

Log Linearized Phillips Curve for Simple New Keynesian Model with No Capital

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Objective

- Obtain the log-linearized Phillips curve for New Keynesian model.
- Follows up on equilibrium conditions derived in handout, “Simple New Keynesian Model without Capital”
 - Work with the equilibrium conditions in which $G_t = 0$, so that $C_t = Y_t$.

Equilibrium Conditions

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^\varepsilon K_{t+1} \quad (1), \quad F_t = 1 + \beta \theta E_t \bar{\pi}_{t+1}^{\varepsilon-1} F_{t+1} \quad (2)$$

$$\frac{K_t}{F_t} = \left[\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3),$$

$$p_t^* = \left[(1 - \theta) \left(\frac{1 - \theta \bar{\pi}_t^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta \bar{\pi}_t^\varepsilon}{p_{t-1}^*} \right]^{-1} \quad (4)$$

$$\frac{1}{C_t} = \beta E_t \frac{1}{C_{t+1}} \frac{R_t}{\bar{\pi}_{t+1}} \quad (5), \quad C_t = p_t^* \exp(a_t) N_t \quad (6)$$

Steady State

Conditional on $\bar{\pi}$

$$K = \frac{\frac{\varepsilon}{\varepsilon-1}S}{1 - \beta\theta\bar{\pi}^\varepsilon} \quad (1), \quad F = \frac{1}{1 - \beta\theta\bar{\pi}^{\varepsilon-1}} \quad (2)$$

$$\frac{K}{F} = \left[\frac{1 - \theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta} \right]^{\frac{1}{1-\varepsilon}} \quad (3),$$

$$p^* = \left[(1 - \theta) \left(\frac{1 - \theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta} \right)^{\frac{\varepsilon}{\varepsilon-1}} + \frac{\theta\bar{\pi}^\varepsilon}{p^*} \right]^{-1} \quad (4)$$

$$1 = \beta \exp(-\Delta a) \frac{R}{\bar{\pi}} \quad (5)$$

Log Linearization

- Hat notation:

$$\hat{x}_t = \frac{dx_t}{x} = \frac{x_t - x}{x} \rightarrow dx_t = \hat{x}_t x.$$

- Log linearize equation (1) about steady state:

$$K_t = \frac{\varepsilon}{\varepsilon - 1} s_t + \beta \theta E_t \bar{\pi}_{t+1}^\varepsilon K_{t+1}$$

$$\hat{K}_t K = \frac{\varepsilon}{\varepsilon - 1} \hat{s}_t s + \beta \theta \varepsilon \bar{\pi}^{\varepsilon-1} \hat{\bar{\pi}}_{t+1} \bar{\pi} K + \beta \theta \bar{\pi}^\varepsilon \hat{K}_{t+1} K$$

$$\hat{K}_t = \frac{\varepsilon}{\varepsilon - 1} \hat{s}_t \underbrace{\frac{s}{K}}_{\frac{1 - \beta \theta \bar{\pi}^\varepsilon}{\varepsilon - 1}} + \beta \theta \bar{\pi}^\varepsilon (\varepsilon \hat{\bar{\pi}}_{t+1} + \hat{K}_{t+1})$$

$$\hat{K}_t = (1 - \beta \theta \bar{\pi}^\varepsilon) \hat{s}_t + \beta \theta \bar{\pi}^\varepsilon (\varepsilon \hat{\bar{\pi}}_{t+1} + \hat{K}_{t+1})$$

Phillips Curve

- Linearizing (1), (2) and (3), about steady state,

$$\hat{K}_t = (1 - \beta\theta\bar{\pi}^\varepsilon) \hat{s}_t + \beta\theta\bar{\pi}^\varepsilon E_t (\varepsilon\hat{\pi}_{t+1} + \hat{K}_{t+1}) \quad (\text{a})$$

$$\hat{F}_t = \beta\theta\bar{\pi}^{\varepsilon-1} E_t ((\varepsilon - 1) \hat{\pi}_{t+1} + \hat{F}_{t+1}) \quad (\text{b})$$

$$\hat{K}_t = \hat{F}_t + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \hat{\pi}_t. \quad (\text{c})$$

- Substitute out for \hat{K}_t in (a) using (c) and then substitute out for \hat{F}_t from (b) to obtain the equation on the next slide.

Phillips Curve

- Performing the substitutions described on the previous slide:

$$\begin{aligned} & \beta\theta\bar{\pi}^{\varepsilon-1}E_t \left((\varepsilon - 1) \hat{\pi}_{t+1} + \hat{F}_{t+1} \right) \\ & + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \hat{\pi}_t = (1 - \beta\theta\bar{\pi}^\varepsilon) \hat{s}_t \\ & + \beta\theta\bar{\pi}^\varepsilon E_t \left(\varepsilon \hat{\pi}_{t+1} + \hat{F}_{t+1} + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \hat{\pi}_{t+1} \right). \end{aligned}$$

Phillips Curve

- Collecting terms,

$$\begin{aligned} & \overbrace{\frac{(1 - \theta\bar{\pi}^{(\varepsilon-1)}) (1 - \beta\theta\bar{\pi}^\varepsilon)}{\theta\bar{\pi}^{(\varepsilon-1)}}}^{\text{familiar Phillips curve}} \hat{s}_t + \beta E_t \hat{\pi}_{t+1} \\ & + (1 - \bar{\pi}) (1 - \theta\bar{\pi}^{(\varepsilon-1)}) \beta \\ & \times E_t \left(\hat{F}_{t+1} + \left(\varepsilon + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \right) \hat{\pi}_{t+1} \right). \end{aligned}$$

- Don't actually get standard Phillips curve unless $\bar{\pi} = 1$.
 - More generally, get standard Phillips curve as long as there are no price distortions in steady state.
- Going for the Phillips curve in terms of the output gap.

Linearized Marginal Cost

- Real Marginal Cost:

$$\begin{aligned} s_t &= \frac{(1 - \nu) \exp(\tau_t) C_t N_t^\varphi}{A_t} \\ &= \underbrace{N_t = C_t / (p_t^* A_t)}_{=} (1 - \nu) \frac{\exp(\tau_t)}{(p_t^*)^\varphi} \left(\frac{C_t}{A_t} \right)^{1+\varphi} \\ &= (1 - \nu) \frac{1}{(p_t^*)^\varphi} \left(\frac{C_t}{A_t \exp\left[-\frac{\tau_t}{1+\varphi}\right]} \right)^{1+\varphi} \\ &= (1 - \nu) \frac{1}{(p_t^*)^\varphi} X_t^{1+\varphi}, \end{aligned}$$

where

$$X_t = \frac{\text{Actual consumption}}{\text{Natural consumption}} = \text{"output gap"}$$

Linearized Marginal Cost, cnt'd

- Real Marginal Cost:

$$s_t = (1 - \nu) \frac{1}{(p_t^*)^\varphi} X_t^{1+\varphi},$$

- Let

$$x_t = d\log X_t = \frac{dX_t}{X} = \hat{X}_t$$

- Then,

$$\begin{aligned} \hat{s}_t &= \frac{(1 - \nu) (1 + \varphi) \frac{1}{(p_t^*)^\varphi} X_t^{1+\varphi}}{s} x_t - \frac{\varphi (1 - \nu) \frac{1}{(p_t^*)^\varphi} X_t^{1+\varphi}}{s} \hat{p}_t^* \\ &= (1 + \varphi) x_t - \varphi \hat{p}_t^* \end{aligned}$$

Phillips Curve in Terms of Output Gap

- Collecting terms,

$$\begin{aligned}\hat{\pi}_t &= \frac{(1 - \theta\bar{\pi}^{(\varepsilon-1)}) (1 - \beta\theta\bar{\pi}^\varepsilon)}{\theta\bar{\pi}^{(\varepsilon-1)}} [(1 + \varphi) x_t - \varphi\hat{p}_t^*] + \beta E_t \hat{\pi}_{t+1} \\ &+ (1 - \bar{\pi}) (1 - \theta\bar{\pi}^{(\varepsilon-1)}) \beta \\ &\times E_t \left(\hat{F}_{t+1} + \left(\varepsilon + \frac{\theta\bar{\pi}^{(\varepsilon-1)}}{1 - \theta\bar{\pi}^{(\varepsilon-1)}} \right) \hat{\pi}_{t+1} \right).\end{aligned}$$

- Only looks like the familiar Phillips curve when $\bar{\pi} = 1$.