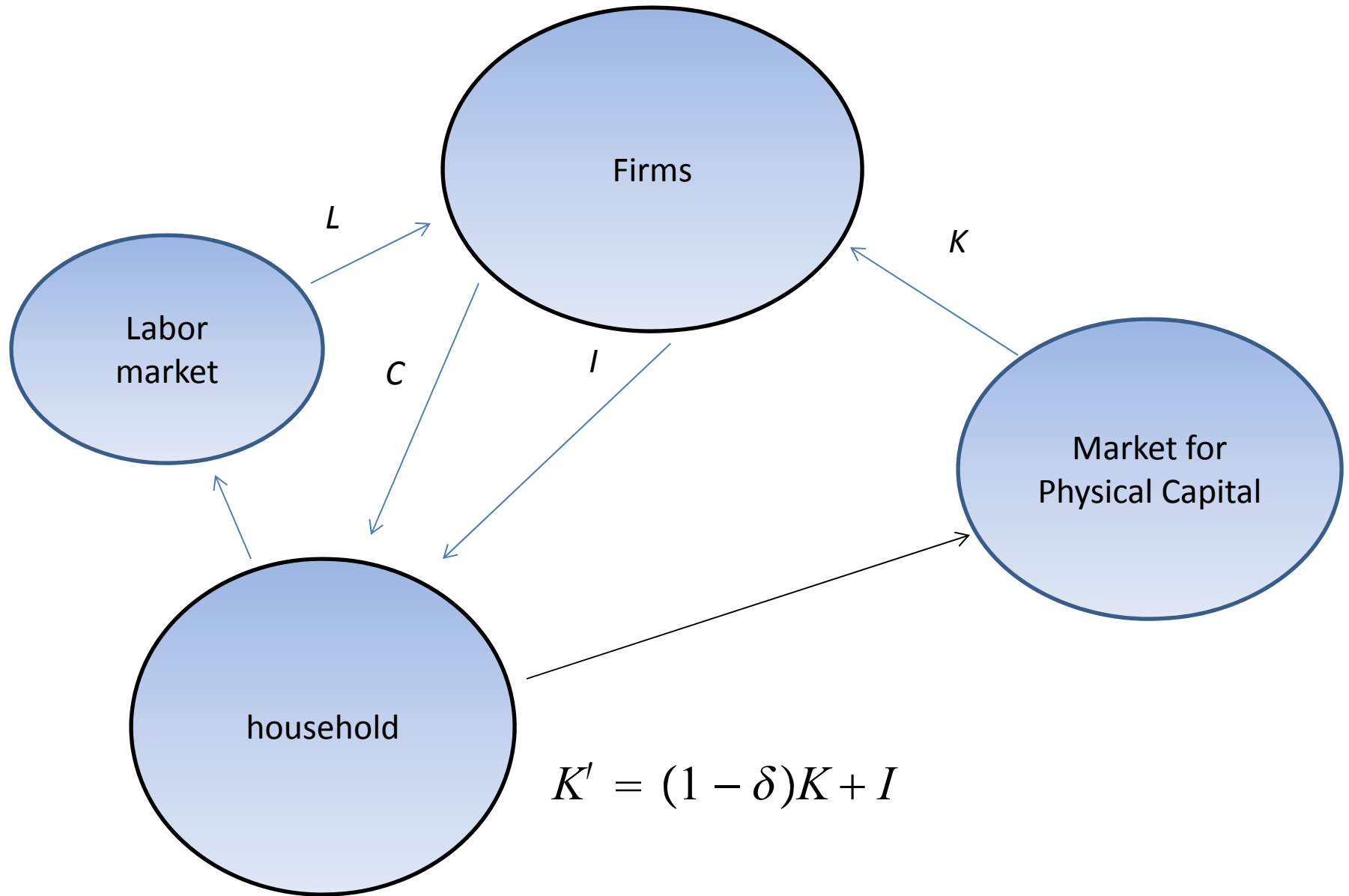
$$u'(c_t) = \beta u'(c_{t+1})(1 + R_t)$$

$$l_t = 1$$

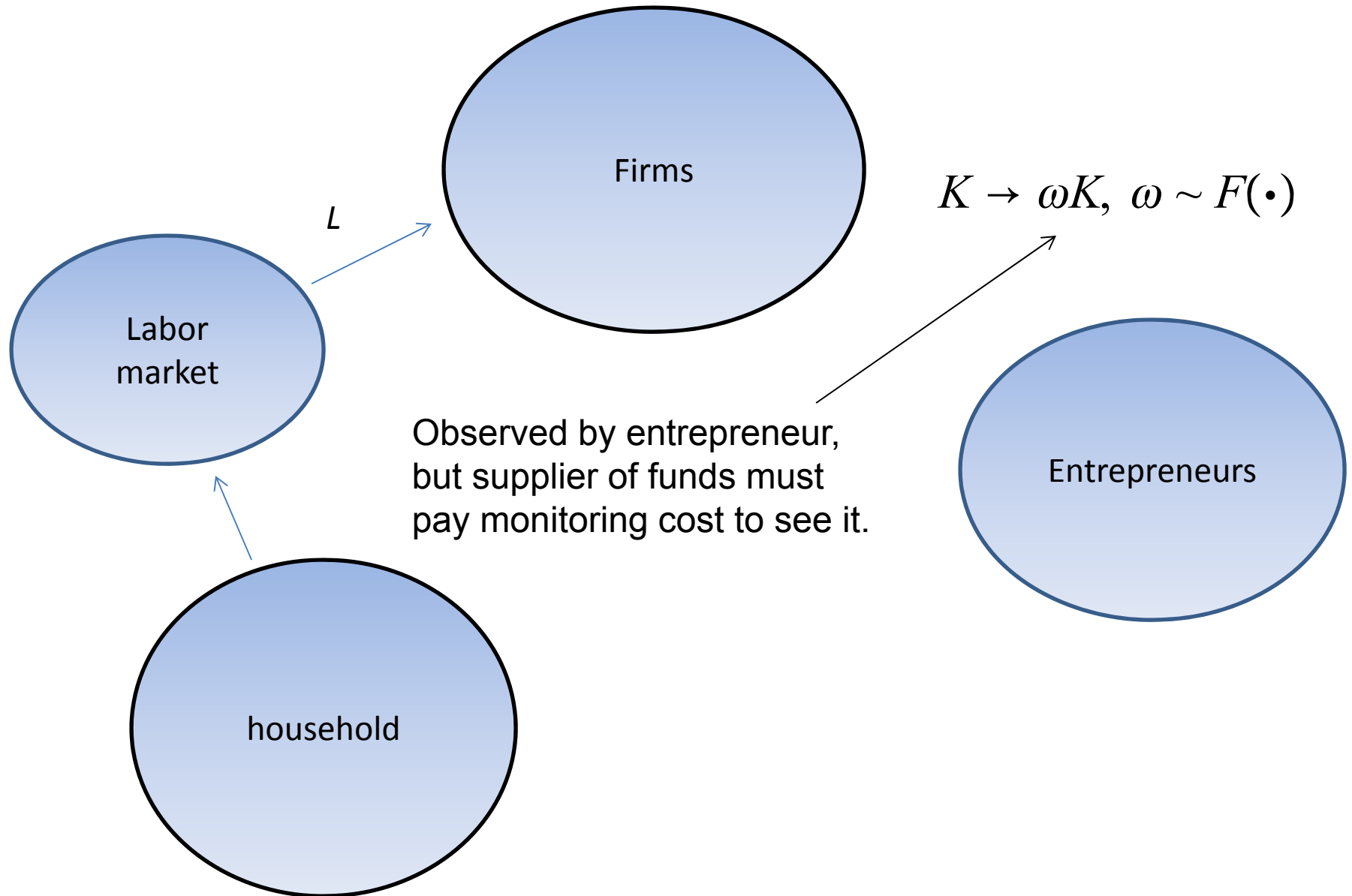
Firms and market clearing:

$$c_t + I_t = K_t^\alpha l_t^{1-\alpha}, \quad B_{t+1} = 0, \quad w_t = (1 - \alpha)K_t^\alpha, \quad r_t = \alpha K_t^{\alpha-1}$$

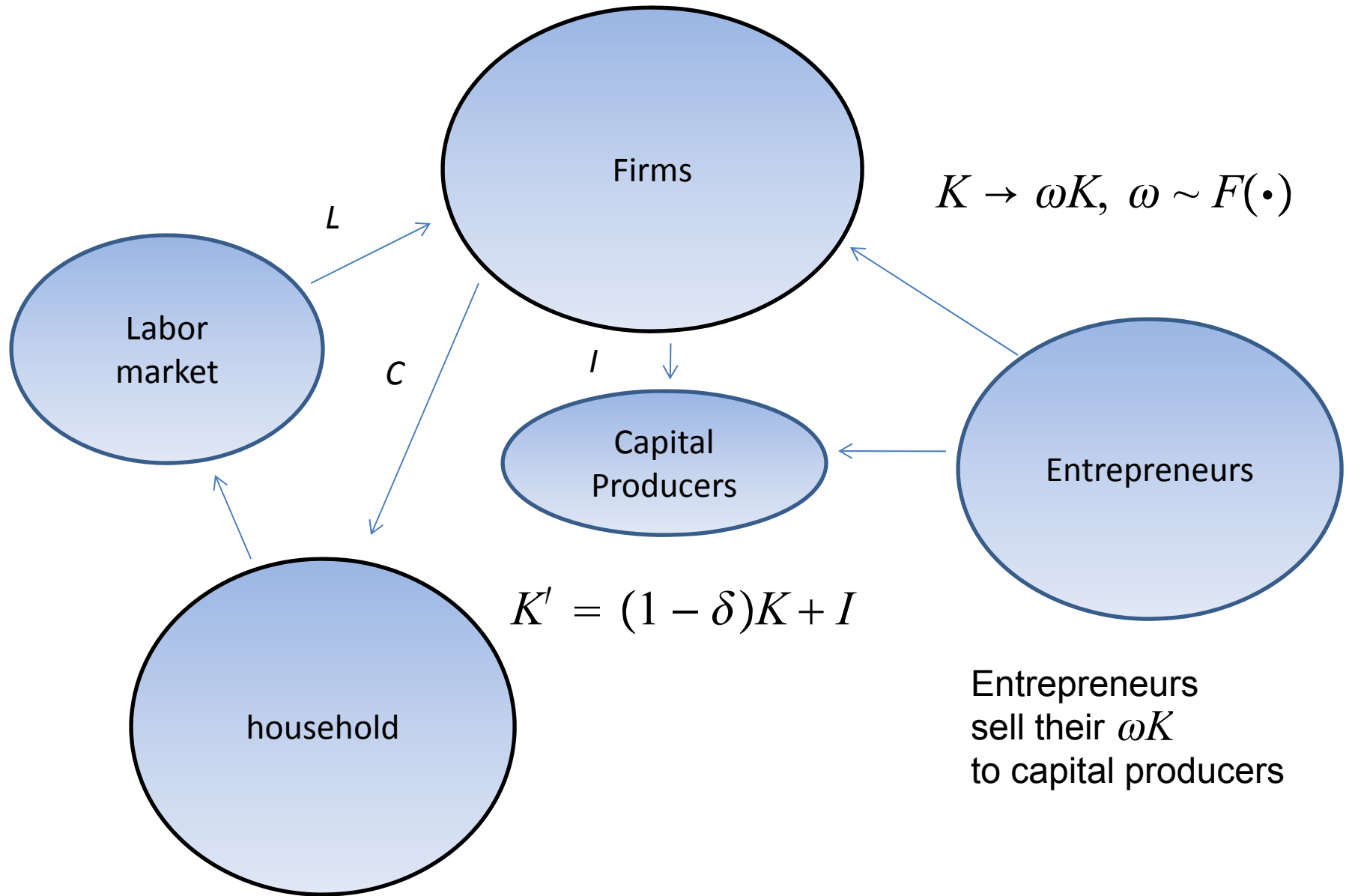
Standard Model



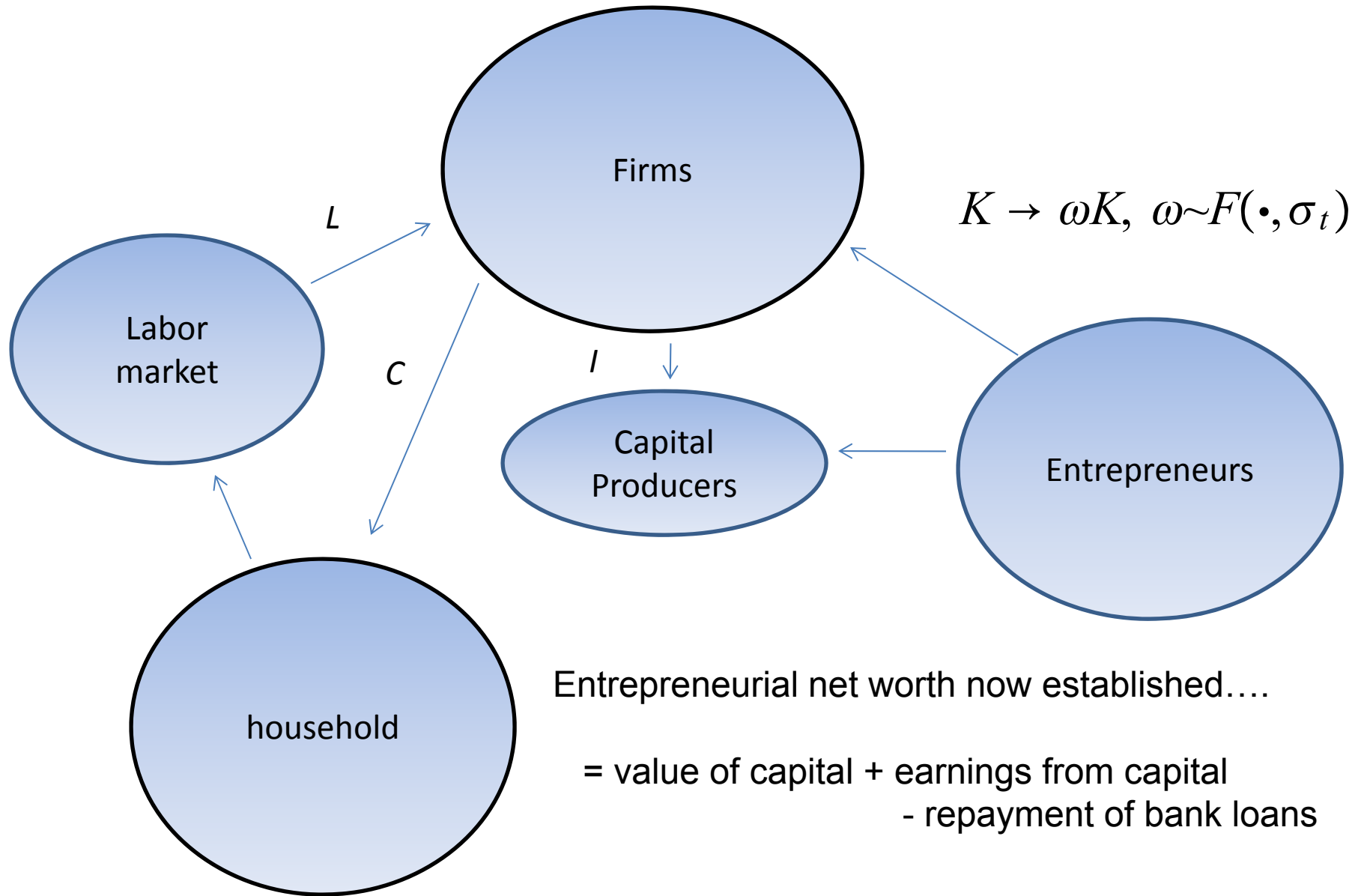
Standard Model with CSV



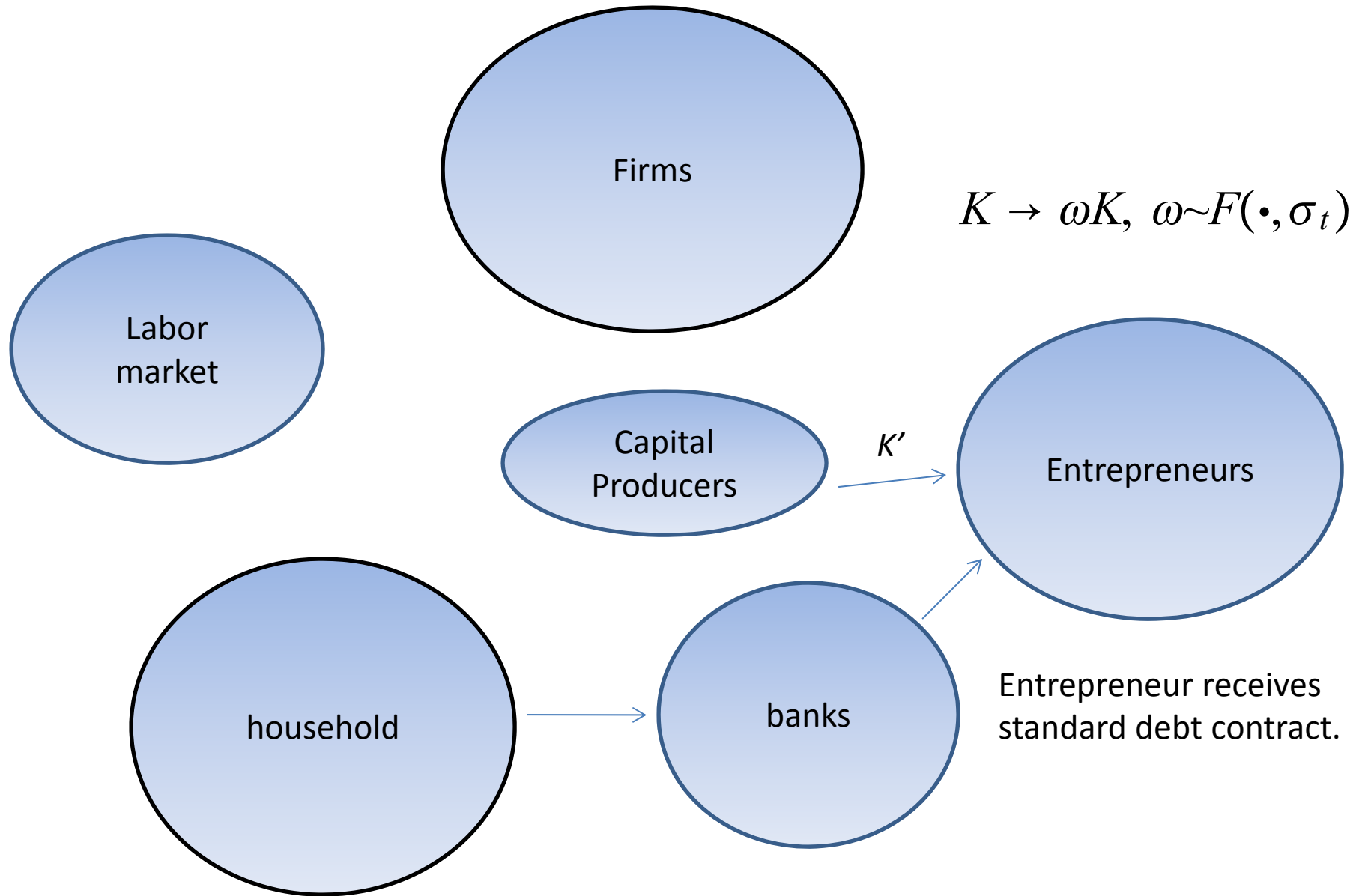
Standard Model with CSV



Standard Model with CSV



Standard Model with CSV



Details About the Entrepreneur

- Begin after period t production, when entrepreneurial net worth is known, and they go to banks for loans.
- End at the point in $t+1$ when net worth in $t+1$ is determined.

Entrepreneur at end of t

- Let $f_t(N)$ denote the number (density, actually) of entrepreneurs with net worth, N , $N \geq 0$.
- An entrepreneur with net worth, N , goes to the bank, receives a loan, B_{t+1}^N , and buys raw, physical capital:

$$K_{t+1}^N = N + B_{t+1}^N$$

- After purchasing the capital, the entrepreneur experiences an idiosyncratic shock, ω , so that physical capital is transformed into effective capital as follows:

$$\omega K_{t+1}^N, \quad \omega \sim F(\cdot, \sigma_t)$$

Entrepreneur in $t+1$

- The entrepreneur with net worth N in t rents

$$\omega K_{t+1}^N$$

in a competitive capital rental market for rental rate, r_{t+1} .

- After goods production, entrepreneur sells undepreciated capital, $\omega K_{t+1}^N (1 - \delta)$, at price unity.

- So, rate of return on capital for entrepreneur is:


$$\omega(1 + R_{t+1}^k), \quad 1 + R_{t+1}^k \equiv r_{t+1} + 1 - \delta$$

Constant rate of return project, just like in micro example

Entrepreneur in $t+1$, cnt'd

- Period $t+1$ net worth of entrepreneur with net worth N in period t and who experiences shock ω :

Gross rate of interest on loan

$$N' = \max\{0, (1 + R_{t+1}^k)K_{t+1}^N \omega - Z_{t+1}B_{t+1}^N\}$$


Entrepreneurial loan contract in t

- The banking system in period t is competitive.
 - The zero profit condition must be satisfied:

$$\text{Leverage} \equiv \frac{K_{t+1}^N}{N} = \frac{1}{1 - \frac{1+R_{t+1}^k}{1+R_t} [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]}$$

– Here:

$$(1 + R_{t+1}^k) K_{t+1}^N \bar{\omega}_{t+1} = Z_{t+1} B_{t+1}^N.$$

- Entrepreneurs treat the zero profit condition as a ‘menu’ of contracts. They select the contract at time t that maximizes expected N' given N .
- Entrepreneurial efficiency condition:

$$\frac{1 - F(\bar{\omega}_{t+1})}{1 - \Gamma(\bar{\omega}_{t+1})} = \frac{\frac{1+R_{t+1}^k}{1+R_t} [1 - F(\bar{\omega}_{t+1}) - \mu \bar{\omega}_{t+1} F'(\bar{\omega}_{t+1})]}{1 - \frac{1+R_{t+1}^k}{1+R_t} [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]}$$

Aggregate Net Worth, Loans and Capital

- The total amount of net worth held by all entrepreneurs at the end of time t :

$$N_{t+1} = \int_0^{\infty} N f_t(N) dN$$

- Total effective capital supplied by all entrepreneurs during $t+1$:

$$\int_0^{\infty} \int_0^{\infty} K_{t+1}^N \omega f_t(N) dF(\omega) dN$$

Must aggregate over all N types and all ω types



Aggregate Net Worth, Loans and Capital

- The total amount of net worth held by all entrepreneurs at the end of time t :

$$N_{t+1} = \int_0^{\infty} N f_t(N) dN$$

- Total effective capital supplied by all entrepreneurs during $t+1$:

$$\int_0^{\infty} \int_0^{\infty} K_{t+1}^N \omega f_t(N) dF(\omega) dN$$

total effective capital of all entrepreneurs with net worth, N

$$= \int_0^{\infty} \underbrace{\left[\int_0^{\infty} K_{t+1}^N \omega dF(\omega) \right]}_{\text{total effective capital of all entrepreneurs with net worth, } N} f_t(N) dN$$

This is just K_{t+1}^N because an entrepreneur with net worth, N , observes ω after selecting the loan contract, and, hence, the quantity of capital purchased.

Aggregate Net Worth, Loans and Capital

- The total amount of net worth held by all entrepreneurs at the end of time t :

$$N_{t+1} = \int_0^{\infty} N f_t(N) dN$$

- Total effective capital supplied by all entrepreneurs during $t+1$:

$$\int_0^{\infty} \int_0^{\infty} K_{t+1}^N \omega f_t(N) dF(\omega) dN$$

total effective capital of all entrepreneurs with net worth, N

$$= \int_0^{\infty} \underbrace{\left[\int_0^{\infty} K_{t+1}^N \omega dF(\omega) \right]}_{\text{total effective capital for all entrepreneurs}} f_t(N) dN$$

$$\text{total effective capital for all entrepreneurs} = \int_0^{\infty} K_{t+1}^N f_t(N) dN$$

Aggregates, cont'd

- Aggregate capital:

$$\begin{aligned}
 K_{t+1} &= \int_0^\infty \int_0^\infty K_{t+1}^N \omega f_t(N) dF(\omega) dN = \int_0^\infty K_{t+1}^N f_t(N) dN \\
 &= \frac{1}{1 - \frac{1+R_{t+1}^k}{1+R_t} [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]} \int_0^\infty N f_t(N) dN \\
 &= \frac{1}{1 - \frac{1+R_{t+1}^k}{1+R_t} [\Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})]} N_{t+1}
 \end{aligned}$$

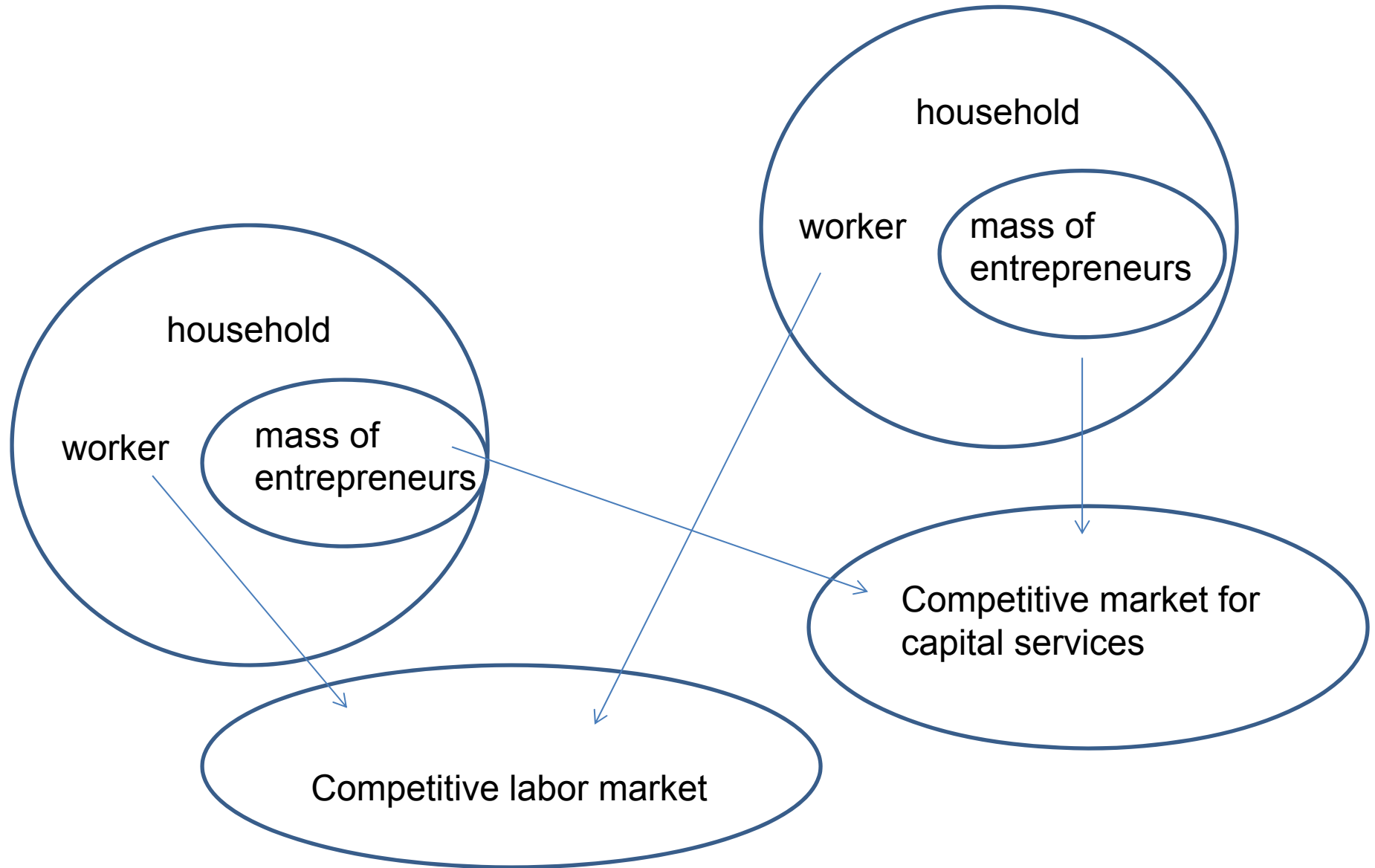
- Aggregate net worth in $t+1$:

$$\begin{aligned}
 &\int_0^\infty \left[\int_0^\infty \max\{0, (1 + R_{t+1}^k) K_{t+1}^N \omega - Z_{t+1} B_{t+1}^N\} dF(\omega) \right] f_t(N) dN \\
 &= [1 - \Gamma(\bar{\omega}_{t+1})] (1 + R_{t+1}^k) K_{t+1}
 \end{aligned}$$

Relationship of Entrepreneur to Household

- Simplest assumption, which avoids a lot of complications, is the ‘large family assumption’.
- There are many identical households
 - Each has a worker.
 - Each has many entrepreneurs....enough so that average net worth in the representative family is always equal to average net worth in the economy as a whole.
 - Entrepreneurs receive perfect consumption insurance from the household:
 - Entrepreneurs and the worker all consume the same amount, C_t .

Large Family Assumption



Entrepreneurs and Households, cnt'd

- Aggregate net worth of all entrepreneurs, after accounting for all their income in $t+1$.

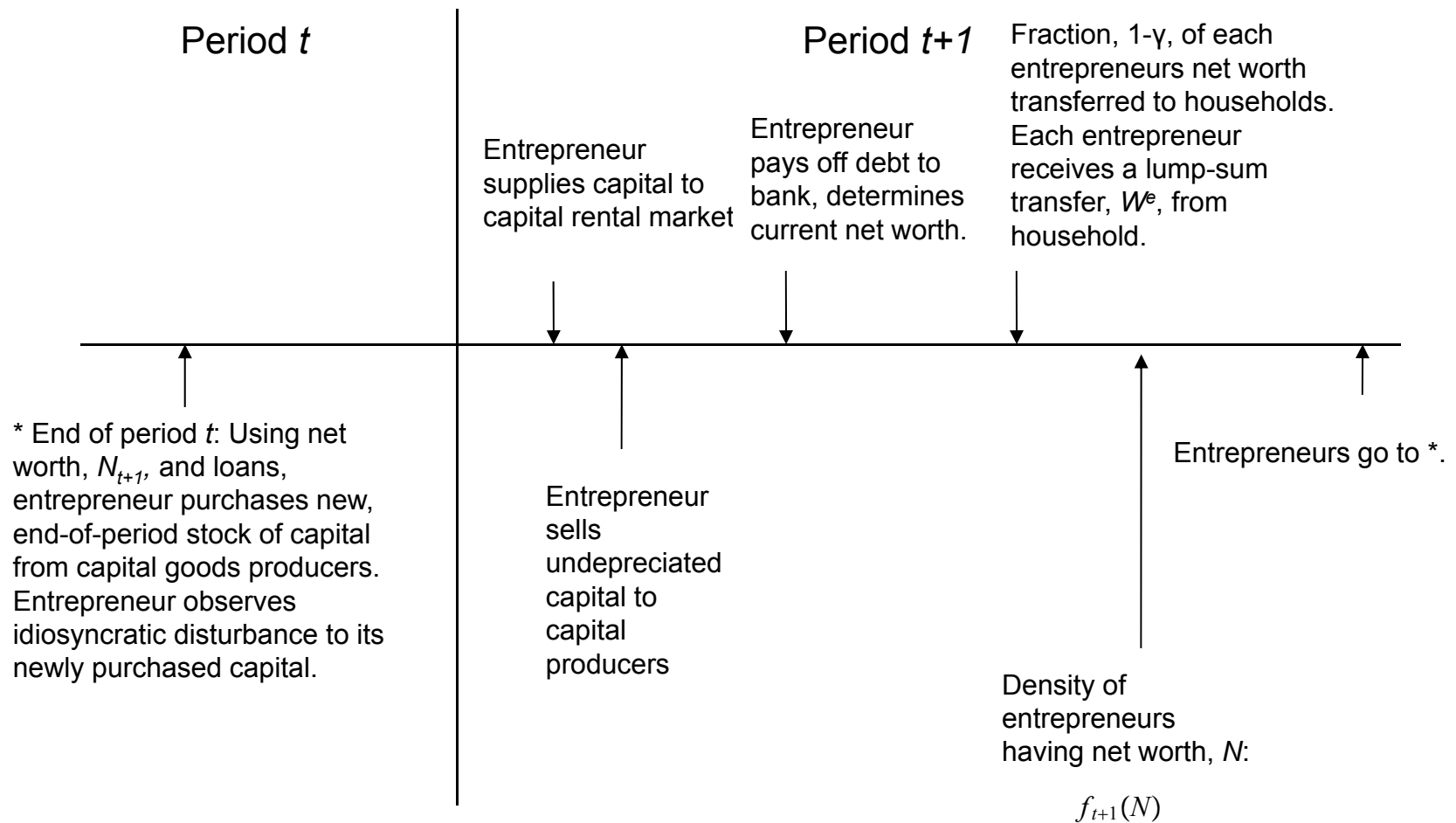
$$[1 - \Gamma(\bar{\omega}_{t+1})](1 + R_{t+1}^k)K_{t+1}$$

- Then,
 - a fraction, $1-\gamma$, of each entrepreneur's net worth is transferred to the household as a lump-sum.
 - the household transfers resources, W_{t+1}^e , as a lump sum to each entrepreneur.
- So, net worth of all entrepreneurs at the end of $t+1$ is:

$$N_{t+2} = \gamma[1 - \Gamma(\bar{\omega}_{t+1})](1 + R_{t+1}^k)K_{t+1} + W_{t+1}^e$$

- The entrepreneurs as a whole take this net worth to the bank in $t+1$, get loans and buy capital, and so on....

One day in life of entrepreneur



Other Equations of the Model

- Goods market clearing (resource constraint):

goods bought by households to feed their entrepreneurs and workers

$$\overbrace{C_t}$$

investment goods bought by capital producers

$$+ \overbrace{I_t}$$

goods bought for monitoring by banks

$$+ \overbrace{\int_0^{\bar{\omega}_t} \omega dF(\omega)(1 + R_t^k)K_t}^{\text{production function}} = \overbrace{K_t^\alpha}$$

- Capital producers' technology:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

Other Equations...

- Household problem:

$$\max_{\{c_t, B_{t+1}, l_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t), \quad u(c_t) = \log(c_t)$$

subject to:

$$c_t + B_{t+1} \leq w_t l_t + (1 + R_{t-1})B_t + \overbrace{=(1-\gamma)[1-\Gamma(\bar{\omega}_t)](1+R_t^k)K_t - W_t^e}^{\text{transfers from entrepreneurs}}$$

$$0 \leq l_t \leq 1$$

Optimization: $u'(c_t) = \beta u'(c_{t+1})(1 + R_t)$, $l_t = 1$

- Bond market clearing:

borrowing by entrepreneurs lending by households

$$\overbrace{K_{t+1} - N_{t+1}} = \overbrace{B_{t+1}}$$

Result

- There are seven aggregate variables among the unknowns:

$$c_t, I_t, \bar{\omega}_t, R_t^k, K_t, N_t, R_t$$

- There are seven dynamic equilibrium conditions that can be used to pin the down (see next slide).
- Solution does not require knowing $f_t(N)$
 - Reflects constant returns to scale in entrepreneurial projects and entrepreneurial objective.
 - Otherwise, leverage and interest rate in standard debt contract would be a function of N . In that case, what entrepreneurs as a group do depends on the distribution of N .
 - Constant return to scale assumptions massively simplify the analysis. Whether this entails significant distortions in conclusions is an interesting issue to explore.

In practice, monitoring costs small

Summary of Equations

Equilibrium conditions familiar from standard neoclassical model	
resource constraint:	$c_t + I_t + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega)(1 + R_t^k)K_t = K_t^\alpha$
household fnc:	$u'(c_t) = \beta u'(c_{t+1})(1 + R_t)$
Capital accumulation:	$K_{t+1} = (1 - \delta)K_t + I_t$
Rate of return on capital:	$1 + R_t^k \leq \alpha K_t^{\alpha-1} + 1 - \delta$

In neoclassical model these are the same.

Financial frictions introduce a wedge.

Equilibrium conditions specific to financial frictions	
entrepreneur fnc:	$\frac{1-F(\bar{\omega}_{t+1})}{1-\Gamma(\bar{\omega}_{t+1})} = \frac{\frac{1+R_{t+1}^k}{1+R_t} [1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F'(\bar{\omega}_{t+1})]}{1-\frac{1+R_{t+1}^k}{1+R_t} [\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})]}$
zero profit condition:	$K_t = \frac{1}{1-\frac{1+R_t^k}{1+R_{t-1}} [\Gamma(\bar{\omega}_t)-\mu G(\bar{\omega}_t)]} N_t$
Aggregate Net Worth:	$N_{t+1} = \gamma [1 - \Gamma(\bar{\omega}_t)] (1 + R_t^k) K_t + W_t^e$

Large family assumption has consequence that model focuses exclusively on the implications of financial frictions for distortions in the intertemporal margin, abstracting from all other implications, such as for distribution of income between entrepreneurs and others.

Financial Friction as Intertemporal Wedge

Equilibrium conditions familiar from standard neoclassical model	
resource constraint:	$c_t + I_t + \mu \int_0^{\bar{\omega}_t} \omega dF(\omega)(1 + R_t^k)K_t = K_t^\alpha$
household func:	$u'(c_t) = \beta u'(c_{t+1})(1 + R_{t+1}^k)(1 - \tau_{t+1})$
Capital accumulation:	$K_{t+1} = (1 - \delta)K_t + I_t$
Rate of return on capital:	$1 + R_t^k = \alpha K_t^{\alpha-1} + 1 - \delta$

Equilibrium conditions specific to financial frictions	
entrepreneur func:	$\frac{1-F(\bar{\omega}_{t+1})}{1-\Gamma(\bar{\omega}_{t+1})} = \frac{\frac{1+R_{t+1}^k}{1+R_t} [1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F'(\bar{\omega}_{t+1})]}{1-\frac{1+R_{t+1}^k}{1+R_t} [\Gamma(\bar{\omega}_{t+1})-\mu G(\bar{\omega}_{t+1})]}$
wedge	$\tau_{t+1} = 1 - \frac{1+R_t}{1+R_{t+1}^k} = 1 - \frac{[1-F(\bar{\omega}_{t+1})-\mu\bar{\omega}_{t+1}F'(\bar{\omega}_{t+1})]}{\frac{1-F(\bar{\omega}_{t+1})}{1-\Gamma(\bar{\omega}_{t+1})} - \Gamma(\bar{\omega}_{t+1}) - \mu G(\bar{\omega}_{t+1})}$
zero profit condition:	$K_t = \frac{1}{1-\frac{1+R_t^k}{1+R_{t-1}^k} [\Gamma(\bar{\omega}_t)-\mu G(\bar{\omega}_t)]} N_t$
Aggregate Net Worth:	$N_{t+1} = \gamma [1 - \Gamma(\bar{\omega}_t)](1 + R_t^k)K_t + W_t^e$

Looks just like neoclassical model with a particular tax on capital income.

Solving the Model

- Perturbation methods can be applied.
 - First, require steady state of the model.
 - Then, linearize equilibrium conditions about the steady state and solve.

- Parameterization:

$$u(c) = \log c, \sigma = 0.26, \mu = 0.21, \gamma = 0.97, \alpha = 1/3, \delta = 0.02, \beta = 1.03^{-1/4}, w^e = 0.01$$
$$\log(\sigma_t/\sigma) = 0.97\log(\sigma_{t-1}/\sigma) + \varepsilon_t$$

- Cross-sectional dispersion evolves according to a first order autoregressive process.

Solving the Model

- Endogenous variables in steady state:

$$\bar{\omega} = 0.5286, G(\bar{\omega}) = 0.0049, F(\bar{\omega}) = 0.010, \Gamma(\bar{\omega}) = 0.5282, n = 11.65$$

$$\Gamma(\bar{\omega}) - \mu G(\bar{\omega}) = 0.5271, \frac{c}{c+i} = 0.77, \frac{k-n}{c+i} = 4.60, \frac{k}{n} = 2.14, \frac{k}{c+i} = 8.62$$

$$400\left(\frac{Z}{1+R} - 1\right) = 1.11, \gamma[1 - \Gamma(\bar{\omega})](1 + R^k)\frac{k}{n} = 0.9991, \text{wedge} = 0.0114$$

- Financial distortion:

$$\text{without financial frictions: } K = \left[\frac{\alpha}{\frac{1}{\beta} - (1 - \delta)} \right]^{\frac{1}{1-\alpha}} = 42.4, c = 2.64$$

$$\text{with financial frictions: } K = \left[\frac{\alpha}{\frac{1}{\beta(1-\text{wedge})} - (1 - \delta)} \right]^{\frac{1}{1-\alpha}} = 25.0, c = 2.42$$

steady state welfare cost of the financial frictions: 8.9 percent of consumption (=100(2.64-2.42)/2.42)

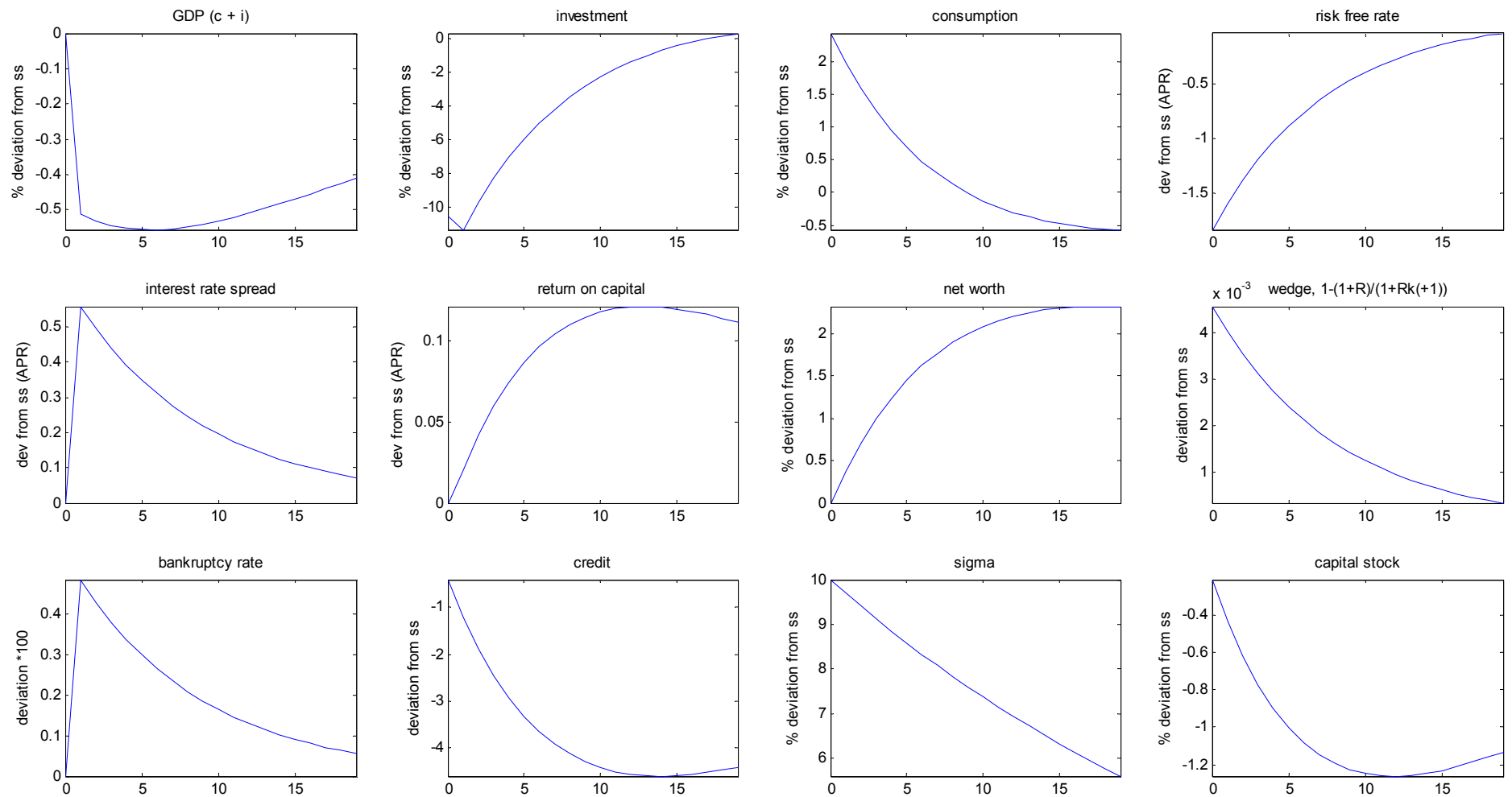
Finding the steady state

- Delete time subscripts from equations.
- Use household fnc to set $1 + R = 1/\beta$
- Fix an initial guess, $R^k \geq R$
 - Compute $\bar{\omega}$ (and, hence, G , F and Γ) from entrepreneur fnc
 - Compute K from rate of return on capital expression:
$$K = \left[\frac{\alpha}{1 + R^k - (1 - \delta)} \right]^{\frac{1}{1-\alpha}}.$$
 - Compute N from net worth equation.
 - Adjust R^k until zero profit condition is satisfied.
- Compute c from the resource constraint.

10% jump

Response to 0.1 jump in ε_t , where $\log(\sigma_t/\sigma) = 0.97 \times \log(\sigma_{t-1}/\sigma) + \varepsilon_t$

model parameters: $\sigma = 0.26$, $\mu = 0.21$, $\gamma = 0.97$, $\alpha = 1/3$, $\delta = 0.02$, $\beta = 1.03^{-1/4}$, $\eta = 1$.

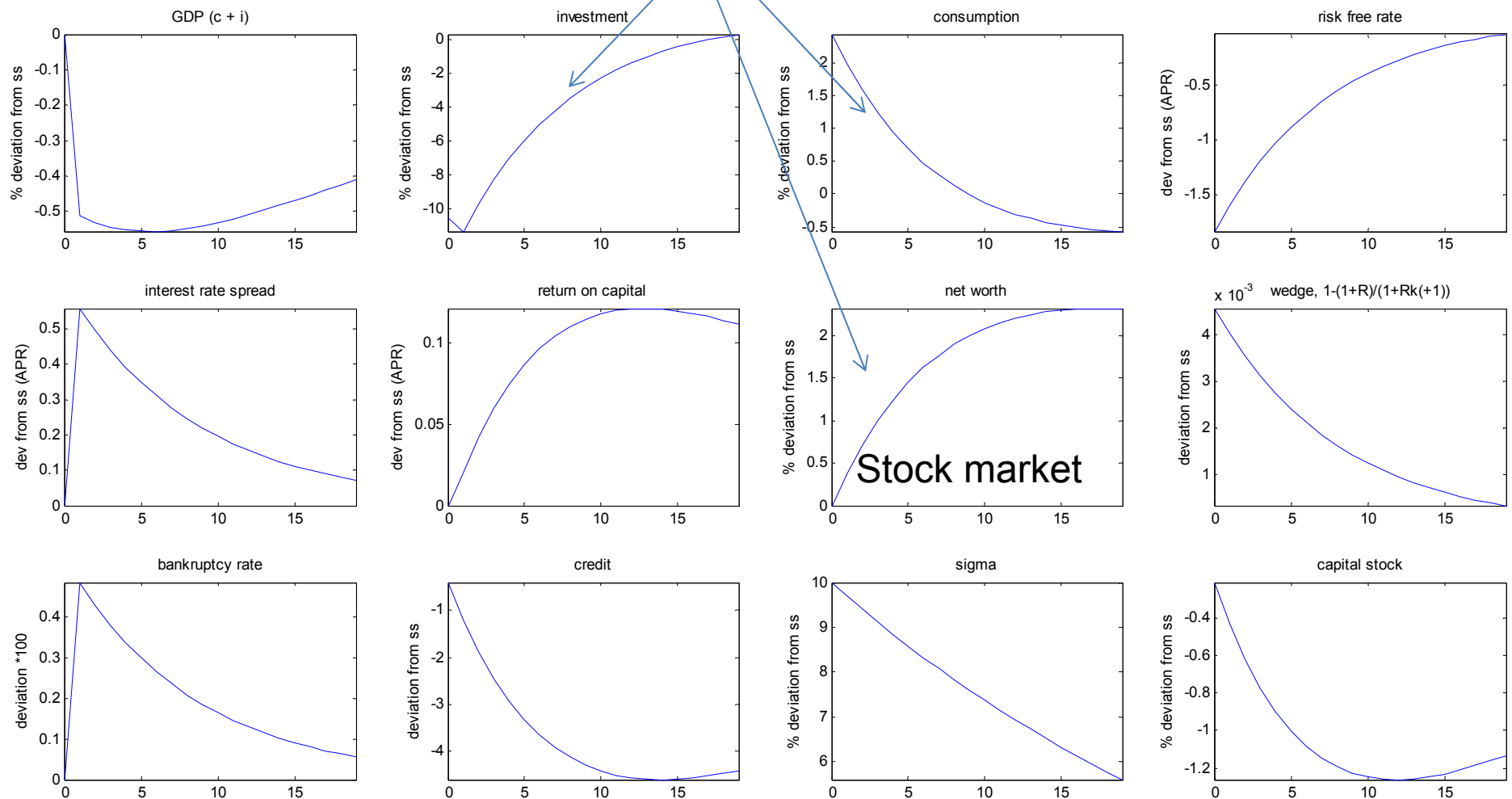


10% jump

Response to 0.1 jump in ε_t , where $\log(\sigma_t/\sigma) = 0.97 \times \log(\sigma_{t-1}/\sigma) + \varepsilon_t$

model parameters: $\sigma = 0.26$, $\mu = 0.21$, $\gamma = 0.97$, $\alpha = 1/3$, $\delta = 0.02$, $\beta = 1.03^{-1/4}$, $\eta = 1$.

Suggests (but see later slide) that risk not likely to be important in business cycles



10% jump

Response to 0.1 jump in ε_t , where $\log(\sigma_t/\sigma) = 0.97 \times \log(\sigma_{t-1}/\sigma) + \varepsilon_t$

model parameters: $\sigma = 0.26$, $\mu = 0.21$, $\gamma = 0.97$, $\alpha = 1/3$, $\delta = 0.02$, $\beta = 1.03^{-1/4}$, $\eta = 1$.

The mechanism by which the shock moves economic variables operates through the intertemporal wedge.

