Properties of New Keynesian Model that Can be Derived Analytically

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Outline

- Fisherian vs anti-Fisherian Debate:
  - How do you get inflation down (or, up)?
    - Fisherian answer: cut the nominal rate of interest.
    - Anti Fisherian answer: raise the interest rate.
  - How do we think about these seemingly contradictory answers?
    - NK model gives us a way to think about this.
  - Draw on Erceg and Levin, 2003 JME paper, "Imperfect credibility and inflation persistence"
- Forward Guidance Puzzle
- How does the Taylor Principle work to stabilize inflation in the equilibrium local to steady state?

Fisherian versus Anti Fisherian Policy

- *Fisherian effect*
  - Interest rate and inflation move in the same direction.
- *Anti-Fisherian effect*
  - Interest rate and inflation move in opposite direction.

Intuition

- Monetary policy rule (inflation target, $\bar{\pi}_t$):

$$r_t = \pi_t + \phi\left(\pi_t - \bar{\pi}_t\right)$$

- Temporary cut in $\bar{\pi}_t$ (anti-Fisherian effect)
  - actual inflation, π_t, responds very little because price setters focus on long-run conditions.
  - r_t rises and $r_t - \pi_{t+1}$ rises too.
  - output, inflation fall: $cov\left(\pi_t, r_t\right) < 0$.
- Permanent cut in $\bar{\pi}_t$ (Fisherian effect)
  - π_t drops strongly
  - r_t falls
  - not much change in $r_t - \pi_{t+1}$ so little change in output.
  - $cov\left(\pi_t, r_t\right) > 0$

Linearized Equilibrium Conditions

- Monetary policy rule:

$$r_t = \pi_t + \phi\left(\pi_t - \bar{\pi}_t\right).$$

- Law of motion of inflation target, $\bar{\pi}_t$:

$$\bar{\pi}_t = \delta\bar{\pi}_{t-1} + \varepsilon_t.$$

- Phillips curve and output gap:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\ x_t &= E_t x_{t+1} - \left[r_t - E_t \pi_{t+1}\right].\end{aligned}$$

Solving Linearized Equilibrium Conditions

- (Linearized) Equilibrium Conditions of Model:

$$r_t = \pi_t + \phi\left(\pi_t - \bar{\pi}_t\right), \quad \bar{\pi}_t = \delta \bar{\pi}_{t-1} + \varepsilon_t$$

$$\begin{aligned} \pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\ x_t &= E_t x_{t+1} - \left[r_t - E_t \pi_{t+1}\right]. \end{aligned}$$

- Undetermined coefficients method, a_1, a_2, a_3 :

$$\pi_t = a_1 \bar{\pi}_t, \qquad x_t = a_2 \bar{\pi}_t, \qquad r_t = a_3 \bar{\pi}_t$$

- Substitute the solution into the equations and require that they hold for all possible $\bar{\pi}_t$:

$$\begin{aligned} a_3 &= a_1 + \phi\left(a_1 - 1\right) \\ a_1 &= \beta \delta a_1 + \kappa a_2 \\ a_2 &= a_2 \delta - \left[a_3 - a_1 \delta\right]. \end{aligned}$$

- Want to know: a_1, a_3 when $\delta = 0$ and $\delta = 1$.

Solving the Model: Getting the a's

- Substitute the solution into the equations:

$$
\begin{aligned}
a_3 &= a_1 + \phi\,(a_1 - 1) \\
a_1 &= \beta\delta a_1 + \kappa a_2 \\
a_2 &= a_2\delta - [a_3 - a_1\delta]\,.
\end{aligned}
$$

- Now start rearranging stuff

$$
\begin{aligned}
a_3 &= (1+\phi)\,a_1 - \phi \\
a_1 &= \frac{\kappa}{1-\beta\delta} a_2 \\
a_2 &= a_2\delta - (1+\phi-\delta)\,a_1 + \phi
\end{aligned}
$$

- $\delta = 1$ result now obvious $(a_1 = a_3 = 1)$; $\delta = 0$ easy.

Solving the Model: Getting the a's

- Model:

$$r_t = \pi_t + \phi\left(\pi_t - \bar{\pi}_t\right), \quad \bar{\pi}_t = \delta\bar{\pi}_{t-1} + \varepsilon_t$$

$$\pi_t = \beta\pi_{t+1} + \kappa x_t$$
$$x_t = x_{t+1} - \left[r_t - \pi_{t+1}\right].$$

- Undetermined coefficients, a_1, a_2, a_3 :

$$\pi_t = a_1\bar{\pi}_t, \qquad x_t = a_2\bar{\pi}_t, \qquad r_t = a_3\bar{\pi}_t$$

- Solution ▸ derivation

$$a_1 = \frac{\phi}{\left[\frac{1-\beta\delta}{\kappa} + 1\right](1-\delta) + \phi}, \qquad a_3 = (1+\phi)\,a_1 - \phi.$$

  - Permanent case, $\delta = 1 : a_1 = a_3 = 1$
  - Temporary case, $\delta = 0$:
    - $a_1 = \frac{\phi}{\frac{1}{\kappa}+1+\phi} > 0, \qquad a_3 = -\frac{\phi/\kappa}{\frac{1}{\kappa}+1+\phi} < 0.$

Erceg and Levin Combine the two Effects to Explain Volcker Recession

- Volcker reduced the inflation target, $\bar{\pi}_t$, permanently.
- But, what matters is people's *beliefs*, and they were convinced the reduced target was only temporary.
  - The 1970s had witnessed numerous episodes in which the Fed reduced the target 'permanently', only to raise it again soon after.
- So, the public thought Volcker was 'business as usual' and interpreted the decline in the target as temporary.
  - Interest rates went way up and output, down.
  - Forecasts of inflation remained stubbornly high.
  - Eventually, everyone realized that $\bar{\pi}_t$ was down permanently.
    - Fisherian effects kicked in and both interest rates and inflation fell.
    - Output returned to potential.

Forward Guidance Puzzle

- When interest rates became very low after 2008, monetary policy authorities resorted to ‘forward guidance’:
  - Announcements that interest rates in the future will be low.

- Studying forward guidance in models, researchers stumbled on what came to be called the ‘forward guidance puzzle’:
  - Announcements about a cut in the interest rate in the distant future have a bigger impact that a current reduction in the interest rate.

- People felt this was implausible (though we have no empirical evidence on the issue) and so called it a puzzle.

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- People feel this property is implausible (though we have no empirical evidence on the issue) and so called it a *puzzle*.

Characterizing the Puzzle

- Phillips curve and output gap:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t$$
$$x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1}].$$

- Two scenarios, each followed by Taylor rule in $t + j + s$, $s \geq 0$.
  - j Period Forward guidance:
    - $r_{t+s} = 0$ for $s = 0, ..., j-1$, $r_{t+j} = \theta$.
  - Immediate policy:
    - $r_t = \theta$, $r_{t+s} = 0$, for $s = 1, 2, ..., j$.
- Taylor rule:

$$r_t = \overbrace{\phi}^{\text{Taylor principle, } \phi > 1} \pi_t.$$

- Result: impact on date t variables greater from forward guidance than from immediate policy.

One-period Forward Guidance ($j = 2$)

- Announcement at time t : $r_t = 0$, $r_{t+1} = \theta$, Taylor rule thereafter.
- Because (i) there are no shocks, (ii) the model is purely forward looking and (iii) Taylor rule with $\phi > 1$ in place after $t + 1$:

$$r_{t+s} = x_{t+s} = \pi_{t+s} = 0,\ s > 1.$$

- In period $t + 1$

$$\begin{aligned} r_{t+1} &= \theta \\ x_{t+1} &= x_{t+2} - [r_{t+1} - \pi_{t+2}] = 0 - [r_{t+1} - 0] = -r_{t+1} \\ \pi_{t+1} &= \beta \pi_{t+2} + \kappa x_{t+1} = \kappa x_{t+1} = -\kappa r_{t+1} \end{aligned}$$

- So, in $t + 1$:

$$r_{t+1} = \theta,\ x_{t+1} = -\theta,\ \pi_{t+1} = -\kappa\theta.$$

- What happens in period t?

One-period Forward Guidance ($j = 2$)

- Effect, in the period $t+1$, of $t+1$ policy action announced in t:

$$r_{t+1} = \theta,\ x_{t+1} = -\theta,\ \pi_{t+1} = -\kappa\theta.$$

- In period t:

$$\begin{aligned} r_t &= 0 \\ x_t &= x_{t+1} - [r_t - \pi_{t+1}] = -\left(\overbrace{1}^{\text{direct effect}} + \overbrace{\kappa}^{\text{indirect effect}} \right) r_{t+1} \\ \pi_t &= \beta\pi_{t+1} + \kappa x_t = -\beta\kappa r_{t+1} + \kappa x_t \end{aligned}$$

so,

$$\begin{aligned} \pi_t &= -\beta\kappa r_{t+1} + \kappa \overbrace{x_t}^{=-(1+\kappa)r_{t+1}} \\ \rightarrow\ \pi_t &= -[1+\beta+\kappa]\,\kappa\theta, \quad x_t = -(1+\kappa)\,\theta \end{aligned}$$

Immediate Policy

- Announcement at time t : $r_t = \theta \neq 0$, $r_{t+1} = 0$ and Taylor rule thereafter.
- Because the model is completely forward looking,

$$r_{t+s} = x_{t+s} = \pi_{t+s} = 0, \; s > 0.$$

- Then,

$$\begin{aligned} r_t &= \theta \\ x_t &= x_{t+1} - [r_t - \pi_{t+1}] = 0 - [r_t - 0] = -r_t \\ \pi_t &= \beta \pi_{t+1} + \kappa x_t = \beta \times 0 + \kappa x_t \end{aligned}$$

- So,

$$\rightarrow r_t = \theta, \quad x_t = -\theta, \quad \pi_t = -\kappa\theta.$$

which is smaller than with one-period forward guidance:

$$\pi_t = -[1 + 2\kappa]\, \kappa\theta, \quad x_t = -(1 + \kappa)\, \theta$$

Intuition

- Consider j Period Forward Guidance.
  - Announcement at time t : $r_{t+j} = \theta \neq 0$ and $r_{t+s} = 0$ for $s = 0, 1, ..., j-1$. Switch to Taylor rule after $t+j$.
- IS equation (recall, $r_t = ... = r_{t+j-1} = 0$) :

$$
\begin{aligned}
x_{t+j} &= x_{t+j+1} - \left(r_{t+j} - \pi_{t+j+1}\right) = -r_{t+j} \\
x_{t+j-1} &= x_{t+j} - \left(r_{t+j-1} - \pi_{t+j}\right) = -\left(r_{t+j-1} - \pi_{t+j}\right) - r_{t+j} \\
&\vdots \\
x_t &= -\left(r_t - \pi_{t+1}\right) - \left(r_{t+1} - \pi_{t+2}\right) \\
&\quad -... - \left(r_{t+j-1} - \pi_{t+j}\right) - r_{t+j}
\end{aligned}
$$

Intuition, cnt'd

- IS equation (recall, $r_t = ... = r_{t+j-1} = 0$) :

$$\begin{aligned} x_t &= -(r_t - \pi_{t+1}) - (r_{t+1} - \pi_{t+2}) \\ &\quad -... - (r_{t+j-1} - \pi_{t+j}) - r_{t+j} \end{aligned}$$

- Change in r_{t+j} has a *direct* effect on x_t and an *indirect* effect.
  - Direct: change in r_{t+j} moves x_{t+j} and (by consumption smoothing channel) that leads to an equal change in earlier output gaps, including x_t.
    - This channel holds fixed the real interest rates, $(r_{t+s} - \pi_{t+s+1})$, $s = 0, ..., j-1$.
  - Indirect: change in r_{t+j} affects $(r_{t+s} - \pi_{t+s+1})$, $0 \leq s \leq j-1$ in each date between now and $t+j$ by reducing inflation in each date.
    - The impact on x_t of the indirect effect is the *cumulative sum* (increasing in j) of the changes in the real interest rate.

Forward Guidance: Conclusion

- Forward Guidance Puzzle is generally attributed to Del Negro, Giannoni and Patterson, 'The Forward Guidance Puzzle', NYFed Staff Report No. 574, 2012, revised manuscript in 2017.
- Sparked a large literature to 'solve' the problem.
  - Gabaix, "A Behavioral New Keynesian Model", NBER WP 22954, June 2019.
  - Farhi and Werning, "Monetary Policy, Bounded Rationality, and Incomplete Markets," NBER Working Paper No. 23281, 2017.
  - Angeletos, and Lian, "Forward guidance without common knowledge," American Economic Review, 2018.
  - Campbell, Fisher, Justiniano, and Melosi, "Forward Guidance and Macroeconomic Outcomes since the Financial Crisis," NBER Macroeconomics Annual, 2017, 31 (1), 283–357.
    - This offers what is perhaps the simplest resolution: in practice, announcements about policy actions far in the future have little impact on behavior because they are not credible.
    - In my presentation, I assumed 100% credibility.

How Does the Taylor Principle Work to Stabilize Inflation?

- Model

$$
\begin{aligned}
x_t &= E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r_t^*] \\
r_t &= \phi_\pi \pi_t, \quad \phi_\pi > 1 \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
\Delta a_t &= \rho \Delta a_{t-1} + \varepsilon_t \\
r_t^* &= E_t (a_{t+1} - a_t) = \rho \Delta a_t.
\end{aligned}
$$

- Unique non-explosive solution:

$$
\pi_t = \gamma_1 \Delta a_t, x_t = \gamma_2 \Delta a_t, r_t = \gamma_3 \Delta a_t
$$

  – γ_i's ˜ undetermined coefficients.

Solving the Model

- Model and solution

$$
\begin{aligned}
x_t &= E_t x_{t+1} - [r_t - E_t \pi_{t+1} - r_t^*] \\
r_t &= \phi_\pi \pi_t, \quad \phi_\pi > 1 \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
\Delta a_t &= \rho \Delta a_{t-1} + \varepsilon_t \\
r_t^* &= E_t (a_{t+1} - a_t) = \rho \Delta a_t \\
\pi_t &= \gamma_1 \Delta a_t, x_t = \gamma_2 \Delta a_t, r_t = \gamma_3 \Delta a_t
\end{aligned}
$$

- Substitute solution into model:

$$
\begin{aligned}
\gamma_2 &= \rho \gamma_2 - \gamma_3 + \rho \gamma_1 + \rho \\
\gamma_3 &= \phi_\pi \gamma_1 \\
\gamma_1 &= \beta \gamma_1 \rho + \kappa \gamma_2
\end{aligned}
$$

- Real rate: $\tilde{r}_t = r_t - E_t \pi_{t+1} = \gamma_4 \Delta a_t$,

$$
\gamma_4 = \gamma_3 - \gamma_1 \rho.
$$

Solving the Model

- Each to verify:

$$r_t - E_t\pi_{t+1} = \overbrace{\psi}^{=\gamma_4}\Delta a_t, x_t = \overbrace{\frac{(1-\beta\rho)}{\kappa(\phi_\pi - \rho)}\psi}^{=\gamma_2}\Delta a_t, \pi_t = \overbrace{\frac{\psi}{\phi_\pi - \rho}}^{=\gamma_1}\Delta a_t$$

  where

$$\psi \equiv \frac{\rho}{\frac{(1-\beta\rho)(1-\rho)}{\kappa(\phi_\pi-\rho)} + 1}.$$

- For ϕ_π sufficiently large,

$$\psi \simeq \rho,\ r_t - E_t\pi_{t+1} \simeq r_t^*,\ \pi_t \simeq 0,\ x_t \simeq 0.$$

- Big value of ϕ stabilizes equilibrium around first best.
  - However, requires very large value of ϕ_π.
  - For practical values, Taylor rule too weak, $\psi < \rho$ and $\gamma_2 > 0$.
- Taylor principle:
  - real rate of interest increases when π_t high ($\psi > 0$ and $\phi > \rho$).
  - effects bigger with bigger ϕ_π.

Solving the Model

- The equations:

$$r_t = \pi_t + \phi\left(\pi_t - \bar{\pi}_t\right)$$
$$\pi_t = \beta\pi_{t+1} + \kappa x_t$$
$$x_t = x_{t+1} - \left[r_t - \pi_{t+1}\right].$$

- Substitute the solution in here:

$$a_3 = a_1 + \phi\left(a_1 - 1\right)$$
$$a_1 = \beta\delta a_1 + \kappa a_2$$
$$a_2 = a_2\delta - \left[a_3 - a_1\delta\right].$$

- Rearranging:

$$a_3 = (1+\phi)\, a_1 - \phi$$
$$a_1 = \frac{\kappa}{1-\beta\delta} a_2$$
$$a_2 = a_2\delta - \left[a_3 - a_1\delta\right] = a_2\delta - (1+\phi-\delta)\, a_1 + \phi$$
$$\rightarrow a_2 = -\frac{1+\phi-\delta}{1-\delta} a_1 + \frac{\phi}{1-\delta}$$

Solving the Model

- Working on the second equation,

$$a_1 \frac{1-\beta\delta}{\kappa} = -\frac{(1+\phi-\delta)}{1-\delta} a_1 + \frac{\phi}{1-\delta}$$

then,

$$a_1 = \frac{\frac{\phi}{1-\delta}}{\frac{1-\beta\delta}{\kappa} + \frac{1+\phi-\delta}{1-\delta}} = \frac{\frac{\phi}{1-\delta}}{\frac{1-\beta\delta}{\kappa} + 1 + \frac{\phi}{1-\delta}} = \frac{\phi}{\left[\frac{1-\beta\delta}{\kappa} + 1\right](1-\delta) + \phi}$$

- Then,

$$a_3 = \frac{(1+\phi)\,\phi}{\left[\frac{1-\beta\delta}{\kappa} + 1\right](1-\delta) + \phi} - \phi$$

- So, when $\delta = 1 : a_1 = a_3 = 1$. When $\delta = 0$, get formulas for a_1, a_3 in main presentation. Go Back