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When Is the Government Spending Multiplier Large?

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We argue that the government-spending multiplier can be much larger than one when the zero lower bound on the nominal interest rate binds. The larger the fraction of government spending that occurs while the nominal interest rate is zero, the larger the value of the multiplier. After providing intuition for these results, we investigate the size of the multiplier in a dynamic, stochastic, general equilibrium model. In this model the multiplier effect is substantially larger than one when the zero bound binds. Our model is consistent with the behavior of key macro aggregates during the recent financial crisis.

I. Introduction

A classic question in macroeconomics is, what is the size of the government-spending multiplier? There is a large empirical literature that grapples with this question. Authors such as Barro (1981) argue that the multiplier is around 0.8 whereas authors such as Ramey (2011) estimate

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the multiplier to be closer to 1.2.[1] There is also a large literature that uses general equilibrium models to study the size of the government-spending multiplier. In standard new-Keynesian models the government-spending multiplier can be somewhat above or below one depending on the exact specification of agents' preferences (see Gali, López-Salido, and Vallés 2007; Monacelli and Perotti 2008). In frictionless real business cycle models this multiplier is typically less than one (see, e.g., Aiyagari, Christiano, and Eichenbaum 1992; Baxter and King 1993; Ramey and Shapiro 1998; Burnside, Eichenbaum, and Fisher 2004; Ramey 2011). Viewed overall, it is hard to argue, on the basis of the literature, that the government-spending multiplier is substantially larger than one.

In this paper we argue that the government-spending multiplier can be much larger than one when the nominal interest rate does not respond to an increase in government spending. We develop this argument in a model in which the multiplier is quite modest if the nominal interest rate is governed by a Taylor rule. When such a rule is operative, the nominal interest rate rises in response to an expansionary fiscal policy shock that puts upward pressure on output and inflation.

There is a natural scenario in which the nominal interest rate does not respond to an increase in government spending: when the zero lower bound on the nominal interest rate binds. We find that the multiplier is very large in economies in which the output cost of being in the zero-bound state is also large. In such economies it can be socially optimal to substantially raise government spending in response to shocks that make the zero lower bound on the nominal interest rate binding.

We begin by considering an economy with Calvo-style price frictions, no capital, and a monetary authority that follows a standard Taylor rule. Building on Eggertsson and Woodford (2003), we study the effect of a temporary, unanticipated rise in agents' discount factor. Other things equal, the shock to the discount factor increases desired saving. Since investment is zero in this economy, aggregate saving must be zero in equilibrium. When the shock is small enough, the real interest rate falls and there is a modest decline in output. However, when the shock is large enough, the zero bound becomes binding before the real interest rate falls by enough to make aggregate saving zero. In this model, the only force that can induce the fall in saving required to reestablish equilibrium is a large, transitory fall in output.

Why is the fall in output so large when the economy hits the zero bound? For a given fall in output, marginal cost falls and prices decline. With staggered pricing, the drop in prices leads agents to expect future

[1] For recent contributions to the vector autoregression (VAR) based empirical literature on the size of the government-spending multiplier, see Ilzetzki, Mendoza, and Vegh (2009) and Fisher and Peters (2010). Hall (2009) provides an analysis and review of the empirical literature.

deflation. With the nominal interest rate stuck at zero, the real interest rate rises. This perverse rise in the real interest rate leads to an increase in desired saving, which partially undoes the effect of a given fall in output. So, the total fall in output required to reduce desired saving to zero is very large.

This scenario resembles the paradox of thrift originally emphasized by Keynes (1936) and recently analyzed by Krugman (1998), Eggertsson and Woodford (2003), and Christiano (2004). In the textbook version of this paradox, prices are constant and an increase in desired saving lowers equilibrium output. But, in contrast to the textbook scenario, the zero-bound scenario studied in the modern literature involves a deflationary spiral that contributes to and accompanies the large fall in output.

Consider now the effect of an increase in government spending when the zero bound is strictly binding. This increase leads to a rise in output, marginal cost, and expected inflation. With the nominal interest rate stuck at zero, the rise in expected inflation drives down the real interest rate, which drives up private spending. This rise in spending leads to a further rise in output, marginal cost, and expected inflation and a further decline in the real interest rate. The net result is a large rise in output and a large fall in the rate of deflation. In effect, the increase in government consumption counteracts the deflationary spiral associated with the zero-bound state.

The exact value of the government-spending multiplier depends on a variety of factors. However, we show that this multiplier is large in economies in which the output cost associated with the zero-bound problem is more severe. We argue this point in two ways. First, we show that the value of the government-spending multiplier can depend sensitively on the model's parameter values. But parameter values that are associated with large declines in output when the zero bound binds are also associated with large values of the government-spending multiplier. Second, we show that the value of the government-spending multiplier is positively related to how long the zero bound is expected to bind.

An important practical objection to using fiscal policy to counteract a contraction associated with the zero-bound state is that there are long lags in implementing increases in government spending. Motivated by this consideration, we study the size of the government-spending multiplier in the presence of implementation lags. We find that a key determinant of the size of the multiplier is the state of the world in which new government spending comes on line. If it comes on line in future periods when the nominal interest rate is zero, then there is a large effect on current output. If it comes on line in future periods in which the nominal interest rate is positive, then the current effect on government spending is smaller. So our analysis supports the view that, for fiscal policy to be effective, government spending must come on line in a timely manner.

In the second step of our analysis we incorporate capital accumulation into the model. For computational reasons we consider temporary shocks that make the zero bound binding for a deterministic number of periods. Again, we find that the government-spending multiplier is larger when the zero bound binds. Allowing for capital accumulation has two effects. First, for a given size shock it reduces the likelihood that the zero bound becomes binding. Second, when the zero bound binds, the presence of capital accumulation tends to increase the size of the government-spending multiplier. The intuition for this result is that, in our model, investment is a decreasing function of the real interest rate. When the zero bound binds, the real interest rate generally rises. So, other things equal, saving and investment diverge as the real interest rate rises, thus exacerbating the meltdown associated with the zero bound. As a result, the fall in output necessary to bring saving and investment into alignment is larger than in the model without capital.

The simple models discussed above suggest that the multiplier can be large in the zero-bound state. The obvious next step would be to use reduced-form methods, such as identified VARs, to estimate the government-spending multiplier when the zero bound binds. Unfortunately, this task is fraught with difficulties. First, we cannot mix evidence from states in which the zero bound binds with evidence from other states because the multipliers are very different in the two states. Second, we have to identify exogenous movements in government spending when the zero bound binds.[2] This task seems daunting at best. Almost surely government spending would rise in response to large output losses in the zero-bound state. To know the government-spending multiplier we need to know what output would have been had government spending not risen. For example, the simple observation that output did not grow quickly in Japan in the zero-bound state, even though there were large increases in government spending, tells us nothing about the question of interest.

Given these difficulties, we investigate the size of the multiplier in the zero-bound state using the empirically plausible dynamic stochastic general equilibrium (DSGE) model proposed by Altig et al. (2011). This model incorporates price- and wage-setting frictions, habit formation in consumption, variable capital utilization, and investment adjustment costs of the sort proposed by Christiano, Eichenbaum, and Evans (2005).

[2] To see how critical this step is, suppose that the government chooses spending to keep output exactly constant in the face of shocks that make the zero bound bind. A naive econometrician who simply regressed output on government spending would falsely conclude that the government-spending multiplier is zero. This example is, of course, just an application of Tobin's (1970) *post hoc, ergo propter hoc* argument.

Altig et al. estimate the parameters of their model to match the impulse response function of 10 macro variables to a monetary shock, a neutral technology shock, and a capital-embodied technology shock.

Our key findings based on the Altig et al. model can be summarized as follows. First, when the central bank follows a Taylor rule, the value of the government-spending multiplier is generally less than one. Second, the multiplier is much larger if the nominal interest rate does not respond to the rise in government spending. For example, suppose that government spending goes up for 12 quarters and the nominal interest rate remains constant. In this case the impact multiplier is roughly 1.6 and has a peak value of about 2.3. Third, the value of the multiplier depends critically on how much government spending occurs in the period during which the nominal interest rate is constant. The larger the fraction of government spending that occurs while the nominal interest rate is constant, the smaller the value of the multiplier. Consistent with the theoretical analysis above, this result implies that for government spending to be a powerful weapon in combating output losses associated with the zero-bound state, it is critical that the bulk of the spending come on line when the lower bound is actually binding. Fourth, we find that the model generates sensible predictions for the current crisis under the assumption that the zero bound binds. In particular, the model does well at accounting for the behavior of output, consumption, investment, inflation, and short-term nominal interest rates.

As emphasized by Eggertsson and Woodford (2003), an alternative way to escape the negative consequences of a shock that makes the zero bound binding is for the central bank to commit to future inflation. We abstract from this possibility in this paper. We do so for a number of reasons. First, this theoretical possibility is well understood. Second, we do not think that it is easy in practice for the central bank to credibly commit to future high inflation. Third, the optimal trade-off between higher government purchases and anticipated inflation depends sensitively on how agents value government purchases and the costs of anticipated inflation. Studying this issue is an important topic for future research.

Our analysis builds on the work by Christiano (2004) and Eggertsson (2004), who argue that increasing government spending is very effective when the zero bound binds. Eggertsson (2011) analyzes both the effects of increases in government spending and transitory tax cuts when the zero bound binds. The key contributions of this paper are to analyze the size of the multiplier in a medium-size DSGE model, study the model's performance in the financial crisis that began in 2008, and quantify the importance of the timing of government spending relative to the timing of the zero bound.

Our analysis is related to several recent papers on the zero bound. Bodenstein, Erceg, and Guerrieri (2009) analyze the effects of shocks to open economies when the zero bound binds. Braun and Waki (2006) use a model in which the zero bound binds to account for Japan's experience in the 1990s. Their results for fiscal policy are broadly consistent with our results. Braun and Waki (2006) and Coenen and Wieland (2003) investigate whether alternative monetary policy rules could have avoided the zero-bound state in Japan.

In online Appendix B, we analyze the sensitivity of our conclusions to the presence of distortionary taxes on labor and capital. Eggertsson (2010, 2011) shows that the effects of distortionary taxes can be very different depending on whether the zero lower bound binds or not. Indeed, some distortionary taxes that lower output when the zero lower bound does not bind actually raise output when the zero bound does bind. Of course, if the tax that finances government spending actually increases output, then the government-spending multiplier is actually increased. We quantify the effects of distortionary labor taxes in Altig et al.'s study when the zero lower bound binds. In addition, we discuss the effects of different types of capital income taxes. We argue that our conclusions are robust to allowing for different types of distortionary taxes.

Our paper is organized as follows. In Section II, we analyze the size of the government-spending multiplier when the interest follows a Taylor rule in a standard new-Keynesian model without capital. In Section III, we modify the analysis to assume that the nominal interest rate does not respond to an increase in government spending, say because the lower bound on the nominal interest rate binds. In Section IV, we extend the model to incorporate capital. In Section V, we discuss the properties of the government-spending multiplier in the medium-size DSGE model proposed by Altig et al. (2011) and investigate the performance of the model during the recent financial crisis. Section VI presents conclusions.

II. The Standard Multiplier in a Model without Capital

In this section we present a simple new-Keynesian model and analyze its implications for the size of the "standard multiplier," by which we mean the size of the government-spending multiplier when the nominal interest rate is governed by a Taylor rule.

Households.—The economy is populated by a representative household, whose lifetime utility, U, is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[C_t^{\gamma}(1 - N_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} + v(G_t) \right\}. \quad (1)$$

Here E_0 is the conditional expectation operator, and C_t, G_t, and N_t denote time t consumption, government consumption, and hours worked, respectively. We assume that $\sigma > 0$, $\gamma \in (0, 1)$, and $v(\cdot)$ is a concave function.

The household budget constraint is given by

$$P_tC_t + B_{t+1} = B_t(1 + R_t) + W_tN_t + T_t, \tag{2}$$

where T_t denotes firms' profits net of lump-sum taxes paid to the government. The variable B_{t+1} denotes the quantity of one-period bonds purchased by the household at time t. Also, P_t denotes the price level and W_t denotes the nominal wage rate. Finally, R_t denotes the one-period nominal rate of interest that pays off in period t. The household's problem is to maximize utility given by equation (1) subject to the budget constraint given by equation (2) and the condition

$$E_0 \lim_{t\to\infty} B_{t+1}/[(1 + R_0)(1 + R_1) \cdots (1 + R_t)] \geq 0.$$

Firms.—The final good is produced by competitive firms using the technology

$$Y_t = \left[\int_0^1 Y_t(i)^{(\varepsilon-1)/\varepsilon} di\right]^{\varepsilon/(\varepsilon-1)}, \quad \varepsilon > 1, \tag{3}$$

where $Y_t(i)$, $i \in [0, 1]$, denotes intermediate good i.

Profit maximization implies the following first-order condition for $Y_t(i)$:

$$P_t(i) = P_t\left[\frac{Y_t}{Y_t(i)}\right]^{1/\varepsilon}, \tag{4}$$

where $P_t(i)$ denotes the price of intermediate good i and P_t is the price of the homogeneous final good.

The intermediate good, $Y_t(i)$, is produced by a monopolist using the following technology:

$$Y_t(i) = N_t(i),$$

where $N_t(i)$ denotes employment by the ith monopolist. We assume that there is no entry or exit into the production of the ith intermediate good. The monopolist is subject to Calvo-style price-setting frictions and can optimize its price, $P_t(i)$, with probability $1 - \theta$. With probability θ the firm sets

$$P_t(i) = P_{t-1}(i).$$

The discounted profits of the ith intermediate-good firm are

$$E_t \sum_{j=0}^{\infty} \beta^j v_{t+j}[P_{t+j}(i)Y_{t+j}(i) - (1 - \nu)W_{t+j}N_{t+j}(i)], \tag{5}$$

where $\nu = 1/\varepsilon$ denotes an employment subsidy that corrects, in steady state, the inefficiency created by the presence of monopoly power. The variable v_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem. The variable W_{t+j} denotes the nominal wage rate.

Firm i maximizes its discounted profits, given by equation (5), subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t(i)$, given by equation (4).

Monetary policy.—We assume that monetary policy follows the rule

$$R_{t+1} = \max(Z_{t+1}, 0), \tag{6}$$

where

$$Z_{t+1} = (1/\beta)(1+\pi_t)^{\phi_1(1-\rho_R)}(Y_t/Y)^{\phi_2(1-\rho_R)}[\beta(1+R_t)]^{\rho_R} - 1.$$

Throughout the paper a variable without a time subscript denotes its steady-state value; for example, the variable Y denotes the steady-state level of output. The variable π_t denotes the time t rate of inflation. We assume that $\phi_1 > 1$ and $\phi_2 \in (0, 1)$.

According to equation (6), the monetary authority follows a Taylor rule as long as the implied nominal interest rate is nonnegative. Whenever the Taylor rule implies a negative nominal interest rate, the monetary authority simply sets the nominal interest rate to zero. For convenience we assume that steady-state inflation is zero. This assumption implies that the steady-state net nominal interest rate is $1/\beta - 1$.

Fiscal policy.—As long as the zero bound on the nominal interest rate is not binding, government spending evolves according to

$$G_{t+1} = G_t^{\rho} \exp(\eta_{t+1}). \tag{7}$$

Here G is the level of government spending in the nonstochastic steady state and η_{t+1} is an independent and identically distributed shock with zero mean. To simplify our analysis, we assume that government spending and the employment subsidy are financed with lump-sum taxes. The exact timing of these taxes is irrelevant because Ricardian equivalence holds under our assumptions. We discuss the details of fiscal policy when the zero bound binds in Section III.

Equilibrium.—The economy's resource constraint is

$$C_t + G_t = Y_t. \tag{8}$$

A "monetary equilibrium" is a collection of stochastic processes

$$\{C_t, N_t, W_t, P_t, Y_t, R_t, P_t(i), Y_t(i), N_t(i), v_t, B_{t+1}, \pi_t\}$$

such that for given $\{G_t\}$ the household and firm problems are satisfied, the monetary and fiscal policy rules are satisfied, markets clear, and the aggregate resource constraint is satisfied.

To solve for the equilibrium we use a linear approximation around the nonstochastic steady state of the economy. Throughout, $\hat{Z}_t$ denotes the percentage deviation of Z_t from its nonstochastic steady-state value, Z. The equilibrium is characterized by the following set of equations.

The Phillips curve for this economy is given by

$$\pi_t = E_t(\beta\pi_{t+1} + \kappa\widehat{MC}_t), \tag{9}$$

where $\kappa = (1-\theta)(1-\beta\theta)/\theta$. In addition, MC_t denotes the real marginal cost, which, under our assumptions, is equal to the real wage rate. Without labor market frictions, the percentage deviation of real marginal cost from its steady-state value is given by

$$\widehat{MC}_t = \hat{C}_t + \frac{N}{1-N}\hat{N}_t. \tag{10}$$

The linearized intertemporal Euler equation for consumption is

$$\begin{aligned}[\gamma(1-\sigma)-1]\hat{C}_t - (1-\gamma)(1-\sigma)\frac{N}{1-N}\hat{N}_t = \\ E_t\left\{\beta(R_{t+1}-R) - \pi_{t+1} + [\gamma(1-\sigma)-1]\hat{C}_{t+1} - (1-\gamma)(1-\sigma)\frac{N}{1-N}\hat{N}_{t+1}\right\}.\end{aligned} \tag{11}$$

The linearized aggregate resource constraint is

$$\hat{Y}_t = (1-g)\hat{C}_t + g\hat{G}_t, \tag{12}$$

where $g = G/Y$.

Combining equations (9) and (10) and using the fact that $\hat{N}_t = \hat{Y}_t$, we obtain

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t - \frac{g}{1-g}\hat{G}_t\right]. \tag{13}$$

Similarly, combining equations (11) and (12) and using the fact that $\hat{N}_t = \hat{Y}_t$, we obtain

$$\begin{aligned}\hat{Y}_t - g[\gamma(\sigma-1)+1]\hat{G}_t = \\ E_t\{-(1-g)[\beta(R_{t+1}-R) - \pi_{t+1}] + \hat{Y}_{t+1} - g[\gamma(\sigma-1)+1]\hat{G}_{t+1}\}.\end{aligned} \tag{14}$$

As long as the zero bound on the nominal interest rate does not bind, the linearized monetary policy rule is given by

$$R_{t+1} - R = \rho_R(R_t - R) + \frac{1-\rho_R}{\beta}(\phi_1\pi_t + \phi_2\hat{Y}_t).$$

Whenever the zero bound binds, $R_{t+1} = 0$.

We solve for the equilibrium using the method of undetermined coefficients. For simplicity, we begin by considering the case in which $\rho_R = 0$. Under the assumption that $\phi_1 > 1$, there is a unique linear equilibrium in which π_t and $\hat{Y}_t$ are given by

$$\pi_t = A_\pi \hat{G}_t \tag{15}$$

and

$$\hat{Y}_t = A_Y \hat{G}_t. \tag{16}$$

The coefficients A_π and A_Y are given by

$$A_\pi = \frac{\kappa}{1-\beta\rho}\left[\left(\frac{1}{1-g} + \frac{N}{1-N}\right)A_Y - \frac{g}{1-g}\right] \tag{17}$$

and

$$A_Y = g\frac{(\rho-\phi_1)\kappa - [\gamma(\sigma-1)+1](1-\rho)(1-\beta\rho)}{(1-\beta\rho)[\rho-1-(1-g)\phi_2] + (1-g)(\rho-\phi_1)\kappa[1/(1-g) + N/(1-N)]}. \tag{18}$$

The effect of an increase in government spending.—Using equation (12), we can write the government-spending multiplier as

$$\frac{dY_t}{dG_t} = \frac{1}{g}\frac{\hat{Y}_t}{\hat{G}_t} = 1 + \frac{1-g}{g}\frac{\hat{C}_t}{\hat{G}_t}. \tag{19}$$

This equation implies that the multiplier is less than one whenever consumption falls in response to an increase in government spending. Equation (16) implies that the government-spending multiplier is given by

$$\frac{dY_t}{dG_t} = \frac{A_Y}{g}. \tag{20}$$

To analyze the magnitude of the multiplier outside of the zero bound, we consider the following baseline parameter values:

$$\begin{gathered}\theta = 0.85, \beta = 0.99, \phi_1 = 1.5, \phi_2 = 0, \gamma = 0.29, \\ g = 0.2, \sigma = 2, \rho_R = 0, \rho = 0.8.\end{gathered} \tag{21}$$

These parameter values imply that $\kappa = 0.03$ and $N = 1/3$. Our baseline parameter values imply that the government-spending multiplier is 1.05.

In our model Ricardian equivalence holds. From the perspective of the representative household, the increase in the present value of taxes equals the increase in the present value of government purchases. In a typical version of the standard neoclassical model we would expect some rise in output driven by the negative wealth effect on leisure of the tax increase. But in that model the multiplier is generally less than one because the wealth effect reduces private consumption. From this perspective it is perhaps surprising that the multiplier in our baseline model is greater than one. This perspective neglects two key features of our model: the frictions in price setting and the complementarity between consumption and leisure in preferences. When government purchases increase, total demand, $C_t + G_t$, increases. Since prices are sticky, price over marginal cost falls after a rise in demand. As emphasized in the literature on the role of monopoly power in business cycles, the fall in the markup induces an outward shift in the labor demand curve. This shift amplifies the rise in employment following the rise in demand. Given our specification of preferences, $\sigma > 1$ implies that the marginal utility of consumption rises with the increase in employment. As long as this increase in marginal utility is large enough, it is possible for private consumption to actually rise in response to an increase in government purchases. Indeed, consumption does rise in our benchmark scenario, which is why the multiplier is larger than one.

To assess the importance of our preference specification, we redid our calculations using the basic specification for the momentary utility function commonly used in the new-Keynesian DSGE literature:

$$u = (C_t^{1-\varsigma} - 1)/(1 - \varsigma) - \eta N_t^{1+\vartheta}/(1 + \vartheta), \tag{22}$$

where ς, η, and ϑ are positive. The key feature of this specification is that the marginal utility of consumption is independent of hours worked. Consistent with the intuition discussed above, we found that, across a wide set of parameter values, dY/dG is always less than one with this preference specification.[3]

To provide additional intuition for the determinants of the multiplier, we calculate dY/dG for various parameter configurations. In each case we perturb one parameter at a time relative to the benchmark parameter values. Our results can be summarized as follows. First, we find that the multiplier is an increasing function of σ. This result is consistent with the intuition above, which builds on the observation that the marginal utility of consumption is increasing in hours worked. This dependence is stronger the higher σ is.

Second, the multiplier is a decreasing function of κ. In other words, the multiplier is larger the higher the degree of price stickiness. This result reflects the fall in the markup when aggregate demand and marginal cost rise. This effect is stronger the stickier prices are. The multiplier exceeds one for all $\kappa < 0.13$. In the limiting case in which prices are perfectly sticky ($\kappa = 0$), the multiplier is given by

$$\frac{dY_t}{dG_t} = \frac{[\gamma(\sigma - 1) + 1](1 - \rho)}{1 - \rho + (1 - g)\phi_2} > 0.$$

Note that when $\phi_2 = 0$, the multiplier is greater than one as long as σ is greater than one.

When prices are perfectly flexible ($\kappa = \infty$), the markup is constant.

[3] See Monacelli and Perotti (2008) for a discussion of the impact of preferences on the size of the government-spending multiplier in models with Calvo-style frictions when the zero bound is not binding.

In this case the multiplier is less than one:

$$\frac{dY_t}{dG_t} = \frac{1}{1 + (1 - g)[N/(1 - N)]} < 1.$$

This result reflects the fact that with flexible prices an increase in government spending has no impact on the markup. As a result, the demand for labor does not rise as much as in the case in which prices are sticky.

Third, the multiplier is a decreasing function of ϕ_1. The intuition for this effect is that the expansion in output increases marginal cost, which in turn induces a rise in inflation. According to equation (6), the monetary authority increases the interest rate in response to a rise in inflation. The rise in the interest rate is an increasing function of ϕ_1. Higher values of ϕ_1 lead to higher values of the real interest rate, which are associated with lower levels of consumption. So higher values of ϕ_1 lead to lower values of the multiplier.

Fourth, the multiplier is a decreasing function of ϕ_2. The intuition underlying this effect is similar to that associated with ϕ_1. When ϕ_2 is large, there is a substantial increase in the real interest rate in response to a rise in output. The contractionary effects of the rise in the real interest rate on consumption reduce the size of the multiplier.

Fifth, the multiplier is an increasing function of ρ_R. The intuition for this result is as follows. The higher ρ_R, the less rapidly the monetary authority increases the interest rate in response to the rise in marginal cost and inflation that occurs in the wake of an increase in government purchases. This result is consistent with the traditional view that the government-spending multiplier is greater in the presence of accommodative monetary policy. By accommodative we mean that the monetary authority raises interest rates slowly in the presence of a fiscal expansion.

Sixth, the multiplier is a decreasing function of the parameter governing the persistence of government purchases, ρ. The intuition for this result is that the present value of taxes associated with a given innovation in government purchases is an increasing function of ρ. So the negative wealth effect on consumption is an increasing function of ρ.[4]

Our numerical results suggest that the multiplier in a simple new-Keynesian model can be above one for reasonable parameter values. However, it is difficult to obtain multipliers above 1.2 for plausible parameter values.

[4] We redid our calculations using a forward-looking Taylor rule in which the interest rate responds to the one-period-ahead expected inflation and output gap. The results that we obtained are very similar to the ones discussed in the text.

III. The Constant–Interest Rate Multiplier in a Model without Capital

In this section we analyze the government-spending multiplier in our simple new-Keynesian model when the nominal interest rate is constant. We focus on the case in which the nominal interest rate is constant because the zero bound binds. Our basic analysis of the multiplier builds on the work of Eggertsson and Woodford (2003), Christiano (2004), and Eggertsson (2004). As in these papers, the shock that makes the zero bound binding is an increase in the discount factor. We think of this shock as representing a temporary rise in agents' propensity to save.

A discount factor shock.—We modify agents' preferences, given by (1), to allow for a stochastic discount factor

$$U = E_0 \sum_{t=0}^{\infty} d_t \left\{ \frac{[C_t^{\gamma}(1 - N_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma} + v(G_t) \right\}. \tag{23}$$

The cumulative discount factor, d_t, is given by

$$d_t = \begin{cases} \dfrac{1}{1 + r_1} \dfrac{1}{1 + r_2} \cdots \dfrac{1}{1 + r_t} & t \geq 1 \\ 1 & t = 0. \end{cases} \tag{24}$$

The time t discount factor, r_t, can take on two values, r and r^l, where $r^l < 0$. The stochastic process for r_t is given by

$$\begin{aligned} \Pr[r_{t+1} = r^l | r_t = r^l] &= p, \\ \Pr[r_{t+1} = r | r_t = r^l] &= 1 - p, \\ \Pr[r_{t+1} = r^l | r_t = r] &= 0. \end{aligned} \tag{25}$$

The value of r_{t+1} is realized at time t. We define $\beta = 1/(1 + r)$, where r is the steady-state value of r_{t+1}.

We consider the following experiment. The economy is initially in the steady state, so $r_t = r$. At time 0, r_1 takes on the value r^l. Thereafter, r_t follows the process described by equation (25). The discount factor remains high with probability p and returns permanently to its normal value, r, with probability $1 - p$. In what follows we assume that r^l is sufficiently high that the zero-bound constraint on nominal interest rates binds. We assume that $\hat{G}_t = \hat{G}^l \geq 0$ in the lower bound and $\hat{G}_t = 0$ otherwise.

To solve the model we suppose (and then verify) that the equilibrium is characterized by two values for each variable: one value for when the zero bound binds and one value for when it does not. We denote the values of inflation and output in the zero bound by π^l and $\hat{Y}^l$, respectively. For simplicity we assume that $\rho_R = 0$, so there is no interest rate smoothing in the Taylor rule, (6). Since there are no state variables and

$\hat{G}_t = 0$ outside of the zero-bound state, as soon as the zero bound is not binding, the economy jumps to the steady state.

We can solve for $\hat{Y}^l$ using equation (13) and the following version of equation (14), which takes into account the discount factor shock:

$$\hat{Y}_t - g[\gamma(\sigma - 1) + 1]\hat{G}_t = \tag{26}$$
$$E_t\{\hat{Y}_{t+1} - g[\gamma(\sigma - 1) + 1]\hat{G}_{t+1} - \beta(1 - g)(R_{t+1} - r_{t+1}) + (1 - g)\pi_{t+1}\}.$$

We focus on the case in which the zero bound binds at time t, so $R_{t+1} = 0$. Equations (13) and (26) can be rewritten as

$$\hat{Y}^l = g[\gamma(\sigma - 1) + 1]\hat{G}^l + \frac{1 - g}{1 - p}(\beta r^l + p\pi^l) \tag{27}$$

and

$$\pi^l = \beta p \pi^l + \kappa\left(\frac{1}{1 - g} + \frac{N}{1 - N}\right)\hat{Y}^l - \frac{g}{1 - g}\kappa\hat{G}^l. \tag{28}$$

Equations (27) and (28) imply that π^l and $\hat{Y}^l$ are given by

$$\pi^l = \frac{(1 - g)\kappa[1/(1 - g) + N/(1 - N)]\beta r^l}{\Delta} \tag{29}$$
$$+ g\kappa(1 - p)\frac{[1/(1 - g) + N/(1 - N)]\gamma(\sigma - 1) + N/(1 - N)}{\Delta}\hat{G}^l$$

and

$$\hat{Y}^l = \frac{(1 - \beta p)(1 - g)\beta r^l}{\Delta} + \frac{(1 - \beta p)(1 - p)[\gamma(\sigma - 1) + 1] - p\kappa}{\Delta}g\hat{G}^l, \tag{30}$$

where

$$\Delta = (1 - \beta p)(1 - p) - p\kappa\left[1 + \frac{N}{1 - N}(1 - g)\right].$$

Since r^l is negative, a necessary condition for the zero bound to bind is that $\Delta > 0$. If this condition did not hold, inflation would be positive and output would be above its steady-state value. Consequently, the Taylor rule would call for an increase in the nominal interest rate so that the zero bound would not bind.

Equation (30) implies that the drop in output induced by a change in the discount rate, which we denote by Θ, is given by

$$\Theta = \frac{(1 - \beta p)(1 - g)\beta r^l}{\Delta}. \tag{31}$$

By assumption $\Delta > 0$, so $\Theta < 0$. The value of Θ can be a large negative number for plausible parameter values. The intuition for this result is as follows. The basic shock to the economy is an increase in agents' desire to save. We develop the intuition for this result in two steps. First,

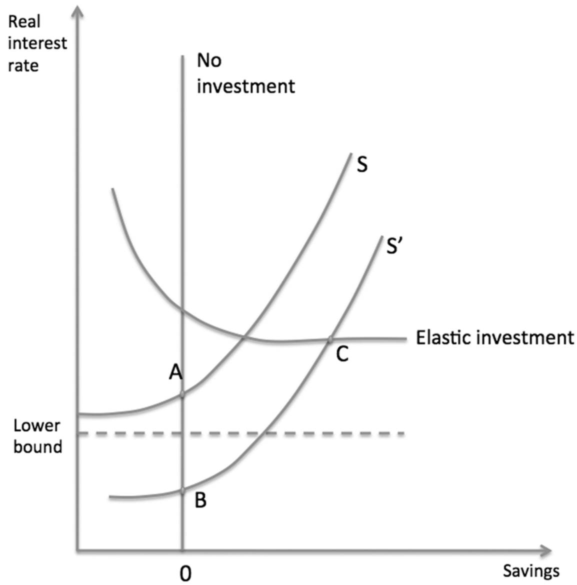

FIG. 1.—Simple diagram

we provide intuition for why the zero bound binds. We then provide the intuition for why the drop in output can be very large when the zero bound binds.

To understand why the zero bound binds, recall that in this economy saving must be zero in equilibrium. With completely flexible prices the real interest rate would simply fall to discourage agents from saving. There are two ways in which such a fall can occur: a large fall in the nominal interest rate and/or a substantial rise in the expected inflation rate. The extent to which the nominal interest rate can fall is limited by the zero bound. In our sticky-price economy a rise in the rate of inflation is associated with a rise in output and marginal cost. But a transitory increase in output is associated with a further increase in the desire to save, so that the real interest rate must rise by even more. Given the size of the shock to the discount factor, there may be no equilibrium in which the nominal interest rate is zero and inflation is positive. So the real interest rate cannot fall by enough to reduce desired saving to zero. In this scenario the zero bound binds.

Figure 1 illustrates this point using a stylized version of our model. Saving (S) is an increasing function of the real interest rate. Since there is no investment in this economy, saving must be zero in equilibrium. The initial equilibrium is represented by point A. But the increase in the discount factor can be thought of as inducing a rightward shift in the saving curve from S to S'. When this shift is large, the real interest rate cannot fall enough to reestablish equilibrium because the lower bound on the nominal interest rate becomes binding prior to reaching that point. This situation is represented by point B.

To understand why the fall in output can be very large when the zero bound binds, recall that equation (29) shows how the rate of inflation, π^l, depends on the discount rate and on government spending in the zero-bound state. In this state Δ is positive. Since r^l is negative, it follows that π^l is negative, and so too is expected inflation, $p\pi^l$. Since the nominal interest rate is zero and expected inflation is negative, the real interest rate (nominal interest rate minus expected inflation rate) is positive. Both the increase in the discount factor and the rise in the real interest rate increase agents' desire to save. There is only one force remaining to generate zero saving in equilibrium: a large, transitory fall in income. Other things equal, this fall in income reduces desired saving as agents attempt to smooth the marginal utility of consumption over states of the world. Because the zero bound is a transitory state of the world, this force leads to a decrease in agents' desire to save. This effect has to exactly counterbalance the other two forces, which are leading agents to save more. This reasoning suggests that there is a very large decline in income when the zero bound binds. In terms of figure 1, we can think of the temporary fall in output as inducing a shift in the saving curve to the left.

We now turn to a numerical analysis of the government-spending multiplier, which is given by

$$\frac{dY^l}{dG^l} = \frac{(1-\beta p)(1-p)[\gamma(\sigma-1)+1]-p\kappa}{\Delta}. \tag{32}$$

In what follows we assume that the discount factor shock is sufficiently large to make the zero bound binding. Conditional on this bound being binding, the size of the multiplier does not depend on the size of the shock. In our discussion of the standard multiplier, we assume that the first-order serial correlation of government spending shocks is 0.8. To make the experiment in this section comparable, we choose $p = 0.8$. This choice implies that the first-order serial correlation of government spending in the zero bound is also 0.8. All other parameter values are given by the baseline specification in (21).

For our benchmark specification the government-spending multiplier is 3.7, which is roughly three times larger than the standard multiplier. The intuition for why the multiplier can be large when the nominal interest rate is constant, say because the zero bound binds, is as follows. A rise in government spending leads to a rise in output, marginal cost, and expected inflation. With the nominal interest rate equal to zero, the rise in expected inflation drives down the real interest rate, leading to a rise is private spending. This rise in spending generates a further rise in output, marginal cost, and expected inflation and a further decline in the real interest rate. The net result is a large rise in inflation and output.

The increase in income in states in which the zero bound binds raises permanent income, which raises desired expenditures in zero-bound states. This additional channel reinforces the intertemporal channel stressed above. Since the zero-bound problem is temporary, we expect that the importance of this channel is relatively small.

We now consider the sensitivity of the multiplier to parameter values. The first row of figure 2 displays the government-spending multiplier and the response of output to the discount rate shock in the absence of a change in government spending as a function of the parameter κ. The circle indicates results for our benchmark value of κ. This row is generated assuming a discount factor shock such that r^l is equal to -2 percent on an annualized basis. We graph only values of κ for which the zero bound binds, so we display results for $0.02 \leq \kappa \leq 0.036$. Three key features of this figure are worth noting. First, the multiplier can be very large. Second, without a change in government spending, the decline in output is increasing in the degree of price flexibility; that is, it is increasing in κ as long as the zero bound binds. This result reflects that, conditional on the zero bound binding, the more flexible prices are, the higher the expected deflation and the higher the real interest rate. So, other things equal, higher values of κ require a large transitory fall in output to equate saving and investment when the zero bound binds.[5] Third, the government-spending multiplier is also an increasing function of κ.

The second row of figure 2 displays the government-spending multiplier and the response of output to the discount rate shock in the absence of a change in government spending as a function of the parameter p. The asterisk indicates results for our benchmark value of p. We graph only values of p for which the zero bound binds, so we display results for $0.75 \leq p \leq 0.82$. Two key results are worth noting. First, without a change in government spending, the decline in output is increasing in p. So the longer the expected duration of the shock, the worse the output consequences of the zero bound being binding. Second, the value of the government-spending multiplier is an increasing function of p.

Figure 2 shows that the precise value of the multiplier is sensitive to the choice of parameter values. But looking across parameter values, we see that the government-spending multiplier is large in economies

[5] The basic logic here is consistent with the intuition in De Long and Summers (1986) about the potentially destabilizing effects of marginal increases in price flexibility.

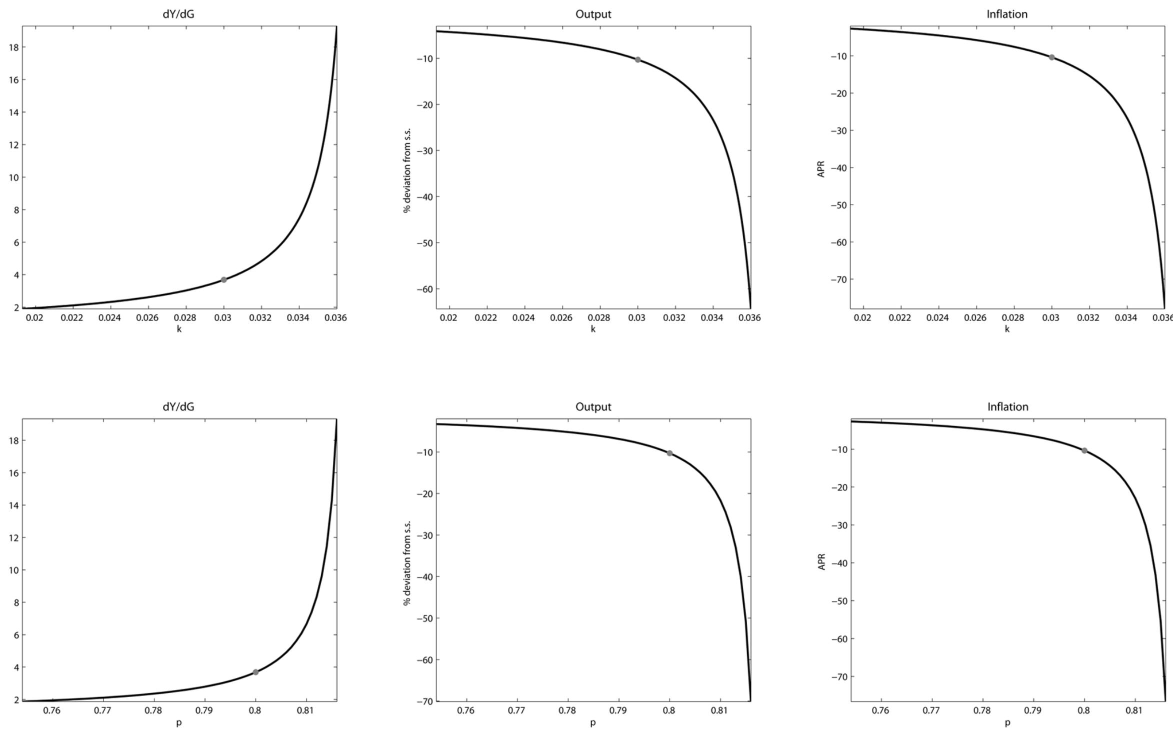

Fig. 2.—Government-spending multiplier when the zero bound is binding (model with no capital)

in which the drop in output associated with the zero bound is also large. Put differently, fiscal policy is particularly powerful in economies in which the zero-bound state entails large output losses. One more way to see this result is to analyze the impact of changes in N, which governs the elasticity of labor supply, on dY^l/dG^l and Θ. Equations (31) and (32) imply that

$$\frac{dY^l}{dG^l} = \frac{(1-\beta p)(1-p)[\gamma(\sigma-1)+1] - p\kappa}{(1-\beta p)(1-g)\beta r^l}\Theta. \quad (33)$$

From equation (31) we see that changes in N that make Δ converge to zero imply that Θ, the impact of the discount factor shock on output, converges to minus infinity. It follows directly from equation (33) that the same changes in N cause dY^l/dG^l to go to infinity. So, again we conclude that the government-spending multiplier is particularly large in economies in which the output costs of being in the zero-bound state are very large.[6]

Sensitivity to the timing of government spending.—In practice, there is likely to be a lag between the time at which the zero bound becomes binding and the time at which additional government purchases begin. A natural question is, how does the economy respond at time t to the knowledge that the government will increase spending in the future? Consider the following scenario. At time t the zero bound binds. Government spending does not change at time t, but it takes on the value $G^l > G$ from time $t+1$ on, as long as the economy is in the zero bound. Under these assumptions, equations (13) and (26) can be written as

$$\pi_t = \beta p \pi^l + \kappa\left(\frac{1}{1-g} + \frac{N}{1-N}\right)\hat{Y}_t \quad (34)$$

and

$$\hat{Y}_t = (1-g)\beta r^l + p\hat{Y}^l - g[\gamma(\sigma-1)+1]p\hat{G}^l + (1-g)p\pi^l. \quad (35)$$

Here we use the fact that $\hat{G}_t = 0$, $E_t(\pi_{t+1}) = p\pi^l$, $E_t(\hat{G}_{t+1}) = p\hat{G}^l$, and $E_t(\hat{Y}_{t+1}) = p\hat{Y}^l$. The values of π^l and $\hat{Y}^l$ are given by equations (29) and (30), respectively. Using equation (30) to replace $\hat{Y}^l$ in equation (35), we obtain

$$\frac{dY_{t,1}}{dG^l} = \frac{1-g}{g}\frac{p}{1-p}\frac{d\pi^l}{d\hat{G}^l}. \quad (36)$$

Here the subscript 1 denotes the presence of a one-period delay in implementing an increase in government spending. So $dY_{t,1}/dG^l$ represents the impact on output at time t of an increase in government spending at time $t+1$. One can show that the multiplier is increasing

[6] An exception pertains to the parameter σ. The value of dY^l/dG^l is monotonically increasing in σ, but $d\hat{Y}^l/dr^l$ is independent of σ.

in the probability, p, that the economy remains in the zero bound. The multiplier operates through the effect of a future increase in government spending on expected inflation. If the economy is in the zero bound in the future, an increase in government purchases increases future output and therefore future inflation. From the perspective of time t, this effect leads to higher expected inflation and a lower real interest rate. This lower real interest rate reduces desired saving and increases consumption and output at time t.

Evaluating equation (36) at the benchmark values, we obtain a multiplier equal to 1.5. While this multiplier is much lower than the benchmark multiplier of 3.7, it is still large. Moreover, this multiplier pertains to an increase in today's output in response to an increase in future government spending that occurs only if the economy is in the zero-bound state in the future.

Suppose that it takes two periods for government purchases to increase in the event that the zero bound binds. It is straightforward to show that the impact on current output of a potential increase in government spending that takes two periods to implement is given by

$$\frac{dY_{t,2}}{dG^l} = p\frac{1-g}{g}\left(\frac{d\pi_{t,1}}{d\hat{G}^l} + \frac{1}{1-p}\frac{d\pi^l}{d\hat{G}^l}\right).$$

Here the subscript 2 denotes the presence of a two-period delay. With our benchmark parameters, the value of this multiplier is 1.44, so the rate at which the multiplier declines as we increase the implementation lag is relatively low.

Consider now the case in which the increase in government spending occurs only after the zero bound ends. Suppose, for example, that at time t the government promises to implement a persistent increase in government spending at time $t+1$ if the economy emerges from the zero bound at time $t+1$. This increase in government purchases is governed by $\hat{G}_{t+j} = 0.8^{j-1}\hat{G}_{t+1}$ for $j \geq 2$. In this case the value of the multiplier, dY_t/dG_{t+1}, is only 0.46 for our benchmark values.

The usual objection to using fiscal policy as a tool for fighting recessions is that there are long lags in gearing up increases in spending. Our analysis indicates that the key question is, in which state of the world does additional government spending come on line? If it comes on line in future periods when the zero bound binds, there is a large effect on current output. If it comes on line in future periods when the zero bound is not binding, the current effect on government spending is smaller.

Optimal government spending.—The fact that the government-spending multiplier is so large in the zero bound raises the following question: taking as given the monetary policy rule described by equation (6), what is the optimal level of government spending when the representative agent's discount rate is higher than its steady-state level? In what follows we use the superscript L to denote the value of variables in states of the world in which the discount rate is r^l. In these states of the world the zero bound may or may not be binding, depending on the level of government spending. From equation (29) we anticipate that the higher government spending is, the higher expected inflation is and the less likely the zero bound is to bind.

We choose G^L to maximize the expected utility of the consumer in states of the world in which the discount factor is high and the zero bound binds. For now we assume that in other states of the world $\hat{G}$ is zero. So we choose G^L to maximize

$$\begin{aligned} U^L &= \sum_{t=0}^{\infty}\left(\frac{p}{1+r^l}\right)^t\left\{\frac{[(C^L)^\gamma(1-N^L)^{1-\gamma}]^{1-\sigma}-1}{1-\sigma}+v(G^L)\right\} \\ &= \frac{1+r^r}{1+r^l-p}\left\{\frac{[(C^L)^\gamma(1-N^L)^{1-\gamma}]^{1-\sigma}-1}{1-\sigma}+v(G^L)\right\}. \end{aligned} \tag{37}$$

To ensure that U^L is finite, we assume that $p<1+r^l$.

Note that

$$Y^L = N^L = Y(\hat{Y}^L+1),$$

$$C^L = Y(\hat{Y}^L+1) - G(\hat{G}^L+1).$$

Substituting these expressions into equation (37), we obtain

$$\begin{aligned} U^L = {}& \frac{1+r^r}{1+r^l-p}\left(\frac{\{[N(\hat{Y}^L+1)-Ng(\hat{G}^L+1)]^\gamma[1-N(\hat{Y}^L+1)]^{1-\gamma}\}^{1-\sigma}-1}{1-\sigma}\right) \\ &+ \frac{1+r^r}{1+r^l-p}v[Ng(\hat{G}^L+1)]. \end{aligned}$$

We choose the value of $\hat{G}^L$ that maximizes U^L subject to the intertemporal Euler equation (eq. [14]), the Phillips curve (eq. [13]), and $\hat{Y}_t = \hat{Y}^L$, $\hat{G}_t = G^L$, $E_t(\hat{G}_{t+1}) = pG^L$, $\pi_{t+1} = \pi^L$, $E_t(\pi_{t+1}) = p\pi^L$, and $R_{t+1} = R^L$, where

$$R^L = \max(Z^L, 0)$$

and

$$Z^L = \frac{1}{\beta} - 1 + \frac{1}{\beta}(\phi_1\pi^L + \phi_2\hat{Y}^L).$$

The last constraint takes into account that the zero bound on interest rates may not be binding even though the discount rate is high.

Finally, for simplicity we assume that $v(G)$ is given by

$$v(G) = \psi_g \frac{G^{1-\sigma}}{1-\sigma}.$$

We choose ψ_g so that $g = G/Y = 0.2$.

Since government purchases are financed with lump-sum taxes, the optimal level of G has the property that the marginal utility of G is equal to the marginal utility of consumption:

$$\psi_g G^{-\sigma} = \gamma C^{\gamma(1-\sigma)-1} N^{(1-\gamma)(1-\sigma)}.$$

This relation implies

$$\psi_g = \gamma\{[N(1-g)]\}^{\gamma(1-\sigma)-1} N^{(1-\gamma)(1-\sigma)} (Ng)^{\sigma}.$$

Using our benchmark parameter values, we obtain a value of ψ_g equal to 0.015.

Figure 3 displays the values of U^L, $\hat{Y}^L$, Z^L, $\hat{C}^L$, R^L, and π^L as a function of $\hat{G}^L$. The asterisk indicates the level of a variable corresponding to the optimal value of $\hat{G}^L$. The circle indicates the level of a variable corresponding to the highest value of $\hat{G}^L$ that satisfies $Z^l \leq 0$. A number of features of figure 3 are worth noting. First, the optimal value of $\hat{G}^L$ is very large: roughly 30 percent (recall that in the steady state government purchases are 20 percent of output). Second, for this particular parameterization the increase in government spending more than undoes the effect of the shock that made the zero-bound constraint bind. Here, government purchases rise to the point where the zero bound is marginally nonbinding and output is actually above its steady-state level. These last two results depend on the parameter values that we chose and on our assumed functional form for $v(G_t)$. What is robust across different assumptions is that it is optimal to substantially increase government purchases and that the government-spending multiplier is large when the zero-bound constraint binds.[7]

The zero bound and interest rate targeting.—Up to now we have emphasized the economy being in the zero-bound state as the reason why the nominal interest rate might not change after an increase in government spending. Here we discuss an alternative interpretation of the constant–interest rate assumption. Suppose that there are no shocks to the economy but that, starting from the nonstochastic steady state, government spending increases by a constant amount and the monetary authority deviates from the Taylor rule, keeping the nominal interest rate equal to its steady-state value. This policy shock persists with probability p. It is easy to show that the government-spending multiplier is given by

[7] We derive the optimal fiscal policy taking monetary policy as given. Nakata (2009) argues that it is also optimal to raise government purchases when monetary policy is chosen optimally. He does so using a second-order Taylor approximation to the utility function in a model with separable preferences in which the natural rate of interest follows an exogenous stochastic process.

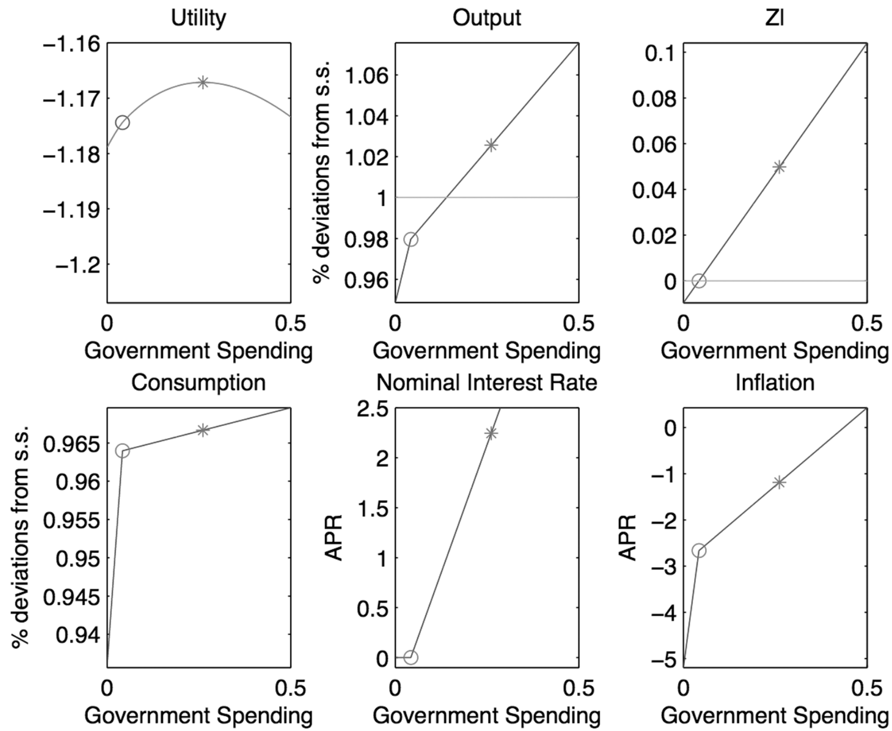

FIG. 3.—Optimal level of government spending in the zero bound

equation (32). So the multiplier is exactly the same as in the case in which the nominal interest rate is constant because the zero bound binds. Of course there is no reason to think that it is sensible for the central bank to pursue a policy that sets the nominal interest rate equal to a positive constant. For this reason, a binding zero bound is the most natural interpretation for why the nominal interest rate might not change after an increase in government spending.

IV. A Model with Capital

In the previous section we use a simple model without capital to argue that the government-spending multiplier is large whenever the output costs of being in the zero-bound state are also large. Here we show that this basic result extends to a generalized version of the previous model in which we allow for capital accumulation. As above we focus on the effect of a discount rate shock.[8]

The model.—The preferences of the representative household are given by equations (23) and (24). The household's budget constraint is given by

$$P_t(C_t + I_t) + B_{t+1} = B_t(1 + R_t) + W_tN_t + P_tr_t^kK_t + T_t, \tag{38}$$

where I_t denotes investment, K_t is the stock of capital, and r_t^k is the real rental rate of capital. The capital accumulation equation is given by

$$K_{t+1} = I_t + (1 - \delta)K_t - D(I_t, I_{t-1}, K_t), \tag{39}$$

where the function $D(I_t, I_{t-1}, K_t)$ represents investment adjustment costs. To assess robustness we consider two specifications for these adjustment costs. The first specification is the one considered in Lucas and Prescott (1971):

$$D(I_t, I_{t-1}, K_t) = \frac{\sigma_I}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t. \tag{40}$$

The parameter $\sigma_I > 0$ governs the magnitude of adjustment costs to capital accumulation. As $\sigma_I \to \infty$, investment and the stock of capital become constant. The resulting model behaves in a manner very similar to the one described in the previous section.

The second specification is the one considered in Christiano et al. (2005) and in Section V:

$$D(I_t, I_{t-1}, K_t) = \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t. \tag{41}$$

[8] In a previous version of this paper, available on request, we also analyze the effect of a neutral and an investment-specific technology shock.

Here the function S is increasing and convex and satisfies the following conditions: $S(1) = S'(1) = 0$.

The household's problem is to maximize lifetime expected utility, given by equations (23) and (24), subject to the resource constraints given by equations (38) and (39) and the condition

$$E_0 \lim_{t\to\infty} B_{t+1}/[(1+R_0)(1+R_1)\cdots(1+R_t)] \geq 0.$$

It is useful to derive an expression for Tobin's q, that is, the value in units of consumption of an additional unit of capital. We denote this value by q_t. For simplicity we derive this expression using the adjustment costs specification (40). Equation (40) implies that increasing investment by one unit raises K_{t+1} by $1 - \sigma_I(I_t/K_t - \delta)$ units. It follows that the optimal level of investment satisfies the following equation:

$$1 = q_t\left[1 - \sigma_I\left(\frac{I_t}{K_t} - \delta\right)\right]. \tag{42}$$

Firms.—The problem of the final-good producers is the same as in the previous section. The discounted profits of the ith intermediate-good firm are given by

$$E_t\sum_{j=0}^{\infty} \beta^{t+j}v_{t+j}\{P_{t+j}(i)Y_{t+j}(i) - (1-\nu)[W_{t+j}N_{t+j}(i) + P_{t+j}r^k_{t+j}K_{t+j}(i)]\}. \tag{43}$$

Output of good i is given by

$$Y_t(i) = [K_t(i)]^{\alpha}[N_t(i)]^{1-\alpha},$$

where $N_t(i)$ and $K_t(i)$ denote the labor and capital employed by the ith monopolist.

The monopolist is subject to the same Calvo-style price-setting frictions described in Section II. Recall that $\nu = 1/\varepsilon$ denotes a subsidy that is proportional to the costs of production. This subsidy corrects the steady-state inefficiency created by the presence of monopoly power. The variable v_{t+j} is the multiplier on the household budget constraint in the Lagrangian representation of the household problem. Firm i maximizes its discounted profits, given by equation (43), subject to the Calvo price-setting friction, the production function, and the demand function for $Y_t(i)$, given by equation (4).The monetary policy rule is given by equation (6).

Equilibrium.—The economy's resource constraint is

$$C_t + I_t + G_t = Y_t. \tag{44}$$

A "monetary equilibrium" is a collection of stochastic processes,

$$\{C_t, I_t, N_t, K_t, W_t, P_t, Y_t, R_t, P_t(i), r^k_t, Y_t(i), N_t(i), v_t, B_{t+1}, \pi_t\},$$

such that, for given $\{d_t, G_t\}$, the household and firm problems are satisfied, the monetary policy rule given by equation (6) is satisfied, markets clear, and the aggregate resource constraint holds.

Experiment.—At time 0 the economy is in its nonstochastic steady state. At time 1 agents learn that r^L differs from its steady-state value for T periods and then returns to its steady-state value. We consider a shock that is sufficiently large so that the zero bound on the nominal interest rate binds between two time periods that we denote by t_1 and t_2, where $1 \leq t_1 \leq t_2 \leq T$.[9] We solve the model using a shooting algorithm. In practice, the key determinants of the multiplier are t_1 and t_2. To maintain comparability with the previous section, we keep the size of the discount factor shock the same and choose $T = 10$. In this case $t_1 = 1$ and $t_2 = 6$. Consequently, the length for which the zero bound binds after a discount rate shock is roughly the same as in the model without capital.

With the exception of σ_I and δ, all parameters are the same as in the economy without capital. We set δ equal to 0.02. We choose the value of σ_I so that the elasticity of I/K with respect to q is equal to the value implied by the estimates in Eberly, Rebelo, and Vincent (2008).[10] The resulting value of σ_I is equal to 17.

We compute the government-spending multiplier under the assumption that G_t increases by $\hat{G}$ percent for as long as the zero bound binds. In general, the increase in G_t affects the time period over which the zero bound binds. Consequently, we proceed as follows. Guess a value for t_1 and t_2. Increase G_t for the period $t \in [t_1, t_2]$. Check that the zero bound binds for $t \in [t_1, t_2]$. If not, revise the guess for t_1 and t_2.

Denote by $\hat{Y}_t$ the percentage deviation of output from steady state that results from a shock that puts the economy into the zero-bound state holding G_t constant. Let $\hat{Y}_t^*$ denote the percentage deviation of output from steady state that results from both the original shock and the increase in government purchases described above. We compute the government spending multiplier as follows:

$$\frac{dY_t}{dG_t} = \frac{1}{g}\frac{\hat{Y}_t^* - \hat{Y}_t}{\hat{G}}.$$

As a reference point we note that when the zero bound is not binding, the government-spending multiplier is roughly 0.9. This value is lower than the value of the multiplier in the model without capital. This lower value reflects the fact that an increase in government spending tends to increase real interest rates and crowd out private investment. This effect is not present in the model without capital.

[9] The precise timing of when the zero-bound constraint is binding may not be unique.

[10] Eberly et al. (2008) obtain a point estimate of b equal to 0.06 in the regression $I/K = a + b\ln(q)$. This estimate implies a steady-state elasticity of I_t/K_t with respect to Tobin's q of $0.06/\delta$. Our theoretical model implies that this elasticity is equal to $(\sigma_I\delta)^{-1}$. Equating these two elasticities yields a value of σ_I of 17.

We now consider the effect of an increase in the discount factor from its steady-state value of 4 percent (annual percentage rate [APR]) to −1 percent (APR). The solid line in figure 4 displays the dynamic response of the economy to this shock. The zero bound binds in periods 1–6. The higher discount rate leads to substantial declines in investment, hours worked, output, and consumption. The large fall in output is associated with a fall in marginal cost and substantial deflation. Since the nominal interest rate is zero, the real interest rate rises sharply. We now discuss the intuition for how the presence of investment affects the response of the economy to a discount rate shock. We begin by analyzing why a rise in the real interest rate is associated with a sharp decline in investment. Ignoring covariance terms, we can write the household's first-order condition for investment as

$$E_t\left(\frac{1+R_{t+1}}{P_{t+1}/P_t}\right) = \frac{1}{q_t}E_t\alpha K_{t+1}^{\alpha-1}N_{t+1}^{1-\alpha}s_{t+1} + \frac{1}{q_t}E_t\left\{q_{t+1}\left[(1-\delta) - \frac{\sigma_I}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2 + \sigma_I\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)\frac{I_{t+1}}{K_{t+1}}\right]\right\}, \tag{45}$$

where s_t is the inverse of the markup rate. Equation (45) implies that in equilibrium the household equates the returns to two different ways of investing one unit of consumption. The first strategy is to invest in a bond that yields the real interest rate defined by the left-hand side of equation (45). The second strategy involves converting the consumption good into $1/q_t$ units of installed capital. The return to this capital has three components. The first component is the marginal product of capital (the first term on the right-hand side of eq. [45]). The second component is the value of the undepreciated capital in consumption units, $q_{t+1}(1-\delta)$. The third component is the value in consumption units of the reduction in adjustment costs associated with an increase in installed capital.

To provide intuition it is useful to consider two extreme cases, infinite adjustment costs ($\sigma_I = \infty$) and zero adjustment costs ($\sigma_I = 0$). Suppose first that adjustment costs are infinite. Figure 1 displays a stylized version of this economy. Investment is fixed and saving is an increasing function of the real interest rate. The increase in the discount factor can be thought of as inducing a rightward shift in the saving curve. When this shift is very large, the real interest rate cannot fall enough to reestablish equilibrium. The intuition for this result and the role played by the zero bound on nominal interest rates is the same as in the model without capital. That model also provides intuition for why the equilibrium is characterized by a large, temporary fall in output, deflation, and a rise in the real interest rate.

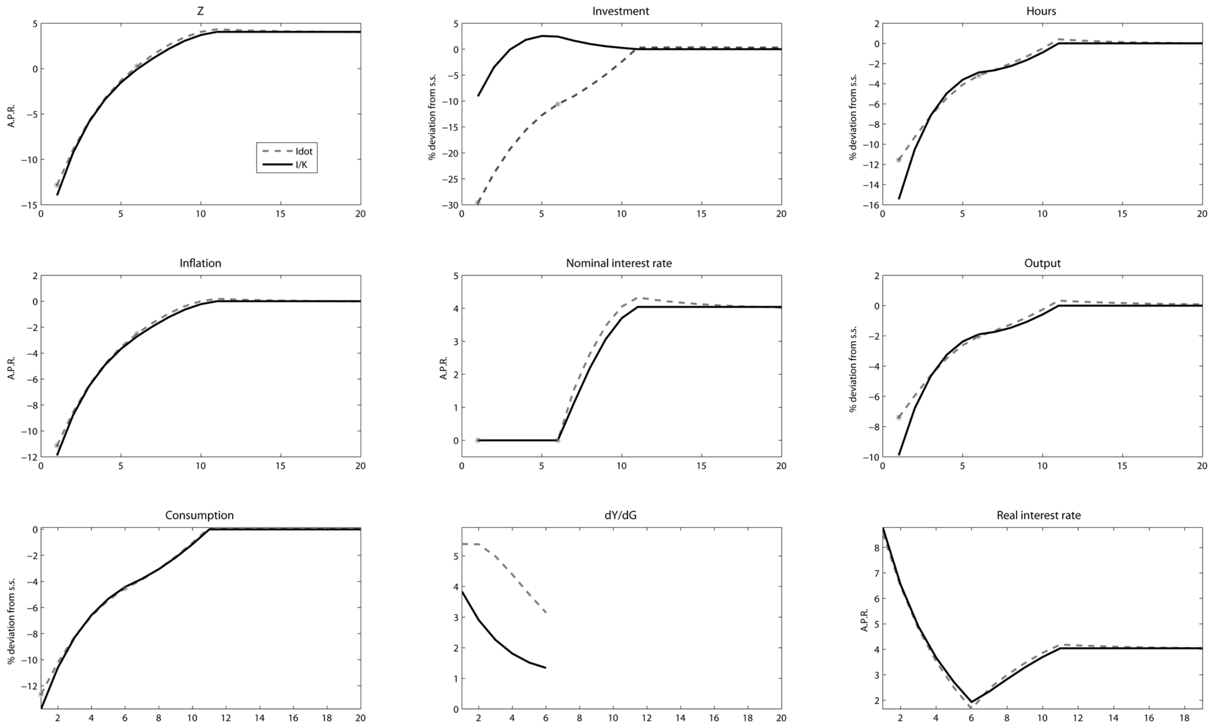

FIG. 4.—Effect of a discount rate shock when the zero bound is binding (model with capital, two types of capital adjustment costs)

Suppose now that there are no adjustment costs ($\sigma_I = 0$). In this case Tobin's q is equal to one and equation (45) simplifies to

$$E_t \frac{1 + R_{t+1}}{P_{t+1}/P_t} = E_t[\alpha K_{t+1}^{\alpha-1} N_{t+1}^{1-\alpha} s_{t+1} + (1 - \delta)].$$

According to this equation, an increase in the real interest rate must be matched by an increase in the marginal product of capital. In general, the latter is accomplished, at least in part, by a fall in K_{t+1} caused by a large drop in investment. In figure 1 the downward-sloping curve labeled "elastic investment" depicts the negative relation between the real interest rate and investment in the absence of any adjustment costs. As drawn, the shift in the saving curve moves the equilibrium to point C and does not cause the zero bound to bind. So the result of an increase in the discount rate is a fall in the real interest rate and a rise in saving and investment.

Now consider a value of σ_I that is between zero and infinity. In this case both investment and q respond to the shift in the discount factor. For our parameter values, the higher the adjustment costs, the more likely it is that the zero bound binds. In terms of figure 1, a higher value of σ_I can be thought of as generating a steeper slope in the investment curve, thus increasing the likelihood that the zero bound binds.

Suppose that the zero bound binds. Other things equal, a higher real interest rate increases desired saving and decreases desired investment. So the fall in output required to equate the two must be larger than in an economy without investment. This larger fall in output is undone by an increase in government purchases.[11] Consistent with this intuition, figure 4 shows that the government-spending multiplier is very large when the zero bound binds (on impact, dY/dG is roughly equal to four). This multiplier, which is computed setting $\hat{G}$ to 1 percent, is actually larger than in the model without capital.

A natural question is what happens to the size of the multiplier as we increase the size of the shock. Recall that in the model without capital, as long as the zero bound binds, the size of the shock does not affect the size of the multiplier. The analogue result here, established using numerical methods, is that the size of the shock does not affect the multiplier as long as it does not affect t_1 and t_2. For a given t_1 the size of the multiplier is decreasing in t_2. For example, suppose that the shock is such that $t_2 = 4$ instead of the benchmark value of 6. In this case the value of the multiplier falls from 3.9 to 2.3. The latter value is

[11] As in the model without capital, the increase in income in states in which the zero bound binds raises permanent income, which raises desired expenditures in zero-bound states. This additional channel reinforces the intertemporal channel stressed in the text.

still much larger than 0.9, the value of the multiplier when the zero bound does not bind.

We conclude by considering the effect of using the adjustment cost specification given by equation (41) rather than equation (40). The dashed line in figure 4 displays the dynamic response of the economy to the discount rate shock. Four key results emerge. First, the response of investment is smaller with the new adjustment cost specification, which directly penalizes changes in investment. Second, while large, the multiplier (6 on impact) is somewhat smaller with the new investment cost specification. This result reflects the smaller response of investment. Third, the dynamic responses of the other variables are similar across the two adjustment cost specifications. Fourth, the values of t_1 and t_2, indicating the period of time over which the zero bound binds, are the same. We conclude that the main results regarding the zero bound are robust across the two.

V. The Multiplier in a Medium-Size DSGE Model

In the previous sections we built intuition about the size of the government-spending multiplier using a series of simple new-Keynesian models. In this section we investigate the determinants of the multiplier in the version of Altig et al.'s (2011) model in which capital is firm specific. The model includes a variety of frictions that are useful for explaining aggregate time-series data. These frictions include sticky wages, sticky prices, variable capital utilization, and the Christiano et al. (2005) investment adjustment cost specification. In what follows all notation is the same as in the previous sections unless noted otherwise.

The final good is produced using a continuum of intermediate goods according to the production function and market structure described in Section II. Intermediate good $i \in (0, 1)$ is produced by a monopolist using the technology

$$y_t(i) = \max\,[\bar{K}_t(i)^{\alpha} N_t(i)^{1-\alpha} - \phi, 0], \tag{46}$$

where $0 < \alpha < 1$. Here, $N_t(i)$ and $\bar{K}_t(i)$ denote time t labor and capital services used to produce the ith intermediate good. The parameter ϕ represents a fixed cost of production. The services of capital, $\bar{K}_t(i)$, are related to the stock of physical capital, $K_t(i)$, by

$$\bar{K}_t(i) = u_t(i) K_t(i).$$

Here $u_t(i)$ is the utilization rate. The cost in investment goods of setting the utilization rate to $u_t(i)$ is given by $a(u_t(i))K_t(i)$, where $a(u_t)$ is increasing and convex. We define $\sigma_a = a''(1)/a'(1) \geq 0$ and impose that $u_t = 1$ and $a(1) = 0$ in steady state.

Intermediate-good firms own their capital, which they cannot adjust within the period. They can change their stock of capital over time only by varying the rate of investment. A firm's stock of physical capital evolves according to equations (39) and (41).

Intermediate-good firms purchase labor services in a perfectly competitive labor market at the wage rate W_t. Firms must borrow the wage bill in advance from financial intermediaries at the gross interest rate, R_t. Profits are distributed to households at the end of each time period.

With one modification, intermediate-good firms set their price subject to the Calvo (1983) frictions described in Section II. The modification is that a firm that cannot reoptimize its price sets $P_t(i)$ according to $P_t(i) = \pi_{t-1}P_{t-1}(i)$. The ith intermediate-good firm's objective function is given by

$$E_t\sum_{j=0}^{\infty}\beta^{t+j}v_{t+j}\{P_{t+j}(i)Y_{t+j}(i) - W_{t+j}R_{t+j}N_{t+j}(i) - [P_{t+j}I_{t+j}(i) + P_{t+j}a(u_{t+j}(i))K(i)_{t+j}]\}. \quad (47)$$

There is a continuum of households indexed by $j \in (0, 1)$. Each household is a monopoly supplier of a differentiated labor service and sets its wage subject to Calvo-style wage frictions as in Erceg, Henderson, and Levin (2000). Household j sells its labor at a wage rate $W_{j,t}$ to a representative competitive firm that transforms it into an aggregate labor input, N_t, using the technology

$$N_t = \left(\int_0^1 N_{j,t}^{1/\lambda_w}dj\right)^{\lambda_w}, \quad 1 \le \lambda_w < \infty.$$

This firm sells the composite labor service to intermediate-good firms at a price W_t.

We assume that there exist complete contingent-claims markets. So in equilibrium all households consume the same amount and have the same asset holdings. Our notation reflects this result. The preferences of the jth household are given by

$$E_t^j\sum_{s=0}^{\infty}\beta^s\left[\log(C_{t+s} - bC_{t+s-1}) - \frac{N_{j,t+s}^2}{2}\right], \quad (48)$$

where E_t^j is the time t expectation operator, conditional on household j's time t information set. The parameter $b > 0$ governs the degree of habit formation in consumption. The household's budget constraint is

$$M_{t+1} = R_t[M_t - Q_t + (x_t - 1)M_t^a] + A_{j,t} + Q_t + W_{j,t}N_{j,t} + D_t - [1 + \eta(\mathcal{V}_t)]P_tC_t - T_t. \quad (49)$$

Here M_t, Q_t, and $W_{j,t}$ denote the household's stock of money at the beginning of period t, cash balances, and the time t nominal wage rate, respectively. Also, T_t denotes period t lump-sum taxes. Each household has a diversified portfolio of claims on all the intermediate-good firms. The variable D_t represents period t firm profits. The variable $A_{i,t}$ denotes the net cash inflow from participating in state-contingent securities at time t. The variable x_t represents the gross growth rate of the economywide per capita stock of money, M_t^a. The quantity $(x_t - 1)M_t^a$ is a lump-sum payment made to households by the monetary authority. The household deposits $M_t - Q_t + (x_t - 1)M_t^a$ with a financial intermediary. The variable $\mathcal{V}_t$ denotes the time t velocity of the household's cash balances: $\mathcal{V}_t = (P_t C_t)/Q_t$. The function $\eta(\mathcal{V}_t)$ captures the role of cash balances in facilitating transactions. This function is increasing and convex. The first-order condition for Q_t implies that the interest semielasticity of money demand in steady state is

$$\epsilon = \frac{1}{4}\left(\frac{1}{R-1}\right)\left(\frac{1}{2 + \eta''\mathcal{V}/\eta'}\right).$$

We parameterize $\eta(\cdot)$ indirectly by choosing steady-state values for ϵ, $\mathcal{V}$, and η.

Financial intermediaries receive $M_t - Q_t + (x_t - 1)M_t$ from the household. Our notation reflects the equilibrium condition, $M_t^a = M_t$. Financial intermediaries lend all of their money to intermediate-good firms, which use the funds to pay the wage bill. Loan market clearing requires that

$$W_t H_t = x_t M_t - Q_t. \tag{50}$$

The aggregate resource constraint is

$$[1 + \eta(\mathcal{V}_t)]C_t + [I_t + a(u_t)K_t] = Y_t. \tag{51}$$

The monetary policy rule is given by equation (6).

Assigning values to model parameters.—In our analysis, we assume that the financial crisis began in the third quarter of 2008. For our experiments we require that the level of the interest rate in the model coincides with that in the data in the second quarter of 2008. A simple way to do this is to suppose that the model is in steady state in the second quarter of 2008 with a nominal interest rate of 2 percent. To this end, we set $\beta = 0.9999$ and $x = 1.0049$.

We assume that intermediate-good firms set their prices once a year ($\xi_P = 0.75$). In conjunction with the other model parameters, the firm-specific capital version of Altig et al. (2011) implies that the coefficient on marginal cost in the new-Keynesian Phillips curve is 0.0026. The low value of this coefficient is consistent with the evidence presented in Altig et al.'s figure 4. We set ξ_W equal to 0.72, Altig et al.'s estimate of this parameter, so that households reoptimize wages roughly once a year. We set the habit formation parameter b to 0.70, a value similar to the point estimates in the models of Altig et al. and Christiano et al. The quarterly rate of depreciation, δ, is 0.02. We set the parameter α

to 0.3. In conjunction with the other parameter values, this value of α generates a steady-state value of $I_t/(C_t + I_t + G_t)$ equal to 0.29, the average value of this ratio in U.S. data over the period 1960Q1–2010Q1. The precise measures of these variables are discussed below.

We set $S''(1)$, ε, and σ_a to the values estimated in Altig et al. (2011) (3.28, 0.80, and 2.02, respectively). We set the parameter ϕ to ensure that the steady-state profits of intermediate-goods firms are zero. We set the steady-state values of V, η, λ_f, and λ_w to the values used in their model (0.45, 0.036, 1.01, and 1.05, respectively). We find that our results are robust to perturbations in this last set of parameters. Finally, we assume that monetary policy is conducted according to the Taylor rule described in equation (6) with $\phi_1 = 0.25$, $\phi_2 = 1.5$, and $\rho = 0$.

The multiplier in the Altig et al. model.—Figure 5 reports the value of the multiplier implied by the model under different scenarios. The first row of figure 5 shows that the value of the government-spending multiplier when monetary policy is governed by a Taylor rule and the zero bound is not binding. We consider the case in which government spending increases by a constant amount for 8 and 12 quarters, respectively. The key result here is that during the first 8 quarters in which the experiments are comparable, the multiplier is higher in the first case than in the second case. This result is consistent with the analysis in Section II, which argues that when the Taylor rule is operative, the magnitude of the multiplier is decreasing in the persistence of the shock to government spending.

The first row of figure 5 also shows the value of the government-spending multiplier when an increase in government spending coincides with a nominal interest rate that is constant, say because the zero bound binds. Recall that the value of the multiplier does not depend on why the nominal interest rate is constant. Given this property, we study the size of the multiplier in the Altig et al. model without specifying either the type or the magnitude of the shock that makes the zero bound binding. Interestingly, when government spending rises for only 8 quarters, the government-spending multiplier is roughly 1.2. When the Taylor rule is operative, the multiplier is smaller. It starts at roughly 1 and declines to about 0.7. When government spending rises for 12 quarters, there is a much larger difference between the Taylor rule case and the zero-bound case. In the latter case the impact multiplier is roughly 1.6. The multiplier rises in a hump-shaped manner, attaining a peak value of roughly 2.3 after five periods. The hump-shaped response of the multiplier reflects the endogenous sources of persistence present in Altig et al.'s model, for example, habit formation in consumption and investment adjustment costs. The zero-bound multiplier is substantially larger when the zero bound binds for 12 periods rather than for 8 periods. This result is consistent with a central finding of this paper:

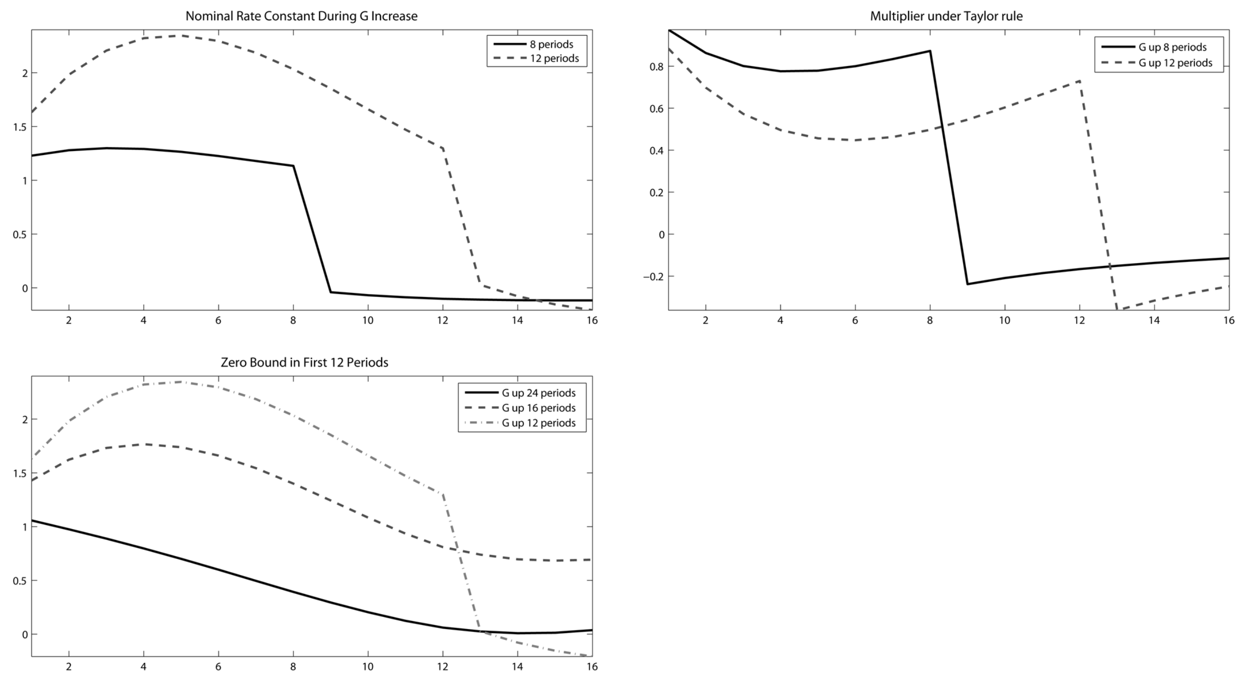

FIG. 5.—Government-spending multiplier in the Altig et al. model

the government-spending multiplier is larger the more severe the zero-bound problem is.[12]

The second row of figure 5 provides information to address the following question: how sensitive is the multiplier to the proportion of government spending that occurs while the nominal interest rate is zero? The figure displays the government-spending multipliers when government spending goes up for 12, 16, and 24 periods. In all cases the nominal interest rate is zero for 12 periods and follows a Taylor rule thereafter. So, in the three cases the proportion of government spending that comes on line while the nominal interest rate is zero is 100, 75, and 50 percent, respectively.

Our basic result is that the multipliers are higher the larger the percentage of the spending that comes on line when the nominal interest rate is zero. This result holds even in the first 12 periods when the increase in government spending is the same in all three cases. For example, the peak multiplier falls from roughly 2.3 to 1.06 as we go from the first to the third case. This decline is consistent with our discussion of the sensitivity of the multiplier to the timing of government spending in Section III. A key lesson from this analysis is that if fiscal policy is to be used to combat a shock that sends the economy into the zero bound, it is critical that the spending come on line when the economy is actually in the zero bound. Spending that occurs after that yields very little bang for the buck and actually dulls the impact of the spending that comes on line when the zero bound binds.

Using a model similar to that of Altig et al., Cogan et al. (2010) study the impact of increases in government spending when the nominal interest rate is set to zero for 1 or 2 years. A common feature of their experiments is that the bulk of the increase in government spending comes on line when the nominal interest rate is no longer constant. Consistent with our results, Cogan et al. find modest values for the government-spending multiplier.

The model's performance during the crisis period.—The Altig et al. model and close variants of it do a good job of accounting for the key properties of U.S. time-series data in the period before the financial crisis (see, e.g., Smets and Wouters 2007; Altig et al. 2011). One natural question is whether the model generates sensible predictions for the current crisis under the assumption that the zero bound binds.

The solid lines in figure 6 display time-series data for the period 2000Q1–2010Q1 for real per capita output, private consumption, investment, government consumption, inflation, and the federal funds

[12] For completeness we also considered the case in which the zero bound binds for only 4 quarters. In this case the zero-bound multiplier and the multiplier when the Taylor rule is operative are very similar.

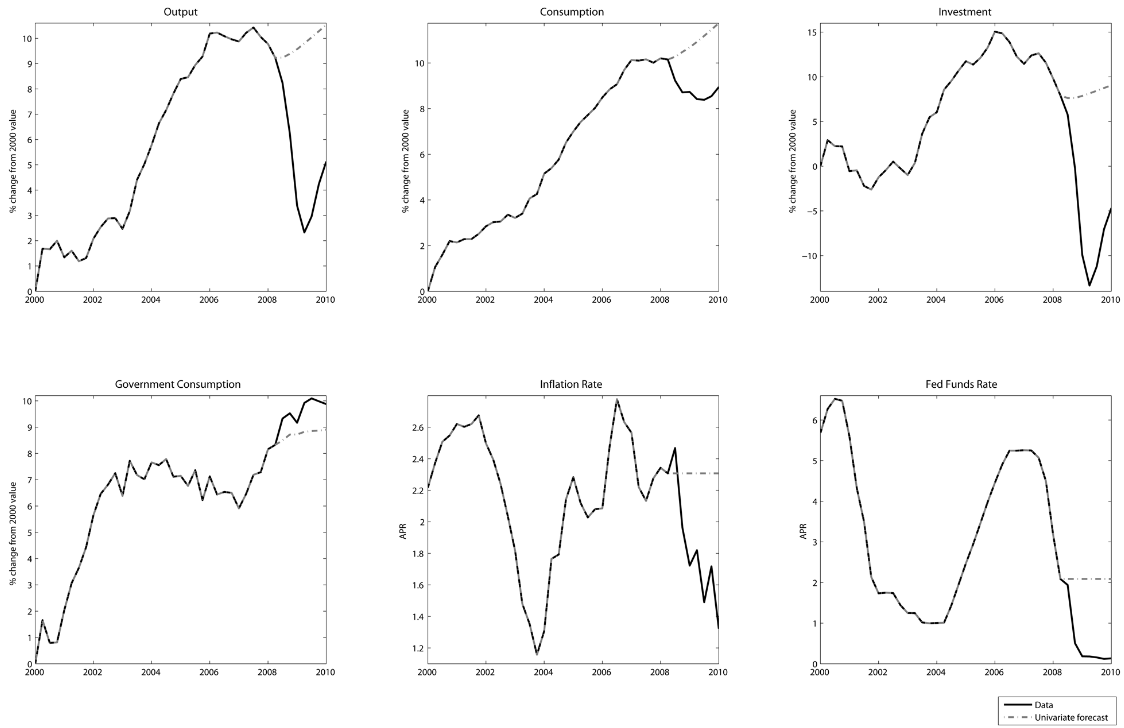

FIG. 6.—Data and forecasts

rate. The data displayed are the percentage change in a variable from its value in 2000Q1. All per capita variables are computed using as a measure of the population the civilian noninstitutional population 16 years and over. All variables with the exception of inflation and the interest rate are seasonally adjusted and computed as real chain-weighted billions of 2005 U.S. dollars. Output is the sum of consumption, investment, and government consumption. We also discuss results when we use real GDP as the measure of output. Private consumption is consumption of nondurables and services. Investment is household purchases of durable goods, federal government investment, and gross private domestic investment. Total government consumption is federal government consumption and state and local expenditures on consumption and investment. Inflation is the year-over-year growth rate in the core consumer price index. The interest rate is the federal funds rate.

We date the beginning of the financial crisis as the third quarter of 2008. This is the quarter during which Lehman Brothers collapsed. We are interested in computing the effect of the financial crisis on the evolution of the U.S. economy. To this end we forecast the variables reported in figure 6 using data up to and including the second quarter of 2008. With the exception of output, inflation, and the interest rate, we compute our forecasts using a four-lag scalar autoregression fit to the level of the data. The output forecast is equal to the weighted sum of the forecasted values of consumption, investment, and government purchases. The forecasts for the interest rate and inflation are equal to the level of these variables in 2008Q2. These forecasts are displayed as the dotted lines in figure 6.

A rough measure of the impact of the crisis on the variables included in figure 6 is the difference between the actual and the forecasted values of these variables. These differences, that is, the impulse response functions to the shocks that precipitated the crisis, are displayed as the solid lines in figure 7. It is evident that the nominal interest rate fell very quickly and hit the zero bound. There was a significant drop in consumption and a very large fall in investment. Output fell by 7 percent. Inflation fell by 1 percent relative to what it would been without the crisis. Despite the fiscal stimulus plan enacted in February 2009 (the American Recovery and Reinvestment Act), total government consumption rose by only 2 percent. Total government purchases, which include both consumption and investment, rose by even less. This result reflects two facts. First, a substantial part of the stimulus plan involved an increase in transfers to households. Second, there was a large fall in

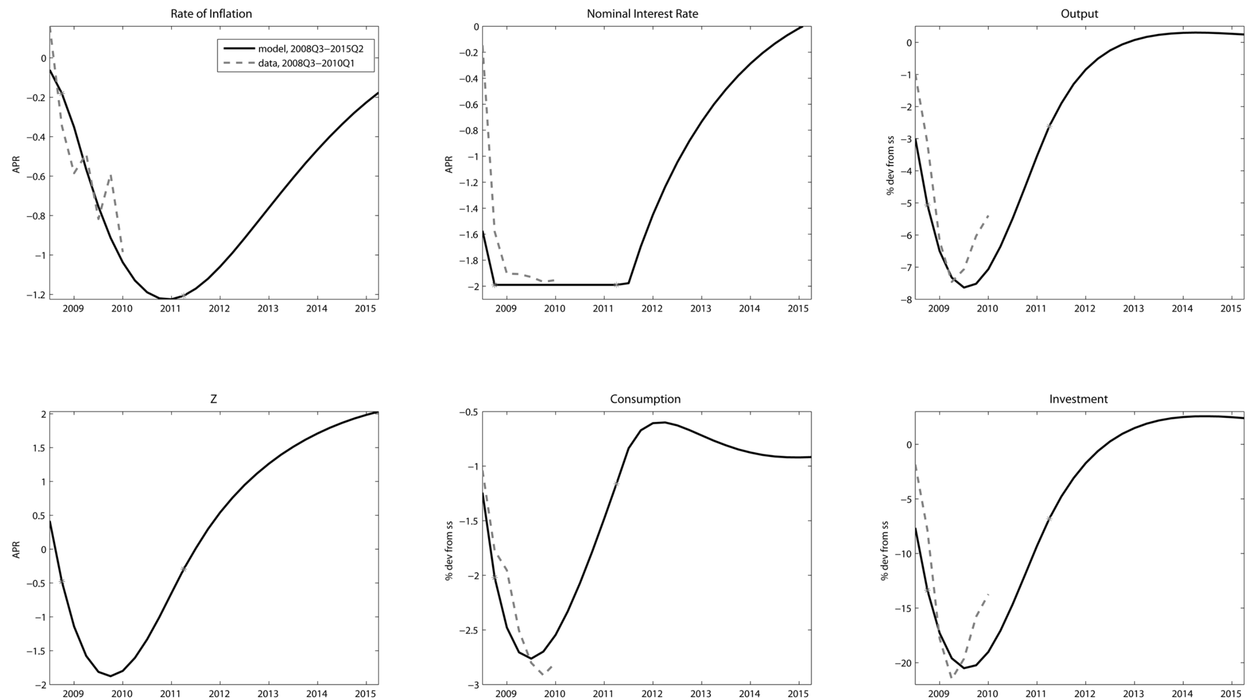

FIG. 7.—Data and model impulse response functions

state and local purchases that offset a substantial part of the increase in federal government purchases.[13]

To assess the model's implication for the crisis period, we need to specify the shocks that made the zero bound binding. In our view the crisis was precipitated by disturbances in financial markets that increased the spread between the return on savings and the return on investment. The financial crisis and the resulting uncertainty led to a large rise in the household's desire to save. Consistent with this view, the personal savings rate (measured by PSAVERT, Federal Reserve Bank of St. Louis) rose sharply from roughly 2 percent in 2007 to a level that stabilized at around 5.5 percent. The Altig et al. model is not sufficiently rich to provide a detailed account of the financial crisis or the steep rise in household saving. We mimic the effects of the crisis by introducing the discount factor shock discussed in the previous sections, as well as a financial friction shock.

The Altig et al. model assumes that firms finance investment out of retained earnings. We imagine that each dollar passing between households and firms goes through the financial system. In normal times every dollar transferred between households and firms uses up τ dollars' worth of final goods. Thus, we replace v_{t+j} in (47) with $v_{t+j}(1-\tau)$. When we abstract from general equilibrium effects on v_{t+j} in (47), the value of τ does not affect the firm's decisions as long as it is constant.[14] We assume that τ is constant until 2008Q2 and that agents expected it to remain constant forever. At the onset of the financial crisis in 2008Q3, agents learn that the costs of intermediation rise. Let

$$1-\tau^k \equiv \frac{1-\tau_t}{1-\tau_{t-1}}. \tag{52}$$

We suppose that $\tau^k > 0$ for t corresponding to the first period of the crisis (i.e., 2008Q3) until the last period, $t = T$, of the crisis. We suppose that $\tau_t^k = 0$ for $t > T$.[15]

The ith intermediate-good firm maximizes the modified version of (47) that accommodates τ_t^k. The necessary first-order condition for investment can be written as follows:

$$u_{ct} = E_t \beta u_{ct+1} R_{t+1}^k(i)(1-\tau_{t+1}^k). \tag{53}$$

Here, u_{ct}, which is taken as given by the firm, is the marginal utility of household consumption:

[13] See Cogan and Taylor (2010) for a detailed analysis of the impact of the American Recovery and Reinvestment Act on government spending.

[14] The general equilibrium effect operates through the impact of τ on the aggregate resource constraint. In our computations, we abstract from this general equilibrium effect on the grounds that it is presumably small.

[15] With this formulation, the constant, postcrisis level of τ is higher than the precrisis level of τ.

$$u_{ct} = \frac{1}{C_t - bC_{t-1}} + E_t\left(\frac{\beta b}{C_{t+1} - bC_t}\right).$$

Let R^k_{t+1} denote the cross-section average return on capital, that is, the average across i of $R^k_{t+1}(i)$. One measure of the interest rate spread in the model is the difference between R^k_{t+1} and the corresponding average return received by households, $R^k_{t+1}(1 - \tau^k_{t+1})$. This difference is equal to $\tau^k_{t+1}R^k_{t+1}$.

Our assumption that τ^k_t rose during the crisis is essentially equivalent to the assumption that our measure of the interest rate spread rose. In reality, interest rate spreads move for many reasons, for example, changes in bankruptcy risk, changes in liquidity, and confidence in the banking system. In the wake of the 2008 financial crisis, virtually all interest rate spreads rose dramatically. Consider, for example, the behavior of the interest spreads on non-AAA corporate bonds relative to AAA bonds. In the case of BAA, BB, B, and "junk," defined as CCC and lower-rated bonds, the average value of the spread is 0.88, 1.75, 2.71, and 5.75 percent, respectively, over the period, 2005–7. These spreads rose to peak values of 3.38, 8.83, 14.10, and 27.72 at the end of 2008.[16] Thereafter, spreads in annual percentage terms declined to values of 1.20, 2.36, 3.87, and 7.88, respectively, by 2010Q3.

With these data as background, we set $\tau^k = 3.6/400$ and $T = 12$. This assumption implies that at the time of the crisis, the interest rate spread on a 3-year bond jumps by 3.6 percentage points at an annual rate and then declines linearly back to zero after 3 years. We focus on the 3-year bond because the work of Barclay and Smith (1995) and Stohs and Mauer (1996) suggests that the average duration of corporate debt is in the range of 3–4 years.[17]

We assume that G_t increases by 2 percent for as long as the zero bound binds. As in Section IV, we compute the time interval $t \in [t_1, t_2]$ during which the zero bound binds. We find that $t_1 = 2$ and $t_2 = 11$, so the zero bound binds from the fourth quarter of 2008 until the third quarter of 2011.

The dashed-dotted line in figure 7 corresponds to the model's predictions for the economy during the crisis. A number of features are worth noting. First, the model accounts for the rapid decline of the

[16] The analysis is based on quarterly averaged data. The peak of the BB and B bond spreads occurs in 2008Q4 and the peak in the junk bond spreads occurs in 2009Q1.

[17] See Stohs and Mauer (1996, table 2). Barclay and Smith do not directly report average durations. Instead, they report the percentage of debt that matures in more than n years, for $n = 1, 2, 3, 4$, and 5. These percentages are 73, 65.7, 58.7, 52.2, and 45.9, respectively. These numbers imply an average duration if one makes an assumption about the mean duration for firms with $n > 5$. For example, if this mean duration is 7.5, then mean duration is 4.2 years, where

$$4.2 = 1.5 \times 0.073 + 2.5 \times 0.070 + 3.5 \times 0.065 + 4.5 \times 0.063 + 7.5 \times 0.459.$$

federal funds rate at the onset of the crisis. Second, the model is consistent with the observed declines in consumption, investment, and output. Third, and perhaps most important, the model also does a good job of accounting for the postcrisis behavior of inflation. According to our estimates, inflation fell by roughly 1 percent as a result of the crisis (see the solid line in fig. 7). The model's predictions are consistent with this decline.

To assess robustness with respect to our output measure, figure 7 reports the difference between the log level and the univariate forecast of per capita real GDP. This difference is displayed as the dashed line in the subplot labeled "Output." Notice that the paths of the two real output measures are very similar. Interestingly, our measure of output falls by somewhat more than per capita real GDP. For example, the maximal impact of the crisis is a 7 percent and a 5.8 percent decline in our measure of output and real GDP, respectively. If we calibrate the model to match the fall in real GDP, we would generate less deflation and smaller declines in consumption and investment.

We conclude by noting that, consistent with the data, in our simulations, government purchases rise by only 2 percent for 11 periods. Recall from figure 5 that the peak value of the multiplier in Altig et al.'s model is 2.3. So the rise in government purchases accounts for, at most, a 0.7 percent rise in annual GDP.[18] The modest contribution of government purchases to the recovery reflects the very modest increase in government spending rather than a small multiplier.

VI. Conclusion

In this paper we argue that the government-spending multiplier can be very large when the nominal interest rate is constant. We focus on a natural case in which the interest rate is constant, which is when the zero lower bound on nominal interest rates binds. In these economies the government-spending multiplier is quite modest when monetary policy is governed by a Taylor rule.

We conclude by noting that an obvious alternative to increasing government spending to deal with the zero-lower-bound problem is to manipulate the demand for goods by varying the time profile of investment tax credits or consumption taxes. Here we briefly comment on the latter.[19] In the context of the Japanese zero-lower-bound episode, Feldstein (2003) proposes raising the value-added tax (VAT) by 1 percent per quarter and simultaneously reducing income tax rates to keep rev-

[18] We base this calculation on the fact that $dY/Y = (dY/dG)(dG/G)(G/Y)$ and the assumption that $G/Y = 0.15$.

[19] See Eggertsson (2011) for a discussion of the effects of investment tax credits.

enue unchanged, continuing this policy for several years until the VAT reaches 20 percent. Correia et al. (2010) argue that if taxes on consumption, labor, and capital income are state contingent, every allocation that can be implemented with a combination of taxes and monetary policy that does not necessarily respect the zero-lower-bound constraint can also be implemented with a different combination of taxes and monetary policy that does respect the zero-lower-bound constraint.

It is evident that the policies envisioned by Feldstein (2003) and by Correia et al. (2010) were not pursued in the United States. Implementing these policies would require introducing a national consumption tax.[20] We are skeptical about introducing a new source of national taxation to deal with rare events like the zero-lower-bound problem. Our skepticism stems from the political economy literature that tries to explain why modern economies do not rely more heavily on consumption taxes (e.g., Brennan and Buchanan 1977; Krusell, Quadrini, and Rios-Rull 1996). A key insight from this literature is that if government revenue is used for redistributive purposes, then consumption taxes may be welfare decreasing by comparison with income taxes. Income taxes are attractive precisely because they are more distortionary. Since it is more costly to raise revenues with income taxes, there are less transfers in equilibrium. Krusell et al. emphasize that in their model, switching from an income to a consumption tax system typically does not make the median voter better off. But changing from income to consumption taxes can make everybody worse off.

Many countries already have VATs, but even here we are skeptical of the feasibility of the policies proposed by Feldstein (2003) and Correia et al. (2010). This skepticism stems from the need to introduce a complicated state-dependent tax policy to deal with the rare occasions in which the zero bound binds. It is possible that a simplified version of the tax policies envisioned by Correia et al. would be desirable. Understanding the quantitative welfare properties of simple tax policies versus increases in government spending as a way of dealing with the zero-bound problem is an important topic that we leave for future research.

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[20] There are, of course, sales taxes at the state and local levels, but, presumably, it would have been difficult and time consuming to coordinate changes in these tax rates. There were programs such as cash for clunkers, but these were small in scale.

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