A Theory of the Non-Neutrality of Money with Banking Frictions and Bank Recapitalization*

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January 2010

Abstract

Policy actions by the Federal Reserve during the recent financial crisis often involve recapitalization of banks. This paper offers a theory of the non-neutrality of money for policy actions taking the form of injecting capital into banks via nominal transfers, in an environment where banking frictions are present in the sense that there exists an agency cost problem between banks and their private-sector creditors. We conduct our analysis in a general equilibrium setting in which a two-sided financial contracting model is embedded. It is shown that even with perfect nominal flexibility, the recapitalization policy has real effects on the economy. This non-neutrality result disappears when banking frictions are absent.

JEL Classification: E44, E52, D82, D86.

Keywords: Banking frictions; money non-neutrality; bank recapitalization; two-sided debt contract.

*I am grateful to Larry Christiano, Marty Eichenbaum, Gadi Barlevy, and Yi Jin for helpful comments and suggestions on an earlier version of the paper. All errors are mine.

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1 Introduction

The Federal Reserve took a variety of unconventional policy actions during the recent financial crisis that started in 2007. As traditional interest rate policy that adjusts the federal funds rate was perceived to be ineffective (Cecchetti, 2009), the Fed adopted various measures of what Reis (forthcoming) classifies as "quantitative policy", i.e., policy that changes the size of the Fed's balance sheet and the composition of its liabilities, as well as "credit policy", policy that manages the composition of its asset holdings. In addition to injecting liquidity into the financial system (Brunnermeier, 2009), some of the Fed's policy measures also have the flavor of providing capital subsidy to banks, a point forcefully made by Cecchetti (2009). During the crisis, lending by the Fed to banks almost always involved a subsidy. By accepting collaterals at prices that were almost surely above their actual market prices (Tett, 2008), lending by the Fed in effect recapitalized the borrowing banks through nominal transfers. In response to the crisis, the Fed attempted to stimulate discount borrowing, which is collateralized, by reducing substantially the premium charged on primary discount lending (relative to the federal funds rate target) and raising the term of lending from overnight to as long as three months. In addition, to remove the stigma attached to discount borrowing[1], the Fed created the Term Auction Facility (TAF) in December 2007 and enlarged it later on in order to better provide funds to banks that need them most. The rules of the TAF allow banks to pledge collaterals that might otherwise have little market value.[2]

In the light of the celebrated Modigliani-Miller theorem, such bank recapitalization efforts would have no real effect in a world where banks can frictionlessly channel funds from investors

[1] Traditionally, banks that borrowed from the discount window might be seen by other banks and institutions as having financial stress.

[2] For details, see Cecchetti (2009). Similar actions were taken by the Fed to help out other financial institutions (e.g., investment banks) through programs such as the Term Securities Lending Facility, the Primary Dealer Credit Facility, and the Term Asset-Backed Securities Loan Facility, etc.

to users of funds, as the capital structure of banks would be irrelevant for their lending activities and the real market value of their loan portfolios. In that kind of world the classical dichotomy holds and the recapitalization of banks by the monetary authority is neutral, despite that it does involve a real transfer that enlarges banks' net worth relative to debt. However, as will be demonstrated in this paper, once an agency cost problem is introduced to the relationship between banks and their private-sector creditors (henceforth "depositors" for ease of exposition), the Modigliani-Miller theorem fails for banks, the classical dichotomy breaks down, and money is no longer neutral when the central bank policy takes the form of injecting money to the banking system to increase bank capital. In particular, a bank recapitalization effort by the monetary authority triggers a redistribution of wealth (nominal and real) in favor of the banks, reduces the cost of banks' external finance, stimulates bank lending, and raises employment and output. Importantly, this non-neutrality of money obtains even without any kind of nominal rigidities.

Needless to say, understanding the mechanism through which policy works is crucial for assessing the effectiveness of central bank reactions to the crisis. It appears highly plausible that there are important channels hinging upon frictions faced by banks. On the asset side of banks' balance sheets, there might exist informational asymmetry regarding the ability of (nonfinancial) firms to repay their loans, giving rise to an agency cost problem between banks and firms as emphasized in the seminal work of Bernanke and Gertler (1989) and a large literature that follows. Frictions of this kind are the literature's main focus thus far. We shall refer to them as "credit market frictions", for the sake of distinguishing it from the informational asymmetry and agency cost problem on the liability side of banks' balance sheets, which we shall call "banking frictions". To introduce the latter kind of frictions we apply the costly-state-verification (CSV) framework of Towsend (1979), Gale and Hellwig (1985), and Williamson (1986) to the bank-depositor relationship. In our model banks face idiosyncratic risks and depositors have to expend monitoring costs in order to verify banks' capacities to repay. As is shown in the paper, bank recapitalization by the monetary authority is neutral when banking frictions are absent, even if the conventionally studied credit market frictions are present. This implies that what credit market frictions do is at best to amplify and propagate the policy's real effects which are brought forth solely by the existence of banking frictions. We are thus compelled to give special attention to the roles banking frictions play.

In our model economy, banks receive both deposits and central bank money injections to finance their lending activities. It should be clarified here that we use the term "deposits" in the broadest sense, referring to all liabilities of banks that are held by the private sector. Meanwhile, we lump all the private-sector creditors of banks, including consumers, nonfinancial businesses, and nonbank financial firms, into a single category of agents called "depositors". At the heart of our story is that the rate of default by banks and the cost of their external finance are positively related to their debt-equity ratios. Recapitalization by the monetary authority induces a real transfer in favor of the banks, no matter how the price level changes. This real transfer is not inconsequential: It lowers the banks' debt-equity ratio, leading to a decline in their default rate and the external finance premium, which in turn stimulates real bank lending and thus employment and output.

To highlight the mechanism at work, our model has abstracted from several aspects of the actual economy that might be considered important in other contexts. First, our analysis is conducted within a framework that allows for perfect nominal flexibility (i.e., there is no price or wage stickiness or adjustment cost on nominal savings). This allows us to isolate the real effects of the recapitalization policy from the non-neutrality produced by nominal rigidities. Second, insurance of deposits is not considered. This does not invalidate our analysis since a large fraction of bank liabilities remain uninsured. Neither are capital adequacy requirements incorporated.

Hence the mechanism in our model does not work through the relaxation of binding capital adequacy requirements. Instead, it works through changing the banks' default rate and their cost of external finance. Third, our model is constructed in such a way that the firms' financial leverage is unaffected by the bank recapitalization policy in equilibrium, which enables us to focus on the role played by the banks' debt-equity ratio. Such a construct is innocuous as neither the non-neutrality result with banking frictions nor the neutrality result without banking frictions (but still with credit market frictions) relies on the fixity of the equilibrium debt-equity ratio of firms. Fourth, the monetary authority's bank recapitalization policy is the only aggregate variable that perturbs the economy. Hence it is not our intention to model the "crisis" per se. Rather, our purpose is to offer a theory on how the policy itself affects the real economy in an otherwise unchanging environment. This allows the policy transmission mechanism to be as transparent as possible.

In a model that allows for perfect nominal flexibility, some other sort of frictions must be employed to generate the non-neutrality of money (shocks). In Lucas' (1972) misperceptions theory it is the imperfect information about the overall price level that temporarily misleads suppliers and generates real effects of money supply shocks. It seems that information on money supply and other policy instruments are available to the public with little delay so there is no serious signal extraction problem to solve. Hence the misperceptions story might not be particularly relevant in our context. In contrast, this paper assumes full information on all aggregate variables but uses a different kind of information problem to generate the non-neutrality of money. The problem here concerns costly revelation of banks' information to the depositors, which leads to the breakdown of the Modigliani-Miller theorem and gives rise to a nontrivial role for banks' capital structure. Although the idea that the Modigliani-Miller theorem might not apply for banks have been put forth by Kashyap and Stein (1995) and Stein

(1998), our non-neutrality result with perfect nominal flexibility is novel.[3]

The rest of the paper is organized as follows. Section 2 presents a model of two-sided financial contracting with idiosyncratic banking risks. A general equilibrium model with consumption/savings and labor supply decisions on the part of households is then developed in Section 3. Section 4 characterizes the equilibrium and presents the non-neutrality result. The last section concludes. All proofs are relegated to the Appendix.

2 Financial Contracting with Banking Risks

2.1 Production and Information Structure

Consider an environment with a unit-mass continuum of regions indexed by i, $i \in [0,1]$. In region i there is one bank, called bank i, and a unit-mass continuum of firms indexed by ij, $j \in [0,1]$. Each firm resides in a distinct location, and operates a stochastic production technology that transforms labor and capital service into a homogeneous final output. The technology of firm ij is represented by the production function

$$y_{ij} = \theta_i \omega_{ij} F\left(k_{ij}, l_{ij}\right), \tag{1}$$

where y_{ij}, k_{ij}, and l_{ij} denote final output, capital input, and labor input for firm ij. $F(\cdot)$ is linearly homogeneous, increasing and concave in its two arguments, and satisfies the usual Inada conditions. All sources of idiosyncratic risks are captured in the productivity factor, with θ_i being the random productivity specific to region i, and ω_{ij} the random productivity specific to location ij. We assume that θ_i is identical and independently distributed across regions, with c.d.f. $\Phi^r(\cdot)$ and p.d.f. $\phi^r(\cdot)$, and that ω_{ij} is identical and independently distributed across all

[3] To be concrete, our model differs from theirs in two major respects. First, we use the CSV framework to model banking frictions, while Stein (1995) uses an adverse selection model, and Kashyap and Stein (1995) use a reduced-form formulation. Second, they rely on exogenously imposed incomplete adjustment of the price level to generate the non-neutrality of money, while our model assumes away all sorts of nominal rigidities.

locations, with c.d.f. $\Phi^l(\cdot)$ and p.d.f. $\phi^l(\cdot)$. Both θ_i and ω_{ij} have non-negative support and unit mean. Furthermore, θ_i and $\omega_{\tau j}$, $i, \tau, j \in [0,1]$, are uncorrelated with each other. The distributions are known by all agents in the economy.

Firms hire labor and rent capital from competitive factor markets at nominal wage rate W and nominal rental rate R^k. Assume that each firm owns the same amount of physical capital K^f, and that each bank owns K^b. Both K^f and K^b are fixed. To simplify matters even further we assume that physical capital is not traded so that capital gains or losses (from changes in the price of capital) are not potential sources of changes in the net worth of the firms and banks. There is, however, a rental market and the rental income of capital constitutes the firms and banks' internal funds.[4] Since the firms' internal funds are generated entirely from the current rental value of the capital stock they own, in a market clearing equilibrium the firms must borrow additional funds to finance their purchase of the labor input supplied by workers and the rental service provided by the stock of physical capital owned by banks. Once firms acquire factor inputs, production takes place, and the region specific and location specific productivities realize. The final output is sold at price P in a competitive goods market.

We use the CSV approach of Towsend (1979), which is later adopted by Gale and Hellwig (1985) and Williamson (1986), to model financial frictions and financial contracting. It is assumed that there is an informational asymmetry regarding borrowers' ex post revenues. In particular, only borrowers themselves can costlessly observe their realized revenues, while lenders have to expend a verification cost in order to observe the same object. In our environment only firm ij can observe at no cost $x_{ij}^f \equiv \theta_i \omega_{ij}$, and only bank i can observe θ_i costlessly. For a bank to observe x_{ij}^f (or ω_{ij}) and for a depositor to observe θ_i, verification costs have to be incurred. Note that by lending to a continuum of firms in a particular region each bank effectively diver-

[4]Note that the assumption of fixed capital stock does not prevent it from generating variable internal funds for the firms and banks, because in the general equilibrium the rental rate responds to aggregate shocks.

sifies away all the firm/location specific risks. But the region specific risk is not diversifiable. Thus it is possible that a bank becomes insolvent when an adverse regional shock occurs.

The concept of "regions" should not be interpreted literally as reflecting geographic areas, albeit this is certainly one of the many possible interpretations. Rather, it is a device designed to generate risks idiosyncratic to individual banks. If banks are subject to risks that cannot be fully diversified, then the kind of agency problem between banks and firms applies equally well to the relationship between banks and depositors. In that case there are needs to "monitor the monitor", in the terminology of Krasa and Villamil (1992a). Bank-level risks might stem from geographic confinement of an individual bank's operation to specific areas, as in the U.S. when out-of-state branching was restricted (see Williamson, 1989). They might also be due to the concentration of a bank's lending activities in specific industries. Savings and loan associations in the U.S., which historically concentrated on mortgage loans, was a good example. It should be noted that even without branching restrictions or regulations on banks' lending and investment activities, an individual bank might optimally choose to limit its scale and/or scope of operation so that the risks associated with its lending activities cannot be fully diversified. An example appears in Krasa and Villamil (1992b), who consider the trade-off involved in increasing the size of a bank's portfolio (i.e., lending to additional borrowers). In their model balancing gains from decreased default risk with losses from increased monitoring costs leads to an optimal scale for banks. Another example is Cerasi and Daltung (2000), who introduce considerations on the internal organization of banks that render scale economies in the banking sector rapidly exhausted.[5] In this paper we follow Krasa and Villamil (1992a) and Zeng (2007) to assume that an individual bank cannot contract with a sufficient variety of borrowers so that credit risks are not perfectly diversifiable.

[5] Specifically, loan officers, who are the ones actually making loans, have to be monitored by the banker.

2.2 The Two-Sided Debt Contract

The three groups of players—firms, banks, and depositors—in the model are connected via a two-sided contract structure. Both sides of the contract, one between firms and banks and the other between banks and depositors—fit into a generic framework we now develop. Here we shall restrict attention to deterministic monitoring, which is actually less restrictive than it appears. Krasa and Villamil (2000) articulates a costly enforcement model that justifies deterministic monitoring when commitment is limited and enforcement is costly and imperfect.[6] It follows that the optimal contract between a generic borrower and a generic lender, both being risk neutral, takes the form of a standard debt contract, in Gale and Hellwig (1985)'s term.

Suppose the borrower's revenue is given by Vx, where V is a common-knowledge component, and x is a risky component that is subject to informational asymmetry, whereby the borrower can costlessly observe x while the lender has to expend a verification cost in order to do so. The verification cost is assumed to be μ times the borrower's revenue, with $\mu \in (0,1)$. The c.d.f. of x, given by $\Phi(\cdot)$, is also common knowledge. The contract specifies a set of realizations of x for which monitoring occurs, together with a payment schedule. An incentive compatible contract must specify a fixed payment for x in the non-monitoring set, otherwise the borrower will always report the value of x for which the payment is lowest among non-monitoring states. A standard debt contract with monitoring threshold $\overline{x}$ is an incentive compatible contract with the following features: (i) the monitoring set is $\{x|x < \overline{x}\}$, (ii) the fixed payment is $V\overline{x}$ for $x \in \{x|x \geq \overline{x}\}$, and

[6] Krasa and Villamil (2000) also show that when there is perfect commitment, stochastic contracts are optimal. They argue, however, that for loan contracts, limited commitment and therefore simple debt seems more appropriate. Boyd and Smith (1994) examine the welfare cost of arbitrarily restricting the set of feasible contracts to standard debt contracts. When model parameters are calibrated to realistic values, the welfare loss from exogenously imposing this restriction is extremely small. Thus, if implementation costs are nontrivial (as seems likely), standard debt contracts will be at least very close to optimal. See also Mookherjee and Png (1989) on deterministic versus stochastic monitoring.

(iii) the payment is Vx for $x \in \{x | x < \overline{x}\}$. The standard debt contract is particularly interesting because it resembles many financial contracts in the real world. It features fixed payment for non-default states and state-contingent payment when default occurs. Requiring the borrower to repay as much as possible in default states allows the fixed payment for non-default states to be minimized, thus minimizing the probability of verification and thus the expected monitoring cost.

Under the standard debt contract, the borrower and the lender each obtains a share of the expected revenue V. The borrower receives $V\Gamma(\overline{x}; \Phi)$ where

$$\Gamma(\overline{x}; \Phi) \equiv \int_{\overline{x}}^{\infty} (x - \overline{x})\, d\Phi(x), \tag{2}$$

reflecting the fact that with x above $\overline{x}$, the borrower gives out the fixed payment $V\overline{x}$ and keeps the remaining, and with x below $\overline{x}$, all revenues are confiscated by the lender. The lender receives $V\Psi(\overline{x}; \Phi)$ where

$$\Psi(\overline{x}; \Phi) \equiv \overline{x}\left[1 - \Phi(\overline{x})\right] + (1 - \mu) \int_{0}^{\overline{x}} x d\Phi(x). \tag{3}$$

When x is larger than or equal to $\overline{x}$, which occurs with probability $1 - \Phi(\overline{x})$, the lender recoups the fixed proportion $\overline{x}$ of the expected revenue V. If x falls below $\overline{x}$, the lender takes all of the realized revenue while expending a verification cost which equals a fraction μ of the revenue. Note that

$$\Gamma(\overline{x}; \Phi) + \Psi(\overline{x}; \Phi) = 1 - \mu \int_{0}^{\overline{x}} x d\Phi(x) < 1,$$

indicating that there is a deadweight loss $\mu \int_{0}^{\overline{x}} x d\Phi(x)$ due to costly monitoring. The following assumption is imposed.

Assumption 1. The p.d.f $\phi(\cdot)$ is bounded, and $x\phi(x) / [1 - \Phi(x)]$ is an increasing function of x.

The assumption that $x\phi(x)/[1-\Phi(x)]$ is increasing is actually weaker than the increasing hazard assumption commonly made in the incentive contract literature, which requires $\phi(x)/[1-\Phi(x)]$ to be monotonically increasing in x. Yet the latter property is already satisfied by a fairly general class of distributions. It can be shown that for $\overline{x}>0$,

$$\Gamma'(\overline{x};\Phi) = -[1-\Phi(\overline{x})] < 0,$$

$$\Psi'(\overline{x};\Phi) = 1-\Phi(\overline{x}) - \mu\overline{x}\phi(\overline{x}) > 0, \text{ if } \overline{x} < \overline{x}^*,$$

and

$$\Gamma'(\overline{x};\Phi) + \Psi'(\overline{x};\Phi) = -\mu\overline{x}\phi(\overline{x}) < 0,$$

where $\overline{x}^*$ satisfies $1-\Phi(\overline{x}^*) - \mu\overline{x}^*\phi^x(\overline{x}^*) = 0$. We rule out the possibility of credit rationing by requiring $V\Psi(\overline{x}^*;\Phi)$ to be no less than the opportunity cost of funds for the lender (see Williamson, 1986). Thus the domain of $\overline{x}$ we are interested in is $[0,\overline{x}^*)$ and $\Psi'(\overline{x};\Phi)>0$ on this interval. It is interesting to note that changes in the monitoring threshold (and hence the default probability) generate redistributions of expected revenues between the borrower and the lender. An increase in $\overline{x}$ reduces the share Γ received by the borrower, while raising the share Ψ received by the lender. The total effect on the returns to the two parties, however, is negative since the marginal increase in the lender's share is less than the marginal increase in the borrower's share, reflecting the additional monitoring cost born by the lender at the margin. Furthermore,

$$\begin{aligned}
\lim_{\overline{x}\to 0}\Gamma(\overline{x};\Phi) &= 1, \quad \lim_{\overline{x}\to 0}\Psi(\overline{x};\Phi) = 0, \quad \lim_{\overline{x}\to 0}[\Gamma(\overline{x};\Phi)+\Psi(\overline{x};\Phi)] = 1, \\
\lim_{\overline{x}\to 0}\Gamma'(\overline{x};\Phi) &= -1, \quad \lim_{\overline{x}\to 0}\Psi'(\overline{x};\Phi) = 1, \quad \lim_{\overline{x}\to 0}\left[\Gamma'(\overline{x};\Phi)+\Psi'(\overline{x};\Phi)\right] = 0,
\end{aligned}$$

whenever the probability density $\phi(\overline{x})$ is bounded as in Assumption 1. These limits indicate that starting from a small default rate, where the borrower grabs virtually all of the revenues, an increase in the monitoring threshold generates a nearly one-for-one transfer of returns from the borrower to the lender *without* producing any discernible effect on the sum of returns (that is, the marginal deadweight loss is practically zero).

We now apply this generic debt contract framework to the bank-firm relationship. The firm's revenue can be written as $V^f\omega$, where $V^f \equiv PF(k,l)\,\theta$ is freely observable to the bank, and ω is the risk that can be observed by the bank only with a cost.[7] The contract between the bank and the firm specifies a monitoring threshold, denoted by $\overline{\omega}$ for the firm/location specific risk ω. Conditional on the region specific productivity θ, the expected return to the firm is then given by $PF(k,l)\,\theta\Gamma^f\left(\overline{\omega};\Phi^l\right)$ and the revenue of the bank from lending to the firms in its region is $PF(k,l)\,\theta\Psi^b\left(\overline{\omega};\Phi^l\right)$, where $\Gamma^f\left(\overline{\omega};\Phi^l\right)$ and $\Psi^b\left(\overline{\omega};\Phi^l\right)$ result from substituting $\left(\overline{\omega};\Phi^l\right)$ for $(\overline{x};\Phi)$ in (2) and (3).[8]

The contract problem between the bank and its depositors specifies a monitoring threshold for the bank risk θ. To fit this into the generic setup, write the bank's revenue as $V^b\theta$, where $V^b \equiv PF(k,l)\,\Psi^b\left(\overline{\omega};\Phi^l\right)$ depends on the monitoring threshold in the bank-firm contract. Let $\overline{\theta}$ represent the monitoring threshold for θ in the bank-depositor contract. Then the expected return to the bank from the contract is $V^b\Gamma^b\left(\overline{\theta};\Phi^r\right)$ and the expected return to depositors is $V^b\Psi^d\left(\overline{\theta};\Phi^r\right)$, where $\Gamma^b\left(\overline{\theta};\Phi^r\right)$ and $\Psi^d\left(\overline{\theta};\Phi^r\right)$ obtain from substituting $\left(\overline{\theta};\Phi^r\right)$ for $(\overline{x};\Phi)$ in (2) and (3).

2.3 Optimal Competitive Contract

To motivate competitive banking assume that in principle a bank is allowed to operate beyond its region. But that entails a fixed cost. It follows that the bank in region i must offer to firms in that region contracts that maximize firms' expected return such that if bank j, $j \neq i$

[7]From the perspective of the bank, monitoring $x^f \equiv \theta\omega$ is equivalent to monitoring ω given its information in θ.

[8]By the law of large numbers, the revenue of the bank from lending to all of the firms in its region is the same as its expected revenue from lending to one firm, the expectation taken over the distribution of ω and conditional on θ.

offers the same contracts to the same firms the expected return earned by bank j will equal the opportunity cost of its funds plus the cost of operating outside region j. Otherwise bank j would offer alternative contracts with terms that are preferable to the firms and make a profit itself. If the out-of-region operating cost goes to zero, then the limit case is perfect competition for the banking industry, where each bank offers contracts that maximize the expected return to firms in its region such that the bank itself at least earns the riskless return on its funds. We focus on this limiting situation and state formally the optimal competitive contract as solving the following problem.

Problem 1.

$$\max_{k,l,\overline{\omega},\overline{\theta},N^d} PF\left(k,l\right)\Gamma^f\left(\overline{\omega};\Phi^l\right)$$

subject to

$$PF\left(k,l\right)\Psi^b\left(\overline{\omega};\Phi^l\right)\Gamma^b\left(\overline{\theta};\Phi^r\right) \geq RN^b, \tag{4}$$

$$PF\left(k,l\right)\Psi^b\left(\overline{\omega};\Phi^l\right)\Psi^d\left(\overline{\theta};\Phi^r\right) \geq RN^d, \tag{5}$$

$$R^k k + Wl \leq N^f + N^b + N^d, \tag{6}$$

where R is the risk-free nominal rate of interest. Here $PF\left(k,l\right)\Gamma^f\left(\overline{\omega};\Phi^l\right)$ is the expected return to the firm, unconditional on θ. Inequality (4) is the individual rationality (IR) constraint for the bank, which says that the bank must obtain at least what it can earn by investing all of its capital (in the financial sense) in riskless securities. The amount of the bank's financial capital equals the rental value of the physical capital stock it owns plus the injection of capital from the central bank, Z. That is, $N^b \equiv R^k K^b + Z$. Inequality (5) is the IR constraint for depositors, which says that the contract guarantees a riskless return R on their deposits. Finally, inequality (6) is the flow-of-funds constraint for firms. The total bill for firms' factor inputs is $R^k k + Wl$, which has to be covered by the internal funds of the firms themselves, $N^f \equiv R^k K^f$, and bank loans which equal the sum of bank capital N^b and deposits N^d. In Problem 1 N^f and N^b are taken as given.

Let λ^b and λ^d be the Lagrangian multipliers associated with (4) and (5), respectively, and define the "debt-equity ratios" for the bank and firms, denoted by ζ^b and ζ^f respectively, as

$$\zeta^b \equiv \frac{N^d}{N^b}, \quad \zeta^f \equiv \frac{N^b + N^d}{N^f}.$$

We place the following assumption:

Assumption 2. Either (1) $\phi^{l\prime}(\overline{\omega}) \geq 0$ and $\phi^{r\prime}(\overline{\theta}) \geq 0$ hold, or (2) $\phi^{l\prime}(\overline{\omega}) < 0$, $\phi^{r\prime}(\overline{\theta}) < 0$, and

$$-\frac{\overline{\omega}\phi^{l\prime}(\overline{\omega})}{\phi^l(\overline{\omega})} - \frac{\overline{\omega}\phi^l(\overline{\omega})}{[1-\Phi^l(\overline{\omega})]} < 1,$$
$$-\frac{\overline{\theta}\phi^{r\prime}(\overline{\theta})}{\phi^r(\overline{\theta})} - \frac{\overline{\theta}\phi^r(\overline{\theta})}{[1-\Phi^r(\overline{\theta})]} < 1$$

hold.

The assumption requires that whenever the p.d.f. $\phi^l(\overline{\omega})$ (resp. $\phi^r(\overline{\theta})$) is negative, its elasticity does not exceed the elasticity of $1-\Phi^l(\overline{\omega})$ (resp. $1-\Phi^r(\overline{\theta})$) by one. Note that (1) implies the two displayed inequalities in (2). The borderline case in between cases (1) and (2) occurs when ω and θ are distributed uniformly so that $\phi^{l\prime}(\overline{\omega}) = 0$ and $\phi^{r\prime}(\overline{\theta}) = 0$. For unimodal p.d.f.s that are increasing before the peaks are reached, Assumption 2, in particular its case (1), is easy to be satisfied whenever the default thresholds ($\overline{\omega}$ and $\overline{\theta}$), and hence the default probabilities, are not unreasonably large. The properties of the solution to Problem 1 are summarized in the following proposition.

Proposition 1. The solution to Problem 1 satisfies the following conditions:

$$F_k(k,l) = q(\overline{\omega},\overline{\theta})\, R\frac{R^k}{P}, \tag{7}$$

$$F_l(k,l) = q\left(\overline{\omega},\overline{\theta}\right) R\frac{W}{P}, \tag{8}$$

$$\frac{\Psi^d\left(\overline{\theta};\Phi^r\right)}{\Gamma^b\left(\overline{\theta};\Phi^r\right)} = \zeta^b, \tag{9}$$

$$q\left(\overline{\omega},\overline{\theta}\right)\Psi^b\left(\overline{\omega};\Phi^l\right)\left[\Gamma^b\left(\overline{\theta};\Phi^r\right)+\Psi^d\left(\overline{\theta};\Phi^r\right)\right] = \frac{\zeta^f}{1+\zeta^f}, \tag{10}$$

where

$$q\left(\overline{\omega},\overline{\theta}\right) \equiv \frac{\lambda^d}{\Gamma^f\left(\overline{\omega};\Phi^l\right)+\lambda^b\Psi^b\left(\overline{\omega};\Phi^l\right)\Gamma^b\left(\overline{\theta};\Phi^r\right)+\lambda^d\Psi^b\left(\overline{\omega};\Phi^l\right)\Psi^d\left(\overline{\theta};\Phi^r\right)}, \tag{11}$$

$$\lambda^b = -\frac{\Gamma^{f\prime}\left(\overline{\omega};\Phi^l\right)}{\Psi^{b\prime}\left(\overline{\omega};\Phi^l\right)}\frac{\Psi^{d\prime}\left(\overline{\theta};\Phi^r\right)}{\Gamma^b\left(\overline{\theta};\Phi^r\right)\Psi^{d\prime}\left(\overline{\theta};\Phi^r\right)-\Gamma^{b\prime}\left(\overline{\theta};\Phi^r\right)\Psi^d\left(\overline{\theta};\Phi^r\right)}, \tag{12}$$

$$\lambda^d = \frac{\Gamma^{f\prime}\left(\overline{\omega};\Phi^l\right)}{\Psi^{b\prime}\left(\overline{\omega};\Phi^l\right)}\frac{\Gamma^{b\prime}\left(\overline{\theta};\Phi^r\right)}{\Gamma^b\left(\overline{\theta};\Phi^r\right)\Psi^{d\prime}\left(\overline{\theta};\Phi^r\right)-\Gamma^{b\prime}\left(\overline{\theta};\Phi^r\right)\Psi^d\left(\overline{\theta};\Phi^r\right)}. \tag{13}$$

The factor $q\left(\overline{\omega},\overline{\theta}\right) > 1$ for all $\overline{\omega},\overline{\theta} > 0$ and $\lim_{\overline{\omega},\overline{\theta}\to 0} q\left(\overline{\omega},\overline{\theta}\right) = 1$. Furthermore,

$$q_{\overline{\omega}}\left(\overline{\omega},\overline{\theta}\right) \equiv \frac{\partial q\left(\overline{\omega},\overline{\theta}\right)}{\partial\overline{\omega}} > 0, \quad q_{\overline{\theta}}\left(\overline{\omega},\overline{\theta}\right) \equiv \frac{\partial q\left(\overline{\omega},\overline{\theta}\right)}{\partial\overline{\theta}} > 0$$

under Assumption 2.

Conditions (7)-(10) capture the notion that monetary frictions *and* financial frictions lead to inefficient use of resources. Equations (7) and (8) are the first-order conditions for factor demand. They state that capital and labor inputs are employed up to the points where their marginal products equal real factor prices, times the gross nominal interest rate R, and times an object called q which is determined by terms of the financial contract, with both R and q larger than or equal to one. In the first best world productive efficiency requires equating the marginal product of factor inputs to their real prices. In our model, however, there are various sources of frictions that prevent the economy from achieving the first best.

The first friction arises from the requirement that factor market transactions must use cash, a friction we call monetary friction. A gross nominal interest rate that is strictly greater than one creates a wedge between the marginal products of factor inputs and their real prices, leading to underemployment of factor inputs. The second and third sources of distortions, measured in combination by the *financial friction indicator* $q\left(\overline{\omega}, \overline{\theta}\right)$, lie in the agency cost problem inherent in financial contracting. It can be shown that $q\left(\overline{\omega}, \overline{\theta}\right) \geq 1$ and if either $\overline{\omega} > 0$ or $\overline{\theta} > 0$ then $q\left(\overline{\omega}, \overline{\theta}\right)$ will be strictly greater than one. Here $\overline{\omega} > 0$ indicates a positive default rate by the firms and reflects the agency cost in the bank-firm relationship. This is what the existing literature on credit market imperfections typically focuses on. On the other hand, $\overline{\theta} > 0$ corresponds to a positive default rate by the banks (to depositors) and reflects the agency cost in the bank-depositor relationship. These financial frictions create additional wedges between the marginal products of factor inputs and their real prices. The variable $q\left(\overline{\omega}, \overline{\theta}\right)$ measures the overall distortions caused by the conventionally considered credit market frictions and the sort of banking frictions that this paper introduces. Again, the presence of financial frictions leads to underemployment of resources. To highlight the role played by financial frictions, the general equilibrium model to be presented in Section 3, in which the two-sided financial contract is embedded, will be constructed such that monetary transfers to the banking system will not have any effect on the nominal risk-free rate R, but will influence the financial friction indicator q.

Equations (9) and (10) reflect the fact that the optimal contract entails binding IR constraints for both the bank and depositors. Essentially, the terms of contract dictate a division of expected revenues between borrowers and lenders. Equation (9) says that in the bank-depositor contract the share of expected revenue received by the depositors, relative to the share received by the bank, is positively related to the bank's debt-equity ratio. Since $\Psi^{d}\left(\overline{\theta}; \Phi^{r}\right) / \Gamma^{b}\left(\overline{\theta}; \Phi^{r}\right)$ is increasing in $\overline{\theta}$, the bank's default probability increases along with $\overline{\theta}$ when it has a larger debt-equity ratio ζ^{b}. Equation (10) says that the total share of expected revenue that goes to the bank *and* the depositors, adjusted for the factor $q\left(\overline{\omega}, \overline{\theta}\right)$, is positively related to the firms' debt-equity ratio ζ^{f}.

3 General Equilibrium

We now embed the two-sided financial contract articulated in the previous section to a full-blown general equilibrium model. The goal is to analyze how a bank recapitalization policy, taking the form of central bank money injection into the banking system, will affect the economy. Our main result is that such a policy is non-neutral, that is, has effects on employment and output as well as the default rates, in a world with banking frictions of the kind introduced above. This stands in sharp contrast to a neutrality result that obtains in a world where banking frictions are absent.

3.1 The Environment

Time is discrete and there is a representative household. Following Lucas (1990), we model the household as a multi-member "family". There are three types of members in the household—workers, shoppers, and investors. Workers supply labor and earn wage income in the labor market, shoppers carry cash to purchase consumption from the goods market, while investors engage in financial transactions (with the banks). At the end of each period these members reconvene and submit all of their income and cash holdings to the family. This multi-member "family" device permits the study of situations in which different agents face different trading opportunities while still retaining the convenience of the representative-household fiction. The investors here correspond to the "depositors" in the previous section. The household as a whole owns all banks and firms in the economy.

The representative household chooses consumption and leisure stream to maximize the following criterion function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_t\right) - \nu \log\left(1 - L_t\right)\right], \tag{14}$$

where C and L are consumption and hours worked, respectively, $\beta \in (0, 1)$ is the discount factor, $\nu > 0$ is a constant that weighs leisure relative to consumption, and E_0 is the expectation operator conditional on time 0 aggregate information. The time endowment is normalized to be 1 for each period. The assumption that utility is logarithmic and separable in consumption and leisure allows us to arrive at a closed-form solution to the model.

Let M_t denote the quantity of money outstanding at the beginning of period t. In equilibrium this is all held by the household. In period t the monetary authority injects $Z_t \equiv M_{t+1} - M_t$ into the economy by means of nominal transfers to the banking system, which effectively recapitalizes the banks. The quantity of money injection is public information so that the model assumes full information on aggregate variables. In the sequel we normalize all nominal quantities and prices by M_t, following the practice of Christiano (1991) and Christiano and Eichenbaum (1992). The resultant variables will be denoted by corresponding lowercase letters. We model $z_t \equiv Z_t/M_t$ as a simple i.i.d. process, so that the complication that would arise from the "anticipated inflation effect" of a money shock can be abstracted away (see Christiano, 1991 and Williamson, 2005 for an exposition). It is reasonable to treat z_t as random as of time $t-1$ since agents, even including the policy makers, did not know exactly what policy is going to be adopted in future time.

After observing the value of z_t, the household chooses its portfolio by dividing the nominal balance m_t between savings n_t^d, to be deposited in the banks, and cash holdings $m_t - n_t^d$ (these quantities obtain after normalization by M_t). We assume that there is always a zero supply of risk-free government bonds, so that in equilibrium all of the household's savings are in the form of deposits in the banks. Nevertheless, the zero-supply risk-free bonds can still be priced (at $1/R_t$). The bank-depositor contract ensures that the risk-free return R_t accrues to household deposits n_t^d. Contrary to the limited participation literature, we assume that there is no cost or other barrier for the household to adjust its nominal savings in response to the shock. Hence our model also abstracts away the "liquidity effect" of a money shock. By removing both the anticipated inflation effect and the liquidity effect, we are able to make the risk-free nominal interest rate fixed in equilibrium, allowing our model to isolate the banking-frictions channel through which monetary policy affects the economy.

There is a cash-in-advance (CIA) constraint, standard in the literature, on the household's purchase of consumption:

$$p_t C_t \leq m_t - n_t^d + w_t L_t, \tag{15}$$

where $p_t \equiv P_t/M_t$ is the normalized price level. This formulation is consistent with our previous assumption that firms must acquire cash to purchase labor inputs (from workers). Implicit in (15) is the notion that the wage income can be used to purchase consumption, along with the cash balance the household set aside at the beginning of period t. Formulation like this allows us to derive a standard quantity equation of money (see the next subsection). The household's cash holdings evolve according to

$$m_{t+1}(1+z_t) = \left(m_t - n_t^d + w_t L_t - p_t C_t\right) + R_t n_t^d + \pi_t, \tag{16}$$

where the term in the parentheses on the right-hand side is the unspent cash of the shoppers, $R_t n_t^d$ is the gross return to the investors from the financial market, and π_t is the profit of the banks and firms, paid out to the household in accordance with ownership.

The household maximizes (14) subject to (15) and (16). Its optimal plan obeys the following conditions:

$$\frac{\nu C_t}{1-L_t} = \frac{w_t}{p_t}, \tag{17}$$

$$E_t\left\{\frac{1}{p_t C_t} - \beta \frac{R_t}{p_{t+1} C_{t+1}(1+z_t)}\right\} = 0, \tag{18}$$

where E_t is the expectation operator conditional on time-t aggregate information. Equation (17) is the first-order condition for labor supply, while equation (18) is the standard consumption/saving Euler equation, modified to the current monetary environment.

Finally, we assume that the production function takes the standard Cobb-Douglas form:

$$F(K, L) = K^{\alpha} L^{1-\alpha}, \quad \alpha \in (0, 1),$$

where we have used K and L to replace k and l in (1) in anticipation of factor-market clearing.

3.2 Competitive Equilibrium Defined

We now define a competitive equilibrium for our model economy with banking frictions and two-sided financial contracting.

Definition 1. A *competitive equilibrium* of the model economy is a policy $\{z_t\}_{t=0}^{\infty}$, an allocation $\left\{C_t, , m_{t+1}, n_t^d, K, L_t\right\}_{t=0}^{\infty}$, a price system $\left\{p_t, w_t, r_t^k, R_t\right\}_{t=0}^{\infty}$, and terms of financial contract $\left\{\overline{\omega}_t, \overline{\theta}_t\right\}_{t=0}^{\infty}$ such that

i. Given the policy and prices, $\left\{C_t, , m_{t+1}, n_t^d, L_t\right\}_{t=0}^{\infty}$ solves the household's problem and satisfies (17), (18), and the equality version of (15), with $m_{t+1} = 1$ for all time.

ii. Given the policy and prices, $\left\{K, L_t, \overline{\omega}_t, \overline{\theta}_t\right\}_{t=0}^{\infty}$ solves the financial contracting problem, Problem 1, and satisfies (7)-(10).

iii. Loan market and goods market clear:

$$w_t L_t = n_t^d + z_t, \tag{19}$$

$$C_t = F(K_t, L_t)\, \varphi\left(\overline{\omega}_t, \overline{\theta}_t\right), \tag{20}$$

where

$$\varphi\left(\overline{\omega}_t, \overline{\theta}_t\right) \equiv \Gamma^f\left(\overline{\omega}_t; \Phi^l\right) + \Psi^b\left(\overline{\omega}_t; \Phi^l\right)\left[\Gamma^b\left(\overline{\theta}_t; \Phi^r\right) + \Psi^d\left(\overline{\theta}_t; \Phi^r\right)\right], \tag{21}$$

and

iii. $R_t \geq 1$ for all time.

In (21), $\varphi\left(\overline{\omega}_t, \overline{\theta}_t\right) < 1$ whenever $\overline{\omega}_t > 0$, or $\overline{\theta}_t > 0$, or both, reflecting the direct deadweight loss due to the agency cost problems.[9] The loan market clearing condition takes the form of (19) because the firms' rental payment on capital are covered by the rental value of the stock of capital owned by the firms and banks. It remains that their wage bills are to be ultimately financed by household deposits and the monetary authority's transfers to the banks.[10]

For analytic purpose it will be especially convenient to look at the behavior of the model economy around a limiting situation where no default by either the banks or the firms occurs. We define such a situation as follows.

Definition 2. A *zero-default limit equilibrium* is the limit of the model economy's competitive equilibrium obtained either as the monitoring cost parameter μ tends to zero or as the distributions for θ and ω tend to degeneracy.

In the neighborhood of the model's zero-default limit the default rates are small. According to Fisher (1999), the historical average of bankruptcy rate is indeed quite small. Using the Dun & Bradstreet dataset, he finds an average quarterly bankruptcy rate of 0.974%. This does not, however, mean that the distortions caused by financial frictions are negligible. In fact, Bernanke, Gertler, and Gilchrist (1999) show that a similar magnitude of bankruptcy rate is consistent with an average external finance premium, or risk spread, of about two hundred basis points per annum.[11] See also Jin, Leung, and Zeng (2010). Therefore the focus of our analysis

[9] Remember that there is also an indirect social loss due to the distortions on the marginal costs of production caused by $q > 1$.

[10] To write the loan market clearing condition in full, we have $r_t^k K_t + w_t L_t = n_t^f + n_t^b + n_t^d = r_t^k \left(K_t^f + K_t^b\right) + n_t^d + z_t$. This simplifies to (19) since $K_t = K_t^f + K_t^b$.

[11] In Bernanke et al. (1999), the empirical measure of the risk spread is taken to be the difference between the prime lending rate and the six-month T-bill rate.

in the neighborhood of the zero-default limit does not entail a large deviation from the reality.

4 The Effects of Bank Recapitalization

4.1 Characterization of Equilibrium

As the policy process z_t is assumed to be stationary, the equilibrium allocation, prices, and contract terms in period t are functions of z_t, the functions being invariant with respect to t. Hence the time subscripts will be dropped in the subsequent analysis whenever possible. To ensure that an equilibrium exists, we place a bound on the strength of the policy.

Assumption 3. z_t is i.i.d., normalized to have zero mean, $z_t > -1$ (positive money supply) and

$$\max\left\{\frac{1}{1-\alpha}\frac{z_t}{1+z_t}, \frac{K}{K^b}\frac{1}{\alpha}\frac{-z_t}{1+z_t}\right\} < \beta E\left(\frac{1}{1+z_{t+1}}\right)$$

for all t, where $E(\cdot)$ denotes unconditional expectations.[12]

Basically what this assumption says is that the realized magnitude of transfers from the monetary authority should not be too large. A policy will be said to be *admissible* if it satisfies Assumption 3. Below we develop an algorithm to solve for the equilibrium. In preparation we note the following.

(1) The loan market clearing condition (19) together with the binding CIA constraint (15) imply the quantity equation:

$$pC = 1 + z. \tag{22}$$

(2) The risk-free nominal interest rate R_t is constant, i.e., independent of z_t. Substitution of the quantity equation (22) into the Euler equation (18) gives

$$R_t = \left[\beta E_t\left(\frac{1}{1+z_{t+1}}\right)\right]^{-1}. \tag{23}$$

[12]Note that $E\left(\frac{1}{1+z_{t+1}}\right) > 1$ even though $E(z_{t+1}) = 1$.

The i.i.d. assumption on z_t then implies the constancy of R_t.

(3) The debt-equity ratio of the firms is constant:

$$\zeta^f = \frac{n^b + n^d}{n^f} = \frac{\left(r^k K^b + z\right) + (wL - z)}{r^k K^f} = \frac{K^b}{K^f} + \frac{wL}{r^k K}\frac{K}{K^f} = \frac{K^b}{K^f} + \frac{1-\alpha}{\alpha}\frac{K}{K^f}. \tag{24}$$

The last equality follows from the Cobb-Douglas form of technology.

(4) The debt-equity ratio of the banks is given by

$$\zeta^b \equiv \frac{N^d}{N^b} = \frac{rK + wL - \left(rK^f + rK^b + z\right)}{rK^b + z} = \frac{wL - z}{rK^b + z}. \tag{25}$$

Absent the term z, ζ^b is also a constant, given by $(1-\alpha)K/\left(\alpha K^b\right)$. Hence by construction our model features a debt-equity ratio of the firms that is unaffected by bank recapitalization policy, along with a debt-equity ratio of the banks that can be perturbed by the policy. This feature allows us to highlight the bank capital/liability side of our story.

(5) The monitoring thresholds $\left(\overline{\omega}, \overline{\theta}\right)$ are functions of ζ^b implicitly defined by (9)-(10). The following proposition summarizes the impact of ζ^b on $\left(\overline{\omega}, \overline{\theta}\right)$ and related variables.

Proposition 2. In equilibrium the banks' debt-equity ratio ζ^b is a *sufficient statistic* for the monitoring thresholds $\left(\overline{\omega}, \overline{\theta}\right)$, as well as the financial friction indicator q and the total direct deadweight loss $(1-\varphi)$. In particular,

$$\frac{d\overline{\theta}}{d\zeta^b} > 0,\ \forall\ \overline{\omega}, \overline{\theta} > 0; \qquad \lim_{\overline{\omega}, \overline{\theta} \to 0} \frac{d\overline{\theta}}{d\zeta^b} > 0;$$

$$\lim_{\overline{\omega} \to 0} \frac{d\overline{\omega}}{d\zeta^b} = 0,\ \forall\ \overline{\theta} > 0; \qquad \lim_{\overline{\theta} \to 0} \frac{d\overline{\omega}}{d\zeta^b} < 0,\ \forall\ \overline{\omega} > 0;$$

$$\lim_{\overline{\omega}, \overline{\theta} \to 0} \frac{dq}{d\zeta^b} > 0; \qquad \lim_{\overline{\omega}, \overline{\theta} \to 0} \frac{d\varphi}{d\zeta^b} = 0.$$

According to the proposition, an increase in the banks' debt-equity ratio results in a higher rate of default by the banks to the depositors, which holds even in the neighborhood of the zero-default limit. This is because the monitoring threshold $\overline{\theta}$ governs the relative return of the depositors to the banks, which must equal the banks' debt-equity ratio according to (9). When the banks become more leveraged, $\overline{\theta}$ must increase to redistribute returns toward the depositors. Note also that the limit case corresponds to the situation where there is no social loss resulting either directly from monitoring (i.e., φ reaches its upper limit 1) or indirectly from distortions to marginal costs of production (i.e., q reaches its lower limit 1). In that case there is still a positive marginal effect on the financial friction indicator q of an increase in the banks' debt-equity ratio ζ^b, even though its marginal effect on φ is zero.

In general, the effect of changes in ζ^b on the monitoring threshold $\overline{\varpi}$ in the bank-firm contract is ambiguous. Given $\overline{\theta}$, condition (10) determines $\overline{\varpi}$, which says that the total share of returns that go to the banks and depositors, adjusted for the factor q, is positively related to the firms' debt-equity ratio ζ^f. To understand the role of q, rewrite the condition as $\Psi^b\left(\Gamma^b + \Psi^d\right) = \left[\zeta^f / \left(1 + \zeta^f\right)\right] / q$. A larger value of q, signifying a greater extent of financial frictions, lowers the total share of returns received by the banks and depositors due to the additional monitoring costs they have to bear. An increase in ζ^b, and hence in $\overline{\theta}$, has two opposing effects on $\overline{\varpi}$. Its positive impact on q means that Ψ^b and therefore $\overline{\varpi}$ should decline, while its negative influence on $\left(\Gamma^b + \Psi^d\right)$ implies that Ψ^b and therefore $\overline{\varpi}$ should increase. For any $\overline{\varpi} > 0$, the latter effect vanishes as $\overline{\theta} \to 0$, hence $\lim_{\overline{\theta}\to 0} d\overline{\varpi}/d\zeta^b < 0$. For any $\overline{\theta} > 0$, both effects are weighed by a zero Ψ^b in the limit as $\overline{\varpi} \to 0$, hence $\lim_{\overline{\varpi}\to 0} d\overline{\varpi}/d\zeta^b = 0$. This also implies $\lim_{\overline{\varpi},\overline{\theta}\to 0} d\overline{\varpi}/d\zeta^b = 0$.[13] In spite of all these different possibilities, the overall effect of an increase in ζ^b is to enlarge the financial friction indicator q in the limit situations (as $\overline{\varpi}, \overline{\theta} \to 0$), indicating that the movement in the monitoring threshold specified in the bank-depositor contract is the dominant force in

[13] Note that $d\overline{\varpi}/d\zeta^b > 0$ for some $\overline{\varpi}, \overline{\theta} > 0$ remains a possibility if the fall in $\left(\Gamma^b + \Psi^d\right)$ due to the increase in $\overline{\theta}$ dominates the rise in q for some $\overline{\varpi}, \overline{\theta} > 0$.

driving the overall change in distortions caused by financial frictions.

Our strategy for solving the equilibrium is to collapse all the equilibrium conditions into one single equation as follows:

$$(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}=q\left(\overline{\omega},\overline{\theta}\right)\varphi\left(\overline{\omega},\overline{\theta}\right)R\nu\frac{K^{\alpha}L^{1-\alpha}}{1-L},$$

Essentially, this equation characterizes equilibrium in the labor market: it is obtained by using the labor supply condition (17) to substitute $\nu C/(1-L)$ for w/p in the labor demand condition (8), and by further substituting $K^{\alpha}L^{1-\alpha}\varphi\left(\overline{\omega},\overline{\theta}\right)$ for C in accordance with the resource constraint. Obviously this condition can be further simplified to

$$\frac{1-L}{L}=\frac{\nu}{1-\alpha}Rq\left(\overline{\omega},\overline{\theta}\right)\varphi\left(\overline{\omega},\overline{\theta}\right). \tag{26}$$

The left-hand side of (26) is a decreasing function of L. In general the right-hand side is also a function of L (and the policy variable z as well), which we now derive in the following steps. First, by substituting the quantity equation (22) into the labor supply condition (17) we have

$$w=\nu\frac{1+z}{1-L}. \tag{27}$$

Second, dividing (7) by (8) yields $r/w=(\alpha/K)/[(1-\alpha)/L]$, which implies

$$r=\frac{\alpha}{1-\alpha}\nu(1+z)\frac{1}{K}\frac{L}{1-L}. \tag{28}$$

Third, substituting of (27) and 28 into $\zeta^{b}=(wL-z)/\left(rK^{b}+z\right)$ gives

$$\zeta^{b}=\frac{\nu(1+z)\frac{L}{1-L}-z}{\frac{\alpha}{1-\alpha}\frac{K^{b}}{K}\nu(1+z)\frac{L}{1-L}+z}. \tag{29}$$

Finally, solve for $\overline{\omega}$ and $\overline{\theta}$ given ζ^{b} using (9)-(10). This also allows us to compute $q\left(\overline{\omega},\overline{\theta}\right)$ and $\varphi\left(\overline{\omega},\overline{\theta}\right)$ as functions of ζ^{b}, and hence as functions of L (and z). The effects of changes in ζ^{b} on these magnitudes were summarized in Proposition 2. The following proposition concerns the existence and uniqueness of equilibrium.

Proposition 3. Under Assumption 3, in the neighborhood of the zero-default limit, a competitive equilibrium of the model economy with banking frictions and two-sided financial contracting exists and is unique.

A unique competitive equilibrium of the model economy with banking frictions exists if and only if a unique solution to condition (26) exists for all admissible policy, that is, for all z satisfying Assumption 3. Figure 1 illustrates the determination of L for given z. As shown in the figure the left-hand side of (26) is a monotonically decreasing function of L, with $\lim_{L\to 0}(1-L)/L=\infty$ and $\lim_{L\to 1}(1-L)/L=0$. To see how the right-hand side depends on L one needs only see how ζ^b depends on L. Assumption 3 guarantees that $\zeta^b>0$. That is, it rules out those extreme policies that would force either household deposits or bank net worth to be nonpositive. It can be shown that the banks' debt-equity ratio ζ^b is a monotonic function of L for all admissible z. In the light of Proposition 2, $\lim_{\overline{\omega},\overline{\theta}\to 0} d(q\varphi)/d\zeta^b>0$. Hence in the neighborhood of the zero-default limit, the right-hand side of (26) is a monotonic, positive, finite-valued, continuous function of L for all z.[14] Therefore the solution to condition (26) exists and is unique for all admissible z, implying that a competitive equilibrium with banking frictions exists and is unique in the neighborhood of the zero-default limit.

[Insert Figure 1 about here.]

4.2 The Non-Neutrality of Money with Banking Frictions

We are now ready to analyze the effects of the bank recapitalization policy of the monetary authority.

[14] Three cases are shown in Figure 1, corresponding to upward sloping, horizontal, and downward sloping right-hand side (RHS) of (26), respectively. All three RHS curves cut the left-hand side (LHS) of (26) from below.

Proposition 4. In the competitive equilibrium of the model economy with banking frictions, an increase in the monetary authority's transfer z to the banking system, i.e., a bank recapitalization policy action, is *non-neutral.* In particular, in the neighborhood of the zero-default limit it raises employment, output, and consumption for all admissible policy. In addition, it lowers the rate of default by the banks $\Phi^r\left(\overline{\theta}\right)$ and the financial friction indicator q for all $z \leq 0$ and for all $z > 0$ sufficiently close to zero.

Although the assumptions we have made allow the exposition to be as clear as possible, the non-neutrality result is actually more general than it appears. It is actually quite straightforward to extend this result to general specifications of preferences, technology, and policy process. To prove by contradiction, suppose that a change in z has no effect on employment, output, real factor prices, and the default thresholds. Then the debt-equity ratio of banks ζ^b must not change according to Proposition 2. But this contradicts (25), according to which ζ^b must change if employment and real factor prices do not change. Hence the bank recapitalization policy must be non-neutral. The direction of change, however, is more easily seen under the simplifying assumptions we have made. Graphically, the curve representing the right-hand side of (26) in Figure 1 slides down when z takes a larger value, resulting in a higher level of employment.

In the real world, financial frictions are often manifested in various measures of interest rate spreads, often referred to as "external finance premia". It is hence useful to come up with a measure of the interest rate spread charged on the banks' external finance. The (gross) interest rate at which the banks borrow from the depositors, denoted by R^b, is simply the non-default payment specified by the bank-depositor contract divided by the amount of deposits:

$$R^b = \frac{PF\left(k,l\right)\Psi^b\left(\overline{\omega};\Phi^l\right)\overline{\theta}}{N^d}.$$

Using the binding IR constraint for depositors (5), we obtain

$$\frac{R^b - R}{R} = \frac{\overline{\theta}}{\Psi^d\left(\overline{\theta};\Phi^r\right)} - 1. \tag{30}$$

Equation (30) implies that the spread between the banks' borrowing rate and the risk-free rate, measured as a fraction of the latter, will be larger when the banks' default threshold $\overline{\theta}$ gets larger. A corollary to Proposition 4 thus obtains.

Corollary 5. For the situations identified in Proposition 4 that feature a negative effect on $\overline{\theta}$ of an increase in z, the increase in z also lowers the premium charged on the banks' external finance as defined in (30).

4.3 The Neutrality of Money without Banking Frictions

The non-neutrality result we just stated depends crucially on the presence of banking frictions. Without such frictions, a neutrality result will obtain. To demonstrate this we can take away banking frictions from the model articulated above simply by assuming that the distribution of the region specific productivity is degenerate, or by assuming that monitoring bank revenues is costless. In this case the optimal competitive contract is to solve the following problem:

Problem 2.

$$\max_{k,l,\overline{\omega},n^d} pF\left(k,l\right)\Gamma^f\left(\overline{\omega};\Phi^l\right)$$

subject to

$$pF\left(k,l\right)\Psi^b\left(\overline{\omega};\Phi^l\right) \geq R\left(n^b + n^d\right), \tag{31}$$

and the flow-of-funds constraint (6) with the nominal quantities and prices normalized by M. The contract is designed to maximize the firms' expected return while ensuring that the banks and depositors taken together receive a return no less than the opportunity cost of their funds. Inequality (31) fully summarizes the IR constraints for both the banks and the depositors.

As long as the return to the banks lies in the interval $\left[Rn^b, pF(k,l)\,\Psi^b\left(\overline{\omega};\Phi^l\right) - Rn^d\right]$, the depositors' IR constraint will be satisfied.

In essence the Modigliani-Miller theorem applies to banks in the world without banking frictions, where the banks' capital structure is irrelevant for the determination of their lending activities. In particular, the banks' debt-equity ratio does not affect in any way the rates of default, the real amount of bank loans, and employment of factor inputs.[15] That is, bank recapitalization by the monetary authority is neutral and the classical dichotomy holds, as indicated in the following proposition.

Proposition 6. When $\Phi^r(\cdot)$ is degenerate or when it is costless to monitor bank revenues, a change in z is neutral in the sense that L, C, $\overline{\omega}$, r/p, and w/p are unaffected by the change.

As shown in the appendix, the financial friction indicator q is now solely a function of the monitoring threshold $\overline{\omega}$ on firm risk, which in turn is determined by the firms' debt-equity ratio ζ^f according to the following condition:

$$q(\overline{\omega})\,\Psi^B\left(\overline{\omega};\Phi^l\right) = \frac{\zeta^f}{1+\zeta^f}.$$

Hence there is no role for the bank debt-equity ratio ζ^b to play. Instead, the bank debt-equity ratio passively adjusts to reflect loan market clearing as real deposits decline to offset the increase in bank capital (in the financial sense) brought about by the increase in z. Since output is unchanged, the price level rises proportionately with the money stock.

Although under our assumptions the firms' debt-equity ratio ζ^f is fixed in equilibrium, the neutrality result holds generally when banking frictions are absent. The fixity of the equilibrium ζ^f is inherited from the earlier model with banking frictions, where the constancy of the equilibrium ζ^f allowed us to focus on the role played by the banks' debt-equity ratio. It is important

[15] The rate of default by the banks is identically zero and the rate of default by the firms is given by $\Phi^l(\overline{\omega})$.

to note that in deriving the neutrality result without banking frictions we still allow for the presence of credit market frictions as conventionally conceived, i.e., the agency cost problem due to costly verification of firm revenues. Bank recapitalization by the monetary authority is nevertheless neutral with the presence of such frictions.[16] Taking this result and the previous non-neutrality result together, we conclude that it is precisely the presence of banking frictions (and the fact that banks are the institutions being recapitalized by the monetary authority) that is responsible for the positive effects on employment and output of the recapitalization policy.

5 Conclusions

Our study has mainly concerned the effects of bank recapitalization by the monetary authority and the analysis has been carried out in a highly stylized model. Nevertheless, the theoretical framework developed in this paper can be extended to study a wide spectrum of issues related to policy and regulation, as well as the monetary transmission mechanism, in perhaps more realistic ways. First, nominal rigidities and richer dynamics can be introduced to allow for a quantitative assessment of the effects of policy. Second, deposit insurance can be incorporated in order to study the effects of raising the limit of deposit insurance, as was implemented in the U.S. in 2008. Third, one can consider situations where some sort of capital adequacy requirements bind. In those situations, bank recapitalization policy may work through relaxing these constraints. Fourth, the model can be extended to allow changes in asset prices to affect the net worth of banks (and firms), as in Bernanke, Gertler, and Gilchrist (1999), Christiano, Motto, and Rostagno (2009) and Jin, Leung, and Zeng (2010).[17] Fourth, our analysis can be extended to include credit rationing as a possible equilibrium outcome as in Williamson (1986)

[16] With only credit market frictions present, some form of nominal rigidities such as price or wage stickiness is needed for the policy to generate real effects.

[17] In models with debt contract, Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2009) consider the price of capital, while Jin, Leung, and Zeng (2010) consider the price of real estate in addition.

so that another dimension in which policy exerts influence on the economy can be explored.[18] Finally, although the purpose of the present paper is to offer a theory on how policy affects the real economy in an otherwise unchanging environment, it would clearly be desirable in future research to model a crisis situation and investigate the optimal policy response to the crisis. We conclude that thorough analysis of frictions in the banking sector should be an integral part of future research on the interaction of money, finance, and the macroeconomy.

[18] In our setup one can imagine two possible types of credit rationing. The first is rationing on the banks' asset side, where firms are unable to obtain the bank loans they desire. This type of credit rationing has been extensively studied in the literature (e.g., Stiglitz and Weiss, 1981 and Williamson, 1986). The second type is rationing on the banks' liability side, where banks are unable to raise the loanable funds they desire. The latter type of credit rationing is an interesting aspect to explore in future research.

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Appendix: Proof of Propositions

In this appendix the arguments of functions, such as $\left(\overline{\omega};\Phi^{l}\right)$, $\left(\overline{\theta};\Phi^{r}\right)$, and $\left(\overline{\omega},\overline{\theta}\right)$, will be omitted to avoid cluttering of notations. No confusion will arise, though.

Proof of Proposition 1.

We first show that the first-order conditions (7)-(10) hold. Let λ^{b} and λ^{d} be the Lagrangian multipliers for (4) and (5), respectively. Then the first-order conditions with respect to $\overline{\omega}$ and $\overline{\theta}$ are

$$\Gamma^{f\prime} + \Psi^{b\prime}\left(\lambda^{b}\Gamma^{b} + \lambda^{d}\Psi^{d}\right) = 0, \tag{A.1}$$

$$\lambda^{b}\Gamma^{b\prime} + \lambda^{d}\Psi^{d\prime} = 0. \tag{A.2}$$

Equations (A.1) and (A.2) imply (12) and (13). The first-order conditions with respect to k and l are given by (7) and (8) with q defined in (11). The linear homogeneity of $f(\cdot)$ together with (7) and (8) imply

$$PF = qR\left(R^{k}k + Wl\right). \tag{A.3}$$

At the optimum constraints (4) and (5) both bind. Substituting (A.3) into the equality version of (4) and (5) yields

$$q\left(R^{k}k + Wl\right)\Psi^{b}\Gamma^{b} = N^{b}, \tag{A.4}$$

$$q\left(R^{k}k + Wl\right)\Psi^{b}\Psi^{d} = N^{d}. \tag{A.5}$$

Dividing (A.5) by (A.4) gives (9). Adding (A.4) and (A.5) and using the equality version of (6) gives (10).

We then show that $q > 1$ for all $\overline{\omega}, \overline{\theta} > 0$ and $\lim_{\overline{\omega}, \overline{\theta} \to 0} q = 1$. Note that $\left(-\Psi^{D\prime}/\Gamma^{B\prime}\right) < 1$ and $\left(-\Psi^{B\prime}/\Gamma^{F\prime}\right) < 1$ implies

$$\begin{aligned} q^{-1} &= \Psi^B \Psi^D - \Psi^B \Gamma^B \frac{\Psi^{D\prime}}{\Gamma^{B\prime}} - \Gamma^F \frac{\Psi^{B\prime}}{\Gamma^{F\prime}} \left(\Psi^D - \Gamma^B \frac{\Psi^{D\prime}}{\Gamma^{B\prime}} \right) \\ &< \left(\Gamma^F + \Psi^B\right)\left(\Gamma^B + \Psi^D\right) \\ &< 1 \end{aligned}$$

for all $\overline{\omega}, \overline{\theta} > 0$. Taking limit,

$$\lim_{\overline{\omega}, \overline{\theta} \to 0} q^{-1} = \lim_{\overline{\omega}, \overline{\theta} \to 0} \Gamma^F \frac{\Psi^{B\prime}}{\Gamma^{F\prime}} \Gamma^B \frac{\Psi^{D\prime}}{\Gamma^{B\prime}} = (+1)(-1)(+1)(-1) = 1.$$

Finally we determine the signs of $q_{\overline{\omega}}^{-1} \equiv \partial q^{-1}/\partial \overline{\omega}$ and $q_{\overline{\theta}}^{-1} \equiv \partial q^{-1}/\partial \overline{\theta}$. We obtain from differentiating (11)

$$\begin{aligned} q_{\overline{\omega}}^{-1} &= \Gamma^f \left(\Gamma^b \frac{\Psi^{d\prime}}{\Gamma^{b\prime}} - \Psi^d \right) \frac{\Psi^{b\prime\prime}\Gamma^{f\prime} - \Psi^{b\prime}\Gamma^{f\prime\prime}}{\Gamma^{f\prime 2}}, \\ q_{\overline{\theta}}^{-1} &= \Gamma^b \left(\Gamma^f \frac{\Psi^{b\prime}}{\Gamma^{f\prime}} - \Psi^b \right) \frac{\Psi^{d\prime\prime}\Gamma^{b\prime} - \Psi^{d\prime}\Gamma^{b\prime\prime}}{\Gamma^{b\prime 2}}. \end{aligned}$$

But, $\Gamma^b \frac{\Psi^{d\prime}}{\Gamma^{b\prime}} - \Psi^d < 0$, $\Gamma^f \frac{\Psi^{b\prime}}{\Gamma^{f\prime}} - \Psi^b < 0$, and

$$\Psi^{b\prime\prime}\Gamma^{f\prime} - \Psi^{b\prime}\Gamma^{f\prime\prime} = \mu \phi^l(\overline{\omega}) \left[1 - \Phi^l(\overline{\omega})\right] \left\{ 1 + \frac{\overline{\omega}\phi^{l\prime}(\overline{\omega})}{\phi^l(\overline{\omega})} + \frac{\overline{\omega}\phi^l(\overline{\omega})}{\left[1 - \Phi^l(\overline{\omega})\right]} \right\} > 0,$$

$$\Psi^{d\prime\prime}\Gamma^{b\prime} - \Psi^{d\prime}\Gamma^{b\prime\prime} = \mu \phi^r(\overline{\theta}) \left[1 - \Phi^r(\overline{\theta})\right] \left[1 + \frac{\overline{\theta}\phi^{r\prime}(\overline{\theta})}{\phi^r(\overline{\theta})} + \frac{\overline{\theta}\phi^r(\overline{\theta})}{1 - \Phi^r(\overline{\theta})} \right] > 0$$

under Assumption 2. Hence $q_{\overline{\omega}}^{-1}, q_{\overline{\theta}}^{-1} < 0$, i.e., $q_{\overline{\omega}}, q_{\overline{\theta}} > 0$.

Q.E.D.

Proof of Proposition 2.

We first prove some preliminary results. In the following an expression "$\Upsilon > 0$" means it holds for all $\overline{\omega}, \overline{\theta} > 0$, "$\lim_{\overline{\theta} \to 0} \Upsilon > 0$" means it holds for all $\overline{\omega} > 0$, "$\lim_{\overline{\omega} \to 0} \Upsilon > 0$" means it holds for all $\overline{\theta} > 0$, and similarly for nonpositive expressions. In addition, $\lim_{\overline{\omega}, \overline{\theta} \to 0} \Upsilon > 0$ implies $\lim_{\overline{\theta} \to 0} \Upsilon > 0$ and $\lim_{\overline{\omega} \to 0} \Upsilon > 0$, and similarly for nonpositive expressions.

(1) Results about λ^b and λ^d:

$$\lambda^b = -\frac{\Gamma^{f\prime}}{\Psi^{b\prime}} \frac{\Psi^{d\prime}}{\Gamma^b \Psi^{d\prime} - \Gamma^{b\prime}\Psi^d} > 0, \quad \lambda^d = \frac{\Gamma^{f\prime}}{\Psi^{b\prime}} \frac{\Gamma^{b\prime}}{\Gamma^b \Psi^{d\prime} - \Gamma^{b\prime}\Psi^d} > 0,$$

$$\lim_{\overline{\omega},\overline{\theta}\to 0} \lambda^b = -\lim_{\overline{\omega},\overline{\theta}\to 0} \frac{\Gamma^{f\prime}}{\Psi^{b\prime}\Gamma^b} > 0, \quad \lim_{\overline{\omega},\overline{\theta}\to 0} \lambda^d = \lim_{\overline{\omega},\overline{\theta}\to 0} \frac{\Gamma^{f\prime}}{\Psi^{b\prime}} \frac{\Gamma^{b\prime}}{\Gamma^b \Psi^{d\prime}} > 0.$$

(2) Results about q: Recall the expressions for $q_{\overline{\omega}}^{-1}$ and $q_{\overline{\theta}}^{-1}$ in the proof of Proposition 1. Taking limits, we now have

$$\lim_{\overline{\omega},\overline{\theta}\to 0} q_{\overline{\omega}}^{-1} = \lim_{\overline{\omega},\overline{\theta}\to 0} \Gamma^f \Gamma^b \frac{\Psi^{d\prime}}{\Gamma^{b\prime}} \frac{\Psi^{b\prime\prime}\Gamma^{f\prime} - \Psi^{b\prime}\Gamma^{f\prime\prime}}{\Gamma^{f\prime 2}} < 0,$$

$$\lim_{\overline{\omega},\overline{\theta}\to 0} q_{\overline{\theta}}^{-1} = \lim_{\overline{\omega},\overline{\theta}\to 0} \Gamma^b \Gamma^f \frac{\Psi^{b\prime}}{\Gamma^{f\prime}} \frac{\Psi^{d\prime\prime}\Gamma^{b\prime} - \Psi^{d\prime}\Gamma^{b\prime\prime}}{\Gamma^{b\prime 2}} < 0.$$

Hence,

$$\lim_{\overline{\omega},\overline{\theta}\to 0} q_{\overline{\omega}} > 0, \quad \lim_{\overline{\omega},\overline{\theta}\to 0} q_{\overline{\theta}} > 0.$$

(3) Results about φ:

$$0 < \varphi = \Gamma^f + \Psi^b \left(\Gamma^b + \Psi^d\right) < 1, \quad \lim_{\overline{\omega},\overline{\theta}\to 0} \varphi = 1,$$

$$\varphi_{\overline{\omega}} = \Gamma^{f\prime} + \Psi^{b\prime} \left(\Gamma^b + \Psi^d\right) < 0, \quad \varphi_{\overline{\theta}} = \Psi^b \left(\Gamma^{b\prime} + \Psi^{d\prime}\right) < 0,$$

$$\lim_{\overline{\theta}\to 0} \varphi_{\overline{\omega}} = \Gamma^{f\prime} + \Psi^{b\prime} < 0, \quad \lim_{\overline{\omega}\to 0} \varphi_{\overline{\omega}} = -1 + \left(\Gamma^b + \Psi^d\right) < 0, \quad \lim_{\overline{\omega},\overline{\theta}\to 0} \varphi_{\overline{\omega}} = \lim_{\overline{\omega},\overline{\theta}\to 0} \left(\Gamma^{f\prime} + \Psi^{b\prime}\right) = 0,$$

$$\lim_{\overline{\omega}\to 0} \varphi_{\overline{\theta}} = \lim_{\overline{\theta}\to 0} \varphi_{\overline{\theta}} = 0.$$

We now prove the main results in Proposition 2.

(1) Results about $\overline{\theta}$: From (9),

$$\frac{d\overline{\theta}}{d\zeta^b} = \frac{\Gamma^{b2}}{\Psi^{d\prime}\Gamma^b - \Psi^d \Gamma^{b\prime}} > 0,$$

$$\lim_{\overline{\theta}\to 0} \frac{d\overline{\theta}}{d\zeta^b} = \lim_{\overline{\theta}\to 0} \frac{\Gamma^{b2}}{\Psi^{d\prime}\Gamma^b} > 0, \quad \lim_{\overline{\omega},\overline{\theta}\to 0} \frac{d\overline{\theta}}{d\zeta^b} = \lim_{\overline{\theta}\to 0} \frac{d\overline{\theta}}{d\zeta^b}.$$

(2) Results about $\overline{\omega}$: Given $\overline{\theta}$, condition (10), i.e., $q\Psi^b\left(\Gamma^b+\Psi^d\right)=\zeta^f/\left(1+\zeta^f\right)$ determines $\overline{\omega}$. Total differentiation of this condition yields

$$\frac{d\overline{\omega}}{d\overline{\theta}}=\frac{-\Psi^b\left[q_{\overline{\theta}}\left(\Gamma^b+\Psi^d\right)+q\left(\Gamma^{b\prime}+\Psi^{d\prime}\right)\right]}{\left(q_{\overline{\omega}}\Psi^b+q\Psi^{b\prime}\right)\left(\Gamma^b+\Psi^d\right)}.$$

The sign of this expression is ambiguous as $q_{\overline{\theta}}>0$ and $\left(\Gamma^{b\prime}+\Psi^{d\prime}\right)<0$. But,

$$\lim_{\overline{\omega}\to 0}\frac{d\overline{\omega}}{d\overline{\theta}}=\lim_{\overline{\omega}\to 0}\Psi^b=0,$$

$$\lim_{\overline{\theta}\to 0}\frac{d\overline{\omega}}{d\overline{\theta}}=\lim_{\overline{\theta}\to 0}\frac{-\Psi^b q_{\overline{\theta}}}{q_{\overline{\omega}}\Psi^b+q\Psi^{b\prime}}<0.$$

Hence given the results in (1),

$$\lim_{\overline{\omega}\to 0}\frac{d\overline{\omega}}{d\zeta^b}=0,\qquad \lim_{\overline{\theta}\to 0}\frac{d\overline{\omega}}{d\zeta^b}<0.$$

(3) Results about q: Note that $\lim_{\overline{\theta}\to 0}d\overline{\omega}/d\zeta^b<0$ implies $\lim_{\overline{\theta}\to 0}d\Psi^b/d\zeta^b<0$. The condition $q\Psi^b\left(\Gamma^b+\Psi^d\right)=\zeta^f/\left(1+\zeta^f\right)$ then implies

$$\lim_{\overline{\theta}\to 0}\frac{dq}{d\zeta^b}>0.$$

On the other hand, $\lim_{\overline{\omega},\overline{\theta}\to 0}q_{\overline{\omega}}>0$, $\lim_{\overline{\omega},\overline{\theta}\to 0}q_{\overline{\theta}}>0$, together with $\lim_{\overline{\omega}\to 0}d\overline{\omega}/d\zeta^b=0$ and $\lim_{\overline{\omega},\overline{\theta}\to 0}d\overline{\theta}/d\zeta^b>0$ imply

$$\lim_{\overline{\omega},\overline{\theta}\to 0}\frac{dq}{d\zeta^b}>0.$$

(Remark: Although $\lim_{\overline{\omega},\overline{\theta}\to 0}d\overline{\omega}/d\zeta^b=0$ implies $\lim_{\overline{\omega},\overline{\theta}\to 0}d\Psi^b/d\zeta^b=0$, condition (10) does not imply $\lim_{\overline{\omega},\overline{\theta}\to 0}dq/d\zeta^b=0$, since $\lim_{\overline{\omega}\to 0}\Psi^b=0$.)

(4) Results about φ: Since $\lim_{\overline{\omega},\overline{\theta}\to 0}\varphi_{\overline{\omega}}=\lim_{\overline{\omega},\overline{\theta}\to 0}\varphi_{\overline{\theta}}=0$, we have

$$\lim_{\overline{\omega},\overline{\theta}\to 0}\frac{d\varphi}{d\zeta^b}=0.$$

Q.E.D.

Proof of Proposition 3.

The left-hand side of (26) $(1-L)/L$ is a monotonically decreasing function of L, with $\lim_{L\to 0}(1-L)/L=\infty$ and $\lim_{L\to 1}(1-L)/L=0$. To see how the right-hand side depends on L one needs only see how ζ^b depends on L. Assumption 3 ensures that $\zeta^b>0$ for all values of z. For $\zeta^b>0$ we must have both the numerator and denominator of (29) positive, that is, we must have

$$\frac{L}{1-L}>\max\left\{\frac{1}{\nu}\frac{z}{1+z},\frac{K}{K^b}\frac{1-\alpha}{\alpha}\frac{1}{\nu}\frac{-z}{1+z}\right\}. \tag{A.6}$$

For the limit situation of zero default we have $(1-L)/L=\nu R/(1-\alpha)$ since q and φ both approach one as $\overline{\omega},\overline{\theta}\to 0$. Assumption 3 then guarantees that (A.6) is satisfied under the limit situation. By continuity (A.6) is also satisfied in the neighborhood of the zero-default limit.

From (29) we have

$$\frac{\partial\zeta^b}{\partial L}=\frac{\nu\left(1+\frac{\alpha}{1-\alpha}\frac{K^b}{K}\right)(1+z)\,z}{\left\{\left[\frac{\alpha}{1-\alpha}\frac{K^b}{K}\nu(1+z)-z\right]L+z\right\}^2}\gtreqqless 0\ \text{ if } z\gtreqqless 0.$$

Hence ζ^b is a monotonic function of L for all admissible z. Furthermore, both q and φ are positive and finite for all $L\in(0,1)$. In the light of Proposition 2, $\lim_{\overline{\omega},\overline{\theta}\to 0}d(q\varphi)/d\zeta^b>0$. Hence in the neighborhood of the zero-default limit, the right-hand side of (26) is a monotonic, positive, finite-valued continuous function of L for all z. Therefore the solution to condition (26) exists and is unique for all admissible z, implying that a competitive equilibrium with banking frictions exists and is unique in the neighborhood of the zero-default limit.

Q.E.D.

Proof of Proposition 4.

From (29),

$$\frac{\partial\zeta^b}{\partial z}=\frac{-\nu\frac{L}{1-L}\left(1+\frac{\alpha}{1-\alpha}\frac{K^b}{K}\right)}{\left[\frac{\alpha}{1-\alpha}\frac{K^b}{K}\nu(1+x)\frac{L}{1-L}+x\right]^2}<0.$$

Thus we must have $dL/dz > 0$ for all admissible z in the neighborhood of the zero-default limit, according to condition (26). For $z \leq 0$ we have $\partial\zeta^b/\partial L \leq 0$, implying $d\zeta^b/dz = \partial\zeta^b/\partial z + \left(\partial\zeta^b/\partial L\right)(dL/dz) < 0$ and hence $d\overline{\theta}/dz < 0$ and $dq/dz < 0$. By continuity we have the same result for $z > 0$ as long as z is sufficiently close to zero.

Q.E.D.

Proof of Corollary 5.

Simply note that $\overline{\theta}/\Psi^d\left(\overline{\theta};\Phi^r\right)$ is an increasing function of $\overline{\theta}$.

Q.E.D.

Proof of Proposition 6.

Independent of z (and ζ^b), the variables $L, C, \overline{\omega}, r/p, w/p$, and R are determined by the following system (with K, K^f, and K^b given):

$$F_k(K,L) = q(\overline{\omega})\, R\left(\frac{r}{p}\right),$$

$$F_l(K,L) = q(\overline{\omega})\, R\left(\frac{w}{p}\right),$$

$$q(\overline{\omega})\, \Psi^b\left(\overline{\omega};\Phi^l\right) = \frac{\zeta^f}{1+\zeta^f},$$

$$R = \left[\beta E\left(\frac{1}{1+z'}\right)\right]^{-1},$$

$$-\frac{U_L}{U_C} = \frac{w}{p},$$

$$C = F(K,L)\,\varphi(\overline{\omega}),$$

where

$$q(\overline{\omega}) \equiv \frac{\lambda}{\Gamma^F\left(\overline{\omega};\Phi^l\right) + \lambda\Psi^B\left(\overline{\omega};\Phi^l\right)}, \qquad \lambda = -\frac{\Gamma^{F\prime}\left(\overline{\omega};\Phi^l\right)}{\Psi^{B\prime}\left(\overline{\omega};\Phi^l\right)},$$

$$\varphi(\overline{\omega}) \equiv \Gamma^f\left(\overline{\omega};\Phi^l\right) + \Psi^b\left(\overline{\omega};\Phi^l\right),$$

$$\zeta^f \equiv \frac{(r/p)\,K^b + (w/p)\,L}{(r/p)\,K^f}.$$

Given these variables so determined,

$$p = \frac{1+z}{C},$$

and

$$\frac{n^d}{p} = \left(\frac{w}{p}\right)L - \frac{z}{p} = \left(\frac{w}{p}\right)L - \frac{z}{1+z}C.$$

Q.E.D.

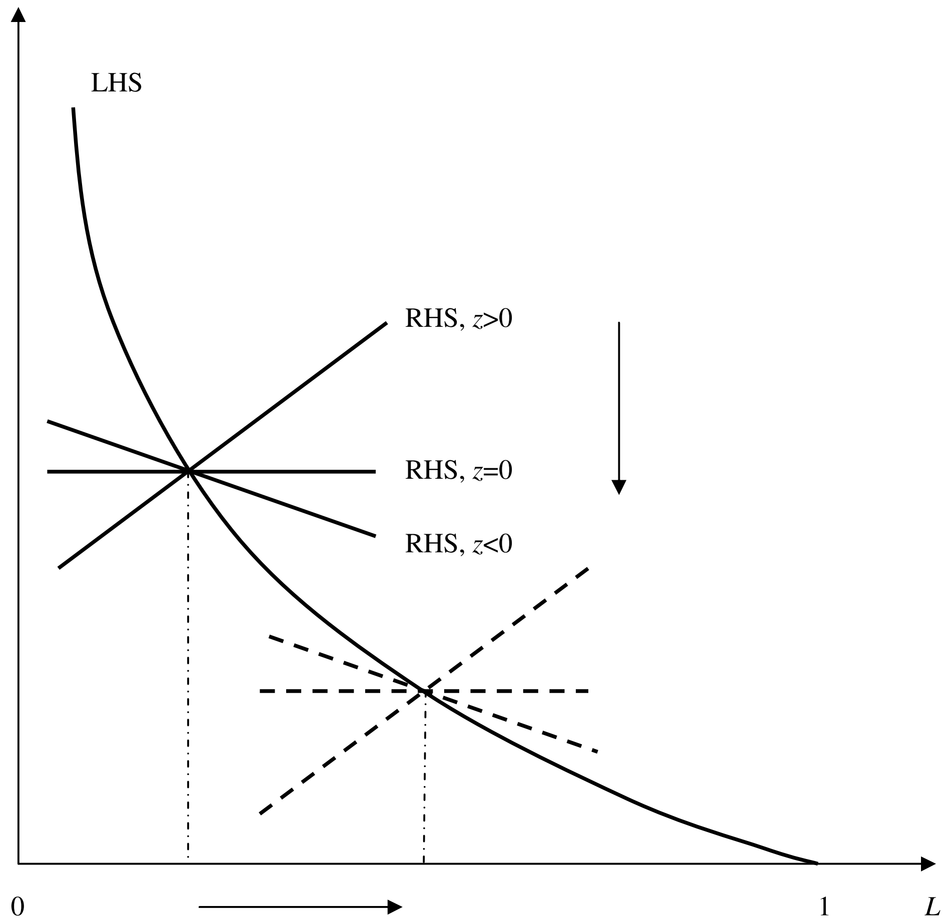

Figure 1. Determination of equilibrium

Note: "LHS" and "RHS" represent the left-hand side and right-hand side, respectively, of condition (26). The dashed lines correspond to a larger value of z compared to the solid lines.