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Equilibrium and government commitment

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Abstract

How should a government use the power to commit to ensure a desirable equilibrium outcome? In this paper, I show a misleading aspect of what has become a standard approach to this question, and I propose an alternative. I show that the complete description of an optimal (indeed, of *any*) policy scheme requires outlining the consequences of paths that are often neglected. The specification of policy along those paths is crucial in determining which schemes implement a unique equilibrium and which ones leave room for multiple equilibria that depend on the expectations of the private sector.

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1. Introduction

How should a government use its limited set of policy instruments to achieve desirable equilibrium outcomes?

While there is a vast literature that studies the features of optimal macroeconomic policy and the characteristics of the equilibria that it can attain,¹ considerably less attention has been devoted to their implementation. Can the government adopt strategies that ensure that its preferred outcome will be the unique equilibrium? Is it instead possible that all

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¹For a recent survey, see [8].

the strategies that support a good outcome imply multiple equilibria, any of which may be triggered by the whims of private expectations?²

In the analysis of optimal policy, the standard framework follows from the pioneering work of Ramsey [26]. The Ramsey problem focuses only on the actual policy the government undertakes within an equilibrium and does not contain a full description of a government strategy. This is justified by the fact that most macroeconomic models assume a large number of agents, whose individual deviations are undetectable: as a consequence, within an equilibrium, each household takes government policy as given and independent of its own actions, and does not need to know the full strategy of the government. The Ramsey outcome is commonly viewed as the benchmark that the government could attain if it were able to commit, and is then compared to the equilibrium outcomes that prevail when commitment is impossible.

In this paper, I describe environments where assuming that the government can commit to unconditional actions (the “Ramsey timing”) contradicts the description of the physical constraints present in the environment. I then describe an alternative view of commitment. Rather than being the choice of an unconditional action, commitment is the choice of the *strategy* that the government will unconditionally follow for the remainder of its interaction with the private sector (the “Schelling timing”). The essential difference between the Ramsey timing and what I will call the “Schelling timing” is captured by the following passage from Schelling [29]:

The threat differs from the ordinary commitment, however, in that it makes one’s course of action *conditional* on what the other player does. While the commitment fixes one’s course of action, the threat fixes a course of reaction, of response to the other player. The commitment is a means of gaining *first move* in a game in which first move carries an advantage; the threat is a commitment to a strategy for *second move*.³

In macroeconomic games, the government faces a continuum of players rather than a single opponent. In such an environment, strategies that (strictly) implement desirable outcomes do not usually take the nature of direct “threats”, but achieve their aim in a more indirect way. For this reason, I will refer to them as “binding promises” instead.

Compared to the standard Ramsey problem, the analysis of binding promises attributes a much more prominent role to the constraints the government is forced to respect away from an equilibrium. These constraints limit the strategies the government can adopt, even when full commitment is assumed.

This alternative way of modeling commitment does not affect the *best* equilibrium that can be attained when the government is able to commit. However, it may dramatically affect the *uniqueness* properties of such an equilibrium. This calls into question whether the Ramsey outcome is truly a benchmark for what the government would be able to attain under commitment.

² The problem of equilibrium indeterminacy and its implications for implementation has received some attention in reference to monetary economics. Some of the early contributions in this area belong to Sargent and Wallace [28] and to Matsuyama [22,23]. Clarida et al. [10] provide a survey of the recent literature on interest-rate rules and their determinacy properties. I will discuss in Section 5.2 the relationship between this literature and the formal treatment of implementation presented here.

³ P. 124. Italics are from the original.

Studying the implementation problem has important policy implications. The appropriate policy advice differs radically depending on whether a strategy for implementing a desirable equilibrium exists. If it does, the appropriate advice calls for ensuring that the government is able to commit: this typically calls for the design of institutions, such as independent central banks, that *constrain* policy and prevent the government from giving in to the temptations coming from time inconsistency. If no such strategy exists, policy advice should instead first explore new instruments that could be used to ensure the favorable outcome: this calls for giving the government *more options* than the ones it currently has.

In Section 2, I introduce my remarks through two simple examples. Section 3 contains the general framework. The notion of a Ramsey outcome is introduced, and the implications of government commitment to actions vs. strategies are explored in detail. Section 4 considers an application to sovereign debt crises, and suggests some policy implications. Section 5 briefly discusses other environments in which the general framework can be fruitfully adopted, and Section 6 concludes.

2. Two simple examples

2.1. Optimal taxation

This example is adapted from Fischer [18] and Chari and Kehoe [7] (CK from now on).

The economy has two periods, 1 and 2.⁴ In the first period, a continuum of households is endowed with ω units of a good that can be either consumed or invested. Each unit invested delivers $R > 1$ units in period 2. Household preferences are $u(c_1, c_2, G)$, where $c_1 \geq 0$ is consumption in period 1, $c_2 \geq 0$ is consumption in period 2, and $G \geq 0$ is a public good that is provided by the government in the second period. We assume that u is strictly concave in private consumption. Fischer [18] and CK assume that the government must finance an exogenously given level of spending G^* in per capita terms, in which case the way u depends on G becomes redundant. To raise resources, the government uses a proportional tax on capital income (τ). The budget constraints for an individual household are thus:

$$\begin{aligned} c_1 &\leq \omega - k, \\ c_2 &\leq Rk(1 - \tau). \end{aligned} \tag{1}$$

A competitive equilibrium for this economy is a vector $(\tilde{c}_1, \tilde{c}_2, \tilde{k}, \tilde{\tau}, \tilde{G})$ such that:

- (i) given $\tilde{\tau}$ and \tilde{G} , $(\tilde{c}_1, \tilde{c}_2, \tilde{k})$ solves the household maximization problem;
- (ii) the government budget constraint holds:

$$\tilde{\tau}R\tilde{k} \geq \tilde{G}. \tag{2}$$

It is assumed that $G^* > 0$ but that it is not too large, so that there exist competitive equilibria that attain the exogenously given level of spending. When this economy has

⁴ CK consider an infinite repetition of this game, in which sustainable plans have the potential of delivering outcomes that are better than what can be achieved without commitment in a one-shot situation. Given that my interest lies with commitment, I only consider the one-shot game.

multiple competitive equilibria that attain G^* (when a Laffer curve is present), the one that achieves the highest welfare is the one with the lowest tax rate, which we call the Ramsey outcome and will denote by $(c_1^*, c_2^*, k^*, \tau^*, G^*)$.

The key question for our purposes is (strict) implementation: can the government behave in such a way to ensure that the Ramsey outcome will prevail and not any other?

Under the assumption of commitment, it is standard to let the government move first and choose τ , taking into account the households' response. We will call this the Ramsey timing.

Formally, the equilibrium of this economy, which we will call Ramsey equilibrium following CK, involves a government policy τ^* and an allocation rule $f(\tau) \equiv (c_1(\tau), c_2(\tau), k(\tau))$ such that:

- (i) for each policy τ' , $f(\tau')$ maximizes the utility of the households given that the tax rate is τ' and given the exogenous level of spending G^* .
- (ii) given f , τ^* is a policy that maximizes $u(c_1(\tau), c_2(\tau), G^*)$ subject to $\tau Rk(\tau) \geq G^*$.

Under the Ramsey timing, the government can implement the Ramsey outcome simply by setting $\tau = \tau^*$. This happens because the household maximization problem has a unique solution for each choice of τ , given G^* . When the government announces a tax rate of τ^* and households anticipate G^* , their maximization problem leads them to choose (c_1^*, c_2^*, k^*) . At this outcome, the government budget constraint holds, which ensures that we are considering a competitive equilibrium.

How does this government policy rule out other equilibrium outcomes? To find all the possible equilibria, we solve for the household's best response to what the government and other households do; we have an equilibrium whenever the best response (as a function of the actions of other households) is at a fixed point. If the policy is truly always characterized by (τ^*, G^*) , no matter what households do, then it is easy to see that the household's best response is always (c_1^*, c_2^*, k^*) , and hence this is the only equilibrium.

But, is adherence to this policy really feasible? As an example, let us consider the optimal course of action should all households choose $k = 0$. In this case, (τ^*, G^*) is physically impossible, because there are no tax revenues. What would the government do? Would G adjust? Would τ adjust? By assumption, all of these possibilities are ruled out. The households' best response is thus computed based on the assumption that the government will pursue a policy that is patently impossible under some scenarios.⁵ This approach to analyzing government policy under commitment is common to many papers in the macroeconomic literature.⁶

⁵ In order to avoid this difficulty, CK in Section V.A assume that the government will receive a large penalty $-M$ whenever the economy attains an outcome in which $G^* > \tau RK$, where K is aggregate capital. In other words, their solution relies on assuming that it is possible for the government, albeit very costly, to pay for spending from resources that come from outside the model. In private conversation, Chari explained to me that CK implicitly assumed that government spending would have to adjust, rather than being financed from resources outside of the model, and that government spending is assumed to be weakly separable from consumption and the labor supply. The implications of these assumptions are studied below.

⁶ Even many papers that do not assume commitment face a similar problem. It is almost universally assumed that the tax rate on labor is set unconditionally before the labor supply is observed, even when no commitment is allowed along many other possible dimensions. This implicit commitment not to revise the tax on labor is open to exactly the same criticism, as the reader can verify e.g. by looking at CK's "no-commitment game".

We now describe the alternative “Schelling” timing, under which the government can make binding promises about its conditional course of action, but must respect its feasibility constraint (2) no matter what households choose. As our previous reasoning shows, if capital-income taxes are the only source of revenues, it is impossible for the government to ensure $G = G^*$.

The following two cases show that the government may or may not be able to ensure that the Ramsey outcome is the unique equilibrium outcome (i.e., that the Ramsey outcome is “Schelling-implementable”).

First, suppose that household preferences are given by $u(v(c_1, c_2), G)$, with $u_1 > 0$ for all levels of G , i.e., government spending does not affect the marginal rate of substitution between current and future consumption.⁷ This case includes the common assumption that government spending is “thrown into the ocean” without affecting household utility.

In this case, the Ramsey outcome is Schelling-implementable. One way to implement it is for the government to set $\tau = \tau^*$, and let G adjust as a residual: $G = \tau^* RK$. Given that the level of G does not affect saving, the optimal choice of k is independent of the choices of other households (that affect aggregate capital K). As an example, if all households choose $k = 0$, the government can still tax capital income at the rate τ^* , and provide no spending. In this case, the optimal choice for an individual would be to save $k^* > 0$, which implies that $k = 0$ cannot be an equilibrium.

Suppose instead preferences are given by $u(c_1, Gc_2)$. As an example, this may happen because G represents expenses to enforce property rights (such as police) that are needed for households to enjoy private consumption in the second period. With these preferences, there is an equilibrium in which all households choose $k = 0$, no matter what the government does. If the aggregate capital is 0, the government budget constraint implies that $G = 0$ independently of the tax rates. When households expect $G = 0$, it is optimal to choose $k = 0$. As a consequence, no matter what the government strategy is, there always exists an equilibrium in which $k = G = 0$.

In this case, the Schelling timing reaches a dramatically different conclusion from the Ramsey timing. In the latter, households believe that $G = G^*$, even when there are no resources for this to happen; in the former, households realize that the government may be forced to $G = 0$ and it is impossible to rule out an equilibrium with no consumption in the second period. It is worth emphasizing that this does not imply that the economy will necessarily be stuck in such an equilibrium. The government can still set its strategy so that taxes and spending are (τ^*, G^*) if aggregate capital is k^* . In this case, the Ramsey outcome is still one of the possible competitive equilibria for the economy. What fails is the ability of government policy to *select* among equilibria and steer the economy to the preferred outcome.

2.2. Production externalities

We consider here an example adapted from Cooper [13].⁸ In this economy, households choose a labor supply $l \in [L, \bar{l}]$. Each household has access to a production technology that

⁷ We can assume that preferences are such that (τ^*, G^*) yields the best competitive equilibrium even when G can vary, or we can still inquire whether the government can implement (τ^*, G^*) , independently of this being the best from the households' perspective.

⁸ See pp. 148–149.

requires labor and is also affected by the average labor L supplied by other households. The amount produced by a household that chooses labor l is given by $z = l^\alpha L^\gamma$, where $\alpha, \gamma > 0$. The household preferences over consumption and labor are given by $c - \eta l^\theta$, where c is the consumption level, $\eta > 0$, and $\theta > \alpha + \gamma$. The government of this economy can set up a tax administration, which requires a fixed cost τ to run. If the administration is set up, the government can collect a lump-sum tax $T \in \mathbb{R}_+$ that can be used to provide a proportional subsidy to production at a rate $s \in \mathbb{R}_+$.

The government budget constraint is

$$\begin{aligned} T &\leq Z, \\ sZ &\leq T - \tau, \end{aligned} \tag{3}$$

where Z is aggregate output of the economy. The first constraint stems from the assumption that taxes are collected before the subsidy is paid out.

Each individual household consumes thus $l^\alpha L^\gamma(1 + s) - T$.

If the government does not set up the tax administration, this economy has a unique equilibrium:⁹

$$\begin{cases} l = \hat{l} \equiv \left(\frac{\alpha}{\eta\theta}\right)^{\frac{1}{\theta-\alpha-\gamma}}, \\ z = \hat{z} \equiv \hat{l}^{\alpha+\gamma}, \\ c = \hat{c}. \end{cases} \tag{4}$$

Depending on the value of τ , the Ramsey outcome either coincides with (4) or is given by

$$\begin{cases} l = l^* \equiv \left(\frac{\alpha+\gamma}{\eta\theta}\right)^{\frac{1}{\theta-\alpha-\gamma}}, \\ z = z^* \equiv (l^*)^{\alpha+\gamma}, \\ c = c^* \equiv z^* - \tau, \\ s = s^* \equiv \frac{\gamma}{\alpha}, \\ T = T^* \equiv \frac{\gamma}{\alpha}z^* + \tau. \end{cases} \tag{5}$$

We assume that (5) is the Ramsey outcome, which will happen whenever τ is not too large.

Cooper studies the Ramsey timing, in which the government sets (s, T) first. In this case, the Ramsey outcome can be implemented by setting (s^*, T^*) . If each household believes that the government will choose (s^*, T^*) independently of the actions of all other households and believes the average labor supply by other households to be L , its maximization problem becomes

$$\max_l l^\alpha L^\gamma(1 + s^*) - T^* - \eta l^\theta. \tag{6}$$

Solving (6) and imposing the equilibrium condition $l = L$, it can be verified that the Ramsey outcome is the unique equilibrium outcome.

⁹ We assume that $0 < \underline{l} < \hat{l} < l^* < \bar{l}$, where $\hat{l} < l^*$ is not an assumption, but a result.

The Ramsey timing is unsatisfactory for two reasons:

- (i) If all households choose low labor (L is sufficiently low), each individual household might not be able to meet its tax obligations T^* , even if it chose $l = \bar{l}$. In this case, the implied consumption would be negative. What should we conclude about the optimal labor supply for a household that believes it will not be able to pay taxes no matter what work effort it chooses? What impact would this have on the actual government revenues in (3)?
- (ii) If all households choose a higher supply than l^* , the government budget constraint (3) fails to hold: the government would not have enough resources to pay its subsidy. However, by assumption households must believe that taxes are still going to be T^* and the subsidy is still going to be s^* , for it is under this assumption that the higher level of work effort was ruled out as an equilibrium.

As in the previous section, the Ramsey timing is not an appropriate representation of the strategic interaction between households and the government in this instance. I now propose a more complete description.

We assume that government taxes must be collected before the subsidy is paid out. The lump-sum nature of the tax is translated into the assumption that the government can inflict a very large nonmonetary penalty P on households that fail to pay the due tax T . We assume $-P < \min\{-\eta \bar{l}^\theta, -T\}$, so the penalty is sufficiently harsh that any household will want to avoid it if at all possible. The government does not gain any direct benefit from imposing the penalty. In order to operate a tax administration and meet penalties out, it is necessary that tax revenues cover at least the fixed cost τ .¹⁰

Government revenues from each household are now $T I_{L^x L^y > T}$, where I is the indicator function: the government collects taxes only if the household output exceeds the tax obligation, otherwise no tax is collected and the penalty is imposed.¹¹ The government budget constraint is thus

$$T I_{L^x L^y > T} \geq sZ - \tau, \tag{7}$$

or $T = 0$ and $s = 0$.

Under the Schelling timing, a strategy for the government is a map from aggregate labor supply L ¹² into pairs of taxes and subsidies $(T, s) \in \mathbb{R}_+$ subject to (3) and (7).¹³

¹⁰ If the policy specifies a smaller tax revenue, the employees of the tax administration will realize that they will not be paid in full and will refuse to work.

¹¹ I make this assumption because it is the closest to lump-sum taxation: the household choice of l is not distorted whether it can or cannot meet the tax obligation, and a household will choose to meet the obligation whenever feasible. Other assumptions could be made; while those assumptions will change the nature of the second equilibrium that the government cannot avoid, they typically share the feature that such a second equilibrium exists.

¹² A complete description of the environment would specify a government strategy as a function of the distribution of actions taken by private households. Appendix A.1 explains in detail why it is not necessary to consider nondegenerate distributions.

¹³ Christiano and Harrison [9] study an implementation problem in an environment similar to this one. While they do not fully consider the details of how the government raises lump-sum taxes, their mechanism relies on an explicit government strategy and is thus more similar to the approach I advocate here.

Now suppose that $\hat{z} < \tau$.¹⁴ In this case, if households do not expect the government to provide any subsidies, they will all choose the work level \hat{l} , leading to a production level \hat{z} . If production is \hat{z} , there are not enough resources for the government to set up a tax administration: even with commitment, the government will be unable to intervene, validating the expectations that led households to choose \hat{l} . Therefore, if $\hat{z} < \tau$, the Ramsey outcome is not Schelling-implementable.

Once feasibility restrictions are taken into account, the subsidization scheme cannot ensure that the economy will attain the Ramsey outcome. The best the government can do is to commit to a strategy that admits the Ramsey outcome as one of the equilibria in the game played by the households; the option to subsidize output offers the government the chance to improve upon the laissez-faire equilibrium, but not the definite ability to coordinate the economy to a welfare-improving equilibrium.

3. A general setup

In this section, I introduce a general setup to formally analyze the implications of commitment. In Section 3.1, I introduce the notation and review the standard definition of commitment, based on the Ramsey timing. In Section 3.2, I study the Schelling timing and I explain its advantages. Section 3.3 generalizes the discussion to a multiperiod environment.

3.1. Ramsey timing

We follow here the notation in Stokey [31]. The players of the economy are a continuum of identical households and a government.

Households choose an action x from a set X , and the government chooses an action y from a set Y . Given that the focus is on symmetric equilibria, the economy is only described along paths in which all households, except at most a measure 0 set, take the same action. The economy is subject to an aggregate feasibility constraint that requires $(x, y) \in D \subseteq X \times Y$. We will refer to a feasible (x, y) pair as an *outcome*.

Preferences for the households are described by a function $u : X \times D \rightarrow \mathbb{R}$,¹⁵ where the first argument is the individual household choice and the second is the pair of aggregate choices by (almost) all households and the government.

¹⁴ It is worth pointing out that there is an open set of parameter values that satisfies all of the implicit restrictions that we have assumed. As an example, there is a neighborhood of $\alpha = 0.9$, $\gamma = 0.6$, $\theta = 1.6$, $\eta = 0.8$, $\tau = 0.5$, $\epsilon = 0.01$, $\bar{e} = 100$ for which all restrictions hold.

¹⁵ In Stokey [31], individual households are subject to a constraint in their choices, which is represented by a correspondence $H : D \rightarrow X$. This constraint is inconsistent with representing the economy as a game, which is essential for establishing results on implementation. The constraint H would require the choice set of a household to depend on information (the moves of other households) that is not yet available at the moment in which the decision is taken. H is meant to capture individual budget constraints. We will consider applications in which X can be chosen so that $H(x, y) = X \forall (x, y) \in D$, i.e., the households can choose any action $x \in X$ independently of what the government does or other households do. In the first example, this is achieved by assuming that the household choice is about k , so that c_1 and c_2 are determined by (1). Whenever prices are an important part of the economy, we assume they are explicitly formed through a well-specified market mechanism by the actions of the households and the government, as in [2].

Government preferences are described by a function $w : D \rightarrow \mathbb{R}$. It is often assumed, though not necessary for our analysis, that the government is “benevolent”, i.e., that¹⁶

$$w(x, y) = u(x, x, y) \quad \forall (x, y) \in D. \tag{8}$$

A *competitive equilibrium* is a pair (\tilde{x}, \tilde{y}) with the following properties:

- (i) $(\tilde{x}, \tilde{y}) \in D$;
- (ii) $\tilde{x} = \arg \max_{\xi} u(\xi, \tilde{x}, \tilde{y})$.

As expected, a competitive equilibrium is not a strategic notion of an equilibrium; rather, it only involves an outcome at which each household is taking a best response to what the government and other households are doing. Let E be the set of competitive equilibria.

A *Ramsey outcome* is a competitive equilibrium (x^*, y^*) such that

$$(x^*, y^*) = \arg \max_{(x, y) \in E} w(x, y).$$

The Ramsey outcome is the best possible outcome among competitive equilibria. The restriction to a competitive equilibrium follows from the assumption that the government does not have access to a way of enforcing directly the actions of each individual household; rather, it can only affect the households’ behavior through the choice of y . In order to attain this outcome, commitment is often necessary. Without it, the households would have to choose x based on the anticipation of what the government will do, rather than on its actual choice; once x is chosen, the government would no longer perceive $(x, y) \in E$ as a constraint, but would rather take as given the households’ choices. This is the basic time-consistency problem.

Fig. 1 illustrates the Ramsey timing. In this case, the government moves first and commits to an action y . Households then respond, with a choice x , and an outcome is determined.

In order to define a Ramsey game (and study its equilibrium), CK need to overcome an important difficulty. When the government moves first, the feasibility restriction $(x, y) \in D$ cannot be imposed, since:

- (i) by choosing an action y first, the government cannot make sure that the households will choose an action x such that $(x, y) \in D$.
- (ii) the restriction to D is a restriction on aggregate quantities and not on individual households. In the game, each household is free to choose any action $x \in X$.

CK extend the set of possible (symmetric) outcomes to $X \times Y$, but set $w(x, y)$ to a very large penalty when $(x, y) \notin D$, so that the government will act in such a way to ensure an outcome in D whenever possible. This is the key difference between their structure, which represents the view of commitment often adopted in macroeconomics, and the one I advocate. In CK’s game, violating a feasibility constraint is “less impossible” than violating the terms of commitment.¹⁷

¹⁶ The only advantage of (8) is that it provides unambiguous results on the welfare properties of different equilibria. When (8) does not hold, the welfare implications become harder to interpret.

¹⁷ CK formalize a notion of equilibrium with commitment that others have used. However, their focus is on equilibria in which commitment is not assumed; in their work, the inconsistency about the treatment of the feasibility constraint under commitment is only an additional reason for moving to a game without commitment.

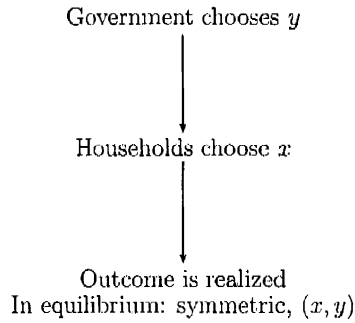


Fig. 1. The Ramsey timing (commitment to actions).

We now turn to the implications of this timing for the ability of the government to attain the best possible competitive equilibrium.

Definition 1 (*Ramsey implementability*). Let (\tilde{x}, \tilde{y}) be a competitive equilibrium. (\tilde{x}, \tilde{y}) is said to be Ramsey implementable if, for all $x \neq \tilde{x}$, $(x, \tilde{y}) \notin E$. If (\tilde{x}, \tilde{y}) is implementable, then \tilde{y} is said to implement it. Similarly, given a set of competitive equilibria $S \subseteq E$, we say that S is implementable if there exists a government action \tilde{y} such that, for any $x \in X$, $(x, \tilde{y}) \notin S \Rightarrow (x, \tilde{y}) \notin E$.

A competitive equilibrium (\tilde{x}, \tilde{y}) is implementable if the government can behave in a way that makes (\tilde{x}, \tilde{y}) the inevitable equilibrium outcome.

The definition of implementability does not address the issue of whether the government will choose a strategy that implements (\tilde{x}, \tilde{y}) : it is silent on the government’s incentives.

With the Ramsey timing, the government policy \tilde{y} that implements an outcome (\tilde{x}, \tilde{y}) does not need to be feasible for all choices of x ; the government can incur the risk of receiving an arbitrarily large penalty if households choose x such that $(x, \tilde{y}) \notin D$, because this will never happen on the equilibrium path.

In many applications, the following proposition shows that all competitive equilibria are implementable with the Ramsey timing.

Proposition 1. *Suppose the set of competitive equilibria E can be represented as a function of y , defined on a subset of Y :*

$$\exists f : Y_0 \subseteq Y \rightarrow X \text{ such that } (x, y) \in E \iff x = f(y). \tag{9}$$

Then any competitive equilibrium (and in particular the Ramsey outcome) is implementable.

Proof. Let (\tilde{x}, \tilde{y}) be a competitive equilibrium. Eq. (9) implies that (x, \tilde{y}) is not a competitive equilibrium unless $x = \tilde{x}$. \square

3.2. Committing to strategies

In many economic examples, the government cannot really take its action first; rather, commitment is best described as the ability to tie its hands with respect to actions that will be taken later. It is thus more fruitful to model commitment as a process of threats and promises that the government is able to bind itself to; after such threats and promises are made, the interaction between households and the government is subject to all the feasibility restrictions that we would impose on the government if the commitment stage of the game was not present. Before introducing binding promises, we thus model the interaction as it would be without any power to commit.

Households move first and choose $x \in X$. We will consider here only the consequences of almost all households taking the same action; we will not consider what happens when households choose actions according to a nondegenerate distribution. Under the assumptions on preferences that we make, Appendix A.1 shows that explicitly considering nondegenerate distributions is irrelevant for implementability.¹⁸

After the households have moved, the government picks an action $y \in Y$ such that $(x, y) \in D$. We need to assume that D is such that $\forall x \in X, \exists y : (x, y) \in D$. In words, the government must always have at least one feasible action no matter what the households played. This action may depend of course on the specific value of x .

Household and government preferences are defined as in Section 3.1. We assume that u is strictly concave in the first argument and that X is convex, which ensures that, in any equilibrium, all households will take the same choice.

The definition of a competitive equilibrium is also the same as previously. The set of competitive equilibria is unaffected by the difference in timing, because competitive equilibrium is a non-strategic notion of equilibrium.

A strategy for the households is now simply a choice of x ; a strategy for the government is a function $\sigma^g : X \rightarrow Y$ such that $(x, \sigma^g(x)) \in D \forall x \in X$. As an example, the choice of setting a fixed tax rate τ^* and letting $G(K) = \tau^*RK$ adjust as a residual is a government strategy in the example of Section 2.1.

We now introduce commitment according to the Schelling timing. In this case, the government can commit to a strategy σ before households choose their action x , though its action y will be carried out after the households have moved. This timing is displayed in Fig. 2.

Definition 2 (*Schelling implementability*). A competitive equilibrium (\tilde{x}, \tilde{y}) is Schelling implementable if there exists $\sigma : X \rightarrow Y$ such that

$$\forall x \in X, x \neq \tilde{x} \implies (x, \sigma(x)) \in D \setminus E.$$

¹⁸ Intuitively, our assumptions ensure that the household problem has a unique optimum. As a consequence, if an outcome envisions a nondegenerate distribution of household choices, it automatically follows that it is not a competitive equilibrium, independently of the government move: some households would be taking a suboptimal choice. Any feasible play by the government after a nondegenerate distribution is enough to rule out such an outcome as an equilibrium path.

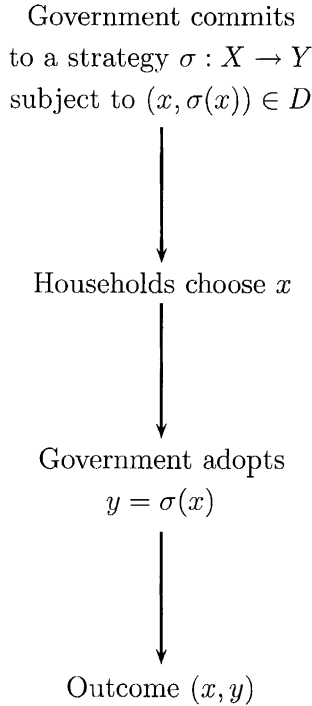


Fig. 2. Schelling timing.

In words, a competitive equilibrium is Schelling-implementable if the government can respond to any choice x by the households with some action $\sigma(x)$ that is feasible when all households choose x but that is not a competitive equilibrium with x .

Ramsey implementability and Schelling implementability are distinct. Suppose (\tilde{x}, \tilde{y}) is Ramsey-implementable, but there exists x' such that $(x', \tilde{y}) \notin D$ and $(x', y) \in E \forall y : (x', y) \in D$. In this case, \tilde{y} is not a feasible government action after the households chose x' ; the Ramsey game lets the government choose such an action ex ante, but this is ruled out by commitment to a strategy. Furthermore, any government response to x' would validate x' as part of an equilibrium, so that there is nothing the government can do to persuade households not to play x' if they believe all other households will play x' : (\tilde{x}, \tilde{y}) is thus not Schelling implementable. This is what happens in the examples of Section 2. In those examples, there is a unique competitive equilibrium associated with the Ramsey policy, but the Ramsey policy is not feasible for all possible households' choices. When we take into account that the government is forced to change its plans of taxes, spending or subsidies in response to what households do, a second competitive equilibrium that the government cannot rule out may emerge, and Schelling implementation fails.

Suppose instead that (\tilde{x}, \tilde{y}) is Schelling-implementable, and suppose there exists $(x', \tilde{y}) \in E$ with $x' \neq \tilde{x}$. Then (\tilde{x}, \tilde{y}) is not Ramsey-implementable. By having to choose \tilde{y} first, the

government cannot respond to x' using some other action y' that does not form a competitive equilibrium with x' ; by contrast, with the Schelling timing, the strategy σ^g that implements (\tilde{x}, \tilde{y}) may prescribe such a response. Notice that this example relies on finding $(x', \tilde{y}) \in E$, which is impossible under the assumptions of Proposition 1.¹⁹

The following proposition formalizes the reasoning above:

Proposition 2. *In order for (\tilde{x}, \tilde{y}) to be Schelling-implementable, it is neither necessary nor sufficient that (\tilde{x}, \tilde{y}) be Ramsey implementable.*

Under the Ramsey timing, by choosing an unconditional action \tilde{y} rather than a conditional response, the government may lose the flexibility that is necessary to implement the desired outcome. In this case, Ramsey implementability is a stronger requirement than Schelling implementability. However, under the Ramsey timing the government has the power to exclude many household choices x as part of the equilibrium outcome simply by setting an action \tilde{y} that is physically impossible when x is chosen. In many applications, this power makes Ramsey implementability a much weaker requirement than Schelling implementability.

The difference between Ramsey and Schelling implementation is very important for policy advice. According to the former, if the set of Ramsey outcomes is implementable, then the government simply needs to set its policy to the preferred outcome; e.g., it only needs to specify a tax rate (or a sequence of tax rates) and a level of spending, or the level of the money supply. Policy advice becomes thus relatively simple. In the latter, implementability simply means that it is possible to design a strategy such that any competitive equilibrium of the game that ensues will be a Ramsey outcome; the role of the policy advisor is thus to design such a strategy, or “reaction function”, that will typically involve different actions in response to different choices by the households. It is necessary to contemplate all possible choices by the households, and to recommend a policy that is consistent with the Ramsey outcome but that makes it optimal for individual households to deviate from what other households do whenever what they do is not a Ramsey outcome.

3.3. Many periods

We now extend the discussion to dynamic, multiperiod environments. We retain the assumption of convexity of the household’s problem which ensures that, in an equilibrium, all households take the same action: this enables me to focus only on histories in which almost all households take the same action and disregard other, more-complex ones.

We characterize the environment in terms of histories, which are defined recursively.²⁰ At each time t , we denote as H^t the set of public histories and H_t^I the set of private histories. A public history contains the record of play for the government and almost all households

¹⁹ When government spending is separable from private consumption in Section 2.1, the flexibility of being able to adjust spending in response to households’ choices is essential for implementation. If we insisted on a fixed level G^* (whenever feasible), with taxes adjusting as a residual, the government might be faced with multiple equilibria if a Laffer curve is present.

²⁰ The structure we introduce here is similar to the definition of a game in [25].

up to time t . A private history contains the same information, plus the record of past play of the individual household facing such history; this record may be important to assess what actions are feasible for the household in period t (e.g., past saving decisions affect the current opportunities for consumption). At time t , the set of feasible actions for a household whose private history is h_I^{t-1} is denoted $S_{I_t}(h_I^{t-1})$, and the set of feasible actions for the government is denoted $S_t(h^{t-1}, x_t)$, where x_t is the action taken by (almost) all households.²¹ S_{I_t} is assumed to be a convex-valued correspondence. Starting from the null (public and private) history $h^{-1} = \emptyset$, the sets of public and private histories are defined recursively as follows:

$$H^t = \{(h^{t-1}, x_t, y_t) | h^{t-1} \in H^{t-1}, x_t \in S_{I_t}(\phi_I(h^{t-1})), y_t \in S_t(h^{t-1}, x_t)\},$$

$$H_I^t = \{(h_I^{t-1}, \xi_t, x_t, y_t) | h_I^{t-1} \in H_I^{t-1}, \xi_t \in S_{I_t}(h_I^{t-1}), x_t \in S_{I_t}(\phi_I(\phi_P(h_I^{t-1}))), y_t \in S_t(\phi_P(h_I^{t-1}), x_t)\},$$

where ϕ_I is a function that maps a public history h^t into the private history of a household that has played the same actions as almost all other households and ϕ_P is the function that extracts the public component of a private history, i.e.,

$$\phi_I((x_t, y_t)_{t=0}^s) \equiv (x_t, x_t, y_t)_{t=0}^s,$$

$$\phi_P((\xi_t, x_t, y_t)_{t=0}^s) \equiv ((x_t, y_t)_{t=0}^s).$$

Let T be the number of periods the game lasts, which could be finite or infinite. Individual outcomes are the elements of H_I^T , and aggregate outcomes are the elements of H^T . Household preferences over outcomes are given by $\sum_{t=0}^T \beta^t u(\xi_t, x_t, y_t)$,²² where u is strictly concave in ξ .

A competitive equilibrium is a public history $(\tilde{x}_t, \tilde{y}_t)_{t=0}^T \in H^T$ such that

$$\{\tilde{x}_t\}_{t=0}^T = \arg \max_{\{\xi_t\}_{t=0}^T} \sum_{t=0}^T \beta^t u(\xi_t, \tilde{x}_t, \tilde{y}_t),$$

subject to $(\xi_t, \tilde{x}_t, \tilde{y}_t)_{t=0}^T \in H_I^T$. In words, a competitive equilibrium is a sequence of actions for the households and the government such that the households' actions are optimal given what the government and the other households are doing. As before, we let E be the set of competitive equilibria.

Competitive equilibria and their welfare properties can be studied without any reference to dynamics: it is a standard result that it does not matter whether all actions are decided

²¹ In the one-shot economy, a crucial difference between Ramsey implementation and Schelling implementation arose from the assumption that the set of feasible actions for the government depends on the actions of the households, so that the government is effectively the second mover within the period. In a multiperiod environment, the same difference may be caused by the fact that the set of feasible actions for the government depends on what households did in previous periods. As a consequence, this important component of the distinction between Ramsey and Schelling implementability would be present even if the government were the first mover within the period. The analysis could thus apply equally well in the case in which S_t only depends on h^{t-1} and S_{I_t} depends on h_I^{t-1} and the government action y_t .

²² We could consider more-general preferences, as long as they are time consistent.

upon at time 0, or they are decided as history unfolds.²³ As an example, this is the basis for the extension of Ramsey’s [26] results on optimal taxation in a static context to dynamic problems.²⁴

Definition 3. A competitive equilibrium $(\tilde{x}_t, \tilde{y}_t)_{t=0}^T$ is Ramsey-implementable if, for all $\{x_t\}_{t=0}^T \neq \{\tilde{x}_t\}_{t=0}^T, (x_t, \tilde{y}_t)_{t=0}^T \notin E$.

Under the Ramsey timing, the government moves first and sets its entire sequence of actions; once the government policy is set, households move competitively. As for competitive equilibria, dynamics play no special role for Ramsey implementation. From a strategic perspective, it does not matter whether the government is setting a single action or a single sequence of actions at time 0.

As in the one-shot economy, Ramsey implementation of a dynamic competitive equilibrium is based on implausible assumptions about household beliefs. Households must believe that the government will play $\{\tilde{y}_t\}_{t=0}^T$ no matter what history will unfold, even though the sequence of government policies is infeasible in some contingencies.

We now turn to Schelling implementation, in which dynamics play an important role: the feasible set of actions for the government evolves over time, which has to be reflected in policy design.

Schelling implementation is based on a government strategy, which is a function σ that specifies a feasible action for the government after any nonterminal public history and after each possible choice by the households:

$$\sigma(h^{t-1}, x_t) \in S_t(h^{t-1}, x_t) \quad t \leq T, \quad \forall h^{t-1}, \quad \forall x_t \in S_{I_t}(\phi_I(h^{t-1})).$$

Let $H(\sigma)$ be the set of all the possible aggregate outcomes for the economy if the government plays according to σ :

$$H(\sigma) = \{(x_t, y_t)_{t=0}^T \in H^T : y_t = \sigma((x_s, y_s)_{s=0}^{t-1}, x_t), t = 0, \dots, T\}.$$

Definition 4. A competitive equilibrium $(\tilde{x}_t, \tilde{y}_t)_{t=0}^T$ is Schelling-implementable if there exists a strategy σ such that $H(\sigma) \cap E = \{(\tilde{x}_t, \tilde{y}_t)_{t=0}^T\}$: $(\tilde{x}_t, \tilde{y}_t)_{t=0}^T$ is the only competitive equilibrium among all possible outcomes when the government is committed to behave according to the strategy σ .

Dynamics play also a second, subtler role in the way the government is able to select among competitive equilibria. In a multiperiod environment, the Schelling timing creates a distinction between Nash implementation, which corresponds to the definition above, and the notion of sequential equilibrium implementation, which is stronger in the case of the games we are studying. This issue is explored in detail in Appendix A.2.

²³ We are assuming here that household preferences are time consistent. If not, this would introduce a strategic element of choice within individual households.

²⁴ Early examples of this vast literature are [6,20,21].

4. Sovereign debt crises

Most models of sovereign debt and/or exchange rate crises that include the government as an optimizing entity have worked under the assumption that the government does not have the ability to commit to its policy.²⁵

I present here a model in which it is not lack of commitment, but rather lack of options that may prevent the government from averting a self-fulfilling economic downturn, whether accompanied by a sovereign default or not.²⁶

We consider a one-period, small open economy, populated by a continuum (of measure 1) of domestic households, whose preferences are $u(c_1, c_2, l)$, where c_1 is consumption of a domestically-produced good, c_2 is consumption of an imported good and l is the labor supply. u is strictly concave, strictly increasing in consumption and strictly decreasing in the labor supply. A household that supplies l units of labor produces output in an amount $l + q$, where q is a random variable, common to all households, with an almost everywhere continuous density function $\psi(q)$ on an interval $[q, \bar{q}]$.²⁷ The total endowment of time is \bar{l} and either $q \geq 0$ or a household must supply at least q units of time. The world price of the domestic good relative to the import is 1. The economy has a government that owes B units (of either good) to some foreign creditors at the end of the period, and uses a proportional tax on output to raise resources to repay its debt. If the government defaults on its debt, foreign trade is disrupted; we make the extreme assumption that households are completely excluded from foreign trade and must thus consume only the domestically-produced output.

The timing of the economy is as follows:

- (i) The government commits to a (measurable) strategy which it will follow in part (iv).
- (ii) Households choose their labor supply. The realization of the shock is not yet known when they make their decision.
- (iii) The production shock q is realized.
- (iv) The government sets a tax rate τ and a repayment rate δ , subject to the budget constraint

$$\tau(L + q) \geq \delta B, \tag{10}$$

where L is the aggregate labor supply. We assume the government does not observe L or q independently, but just aggregate output, and its strategy must thus be a function of output.

- (v) If $\delta \geq 1$, households trade in the foreign markets at the world price of 1.

²⁵ Some examples include [1,4,12,19,27].

²⁶ The economic forces at work in this example are similar to those that operate in [15].

²⁷ The presence of the shock and the assumption that the government only observes total output prevent the use of discontinuous strategies to “jump over” undesired equilibria. This restricts the government’s ability to implement outcomes more than in previous sections. The results in the text apply for intervals $[q, \bar{q}]$ that are arbitrarily small and close to 0.

A competitive equilibrium is given by a labor supply \tilde{l} and a random vector $(\tilde{c}_1, \tilde{c}_2, \tilde{\tau}, \tilde{\delta})$, adapted to q , such that:²⁸

- (i) Households are maximizing their utility given the technology, taxes, and the opportunity to trade:²⁹

$$\begin{aligned} u_l(\tilde{c}_1, \tilde{c}_2, \tilde{l}) &= E[(1 - \tilde{\tau})u_1(\tilde{c}_1, \tilde{c}_2, \tilde{l})], \\ u_1(\tilde{c}_1, \tilde{c}_2, \tilde{l}) &= u_2(\tilde{c}_1, \tilde{c}_2, \tilde{l}) \quad \text{if } \tilde{\delta} \geq 1, \\ \tilde{c}_2 &= 0 \quad \text{if } \tilde{\delta} < 1, \\ \tilde{c}_1 + \tilde{c}_2 &= (1 - \tilde{\tau})(\tilde{l} + q). \end{aligned}$$

- (ii) The government budget constraint (10) is satisfied.

We assume the government only cares about domestic households, so we define the Ramsey outcome as the competitive equilibrium that has the highest welfare for them. We assume that parameter values are such that the Ramsey outcome $(l^*, c_1^*, c_2^*, \tau^*, \delta^*)$ involves no default, independently of the realization of q : $\delta^* \equiv 1$. According to the Ramsey outcome, output will for sure be contained in $[l^* + q, l^* + \bar{q}]$, hence the Ramsey policy only specifies taxes and repayment rates for levels of output that fall in this range.

Whether the government can implement the Ramsey outcome relies crucially on the properties of the labor supply. To show this, we first define the best reply for a household when the government is committed to a strategy σ and other households choose a labor supply L . We denote the best reply as $(\hat{c}_1, \hat{c}_2, \hat{l})(L, \sigma)$, which is given by

$$\begin{aligned} (\hat{c}_1, \hat{c}_2, \hat{l})(L, \sigma) &= \arg \max_{c_1, c_2, l} E[u(c_1, c_2, l)] \quad \text{subject to} \\ c_1 + c_2 &\leq [1 - \tau_\sigma(L + q)](l + q), \\ c_2 &= 0 \quad \text{if } \delta_\sigma(L + q) < 1, \end{aligned}$$

where τ_σ and δ_σ are the tax rate and repayment rate that form the strategy σ .

Proposition 3. *The Ramsey outcome is implementable if and only if there is a government strategy σ^* such that $\hat{l}(l^*, \sigma^*) = l^*$ and*

$$\hat{l}(L, \sigma^*) > L \quad \forall L \in [0, l^*]. \tag{11}$$

The proof of the proposition is presented in the appendix. We discuss here the intuition. The crucial problem for the government is how to respond to low levels of output. In order to repay foreign debt, the government would need to raise the tax rate above the Ramsey level; however, this discourages production and may be a source of a self-fulfilling downturn when preferences are such that a Laffer curve arises. Alternatively, the government may respond by defaulting on foreign debt; however, the anticipated inability to engage in foreign trade can also be a disincentive for households to work, validating once more the expectations

²⁸ There is no contradiction between the previous assumption that the government cannot set its strategy as a function of q and the definition of a competitive equilibrium, in which $\tilde{\tau}$ and $\tilde{\delta}$ are adapted to q . Within a competitive equilibrium, \tilde{l} is known, and hence a function of aggregate output is equivalent to a function of the shock.

²⁹ u_1, u_2 , and u_l are the marginal utilities with respect to c_1, c_2 , and l .

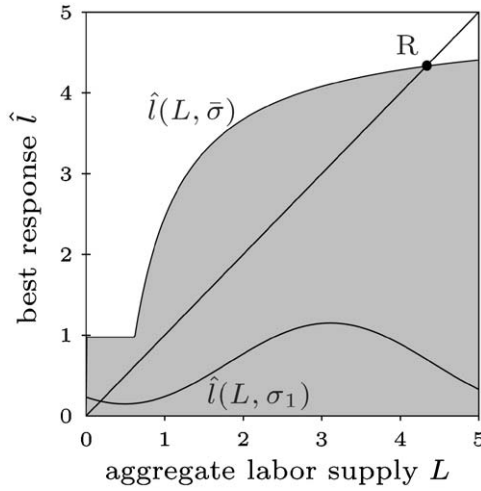


Fig. 3. Policy design with $B = 0.5$.

that led to a low output level. This situation is reminiscent of the crisis recently faced by Argentina.

To illustrate this point, we consider a numerical example. We consider the following preferences:

$$u(c_1, c_2, l) = \frac{a_1}{1 - \rho} c_1^{1-\rho} + \frac{a_2}{1 - \rho} c_2^{1-\rho} - l,$$

with $a_1 = 1$, $a_2 = 2$, $\rho = 0.5$, and q uniformly distributed between 0 and +0.01. Simple algebra shows that the function \hat{l} is implicitly defined as follows:

$$E \left\{ \left[a_1^{1/\rho} + a_2^{1/\rho} I_{\delta_\sigma(L+q) \geq 1} \right]^\rho (\hat{l} + q)^{-\rho} [1 - \tau_\sigma(L + q)]^{1-\rho} \right\} = 1. \tag{12}$$

Fig. 3 displays what happens for low levels of debt. Equilibria of the economy occur along the 45-degree line, where the individual best response coincides with the aggregate labor supply. The Ramsey outcome is the point R.³⁰ By its choice of a strategy, the government

³⁰ The Ramsey outcome is the solution to the following problem:

$$\max_{\{\tau(q), \delta(q), L\}} E \left\{ \frac{1}{1 - \rho} (a_1^{1/\rho} + a_2^{1/\rho} I_{\delta(q) \geq 1})^\rho (1 - \tau(q))^{1-\rho} (L + q)^{1-\rho} \right\} - L$$

subject to

$$E \left\{ [a_1^{1/\rho} + a_2^{1/\rho} I_{\delta(q) \geq 1}]^\rho (L + q)^{-\rho} (1 - \tau(q))^{1-\rho} \right\} = 1,$$

$$\delta(q)B = \tau(q)(L + q),$$

where $\tau(q)$ and $\delta(q)$ are the equilibrium tax and repayment rate as a function of the shock.

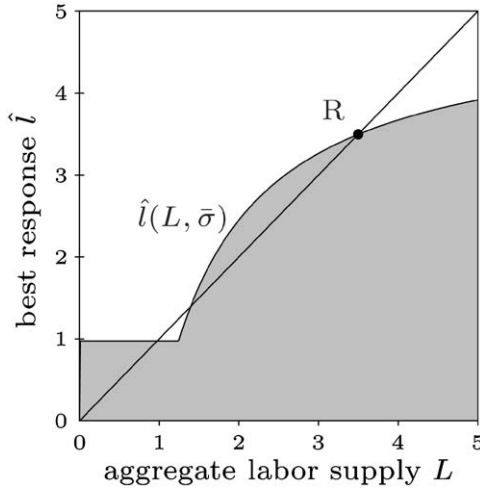


Fig. 4. Policy design with $B = 1$.

dictates the shape of the best response, which will be some continuous function within the shaded area.³¹ Fig. 3 plots the best response for some strategy σ_1 as an example.

The upper contour of the region is given by the strategy $\bar{\sigma}$ that maximizes \hat{l} in (12), which is given by

$$(\tau_{\bar{\sigma}}, \delta_{\bar{\sigma}})(Z) = \begin{cases} (B/Z, 1) & \text{if } (a_1^{1/\rho} + a_2^{1/\rho})^\rho (1 - B/Z)^{1-\rho} > a_1, \\ (0, 0) & \text{otherwise.} \end{cases} \quad (13)$$

This strategy compares the consequences of a tax rate that is just enough to repay the debt vs. full default and no taxes, and it chooses the option that offers greater work incentives. In Fig. 3, this strategy is successful at strictly implementing the Ramsey outcome: the upper contour of the shaded area has a unique intersection with the 45-degree line.

Fig. 4 displays what happens for higher levels of debt, such as $B = 1$. In this case, the Ramsey outcome still involves full repayment; however, the upper-contour strategy (13) now has two more equilibria, one on the wrong side of the Laffer curve, and the other with no taxes and default. For this level of debt, any strategy is bound to imply a fixed point below R, and multiple equilibria are unavoidable.³²

When the Ramsey outcome is implementable, as in the first case, the appropriate policy advice calls for building institutions capable of ensuring adherence to commitment, avoiding

³¹ Within the shaded area, the government only faces a constraint on the steepness of the best response. As the size of the interval $[\underline{q}, \bar{q}]$ shrinks to 0, any continuous function contained in the shaded area would become possible.

³² If debt is pushed even higher, eventually it surpasses the peak of the Laffer curve; in this case, it is impossible for the government to repay its debt for sure. Up to a point, it is then possible and desirable to sustain a random policy of repaying with some probability, rather than defaulting for sure.

the potential time-inconsistency problem.³³ As an example, the country could delegate the choice of a tax rate to an independent authority, shielded from political pressure in the short run.³⁴

When the Ramsey outcome is not implementable, commitment does not help to overcome the risk of a self-fulfilling crisis. In this case, optimal policy design must confront the existence of multiple equilibria. As Ennis and Keister [16,17] point out, if the policy choice interacts with the likelihood that one equilibrium will prevail over another, the optimal choice will be altered and will not coincide in general with the simple Ramsey prescription.³⁵

Unless the government has reason to believe that the “good” equilibrium will always be selected by the private sector, it is useful to explore ways of providing the policymaker with new policy instruments, that could avoid entirely the potential for unwanted equilibria. In this application, an example of such an instrument would be the ability to own securities whose payoff is contingent to the realization of GDP. This would allow the government to prevent undesirable equilibria, much in the same way as deposit insurance prevents bank runs in [14]. To illustrate this mechanism simply, consider the case in which $q = 0$ for sure.³⁶ Since we assume that the Ramsey outcome involves debt repayment, $\tau^*l^* = B$. Suppose the government has securities that pay off $\tau^*(l^* - L)$ if output (i.e., the labor supply) is L , thereby effectively insuring against fluctuations in tax revenues. With these, the government would be able to set the tax rate at τ^* and still repay the debt, no matter what output turns out to be. When this is anticipated, the households’ best response implies a unique solution, with the labor supply at l^* . Since the equilibrium payoff of the securities would be $\tau^*(l^* - l^*) = 0$, this insurance would be free to provide for an external agency acting as a lender of last resort (the IMF?).³⁷

As the policy prescriptions above suggest, analyzing implementability has important implications when thinking of ways to avoid sovereign debt crises such as the one that has affected Argentina recently.

³³ Time inconsistency arises in the example because the government can vary its tax rate on output after households have made their labor choice.

³⁴ Holding infrequent elections as opposed to continuous referenda may be one way societies deal with the problem.

³⁵ Carlson and Van Damme [5] and Morris and Shin [24] suggest a specific selection criterion to pick a unique equilibrium in environments in which multiple equilibria arise from complementarities in the players’ actions, as is the case here. While multiplicity of equilibria is then avoided, it can be shown that in general the limit of the Ramsey outcomes in a sequence of global games in which idiosyncratic information vanishes is not the Ramsey outcome under perfect information. It is therefore still extremely important to fully specify the government strategy, so that the emergence of complementarities in the households’ actions is revealed.

³⁶ The same reasoning applies when q is random, at least if the support of the distribution of aggregate output does not overlap across the 3 equilibria. However, when q is random, the presence of this additional market would change the Ramsey outcome itself.

³⁷ To pursue the analogy with bank runs further, the insurance scheme described above is likely to generate severe moral hazard problems in a richer environment, which might justify monitoring the government’s actions.

5. Other applications

5.1. Fiscal theory of the price level

The fiscal theory of the price level³⁸ is based on the assumption that the government can commit to a sequence of real taxes and spending $\{(T_t, G_t)\}_{t=0}^{\infty}$ that can violate the present-value budget constraint for some price levels.

Compared to the previous examples, the fiscal theory of the price level is much more complex. This is partly of course because the environment is much richer, but mainly because the definition of a competitive equilibrium includes a sequence of prices that are not a choice of any player in the economy.

Introducing prices (p) in the definition of a competitive equilibrium does not require an explicit account of the way they form out of the actions of households (x) and of the government (y). In the equilibrium, households take prices as given, just as government policy.

Ramsey implementation shares the simplicity of competitive equilibria. To check whether a competitive equilibrium $(\tilde{x}, \tilde{y}, \tilde{p})$ is Ramsey-implementable, it is sufficient to know whether there exists a unique competitive equilibrium when households take as given the government policy \tilde{y} .

Unfortunately, Ramsey implementation does not allow a proper distinction between equilibrium conditions and restrictions on government policy, such as its budget constraint. The failure of this distinction is the main cause the debate on the fiscal theory has been so contentious.

To study Schelling implementation, it becomes crucial to establish which actions each of the players in the economy can take in all contingencies. This is impossible without a full description of the strategic environment as an anonymous game, that must include an explicit account of the way prices form out of the actions of the government and the households. In the case of the fiscal theory of the price level, I analyze such a game in detail in [2].

5.2. Monetary policy rules

In recent years, there has been significant research on the welfare and equilibrium determinacy properties of monetary policy rules.³⁹ The goal of this research is to find how monetary policy can be used to implement desirable equilibrium outcomes.

Svensson [32] and Svensson and Woodford [33] distinguish between a policy rule and a reaction function. A policy rule is a “prescribed guide for monetary-policy conduct”. It usually takes the form of a simple relationship that may involve government policy, exogenous variables and endogenous variables, both past, present and future (in expectation). A reaction function “specifies the central bank’s instrument as a function of predetermined endogenous or exogenous variables observable to the central bank at the time that it sets the instrument”. The solution to any implementation problem should thus include a reaction

³⁸ See [11,30,34].

³⁹ See e.g. [10] and references therein.

function and not simply a policy rule. A policy rule may only be useful as a simple tool to evaluate whether the government is adhering to its commitment. For this purpose, it might have the advantage of being much simpler and hence more transparent than the reaction function itself.

A reaction function corresponds to what I called in this paper a government strategy, so the analysis I developed here supports the view that implementation should be studied at the level of government reaction functions. Svensson and Woodford [33] are mainly focused on interest-rate rules, in which the instrument used by the central bank is the nominal interest rate. The only restriction they identify is that this instrument should be a function of predetermined variables only, a necessary requirement for a strategy in a game. A fuller description of the economy as a game is likely to offer new insights in the way monetary policy can be used to select desirable equilibria. As an example, in [3] I adopt such an approach to analyze whether the nonnegativity constraint on nominal interest rates is a constraint on monetary policy or an equilibrium condition that could be exploited to rule out unwanted outcomes.

6. Conclusion

In many macroeconomic problems, the policy actions available to the government depend on previous choices made by the private sector. In this paper, I have shown that modeling government commitment as the ability to take (or set) its actions before any other player moves is unsatisfactory from the perspective of designing optimal policy schemes. When the potential for multiple equilibria is present, this approach may also paint a misleading picture of the ability of the government to determine the equilibrium outcome that will prevail. I have presented an alternative description of the strategic interaction between the government and private agents, one in which the ability of the government to commit corresponds to the ability to set a *strategy* that it will adhere to.

In the examples I have presented, the restrictions on government policy came from clear and unavoidable physical constraints, such as the impossibility of spending resources that do not exist. The approach developed here can also be useful for constitutional design, in which institutions with limited commitment power are studied. Rather than merely reflecting physical constraints, the restrictions on the strategies available to the government would then come from the explicit model of these further limits.

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Appendix A.

A.1. Nonsymmetric outcomes

The environment we described in Section 3.2 is not a full description of a game, in that only symmetric histories are considered. We show here that, under our assumptions, considering nonsymmetric outcomes is redundant to establish what can be implemented.

To fully describe an anonymous game, let (X, \mathcal{X}) be a measurable space, and let \mathcal{M} be the set of probability measures over (X, \mathcal{X}) . After the households have moved, the government observes the distribution of their choices (but not the individual actions), which can be any element $\mu \in \mathcal{M}$. In the main text, we only focus on the case in which μ is degenerate, attributing probability 1 to a specific element of X . The set of possible aggregate outcomes for the game is now expanded to be $\hat{D} \subseteq \mathcal{M} \times Y$: it contains a distribution of actions for the households μ and a government action y . The relationship between \hat{D} and D is the following:

$$(x, y) \in D \Leftrightarrow (\delta_x, y) \in \hat{D},$$

where δ_x is the measure that attributes probability 1 to $\{x\}$. As we did for D , we need to assume that the government has at least one feasible action after any possible history of the game: $\forall \mu \in \mathcal{M}, \exists y : (\mu, y) \in \hat{D}$.

It is likewise necessary to extend household and government preferences to $X \times \hat{D}$ and \hat{D} , respectively. Crucially, we assume that the household utility function u is strictly concave in its first argument for each $(\mu, y) \in \hat{D}$.

A government strategy is now a mapping $\sigma : \mathcal{M} \rightarrow Y$ such that $(\mu, \sigma(\mu)) \in \hat{D} \forall \mu \in \mathcal{M}$.

Schelling implementation corresponds to a specific mechanism design problem. Once the government has committed to play a specific strategy σ , what remains is an (anonymous) game among households only, whose payoff structure depends on the way the government is committed to respond to those actions. A choice of σ corresponds thus to choosing a specific game that the households will play.

The following proposition shows the connection between competitive equilibria as described in the main text and the Nash equilibria of the games the government can design by its choice of σ .

Proposition A.1. (i) *Let $(x, y) \in E$. Then (δ_x, y) is a Nash equilibrium outcome for the game that ensues whenever the government commits to a strategy such that $\sigma(\delta_x) = y$.*

(ii) *Let (μ, y) be a Nash equilibrium outcome in a game after the government has committed to a strategy σ . Then μ attributes probability 1 to a single point $x \in X$, and (x, y) is a competitive equilibrium.*

Proof. (i) Immediate from the definition of Nash equilibrium and competitive equilibrium.

(ii) By strict concavity of u , we know that $\arg \max_{\xi} u(\xi, \mu, y)$ is unique. This implies that the support of an equilibrium distribution μ must be a singleton. The fact that a Nash equilibrium (δ_x, y) coincides with a competitive equilibrium (x, y) follows immediately from the appropriate definitions. \square

The relationship between Nash equilibria and competitive equilibria has an immediate counterpart for the implementation problem. A competitive equilibrium (x, y) is Schelling implementable if and only if the government can design a game among households (by choice of σ) that has (δ_x, y) as its unique Nash equilibrium outcome. Since nondegenerate distributions μ cannot be an equilibrium outcome no matter what choice the government takes, it is unnecessary to specify how the government would react in such a case.

A.2. Nash vs. sequential equilibrium implementation in multiperiod settings

As for the one-period version of the game, the multiperiod environment of Section 3.3 can also be described as an anonymous game, if a full description of the consequences of nondegenerate distributions is included. However, the assumption of strict concavity of the households' utility function implies that the results of Section A.1 still hold.

When households are called to move more than once, a distinction arises between Nash and sequential equilibria.⁴⁰ Accordingly, we distinguish between Nash implementation, in which the government designs a game that has a unique Nash equilibrium outcome, and sequential equilibrium implementation, in which the government designs a game that has a unique sequential-equilibrium outcome. Unlike what happens in games with a finite number of players, implementation in sequential equilibrium is here necessarily a stronger notion than Nash implementation. In games with a finite number of players, implementation in sequential equilibrium allows for the design of games that have a unique sequential equilibrium but more than one Nash equilibrium. This does not happen in anonymous games with a continuum of players, in which each player's individual move does not have any impact on other players' actions. Both in games with a finite number of players and in anonymous games, Nash implementation allows for the design of games that have a unique Nash equilibrium but potentially no sequential equilibria.⁴¹

The following example clarifies the distinction: in the example, the Ramsey outcome is Nash implementable but not implementable in sequential equilibrium.

Consider repeating twice the first example of Section (2). To distinguish between the timing within each repetition and across repetitions, we now call stages the "periods" within each repetition, and we use the word "period" to refer to the repetition number. Neither the

⁴⁰ In an anonymous game, we cannot properly speak of subgame-perfect equilibria, because the actions of each individual household are never observed. However, the distribution of actions of the households is observed; for each household and for the government, this is the only payoff-relevant aspect of the actions of other households. We only analyze equilibria in which a household is thus indifferent among all nodes of an information set and will take the same choice independently of the belief over the specific node the game is at within the information set. For this reason, we can omit specifying beliefs.

⁴¹ In standard mechanism-design problems, a common trick to obtain Nash implementation is to design a game in which an "integer game" with no equilibrium follows any undesired outcome.

households nor the government are allowed to save from one repetition to the next, so that the two periods are independent, unless the strategies of the government and/or the households dictate second-period choices that depend on first-period actions. We assume that preferences in the first period are given by $u(c_{11}, G_1 c_{21})$ and preferences in the second period are given by $u(v(c_{12}, c_{22}), G_2)$, where c_{ij} is the consumption in stage i of period j and G_i is the provision of the public good in period i . Let τ_i be the tax rate in period i . We use * superscripts to denote Ramsey allocations and policies. Given that the two periods are independent of each other, we might expect that the government can guarantee the Ramsey outcome in the second period, but not in the first. We now show that this is true for sequential equilibrium implementation, but that the Ramsey outcome is implementable in Nash equilibrium.

Let σ be a government strategy such that, if the government is committed to follow σ , the game among households has a sequential equilibrium whose outcome is Ramsey. Let σ^h be the household strategy in such an equilibrium. We show that there also is a sequential equilibrium in which $c_{21} = G_1 = 0$. The new equilibrium has the same strategy in period 2, but it prescribes households to choose $c_{11} = \omega$ in the first period. We now work backwards to check that this is indeed a sequential equilibrium. In the second period, independently of the history h_1 up to that point, $\sigma^h(h_1)$ must prescribe an action that is optimal for the households given that other households will also play $\sigma^h(h_1)$ and the government will respond by $\sigma(h_1, \sigma^h(h_1))$. This is true in particular even after the history in which almost all households chose $c_{11} = \omega$, even though that history is not on the equilibrium path for the original strategy σ^h . In the first period, it is optimal for a household to choose $c_{11} = \omega$ whenever it expects $G_1 = 0$. As we already observed, G_1 is necessarily 0 if all households choose $c_{11} = \omega$. It thus follows that the modified strategy is indeed a sequential equilibrium.

We now display a government strategy that implements the Ramsey outcome in a Nash equilibrium:

$$\sigma(c_{11}) = \tau_1^* \quad \forall c_{11} \in [0, \omega],$$

$$\sigma(c_{11}, \tau_1, c_{12}) = \begin{cases} 0 & \text{if } c_{11} \neq c_{11}^* \text{ and } c_{12} = c_{12}^*, \\ \tau_2^* & \text{otherwise.} \end{cases}$$

If the government adopts this strategy and households do not play $c_{11} = c_{11}^*$ in the first period, there is no equilibrium in the second period: the government strategy is designed so that a household would always have an incentive to deviate from what other households do. By contrast, if $c_{11} = c_{11}^*$ in the first period, then in the second period there is only one equilibrium, in which the second-period Ramsey outcome is attained.

The definition of Schelling implementability in the main text corresponds to Nash implementation in this mechanism design problem, and was chosen because of its simplicity.

A.3. Proof of Proposition 3

First, suppose a strategy such that (11) holds can be found; let such a strategy be σ . We prove that the following strategy implements the Ramsey outcome:

$$\sigma'(Z) \equiv (\tau_{\sigma'}, \delta_{\sigma'})(Z) = \begin{cases} \sigma(Z) & \text{if } Z \leq l^* + \bar{q}, \\ (B/Z, 1) & \text{otherwise.} \end{cases} \tag{14}$$

σ' and σ coincide for all the levels of output that are possible when the labor supply is at most l^* . As a consequence $\hat{l}(L, \sigma) = \hat{l}(L, \sigma')$ if $L \leq l^*$, which implies that there can be no equilibria with a labor supply below l^* if the government commits to σ' . With this strategy, we also prove by contradiction that there can be no equilibrium with $l' > l^*$. If such equilibrium existed, Eq. (14) implies that it would entail no default and a tax rate that is lower than Ramsey's for each realization of q . However, it is easy to show that household welfare is a decreasing function of the tax rate for a given δ ; as a consequence, the equilibrium at l' would have higher welfare than the one at l^* , which contradicts l^* being part of the Ramsey outcome.

Now, suppose instead that there is no strategy such that (11) holds. We first prove that, given any strategy σ , $\hat{l}(L, \sigma)$ is continuous in L . Define

$$v(l, q, \tau, \delta) \equiv \max_{c_1, c_2, l} u(c_1, c_2, l)$$

subject to $c_1 + c_2 \leq (1 - \tau)(l + q)$ and to $c_2 = 0$ if $\delta < 1$. v is the indirect function after uncertainty is resolved for a household that has chosen a labor supply l . v is continuous in (l, q, τ) , but it is discontinuous in δ ; it is also strictly concave in l . Given a government strategy σ , the function \hat{l} is defined as

$$\begin{aligned} \hat{l}(L, \sigma) &= \arg \max_l \int_{\underline{q}}^{\bar{q}} v(l, q, \tau_\sigma(L + q), \delta_\sigma(L + q)) \psi(q) dq \\ &= \arg \max_l \int_{L + \underline{q}}^{L + \bar{q}} v(l, Z - L, \tau_\sigma(Z), \delta_\sigma(Z)) \psi(Z - L) dZ. \end{aligned}$$

Continuity of \hat{l} in L follows thus from the theorem of the maximum. Given that the labor supply must be nonnegative, it follows that $\hat{l}(0, \sigma) \geq 0$. If Eq. (11) does not hold, it follows that there must be a fixed point of the function \hat{l} at a level that is strictly smaller than l^* , which implies that l^* is not the unique equilibrium. The exact location of the second equilibrium (or of the other equilibria, if there are more than 1) depends on the government strategy σ , and will usually not happen at $L = 0$. \square

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