Christiano
D-16, Spring 2000
Homework 1.
Due: Friday, April 28.

1. (Stefania Albanesi) Consider an economy in which a unitary measure of identical households value cash goods, c_1, credit goods, c_2, and leisure, $1-n$, according to the utility function:

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_{1t} + \alpha \log c_{2t} + \log\left(1 - n_t\right)\right].$$

They enter each period with nominal wealth holdings A_t, which they allocate between money holdings M_t and bond holdings B_t. They then participate in the goods market, where they buy cash goods subject to the constraint:

$$P_t c_{1t} \leq M_t,$$

and credit goods, and supply labor. At the end of the period they receive labor income, a lump-sum transfer from the government and bond payments and they pay for credit good purchases. Their end of period nominal wealth is given by:

$$A_{t+1} = M_t - P_{1t}c_{1t} - P_{2t}c_{2t} + B_t R_t + W_t n_t + T_t,$$

where R_t is the gross nominal interest rate and T_t is the government transfer.

The benevolent monetary authority sets policy g_t subject to its budget constraint:

$$M_{t+1} = M_t + T_t,$$

where $M_{t+1} = g_t M_t$, and g_t is the gross rate of money growth. Bonds are privately issued and in 0 net supply.

Production of cash and credit goods occurs in different sectors, indexed by $i = 1, 2$. Each sector i is composed of one-period lived final and intermediate goods producers. Final goods producers are perfectly competitive and their production is given by:

$$y_i = \left[\int_0^1 y_i\left(x\right)^{\lambda} dx\right]^{\frac{1}{\lambda}},\ i = 1, 2,$$

where $0 < \lambda < 1$ and $y_i(x)$ is intermediate good x is sector i, with $0 < x < 1$. Intermediate goods producers' technology is given by:

$$y_i(x) = n_i(x), \text{ for } i = 1, 2,$$

where $n_i(x)$ is employment of labor by intermediate producer x in sector i. They are perfectly competitive in the market for labor but monopolistically competitive in their own output market. A fraction $0 < \mu < 1$ of intermediate goods producers in each sector sets their price before the monetary authority chooses policy. The remaining $(1 - \mu)$ producers set their prices after current period monetary policy has been set. We index intermediate good producers in each sector so that producers indexed by x, with $0 < x < \mu$, set their prices "in advance".

a Write out the households' first order conditions for a given price sequence.

b Write out the necessary conditions for optimality for final goods producers and for sticky- and flexible-price intermediate goods producers in each sector. Consider symmetric equilibria in which all advance price-setters set the same price, P^e, and all flexible-price producers set the same price, $\hat{P}$. Derive an expression for the equilibrium final goods price level, P_i, as a function of $\hat{P}$ and P^e, for $i = 1, 2$.

c Define a momentary private sector equilibrium as an allocation $\{c_1, c_2, n, y_1, y_2\}$ and prices R, P_1, P_2, $\hat{P}$ and P^e such that the households' Euler equations are satisfied at a given value of initial money holdings M, final goods firms optimality conditions are satisfied at the given prices and such that the flexible- and sticky-price intermediate producers' optimality conditions are satisfied at $\hat{P}$ and P^e. Solve for the momentary equilibrium as a function of $\hat{P}$ for a given P^e.

d Assume P^e is given an that the monetary authority optimally chooses $\hat{P}$. Define and characterize a stationary Markov equilibrium in this environment. Is the Markov equilibrium unique?

2. Consider the cash-credit good economy discussed in the lectures on monetary policy in the absence of commitment. The economy is analyzed in detail in the manuscript, 'How Big is the Time Consistency Problem in Monetary Policy?', which is on the web site for this class.

1. Household preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \; c_t = \left[\int_0^1 c_t(\omega)^\rho d\omega \right]^{\frac{1}{\rho}},$$

with

$$u(c,l) = \frac{\left[c(1-l)^\psi \right]^{1-\sigma}}{1-\sigma}$$

Households have the cash in advance constraint and the asset evolution equation specified in class (and in the manuscript). Each good is produced by a monopolist using only labor, using a production function with constant marginal product of labor, θ. As is standard in this type of setting, monopolists set prices as a fixed markup over marginal cost. A fraction, μ, of monopolists set prices before the monetary authority moves, and the rest set prices after the monetary authority moves. The fraction of goods that are cash goods is z, and the fraction that is credit goods is $1-z$.

In class, we expressed the monetary authority's first order condition evaluated at $\hat{P} = P^e$ as the sum of a monopoly distortion and an inflation distortion. These are also given in equation (25) in the manuscript, and can be expressed as a function of c_1/c_2, where c_1 is the quantity of each cash good consumed and c_2 the quantity of each credit good consumed, when $\hat{P} = P^e$.

   1. Graph, for $c_1/c_2 \in (0,1)$, the inflation distortion and the monopoly distortion. Do so for the parameter values, $z = 0.3$, $\mu = 0.1$, $\rho = 0.83$, $\psi = 4$, $\theta = 1$.[1] Show that there are two Markov equilibria. Report what c_1, c_2, R, n, P/M at each equilibrium, where M denotes the beginning of period stock of money (hint: use the various first order conditions and other conditions that must hold for $q = 1$, to go from c_1/c_2 to these other variables.)
   2. Do the same as above, with z set to 0.6 instead. Show that there are still two equilibria, but now one involves $R = 1$. Compute all the other equilibrium prices and allocations in the two equilibria.

[1] In the model in the paper, government spending is potentially non-zero (this is financed by lump-sum taxes). In the homework, just set government spending to zero.